Asset Prices, Wealth Inequality, and Taxation*

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Abstract

We investigate the interplay between asset prices, wealth inequality, and taxation in a dynamic general equilibrium economy populated by multiple agents with heterogeneous risk aversions. Tax revenues are collected from consumption taxes and are equally redistributed to all investors through non-pledgeable government transfers. Taxes address wealth inequality by ensuring stationarity of consumption share distributions and preventing consumption shares of less affluent investors from diminishing toward zero. Higher taxes increase stock risk premia and volatilities by shifting wealth toward poorer risk-averse investors, and tend to decrease stock price-dividend ratios and interest rates. The rise in risk premia and decrease in interest rates benefit more affluent, less risk-averse investors, partially offsetting the impact of higher taxes on their wealth, albeit to a small extent. We find that taxes do not prevent high concentrations of wealth at the top of the wealth distribution due to the investment decisions and tax responses of more affluent investors. We also extend the model to incorporate restricted stock market participation and show that its interplay with taxation increases stock risk premia and volatility, and decreases interest rates.

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1. Introduction

Taxation is a crucial instrument for financing public services and addressing wealth inequality through the redistribution of resources within an economy. Its influence on savings and wealth distribution directly impacts investors' asset holdings, and consequently asset prices. While existing literature has extensively examined the economic implications of various financial taxes, such as corporate taxes, on investment and stock prices, the effects of consumption taxes remain unexplored despite the fact that consumption taxes represent a significant proportion of tax revenue, accounting for approximately 30% of total collections, as opposed to a mere 10% for corporate taxes in OECD countries (Jacob, Michaely, and Müller, 2018; OECD, 2022).[1] Given the prominence of consumption taxes as a revenue source, understanding their influence on asset prices and wealth distribution is of paramount importance for informing policy and addressing wealth disparities in society.

We provide a tractable dynamic model that captures the interplay between taxation, wealth inequality, and asset prices. We demonstrate that government transfers to investors (funded by consumption tax) play a key role in confronting wealth inequality by preventing consumption shares of the poorer investors in the aggregate consumption from diminishing to zero. Our findings also underscore the substantial impact of taxes on asset prices and reveal a feedback loop between inequality and asset prices. However, we find that taxes overall do not prevent high concentration of wealth by the most affluent investors due to their optimal investment choices and responses to taxes.

In particular, we study a general equilibrium continuous-time economy with taxation populated by investors with heterogeneous constant relative risk aversions (CRRA). The investors allocate their wealth across a riskless bond and a stock, pay consumption taxes, and receive government transfers (e.g., social security, unemployment benefits, education grants, healthcare subsidies), which are financed by tax revenues and are equally redistributed back to investors. Consequently, the investors are effectively subject to taxation on their consumption relative to the average consumption in the economy. Generally, consumption levels exhibit a positive correlation with wealth levels. Therefore, this taxation system promotes equality by providing greater benefits to less affluent investors, akin to other progressive tax systems devised to tackle inequality. To focus on the impact of taxes on inequality and asset prices more effectively, we treat taxes as an exogenous variable in

[1]The proportion of total tax revenue derived from consumption taxes (such as value added taxes, sales taxes, excises, customs and import duties) varies across OECD countries from 15% in the USA to 50.5% in Chile (OECD, 2022).

our analysis. This allows us to isolate the influence of taxes on the economic variables of interest, while minimizing confounding effects of other factors. We provide novel predictions regarding the effects of taxes on asset prices and inequality, as elaborated below. There is, however, limited empirical research available in this area, possibly due to the challenges in identifying a suitable natural experiment with exogenous tax changes. This difficulty of obtaining empirical evidence underscores the importance of theoretical frameworks for examining and understanding the impact of taxes on asset prices and wealth inequality. Our theoretical analysis may serve as a starting point for further empirical research in this field.

Our analysis highlights the importance of taxes and economic frictions in addressing wealth inequality. In a frictionless economy, inequality escalates without restraint, leading to a situation where the most risk-tolerant investors eventually hold nearly all wealth and consumption, even when taxation is in place. This occurs because investors in such economies can negate the impact of government transfers by engaging in financial transactions. Government transfers in our economy are perfectly correlated with the aggregate output, and hence are equivalent to holding non-tradable shares in the aggregate output, which can be easily offset by a reduction of stock holdings or shorting. Only a combination of taxes and frictions that prevent investors from offsetting government transfers leads to the reduction of inequality and stationary distributions of the wealth and consumption shares of investors in the economy. We argue that one such friction is non-pledgeability for government transfers, preventing investors from taking risky positions in asset markets backed by government transfers. Consequently, investors are unable to counterbalance the effects of government transfers via financial transactions. As a result, during times when investors' wealth is diminished, receiving government transfers enables them to rebuild their wealth, giving rise to the stationarity.

To solve the model analytically, we endow investors with myopic time-separable CRRA preferences defined over current consumption and next-period financial wealth, following the literature.[2] The CRRA preferences over next-period financial wealth induce non-negativity of financial wealth, making investors averse to default. As shown in the related literature, this constraint on financial wealth can be interpreted as a collateral constraint in settings with extraneous non-pledgeable sources of income and moral hazard (Detemple and Serrat, 2003; Chien and Lustig, 2010; Chabakauri and Han, 2020). Consequently, our model captures

[2] Myopic preferences have been widely employed in the literature (DeLong, Shleifer, Summers, and Waldman, 1990; Acharya and Pedersen, 2005; He and Krishnamurthy, 2013; Basak, Chabakauri, and Yavuz, 2019), and are further discussed in Section 2.1 below. Our formulation of investor preferences can be further motivated by economic settings in which investors live for one period and bequeath their next-period financial wealth to the subsequent generation.

in reduced form the effect of non-pledgeability of transfers.

We obtain closed-form expressions for the equilibrium interest rates, price-dividend ratios, stock risk premia, and return volatilities. These expressions feature cross-sectional expectations and covariances in which investors' consumption shares in the aggregate consumption play the role of probabilities. These consumption shares act as endogenous state variables in our model. We also obtain closed-form stationary distribution of consumption shares in an economy with two investor types. The tractability of our setup overcomes a common challenge of handling numerous state variables in models featuring multiple investors and economic frictions, which significantly complicates solving these models. Our approach circumvents the use of approximate dynamics of state variables in numerical methods, such as those developed in Krusell and Smith (1998), by providing closed-form dynamics of state variables, which enables straightforward simulation of their paths.

We analyze the impact of taxation on equilibrium processes by dissecting it into direct and indirect effects. Direct effects pertain to the influence of taxes on equilibrium processes holding consumption shares fixed, and capture the effects immediately following unforeseen tax alterations. Indirect effects, on the other hand, emerge through the consequences of taxation on inequality, particularly regarding consumption shares that shift due to taxation. While direct effects can be investigated analytically, indirect and overall effects of taxation are evaluated through economic simulations and subsequent calculation of unconditional expectations of equilibrium processes.

We first assess the direct impact of taxation, which hinges on how investors' investment and saving behaviors respond to changes in taxes. We show that the price-dividend ratio, risk premium, and stock return volatility are all increasing functions of the tax rate, holding consumption shares fixed. This is because as the tax rate increases, investors consume less and save more in our model, consistent with standard models in public economics (Atkinson and Stiglitz, 2015). As a result, the stock investments also grow, leading to higher price-dividend ratios. Moreover, as less risk-averse investors tend to invest more in stocks as compared to their more risk-averse counterparts, the stock return volatility escalates when taxes increase. This occurs because the wealth of less risk-averse investors is more volatile. For the same reason, stock return volatility exceeds dividend volatility, consistent with the evidence (Shiller, 1981). Risk premium in our economy is proportional to the stock return volatility (as in standard models), and hence, also increases as taxes go up. The interest rate can either increase or decrease with higher taxes depending on the relative strength of two effects that partially offset each other. On the one hand, higher taxes raise consumption costs and hinder savings, reducing riskless investments and lowering interest rates. On the other hand, government transfers boost disposable income, enabling increased investments and raising interest rates.

We next analyze the overall impact of taxation on equilibrium, also accounting for its indirect effects on wealth and consumption distribution. Higher taxes tend to reduce interest rates and price-dividend ratios, though non-monotonicity may occur due to conflicting direct and indirect effects, while also raising stock risk premia and volatilities. Intuitively, higher taxes shift consumption towards poorer more risk-averse investors, raising the representative investor's risk aversion. This causes interest rates to decline as risk-averse investors favor safer assets, outweighing the direct effects discussed previously. Increased risk aversion of the representative investor also causes risk premia to rise. Furthermore, higher taxes also lead to larger consumption shares for risk-averse investors with lower wealth-consumption ratios. As the price-dividend ratio reflects the economy's weighted average of wealth-consumption ratios, it may decrease if the indirect effect surpasses the direct effect. The volatility also increases through its positive dependence on the representative risk aversion and inverse dependence on the price-dividend ratio.

We next examine the impact of taxation on inequality. Higher taxes and non-pledgeable government transfers ensure the stationarity of consumption shares so that they do not diminish to zero over time, unlike in economies without taxes. Although our model may not match quantitatively the stylized facts about wealth inequality, it generates extreme inequality at the top of the distribution, consistent with the evidence in Saez and Zuckman (2016). We show that the least risk averse investors ultimately possess disproportionately large share of aggregate wealth. Taxation falls short in addressing wealth concentration due to consumption and investment choices of more affluent less risk averse investors and their response to tax increases. First, they save more and consume less when taxes increase. Second, they maintain higher wealth-consumption ratios as compared to poorer more risk-averse investors who invest less, consume more, and hence lose wealth over time. Third, they invest more in high-yielding risky assets than their more risk-averse counterparts. Lastly, we find that lower interest rates and higher risk premia resulting from higher taxes create a small welfare gain for less risk averse investors because they borrow and invest in risky assets more than their risk averse counterparts.

Finally, we extend the model to incorporate restricted stock market participation, a prominent feature of financial markets that helps explain stock risk premia, volatilities, and interest rates (Mankiw and Zeldes, 1991; Basak and Cuoco, 1998; Guvenen, 2009). We find that the interplay of restricted participation and taxation elevates stock risk premia and volatilities, whilst driving down interest rates. This extension helps untangle the portfolio choice and saving decisions of investors, which in the main model are driven by the risk aversion parameter. Specifically, we consider an economy in which the investors have identical savings rates but different stock holdings and confirm portfolio heterogeneity as an important source of inequality. Additionally, we verify that the findings of our main analysis remain valid in this extended setup.

Literature Review. Our paper belongs to the literature studying the impact of investor heterogeneity on wealth inequality and asset prices. We contribute to this literature by exploring the roles of taxes and non-pledgeable government transfers in addressing wealth inequality. We provide novel insights into how inequality and taxes influence asset prices and investigate the feedback effects between inequality and asset prices. Below, we review the literature closely related to our research.

Pástor and Veronesi (2016) investigate the impact of production taxation on inequality and asset prices in a general equilibrium framework. In their model, investors exhibit heterogeneous risk aversions and skills, with their decision to become entrepreneurs being influenced by the level of taxation. This choice, in turn, endogenously contributes to inequality. Pástor and Veronesi (2020) explore political cycles in a model characterized by investors with varying risk aversions and taxation on aggregate output. Our model differs from these works in several significant ways. Taxes in our economy are imposed on consumption rather than production, our economy is populated by multiple groups of investors, and the distribution of consumption shares remains stationary. Veronesi (2019) studies asset prices and wealth inequality in a complete-market economy, inhabited by a continuum of investors with heterogeneous risk aversions, and characterizes asset prices based on cross-sectional expectations, utilizing consumption shares as probabilities. Our work diverges from Veronesi in two main aspects: we emphasize the influence of taxation, and our economy is stationary, ensuring that even risk-averse investors survive in the long run.

Gomez (2022a) investigates wealth inequality in an overlapping generations economy with two investor risk aversion levels. He analyzes the feedback effect on asset prices and derives tails of the wealth distribution. In contrast, our study focuses on taxation, non-pledgeability of transfers, and features a wide range of investor risk aversions. The stationarity of wealth and consumption shares in our model is due to taxes and non-pledgeability of transfers, unlike the OLG structure in Gomez (2022a). Gomez (2022b) explores the determinants of wealth inequality and characterizes the growth rates of aggregate wealth shares, but does not examine the influence of wealth inequality on asset prices, as we do in our paper.

Bewley (1986), Aiyagari (1994), and Huggett (1996) examine the influence of idiosyncratic income shocks on wealth inequality within an incomplete-market economy populated by ex ante identical investors who only invest in risk-free assets and are subject to borrowing constraints. These models have been extended in various directions. For example, Wang (2007) investigates the distribution of income and wealth in an incomplete-market economy with CARA investors, whose rate of time preferences is determined recursively based on their consumption levels. Benhabib, Bisin, and Zhu (2011, 2015) extend Bewley (1986) to account for idiosyncratic shocks to labor and capital incomes and show that wealth distributions have fat tails. Hubmer, Krusell, and Smith (2021) study further realistic extensions, keeping the excess returns on capital exogenous, that help explain wealth inequality at the very top. Our paper also has ties to the mean-field games literature, as reviewed by Lasry and Lions (2007). Similarly to this literature, we also focus on the behavior of numerous interacting agents whose actions shape the overall dynamics of equilibrium processes. Achdou, Han, Lasry, Lions, and Moll (2022) apply the mean-field game theory to analyze a continuous-time version of the models in Bewley (1986), Aiyagari (1994), and Huggett (1996). Our study, like those discussed above, incorporates a friction but assumes non-pledgeable government transfers instead of borrowing constraints. We extend the literature by examining the impact of taxes on asset prices and the effect of a single systematic shock, revealing that such a shock can lead to extreme wealth inequality. Our model generates wealth heterogeneity through diverse saving and investment decisions influenced by systematic shocks and taxes.

Piketty (1997) studies the joint dynamics of wealth distribution, inequality, and interest rates in a production economy with credit rationing. Gabaix, Lasry, Lions, and Moll (2016) examine reduced-form models of inequality with stabilizing forces like minimum income and death/replacement rates, resulting in stationary wealth distributions. We complement these works by exploring taxation and non-pledgeable government transfers as stabilizing forces and examining the impact of inequality and taxation on asset prices. Furthermore, the inequality in our model is driven by systematic shocks rather than idiosyncratic ones.

Our paper further contributes to the extensive literature on asset pricing with frictions such as borrowing, liquidity, and various portfolio constraints (Detemple and Murthy, 1997; Basak and Cuoco, 1998; Alvarez and Jermann, 2000; Detemple and Serrat, 2003; Chien and Lustig, 2010; Chabakauri, 2013; Rytchkov, 2014; Chabakauri, 2015). The primary friction in our model arises from the fact that government transfers are non-pledgeable and cannot be utilized as collateral for risky positions. This aspect links our work to the broad literature on collateral constraints (Fostel and Geanakoplos, 2008; Geanakoplos, 2009; Chabakauri and Han, 2020). Moreover, our paper is connected to research that investigates frictionless economies where investors differ in their risk aversions, focusing on models with two investors (Dumas, 1989; Chan and Kogan, 2002; Gârleanu and Panageas, 2015; Longstaff and Wang, 2012; Bhamra and Uppal, 2014) and multiple investors (Xiouros and Zapatero, 2010; Veronesi, 2019).

Our paper is also related to the empirical literature on inequality and the effects of taxation. Specifically, Saez and Zuckman (2016) investigate wealth distribution in the United States, and find high wealth concentration at the top of the distribution. They document that the top 0.1% wealth share increased significantly from 7% in 1978 to 22% in 2012. Meanwhile, Jacob, Michaely, and Müller (2018) presents evidence suggesting that higher consumption taxes lead to a reduction in corporate investment, demonstrating the significant influence that consumption taxes have on the economy.

The remainder of the paper is organized as follows. Section 2 presents our model with taxation and non-pledgeability of government transfers. Section 3 provides the dynamic equilibrium with taxation and Section 4 discusses our results on the interplay between asset prices, wealth inequality, and taxes. Section 5 concludes. Appendix provides all the proofs.

2. The Economy with Taxation

We consider a pure-exchange continuous-time infinite-horizon economy with a representative firm that produces one consumption good and N groups of investors with heterogeneous risk aversions. In this Section, we discuss the financial markets and investor optimizations in the presence of taxes.

2.1. Financial Markets, Investors, and Taxes

We consider a dynamic continuous-time economy in which uncertainty is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$, on which is defined a Brownian motion w. All stochastic processes are adapted to the filtration $\{\mathcal{F}_t, t \in [0, \infty]\}$, generated by w. There is one representative firm in the economy that produces an exogenous stream of output D, which follows a geometric Brownian motion (GBM) process:

$$dD_t = D_t\big[\mu_D dt + \sigma_D dw_t\big], \tag{1}$$

with constant output mean-growth rate μ_D and volatility σ_D.

The economy is populated by N types of competitive investors with heterogeneous constant relative risk aversion (CRRA) coefficients $\gamma_N > \cdots > \gamma_2 > \gamma_1 \geq 1$. Each group of investors has mass $\pi_i > 0$ and all masses sum up to one. The investors trade continuously in two securities, a riskless bond in zero net supply paying an instantaneous riskless rate r, and a stock in net supply of one unit. The stock represents a claim to the stream of firm's output D paid continuously as dividend to the investors. We study Markovian equilibria in which the bond price, B, and stock price, S, follow dynamics

$$dB_t = B_t r_t dt, \tag{2}$$

$$dS_t + D_t dt = S_t\big[\mu_t dt + \sigma_t dw_t\big], \tag{3}$$

where the risk-free interest rate r, stock mean-return μ, and volatility σ are endogenously determined in equilibrium, and the bond price at time 0 is normalized so that $B_0 = 1$.

At each date t the investors choose consumption c_{it}, allocate fraction of wealth θ_{it} to stocks and the remaining fraction of wealth $\alpha_{it} = 1 - \theta_{it}$ to bonds. All investors pay consumption taxes τc_{it}, where τ is an exogenous tax rate in the economy. The taxes finance public goods and government transfers (e.g., education, healthcare, energy subsidies, unemployment benefits), which are modeled as direct payments to the individuals.[3]

A fair tax system is one that levies taxes on individuals and businesses according to their ability to pay and distributes public goods and government transfers in a way that benefits the poor more than the wealthy. In line with this principle, the government transfers in our economy are based on tax revenue per capita and are distributed equally to all individuals. As a result, those with lower-than-average consumption (usually the less affluent) see an increase in their wealth, while those with higher-than-average consumption experience a decrease. Due to the market clearing in consumption good (formally introduced below), aggregate consumption and output are equal in equilibrium, and hence, tax revenue per capita is equal to τD_t. Therefore, net individual gain or loss due to taxation is equal to $\tau(c_{it} - D_t)$. The taxation system operates through redistribution, where the wealth of some individuals is transferred to others. This system promotes equality and also ensures that all economic output remains in the economy without distorting the market clearing in consumption good. We make the assumption that a reduction in wealth inequality is a desirable goal. Our model does not examine the reasons why wealth inequality is considered detrimental to society.

[3]Our model can be extended to incorporate income taxes when investors receive a fraction of output D_t as their labor income. However, such taxes have only indirect effects on equilibrium through the redistribution of wealth towards less affluent investors. This redistribution affects the equilibrium processes in the same way as in the case of consumption taxes. The latter taxes, however, have important direct effects, studied in Section 3, which are absent for income taxes. The model can also incorporate wealth taxes τW_{it}, where W_{it} is the wealth of investor i. The economic implications of such taxes are similar to consumption tax in our model because consumption is proportional to aggregate wealth, as shown below.

2.2. Non-pledgeability of Taxes and Investor's Optimization

We observe that in frictionless dynamic economies with pledgeable government transfers consumption taxes do not affect the long-term distribution of consumption. In such economies this distribution is degenerate and concentrated in the hands of the least risk-averse investor, and hence, taxes cannot prevent extreme wealth inequality in the long run. Moreover, taxes do not have any direct effect on equilibrium processes. The intuition is that in complete markets investors can offset future cash flows by taking risky positions. The government transfers τD_t are perfectly correlated with the aggregate output D_t, and hence, are equivalent to holding a non-tradable share τ in the aggregate output. The investors may then find it optimal to short stocks, using future transfers as collateral, to reduce their overall effective share of the aggregate output.

Consequently, the non-pledgeability of government transfers is essential for addressing wealth inequality. The government in our economy imposes non-pledgeability of future transfers to discourage risky investments and the diversion of transfers from their intended uses, such as meeting basic needs. The non-pledgeability of future income has been addressed in the related literature by imposing non-negativity of next-period financial wealth $W_{i,t+dt} \geq 0$, defined as the value of investors' financial assets (Detemple and Serrat, 2003; Chien and Lustig, 2010; Chabakauri and Han, 2020). The latter restriction represents a no-default collateral constraint on risky positions when future incomes cannot be pledged. The intuition is that it requires portfolio losses to be fully offset by portfolio gains without pledging future income. In contrast, in economies with pledgeable income total collateral of investors is given by the sum of the financial wealth and the present value of future transfers $PV(\tau D)$. The financial wealth then can be negative without triggering default because risky positions are covered by future income and the overall wealth, $W_{i,t+dt} + PV(\tau D)$, remains non-negative.

Incorporating financial frictions such as non-pledgeability in an economy with multiple agents is a challenging task. We adopt a simple setup that captures this friction in a tractable way and admits closed-form solutions that allow straightforward comparative statics. Our approach seamlessly incorporates aversion to default, triggered by violations of the constraint $W_{i,t+dt} \geq 0$, into the investors' preferences, and hence, avoids solving constrained optimization. Specifically, we assume that investors have myopic time-separable preferences represented by a weighted average of constant relative risk aversion (CRRA) utility functions over current consumption c_{it} and next-period wealth $W_{i,t+dt}$, given by:

$$\frac{c_{it}^{1-\gamma_i} - 1}{1-\gamma_i}\rho dt + \mathbb{E}_t\Big[\frac{W_{i,t+dt}^{1-\gamma_i} - 1}{1-\gamma_i}\Big](1-\rho dt), \tag{4}$$

where $0 < \rho < 1$ is a time-preference parameter. The intuition behind preferences (4) is that despite myopia the investors understand that all of their future consumption is reliant on next-period wealth $W_{i,t+dt}$. Accordingly, they place a larger weight $1 - \rho dt$ on the utility of next-period financial wealth and smaller weight ρdt on the utility of current consumption.[4] Additionally, the investors experience extreme disutility when their next-period financial wealth approaches zero, leading them to maintain positive financial wealth at all times. Preferences (4) also arise in economic settings where investors live for a single period and pass on their financial wealth to the next generation.

The investors maximize preferences (4) with respect to consumption c_{it} and portfolio weight θ_{it} subject to the dynamic budget constraint

$$dW_{it} = W_{it}\left(r_t + \theta_{it}(\mu_t - r_t)\right)dt + W_{it}\theta_{it}\sigma_t dw_t - c_{it}dt - \tau(c_{it} - D_t)dt. \tag{5}$$

The last term in equation (5) captures the net effect of taxation on the dynamics of wealth.

3. Dynamic Equilibrium with Taxation

In this Section, we analytically derive the economic equilibrium and examine the comparative statics of equilibrium processes with respect to the tax rate τ. Our analysis demonstrates that price-dividend ratios, stock risk premia, and stock return volatilities are increasing functions of the tax rate, holding the distribution of consumption across investors fixed. We establish that taxes render the distribution of investor consumption shares stationary. This stationarity implies that taxes serve as a crucial instrument in mitigating inequality and averting the long-term convergence of poor investors' consumption shares to zero.

We define the equilibrium as a set of processes r_t, μ_t, and σ_t, and optimal consumptions c^*_{it} and portfolios θ^*_{it} that solve investors' optimization problems such that the following market

[4] Myopic preferences have been widely employed in the literature (DeLong, Shleifer, Summers, and Waldman, 1990; Acharya and Pedersen, 2005; He and Krishnamurthy, 2013; Basak, Chabakauri, and Yavuz, 2019). Moreover, utility weights in preferences (4) endogenously arise in dynamic optimization problems with logarithmic preferences in which optimization is equivalent to a myopic problem with the objective function $\ln(c_t)\rho dt + \mathbb{E}_t[\ln(W_{t+dt})](1 - \rho dt)$ that corresponds to the objective (4) with $\gamma_i = 1$. In Section 4.2, we also demonstrate that the model reproduces several key dynamic aspects of asset prices such as countercyclicality of Sharpe ratios, risk premia and stock return volatilities, and procyclicality of price-dividend ratios, much like its non-myopic counterparts.

clearing conditions in consumption good, stock, and bond markets are satisfied:

$$\pi_1 c^*_{1t} + \pi_2 c^*_{2t} + \cdots + \pi_N c^*_{Nt} = D_t, \tag{6}$$

$$\pi_1 \theta^*_{1t} W_{1t} + \pi_2 \theta^*_{2t} W_{2t} + \cdots + \pi_N \theta^*_{Nt} W_{Nt} = S_t, \tag{7}$$

$$\pi_1 \alpha^*_{1t} W_{1t} + \pi_2 \alpha^*_{2t} W_{2t} + \cdots + \pi_N \alpha^*_{Nt} W_{Nt} = 0. \tag{8}$$

Summing up the market clearing conditions for stocks (7) and bonds (8) and taking into account that the weights on stocks and bonds sum up to one, $\theta_{it} + \alpha_{it} = 1$, we find that the aggregate wealth in the economy equals the stock value:

$$\pi_1 W_{1t} + \cdots + \pi_N W_{Nt} = S_t. \tag{9}$$

Similar to the related literature on economies with heterogeneous investors (Chabakauri, 2013, 2015), we derive all equilibrium processes as functions of investors' consumption shares in the aggregate consumption, weighted by masses π_i of investor types, $y_{it} = \pi_i c^*_{it}/D_t$. These consumption shares are endogenous state variables of the model and sum up to one due to the consumption good clearing (6). We conjecture and verify that these consumption shares follow Markovian processes

$$dy_{it} = \mu_{y,it} dt + \sigma_{y,it} dw_t, \tag{10}$$

where the drift $\mu_{y,it}$ and volatility $\sigma_{y,it}$ are themselves functions of consumption shares y_{it}.

We start by solving for optimal consumptions and asset allocations of investors. Applying Itô's Lemma to the utility of wealth, we observe that the maximization of preferences (4) conveniently breaks down into two separate optimizations. In particular, the optimal portfolio weight solves a myopic mean-variance optimization problem. Lemma 1 presents optimal consumptions and portfolio weights.

Lemma 1. *Optimal consumption and portfolio of investor i are given by:*

$$c^*_{it} = \left(\frac{\rho}{1+\tau}\right)^{1/\gamma_i} W_{it}, \tag{11}$$

$$\theta^*_{it} = \frac{\mu_t - r_t}{\gamma_i \sigma_t^2}. \tag{12}$$

Optimal consumptions and portfolio weights have familiar tractable structures. From equation (11), we observe that more risk-averse investors consume a larger fraction of their wealth, and hence, invest less. The consumption of all investors is a decreasing function of the tax rate because higher taxes make consumption more costly. The investors hold a simple mean-variance portfolio, and their stock investment is a decreasing function of their risk aversion γ_i.

We solve for the equilibrium by substituting optimal consumptions (11) into the consumption good clearing (6) and then applying Itô's Lemma to both sides. The equilibrium processes are given by closed-form expressions in which consumption shares y_{it} act as cross-sectional probabilities. Proposition 1 provides a complete characterization of equilibrium and corresponding comparative statics with respect to consumption tax τ.

Proposition 1. *The equilibrium interest rate r, stock price-dividend ratio Ψ, risk premium $\mu - r$, and stock return volatility σ, in the economy with taxation are given by:*

$$
\begin{aligned}
r_t &= \mu_D - \sigma_D^2\Gamma_t + (1+\tau)\widehat{\mathbb{E}}_t\Big[\Big(\frac{\rho}{1+\tau}\Big)^{1/\gamma_i}\Big] - \tau\bar{\mathbb{E}}\Big[\Big(\frac{\rho}{1+\tau}\Big)^{1/\gamma_i}\Big], && (13)\\
\Psi_t &= \widehat{\mathbb{E}}_t\Big[\Big(\frac{\rho}{1+\tau}\Big)^{-1/\gamma_i}\Big], && (14)\\
\mu_t - r_t &= \sigma_t\sigma_D\Gamma_t, && (15)\\
\sigma_t &= \sigma_D + \widehat{\text{cov}}_t\Big(\frac{1}{\gamma_i}, \Big(\frac{\rho}{1+\tau}\Big)^{-1/\gamma_i}\Big)\frac{\sigma_D\Gamma_t}{\Psi_t}, && (16)
\end{aligned}
$$

where $\widehat{\text{cov}}_t(\cdot,\cdot)$ and $\widehat{\mathbb{E}}_t[\cdot]$ are time-t cross-sectional covariance and expectation, respectively, that use consumption shares y_{it} as probabilities, $\bar{\mathbb{E}}[\cdot]$ is an expectation that uses investors' masses π_i as probabilities, and Γ_t is the risk-aversion of the representative agent, given by

$$\Gamma_t = \frac{1}{\widehat{\mathbb{E}}_t[1/\gamma_i]}. \tag{17}$$

Consequently, the stock price-dividend ratio Ψ_t, risk premium $\mu_t - r_t$, and volatility σ_t are all increasing functions of tax rate τ, holding consumption shares fixed, and the stock return volatility exceeds dividend volatility, $\sigma_t > \sigma_D$.

The drifts and volatilities of the consumption share dynamics (10) are given by:

$$
\begin{aligned}
\mu_{y,it} &= y_{it}\Big(r_t + \frac{\sigma_D^2\Gamma_t(\Gamma_t-1)}{\gamma_i} - (1+\tau)\Big(\frac{\rho}{1+\tau}\Big)^{1/\gamma_i} - \mu_D + \sigma_D^2\Big) + \tau\pi_i\Big(\frac{\rho}{1+\tau}\Big)^{1/\gamma_i}, && (18)\\
\sigma_{y,it} &= \frac{y_{it}\sigma_D}{\gamma_i}(\Gamma_t - \gamma_i). && (19)
\end{aligned}
$$

The equilibrium processes in Proposition 1 are in closed form and admit simple comparative statics with respect to the tax rate τ. The explicit expressions for the price-dividend ratio and the stock return volatility are not available in dynamic frictionless economies. The equilibrium processes also feature the risk aversion of a representative agent Γ_t, which is a weighted harmonic average of investors' risk aversions given by (17), as in the related literature (Chabakauri, 2013, 2015; Veronesi, 2019).

The interest rate (13) is comprised of four terms. The first and second terms capture the effects of consumption smoothing and precautionary savings. In particular, higher mean output growth rate μ_D increases interest rates because investors are willing to borrow more when they expect higher consumption next period, whereas higher risk aversion Γ_t pushes interest rates down because investors are more willing to hold a riskless asset. The third term in (13) arises because higher consumption and taxes make investors less able to save and more willing to borrow. The last term arises due to government transfer τD_t, which increases disposable income and eases borrowing needs. The latter two effects push interest rates in opposite directions, and hence, the overall impact of taxes depends on which of these effects is stronger.

The stock price-dividend ratio (14) is a weighted average of investors' wealth-consumption ratios $W_{it}/c^*_{it} = (\rho/(1+\tau))^{-1/\gamma_i}$, implied by equation (11) for investors' optimal consumptions. The ratio Ψ is increasing with the tax rate τ, holding consumption shares fixed. The intuition is that as the tax rate increases, investors consume a smaller fraction of their wealth and invest more in stocks, giving rise to higher stock prices and price-dividend ratios.

The risk premium (15) is given by a standard expression that also holds in dynamic frictionless economies. This expression can be rewritten as $(\mu_t - r_t)dt = \Gamma_t \operatorname{cov}_t(dS_t/S_t, dD_t/D_t)$, and hence is equivalent to a consumption CAPM. The risk premium is increasing with the tax rate τ due to the effect of taxes on stock return volatility σ_t, explained below.

Equation (16) reports the stock return volatility. It sheds light on the excess volatility $\sigma_t - \sigma_D > 0$ and the impact of taxes. The excess volatility arises due to a positive covariance term in equation (16) between risk tolerance $1/\gamma_i$ and an increasing function of this risk tolerance $(\rho/(1+\tau))^{-1/\gamma_i}$, which equals the wealth-consumption ratio, as explained previously. To provide intuition, we rewrite equation (16) in terms of the ratio W_{it}/c^*_{it} as follows:

$$\sigma_t = \sigma_D + \widehat{\operatorname{cov}}_t\Big(\frac{1}{\gamma_i}, \frac{W_{it}}{c^*_{it}}\Big)\frac{\sigma_D \Gamma_t}{\Psi_t}. \tag{20}$$

Positive covariance $\widehat{\operatorname{cov}}_t\Big(1/\gamma_i, W_{it}/c^*_{it}\Big)$ implies that more risk tolerant investors consume a smaller fraction of their wealth and invest more in stocks. We also note that the volatility of an investor's wealth (5) is an increasing function of risk tolerance $1/\gamma_i$. Consequently, the aggregate wealth is more exposed to economic shocks and more volatile in economies where the latter covariance is larger. Moreover, the latter effect is stronger when taxes are higher because, as explained previously, higher taxes increase stock investments of more risk tolerant investors. The volatility of aggregate wealth equals the stock return volatility due to the market clearing condition (9). Hence, the volatility exceeds dividend volatility and is increasing with the tax rate τ.

The drift (18) and volatility (19) of consumption shares are affected by taxation due to its effect on consumption. Therefore, in addition to the direct effects of taxation on equilibrium processes, discussed above, there are also indirect effects through the effects of taxation on consumption shares of investors, which act as state variables in the economy. We study these indirect effects in Section 4 below.

The volatility of consumption share $\sigma_{y,it}$ reveals the direction of the reallocation of consumption and wealth across investors in response to economic shocks. This volatility is negative when $\gamma_i > \Gamma_t$. Therefore, a positive output shock dw_t transfers consumption and wealth from investors who are more risk averse than the representative investor, $\gamma_i > \Gamma_t$, towards less risk-averse investors with $\gamma_i < \Gamma_t$. We observe also that in economies where less risk averse investors have larger consumption shares, and hence, the representative risk aversion Γ_t is lower, more investors have risk aversions exceeding Γ_t. As a result, in such economies more investors lose and fewer investors gain consumption and wealth following a positive shock to aggregate consumption.

3.1. Stationarity and Two-Agent Economy

The expressions for the drift (18) and volatility (19) of consumption shares suggest that the consumption shares have repulsive boundaries at $y_{it} = 0$ and stationary distributions. At these boundaries, while the volatilities $\sigma_{i,y}$ are zero, the drifts $\mu_{i,y}$ are strictly positive, meaning that the consumption shares are repulsed from the boundaries. As a result, it is not possible for one of the consumption shares to converge to zero in an economy with taxation. Proposition 2 provides a closed-form expression for the stationary distribution in the economy with two investor types and demonstrates that stationarity is not attainable without taxation.

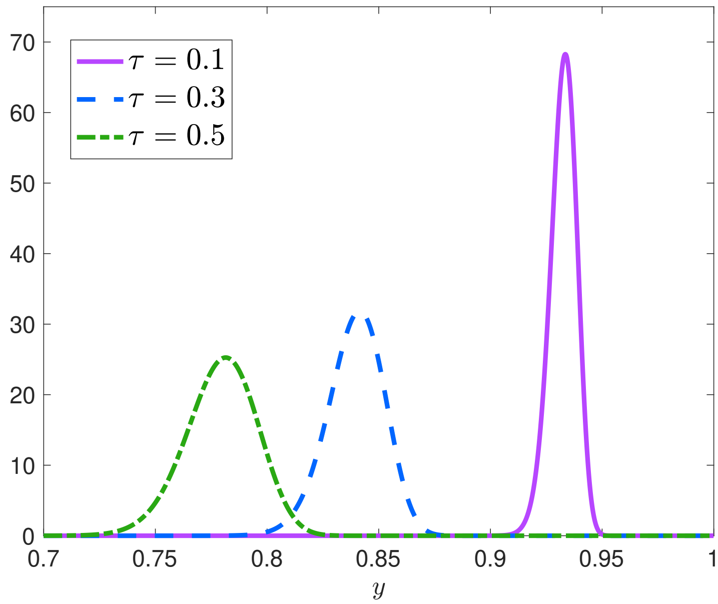

Figure 1
Stationary distributions of consumption share y in economy with two investor types
This Figure shows the stationary probability density functions for consumption share $y = \pi_1 c_1^*/D$ in the economy with two investor types for different levels of consumption taxes τ. The model parameters are $\gamma_1 = 2$, $\gamma_2 = 4$, $\rho = 0.02$, $\sigma_D = 13\%$, $\pi_1 = \pi_2 = 0.5$.

Proposition 2. *In the economy with two groups of investors, the stationary probability density function of consumption share y_t of the less risk averse group is given by*

$$f(y) = \frac{y^{a_1}(1-y)^{a_2}}{\Gamma^2} \exp\Big(b_0 + b_1 y - \frac{c_1}{y} - \frac{c_2}{1-y}\Big), \tag{21}$$

where a_i, b_i, and $c_i > 0$ are coefficients given by equations (A.10) in the Appendix. Moreover, in the economy without taxation, consumption share y_t converges towards 1.

Proposition 2 suggests that taxation is an indispensable tool for addressing inequality. It demonstrates that a positive tax rate leads to a stationarity ensuring that both types retain non-diminishing consumption share. It also shows that without taxation inequality becomes extreme in that more risk-averse investors hold an ever-diminishing share of consumption $1 - y_t$ that converges to zero over time. The reason for this persistent inequality is that the less risk-averse investors consume less and invest more in the high-return risky asset than their counterparts. As a result, their wealth and consumption continue to grow over time.

Taxation leads to a more equitable distribution of wealth and consumption in two ways. Firstly, it redistributes wealth from the wealthier investors to the less affluent. Secondly, non-pledgeability of government transfers restricts less wealthy investors from taking on wealth-diminishing positions such as short positions (backed by future transfers) to reduce their effective exposure to risk. As a result, they effectively hold a larger share of the high-yielding risky claim on aggregate output, given by $\theta^*_{2t} W_{2t}/S_t + \tau$, than in a tax-free economy.

Figure 1 illustrates the stationary probability density functions (pdf) of the consumption share y_t of the less risk-averse investors in an economy with two equal-sized investor groups that have risk aversions $\gamma_1 = 2$ and $\gamma_2 = 4$. We also set time-discount parameter to 0.02 and dividend volatility to 13%. The plot shows that higher taxes lead to a reduction in inequality, resulting in the less affluent investors having a larger share of consumption. However, despite the presence of high taxes and the stationarity of the distribution, inequality remains pronounced. This means that the investment strategy of less risk averse investors creates more wealth despite taxation.

4. Asset Prices, Taxes, and Inequality

In this Section, we delve into the further implications of taxation on asset prices and inequality. We derive equilibrium processes based on calibrated parameters and study how they are affected by changes in taxation. In Section 3, our analysis focused on the direct effect of taxes, holding the distribution of consumption across investors fixed. This effect represents an unanticipated change in taxes that occurs before the consumption shares adjust to new equilibrium. In this Section, we also consider the effect of taxation on consumption shares themselves, which we label as the indirect effect, and study the overall effect of taxes on asset prices and inequality. We find that as taxes increase, interest rates generally decline, and price-dividend ratios may also decline due to the indirect effect outweighing the direct effect. Higher taxes also raise stock risk premia and market volatilities. Additionally, although taxation ensures that the wealth of less affluent investors does not converge to zero, it does not prevent extreme concentration of wealth, and hence, inequality.

Following Veronesi (2019), we specialize the distribution of investor risk aversions to be given by the truncated normal distribution, given by

$$g(\gamma) = Ce^{-0.5(\gamma-m)^2/s^2} 1_{\{1\le\gamma\le\bar{\gamma}\}}, \tag{22}$$

where C is a constant that normalizes the mass of all investors to one. Larger parameter m corresponds to larger average risk aversion and implies higher mass of relatively risk averse investors in the economy. For the numerical investigations, we discretize the distribution (22) into N groups of investors with different risk aversions, where a group with risk aversion γ_i has mass $\pi_i = g(\gamma_i)/\sum_{i=1}^N g(\gamma_i)$.

We solve the model by employing Monte Carlo simulations. The consumption share dynamics (10) are in closed-form, which allows us to simulate them easily. We generate numerous consumption share paths and determine terminal values y_{iT} for a distant terminal date T and varying tax rates τ. We then compute the unconditional expectations of equilibrium processes (13)–(16) by averaging these processes evaluated at the terminal consumption shares y_{iT}, accounting for both direct and indirect effects of taxation. To ensure equitable consumption distribution among investors at the initial date $t = 0$, we set the consumption shares y_{i0} equal to group sizes π_i. Consequently, any wealth inequality that emerges in the model is solely attributable to the diverse asset allocation choices and saving behaviors of the investors.

Figure 2 presents the unconditional expected interest rates, price-dividend ratios, risk premia, and excess stock return volatilities as functions of the tax rate τ for different investor risk aversion distributions (22) with varying mean parameter m and calibrated other parameters.[5] As discussed in Section 3, the direct impact of taxes is that higher taxes can push interest rates in either direction, and lead to higher price-dividend ratios, risk-premia, and stock-return volatilities. Taxation, however, also changes the distributions of consumption and wealth in the economy, which has further effects on asset prices and their moments. In our economy, wealth and consumption distributions are skewed towards less risk averse investors because these investors save and invest more in higher yielding risky assets than more risk averse investors. Higher taxes partially offset the resulting wealth inequality by shifting wealth towards more risk averse investors. This shift in wealth increases the risk aversion of the representative investor Γ, which affects the equilibrium processes (13)–(16). We next discuss the impact of the indirect and direct effects of taxation on equilibrium processes.

Figure 2a presents the average riskless rate r. This rate is a decreasing function of the tax rate, which we attribute to the indirect effect of taxation through consumption distribution, discussed above. As tax rate increases and the representative investor becomes more risk averse, the interest rate goes down because risk averse investors invest more in riskless assets. There is also a non-monotonic relationship between the interest rate and mean risk aversion

[5] We set $\mu_D = 1.5\%$ and $\sigma_D = 13\%$ (Brennan and Xia, 2001; Dumas, Kurshev, and Uppal, 2009), and $\rho = 0.02$. The number of investors in the economy is $N = 200$, their risk aversions are equally spaced between 1 to 15. We sample 1000 consumption shares (10) from the stationary distribution, which is obtained by simulating consumption shares over a large horizon of $T = 1000$ years. Then, we compute unconditional expectations by averaging the equilibrium processes evaluated at these consumption shares. We compute the equilibrium processes for three distributions of risk aversions (22) with the volatility parameter $s = 2$ and mean parameters $m = 2, 3, 4$. The latter three pairs of distribution parameters m and s correspond to the following pairs of means and standard deviations of risk aversions: i) $\mu_\gamma = 3$, $\sigma_\gamma = 1.4$; ii) $\mu_\gamma = 3.6$, $\sigma_\gamma = 1.6$; and iii) $\mu_\gamma = 4.3$, $\sigma_\gamma = 1.75$, respectively.

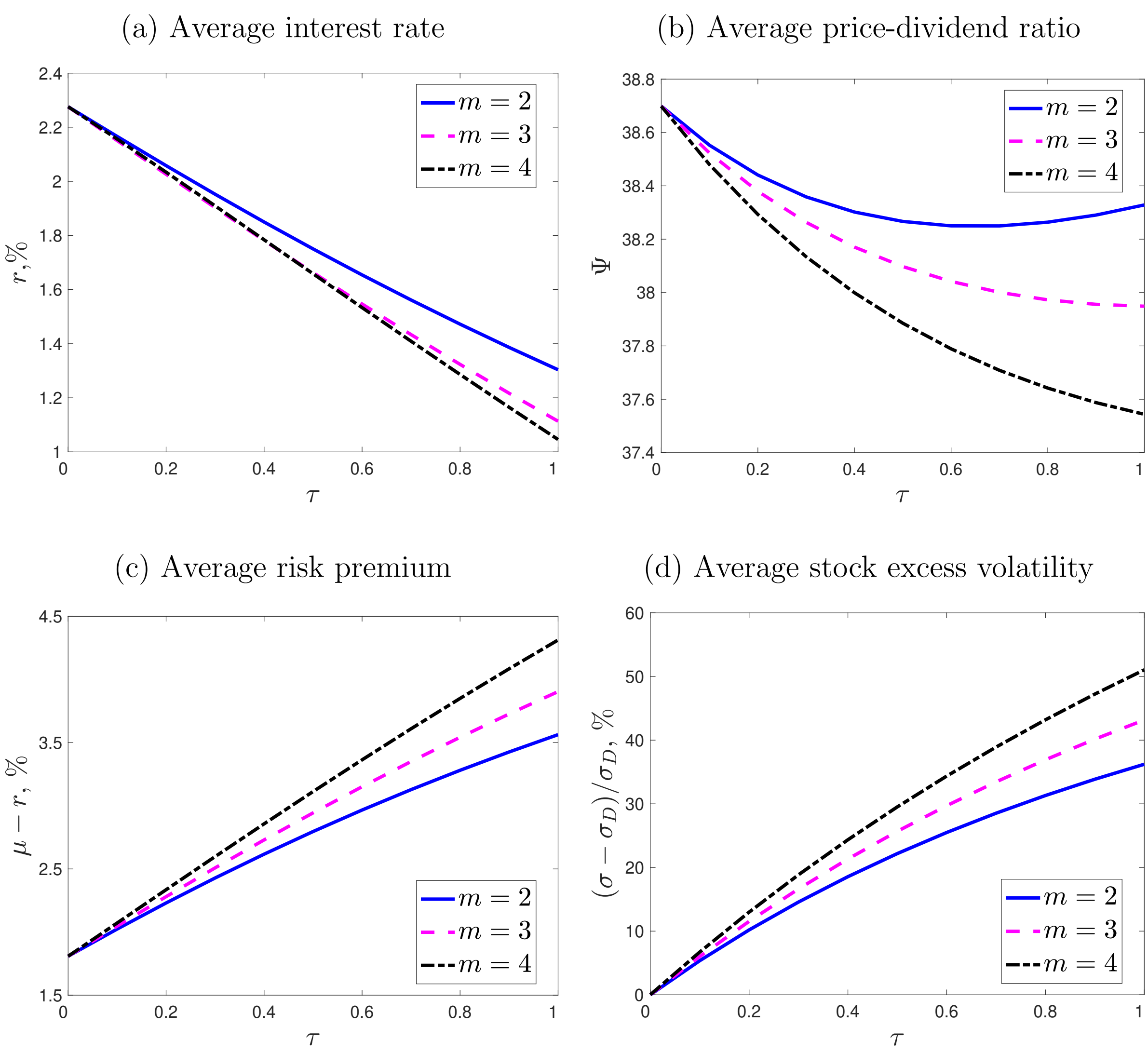

Figure 2
Overall impact of taxes and mean risk aversion on equilibrium processes
This Figure reports the equilibrium unconditional expected interest rate r, price-dividend ratio Ψ, risk premium $\mu - r$, and volatility σ as functions of tax rate τ. The results are reported for varying mean investor risk aversion parameter m and the risk aversion volatility parameter fixed at $s = 2$.

parameter m because the second and third terms in equation (13) for the riskless rate non-linearly depend on risk aversions and partially offset each other. The second term is a decreasing function of the representative risk aversion, and goes down when the mass of risk averse investors increases. The third term can be interpreted as the cross-sectional mean consumption to wealth ratio c/W, where consumption shares replace probabilities. This term increases with higher m because more risk averse investors have larger c/W ratios, as discussed in Section 3. As a result, higher mean risk aversion parameter m can push interest rates in either direction depending on which of the latter economic effects is stronger.

Figure 2b illustrates the influence of taxation on the price-dividend ratio Ψ. The price-dividend ratio may exhibit a non-monotonic relationship with taxes, as the direct and indirect effects can partially counterbalance one another. Ultimately, the overall impact of taxes on the ratio is determined by the prevailing effect. On one hand, the direct effect of taxation, as outlined in Proposition 1, leads to an increase in price-dividend ratios when taxes rise. On the other hand, higher taxes result in larger consumption shares of more risk-averse investors, who exhibit lower W/c ratios compared to their less risk-averse counterparts. Consequently, because the price-dividend ratio is a weighted average of W/c ratios given by $\Psi = \widehat{\mathbb{E}}[W_i/c_i]$, it may decrease due to the indirect effect. Consistent with the latter intuition, we observe that Ψ ratio is a decreasing function of taxes with a higher mean parameter m, which implies a larger mass of more risk-averse investors in the economy, and hence the indirect effect is stronger. Overall, the analysis depicted in Figure 2b underscores the importance of indirect effects when assessing the influence of taxation on asset prices.

Figure 2c presents the risk premia and reveals that they are increasing with both higher tax rates and higher mean parameter m because the direct and indirect effects are aligned, and hence move risk premia in the same direction. As taxes increase, the direct effect discussed in Proposition 1 increases the risk premia and also shifts the distribution of consumption towards more risk averse investors. Consequently, the risk aversion of the representative investor also increases, giving rise to higher risk premia.

Figure 2d exhibits stock excess volatility relative to dividend volatility. Similarly to risk premia, excess volatility is an increasing function of taxes and the mean parameter m. Equation (16) shows that the volatility is a complex non-linear function of consumption shares, taxes, and risk aversions. As shown in Proposition 1, it is an increasing function of taxes holding consumption shares fixed. It is also an increasing function function of the representative risk aversion Γ and a decreasing function of the price-dividend ratio Ψ. As taxes increase and consumption distribution shifts towards more risk averse investors, the risk aversion Γ increases and ratio Ψ tends to be a decreasing function of taxes (as shown on Figure 2c). These effects make the volatility an increasing function of taxes.

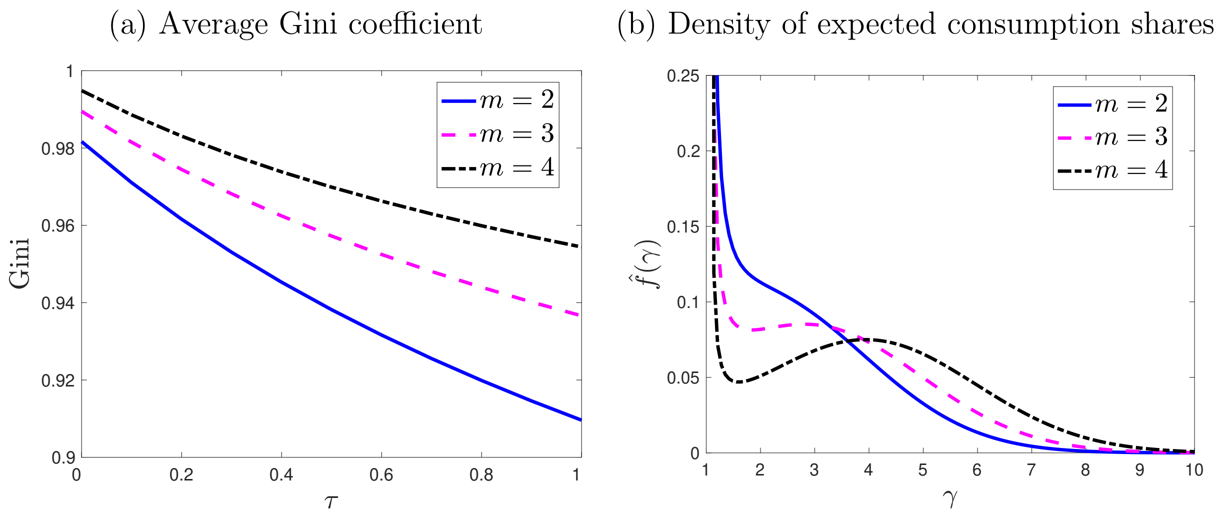

Figure 3
Impact of taxes on wealth and consumption inequality
Panel (a) shows average Gini coefficients as functions of taxes. Panel (b) shows the density of expected consumption shares as functions of risk aversion γ for tax rate $\tau = 0.5$. The results are reported for varying mean investor risk aversion parameter m with the volatility parameter $s = 2$.

4.1. Wealth Inequality

We here explore the effect of taxes on wealth inequality. We measure inequality using the widely-employed Gini coefficient, where a higher Gini coefficient indicates higher inequality.[6] As depicted in Figure 3a, increasing tax rates reduce wealth inequality, as higher taxes redistribute wealth from richer to poorer investors. We also observe that when m is higher and there are more risk-averse investors in the economy, inequality tends to be greater. This occurs because in such an economy there are more risk averse investors that lag behind less risk averse investors in terms of wealth and consumption. Furthermore, the inequality increases because the net tax revenues generated by more affluent investors have to be shared equally among a larger population of poorer investors.

In Figure 3b, we present the density of expected consumption shares $\widehat{f}(\gamma)$ for varying mean risk aversion parameter m and tax rate $\tau = 0.5$. We define this density such that the area under $\widehat{f}(\gamma)$ between two values of γ (γ_A and γ_B) corresponds to the unconditional mean cumulative consumption share of all investors with risk aversions within the range $[\gamma_A, \gamma_B]$. Figure 3b shows an extreme concentration of consumption within a small group of investors

[6]The Gini coefficient is calculated as the area between the Lorenz curve (a graph showing the cumulative percentage of the total income or wealth held by the bottom $x\%$ of the population) and the line of perfect equality (where the bottom $x\%$ of the population would have $x\%$ of the total income or wealth), divided by the total area under the line of perfect equality.

with low risk aversions.

Figures 3a and 3b suggest that while taxes ensure that poorer investors maintain a certain minimum level of consumption, they may not be as effective in reducing extreme inequality due to several reasons. First, investors who are less risk averse tend to consume a smaller portion of their wealth and allocate more of their funds towards high-yield investments. Second, these investors respond to taxation by further reducing their consumption and increasing their investments in risky assets. Finally, changes in interest rates and risk premia resulting from taxation may indirectly benefit wealthier investors. Figures 2a and 2c demonstrate that higher taxes lead to lower interest rates and higher risk premia. As less risk-averse investors borrow and invest in stocks, this indirect equilibrium effect partially offsets their losses due to taxation.

We now examine unintended welfare gains that less risk-averse investors may experience due to lower interest rates and higher risk premia resulting from an increase in taxes. We analyze a partial equilibrium economy with a tax rate of τ on relative consumption $c_i - (\pi_1 c_1 + \cdots + \pi_N c_N)$, where the exogenous interest rates and stock prices are drawn from a general equilibrium economy with a lower tax rate of τ_0. By doing so, we eliminate the welfare effect of taxation through the equilibrium interest rates and risk premia. We simulate the two latter economies side by side using the same shocks dw_t and feed interest rates, stock risk premia, and volatilities from the general equilibrium model with tax rate τ_0 into the partial equilibrium economy with tax rate $\tau > \tau_0$. Proposition A1 in the Appendix outlines the consumption share dynamics, $\widehat{y}_{it}$, in economies with exogenous asset prices.

We simulate the two economies over a long period, and calculate investors' wealths in the general equilibrium economy with tax $\tau_0 = 10\%$ (W_{GE}) and in the economy with tax rate τ but exogenous asset prices taken from the latter economy (W_{PE}). We then compute the unconditional expected percentage gain due to the changes in investment opportunities resulting from taxation as $\delta = \mathbb{E}[(W_{GE} - W_{PE})/W_{PE}]$. Our analysis reveals a small welfare gain that benefits the most risk tolerant investor at the expense of other investors. The percentage gain δ for that investor is an increasing function of the tax rate, and grows monotonically from 0 to 0.8% as consumption tax rises from 10% to 90%.

4.2. Dynamic Properties of Asset Prices

Finally, we further explore dynamic properties of asset prices. We define a process X_t as procyclical if $\text{cov}_t(dX_t, dD_t) > 0$ and countercyclical if $\text{cov}_t(dX_t, dD_t) > 0$, following the related literature (Longstaff and Wang, 2012; Chabakauri, 2015). We next show that the

Sharpe ratio is countercyclical and price-dividend ratio is procyclical, consistent with the evidence in the literature (Shiller, 1981; Campbell and Shiller, 1988). From the risk premium expression (15), we see that the Sharpe ratio is given by $\kappa_t = \sigma_D \Gamma_t$. Applying Itô's Lemma to the Sharpe ratio κ_t and the price-dividend ratio Ψ_t given by equation (14), we obtain:

$$d\kappa_t = \mu_{\kappa t} dt - \sigma_D \Gamma_t^3 \widehat{\text{var}}_t[1/\gamma_i] dw_t, \tag{23}$$

$$d\Psi_t = \mu_{\Psi t} dt + \sigma_D \Gamma_t \widehat{\text{cov}}_t\Big(\frac{1}{\gamma_i}, \Big(\frac{\rho}{1+\tau}\Big)^{-1/\gamma_i}\Big) dw_t, \tag{24}$$

where $\mu_{\kappa t}$ and $\mu_{\Psi t}$ are drifts of the latter processes which are known to investors at date t. The processes (23) for the Sharpe ratio and (1) for the dividend imply that positive shocks decrease the Sharpe ratio and increase the dividend. Consequently, the innovations $d\kappa_t$ and dD_t are negatively correlated, and hence, the Sharpe ratio is countercyclical. The process (24) for the price-dividend ratio implies that positive shocks increase this ratio, because the covariance term in front of shock dw_t is positive, as discussed in Section 3 and demonstrated in the proof of Proposition 1. Therefore, the price-dividend ratio is procyclical.

The countercyclicality of the Sharpe ratio and the procyclicality of the price-dividend ratio can be explained by the fact that positive dividend shocks disproportionately benefit less risk-averse investors because they invest more funds in stocks. Holding a tax rate fixed, positive shocks transfer consumption and wealth towards less risk averse investors. As a consequence of this transfer, the risk aversion of the representative investor in the economy declines, leading to a lower Sharpe ratio. Additionally, less risk-averse investors tend to invest more in stocks and consume smaller fraction of their wealth than their more risk-averse counterparts. This behavior causes stock prices to rise more during times when less risk-averse investors are wealthier due to positive dividend shocks.

The variations of Sharpe ratios and price-dividend ratios in relation to consumption cycles can also be derived from other models that incorporate habit-based preferences, economies with frictions, and heterogeneous investors (Campbell and Shiller, 1988; Basak and Cuoco, 1998; Chabakauri, 2015). Our framework allows examining how taxation influences the strength of these patterns. The sensitivities of both the Sharpe ratio and the price-dividend ratio to the dividend shock dw in processes (23) and (24) are dependent on consumption shares. Tax rates influence these sensitivities through their impact on consumption shares. We discuss the effect of taxes on cyclicality properties of equilibrium processes below.

The conditional covariances of the risk premium $\mu - r$, volatility σ, and interest rate r with dividend shocks encompass various terms that represent distinct economic effects that partially counterbalance each other. Consequently, to better understand their dynamics, we analyze their unconditional covariances with the representative investor's risk aversion Γ by employing Monte Carlo simulations. Processes demonstrating positive correlations with Γ are classified as countercyclical, whereas those exhibiting negative correlations are characterized as procyclical. The risk aversion Γ is a convenient indicator of cyclicality because it tends to increase following negative shocks and decrease following positive shocks. This is because negative shocks transfer both consumption and wealth towards more risk-averse investors, leading to an increase in Γ. Conversely, positive shocks transfer both consumption and wealth towards less risk-averse investors, leading to a decrease in risk aversion Γ.

We find that Sharpe ratios $(\mu - r)/\sigma$ are countercyclical and price-dividend ratios Ψ are procyclical, using their correlations with risk aversion Γ as a criterion of cyclicality. We also find that risk premia and stock return volatilities are countercyclical, consistent with the empirical findings (Ferson and Harvey, 1991; Schwert, 1989). The intuition is similar to that for Sharpe ratios, discussed above.

The interest rate r is also countercyclical. The second term in equation (13) for r is procyclical because it is negatively correlated with the risk aversion Γ. The countercyclicality of rate r arises because of the third term in (13) that outweighs the effect of the second term. The third term is countercyclical because, as discussed in Section 3, it can be rewritten in terms of investors' consumption to wealth ratios, $\widehat{\mathbb{E}}[c_i/W_i]$. The latter term increases following negative dividend shocks that transfer wealth to more risk averse investors because these investors consume larger fraction of their wealth, and hence, have higher c/W ratios.

The absolute values of all correlations between equilibrium processes and aggregate risk aversion Γ are near perfect, and decrease as the tax rate increases. This can be attributed to the increased variability in consumption shares that determine the sensitivities of these processes to dividend shocks. Higher variability of consumption shares then makes equilibrium processes less correlated.

5. Restricted Stock Market Participation

In this Section, we extend our main model of Section 2 to the case of restricted stock market participation, whereby some investors do not invest in the stock market for exogenous reasons (such as perceived complexity, lack of expertise and financial literacy). Restricted stock market participation is a salient feature of financial markets that helps explain stock risk premia, interest rates, and volatilities (Mankiw and Zeldes, 1991; Basak and Cuoco, 1998; Guvenen, 2009). We first show that the findings in our main analysis remain valid with this friction. We also find a new effect where restricted participation and taxation amplify each other, giving rise to higher risk premia and stock return volatilities, alongside lower interest rates. More notably, this extension allows us to disentangle the effects of heterogeneous portfolio holdings and saving rates, defined as the fraction of wealth invested in stocks and bonds, on wealth inequality. Both portfolio choice and saving decisions in our model are guided by the risk aversion parameter, and hence, are intertwined. We untangle their effects here by considering an economy in which investors have identical risk aversions and saving rates but one group abstains from the stock market. We show that portfolio differences significantly contribute to wealth inequality.

To facilitate clarity, we consider a parsimonious economic setup featuring only two representative investor groups with risk aversions γ_1 and γ_2, respectively. The first group is unconstrained, while the second group invests only in bonds. The consumptions of both investor groups are given by equation (11), and the stock holding of the unconstrained investors is given by (12), as in the main analysis. The stock holding of the constrained investors is $\theta_2^* = 0$. The constrained investors are assumed to have higher risk aversions, $\gamma_2 \geq \gamma_1$, because more risk averse investors might be less willing to take on risk by investing in the stock market, in line with the evidence (Mankiw and Zeldes, 1991). The definition of equilibrium remains as in our main analysis except that the stock holding of the constrained investors is set to zero in the market clearing condition (7). The derivation of equilibrium follows the same steps as in the main analysis, and consumption share of the unconstrained investor $y_t = \pi_1 c_{1t}^*/D_t$ is the endogenous state variable. Proposition 3 reports the equilibrium processes in closed form.

Proposition 3. *The equilibrium interest rate r, stock price-dividend ratio Ψ, risk premium $\mu - r$, and stock return volatility σ, in the economy with taxation and restricted stock market participation are given by:*

$$
\begin{aligned}
r_t &= \mu_D - \frac{\sigma_D^2 \gamma_1}{y_t} + (1+\tau)\Big(y_t\Big(\frac{\rho}{1+\tau}\Big)^{1/\gamma_1} + (1-y_t)\Big(\frac{\rho}{1+\tau}\Big)^{1/\gamma_2}\Big) \\
&\quad - \tau\Big(\pi_1\Big(\frac{\rho}{1+\tau}\Big)^{1/\gamma_1} + \pi_2\Big(\frac{\rho}{1+\tau}\Big)^{1/\gamma_2}\Big), \qquad (25)
\end{aligned}
$$

$$\Psi_t = y_t\Big(\frac{\rho}{1+\tau}\Big)^{-1/\gamma_1} + (1-y_t)\Big(\frac{\rho}{1+\tau}\Big)^{-1/\gamma_2}, \qquad (26)$$

$$\mu_t - r_t = \frac{\sigma_t \sigma_D \gamma_1}{y_t}, \qquad (27)$$

$$\sigma_t = \sigma_D + \Big(\Big(\frac{\rho}{1+\tau}\Big)^{-1/\gamma_1} - \Big(\frac{\rho}{1+\tau}\Big)^{-1/\gamma_2}\Big)\frac{\sigma_D(1-y_t)}{\Psi_t}. \qquad (28)$$

Consequently, the stock price-dividend ratio Ψ_t, risk premium $\mu_t - r_t$, and volatility σ_t are all increasing functions of tax rate τ, holding consumption shares fixed, and the stock return volatility exceeds dividend volatility, $\sigma_t > \sigma_D$. Furthermore, the interest rate is lower, price-dividend ratio is the same, market price of risk and stock return volatility are higher, as compared with our main economy of Proposition 1, holding consumption share y the same in the two economies.

The drifts and volatilities of the consumption share dynamics (10) are given by:

$$\mu_{yt} = y_t\Big(\frac{\gamma_1\sigma_D^2}{y_t^2} - \frac{(1+\gamma_1)\sigma_D^2}{y_t} + \sigma_D^2 + (1+\tau)(1-y_t)\Big(\Big(\frac{\rho}{1+\tau}\Big)^{-1/\gamma_1} - \Big(\frac{\rho}{1+\tau}\Big)^{-1/\gamma_2}\Big)\Big) \quad (29)$$

$$+\tau(1-y_t)\pi_1\Big(\frac{\rho}{1+\tau}\Big)^{1/\gamma_1} - \tau y_t\pi_2\Big(\frac{\rho}{1+\tau}\Big)^{1/\gamma_2}, \quad (30)$$

$$\sigma_{yt} = (1-y_t)\sigma_D. \quad (31)$$

The equilibrium processes (25)–(31) have similar structure and intuition as those in Proposition 1 for the main economy. Moreover, the equilibrium processes here can be obtained from those in Proposition 1 by specializing to the case of two investors, rewriting the cross-sectional expectations explicitly in terms of consumption shares, setting the aggregate risk aversion (17) to $\Gamma = \gamma_1/y$, and setting to zero the direct occurrences of $1/\gamma_2$ when it is not part of the wealth-consumption ratio $(\rho/(1+\tau))^{1/\gamma_2}$. The latter aggregate risk aversion Γ obtains when the second group has infinite risk aversion, albeit this group's wealth-consumption ratio is $(\rho/(1+\tau))^{1/\gamma_2}$, as for investors with finite risk aversion γ_2. Consequently, our model is able to separate the effect of the risk aversion parameter on saving rates and portfolio choice.

Proposition 3 also shows that, holding consumption share y fixed, restricted participation decreases interest rates and increases risk premia and volatilities. The intuition for these effects is similar to that in Basak and Cuoco (1998). Risk premia and volatility increase, especially in bad times, because the constrained investors act as if they were infinitely risk averse in the stock market, driving up the aggregate risk aversion Γ. The latter group also invests more in bonds, driving interest rates down. Similar to the main analysis, consumption share y increases (decreases) following positive (negative) dividend shocks, and hence, is procyclical. Consequently, market price of risk and stock return volatility are countercyclical and the price-dividend ratio is procyclical. The interest rate is procyclical due to second term in equation (25) that pushes interest rates down in bad times when consumption share y is low. This is because in bad times constrained investors hold more wealth than the unconstrained investors, driving down the interest rates by investing all their wealth is bonds.

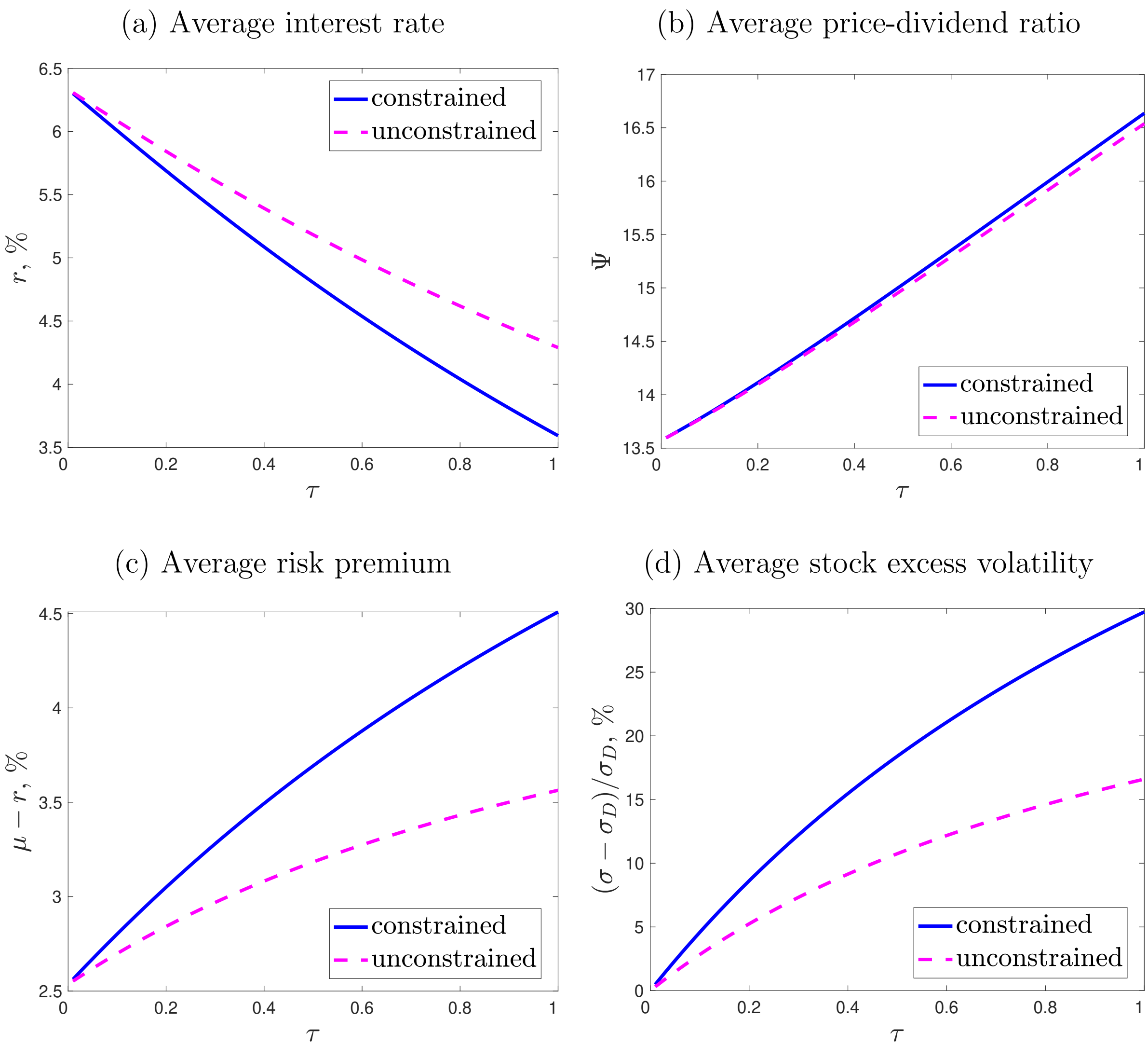

Figure 4
Overall impact of taxes on equilibrium processes
This Figure reports the equilibrium unconditional expected interest rate r, price-dividend ratio Ψ, risk premium $\mu - r$, and volatility σ as functions of tax rate τ in the economy with restricted stock market participation.

The novel feature of our analysis is combining restricted participation with the consumption tax. Figure 4 shows the average equilibrium processes as functions of the tax rate τ in the economy with and without restricted participation, for calibrated parameters.[7] Consistent with the discussion above, we find that restricted participation decreases interest rates and increases risk premia and volatilities. It also increases the price-dividend ratio, albeit the

[7]We set $\mu_D = 1.5\%$ and $\sigma_D = 13\%$ (Brennan and Xia, 2001; Dumas, Kurshev, and Uppal, 2009), $\rho = 0.02$, $\gamma_1 = 1.5$, $\gamma_2 = 4$, $\pi_1 = \pi_2 = 0.5$.

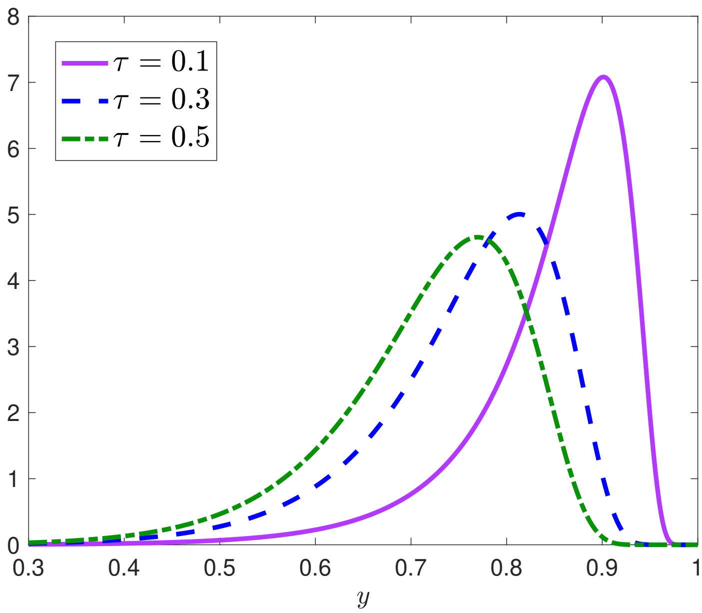

Figure 5
Stationary distributions of consumption share y in economy with two investor types and restricted participation
This Figure shows the stationary probability density functions for consumption share $y = \pi_1 c_1^*/D$ in the economy with two investor types for different levels of consumption taxes τ. The model parameters are $\gamma_1 = \gamma_2 = 1.5$, $\rho = 0.02$, $\sigma_D = 13\%$, $\pi_1 = \pi_2 = 0.5$.

effect is small. We observe that taxation amplifies the effects of restricted participation by pushing interest rates further down, and market risk premia, volatilities, and price-dividend ratios further up. The intuition is that, similar to the main model, higher taxes shift wealth and consumption towards more risk averse constrained investors. Consequently, the interest rate drops as more wealth flows into the bond market, and the risk premium increases because the unconstrained investors have less wealth and require higher risk premium to clear the market. The more volatile wealth of the unconstrained investors translates into higher stock market volatility through the market clearing condition (7). Price-dividend ratio increases because higher tax induces unconstrained investors to invest more in the stock, and the indirect effect of wealth shifting towards constrained investors is not strong enough to overcome the direct effect.

We next investigate the stationary distribution of consumption share y. The expression for the probability density function has the same structure as in Proposition 2 of the main analysis, albeit with different parameter values. Proposition 4 reports the results.

Proposition 4. *In the economy with two groups of investors, the stationary probability* *density function of consumption share y_t of the less risk averse group is given by equation (21) in which $c_1 = 0$, and coefficients a_i, b_i, and $c_2 > 0$ are given by equations (A.24) in the Appendix.*

We use Proposition 4 to study the impact of portfolio heterogeneity on inequality. Figure 5 presents the stationary distributions for varying tax rates when investors have identical risk aversions. Identical risk aversions imply that the investors' wealth-consumption ratios are the same, and hence, the consumption share y of the unconstrained investors coincides with their wealth share in the aggregate wealth. We observe that the distributions are skewed towards the unconstrained investors, so that they have larger consumption share on average, as in the main economy. Consequently, portfolio heterogeneity is a significant contributor towards inequality even when investors have identical saving rates. The distributions on Figure 5 have wider support than those on Figure 1 in the main analysis because the consumption share of the unconstrained investors is more volatile than in the main economy as they take more risk to clear the entire stock market.

6. Conclusion

In this paper, we lay out analysis that sheds light on the role of consumption taxes and non-pledgeable government transfers in shaping asset prices and wealth inequality in a dynamic general equilibrium economy populated by multiple agents with heterogeneous risk aversions. Our findings reveal that taxes play a significant role in addressing wealth inequality by ensuring stationarity of consumption share distributions and preventing the consumption shares of less affluent investors from diminishing toward zero. Moreover, we demonstrate that higher taxes increase stock risk premia and volatilities by shifting wealth toward poorer more risk-averse investors, and generally decreasing stock price-dividend ratios and interest rates. However, we also find that taxes do not prevent high concentration of wealth at the top of the distribution due to the investment decisions and tax responses of more affluent, less risk-averse investors. Finally, we extend the model to incorporate restricted stock market participation, which allows us to disentangle the portfolio choice and saving decisions of investors and demonstrate that portfolio heterogeneity is an important source of inequality. We also show that combining restricted participation and taxation leads to higher stock risk premia and volatilities and lower interest rates.

This paper contributes to the literature by offering a tractable model that accounts for the interplay between taxation, wealth inequality, and asset prices. Our results have implications for addressing wealth disparities while simultaneously considering the potential impacts on financial markets. While taxes play a crucial role in mitigating wealth inequality, they do not resolve the issue of high wealth concentration at the top. Therefore, it is essential to consider a combination of taxes, economic frictions, and other policy tools to more effectively address wealth inequality.

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Appendix: Proofs

Proof of Lemma 1. Optimal consumption and portfolio weight solve the following optimization problem:

$$\max_{c_{it},\theta_{it}} \frac{c_{it}^{1-\gamma_i}-1}{1-\gamma_i}\rho dt + \mathbb{E}_t\Big[\frac{dW_{it}^{1-\gamma_i}-1}{1-\gamma_i}\Big](1-\rho dt). \tag{A.1}$$

Applying Itô's Lemma to the utility of wealth term $dW_{it}^{1-\gamma_i}$, we obtain:

$$\frac{dW_{it}^{1-\gamma_i}}{1-\gamma_i} = W_{it}^{1-\gamma_i}\Big[\Big(r_t+\theta_{it}(\mu_t-r_t)-0.5\gamma_i\theta_{it}^2\sigma_t^2\Big)dt+\sigma_t\theta_{it}dw_t\Big] - W_{it}^{-\gamma_i}[(1+\tau)c_{it}-\tau D_t]dt.$$

Substituting the latter equation into the optimization problem (A.1), we observe that the optimization problem breaks down into two separate problems for determining the optimal consumption and portfolio weight, respectively:

$$\max_{c_{it}} \quad \frac{c_{it}^{1-\gamma_i}-1}{1-\gamma_i}\rho - W_{it}^{-\gamma_i}(1+\tau)c_{it}, \tag{A.2}$$

$$\max_{\theta_{it}} \quad \theta_{it}(\mu_t-r_t)-0.5\gamma\theta_{it}^2\sigma_t^2. \tag{A.3}$$

Solving (A.2) and (A.3), we obtain the optimal consumption (11) and portfolio (12). ■

Proof of Proposition 1. For simplicity, we let $h_i = W_{it}/c_{it}^*$ denote the wealth-consumption ratio of an investor and $\kappa_t = (\mu_t - r_t)/\sigma_t$ denote the Sharpe ratio. Equation (11) implies that $h_i = (\rho/(1+\tau))^{-1/\gamma_i}$. Then, substituting $W_{it} = c_{it}^* h_i$ into the market clearing condition (9) and dividing both sides by output D_t, we obtain the price-dividend ratio (14). Substituting $W_{it} = c_{it}^* h_i$ and portfolio (12) into the dynamic budget constraint (5), we obtain the following dynamics of investor i's equilibrium consumption:

$$dc_{it}^* = c_{it}^*\Big[\Big(r_t+\frac{\kappa_t^2}{\gamma_i}-\frac{1+\tau}{h_i}\Big)+\frac{\kappa_t}{\gamma_i}dw_t\Big]+\frac{\tau}{h_i}D_t dt. \tag{A.4}$$

Applying Itô's Lemma to both sides of the consumption clearing condition (6), using the consumption processes (A.4), dividing both sides of the equation by D_t, and then matching dt and dw_t terms, we obtain:

$$\begin{aligned} r_t+\frac{\kappa_t^2}{\Gamma_t}-(1+\tau)\widehat{\mathbb{E}}_t\Big[\frac{1}{h_i}\Big]+\tau\bar{\mathbb{E}}\Big[\frac{1}{h_i}\Big] &= \mu_D, \\ \frac{\kappa_t}{\Gamma_t} &= \sigma_D, \end{aligned}$$

where $\widehat{\mathbb{E}}_t[\cdot]$ and $\bar{\mathbb{E}}[\cdot]$ are cross-sectional expectations that use consumption shares y_{it} and investor masses π_i as probabilities, respectively. Solving the latter system of equations for κ_t and r_t, we obtain the riskless rate (13) and Sharpe ratio

$$\kappa_t = \sigma_D \Gamma_t. \tag{A.5}$$

We obtain the drift (18) and volatility (19) of consumption shares by applying Itô's Lemma to $y_{it} = \pi_i c^*_{it}/D_t$, where c^*_{it} follows the dynamics (A.4). The volatility (16) is obtained by applying Itô's Lemma to the price-dividend ratio (14).

The excess volatility $\sigma_t - \sigma_D > 0$ emerges because the covariance term in equation (16) is positive. This covariance term can be rewritten as $\widehat{\text{cov}}_t(\psi_i, g(\psi_i))$, where $\psi_i = 1/\gamma_i$ and $g(\psi_i)$ is an increasing function of ψ_i (because $\rho/(1+\tau) < 1$). Then, the covariance can be rewritten as follows:

$$\widehat{\text{cov}}_t(\psi_i, g(\psi_i)) = \widehat{\mathbb{E}}[(\psi_i - \widehat{\mathbb{E}}[\psi_i])g(\psi_i)] = \widehat{\mathbb{E}}[(\psi_i - \widehat{\mathbb{E}}[\psi_i])(g(\psi_i) - g(\widehat{\mathbb{E}}[\psi_i])] > 0,$$

where the inequality follows from the fact that $(\psi_i - \widehat{\mathbb{E}}[\psi_i])(g(\psi_i) - g(\widehat{\mathbb{E}}[\psi_i]) > 0$ for all ψ_i because $g(\psi_i)$ is an increasing function.

Finally, we derive the comparative statics of the price-dividend ratio and the volatility. Differentiating the price-dividend ratio (14) with respect to tax rate τ, holding consumption shares fixed, we obtain:

$$\frac{\partial \Psi_t}{\partial \tau} = \frac{1}{1+\tau}\widehat{\mathbb{E}}_t\Big[\frac{1}{\gamma_i}\Big(\frac{\rho}{1+\tau}\Big)^{-1/\gamma_i}\Big] > 0.$$

Differentiating the volatility (16) with respect to tax rate τ, holding consumption shares fixed, using the fact that $\Psi = \widehat{\mathbb{E}}[h_i]$, we obtain:

$$\begin{aligned}
\frac{\partial \sigma_t}{\partial \tau} &= \widehat{\text{cov}}_t\Big(\frac{1}{\gamma_i}, \frac{\partial h_i}{\partial \tau}\Big)\frac{\kappa_t}{\Psi_t} - \widehat{\text{cov}}_t\Big(\frac{1}{\gamma_i}, h_i\Big)\frac{\kappa_t}{\Psi_t^2}\frac{\partial \Psi_t}{\partial \tau} \\
&= \frac{1}{1+\tau}\widehat{\text{cov}}_t\Big(\frac{1}{\gamma_i}, \frac{h_i}{\gamma_i}\Big)\frac{\kappa_t}{\Psi_t} - \frac{1}{1+\tau}\widehat{\text{cov}}_t\Big(\frac{1}{\gamma_i}, h_i\Big)\frac{\kappa_t}{\Psi_t^2}\widehat{\mathbb{E}}\Big[\frac{h_i}{\gamma_i}\Big] \\
&= \frac{1}{1+\tau}\frac{\kappa_t}{\Psi_t^2}\Big[\widehat{\text{cov}}_t\Big(\frac{1}{\gamma_i}, \frac{h_i}{\gamma_i}\Big)\widehat{\mathbb{E}}[h_i] - \widehat{\text{cov}}_t\Big(\frac{1}{\gamma_i}, h_i\Big)\widehat{\mathbb{E}}\Big[\frac{h_i}{\gamma_i}\Big]\Big] \\
&= \frac{1}{1+\tau}\frac{\kappa_t}{\Psi_t^2}\Big[\widehat{\mathbb{E}}\Big[\frac{h_i}{\gamma_i^2}\Big]\widehat{\mathbb{E}}[h_i] - \widehat{\mathbb{E}}\Big[\frac{h_i}{\gamma_i}\Big]^2\Big].
\end{aligned}$$

It remains to show that the last expression is positive. Let $X = \sqrt{h_i}/\gamma_i$ and $Y = \sqrt{h_i}$. Then, using Cauchy-Schwartz inequality we obtain:

$$\widehat{\mathbb{E}}\Big[\frac{h_i}{\gamma_i^2}\Big]\widehat{\mathbb{E}}[h_i] - \widehat{\mathbb{E}}\Big[\frac{h_i}{\gamma_i}\Big]^2 = \widehat{\mathbb{E}}[X^2]\widehat{\mathbb{E}}[Y^2] - (\widehat{\mathbb{E}}[XY])^2 \geq 0. \blacksquare$$

Proof of Proposition 2. The stationary pdf in our economy can be derived as in Karlin and Taylor (1981, p. 221), and is given by:

$$f(y) = \frac{\exp\{b_0 + \int_{0.5}^{y} 2\mu_y(z)/\sigma_y(z)^2 dz\}}{\sigma_y^2}, \tag{A.6}$$

where b_0 is a normalization constant for the pdf.

Substituting the interest rate r_t from (13) and the aggregate risk aversion (17) into the expression for the drift (18) and volatility (19) of consumption share y for the case of two investor groups, after some algebra, we obtain:

$$\mu_y = y(1-y)\Big[\sigma_D^2\Gamma(\Gamma-1)\Big(\frac{1}{\gamma_1}-\frac{1}{\gamma_2}\Big) + (1+\tau)\Big(\frac{1}{h_2}-\frac{1}{h_1}\Big)\Big] + \frac{\tau\pi_1}{h_1}(1-y) - \frac{\tau\pi_2}{h_2}y, \tag{A.7}$$

$$\sigma_y = y(1-y)\Gamma\Big(\frac{1}{\gamma_1}-\frac{1}{\gamma_2}\Big), \tag{A.8}$$

where $h_i = (\rho/(1+\tau))^{-1/\gamma_i}$ is investor i's wealth-consumption ratio.

Using the above equations for μ_y and σ_y, after further algebra, we find that

$$\frac{2\mu_y}{\sigma_y^2} = b_1 + \frac{a_1+2}{y} - \frac{a_2+2}{1-y} + \frac{c_1}{y^2} - \frac{c_2}{(1-y)^2}, \tag{A.9}$$

where the coefficients are given by:

$$\begin{aligned}
a_1 &= \frac{2(1-1/\gamma_1)}{1/\gamma_1-1/\gamma_2} + \frac{2(1+\tau)(1/h_2-1/h_1)}{\sigma_D^2\gamma_2^2(1/\gamma_1-1/\gamma_2)^2} - \frac{2\tau\pi_2}{h_2}\frac{1}{\sigma_D^2\gamma_2^2(1/\gamma_1-1/\gamma_2)^2}, \\
&\quad + \frac{2\tau\pi_1}{h_1}\frac{1}{\sigma_D^2(1/\gamma_1-1/\gamma_2)^2}\Big[\frac{1}{\gamma_1^2} - \Big(\frac{1}{\gamma_1}-\frac{1}{\gamma_2}\Big)^2\Big], \\
a_2 &= -\frac{2(1-1/\gamma_2)}{1/\gamma_2-1/\gamma_1} + \frac{2(1+\tau)(1/h_1-1/h_2)}{\sigma_D^2\gamma_1^2(1/\gamma_1-1/\gamma_2)^2} - \frac{2\tau\pi_1}{h_1}\frac{1}{\sigma_D^2\gamma_1^2(1/\gamma_1-1/\gamma_2)^2} \\
&\quad + \frac{2\tau\pi_2}{h_2}\frac{1}{\sigma_D^2(1/\gamma_1-1/\gamma_2)^2}\Big[\frac{1}{\gamma_2^2} - \Big(\frac{1}{\gamma_1}-\frac{1}{\gamma_2}\Big)^2\Big], \qquad \text{(A.10)} \\
b_1 &= -\frac{2(1+\tau)}{\sigma_D^2}\Big(\frac{1}{h_2}-\frac{1}{h_1}\Big), \\
c_1 &= \frac{2\tau\pi_1}{h_1}\frac{1}{\sigma_D^2\gamma_2^2(1/\gamma_1-1/\gamma_2)^2}, \\
c_2 &= \frac{2\tau\pi_2}{h_2}\frac{1}{\sigma_D^2\gamma_1^2(1/\gamma_1-1/\gamma_2)^2}.
\end{aligned}$$

Substituting (A.9) and (A.8) into the expression for the pdf function (A.6) and integrating, we obtain the pdf function (21).

We next investigate the behavior of y_t at the boundaries $y = 0$ and $y = 1$ when $\tau = 0$. In this case, the coefficients (A.10) become:

$$\begin{aligned} a_1 &= \frac{2(1-1/\gamma_1)}{1/\gamma_1 - 1/\gamma_2} + \frac{2(1/h_2 - 1/h_1)}{\sigma_D^2 \gamma_2^2 (1/\gamma_1 - 1/\gamma_2)^2}, \\ a_2 &= -\frac{2(1-1/\gamma_2)}{1/\gamma_2 - 1/\gamma_1} + \frac{2(1/h_1 - 1/h_2)}{\sigma_D^2 \gamma_1^2 (1/\gamma_1 - 1/\gamma_2)^2}, \\ b_1 &= -\frac{2}{\sigma_D^2}\Big(\frac{1}{h_2} - \frac{1}{h_1}\Big), \\ c_1 &= c_2 = 0. \end{aligned} \tag{A.11}$$

We observe that coefficient a_2 can be rewritten in terms of coefficient a_1 as follows:

$$a_2 = -a_1 - 2 + \frac{2(1/h_2 - 1/h_1)}{\sigma_D^2 (1/\gamma_1 - 1/\gamma_2)^2}\Big(\frac{1}{\gamma_2^2} - \frac{1}{\gamma_1^2}\Big). \tag{A.12}$$

The latter equation and the equation for a_1 in (A.11) imply that for $\gamma_1 < \gamma_2$, we have:

$$a_1 > 0, \quad a_2 < -2. \tag{A.13}$$

Following Karlin and Taylor (1981, p. 226), consider the following function:

$$F[x_1, x_2] = \int_{x_1}^{x_2} \exp\Big\{-\int_{\xi_0}^{\xi} 2\mu_y(\eta)/\sigma_y^2(\eta) d\eta\Big\} d\xi. \tag{A.14}$$

Substituting (A.9) with coefficients (A.11) into (A.14) and integrating, we obtain:

$$F[x_1, x_2] = \int_{x_1}^{x_2} \exp\{-b_1\xi\} \xi^{-(a_1+2)} (1-\xi)^{-(a_2+2)} d\xi. \tag{A.15}$$

Inequalities (A.13) and equation (A.15) imply that $F[0, x] = +\infty$ and $F[x, 1] < +\infty$ for all $x \in (0, 1)$. Consequently, boundary $y = 0$ is not attracting and $y = 1$ is attracting (see Karlin and Taylor, 1981, pp. 226-228). ■

Proposition A.1. *Consider a partial equilibrium economy with exogenous processes for the interest rate r_t and the Sharpe ratio κ_t, which are adapted to the filtration $\{\mathcal{F}_t, t \in [0, \infty]\}$, generated by w. Let τ be the tax levied on the relative consumption of investor i, so that net tax payment/benefit is given by $\tau(c_{it} - \bar{c}_t)$, where $\bar{c}_t = \pi_1 c_{1t} + \cdots + \pi_N c_{Nt}$ is the average* *consumption. Then, consumption shares* $\widehat{y}_{it} = \pi_i c_{it}/\bar{c}_t$ *follow dynamics:*

$$\begin{aligned} d\widehat{y}_{it} &= \widehat{y}_{it}\Big(\kappa_t^2\Big(1-\frac{1}{\Gamma_t}\Big)\Big(\frac{1}{\gamma_i}-\frac{1}{\Gamma_t}\Big)+\widehat{\mathbb{E}}_t\Big[\frac{1+\tau}{h_i}\Big]-\frac{1+\tau}{h_i}-\bar{\mathbb{E}}\Big[\frac{\tau}{h_i}\Big]\Big)dt \\ &\quad +\frac{\tau\pi_i}{h_i}dt+\kappa_t\Big(\frac{1}{\gamma_i}-\frac{1}{\Gamma_t}\Big)dw_t, \end{aligned} \tag{A.16}$$

where $h_i = (\rho/(1+\tau))^{-1/\gamma_i}$ *is the wealth-consumption ratio of investor* i.

Proof of Proposition A.1. Substituting $W_{it} = c_{it}h_i$ into the dynamic budget constraint of investor i, we obtain the following dynamics for the investor's consumption:

$$dc_{it} = c_{it}\Big(r_t + \frac{\kappa_t^2}{\gamma_i} - \frac{1+\tau}{h_i}\Big)dt + \frac{\tau\bar{c}_t}{h_i}dt + c_{it}\frac{\kappa_t}{\gamma_i}dw_t, \tag{A.17}$$

where $\bar{c}_t = \pi_1 c_{1t} + \cdots + \pi_N c_{Nt}$ is the average consumption. Differentiating both sides of the aggregate consumption, we find that

$$d\bar{c}_t = \bar{c}_t\Big(r_t + \frac{\kappa_t^2}{\Gamma_t} - \widehat{\mathbb{E}}_t\Big[\frac{1+\tau}{h_i}\Big] + \bar{\mathbb{E}}\Big[\frac{\tau}{h_i}\Big]\Big)dt + \bar{c}_t\frac{\kappa_t}{\Gamma_t}dw_t. \tag{A.18}$$

Applying Itô's Lemma to consumption share $\widehat{y}_{it} = \pi_i c_{it}/\bar{c}_{it}$, we obtain the dynamics of consumption shares (A.16). ■

Proof of Proposition 3. Let $h_i = W_{it}/c_{it}^*$ denote the wealth-consumption ratio of an investor and $\kappa_t = (\mu_t - r_t)/\sigma_t$ denote the Sharpe ratio. Equation (11) implies that $h_i = (\rho/(1+\tau))^{-1/\gamma_i}$. The dynamics of investor 1's equilibrium consumption is given by equation (A.4), as in the main economy.

Investor 2 does not invest in the stock market, and hence, we set $\theta_{2t}^* = 0$ in the dynamic budget constraint (5). Consequently, the dynamics of investor 2's wealth is given by:

$$dW_{2t} = W_{2t}\Big(r_t - \frac{1+\tau}{h_2}\Big)dt + \tau D_t dt. \tag{A.19}$$

Substituting $W_{2t} = c_{2t}^* h_2$ into equation (A.19), we find investor 2's consumption dynamics:

$$dc_{2t}^* = c_{2t}^*\Big(r_t - \frac{1+\tau}{h_2}\Big)dt + \frac{\tau}{h_2}D_t dt. \tag{A.20}$$

Differentiating both sides of the market clearing condition $c_{1t}+c_{2t} = D_t$, matching terms, and dividing both sides of the resulting equations by D_t, we obtain the following system of equations for the interest rate r_t and the Sharpe ratio κ_t:

$$\begin{aligned} r_t + \frac{y_t \kappa_t^2}{\gamma_1} - (1+\tau)\Big(\frac{y_t}{h_1} + \frac{1-y_t}{h_2}\Big) + \tau\Big(\frac{\pi_1}{h_1} + \frac{\pi_2}{h_2}\Big) &= \mu_D, \\ \frac{y_t \kappa_t}{\gamma_1} &= \sigma_D, \end{aligned}$$

where $y_t = \pi_1 c_{1t}/D_t$. Solving this system of equations, we find the interest rate (25) and the Sharpe ratio $\kappa_t = \gamma_1 \sigma_D / y_t$, which implies the risk premium given by (27).

The drift (30) and volatility (31) of the consumption share y_t can be found by applying Itô's Lemma to $y_t = \pi_1 c_{1t}/D_t$ and matching dt and dw terms. Stock return volatility volatility (28) can be obtained by applying Itô's Lemma to $S_t = \Psi_t D_t$, where Ψ_t is given by (26) and dividend dynamics is given by (1). The comparative statics with respect to the tax rate τ can be derived following the same steps as in the proof of Proposition 1.

Finally, we compare the equilibrium processes with those in the main economy, holding consumption share y the same across the two economies. Using equations (13)–(16), we derive the equilibrium processes in the main economy for the case of two investors. It can then be directly observed that the interest rate (25) is smaller than its unconstrained counterpart, and the price-dividend ratio is the same in the two economies. Using equation (28), we rewrite the stock return volatility in the following equivalent form:

$$\begin{aligned} \sigma_t &= \sigma_D + \Big(h_1 - h_2\Big)\frac{1}{\gamma_1}\frac{\gamma_1}{y_t}\frac{\sigma_D(1-y_t)y_t}{\Psi_t} \\ &> \sigma_D + \Big(h_1 - h_2\Big)\Big(\frac{1}{\gamma_1} - \frac{1}{\gamma_2}\Big)\Gamma_t \frac{\sigma_D(1-y_t)y_t}{\Psi_t} \\ &= \sigma_t^{unc}, \end{aligned} \tag{A.21}$$

where σ_t^{unc} is the stock return volatility in the main economy, that can be obtained from equation (16) for the case of two investor groups. The inequality (A.21) follows from inequality $\gamma_1/y > \Gamma$, where $\Gamma = 1/(y/\gamma_1 + (1-y)/\gamma_2)$. Using the latter inequality and the inequality (A.21), for the risk premium we obtain:

$$\mu_t - r_t = \frac{\sigma_t \sigma_D \gamma_1}{y_t} > \sigma_t^{unc}\sigma_D \Gamma_t = \mu_t^{unc} - r_t^{unc}, \tag{A.22}$$

where $\mu_t^{unc} - r_t^{unc}$ is the market price of risk in the main model given by (15). ■

Proof of Proposition 4. Using the equations for μ_y in (30) and σ_y in (31), after some algebra, we find that

$$\frac{2\mu_y}{\sigma_y^2} = b_1 + \frac{a_1}{y} - \frac{a_2+2}{1-y} - \frac{c_2}{(1-y)^2}, \tag{A.23}$$

where the coefficients are given by:

$$
\begin{aligned}
a_1 &= 2\gamma_1, \\
a_2 &= -2\gamma_1 - \frac{2(1+\tau)}{\sigma_D^2}\Big(\frac{1}{h_1} - \frac{1}{h_2}\Big) - \frac{2\tau}{\sigma_D}\Big(\frac{\pi_1}{h_1} + \frac{\pi_2}{h_2}\Big) \\
b_1 &= \frac{2(1+\tau)}{\sigma_D^2}\Big(\frac{1}{h_1} - \frac{1}{h_2}\Big), \\
c_2 &= \frac{2\tau}{\sigma_D^2}\frac{\pi_2}{h_2}.
\end{aligned}
\tag{A.24}
$$

Substituting (A.23) into the equation (A.6) for the stationary distribution and integrating, we obtain the stationary distribution for the restricted participation model that has form (21) with $c_1 = 0$ and the other coefficients given by (A.24). The rest of the analysis is as in the proof of Proposition 2. ■