Shaping Inequality and Intergenerational Persistence of Poverty: Free College, Better Schools or Transfers?*

Preliminary and Incomplete Please Do Not Cite

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December 28, 2023

Abstract

This paper constructs a general equilibrium overlapping generations model with intergenerational linkages and a multi-stage human capital production process during childhood and adolescence with both parental time and resource investments as well as government schooling inputs investments. The model features rich cross-sectional heterogeneity, and specifically, distinguishes between single and married parents, and is disciplined by US household survey data on income, wealth, education and time use. The purpose of the model it to study alternative government education policies to encourage college attendance and completion (such as making college free, improving funding for primary and secondary public schooling, and strengthening social insurance policies for children from disadvantaged socio-economic backgrounds). More broadly, we investigate which policies efficiently improve outcomes of socially disadvantaged children, increase upward intergenerational mobility, and improve aggregate welfare.

Keywords: education spending, public transfers, welfare benefits, inequality, poverty, intergenerational persistence

J.E.L. Codes: D15, D31, E24, I24

^{*}Computations in this paper were carried out using resources of the Goethe-HLR high performance computing cluster and Amazon Web Services EC2 Cloud Computing.

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1 Introduction

In international comparison, the U.S. displays low intergenerational socio-economic mobility, with especially high persistence at the bottom of the income distribution. Achievement gaps between children of different socio-economic backgrounds appear early in life and persist into adulthood. Skill and achievement gaps during adolescence (i.e. before labor market entry) have been have been identified as a key factor determining differences in later life economic outcomes (see, e.g., Keane and Wolpin (1997); Carneiro and Heckman (2002); Huggett et al. (2011)), and a variety of policies have been proposed to lessen these disparities, most recently, a proposal by the current administration to make at least community college free. If however, the main problem holding poor children back from successfully pursuing a college education is not that they cannot pay for it (i.e., the prevalence of binding credit constraints), but rather that they arrive at college age ill-equipped to successfully apply and ultimately graduate from college, policies tackling this problem (such as public funding to improve primary and secondary schools, or transfer payments to poor families) might be more appropriate tools to increase the average and reduce the crosssectional dispersion and intergenerational persistence of college attendance, earnings, wealth and welfare. This paper seeks to develop a dynamic general equilibrium model of childhood human capital and higher education decisions to assess the positive and normative implications of policies designed to boost college attendance, especially those children from disadvantaged socio-economic backgrounds.

We develop a general equilibrium overlapping generations framework with intergenerational links through altruistically-motivated education and wealth transfers, in the spirit of Barro and Becker (1988), and with rich cross-sectional heterogeneity with respect to labor productivity (and thus earnings), human capital, wealth and marital status. Human capital is accumulated at different stages of a child's development, depending on parental resource and time investments as well as public education funding. Crucially, human capital acquired at earlier stages of child development determines the productivity of all future human capital investments, and the human capital acquired prior to the higher education (college) stage determines both the chances to succeed in college and the expected returns from a college education. Altruistically motivated and rationally forward-looking parents respond to policies affecting the labor market stage of their children's life cycle. At the same time, parents react to redistributive tax and transfer reforms by adjusting both their own education choices and investments in their children. These interactions between education subsidies targeted at different stages of child and adolescent development and progressive taxation suggest that these policy instruments must be studied jointly.

The dynastic modeling framework with intergenerational linkages allows us to evaluate the implications of policy reforms not only for the cross-sectional inequality statistics but also for

intergenerational earnings and education mobility. The explicit consideration of families with single parents allows us to analyze both short-run policies targeting families located at the lower end of the socio-economic spectrum, such as progressive taxation and redistributive transfers, and long-run education policies that shape the distribution of human capital and thus market earnings inequality.

Our policy analysis starts from an initial equilibrium calibrated to the status quo of the US economy in 2010s. We then investigate the impact of policy reforms along the transition of the economy towards a final steady state. Along this transition, the government may issue new government debt to finance redistributive policies. This is important because along the transition through the endogenous human capital accumulation the allocation in the economy will be more efficient, which increases the tax base—a fiscal externality—and makes it possible to finance higher government debt. By issuing debt the government may thus smooth out the transitional costs of the policy reforms.

We evaluate a set of once-and-for-all policy reforms that are motivated by the current political discussion and that are comparable in their short-run government expenditure requirements. The first reform we consider is making college education free (motivated by President Biden's proposal to provide for universal free community college) which in our model is implemented by a 100% tertiary education subsidy by the government. This reform is then used as benchmark to determine the size of the other reforms to make those fiscally comparable. The alternative policy reforms we consider either focus on human capital accumulation of younger children, such as increasing public spending for primary or secondary schools, and subsidizing parental pre-college human capital investments in children, or tackle poverty induced by the lack of (higher) education directly through the progressive tax-transfer system.

Our findings can be summarized as follows. In terms of aggregates, the college subsidy reform increases the share of a cohort going to college most strongly, but the overall expansion of human capital is much more pronounced under the pre-college school spending reform. This in turn also implies a very different composition of the pool of college graduates under the two reforms. Because of the stronger increase in human capital in the long run, the net present discounted value of government revenues rises more substantially under the school expenditure expansion than under the 100% college subsidy reform. Both reforms generate significant welfare gains (in the order of 2-4% of permanent consumption, when measured as consumption equivalent variation of newborn agents), but consistent with the more favorable human capital and tax revenue expansions, is ca. 0.6 percentage points higher in the "better schools" reform than in the free college reform in the long run. The expansion of the tax base is more substantial in the required

(permanent) increase in the labor income tax rate to balance the intertemporal government budget

Second, the two policy reforms have vastly different distributional consequences. Most crucially, the pre-college expenditure reform also benefits children from households who will not go to college even if it is free. However, children with single parents benefit relatively little from both education spending reforms, due to the fact that their pre-reform level of human capital tends to be low, and thus these children are not prepared to go to college, resulting in high dropout rates for this segment of the population, low expected returns from college even if they succeed to graduate, and limited scope for single parents to adjust their pre-college time- and resource investments in response to the reforms. These results suggest that in order to improve lifetime outcomes for these children, reforms that target poor, single parent households more directly are necessary.

Third, the reforms also impact intergenerational persistence of earnings and educational attainment. Perhaps the most striking contrast between the reforms is the differential impact on the educational attainment of the poorest children, which tend to be children growing up in a household with a single parent and low (less than high school) educational attainment. The free college reform hardly changes the educational attainment of these children, primarily because their accumulated human capital during childhood makes them very unlikely to go to college and succeed there. In contrast, the additional human capital accumulation these children obtain with the school expenditure reform, although insufficient in most cases to push them above the college threshold, strongly increases the chances of these children to complete high school, therefore strongly reducing the intergenerational persistence of dropping out of high-school.

To isolate the importance of changes in endogenous interest rates and (relative) wages we also conduct a sequence of partial equilibrium exercises in which we hold these endogenous prices constant. We show, broadly speaking, that qualitatively, the aggregate and distributional conclusions discussed above also emerge in the absence of equilibrium price adjustments. In fact, endogenous interest and (relative) wage adjustments dampen the effects of, and the difference between the two reforms. The most important generation equilibrium effect stems from the fact that inflow of more college-educated workers into the labor market (induced by the education reforms) lowers the college wage premium, muting the increase in the college share in general equilibrium relative to partial equilibrium. This effect is stronger in the school expenditure expansion reform since college enrollment decisions are more sensitive to the college wage premium in that thought experiment. An increase in the interest rate induced by the reduction of the capital-labor ratio (in turn due to the increase in effective labor as well reduced savings incentives for privately funded education expenditures and inter-vivos transfers) in general equilibrium mutes the crowding-out effect of the free college reform on inter-vivos transfers and results in a larger crowding-in under the school expenditure reform. As a result of these general price movements, the welfare gains are somewhat smaller, and the *difference* between the two reforms is also slightly less in general equilibrium relative to partial equilibrium, still amounting to a gap of 0.6% of permanent consumption.

1.1 Related Literature

Our paper seeks to connect two broad literatures and exploit that connection for the study of currently proposed education finance and fiscal policy reforms. The first, and perhaps older literature in quantitative macro public finance, is concerned with (optimal) redistributive tax-transfer and education policies; see Benabou (2002), Hanushek et al. (2003) and Bovenberg and Jacobs (2005) for foundational papers. Recent papers in this genre focusing on education (financing) reform include Abbott et al. (2019), Caucutt and Lochner (2020), Stantcheva (2017), Capelle (2020), Fu et al. (2023) and also Athreya et al. (2019) as well as our own work, Krueger and Ludwig (2016a). An important part of this literature studies the impact of tax- and education policy on intergenerational mobility,¹ see e.g. Holter (2015), Lee and Seshadri (2019), Koeniger and Prat (2018) and Koeniger and Zanella (2022), and a complementary and equally relevant literature studies (optimal) tax-transfer and poverty alleviation policy (transitions), see, e.g., Boar and Midrigan (2022), Dyrda and Pedroni (2023), Daruich and Fernandez (2023), Floden (2001), Ortigueira and Siassi (2023), Guner et al. (2020) and Guner et al. (2021).² In contrast to most of this existing literature, this project takes as central tenet that the heterogeneity in initial conditions at labor market entry with respect to human capital and wealth is an endogenous objects that can be affected by education and fiscal policies. Thus, it considers education policies as additional means of redistribution, by reducing education and achievement gaps of children from different socio-economic backgrounds and at different stages of the skill formation process. In addition, we seek to contribute to the literature cited above by developing a framework that can distinguish between the incidence of pre-college versus college subsidies while explicitly modelling the complementarity between ability and educational attainment for wages (see Jacobs and Bovenberg (2011) or Stantcheva (2017)) and the dynamic complementarities in child human capital accumulation recently stressed by Cunha et al. (2010).

Therefore, into the above literature we seek to integrate an explicit modelling of life cycle choices with an explicit production function for human capital at different stages of child devel-

¹Since intergenerational persistence in outcomes is impacted by intergenerational transfers, the empirical literature on these transfers in, e.g., Gale and Scholz (1994), Altonji et al. (1997) and especially Yang and Ripoll (2023) provides important references for the calibration of our the model.

 $^{^{2}}$ A complementary empirical literature studies the interaction of welfare programs and the education and human capital accumulation of children, see, e.g., Del Boca et al. (2014), Del Boca et al. (2016), National Academies of Sciences and Medicine (2019) and Bailey et al. (2023)

opment. In this regard the proposal builds on a recent literature in empirical microeconomics and quantitative macroeconomics that models child skill formation and human capital accumulation endogenously, see, e.g., Cunha et al. (2006), Cunha and Heckman (2007), Cunha et al. (2010), Caucutt et al. (2020), Bolt et al. (2023), Daruich (2022), Yum (2023) and our own work, Fuchs-Schündeln et al. (2022), to study the dynamic interactions between parental borrowing constraints and public education spending.³ On the modelling side we extend this literature by considering the endogenous time allocation choice for both parents between work, leisure and spending time with children of different ages. On the applied policy side, our main focus lies on the impact of optimal policy transitions (permitting government debt) on cross-sectional inequality and intergenerational persistence of economic outcomes, especially those at the lower end of the income and wealth distribution. This in turn requires the explicit model with intergenerational linkages and rich household heterogeneity especially with respect to family marital structure that we provide in this paper.

2 Model

2.1 Overview

The model is a general equilibrium OLG model in which generations are linked through the intergenerational transmission of innate ability and financial wealth transfers. Parents are altruistic towards their children and can invest their time and monetary resources into the human capital accumulation of children when the latter are still living in the parental household. In addition, parents can transfer wealth to children directly when they leave the household. The government collects taxes, runs a PAYGO social security system and finances exogenous government spending and endogenous education spending with taxes and government debt, subject to an intertemporal budget constraints. In general equilibrium the goods-, labor- and asset markets have to clear in every period along a policy-reform induced transition.

Relative to the standard quantitative life cycle literature our model contains three key additional features. First, households have children whose human capital accumulation during the transition from childhood to adolescence is endogenous and depends both on public and private parental inputs. This element of the model is crucial for a study of education policies that differ in the extent to which primary/secondary and tertiary education is altered and fiscal policies that

³The *empirical* literature on the impact of day care- and education spending and financing on education and economic outcomes, see, e.g., Havnes and Mogstad (2011), Abramitzky and Lavy (2014), Jackson et al. (2015), Deming and Walters (2017), Johnson and Jackson (2019), Jackson and Mackevicius (2021), Black et al. (2020), Duncan et al. (2022) and Flood et al. (2022) (as well as the survey by Handel and Hanushek (2022)) will provide key targets for our structural model.

impact the trade-off between market work (including participation) and time investment into children.⁴ Second, generations in our model are linked through "brains and bucks", that is, human capital inputs and financial transfers from parents to children. With this model element, parents have endogenous margins of adjustment in direct response to both headline policy reforms. If college will be free, private inter-vivos transfers (and the accumulation of parents assets to make these transfers) will endogenously adjust. When public schools become better, private time and resource inputs can respond as well. Third, modelling both for married households but also single mothers allows us to explicitly account for a group of children that disproportionally grow up in poverty and are least likely to go to college. We now describe the model in greater detail.

2.2 Individual State Variables

In order to meaningfully study the distributional consequences of the proposed policy reforms the model features rich cross-sectional heterogeneity, best described in terms of the individual state variables that characterize households. These are summarized in Table 1, including the range of values these state variables can take. Individuals differ by age j and young households start

State Var.	Values	Interpretation	
j	$j \in \{0, 1, \dots, J\}$	Model Age	
g	$g \in \{wo, ma\}$	Gender	
h	h > 0	Human Capital	
a	$a \ge -\underline{a}(j,s)$	Financial Assets	
s, s_p	$s \in \{hsd, hs, cod, co\}$	Higher Education	
γ	$\gamma \in \{\gamma_l(s), \gamma_h(s)\}$	Fixed Productivity Component	
η	$\eta \in \{\eta_l, \eta_h\}$	Persistent Productivity Shock	
q	$q \in \{si, cpl\}$	Marital Status	

 Table 1: Individual State Variables

their independent economic life as singles and with four ex-ante predetermined state variables: gender (either being a woman or a man, $g \in \{wo, ma\}$), the education of their parents s_p (which determines their cost of attending college) initial human capital (h) and initial assets (a). After an individual has taken its own higher education decision s, the highest completed education level also becomes a state variable (and that of the parent ceases to be relevant), and upon labor market entry, the acquired human capital stochastically translates into a discrete-valued fixed

⁴The presence of government transfer- and social assistance programs whose importance varies by family structure renders the explicit modelling of an extensive margin, for both partners in a married household, important, as the recent work by Guner et al. (2012), Bick and Fuchs-Schündeln (2017) and Holter et al. (2023) suggests

effect γ with education-specific support.⁵ Labor productivity is also impacted by a persistent stochastic component η which is part of the state space. Finally, one period before children are born into the household, the marital status q of a household realizes and becomes a state variable, as a fraction of single households marry (q = cpl for "couples") while the rest remains single (q = si). When children are born into household, their human capital h becomes a state variable as well.⁶ In terms of notation, for married households the education and labor productivity of both partners are state variables, and the notation s_{-g} and γ_{-g} will be used to denote the state variable of the "other" spouse. We now go on to describe the life cycle decision problems of households, including selected dynamic programming problems of households.

2.3 Demographics, Timing and Economic Decisions

Time is discrete, indexed by t and extends to infinity. In every period t the economy is populated by J overlapping generations indexed by j. Individuals survive from age j to age j + 1 with probability ϕ_{j+1} . Before retirement survival is certain while from the retirement age j_r onwards survival risk becomes relevant. Assets of households that die at age j are distributed in a lumpsum fashion among all working age households⁷. Transfers from accidental bequests are denoted by $Tr_{t,j}$.

Children are born at age j = 0 (biological age 2; the first two years of child lifecycle are discarded). Parental fertility age is denoted by j_f . The number of children per household (fertility rate) is denoted by $\varsigma(s(wo))$ and is a function of the mother's education level. At this age parents draw an initial child human capital level from a distribution that depends on their education level. Children stay in the parental household and accumulate human capital depending on their initial human capital and the time and resource input of their parents as described below. These parental investments are referred to as private human capital investments. When children leave the parental household parents give them (non-negative) inter-vivos transfers *b* which can be used for consumption financing and/or for covering college expenses.

At model age j_a (biological age 18) a college education decision takes place. Those children who choose college spend one model period for education, the other group starts working directly at age j_a . Dropping out of college takes place stochastically with the dropout shock being realized directly before the college education starts⁸. College dropouts are assumed to have to

⁵The stochastic mapping from human capital h to the fixed labor productivity γ replaces a continuous state variable (h) with a discrete-valued one (γ) which reduces the state space.

⁶Since at that time fixed productivity γ has replaced parental human capital, and thus there is no scope for confusion between parental and child human capital.

⁷To be more precise, what is redistributed among surviving households are the accidental bequests that remain after the amount needed to finance private college subsidies is deducted.

⁸TBC: alternatively, dropping out of college can be modelled as an endogenous decision.

pay two times smaller tuition costs than college graduates, and they also face a (two times) tighter borrowing limit.

At model age j_a all education groups draw a fixed productivity component $\gamma(s, h)$ which has only two realizations - high and low. The probability of drawing a high realization of the fixed effect is an increasing function of acquired human capital. College students (both those who will graduate and those who will drop out) are assumed to work at high school wages⁹.

After education is completed all households enter the labor market. When the labor market entry happens acquired human capital seizes to be a state variable for all education groups. College graduates and college dropouts redraw their fixed productivity component based on a newly obtained higher education level.

During the working life, households make a discrete decision whether to work, and conditional on employment endogenously choose hours worked subject to a time endowment constraint. One period before the fertility age households face an exogenous (education specific) probability of marriage, and depending on the realization of the marriage shock continue living to the next period either as singles or as couples. The marriage age is denoted j_m . Retirement takes place exogenously at the model age j_r . The maximum possible lifespan is J.

All choice variables are summarized in Table 2.

Table 2: Per Period 1	Decision Variables
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Control Var.	Values	Decision Period	Interpretation
с	c > 0	$j \ge j_a$	Consumption
l	$\ell \ge 0$	$j \ge j_a$	Hours worked (for couples $\ell(wo)$ and $\ell(ma)$)
a'	$a' \ge -\underline{a}(j,s)$	$j \ge j_a$	Asset Accumulation
i^t	$i^t \ge 0$	$j \in \{j_f,, j_f + j_a\}$	Time Investments (for couples $i^t(wo)$ and $i^t(ma)$)
i^m	$i^m \ge 0$	$j \in \{j_f,, j_f + j_a\}$	Monetary Investments
b	$b \ge 0$	$j = j_f + j_a$	Monetary Inter-vivos Transfer
s	$s \in \{hsd, hs, cod, co\}$	$j = j_a$	(Higher) Education

Notes: List of decision variables of the economic model.

2.4 Human Capital

Human Capital Accumulation during Childhood. In every period during childhood human capital accumulation takes places according to the following production function:

$$h' = g\left(j, h, i^m, i^t, i^g\right),\tag{1}$$

⁹This means that both the aggregate wage level as well as the fixed productivity component are the same as for high school graduate workers.

where i^t and i^m denote parental time and monetary investment, while i^g denotes public (time) investment. Some of the parameters of the human capital production function are age-dependent for calibration purposes to capture differences in the relative importance of inputs at different stages of childhood.

For married households the time investment i^t is a composite of the time inputs of both parents which are assumed to be perfectly substitutable:

$$i^t = i^t(wo) + i^t(ma) \tag{2}$$

where $i^t(wo)$ and $i^t(ma)$ denote the time inputs of a *woman* and of a *man*, respectively.

2.5 Higher Education Decision

After leaving the parental household, the first economic decision of children is a discrete choice whether to attend college.

At the beginning of the college period the college completion shock is realized which together with the fixed productivity component determines the household wages for the rest of the lifcycle. The effective cost of college education and borrowing conditions also depend on the realization of the college completion shock - college dropouts pay smaller tuition costs and face a tighter borrowing limit than college graduates.

2.6 Labor Productivity

The wage of a single household at age j, of gender g with an education level s and with a fixed productivity component realization $\gamma(s)$ is given by:

$$w(s,\gamma(s),g,j) = w(s) \cdot \gamma(s) \cdot \epsilon(s,g,j) \cdot \eta \tag{3}$$

where w(s) is the aggregate wage component, $\gamma(s)$ is a fixed household productivity component, $\epsilon(s, g, j)$ is a deterministic gender- and education-specific productivity profile, and η and ε denote persistent and transitory productivity shocks, respectively.

For couples, the household wage is a sum of male and female wages:

$$w(s(wo), s(ma), \gamma(s(wo)), \gamma(s(ma)), j) =$$
(4)

$$w(s(wo)) \cdot \gamma(s(wo)) \cdot \epsilon(s(wo), g = wo, j) \cdot \eta + w(s(ma)) \cdot \gamma(s(ma)) \cdot \epsilon(s(ma), g = ma, j) \cdot \eta$$
(5)

2.7 Decision Problems

Below household decision problems are stated using a recursive formulation. All variables are expressed in per capita terms and detrended by the rate of technological progress.

2.7.1 Children

Children are born at age j = 0 and are themselves economically inactive until age $j = j_a - 1$; they stay in the parental household and accumulate human capital depending on their initial human capital and the time and resource input of their parents as described below.

2.7.2 Young Adults and the Education Decision at Age j_a

At model age j_a (biological age 18) children have become young adults and form an independent household with initial state (g, s_p, a, h) given by gender, parental education, financial assets and human capital. Now the tertiary education level is determined, partially by choice and partially by chance. It takes four values, as individuals can be high-school dropouts (hsd), high-school graduates (hs), college dropouts (cod) and college graduates (co). First, high school graduation is exogenous from the perspective of the newly founded household but stochastic: with probability $\pi^{hs}(h)$ (which depends positively on human capital h of the individual and thus is influenced by choices parents took during childhood) the individual obtains a high-school diploma and with complementary probability it becomes a high-school dropout, with continuation lifetime utility $V_t(j_a, si, g, hsd, a, h)$ of an age j_a single si of gender g and assets a as well as human capital h.

A high-school graduate can then choose to attend college. Attending college is costly, both in terms of tuition (which is potentially subsidized by the government and can be financed by student loans) as well as in terms of the opportunity cost of time, and subject to exogenous (but human-capital dependent) drop-out risk: individuals succeed in college only with probability $\pi^{co}(h)$. Individuals weigh these costs against the benefits of higher wages upon college graduation.¹⁰ The college attendance choice can then be written as

$$s = \begin{cases} hs & \text{if } V_t(j_a, si, g, hs; a, h) \ge V_t(j_a, si, g, ce; s_p, a, h) \\ ce & \text{if } V_t(j_a, si, g, ce; s_p, a, h) > V_t(j_a, si, g, hs, s_p; a, h), \end{cases}$$
(6)

¹⁰College students (both those who will graduate and those who will drop out) can work-part time at high school wages. Additionally, students experience a utility cost of attending college that depend on their acquired human capital h and on the education of their parents s_p . Finally, college dropouts pay smaller tuition costs and face a tighter borrowing limit than college graduates.

where $V_t(j_a, g, ce; s_p, a, h)$ is the pre-dropout college *attendance* value function given by:

$$V_t(j_a, si, g, ce; s_p, a, h) =$$

$$\pi^{co}(h) \cdot V_t(j_a, si, g, co; s_p, a, h) + (1 - \pi^{co}(h)) \cdot V_t(j_a, si, g, cod; s_p, a, h).$$
(7)

and the pre-college *decision*, age j_a value function is given by

$$V_t(j_a, si, g, s_p; a, h) = (1 - \pi^{hs}(h)) \cdot V_t(j_a, si, g, hsd; a, h)$$

$$+ \pi^{hs}(h) \cdot (\max_{s \in \{hs, ce\}} \{V_t(j_a, si, g, hs; a, h), V_t(j_a, si, g, ce, s_p; a, h)\}).$$
(8)

2.7.3 First Period of Working Life / College Period

At the beginning of the first period of independent economic life, realizations of the fixed productivity component and idiosyncratic productivity shocks are drawn. Thus, in the fist period of economic life the decision problem can be split in two subperiods. In the first subperiod, the fixed productivity component and the persistent income shock are drawn:

$$V_t(j_a, si, g, s, s^p, h, a) = \sum_{\gamma} \pi^{\gamma}(s, h) \sum_{\eta} \Pi(\eta) V_t(j_a, q = si, g, s, s^p, h, \gamma, \eta, a)$$

where γ denotes education-specific realizations of the fixed productivity component, and η is the persistent productivity shock realization. The probability of drawing a high fixed effect realization is denoted by $\pi^h(s,h)$ and is a function of acquired human capital. For households that neither enroll in college nor complete it, from this point in time onward acquired human capital seizes to be a state variable and is replaced by the fixed effect $\gamma(s)$. Recall, college students work at high school wages during the college phase, therefore for them γ has to be redrawn upon college completion.

After the fixed effect is drawn, a standard consumption-savings problem with endogenous labor supply is solved. For households that choose not to enroll in college, the decision problem is identical to the one described in the next subsection 2.7.4. For households that complete college, the decision problem is slightly modified because they redraw the fixed productivity component given their newly obtained higher education level, incur psychological and financial

costs of attending college, can work only up to maximum $\bar{\ell}^{ce}$ and also are allowed to borrow:

$$V_{t}(j_{a}, si, g, s, s^{p}, \gamma(s < co, h), h, \eta, a) = \max_{c, a', \ell \leq \bar{c}^{co}} \left\{ u(c, \ell) - F(g)_{\ell > 0} - p(s, s^{p}; h) \right.$$
$$\left. + \beta \sum_{\gamma'} \pi^{\gamma'}(s, h) \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j+1, si, g, s, \gamma', \eta', a') \right\}$$

subject to

$$\begin{aligned} a' + c(1 + \tau^{c}) + T(y(1 - 0.5\tau^{p})) + \iota(1 - \varrho - \varrho^{pr}) &= (a + Tr_{t,j})(1 + r(1 - \tau^{k})) + y(1 - \tau^{p}) \\ y &= w(s)\gamma(s)\epsilon(s, g, j)\eta\ell \\ a' &\ge -\underline{a}(s, j) \\ c &\ge 0 \\ \ell \in [0, \bar{\ell}^{ce}]. \end{aligned}$$

where $p(s, s_p; h)$ is the psychological (utility) cost of attending college, and $\iota(1 - \varrho - \varrho^{pr})$ is the tuition cost net of public and private subsidies. $F(g)_{\ell>0}$ denotes a fixed utility cost of working positive hours which depends on the household gender.

Households that enroll in college but drop out solve the same problem as above with the only difference that they are assumed to pay only half of the tuition costs.

2.7.4 Working Life Before Marriage

After completing (or not) their tertiary education single individuals enter the labor market and make labor supply as well as consumption-saving choices (c, a', ℓ) , in light of their labor productivity, which is determined by an individual fixed effect γ , a deterministic education-, genderand age-specific life cycle profile $\epsilon(s, g, j)$ and a persistent stochastic component η . The permanent labor productivity type γ is drawn at the beginning of labor market entry.¹¹. With probability $\pi^{\gamma}(s; h)$ permanent productivity is $\gamma = \gamma_l(s)$ and with complementary productivity it is $\gamma = \gamma_h(s)$. The wage of a single individual is then given by $w(s) \cdot \gamma(s) \cdot \epsilon(s, g, j) \cdot \eta$, where w(s) is the education-specific aggregate wage per efficiency unit of labor.

During working life, households make the discrete decision whether to work, and conditional on employment endogenously choose hours worked subject to a time endowment constraint. The

¹¹Since college students can also work part-time, they also draw their fixed effect prior to college entry.

decision problem of singles can then be written as

$$V_{t}(j, si, g, s, \gamma, \eta, a) = \max_{c, a', \ell} \left\{ u(c, \ell) - F(g)_{\ell > 0} + \beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j+1, si, g, s, \gamma, \eta', a') \right\}$$
(9)

subject to

$$\begin{aligned} a' + c(1 + \tau^{c}) + T(y(1 - 0.5\tau^{p})) &= (a + Tr_{t,j})(1 + r(1 - \tau^{k})) + y(1 - \tau^{p}) \\ y &= w(s)\gamma(s)\epsilon(s, g, j)\eta\ell \\ a' &\ge -\underline{a}(s, j) \\ c &\ge 0 \\ \ell \in [0, \Gamma^{si}]. \end{aligned}$$

where $\underline{a}(s, j)$ is an age- and education-specific borrowing limit, and Γ^{si} denotes the per period time endowment of a single household; $F(g)_{\ell>0}$ denotes a fixed, gender-specific utility cost of working positive hours. The household takes as given aggregate wages and interest rates (w(s), r) as well as the proportional tax rates on consumption, asset income and labor income for social security (τ^c, τ^k, τ^p) , the nonlinear labor income tax schedule T(.) and well as the transfers Tr_j . Labor income taxes are levied on labor income net of employer contributions to social security $y(1-0.5\tau^p)$.

2.7.5 Marriage

Individuals remain single, until at age j_m (and at that age only) they face an exogenous, educationspecific probability $\pi^m(s)$ of marriage. Depending on the realization of the marriage shock individuals continue to live as singles or form a new married household. Since a married household is characterized by the education and wage fixed effect of both spouses as well as their combined financial asset positions (all of which are at least partially the result of endogenous choices), at age $j_m - 1$ a single individual has to form expectations over the type of spouse it might marry (and these expectations have to be confirmed in a rational expectations equilibrium, inducing an additional equilibrium fixed point problem). Recalling that state variables of the spouse of the opposite gender are indexed by -g, the decision problem at model age $j_m - 1$, in anticipation of potential marriage next period, is given by:

$$V_{t}(j, si, g, s, \gamma, \eta, a) = \max_{c, a', \ell} \left\{ u(c, \ell) - F(g)_{\ell > 0} + \beta(\pi^{m}(s)E_{a'^{-g}, s^{-g}, \gamma^{-g}} \sum_{\eta'(wo)} \Pi(\eta'(wo)) \sum_{\eta'(ma)} \Pi(\eta'(ma)) \times V_{t+1}(j+1, cpl, s(wo), s(ma), \gamma(s(wo)), \gamma(s(ma)), \eta'(wo), \eta'(ma), a'(wo) + a'(ma)) + (1 - \pi^{m}(s)) \cdot \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j+1, g, s, \gamma, \eta', a') \right\}$$

where $E_{a'^{-g},s^{-g},\gamma^{-g}}(\cdot) = \int (\cdot)d\Phi(j_m - 1, q = si, -g, s, \gamma, \eta; a)$, i.e. the expectation over the characteristics of potential spouses is determined by the cross-sectional measure of the opposite gender households in period $j_m - 1$. $V_{t+1}(j+1, cpl, s, s_{-g}, \gamma, \gamma_{-g}, \eta'(wo), \eta'(ma), a' + a'_{-g})$ is the continuation value function of the newly formed couple. The maximization problem is subject to the following constraints

$$\begin{aligned} a' + c(1 + \tau^{c}) + T(y(1 - 0.5\tau^{p})) &= (a + Tr_{t,j})(1 + r(1 - \tau^{k})) + y(1 - \tau^{p}) \\ y &= w(s)\gamma(s)\epsilon(s, g, j)\eta\ell \\ a' &\ge -\underline{a}(s, j) \\ c &\ge 0 \\ \ell \in [0, \Gamma^{si}]. \end{aligned}$$

2.7.6 Parenthood and Child Human Capital Accumulation

At age $j_f > j_m$ children enter single women- and married households (single men do not live with children). The number of children per household is a function of the mother's marital status and education level, and is denoted by $\varsigma(q, s(wo))$. All children of a household are assumed to be identical and characterized initially by a level of human capital h that depends on parental education and marriage status (s, q). As long as children are present, parents invest time and resources (i^m, i^t) into the production of new child human capital; we term these *private* human capital investments. For married couples, time investment is the sum of time devoted to their children by both partners, $i^t = i^t(wo) + i^t(ma)$. Finally, when children leave the household, parents can give them non-negative inter-vivos transfers b to finance tertiary education (or their consumption).

In every period during childhood private human capital investments are combined with *public* investment into schooling to transform existing child human capital h into new human capital h' according to the following age-dependent production function $h' = g(j, h, i^m, i^t, i^g)$. For single

women, the decision problem during this stage of the life cycle then is

$$V_{t}(j, si, wo, s, \gamma, \eta; a, h) = \max_{c, i^{m}, i^{t}, a', h', \ell} \left\{ u\left(c, \ell, i^{t}\right) - F(wo)_{\ell > 0} + \beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j+1, si, wo, s, \gamma, \eta'; a', h') \right\}$$
(10)

$$\begin{aligned} a' + c(1 + \tau^c) + \varsigma(si, s) \cdot i^m + T(y(1 - 0.5\tau^p)) &= (a + Tr_{t,j})(1 + r(1 - \tau^k)) + y(1 - \tau^p) \\ y &= w(s)\gamma(s)\epsilon(s, g, j)\eta\ell \\ a' &\geq -\underline{a}(j, s), c \geq 0 \\ \ell + \varsigma(si, s) \cdot i^t + \xi(j - j_f + 1, q, s) &\leq \Gamma^{si} \\ h' &= g(j, h, i(i^m, i^t, i^g)), \end{aligned}$$

where $\xi(j - j_f + 1, q, s)$ denotes a fixed childcare time requirement which is a function of the age of the child as well of marital status and education of the mother. Since single men are assumed not to have children present in the household, they solve the same maximization problem as in (9).

For couples, participation, hours worked and the time investment of both spouses are choice variables, and thus the dynamic programming problem of the household reads as:

$$\begin{aligned} V_{t}(j, cpl, s(wo), s(ma), \gamma((wo)), \gamma((ma)), \eta(wo), \eta(ma); a, h) &= \\ \max_{c, i^{m}, i^{t}(wo), i^{t}(ma), a', h', \ell(wo), \ell(ma)} \left\{ u\left(c, \ell(wo), \ell(ma), i^{t}(wo), i^{t}(ma)\right) - F(wo)_{\ell(wo)>0} - F(ma)_{\ell(ma)>0} \right. \\ &+ \beta \sum_{\eta'(wo)} \pi(\eta'(wo)|\eta(wo)) \sum_{\eta'(ma)} \pi(\eta'(ma)|\eta(ma)) \times \\ & V_{t+1}(j, cpl, s(wo), s(ma), \gamma(s(wo)), \gamma(s(wo)), \eta'(wo), \eta'(ma); a', h') \end{aligned}$$

subject to

$$\begin{aligned} a' + c(1 + \tau^{c}) + \varsigma(s(wo)) \cdot i^{m} + T^{cpl}(y(1 - 0.5\tau^{p})) &= (a + 2 \cdot Tr_{t,j})(1 + r(1 - \tau^{k})) + y(1 - \tau^{p}) \\ y &= w(s(wo))\gamma(s(ma))\epsilon(s(wo), g = wo, j)\eta(wo)\ell(wo) + w(s(ma))\gamma(s(ma))\epsilon(s(ma), g = ma, j)\eta(ma)\ell(ma) \\ a' &\geq -\underline{a}(j, \max(s(wo), s(ma))) \\ c &\geq 0 \\ \ell(wo) + \ell(ma) + \varsigma(s(wo)) \cdot (i^{t}(wo) + i^{t}(ma)) + \xi(j - j_{f} + 1, q, s(wo), s(ma)) \leq \Gamma^{cpl} \\ h' &= g(j, h, i(i^{m}, i^{t}, i^{g})) \\ i^{t} &= i^{t}(wo) + i^{t}(ma), \end{aligned}$$

where Γ^{cpl} denotes the couple's time endowment, and $T^{cpl}(.)$ is the labor income tax function it faces.

2.7.7 Children Leaving the Household and Inter-Vivos Transfers

At age $j_f > j_m$ children enter single women- and married households (single men do not live with children). The number of children per household is a function of the mother's marital status and education level, and is denoted by $\varsigma(q, s(wo))$. All children of a household are assumed to be identical and characterized initially by a level of human capital h that depends on parental education and marriage status (s, q). As long as children are present, parents invest time and resources (i^m, i^t) into the production of new child human capital; we term these *private* human capital investments. For married couples, time investment is the sum of time devoted to their children by both partners, $i^t = i^t(wo) + i^t(ma)$. Finally, when children leave the household, parents can give them non-negative inter-vivos transfers b to finance tertiary education (or their consumption).

In every period during childhood private human capital investments are combined with *public* investment into schooling to transform existing child human capital h into new human capital h' according to the following age-dependent production function $h' = g(j, h, i^m, i^t, i^g)$. For single women, the decision problem during this stage of the life cycle then is

$$\begin{split} V_t(j_a + j_f, si, wo, s, \gamma, \eta; a, h) &= \max_{c, b, a', \ell} \left\{ u\left(c, \ell\right) - F(g)_{\ell > 0} \right. \\ &+ \beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j_a + j_f + 1, si, wo, s, \gamma, \eta'; a') \\ &+ \nu_{\varsigma}(s) E_{g^{ch}} V_t\left(j_a, g^{ch}, s; \frac{b}{1 + r(1 - \tau^k)}, h\right) \bigg\} \,, \end{split}$$

where $V_t\left(j_a, g^{ch}, s; \frac{b}{1+r(1-\tau^k)}, h\right)$ denotes the pre-education decision value function of children. Maximization is subject to

$$\begin{aligned} a' + c(1 + \tau^c) + \varsigma(s) \cdot b + T(y(1 - 0.5\tau^p)) &= (a + Tr_{t,j})(1 + r(1 - \tau^k)) + y(1 - \tau^p) \\ y &= w(s)\gamma(s)\epsilon(s, g, j)\eta\ell \\ a' &\ge -\underline{\mathbf{a}}(s, j) \\ c &\ge 0 \\ \ell \in [0, \Gamma^{si}]. \end{aligned}$$

After children have left the household, parental households continue solving a consumptionsavings problem with endogenous labor supply they reach retirement:

$$V_{t}(j, si, wo, s, \gamma, \eta, a) = \max_{c, a', \ell} \left\{ u(c, \ell) - F(g)_{\ell > 0} + \beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j+1, si, wo, s, \gamma, \eta', a') \right\}$$

subject to

$$\begin{aligned} a' + c(1 + \tau^{c}) + T(y(1 - 0.5\tau^{p})) &= (a + Tr_{t,j})(1 + r(1 - \tau^{k})) + y(1 - \tau^{p}) \\ y &= w(s)\gamma(s)\epsilon(s, g, j)\eta\ell \\ a' &\ge -\underline{a}(s, j) \\ c &\ge 0 \\ \ell \in [0, \Gamma^{si}]. \end{aligned}$$

2.7.8 Retirement and Death

In retirement, that is after reaching the model age j_r , households solve a standard consumptionsaving problem, receive social security benefits $pen(s, \gamma, \eta(j_r - 1))$ and face mortality risk (until they die for sure at maximal lifetime J). The problem reads as

$$V_t(j, si, g, s, \gamma, \eta; a) = \max_{c, a' \ge 0} \{ u(c) + \beta \phi(j) V_{t+1}(j+1, si, g, s, \gamma, \eta; a') \}$$
 s.t.
$$a' + c(1 + \tau^c) = (a + Tr_{t,j})(1 + r(1 - \tau^k)) + pen(s, \gamma, \eta)$$

where $pen(s, \gamma, \eta(j_r - 1))$ is retirement income which depends on education-specific wages w(s), the persistent shock realization in the last working period¹² before retirement η , the education level s and the fixed productivity component γ .

We now embed this life cycle model with altruistically linked generations into a general equilibrium neoclassical production economy with a government that sets potentially time-varying tax-transfer and education policies. Time is indexed by t.

¹²This construction allows us to capture the progressivity embedded in the actual US social security benefit formula without carrying around another continuous state variable during working age.

2.8 Production

Final output Y_t is produced according to a Cobb-Douglas production function $Y_t = K_t^{\alpha}(L_t)^{1-\alpha}$, where α determines the elasticity of output with respect to capital. In order to permit the possibility that a policy-induced change in the share of college graduates changes their relative wages we assume that non-college labor (including college dropouts) and college labor (i.e., college graduates) are imperfectly substitutable in production. Total labor efficiency units L_t at time t are given by:

$$L_{t} = (L_{t,nc}^{\rho} + L_{t,co}^{\rho})^{\frac{1}{\rho}}$$
(11)

where ρ governs the elasticity of substitution between college $L_{t,co}$ and non-college labor efficiency units and $L_{t,nc} = L_{t,hsd} + L_{t,hs} + L_{t,cod}$ are the labor efficiency units jointly supplied by high-school dropouts, high-school graduates and college dropouts supplied in period t.

2.9 Government

The government administers a progressive labor income tax code, pays transfers to households and collects linear taxes on consumption and capital income. Aggregate labor income tax revenues net of transfers are denoted by T_t . In addition, the government spends $\alpha_j i^g$ per child on primary and secondary school education. The age profile α_j permits us to differentiate between the cost of primary and secondary school and i^g measures the scale of public education spending, and will be one key policy choice by the government. Total spending on primary and secondary public schools is denoted by E_t . The government also subsidizes tertiary education, with the share ρ of tuition covered by the government; ρ is the second crucial policy choice variable, and a choice of $\rho = 1$ represents free college. We denote by E_t^{CL} the aggregate cost of college subsidies.

In addition to the endogenous streams of education expenditures (E_t, E_t^{CL}) for primary, secondary and tertiary education the government also needs to finance an exogenous stream of non-education related expenditures G_t . To do so, the government raises revenues from taxing labor- and capital income as well as consumption, and from issuing government debt B_t . The period t flow government budget constraint then reads as

$$E_t + E_t^{CL} + G_t + (1+r_t)B_t = (1+\mu)(1+n)B_{t+1} + T_t + \tau_{c,t}C_t + \tau_{k,t}r_t(K_t + B_t)$$
(12)

The initial stock of government debt B_0 is an exogenously given initial condition (as is the initial aggregate capital stock K_0). Finally, the government also runs a pure pay-as-you-go social security system whose budget equates payroll taxes (with tax rate τ^p) to all pension benefits paid out according to the benefit formula $pen(s, \gamma, \eta)$.

3 Equilibrium Definition and Computation

The key equilibrium object in our model is the cross-sectional measure Φ_t over household characteristics¹³ $(j, q, \gamma, \eta, a, h)$. For each time period t and age j we normalize the total measure $\Phi_t(j, \cdot)$ to 1 and denote by N_j the (time-invariant) size of age cohort j.

$$\int d\Phi_t(j,si,g,s,\gamma,\eta,a,h) + \int d\Phi_t(j,cpl,s(ma),\gamma(wo),\gamma(ma),\eta(wo),\eta(ma),a,h) = 1$$
(13)

In order to clarify the distinction between the partial- and the general equilibrium versions of the model it is necessary to give a somewhat formal definition of equilibrium.

3.1 Equilibrium Definition

For given initial physical capital stock and government debt (K_0, B_0) and initial cross-sectional distributions of singles $\{\Phi_0(j, si, \cdot)\}_{ja}^J$ and couples $\{\Phi_0(j, cpl, \cdot)\}_{jm}^J$ a competitive equilibrium is given by sequences of household value and policy functions (for consumption, assets, labor supply, child human capital investments and bequests), aggregate capital and labor inputs, tax and transfer policies and government debt levels, aggregate prices, accidental bequests as well as household measures such that

- 1. In each period, household value and policy functions solve the household optimization problems, given factor prices, government policies and accidental bequests.
- 2. Defining the capital-labor ratio as $k_t = K_t/L_t$, factor prices for capital and college- as well as non-college labor (per efficiency unit) satisfy

$$r_t = \alpha k_t^{\alpha - 1} - \delta \tag{14}$$

$$w_{co,t} = (1-\alpha)k_t^{\alpha} \left(\frac{L_t}{L_{t,co}}\right)^{1-\rho} \quad \text{and} \quad w_{nc,t} = (1-\alpha)k_t^{\alpha} \left(\frac{L_t}{L_{t,nc}}\right)^{1-\rho}$$
(15)

3. Government budget constraint (12) and social security system budget constraint holds $\forall t$

$$\tau_t^p(w_{co,t}L_{co,t} + w_{nc,t}L_{nc,t}) = \sum_{j=j_r}^J N_j \int pen_t(s,\gamma,\eta) d\Phi_t$$
(16)

¹³It is understood that, depending on the age j of the household as well as its marital status q, the household state space changes; for example, for couples it includes the education and fixed effect of both partners.

4. Markets clear in all periods *t*:

$$L_{co,t} = \sum_{j_a}^{j_r - 1} N_j \int \gamma \epsilon(co, g, j) \eta \ell_t(j, co, \cdot) d\Phi_t$$
(17)

$$L_{nc,t} = L_{t,hsd} + L_{t,hs} + L_{t,cod} = \sum_{j_a}^{j_r - 1} N_j \sum_{s \in \{hsd, hs, cod\}} \int \gamma \epsilon(s, g, j) \eta \ell_t(j, s, \cdot) d\Phi_t(18)$$

$$K_{t+1} + B_{t+1} = \sum N_{j=ja}^{J} \int a'_{t}(j, \cdot) d\Phi_{t}(j, \cdot)$$
(19)

$$C_t + K_{t+1} + CE_t + E_t + G_t = K_t^{\alpha} (L_t)^{1-\alpha} + (1-\delta)K_t$$
(20)

where L_t was defined in (11) and CE_t are aggregate private education expenditures.

- 5. Marriage market equilibrium: for each men education type s(ma), the share of women married to this type is equal to the share of married men with s(ma). The same is true for women. ¹⁴
- 6. The total accidental bequests received by the working age population in period t + 1 are equal to the total assets of the dead in period t net of private college subsidies

$$\sum N_{t+1,j} \sum_{j=j_a}^{j_r-1} Tr_{t+1,j} = \sum N_{t,j} \sum_{j=j_r}^{J} \int (1-\phi_j) a_t'(j,\cdot) d\Phi_t(j,\cdot)$$
(21)

- 7. The cross-sectional measures of households evolve according to the laws of motion induced by exogenous population dynamics, the exogenous Markov processes for idiosyncratic labor productivity, the exogenous transitory shocks law of motion, endogenous asset and child human capital (when children are present in the household) accumulation, higher education and inter-vivos transfer decisions, both at the age of marriage and at all other ages.
- 8. The initial measure of newly formed households $\Phi_t(j_a, si, \cdot)$ at age j_a is consistent with inter-vivos transfers and human capital investment decisions of parents and the measure of economic newborns at age j_a after the higher education choice is made.

$$\sum_{s(wo)} \pi^m(s(wo)|s(ma)) \Phi_t(j_m - 1, si, wo, s(wo), \cdot) = \sum_{s(wo)} \pi^m(s(ma)|s(wo)) \Phi_t(j_m - 1, si, ma, s(ma), \cdot)$$
$$\sum_{s(ma)} \pi^m(s(ma)|s(wo)) \Phi_t(j_m - 1, si, ma, s(ma), \cdot) = \sum_{s(ma)} \pi^m(s(wo)|s(ma)) \Phi_t(j_m - 1, si, wo, s(wo), \cdot)$$

¹⁴Formally,

9. At age $j_m - 1$ prior to marriage expectations of singles about characteristics of future spouses are consistent with the cross-sectional distribution of the opposite gender at age $j_m - 1$.

3.2 Solution Algorithm

We propose to solve for (optimal) policy transitions in a model characterized by non-convex household maximization problems involving discrete and continuous decision variables as well as a sizeable individual state space, and in which there are two nested fixed-point problems even in partial equilibrium, one emerging from the intergenerational linkages (the value function of children enters lifetime utility of their altruistic parents) and one from the marriage market equilibrium (types of pre-marriage singles are endogenous and have to match and conform to household expectations). The solution of market clearing prices in steady state and along the transition path is then relatively standard; here we focus on the more novel fixed point problems in steady state.¹⁵

Modeling of marriage requires that the marriage market clears which results in a fixed point problem in distributions. Assuming rational expectations implies that before the marriage period expectation of assets and productivity of a future spouse should be consistent with the crosssectional distribution of assets and productivity of the opposite gender (for a given education level), $\Phi(j_{m-1}, si, g, s, a, \gamma(s))$. Recall that due to explicitly modelled intergenerational altruism, the initial measure of economic newborns $\Phi(j_a, si, \cdot)$ must be consistent with inter-vivos transfers and human capital investment decisions of parents. This implies a second fixed point problem in distributions. Additionally, the value function of the child generation at age j_a should be consistent with the value function of the parental generation at age j_a which turns the finite horizon life cycle problem of each generation into an infinite horizon problem over time. Given that each iteration of the latter fixed point problem is affected by $\Phi(j_{m-1}, si, g, s, a, \gamma(s))$ the three fixed point problems (one in value functions and two in measures) have to be solved jointly.

Thus, aggregation of the model requires solving the two fixed point problems in distributions one of which interacts with the household problem solution because the cross-sectional distribution at model age j_m determines the continuation value before the marriage period. To deal with this multi-layer fixed point problem, we propose the following algorithm:

¹⁵The algorithm for the household problem is a combination of the discrete-continuous endogenous grid method described in Iskhakov et al. (2017)), embedded in a value function iteration algorithm that draws on Druedahl (2021).

- 1: Step 1: Guess distribution of assets, fixed productivity and education for both genders at the end of period j_{m-1} (for a given skill level s), $\Phi(j_m 1, si, g, s, a, \gamma(s))$
- 2: Step 2: For given $\Phi(j_{m-1}, si, g, s, a, \gamma(s))$, solve for intergenerational RE equilibrium:
- 3: **2.1:** Solve fixed point problem in value functions (guess $V(j_a, si, \cdot)$, iterate till convergence)
- 4: **2.1:** Solve fixed point problem in distributions (guess $\Phi(j_a, si, \cdot)$ iterate till convergence)
- 5: Step 3: If $||\Phi(j_{m-1}, si, g, s, a, \gamma(s))^{\text{update}} \Phi(j_{m-1}, si, g, s, a, \gamma(s))^{\text{guess}}|| < \epsilon$, EXIT, else go back to Step 1 and continue until convergence.

4 Calibration

The model is calibrated to US aggregate and cross-sectional data, following an approach that is similar in spirit to our previous related work Fuchs-Schündeln et al. (2022). Specifically, while most demographic, aggregate technology and fiscal policy parameters as well as individual labor productivity parameters are set exogenously or directly estimated from the data, the key parameters governing preferences and the child human capital production function are calibrated internally so that the initial steady state general equilibrium of the model is consistent with the (child) age profile of parental time and resources investments as well as average hours worked and labor force participation.

Table 3 summarizes the subset of parameters calibrated exogenously outside the model, and Table 4 provides an overview of the second stage parameters that are calibrated endogenously within the model.

4.1 Demographics

Population growth rate n is assumed to be 1% which is an average of the US annual population growth rate values in 2000s. The number of children (fertility rate) differs by education level and is 30% higher for households without a college degree (based on the four most recent PSID waves).

4.2 Technology

The capital share parameter α is set to 1/3 which is a standard value in the literature, and the annual physical capital depreciation rate equals 5%.

The substitution elasticity between skilled (college) and non-skilled (high school graduates and college dropouts) labor is set to 3.3, following Abbott et al. (2019).

The rate of technological progress g equals 1%.

Parameter	Interpretation	Value	Source (data/lit)		
	Population				
j = 0	Age at economic birth (age 2)	0			
j_a	Age at beginning of econ life (age 18)	4			
j_c	Age at finishing college (age 24)	5			
j_f	Fertility Age (age 32)	7			
j_r	Retirement Age (age 66)	16			
J	Max. Lifetime (age bin 98-101)	24			
$\{\phi_j\}$	Survival Probabilities	see main text	Life Tables SSA		
n	Population Growth Rate	1%			
$\frac{\varsigma(s < co)}{\varsigma(s = co)}$	Fertility Education Gradient	1.3	PSID 2011-2017		
$\pi^{m}(s)$	Marriage probability (tbc: education- specific)	0.5	Preliminary value / PSII 2011 - 2017		
	Preferences				
σ	Relative risk aversion parameter	1			
φ	Frish elasticity	0.6			
Ŷ	Labor Productivity	0.0			
$\{\epsilon(j,s\}$	Age Profile	see main text	PSID 1968-2012		
$[\varepsilon_l, \varepsilon_h]$	Realizations of Transitory Shock	[0.881, 1.119]	PSID 1968-2012		
$[\eta_l, \eta_h]$	States of Markov process	[0.8226, 1.1774]	PSID 1968-2012		
	Transition probability of Markov process	0.0431	PSID 1968-2012		
π_{hl}	Ability/Human Capital and Education				
L	College tuition costs (annual, net of grans	14,756\$	Krueger and Ludwig		
ι	and subsidies)	14,7500	(2016b)		
$\underline{\mathbf{a}}(j \in [j_a], co)$	College borrowing limit	45,000\$	Krueger and Ludwig (2016b)		
σ^h	Elast of subst b/w human capital and CES inv. aggr.	1	Cunha et al. (2010)		
σ^g	Elast of subst b/w public inv. and CES aggr. of private inv.	2.43	Kotera and Seshadri (2017)		
σ^m	Elast of subst b/w monetary and time inv.	1	Lee and Seshadri (2019)		
κ_3^m	CES share parameter of monetary and time	0.5	normalization		
$\Phi(h(j=0) s_p)$	inv. (age bin 10-14) Innate ability dist-n of children by parental education	see main text	PSID CDS I		
\underline{h}_0	Normalization parameter of initial dist-n of initial ability	0.1248	PSID CDS I-III		
	Baseline Government po	olicy			
Q	Public subsidy of college education	38.8%	Krueger and Ludwig (2016b)		
ϱ^{pr}	Private subsidy of college education	16.6%	Krueger and Ludwig (2016b)		
i_j^g	Public high school education spending by age	$\approx 14,000\$$	NCES (2000-2018)		
$ au_c$	Consumption Tax Rate	5.0%	legislation		
$ au_k$	Capital Income Tax Rate	36%	Trabandt and Uhlig (2011)		
ξ^q	Labor Income Tax Progressivity	0.18 [TBC: by mar- ital status and $\#$	Heathcote et al. (2017)		
ω	Income of non-working households	kids] 20.2% of average earnings	CEX 2001-2007 (see Holter et al. 2023)		
$ au^p$	Soc Sec Payroll Tax	12.4%	legislation		
G/Y	Government consumption to GDP	13.8%	current value		

Table 3: First Stage Calibration Parameters

Notes: First stage parameters calibrated exogenously by reference to other studies and data.

Parameter	Interpretation	Value		
Preferences				
β	Time discount rate (target: interest rate)	0.9904		
ν	Altruism parameter $\overline{(target: average IVT transfer per child)}$	0.5944		
ϕ	Weight on hours $disutility^{16}$ (target: average hours per hh)	10.32		
F	Fixed cost of working positive hours (target: employment rate)	0.06		
	Labor Productivity			
$ ho_0(s)$	Normalization parameter (target: $\mathbb{E}\gamma(s,h) = 1$)	[0.1890, 0.0034, -0.2015]		
	Human Capital and Education			
κ	Utility weight on time inv. ¹⁷ (target: average time inv.)	0.3032		
κ_j^h	Share of human capital (target: average monetary inv. & slope cf. Figures in main text			
J	of time inv.)			
$\kappa_j^g = \bar{\kappa}^g, j > 0$	Share of government input for ages 6 and older (target: Jackson	0.871		
5	et al. estimates)			
κ_i^m	Share of monetary input (target: slope of money inv.)	cf. Figures in main text		
$rac{\kappa_j^m}{\kappa_0^g}$	Share of government input for age bin 4-6 (target: average time 0.4437			
	inv. age bin 2-6)			
Ā	Investment scale parameter (target: average HK at age j_a)	1.1989		
$\varrho(s^p < co)$	utility costs $s = co, s^p < co$ (target: fraction of group $s = co$) 3.4301			
$\varrho(s^p = co)$	utility costs $s = co, s^p = co$ (target: conditional fraction of	0.4929		
	group $s = co$)			
	$Government \ policy$			
τ	Level parameter of HSV tax function (balance gvt budget -	0.22		
	match B/Y of 100%)			
ρ^p	Pension replacement rate (balance socsec budget)	0.1893		

Table 4: Second Stage Calibration Parameters

Notes: Second stage parameters calibrated endogenously by targeting selected data moments.

4.3 Preferences

For single households, the per period utility function takes the following functional form:

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \phi(g) \frac{\ell(g)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}$$
(22)

where $\sigma = 1$, i.e. assume logarithmic utility¹⁸. Parameter ψ that can directly be interpreted as the Frisch elasticity of labor supply is set to 0.6 following Kindermann and Krueger (2014)¹⁹. Finally, gender-specific parameters $\phi(g)$ are calibrated endogenously to match the average hours worked of 1/3 of the time endowment.

For couple households, there is disutility of hours worked of both partners:

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \phi(wo)\frac{\ell(wo)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \phi(ma)\frac{\ell(ma)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}.$$
(23)

 $^{^{18}{\}rm Given}$ the logarithmic utility assumption, the child equivalence scale parameter is irrelevant for the household problem and for brevity considerations is omitted.

¹⁹As Kindermann and Krueger (2014) point out this value is based on average estimates for both men and women.

The terms capturing fixed costs of working positive hours $F(g)_{\ell>0}$ are also calibrated endogenously to match the average share of non-participating and unemployed households of 25%.

During the model periods when children live in the parental household also time spent with children affects parental utility. We assume that the disutility from time with children enters the utility function of parents in an additively separable manner²⁰:

$$u(c,n) = \frac{c^{1-\theta}}{1-\theta} - \phi(g)\frac{\ell^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \kappa \frac{\varsigma \cdot i^{t^{1+\frac{1}{\psi}}}}{1+\frac{1}{\psi}}$$
(24)

where κ is calibrated to match the average household time investment into children (per week per child), and ς is the average number of children per household.

For couple households, accordingly, there is are additional terms capturing disutility from hours worked and time with children of the second partner:

$$u(c,n) = \frac{c^{1-\theta}}{1-\theta} - \phi(wo)\frac{\ell(wo)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \phi(ma)\frac{\ell(ma)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \kappa\frac{\varsigma \cdot i^t(wo)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \kappa\frac{\varsigma \cdot i^t(ma)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}$$
(25)

When children attend college, they experience utility costs according to the cost function

$$p(s, s_p; h) = \varrho(s_p) + \frac{1}{h}$$

where $\varrho(s_p)$ is a calibration parameter which depends on parental education. Particularly, $\varrho(s_p < co)$ is calibrated to match the average college share while $\varrho(s_p = co)$ is chosen such that the share of college graduates conditional on their parents being college graduates equals 63.3% (see Krueger and Ludwig (2016b)). Observe that the psychological cost specification above implies that the utility costs are monotonically decreasing and convex in the acquired human capital h.

Households discount utility at rate β which is chosen such that in general equilibrium the implied interest rate equals 2.5%. Utility of future generations is discounted at rate ν which governs the degree of parental altruism. Parameter ν is chosen so that average per child intervivos transfer is ca. 61,200\$, as implied by the Rosters and Transfers supplement to the PSID (based on monetary transfers from parents to children until age 26, see ?).

 $^{^{20}}$ Bastian and Lochner (2020) based on females responses to EITC expansions point out that mothers increase their time with children not at the cost of hours worked but rather via reallocating their leisure time.

4.4 Human Capital Production Function

Initial Child Human Capital In the model innate human capital (at biological age 2) depends only on parental education and for given parental education is defined exogenously. The dependency of innate child ability on parental education is disciplined using child test score data from the Child Development Supplement (CDS) to PSID.

Human Capital Production Function At ages j_0, \ldots, j_a-1 , children then receive parents' human capital investments through money and time $i^m(j), i^t(j)$ and governmental input i^g , respectively. Human capital is accumulated according to a multi-layer human capital production function with imperfectly substitutable inputs:

$$h'(j) = \left(\kappa_j^h h^{1-\frac{1}{\sigma^h}} + (1-\kappa_j^h)i(j)^{1-\frac{1}{\sigma^h}}\right)^{\frac{1}{1-\frac{1}{\sigma^h}}}$$
(26a)

$$i(j) = \bar{A} \left(\kappa_j^g \left(\frac{i^g}{\bar{i}^g} \right)^{1 - \frac{1}{\sigma^g}} + (1 - \kappa_j^g) \left(\frac{i^p(j)}{\bar{i}^p} \right)^{1 - \frac{1}{\sigma^g}} \right)^{\frac{1}{1 - \frac{1}{\sigma^g}}}$$
(26b)

$$i^{p}(j) = \left(\kappa_{j}^{m}\left(\frac{i^{m}(j)}{\bar{i}^{m,d}}\right)^{1-\frac{1}{\sigma^{m}}} + (1-\kappa_{j}^{m})\left(\frac{i^{t}(j)}{\bar{i}^{t,d}}\right)^{1-\frac{1}{\sigma^{m}}}\right)^{\frac{1}{1-\frac{1}{\sigma^{m}}}}.$$
(26c)

The production function features partially age dependent parameters for calibration purposes - to reflect relative differences in importance of different inputs at different stages of childhood. All inputs are divided by their respective unconditional means which allows to achieve unit independence (see Cantore and Levine (2012)).

In the outermost (first) nest human capital at age j is combined with aggregate investment at age j. The substitution elasticity σ^h is set exogenously to 1 for all ages (implying a Cobb-Douglas specification) while the age-specific weight parameter $\kappa^j(j)$ is calibrated to match the age profile of (per child) parental time investment.

In the inner (second) nest public and private inputs are combined with the substitution elasticity being denoted by σ^g and the age-specific weight parameter $\kappa^g(j)$. The substitution elasticity is set exogenously to 2.43 using the estimate provided in Kotera and Seshadri (2017). The weight parameter is also fixed exogenously for all ages apart from the kindergarten age, i.e. age bin 2-6. Thus, for school aged children the parameter κ^g is set to 0.676 while for the age bin 2-6 it is calibrated endogenously to match average parental time investment at that age. \overline{A} is a computational normalization parameter which is chosen such that average acquired human capital at age 18 is equal to 1.

Finally, in the innermost (third) nest parental time and money inputs are combined with the substitution elasticity being denoted by σ^m and the age-dependent weight parameter $\kappa^m(j)$. The

substitution elasticity σ^m is fixed exogenously at the value of 1 using the estimate provided in Lee and Seshadri (2019) while the weight parameter $\kappa^m(j)$ is calibrated endogenously to match the age profile of the parental monetary input.

4.5 College Dropout

The probability of finishing college takes the following functional form:

$$\pi^c(h) = 1 - \exp(-\lambda^c h) \tag{27}$$

where λ^c is a parameter calibrated endogenously to match the average share of college dropouts in PSID data²¹. Observe that for $\lambda^c > 0$ this functional form specifiation implies that the probability of finishing college is increasing in acquired human capital.

4.6 College Tuition Costs & Borrowing Constraint of Students

As in Krueger and Ludwig (2016b) the net tuition cost ι (tuition, fees, room and board net of grants and education subsidies) for one year of college in constant 2005 dollars is 13,213\$. In 2008 dollars, the maximum amount of publicly provided students loans per year is given by 11,250\$, which is the borrowing limit for college students in the model. For college dropouts I assume that the borrowing limit is 50% of the borrowing limit for college graduates. For all ages after the college period (i.e. for all $j > j_a$) we let

$$\underline{\mathbf{a}}(j,s > hs) = \underline{\mathbf{a}}(j-1,s > hs)(1+r) - rp$$

and compute rp such that the terminal condition $\underline{a}(j_r, s) = 0$ is met.

4.7 Education Spending

The government spends on schooling for children and pays the college subsidy for college students. The former we approximate as 5000 per pupil based on UNESCO (1999-2005) data, as for example in Holter (2015). The latter is set to 38.8% of average gross tuition costs, as in Krueger and Ludwig (2016b). Additionally, we also explicitly model private subsidies that are paid from accidental bequests and constitute 16.6% of the gross tuition cost in the baseline (see Krueger and Ludwig (2016b)).

²¹Education shares are based on the four recent waves of PSID: 2011, 2013, 2015 and 2017.

4.8 Productivity

We use PSID data to regress by education of the household head log wages measured at the household level on a cubic in age of the household head, time dummies, family size, a dummy for marital status, and person fixed effects. Predicting the age polynomial (and shifting it by marital status) gives our estimates of $\epsilon(e, m, j)$. We next compute log residuals and estimate moments of the earnings process by GMM and pool those across education categories and marital status. We assume a standard process of the log residuals according to a permanent and transitory shock specification, i.e., we decompose log residual wages $\ln(y_t)$ as

$$\ln (y_t) = \ln (z_t) + \ln (\varepsilon_t)$$
$$\ln (z_t) = \rho \ln (z_{t-1}) + \ln (\nu_t)$$

where $\varepsilon_t \sim_{i.i.d} \mathcal{D}_{\varepsilon}(0, \sigma_{\varepsilon}^2)$, $\nu_t \sim_{i.i.d} \mathcal{D}_{\nu}(0, \sigma_{\nu}^2)$ for density functions \mathcal{D} , and estimate this process pooled across education and marital status. To approximate the persistent component in our model, we translate it into a 2-state Markov process targeting the conditional variance of z_t , conditional on z_{t-2} , $(1 + \rho^2)\sigma_{\nu}^2$ (accounting for the two year frequency of the model). The transitory component is in turn approximated in the model by two realizations with equal probability with the spread chosen to match the respective variance σ_{ε}^2 . The estimates and the moments of the approximation are reported in Table 5.

 Table 5: Stochastic Wage Process

Estimates			Mar	kov Chain	Transitory Shock	
Parameter	ρ	σ_{ν}^2	σ_{ε}^2	$\pi_{hh} = \pi_{ll}$	$[\eta_l,\eta_h]$	$[\varepsilon_l, \varepsilon_h]$
Estimate	0.9559	0.0168	0.0566	0.9569	[0.8226, 1.1774]	[0.881, 1.119]

Notes: This table contains the estimated parameters of the residual log wage process.

Acquired Human Capital and Wages The mapping of human capital into a fixed productivity component is probabilistic. The fixed effect $\gamma(s)$ can take two values - high and low $(\gamma^h(s) \text{ and } \gamma^l(s), \text{ respectively})$ - for each education group. The probability of drawing a high realization $\gamma^h(s)$ is given by

$$\pi^{h}(s,h) = \min\left\{1,\frac{h}{\bar{h}}\right\}$$
(28)

where h is the child acquired human capital (at age 18) and \bar{h} is a scaling parameter fixed exogenously for all education groups, and informed by the test score data (maximum observed test score outcomes by age 18).

Setting h this way means using a cardinal interpretation of test scores, which according to the CDS PSID documentation is fine. But given a small number of observations, I'm not sure if these maximum test scores in the data are very informative.

Using for example this functional form instead $\pi^h(s,h) = 1 - b \cdot \exp(-h)$ (which is concave in h for b > 0) implies decreasing returns to h in terms of expected earnings (so effect on earnings of the high HK kids who always go to college will be more muted), but setting parameter b would be as arbitrary is currently setting \bar{h} .

So, I'm not necessarily suggesting to change the functional form, I'm just reflecting on the role of \bar{h} .

The empirical estimates from Jackson papers are about effects of i^g on test scores, college going and college completion, but not earnings. So, there is also no empirical guidance on how much school spending affects earnings of those who anyway go to college.

But the functional form and parameterization of this probability function directly determine the extent to which i^g reforms pay for themselves!

 $\gamma^h(s)$ and $\gamma^l(s)$ are calibrated endogenously to ensure that for each education group the average $\gamma(s)$ is equal to one²², i.e.

$$\int \left(\pi^h(s,h)\gamma^h(s) + \left(1 - \pi^h(s,h)\right)\gamma^l(s)\right)\Phi(dh,s) = 1.$$

The education-specific spreads between $\gamma^h(s)$ and $\gamma^l(s)$ are calibrated to match (in expectation, i.e. from the ex ante perspective) education-specific ability gradients of (lifetime) wages estimated using NLSY79 data. Specifically, estimates of ability gradients $\hat{\rho}(s)$ are obtained by running the following regressions:

$$\ln\left(\omega(s)\right) = \rho(s) \cdot \frac{e}{\bar{e}} + \upsilon(s),$$

where $\omega(s)$ denotes age-free education-specific wages and e measures test scores of the Armed Forces Qualification Test (AFQT) which are normalized by their mean \bar{e} . Finally, v(s) is an education group specific error term.

 $^{^{22}\}mathrm{This}$ ensures that the skill premia are matched.

So, $\gamma^h(s)$ and $\gamma^l(s)$ are defined as $\bar{\gamma}(s) + \Delta(s)$ and $\bar{\gamma}(s) - \Delta(s)$, respectively, with $\bar{\gamma}(s)$ being adjusted to get the average $\gamma(s)$ for each education group equal to 1.

An equally justifiable alternative is to fix all $\bar{\gamma}(s)$ to 1, and rescale the probability functions instead to target the average $\gamma(s)$ of 1.

Table 6 shows the resulting estimates $\hat{\rho}(s)$. The estimated ability (human capital) gradient is strictly increasing in education reflecting a pronounced complementarity between ability (human capital) and education.

Table 6: Ability Gradient by Education Level

Education Level	Ability Gradient
(HS- & HS)	0.4248(0.0481)
(CL-)	$0.5786\ (0.0245)$
(CL & CL+)	$0.7298\ (0.0670)$

Notes: Estimated ability gradient $\hat{\rho}(s)$, using NLSY79 as provided in replication files for Abbott et al. (2019). Standard errors in parentheses.

4.9 Government

The government has to balance the budget of the general tax and transfer system as well as the budget of the pension system. Regarding the general tax and transfer system budget, the government has to finance an exogenous stream of (non-education related) expenditures and an endogenous stream of education related expenditures (pre-tertiary and tertiary). The revenue side of the general tax and transfer system is comprised by taxes on consumption, capital income and labor income. The consumption tax rate is set to 5% (see Mendoza et al. (1994)) while the capital income tax rate is fixed at 36%, following Trabandt and Uhlig (2011). Additionally, the government can issue debt.

Households working positive hours face the labor income tax schedule that is approximated using a two-parameter tax function as in Heathcote et al. (2017):

$$T(y, n > 0) = y - (1 - \tau)y^{1 - \xi^q}$$
(29)

where τ is the level parameter, and ξ^q is the (marital status q specific) progressivity parameter. In the preliminary calibration, it is exogenously set to 0.18 for all population groups, following Heathcote et al. (2017), while the former is calibrated endogenously to match the government debt to GDP ratio of 100% in the baseline. The non-participating and unemployed households have no labor income and thus do not pay labor income taxes but only receive government transfers ω that are set to 20.2% of average (fulltime) earnings (CEX 2001-2007; consumption of bottom 10%). Thus, for non-working singles and couples (i.e. both spouses do not work) the tax / transfer functions are given by:

$$T(q = si, 0) = -\omega, \tag{30}$$

and

$$T(q = cpl, 0) = -2\omega. \tag{31}$$

If, however, only one spouse is non-working and the other spouse supplies positive hours, the tax function is as follows:

$$T(q = cpl, n(g) > 0, n(g^{-}) = 0, y) = y - (1 - \tau)y^{1 - \lambda^{cpl}} - \max\{0, 2\omega - y - (1 - \tau)y^{1 - \lambda^{cpl}}\}$$
(32)

In other words, the government guarantees the minimum income of 2ω also to the couples with only one partner supplying positive hours.

Finally, as for the pension system, the payroll tax τ^p is set to the current legislative level of 12.4% and the actual progressivity of the pension system is taken into account.

5 Results

5.1 The Thought Experiment

We now present the results of our main policy reforms. For each transition thought experiment we assume that the economy is in steady state calibrated to 2013-2017, and that the policy reform triggering the transition is completely unexpected (the proverbial MIT shock), but that the government is henceforth fully committed to the policy reform. Our benchmark reform is "Free College", a 100% subsidy of college tuition, financed by permanent increase in labor income tax rate τ . That is, this tax parameter increases to insure that the intertemporal government budget constraint remains satisfied. In order to guarantee that the period-by-period budget constraint holds, government debt endogenously adjusts along the transition from the old steady state to its new steady state value (as a fraction of GDP). The corresponding "Better Schools" reform increases public (primary and secondary) school spending i_g permanently so that the extra

expenditures have the same present discounted value as the "Free College" reform, making both interventions fiscally comparable.

We present our main results in Subsection 5.2, contrasting in turn the aggregate, welfare and distributional consequences of the two reforms in general equilibrium. In order to insulate the importance of endogenous factor price movements (that is, changes in (relative) wages as well as the interest rate), in Subsection ?? we study the same reforms in a partial equilibrium setting where wages and interest rates remain fixed (so that labor markets and the capital market need not clear, i.e., (17)-(19) need not hold). However, the government intertemporal government budget constraint (12) is required to be satisfied in all our thought experiments.²³

5.2 General Equilibrium: Transitional Dynamics

In this section we summarize the transition results from our two main policy exercises; Table 9 in Appendix A provides a summary of comparison of the initial and the final steady states two which the policy transitions converge to.

5.2.1 Aggregate Variables and Welfare

In Figure 1 we display the dynamics of the college share, aggregate labor in efficiency units (L_t) , average human capital at age 18 and aggregate inter-vivos transfers over time. Panel (a) of Figure 1 demonstrate that both policies are successful in inducing more individuals to attend college, although making college free does so to a larger extent (it also leads to many more college dropouts). Furthermore, it takes time for the full impact of the reforms to take hold. Only after the third generation coming of college age after the reform has made its higher education decision (i.e., roughly 60 years after the policy change) has the share of the population with a college degree reached its new (and higher) steady state level. This observation reinforces the need to model transitions explicitly.

Panel (b) and (c) of Figure 1 indicate that college expansion is achieved through very different channels in the two reforms: in the "better schools" reform there is a much more pronounced increase in child human capital accumulation, and some of the now better-schooled 18 year old teenagers choose to attend college when they used not to. Crucially, as panel (c) demonstrates, even those not attending college now tend to have more human capital, and consequently are

 $^{^{23}}$ The progressive labor income tax code is specified as a two-parameter family following Benabou (2002) and Heathcote et al. (2017), and for the current thought experiments we fix the progressivity parameter and adjust the tax level parameter once and for all so that the intertemporal government budget constraint is satisfied. The sequence of government debt ensures that the flow budget constraint is satisfied in every period.

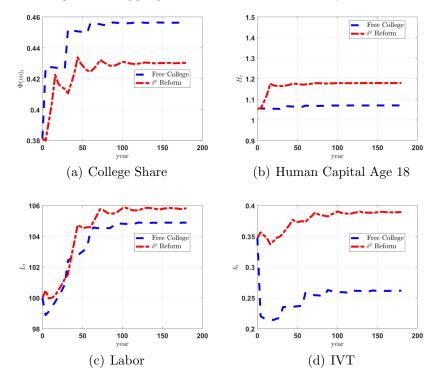


Figure 1: Aggregate Variables: General Equilibrium

more productive in the labor market. Thus, aggregate labor efficiency units actually rise more strongly under the "better schools" reform than under the "free college reform.

Finally, panel (d) demonstrates that private parental adjustments also significantly differ: when college is free, private inter-vivos transfers (which are mainly used for financing college tuition) collapse, which in turn impact parental asset accumulation. The strong response in labor and savings also anticipate the finding that accounting for general equilibrium factor price adjustments will have non-trivial quantitative implications.

The two policy reforms also have vastly different distributional consequences. This is apparent from Figure 2 which complements panel (a) of Figure 1 and shows the evolution of the share of college dropouts (panel (a)), those completing high-school (panel (b)) and those with some, but not complete college (panel (c)). We want to highlight two key observations here.

First, the free college reform does not change the share of children dropping out from high school (see panel (c)). For these children, predominantly from families with low educational background and often with only a single parent, the problem of college attainability prior to the reform is not (primarily) that it too expensive to attend college, but that their human capital acquired during childhood makes it very difficult for them to succeed and very strenuous to study (the utility cost of attending college is very high, given their human capital). In contrast, the "better schools" reform leads to a decline in the share of high-school dropouts in the population by about 1.5 percentage points.

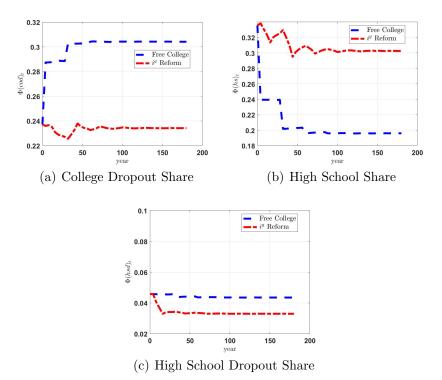


Figure 2: Higher Education Shares: General Equilibrium

Second, many of the additional students drawn to college under the free college reform do not actually complete college (since their human capital from high school is relatively low and thus the chances of dropping out are high). In the long run (see Table 9 in the appendix), although the "free college" reform shifts 14 percentage points of previous high school graduates to college attendance, only about have of these end up with a degree. In contrast, virtually all of the new college attendees under the "better schools reform" (approximately 5 percentage points) graduate from college, suggesting that this reform uniformly shifts up the tertiary school attainment distribution and benefits all segments of the distribution in terms of labor-market relevant skills.

Figure 3 shows the education population shares (as opposed to the education shares of a specific age cohort that was depicted in Figure 1). It displays the recurrent theme that the education reforms studied in this paper take time to materialize their full effect since the education expansion only directly impacts currently young generations that still have to go through school and/or make their higher education choices. Initially, this is a small share of the population, but over time these cohorts make an increasingly large share of the total labor force and thus the share of college-educated workers gradually increases (and that of individuals with only a high school

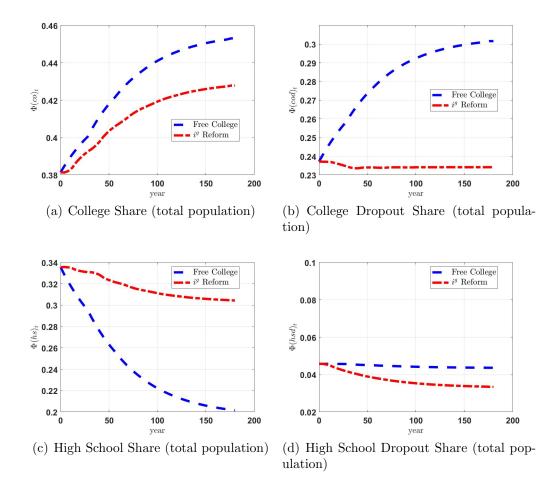
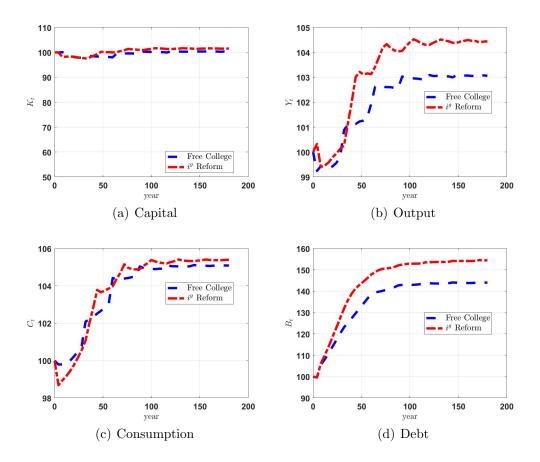


Figure 3: Total Population Education Shares (general equilibrium)

degree declines). The extent to which this happens differs, of course, across the two reforms and is stronger for the free college thought experiment. In contrast, the better schools reform over time almost halves the share of the population without even a high school degree, although this takes three generations. with no such effect from the free college reform.

Figure 4 displays the evolution of aggregate capital, output, consumption and government debt. Since capital is broadly constant along the transition, the time path of output roughly follows that of aggregate labor input; the same is true for aggregate consumption. For the same reason, the tax base increases gradually with labor along the transition, whereas the education cost in both reforms rises immediately. Therefore, the government accumulates debt along the transition, and since the "better schools" reform delivers a larger output in the long run, the capacity to service debt is more substantial in that reform as well. See panel (d) of Figure 4.



We cast our model in general equilibrium, and therefore interest rates, wages and taxes adjust along the transition path to ensure that the labor markets for college-educated labor, non-college labor and the assets market clears. In Figure 5 we display the time paths of these equilibrium factor prices and taxes. We observe that on account of the increase in labor input (in efficiency units) induced by both education reforms, the capital-labor ratio falls, the interest rate increases over time (see panel (a)), and wages per efficiency unit (not shown) fall over time. However, since college- and non-college labor are imperfect substitutes and non-college labor becomes scarcer relative to college labor, the college wage premium falls by ten percentage points (see panel (b)) but wages of those without a college degree actually increase (see panel (c) of Figure 5). In contrast, those with a college degree see their absolute wages decline substantially (relative to the long-run balanced growth path, of course). Finally, panel (e) shows the (once and for all) adjustment in the labor income tax rate τ required to insure that the intertemporal government budget constraint holds. It demonstrates that although both reforms require an increase in

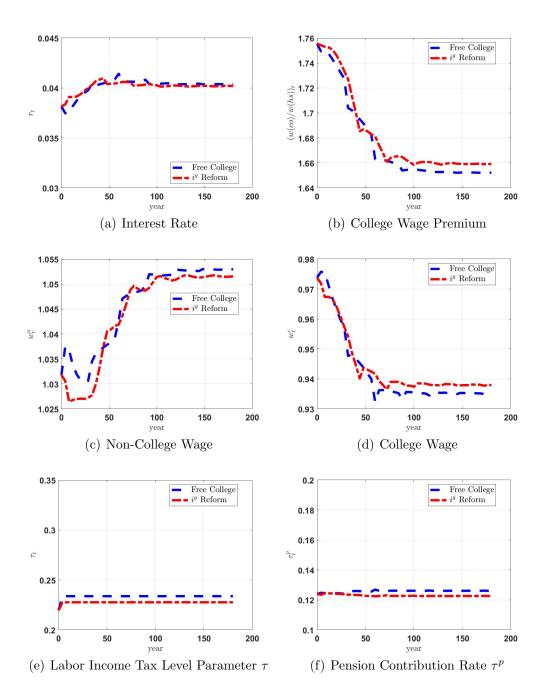


Figure 5: Prices and Taxes in General Equilibrium

5.2.2 Intergenerational Education Transmission

Increased School Funding								
	$s^p = hsd, q = si$ $s^p = hs, q = si$		$s^p = cod, q = si$	$s^p = co, q = si$				
s = hsd	0.051[-0.019]	0.048[-0.018]	0.042[-0.016]	0.028[-0.010]				
s = hs	0.791[+0.051]	0.735[+0.005]	0.681[+0.015]	0.003[-0.010]				
s = cod	0.056[-0.019]	0.078 [-0.001]	0.100[-0.011]	0.352[-0.023]				
s = co	0.102[-0.013]	0.139[+0.015]	0.178[+0.011]	0.617[+0.044]				
Free College								
	$s^p = hsd, q = si$	$s^p = hs, q = si$	$s^p = cod, q = si$	$s^p = co, q = si$				
s = hsd	0.070[+0.001]	0.065[-0.001]	0.059[+0.002]	0.040[+0.001]				
s = hs	0.505[-0.235]	0.503 [-0.227]	0.496[-0.170]	0.002[-0.012]				
s = cod	0.197[+0.121]	0.197 [+0.117]	0.199[+0.088]	0.386[+0.011]				
s = co	0.227[+0.112]	0.235[+0.111]	0.246[+0.080]	0.572[-0.001]				

Table 7: Intergenerational Education Transition Matrix: Single Parents

Table 7 summarizes one key dimension of the distributional consequences of these reforms, the intergenerational education transmission matrices for children with single mothers (that is, the share of children with maternal education $s^p \in \{hsd, hs, cod, co\}$ that end up with own education s). Positive changes relative to the baseline are marked in red, negative changes are marked in blue, and point changes relative to the initial steady state baseline are in square brackets. The results highlight the possibility of very different distributional consequences of the reforms: whereas the free college reform leaves the intergenerational transmission of dropping out of high school almost completely unchanged (see the first row of the lower panel) since the pull of free college is insufficient to change higher education choices of those otherwise dropping out of high school, increased primary/secondary school financing enhances human capital accumulation at younger ages and makes transitions out of this state more likely and transitions into this state less likely.

Table 8 shows the same intergenerational education state transition matrix for children with married parents. The education level of parental households is determined by the highest educational degree obtained by either of the two parents.

Third, the reforms also impact intergenerational persistence of earnings and educational attainment. Perhaps the most striking contrast between the reforms is the differential impact on the educational attainment of the poorest children, which tend to be children growing up in a household with a single parent and low (less than high school) educational attainment. The free college reform hardly changes the educational attainment of these children, primarily because their accumulated human capital during childhood makes them very unlikely to go to college and succeed there. In contrast, the additional human capital accumulation these children obtain with the school expenditure reform, although insufficient in most cases to push them above the col-

Increased School Funding								
	$s^p = hsd, q = si$	$s^p = hs, q = si$	$s^p = cod, q = si$	$s^p = co, q = si$				
s = hsd	0.046[-0.013]	0.043[-0.014]	0.045[-0.012]	0.021[-0.006]				
s = hs	0.688[+0.015]	0.670 [-0.007]	0.713[+0.040]	0.005[+0.000]				
s = cod	0.096[-0.009]	0.103 [+0.001]	0.086[-0.019]	0.334[-0.028]				
s = co	0.170[+0.008]	0.184[+0.021]	0.156[-0.009]	0.641[+0.035]				
Free College								
	$s^p = hsd, q = si$	$s^p = hs, q = si$	$s^p = cod, q = si$	$s^p = co, q = si$				
s = hsd	0.060[+0.001]	0.056[-0.002]	0.055[-0.002]	0.029[+0.001]				
s = hs	0.501[-0.172]	0.513 [-0.164]	0.548[-0.125]	0.003 [-0.002]				
s = cod	0.195[+0.089]	0.187[+0.085]	0.168[+0.064]	0.365[+0.004]				
s = co	0.245[+0.082]	0.244[+0.081]	0.228[+0.063]	0.603[-0.003]				

Table 8: Intergenerational Education Transition Matrix: Married Parents

lege threshold, strongly increases the chances of these children to complete high school, therefore strongly reducing the intergenerational persistence of dropping out of high-school.

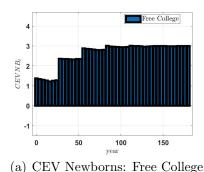
5.2.3 The Welfare Consequences of the Reforms

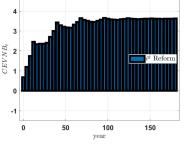
Finally, Figure 6 displays the welfare consequences of both policy reform transitions, measured as consumption equivalent variation of economically newborn individuals (i.e., based on expected lifetime utility at age 18), and plotted as a function of the period of the transition at which these individuals enter the economy (i.e., t = 0 means individuals becoming economically active in the first period of the transition). Expectations (averages) are taken over the state variables by which newborns differ, including their initial assets and human capital as well as the education of their parents.

The left panels are for the free college reform and the right panels are for the expansion of public school funding. The distinction between the upper and the lower panels is that the former uses the actual (endogenous) cross-sectional distribution over initial characteristics of 18-year olds, whereas the lower panel uses the initial steady state distribution, thereby abstracting from the fact that a policy change will change the distribution of human capital, financial wealth and parental education that 18 year-olds enter their economic life with.

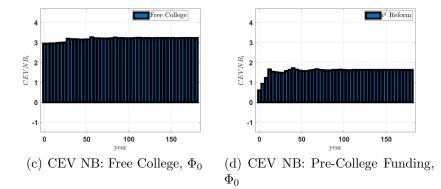
We highlight three qualitative points. First, both reforms entail welfare gains for current newborns and future generations. Second, as a comparison between the upper and the lower left panel reveals, the benefits of free college for 18 year-olds are partially offset by the fact that they enter adult life with fewer assets as their parents respond to the policy by adjusting inter-vivos transfers. That is, the actual welfare gains for the youth are smaller than if the policy were evaluated under a fixed initial distribution of wealth (as well as human capital), especially early in







(b) CEV Newborns: Pre-College Funding



the transition.²⁴ Third, and in sharp contrast, the key part of the welfare gains with better school funding comes from an enhanced human capital (and parental education) distribution and these gains increase over time, as can be seen comparing the top and the bottom right panels. This reinforces the need for studying (debt financed) policy *transitions* when considering fundamental education reforms.

Of course, welfare gains for academically newborn agents might partially come at the const of welfare losses for existing (at the time of the policy reform) generations that have to pay the higher taxes but do not benefit directly from the reforms since their human capital accumulation and tertiary education decisions lie in the past. However, since these generations are altruistically motivated toward their children, these generations might benefit indirectly through higher expected lifetime utility of their offspring,

In Figure 7 we summarize the welfare consequences (again measured in terms of consumption equivalent variation) of these generations (by their age, and again averaged over their relevant state variables). Again left panels are for the free college reform and right panels for the better school reform. The upper panels are aggregating across the entire age group, and the lower panels zoom in on the poorest part of the population (high school dropouts with low labor productivity).

 $^{^{24} {\}rm Later}$ during the transition, the fact that young adults have better educated parents partially offsets this effect.

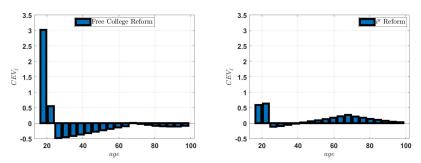
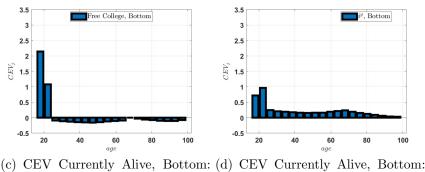


Figure 7: Welfare Gains of Currently Alive Population

(a) CEV Currently Alive, Total: (b) CEV Currently Alive, Total: "Free College" "Better Schools"



"Better Schools"

"Free College"

We observe that older generations (those whose college education is completed) indeed suffer welfare losses from the reforms (i.e., the gains from seeing their children enjoy higher lifetime utility cannot compensate for the higher taxes and thus lower own lifetime utility these generations have to endure to finance the reform). Note, however, that these welfare losses are relatively mild. Thus, although neither reform constitutes a Pareto improvement (since the current old lose), it is conceivable that a reform that phases in the tax increases slowly might be sufficient to avoid the welfare losses for generations older than 22 at the time of the reform that Figure 7 documents.

6 Conclusion

This paper is concerned with evaluating the welfare and distributional implications, both in the short run and in the long run, of government education policy interventions targeted at different stages of childhood and adolescence, and analyzing their interaction with social insurance policies such as progressive taxation and social safety net policies.

Eventually, our objective is to characterize numerically the optimal policy transition along these dimensions. Special emphasis is placed on the performance of children of single mothers that constitute more than 20% of the current US child population²⁵ and are subject to particularly low upward mobility levels.

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 $^{^{25}}$ See e.g. the 2019 Pew Research Center study on "Religion and Living Arrangements Around the World"

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A Additional Results

A.1 Comparison of Steady States

Variable	Initial SS	GE CS	PE+r CS	PE CS	GE i^g	PE+r i^g	PE i^g
$\Phi(j_a, s = co)$	38.14%	7.49% p	8.25% p	8.36% p	4.87%p	8.18%p	8.81%p
$\Phi(j_a, s = cod)$	23.72%	$6.69\%\mathrm{p}$	$6.86\%\mathrm{p}$	$7.17\%\mathrm{p}$	-0.30%p	1.38% p	$1.81\% \mathrm{p}$
$\Phi(j_a, s = hs)$	33.56%	-13.95% p	-14.73%p	-15.19%p	-3.29%p	-8.12%p	-9.15%p
$\Phi(j_a, s = hsd)$	4.58%	-0.22%p	-0.37%p	-0.34%p	-1.28%p	-1.44%p	-1.47%p
HK	1.06	1.42%	2.49%	2.27%	11.71%	13.47%	13.81%
L	9.56	4.89%	5.61%	6.52%	5.81%	8.85%	10.45%
Hours	0.28	-0.36%	-0.78%	0.58%	0.38%	0.46%	2.04%
C	8.29	5.09%	5.56%	2.81%	5.40%	8.09%	5.29%
K	12.50	0.34%	n/a	-19.39%	1.54%	n/a	-21.67%
В	3.65	44.03%	n/a	51.32%	54.45%	n/a	85.03%
Y	14.27	3.06%	n/a	-3.20%	4.44%	n/a	0.17%
r (point change)	0.038	0.0022	0.0022	0	0.0020	0.0020	0
$\frac{w^c}{w^n}$	1.86	-5.88%	0%	0%	-5.49%	0%	0%
\tilde{w}^n (outerloop var)	1.03	2.05%	0%	0%	1.91%	0%	0%
w^c (outerloop var)	0.97	-3.96%	0%	0%	-3.68%	0%	0%
au	0.2195	6.51%	9.59%	9.59%	3.73%	3.90%	3.90%
CEV NB	0	3.00%	3.83%	1.66%	3.63%	5.25%	3.63%
CEV alive	0	0.00%	0.04%	-0.32%	0.14%	0.24%	-0.15%
LSRA	0	tbc	tbc	1.35%	tbc	tbc	1.53%

Table 9: Aggregates, Prices, Taxes and Welfare

Notes: GE refers to the full general equilibrium version of the model where wages and interest rate are endogenous as well as the government budget constraints are balanced. The PE version refers to a lifecycle model with balanced government budgets, with aggregate physical capital being computed as: $K_t = A_t - B_t$, where A_t are total household assets, and B_t is debt stock (which is endogenous and results from rebalancing the intertemporal government budget constraint).

PE+r refers to the PE version where the interest rate path from the GE version is exogenously imposed without any adjustments of other outerloop variables. Therefore, the government budget constraints do not have to hold, and therefore the debt values (as well as the implied physical capital $K_t = A_t - B_t$ and thus also output are not shown, and marked as n/a.