Optimal Fiscal and Monetary Policy with Investment Spillovers and Endogenous Private Information*

Luca Colombo† Gianluca Femminis‡ Alessandro Pavan§

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Abstract

How should firms be incentivized to invest efficiently when the profitability and spillovers of such investments are uncertain? We show that, under flexible prices, efficiency in information acquisition and in investment and employment decisions can be induced through a fiscal policy that pays to the innovating firms a subsidy that resembles a familiar Pigou's correction but accounts for the non-verifiability of firms' acquisition and usage of information. The same fiscal policy also induces efficiency in information acquisition and usage when prices are sticky, under an appropriate monetary policy that induces firms to disregard their endogenous private information when setting prices and only use it for investment purposes.

Keywords: endogenous information, investment spillovers, optimal fiscal and monetary policy, Pigouvian corrections

JEL classification: D21, D62, D83, E60, E62

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†Università Cattolica del Sacro Cuore (lucava.colombo@unicatt.it).

‡Università Cattolica del Sacro Cuore (gianluca.femminis@unicatt.it).

§Northwestern University and CEPR (alepavan@northwestern.edu).

1 Introduction

When deciding whether to adopt a new technology or invest in a new production process, firms face uncertainty about the profitability of their investments. Such uncertainty may reflect limited familiarity with the new investment opportunity, but also the fact that its profitability may depend on whether other firms make similar investments. Importantly, this uncertainty is often endogenous, as firms can collect information about the new investment opportunity before finalizing their decisions.

In such contexts, how should a benevolent government incentivize firms to collect and use information in society's best interest? This question is at the center of an active policy debate as many countries are devoting significant resources to boost investment in innovation and technology adoption in fields such as green technologies, the industrial internet of things, and fintech, to mention a few.[1]

We show that, when prices are flexible and the information the firms possess is dispersed but exogenous, efficiency in investment, pricing, and employment decisions can be induced by combining familiar revenue subsidies correcting for firms' market power with additional subsidies to the innovating firms appropriately designed to make them use the available information efficiently. The subsidies to the innovating firms often take a simple form. We characterize the conditions that such subsidies must satisfy, and identify instances in which they can be invariant to the realized productivity of the new investment opportunities. When, instead, firms must also be incentivized to collect information efficiently prior to investing, typically it becomes necessary to resort to more sophisticated policies that condition the subsidies to the innovating firms on the realized profitability of the new investment opportunities and, when the cost of acquiring information is unknown to the policy maker, on the aggregate investment in the new technology. Such richer policies operate as a Pigouvian correction realigning the private value of investment to its social counterpart, by inducing firms to internalize the externality that their investments impose on the production of intermediate and final goods. Importantly, these policies also realign the private value to acquiring more precise information to its social counterpart, accounting for the fact that neither the acquisition nor the usage of information is verifiable. That Pigouvian taxes/subsidies can correct externalities when information is complete and firms' activities are verifiable is known. The paper's contribution is in showing that a specific version of such policies also creates the right incentives for information acquisition and its subsequent utilization.

Finally, we show that, when prices are sticky (that is, firms set them under dispersed

[1] See, for example, the European Commission policy briefs on advanced technologies for industry – https://ati.ec.europa.eu/reports/Policy-Briefs

information), the same fiscal policies described above remain optimal but must be paired with a monetary policy that induces firms to disregard their endogenous private information when setting prices, and only use it to make their investment decisions.

In our model, the key externality originates in investment spillovers. Policies similar to those characterized in this paper can also be used to correct for other externalities. One example is the adoption of "greener" technologies that reduce pollution, where firms face uncertainty about the technical merits of the new technologies and whether they will be used by a large enough number of firms to make them not only environment-friendly but also economically viable.

Related literature. The paper is related to the literature on incentives for information acquisition and efficient information usage. See, among others, Bergemann and Välimäki (2002) for how to use Vickrey-Clarke-Groves (VCG) transfers in mechanism design to incentivize agents to acquire information prior to participating in a mechanism, and Angeletos and La'O (2020) for optimal monetary policy over the business cycle with dispersed information. Our contribution is in introducing investment spillovers and endogenous private information in an otherwise standard general-equilibrium macro model, and investigating how the interaction between the two shapes optimal fiscal and monetary policy. We show how subsidies resembling Pigouvian interventions can correct for externalities in real activity (such as those originating in investment spillovers) and induce efficiency in both information acquisition and usage, even when neither of these activities is verifiable.

The paper is also related to the literature investigating the interaction between investment under uncertainty, innovation, and the corrective role of taxation in the presence of externalities (see, e.g., Akcigit, Caicedo, Miguelez, Sterzi, and Stantcheva (2018), Akcigit, Grigsby, Nicholas, and Stantcheva (2022), and the references therein). In particular, our work is related to Akcigit, Hanley, and Stantcheva (2022), who investigate how to use policy to stimulate R&D investments in the presence of technology spillovers between firms that are heterogeneous and privately informed about their research productivity.[2] Our contribution is in endogenizing information about both the technical merits of new technologies and the spillovers associated with them, and in showing how an appropriate combination of fiscal and monetary policy can correct for inefficiencies in both the acquisition and usage of information, both when prices are flexible and when they are sticky. To isolate the novel effects, we abstract from forces that naturally arise in dynamic settings (most notably, information externalities) and focus on the implications of the interaction between spillovers and endogenous private information on optimal policy. Related is also Alvarez, Argente, Lippi, Mendez, and Van Patten (2022)

[2]See also Bloom, Griffith, and Van Reenen (2002) for the effects of R&D tax credits on innovation.

who study how to stimulate the adoption of new technologies in the fintech industry. That paper focuses on dynamic spillovers, but does not investigate how to correct inefficiencies in the acquisition of information. The latter topic is investigated in Pavan, Sundaresan, and Vives (2022) who, however, focus on information aggregation in financial markets, and do not consider spillovers in investment decisions or other direct payoff interdependencies among the relevant actors.

Outline. The rest of the paper is organized as follows. Section 2 introduces the model abstracting from nominal rigidities. Section 3 contains the key results about the structure of optimal fiscal policy. Section 4 introduces nominal rigidities (sticky prices), and discusses how the fiscal policies in Section 3 remain optimal when paired with an appropriate monetary policy. Section 5 concludes. All proofs omitted in the main text are in the Appendix at the end of the document.

2 The Model

We consider a static general-equilibrium economy in which firms make their investment, pricing, and employment decisions simultaneously. This permits us to isolate the novel effects on optimal fiscal and monetary policy originating in the interaction between investment spillovers and endogenous private information from the more familiar information externalities that arise in dynamic settings in which firms learn from other firms' investment decisions and prices. We also abstract from financial markets and the familiar role that the latter play in imperfectly aggregating dispersed information. We believe that the power of Pigouvian corrections discussed in the paper extends to these economies, provided that the aggregation of information remains imperfect. However, we expect the structure of the optimal subsidies in these economies to also reflect the government's desire to manipulate the sensitivity of firms' investment decisions to their private information to increase the information content of prices and early investment decisions (in the spirit of Grossman and Stiglitz (1980)).

The economy is populated by (i) a measure-1 continuum of firms, each producing a differentiated intermediate good, (ii) a competitive retail sector producing a final good using the intermediate goods as inputs, (iii) a measure-1 continuum of homogenous workers, and (iv) a benevolent government controlling fiscal and monetary policy.

Each firm is run by a single entrepreneur who must decide whether to operate under an existing technology or adopt a new one. Indexing firms by $i \in [0, 1]$, we denote by $n_i = 1$ (alternatively, $n_i = 0$) the decision by firm i to adopt the new technology (alternatively, to retain the old one). Adopting the new technology costs $k > 0$. Such a cost can be interpreted as the disutility the entrepreneur incurs to familiarize with the new technology. What matters for the results is that such a cost is not mediated by a market that fully aggregates the entrepreneurs' dispersed information.

Let

$$N = \int n_i di$$

denote the aggregate investment in the new technology, and $l_i \in \mathbb{R}_+$ the amount of labor employed by firm i. The amount of the intermediate good produced by firm i is given by

$$y_i = \begin{cases} \gamma\Theta\left(1+\beta N\right)^{\alpha} l_i^{\psi} & \text{if } n_i = 1 \\ \Theta\left(1+\beta N\right)^{\alpha} l_i^{\psi} & \text{if } n_i = 0 \end{cases}, \tag{1}$$

with $\gamma > 1$, $\beta \geq 0$, $\alpha \geq 0$, and $\psi \leq 1$. The variable $\Theta > 0$ proxies for a combination of economic fundamentals that are jointly responsible for the productivity differential

$$(\gamma - 1)\Theta\left(1+\beta N\right)^{\alpha} l^{\psi}$$

across the two technologies and for the uncertainty that firms face at the time they make their investment decisions. The parameter γ scales the return differential between the two technologies, whereas the parameters α and β control for the returns to scale and the intensity of the investment spillovers, respectively. Finally, the parameter ψ controls for the returns to scale of labor. Note that the variable Θ contributes both to the output differential between the two technologies and to the magnitude of the investment spillover, that is, the effect of aggregate investment N on individual output, for given technology. That each entrepreneur benefits from the adoption of the new technology by the other entrepreneurs, both when he adopts the new technology and when he retains the old one is not important for the results. What matters is that the output differential between the two technologies is increasing in both N and Θ.

The final good is produced by a competitive retail sector using the familiar CES technology

$$Y = \left(\int y_i^{\frac{v-1}{v}} di\right)^{\frac{v}{v-1}}, \tag{2}$$

with $v > 1$ denoting the elasticity of substitution between goods. The price of the final good is P and the profits of the competitive retail sector are given by

$$\Pi = PY - \int p_i y_i di,$$

where p_i is the price of the intermediate good paid to firm i.

Let $\theta \equiv \log \Theta$. It is common knowledge among the entrepreneurs and the government that θ is drawn from a Normal distribution with mean 0 and precision π_θ. The realization of θ is not observed by the entrepreneurs at the time they make their investment descisions. Each entrepreneur i chooses the precision π_i^x of an additive private signal

$$x_i = \theta + \xi_i$$

about θ, with ξ_i drawn from a Normal distribution with mean zero and precision π_i^x, independently from θ and independently across i. The cost of information of precision π_i^x is equal to $\mathcal{I}(\pi_i^x)$, with $\mathcal{I}$ continuously differentiable and such that $\mathcal{I}'(0) = 0$, $\mathcal{I}^{'}(\pi_i^x) > 0$ and $\mathcal{I}^{''}(\pi_i^x) \geq 0$ for all $\pi_i^x > 0$. Such a cost can be interpreted as disutility of effort. The results extend to general/flexible information technologies but are best illustrated with the Gaussian structure described above.

After selecting π_i^x and receiving information x_i, entrepreneur i chooses which technology to use. After learning Θ and N, the entrepreneur then chooses the price p_i for his intermediate good. Finally, given Θ, N, and the realized demand for his product, the entrepreneur employs labor l_i on a competitive market to meet the demand for his good. Labor is supplied by the continuum of measure-one workers.

Consistently with the pertinent literature, we assume that each entrepreneur is a member of a representative household whose utility function is given by

$$U = C - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon} - \int \mathcal{I}(\pi_i^x)di - \Upsilon,$$

with $\varepsilon > 0$, where $l^{1+\varepsilon}/(1+\varepsilon)$ denotes the disutility of labor, and Υ is a tax paid to the government, expressed in terms of units of consumption of the final good. Because labor is homogenous and exchanged in a competitive market, each worker provides the same amount of labor (i.e., $l_i = l$ for all i). Being a member of the representative household, each entrepreneur maximizes his firm's market valuation, taking into account that the profits the firm generates are used for the purchase of the final good. This means that each entrepreneur maximizes

$$\mathbb{E}\left[\frac{p_i y_i - W l_i}{P} + T_i \middle| x_i, \pi_i^x\right] - kn_i - \mathcal{I}(\pi_i^x),$$

where W is the nominal wage rate, and T_i is a transfer to the firm in terms of the consumption of the final good based on the firm's revenue $r = p_i y_i / P$, expressed in terms of the consumption of the final good. Naturally, T_i may also depend on whether the firm adopts the new technology or retains the old one.[3]

The representative household collects profits from all firms and wages from all workers, and pays a lump-sum tax Υ to the government. Using the fact that (a) the government budget must be balanced, i.e., $\int T_i di = \Upsilon$, (b) the total labor demand must equal the total labor supply, (c) all entrepreneurs choose the same precision of private information in equilibrium, (d) firms' total revenues coincide with the total expenditure on the final good, and (e) the total consumption of the final good C coincides with its production Y, we have that the government's objective can be expressed as

$$\mathcal{W} = \mathbb{E}\left[C - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon}\right] - \mathcal{I}(\pi^x). \tag{3}$$

The government thus maximizes aggregate consumption, net of the costs to upgrade the technology, the labor costs, and the information-acquisition costs. It does so by designing a fiscal and monetary policy (more on this in the next sections).

Summarizing, the timing of events is the following:

1. Nature draws θ;
2. each entrepreneur i chooses the precision π_i^x of his private information;
3. each entrepreneur i receives a private signal x_i about θ;
4. entrepreneurs simultaneously choose n_i;
5. after θ and N are publicly revealed, entrepreneurs simultaneously set prices p_i;
6. the competitive retail sector chooses how much of each intermediate good to purchase, taking the prices of the intermediate goods and the price P of the final good as given;
7. given the demand y_i for his intermediate good, entrepreneur i hires l_i units of labor to meet his demand, taking N and θ as given;
8. a representative household comprising all workers and entrepreneurs chooses how much of the final good to buy, taking the price of the final good P as given.

The assumptions that firms are differentiated monopolists, that the production function is Cobb-Douglas, and that the technology for producing the final good is iso-elastic are standard

[3] One could also consider other fiscal policies in which the transfers to the firms are a function of employment, profits, or a combination of these and other verifiable variables. Following the pertinent literature, we focus on revenue-based transfers.

in the literature on optimal fiscal and monetary policy under dispersed information. Assuming the same structure facilitates the comparison with previous work and permits us to isolate the novel effects on optimal fiscal and monetary policy originating in the interaction between (a) investment spillovers and (b) endogenous private information, which is the contribution of the paper. This structure is also known to facilitate the computation of the equilibrium allocations in the presence of nominal rigidities (sticky prices), which we address in Section 4.[4]

3 Constrained Efficiency, Equilibrium, and Optimal Fiscal Policy

Subsection 3.1 characterizes constrained efficiency, whereas Subsection 3.2 characterizes properties of the equilibrium allocations. Finally, Subsection 3.3 characterizes optimal fiscal policies. Because prices in the economy under consideration are flexible (i.e., are set by the firms after observing θ), money in this economy has only a nominal effect on prices and plays no other role. We thus omit it for the time being, and introduce it only in Section 4 where we consider optimal fiscal and monetary policy in the presence of nominal rigidities.

3.1 Constrained Efficiency

We assume that the government cannot transfer information across agents. This restriction is standard in the literature on optimal fiscal and monetary policy under dispersed information (see, among others, Vives (1988), Angeletos and Pavan (2007), Colombo, Femminis and Pavan (2014), Angeletos, Jovino and La'O (2016), Angeletos and La'O (2020), and Llosa and Venkateswaran (2022)).

The constrained efficient allocation has three parts: the precision of private information, π^{x*}, a rule specifying how firms should choose between the two technologies based on their private information x, and a rule describing how much labor each firm should employ as a function of θ and x (equivalently, θ and the technology adopted). These three parts are chosen jointly to maximize ex-ante welfare, $\mathcal{W}$, as given in (3). Lemma 1 below focuses on efficient technology adoption. The rule describing the efficient employment of labor is in the proof of

[4]The assumption that U is linear in C is not important for the results. In the Supplementary Material, we consider the case where U is iso-elastic in C, as in Angeletos, Jovino and La'O (2016), and Angeletos and La'O (2020). In this case, the assumption that each entrepreneur is a member of a representative household implies perfect consumption-risk sharing. See also Llosa and Venkateswaran (2022) for a recent paper in which, for simplicity, U is assumed to be linear in C.

Lemma 1, whereas the formula for the efficient precision of private information π^{x*} is in the proof of Lemma 3. The reason for relegating these parts to the Appendix is that they are useful for comparative statics but not essential to the arguments establishing the key results.

Lemma 1. *Let* $\varphi \equiv \frac{v-1}{v-\psi(v-1)}$*, and assume that* $\gamma^{\varphi} \geq 1+\beta$ *and* $\psi < \min\left\{1, \frac{1+\varepsilon}{\varepsilon(v-1)}\right\}$*. For any precision of private information* π^x*, there exists a threshold* $\hat{x}(\pi^x)$ *such that efficiency in technology adoption requires that each firm with signal* $x > \hat{x}(\pi^x)$ *adopts the new technology, whereas each firm with signal* $x < \hat{x}(\pi^x)$ *retains the old technology.*

Proof. See the Appendix.

The parameters' restrictions in the lemma guarantee that the social value of upgrading the technology (net of its disutility cost) is increasing in the fundamental θ and in the mass N of firms adopting the new technology. These monotonicities, in turn, imply that the efficient rule for technology adoption is monotone in the firms' private information. These restrictions are fairly standard. They play a role similar to the one played by the assumption that substitution effects are stronger than income effects in other macro settings. The property that the rule for efficient technology adoption is monotone in signals is not essential for our key results but it facilitates the exposition. In particular, it permits us to fully characterize necessary and sufficient conditions for a fiscal policy to implement the efficient allocation, both when information is exogenous (Lemma 2), and when it is endogenous (Lemma 3). On the other hand, the results in Propositions 1 and 2 below establishing that Pigouvian corrections eliminate any discrepancy between private and social objectives (and hence induce efficiency in both information acquisition and usage, despite the fact that neither of the two activities is verifiable) apply also to economies in which the constrained-efficient allocation is not monotone.

3.2 Equilibrium

The following definition summarizes the key equilibrium conditions.

Definition 1. A (symmetric) **equilibrium** consists of (1) a precision π^x of private information, (2) an investment strategy $n(x;\pi^x)$, and (3) a pair of price functions $p_0(\theta;\pi^x)$ and $p_1(\theta;\pi^x)$, respectively for firms retaining the old technology and for those adopting the new one, such that, when each firm $j \neq i$ chooses a precision of information equal to π^x, chooses its technology according to $n(x;\pi^x)$, and sets its price according to $p_0(\theta;\pi^x)$ and $p_1(\theta;\pi^x)$, each entrepreneur i maximizes his firm's market valuation by doing the same.

The complete description of the equilibrium allocation also entails the specification of the labor $l_0(\theta;\pi^x)$ and $l_1(\theta;\pi^x)$ demanded respectively by those firms retaining the old technology and those adopting the new technology, the total labor supply $L(\theta;\pi^x)$, the wage $W(\theta;\pi^x)$, and the price $P(\theta;\pi^x)$ of the final good, with all the equilibrium variables naturally conditioning on the state θ and the endogenous precision of private information π^x. These functions are standard and described concisely below. They are not included in the equilibrium definition so as to highlight the parts that are most relevant for our results.[5]

As usual, the assumption that the retail sector is competitive implies that, in equilibrium, profits are equal to zero, i.e., $\Pi = 0$, and that the price of the final good is equal to

$$P = \left(\int p_i^{1-v} di\right)^{\frac{1}{1-v}}, \tag{4}$$

with the demand for each intermediate good given by

$$y_i = C\left(\frac{P}{p_i}\right)^v, \tag{5}$$

where $C = Y$. Furthermore, because labor is undifferentiated and the labor market is competitive, the supply of labor is given by

$$\frac{W}{P} = l^{\varepsilon}, \tag{6}$$

where the left-hand side is the "real wage" (that is, the wage in units of consumption of the final good), whereas the right-hand side is the marginal disutility of labor. The labor demand for each entrepreneur i is then given by

$$l_{1i} = \left(\frac{y_i}{\gamma\Theta\left(1+\beta N\right)^{\alpha}}\right)^{1/\psi}, \tag{7}$$

if the entrepreneur adopts the new technology, and by

$$l_{0i} = \left(\frac{y_i}{\Theta\left(1+\beta N\right)^{\alpha}}\right)^{1/\psi}, \tag{8}$$

otherwise. In both cases, the entrepreneur takes N and Θ as given and employs labor so as to be able to produce the amount of intermediate good y_i demanded. Market clearing in the

[5] The dependence of all the equilibrium variables on π^x is meant to highlight the fact that the fraction of firms adopting the new technology in each state θ depends on π^x. Highlighting the dependence on π^x also facilitates the comparison between the equilibrium and the efficient allocations.

labor market then implies that

$$\frac{W}{P} = \left(\int l_i di \right)^{\varepsilon}. \tag{9}$$

3.3 Optimal Fiscal Policy

We first characterize (jointly necessary and sufficient) conditions that any optimal fiscal policy satisfies when the precision of private information π^x is exogenous. Next, we characterize additional conditions that any optimal policy must satisfy when information is endogenous. The comparison between the two sets of conditions permits us to illustrate that policies that are optimal under exogenous information need not be optimal when information is endogenous. Along the the way, we also show that simple state-invariant subsidies to the innovating firms suffice to induce efficiency in the usage of information, but fail to induce efficiency in the acquisition of information. The latter requires that the subsidies co-move with the marginal effect of more precise private information on the measure of firms adopting the new technology, which in turn requires conditioning the subsidies on the realized productivity of the two technologies.

3.3.1 Exogenous Information

Suppose that the precision of private information is exogenous and equal to π^x. Let $\hat{n}(x;\pi^x)$ denote the rule describing the efficient technology adoption, and $\hat{l}_0(\theta;\pi^x)$ and $\hat{l}_1(\theta;\pi^x)$ the rules describing the efficient labor employment, respectively for firms retaining the old technology and for those adopting the new one. Let $\hat{y}_0(\theta;\pi^x)$ and $\hat{y}_1(\theta;\pi^x)$ denote the efficient production of the intermediate goods under the two technologies. Finally, let $\hat{p}_0(\theta;\pi^x)$ and $\hat{p}_1(\theta;\pi^x)$ denote the prices, respectively for firms retaining the old technology and for those adopting the new one, that induce demands equal to $\hat{y}_0(\theta;\pi^x)$ and $\hat{y}_1(\theta;\pi^x)$, and hence employment equal to the efficient levels $\hat{l}_0(\theta;\pi^x)$ and $\hat{l}_1(\theta;\pi^x)$.

Definition 2. Assume the precision of private information is exogenous and equal to π^x. The fiscal policy $\bar{T}$ is **optimal** if it implements the efficient usage of information as an equilibrium. That is, if it induces all firms to adopt the new technology according to the rule $\hat{n}(x;\pi^x)$, and set prices according to the rules $\hat{p}_0(\theta;\pi^x)$ and $\hat{p}_1(\theta;\pi^x)$.

Let $r = py/P$ denote a representative firm's revenue in terms of the consumption of the final good. Next, let $\hat{C}(\theta;\pi^x)$ and $\hat{N}(\theta;\pi^x)$ denote, respectively, the amount of the final good consumed and the measure of firms adopting the new technology in state θ when the precision of private information is π^x, and all firms make all decisions efficiently. Hereafter, we denote by s the differential in the subsidy paid to an innovating firm (relative to a firm retaining the old technology) when the two firms generate the same revenue. We adopt the convention that s is paid to the innovating firms.

The following lemma provides a complete characterization of the policies that, when information is exogenous, implement the efficient use of information.

Lemma 2. *Assume that the precision of private information is exogenous and equal to π^x and that the conditions in Lemma 1 hold. Let*

$$\mathcal{R}(\theta;\pi^x) \equiv \left(\frac{v-\psi\left(v-1\right)}{v-1}\right)\hat{C}\left(\theta;\pi^x\right)^{\frac{1}{v}}\left(\hat{y}_1(\theta;\pi^x)^{\frac{v-1}{v}}-\hat{y}_0(\theta;\pi^x)^{\frac{v-1}{v}}\right)+s\left(\theta;\pi^x\right)-k. \quad (10)$$

Any optimal fiscal policy $\bar{T}$ pays to each firm retaining the old technology a transfer equal to

$$\bar{T}_0\left(r\right)=\frac{1}{v-1}r,$$

and to each firm adopting the new technology a transfer equal to

$$\bar{T}_1\left(r\right)=\frac{1}{v-1}r+s\left(\theta;\pi^x\right),$$

where the additional subsidy $s\left(\theta;\pi^x\right)$ to the innovating firms is such that $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right]<0$ when $x<\hat{x}(\pi^x)$, and $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right]>0$ when $x>\hat{x}(\pi^x)$, where $\hat{x}(\pi^x)$ is the signal threshold for efficient technology adoption defined in Lemma 1.

Proof. See the Appendix.

Any fiscal policy implementing the efficient use of information must combine the familiar revenue subsidy $r/(v-1)$ designed to offset firms' market power with an additional subsidy $s\left(\theta;\pi^x\right)$ to the innovating firms appropriately designed to satisfy the conditions in the lemma. Naturally, firms adopting the new technology expect higher revenues, and hence a higher subsidy $r/(v-1)$. However, this standard subsidy alone is not enough to induce firms to adopt the new technology efficiently. This is because firms do not internalize that, by adopting the new technology, they increase other firms' output. The additional subsidy $s\left(\theta;\pi^x\right)$ to the innovating firms must correct for such an externality. In the proof of the lemma in the Appendix, we show that $\mathcal{R}(\theta;\pi^x)$ is the private benefit of adopting the new technology, net of its cost. Such a benefit is equal to

$$\mathcal{R}(\theta;\pi^x)=\mathcal{Q}(\theta;\pi^x)-\frac{\alpha\beta\hat{C}\left(\theta;\pi^x\right)}{1+\beta\hat{N}\left(\theta;\pi^x\right)}+s\left(\theta;\pi^x\right),$$

where $\mathcal{Q}(\theta;\pi^x)$ is the social benefit, and

$$\frac{\alpha\beta\hat{C}(\theta;\pi^x)}{1+\beta\hat{N}\,(\theta;\pi^x)}$$

is the marginal externality created by the investment spillover. The externality coincides with the increase in the production of the final good that obtains if one increases N by a small amount $\varepsilon > 0$ around the efficient level $\hat{N}\,(\theta;\pi^x)$, holding firms' technology and employment decisions fixed. The subsidy $s\,(\theta;\pi^x)$ must thus be designed to compensate for the fact that firms do not internalize such an externality. Many subsidies $s\,(\theta;\pi^x)$ accomplish this objective. In fact, because efficiency requires that firms invest when $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^x)|x,\pi^x\right] > 0$ and refrain from investing when $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^x)|x,\pi^x\right] < 0$, any subsidy that aligns the sign of the expected private benefit $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right]$ to the sign of the expected social benefit $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^x)|x,\pi^x\right]$ does the job. When the conditions in Lemma 1 hold, $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^x)|x,\pi^x\right] > 0$ turns from negative to positive at $x = \hat{x}(\pi^x)$. Hence, any subsidy that makes the expected private benefit $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right]$ turn from negative to positive at $x = \hat{x}(\pi^x)$ induces all firms to invest efficiently. A particularly simple one entails a constant (i.e., state-invariant) subsidy, as shown in the following corollary.

Corollary 1. *Assume that the precision of private information is exogenous and equal to* π^x *and that the conditions in Lemma 1 hold. A fiscal policy that pays to each firm a standard revenue subsidy* $r/(v-1)$ *irrespective of the type of technology used and, in addition, pays to each innovating firm an extra constant subsidy equal to*

$$\bar{s}_{\pi^x} \equiv \mathbb{E}\left[\left.\frac{\alpha\beta\hat{C}\,(\theta;\pi^x)}{1+\beta\hat{N}\,(\theta;\pi^x)}\right|\hat{x}(\pi^x),\pi^x\right] \tag{11}$$

is optimal.

Proof. See the Appendix.

The constant subsidy $\bar{s}_{\pi^x}$ to the innovating firms is thus the externality expected by the "marginal innovator" with signal equal to the efficient threshold $\hat{x}(\pi^x)$. The advantage of such a simple policy is that it does not require the government to track the fundamental variable θ. When the government promises to pay to the innovating firms a constant subsidy equal to $\bar{s}_{\pi^x}$, a firm with signal equal to $\hat{x}(\pi^x)$ that expects all other firms to invest efficiently (and then set prices according to the rules $\hat{p}_0(\theta;\pi^x)$ and $\hat{p}_1(\theta;\pi^x)$, inducing demands $\hat{y}_0(\theta;\pi^x)$ and $\hat{y}_1(\theta;\pi^x)$, and hence efficient employments $\hat{l}_0(\theta;\pi^x)$ and $\hat{l}_1(\theta;\pi^x)$) is indifferent between retaining the old technology and adopting the new one. Because

$$\mathcal{Q}(\theta;\pi^x) - \frac{\alpha\beta\hat{C}\left(\theta;\pi^x\right)}{1+\beta\hat{N}\left(\theta;\pi^x\right)}$$

is monotone in θ, under the same expectations, any firm with signal above $\hat{x}(\pi^x)$ has incentives to invest, whereas any firm with signal below $\hat{x}(\pi^x)$ has incentives to retain the old technology. This means that the the constant subsidy $\bar{s}_{\pi^x}$ to the innovating firms, along with the familiar revenue subsidy $r/(v-1)$, aligns the private benefit $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right]$ to its social counterpart $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^x)|x,\pi^x\right]$, and hence implements the efficient allocation.

3.3.2 Endogenous Information

We now turn to the case in which firms' information is endogenous. Let π^{x*} denote the precision of the firms' private information that maximizes welfare (its characterization is in the proof of Lemma 3 below). In the presence of endogenous information, optimality is defined as follows.

Definition 3. The fiscal policy T^* is **optimal** if it implements the efficient acquisition and usage of information as an equilibrium. That is, if it induces all firms to (1) choose the efficient precision of private information π^{x*}, (2) follow the efficient rule $\hat{n}(x;\pi^{x*})$ to determine whether or not to upgrade the technology, and (3) set prices $\hat{p}_0(\theta;\pi^{x*})$ and $\hat{p}_1(\theta;\pi^{x*})$ that induce demands for the intermediate products equal to $\hat{y}_0(\theta;\pi^{x*})$ and $\hat{y}_1(\theta;\pi^{x*})$, and hence efficient employment $\hat{l}_0(\theta;\pi^{x*})$ and $\hat{l}_1(\theta;\pi^{x*})$.

Let $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^x$ denote the marginal variation in the measure of firms adopting the new technology at θ that obtains when one varies π^x infinitesimally at $\pi^x=\pi^{x*}$, holding fixed the rule for technology adoption at the efficient level $\hat{n}(x;\pi^{x*})$.

Lemma 3. *Assume that information is endogenous and that the economy satisfies the conditions in Lemma 1. Any optimal fiscal policy T^* pays to each firm retaining the old technology a transfer equal to*

$$T_0^*\left(r\right) = \frac{1}{v-1}r$$

and to each firm adopting the new technology a transfer equal to

$$T_1^*\left(r\right) = \frac{1}{v-1}r + s\left(\theta;\pi^{x*}\right),$$

where the additional subsidy $s\left(\theta;\pi^{x*}\right)$ to the innovating firms satisfies the condition in Lemma

2, applied to $\pi^x = \pi^{x*}$, and in addition satisfies the following condition

$$\mathbb{E}\left[s\left(\theta;\pi^{x*}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\mathbb{E}\left[\frac{\alpha\beta\hat{C}\left(\theta;\pi^{x*}\right)}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]. \tag{12}$$

Proof. See the Appendix.

The lemma provides a complete characterization of the policies that induce efficiency in both information usage and information acquisition. Relative to the case in which information is exogenous (with precision π^{x*}), the subsidy to the innovating firms must satisfy an additional restriction. The restriction is on the co-movement between the subsidy $s\left(\theta;\pi^{x*}\right)$ and the marginal effect $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^{x}$ of more precise private information on the aggregate investment in the new technology under the efficient allocation. The restriction is necessary to realign the private benefit from acquiring more precise information to its social counterpart. Under the conditions of Lemma 1, the externality $\alpha\beta\hat{C}\left(\theta;\pi^{x*}\right)/[1+\beta\hat{N}\left(\theta;\pi^{x*}\right)]$ increases with the state θ. The marginal variation $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^{x}$ in the measure of firms adopting the new technology due to more precise private information is also monotone in θ (it is negative for $\theta<\hat{x}(\pi^{x})$ and positive for $\theta>\hat{x}(\pi^{x})$). The subsidy $s\left(\theta;\pi^{x*}\right)$ must thus change with the state θ so that the co-movement between $s\left(\theta;\pi^{x*}\right)$ and the marginal variation $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^{x}$ is the same as that between the externality $\alpha\beta\hat{C}\left(\theta;\pi^{x*}\right)/[1+\beta\hat{N}\left(\theta;\pi^{x*}\right)]$ and $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^{x}$.

As a result of the additional restriction, policies that are optimal under exogenous information need not be optimal when information is endogenous. For example, the simple policy of Corollary 1, specialized to $\pi^x=\pi^{x*}$, under which the government pays a constant subsidy $\bar{s}_{\pi^{x*}}$ to the innovating firms, fails to induce efficiency in information acquisition, and hence it is not optimal when information is endogenous. This is because a constant subsidy equal to the externality expected by the marginal innovator with signal $\hat{x}(\pi^{x*})$ does not induce the right co-movement between the subsidy $s(\theta;\pi^{x*})$ and the (state-dependent) marginal effect of more precise private information on aggregate investment $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^{x}$, which is necessary to realign the private benefit of information acquisition to its social counterpart. Conversely, a policy that pays, in each state θ, a subsidy to the innovating firms equal to the state-specific externality from the investment spillover satisfies the co-movement condition in (12), and hence it induces efficiency in both information acquisition and information usage.

Proposition 1. *Irrespective of whether the economy satisfies the conditions in Lemma 1, the fiscal policy of Lemma 3 with a state-contingent subsidy to the innovating firms equal to*

$$s(\theta;\pi^{x*})=\frac{\alpha\beta\hat{C}\left(\theta;\pi^{x*}\right)}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)} \tag{13}$$

is optimal.

Proof. Suppose that all other firms (1) acquire information of precision π^{x*}, (2) adopt the new technology when, and only when, it is socially efficient to do so (i.e., if $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^{x*})|x,\pi^{x*}\right] > 0$ and only if $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^{x*})|x,\pi^{x*}\right] \geq 0$), and (3) set the prices $\hat{p}_0(\theta;\pi^{x*})$ and $\hat{p}_1(\theta;\pi^{x*})$ that induce the efficient employment decisions. Then, in each state θ, irrespective of the precision π^x of its private information, each firm finds it optimal to set a price equal to $\hat{p}_0(\theta;\pi^{x*})$ when it retains the old technology, and equal to $\hat{p}_1(\theta;\pi^{x*})$ when it adopts the new technology. Furthermore, the private value $\mathbb{E}\left[\mathcal{R}(\theta;\pi^{x*})|x,\pi^{x}\right]$ to upgrading the technology coincides with the social value $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^{x*})|x,\pi^{x}\right]$, for any x (see the proof of Lemma 2 in the Appendix for the formal arguments). These properties hold irrespective of whether the precision π^x selected by the firm coincides with the efficient level π^{x*}. They also hold irrespective of whether the economy satisfies the conditions in Lemma 1, the sole role of which is to guarantee that, when $\pi^x = \pi^{x*}$, the social benefit $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^{x*})|x,\pi^{x*}\right]$ to upgrading the technology turns from negative to positive at $x = \hat{x}(\pi^{x*})$. The same properties also imply that the gross value that the firm assigns to acquiring information coincides with the social value. Because the private cost of information also coincides with the social one, the above results imply that acquiring information of precision π^{x*} and then using the information efficiently (both when it comes to choosing the technology and setting the prices) is individually optimal for each firm expecting all other firms to do the same. Q.E.D.

As anticipated above, the state-contingent subsidy in (13) operates as a Pigouvian correction that induces each firm to internalize the effect of its technology choice on the production of the final good when all other firms acquire and use information efficiently. To see this, let Λ denote the cross-sectional distribution of firms' technology and employment decisions (n_i, l_i). Let $C_N(\theta, \Lambda)$ denote the marginal change in the production of the final good that obtains when, holding θ and Λ fixed, one changes N in all firms' production functions by a small $\varepsilon > 0$, starting from $N = N_\Lambda$, where N_Λ is the aggregate investment in the new technology under the distribution Λ. Next, let $\hat{\Lambda}(\theta, \pi^{x*})$ denote the cross-sectional distribution of firms' technology and employment decisions (n_i, l_i) under the efficient allocation. Then

$$C_N\left(\theta, \hat{\Lambda}(\theta, \pi^{x*})\right) = \frac{\alpha\beta\hat{C}\left(\theta;\pi^{x*}\right)}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}.$$

That is, the state-dependent subsidy in (13) coincides with the marginal change in the production of the final good that obtains as a result of a marginal change in N, evaluated at $N = \hat{N}\left(\theta;\pi^{x*}\right)$, holding all firms' technology and employment decisions fixed at the efficient level. Such a policy is thus reminiscent of familiar Pigouvian corrections for complete-information economies. These corrections induce firms to collect and use information efficiently even when firms' decisions (how much they invest in information acquisition and how they use their information) is not verifiable.

The Pigouvian policy of Proposition 1 is not the unique one implementing the efficient allocation. Other state-contingent policies do the job. One of the limitations of many of these policies (including the one in Proposition 1) is that they require the government to know what type of information the firms can collect (equivalently, the cost of different information structures). This knowledge is necessary to compute $\hat{C}(\theta;\pi^{x*})$ and $\hat{N}(\theta;\pi^{x*})$, and hence the state-contingent subsidy $s(\theta;\pi^{x*})$ in (13). This knowledge may not be available in many economies of interest. When this is the case, efficiency in both information acquisition and usage can still be induced by conditioning the subsidy to the innovating firms directly on C and N. Alternatively, it can be obtained by conditioning the subsidy s on the cross-sectional distribution of firms' technology and employment decisions, as the next proposition shows.

Proposition 2. *Assume that the government does not know what type of information the firms can collect (equivalently, the cost of different information structures). Efficiency in both information acquisition and usage can be induced through a fiscal policy that pays to the non-innovating firms a transfer equal to*

$$T_0^{\#}(r) = \frac{1}{\upsilon - 1} r,$$

and to the innovating firms a transfer equal to

$$T_1^{\#}(\theta, r, \Lambda) = \frac{1}{\upsilon - 1} r + C_N(\theta, \Lambda),$$

where Λ is the ex-post cross-sectional distribution of firms' technology and employment decisions (n_i, l_i), and where $C_N(\theta, \Lambda)$ is the marginal change in the production of the final good that obtains as a result of a marginal change in N holding all firms' technology and employment decisions fixed at the level specified by Λ.

Proof. Suppose that all other firms (1) acquire information efficiently (with information acquisition taking the form of a private signal $q : \Theta \to \Delta(\mathcal{S})$ mapping θ into a distribution over a Polish space $\mathcal{S}$ of signal realizations, which, without loss of generality can be taken to coincide with $[0, 1]$, (2) use information efficiently to make their technology choice, and (3) in each state θ, given the aggregate investment N in the new technology, set prices so as to induce the efficient employment (and hence production) decisions. Then, each firm has enough knowledge about the economy to compute the efficient allocation, and has incentives to follow the same efficient policies as any other firm. In fact, the revenue subsidy $r/(v-1)$ guarantees that each firm, no matter its technology, after learning θ, has the right incentives to set the price for its intermediate good at a level that induces the efficient demand for its product, and hence the efficient employment decisions (see the proof of Lemma 2 where the result is established without using the specific properties of the firms' information structure). Furthermore, when, in each state θ, the extra subsidy to the innovating firms takes the form of the marginal externality $C_N(\theta, \Lambda)$ exerted by N on the production of the final good (holding all firms' information, technology, and pricing rules fixed), the marginal value that each firm assigns to upgrading its technology coincides with the government's value in each state (see the proof of Lemma 2). The above properties imply that the private value of information acquisition coincides with the social one, no matter the cost of each experiment q. Hence, all firms have the right incentives to acquire and use information efficiently when expecting all other firms to do the same. Q.E.D.

The result in Proposition 2 illustrates the power of the Pigouvian logic. When the policy maker announces that innovating firms will receive a subsidy equal to the *ex-post* (marginal) externality $C_N(\theta, \Lambda)$ that each firm's technology choice exerts on the production of the final good, it re-aligns firms' (marginal) incentives with their social counterpart, not just at the interim stage but ex-post. The government can then delegate to firms the computation of the efficient allocation, while guaranteeing that, in equilibrium, they acquire and use information efficiently. One can also show that the power of the Pigouvian logic extends to economies in which firms are heterogeneous in their cost of acquiring information and/or of replacing the existing technology with the new one. This is because there are no discrepancies between private and social marginal costs. Hence, once the subsidy realigns the private benefits to their social counterparts, it induces efficiency in both information acquisition and usage, irrespective of whether costs are homogenous or heterogenous across firms.

Propositions 1 and 2 complement each other. Proposition 1 shows that, when the government knows the cost of different information structures, efficiency in both information acquisition and usage can be induced with a fiscal policy that conditions the subsidy s to the innovating firms only on the state θ — no further contingencies are necessary. Proposition 2, instead, shows that, when the aforementioned cost is unknown to the government, efficiency in information acquisition and usage can still be induced by expanding the contingencies in the optimal subsidy, for example by conditioning on the cross-sectional distribution of investment and employment decisions.

The policies of Propositions 1 and 2 also resemble VCG transfers, but with the correction operating on the margin instead of the levels.[6] While the VCG transfers eliminate the wedge between the private and the social objectives by making firms' profits (net of the transfers) proportional to their contribution to total welfare, the policies in Propositions 1 and 2 eliminate the wedge between the *marginal* private and social benefit of varying the firms' decisions.[7]

4 Sticky Prices and Optimal Monetary Policy

We now extend the analysis by introducing nominal rigidities. We do so by assuming that firms set prices under their endogenous private information before observing the realization of the fundamental variable θ. Such nominal rigidities introduce a role for monetary policy, in the spirit of Correia, Nicolini, and Telles (2008), and Angeletos and La'O (2020).

To capture the role of these nominal rigidities in the simplest possible terms, we introduce a cash-in-advance constraint. The government provides the representative household with an amount of money M, and the maximal expenditure on the purchase of the final good cannot exceed M, that is,

$$PY \leq M.$$

The timing of events is the same as in the model of Section 2, with the exception that prices are set under dispersed information about θ (i.e., with each p_i based on x_i instead of θ), and that the supply of money is state-dependent and governed by the monetary policy $M(\cdot)$. Each firm knows the monetary policy but does not observe the realized money supply at the time it sets the price for its intermediate good. This economy is consistent with most of the assumptions that are typically made in the pertinent literature.

The presence of price rigidities has no implications for the efficient allocation, which continues to be characterized by the conditions in the proof of Lemmas 1 and 3. The analysis of the equilibrium allocation, instead, must be amended to account for price rigidity. In this economy, the demands for the intermediate products, as well as the labor demands, continue to satisfy the same conditions as in Subsection 3.2. In particular, equilibrium in the labor market requires that Condition (9) holds.

Let $p_1(x;\pi^x)$ and $l_1(x,\theta;\pi^x)$ denote the equilibrium price and employment, respectively, of each firm that invests in the new technology. The corresponding functions for the firms retaining the old technology are $p_0(x;\pi^x)$ and $l_0(x,\theta;\pi^x)$. Because prices are set under (en-

[6] See Bergemann and Välimäki (2002) for the role of VCG payments in mechanism design with endogenous information acquisition.

[7] In our economy with a continuum of infinitesimal firms, VCG payments do not work, as the contribution of each firm's decisions to total welfare is zero.

dogenous) imperfect information about θ, the firms' labor demands $l_1(x, \theta; \pi^x)$ and $l_0(x, \theta; \pi^x)$ depend not only on θ and π^x but also on x.

Definition 4. Given the monetary policy $M(\cdot)$ and the fiscal policy $T(\cdot)$, an **equilibrium** is a precision π^x of private information, along with an investment strategy $n(x; \pi^x)$, and a pair of price functions $p_0(x; \pi^x)$ and $p_1(x; \pi^x)$ such that, when each firm $j \neq i$ chooses a precision of information equal to π^x and then chooses its technology according to $n(x; \pi^x)$ and sets its price according to $p_0(x; \pi^x)$ and $p_1(x; \pi^x)$, each firm i maximizes its market valuation by doing the same.

As in Section 3, the above equilibrium definition abstracts from other conditions (for wages, labor demand and supply, price of the final good) that are standard to isolate the novel and most relevant parts.

The following definition clarifies what it means that $M(\cdot)$ and $T(\cdot)$ are optimal.

Definition 5. The monetary policy $M^*(\cdot)$ along with the fiscal policy $T^*(\cdot)$ are **optimal** if, jointly, they implement the efficient acquisition and usage of information as an equilibrium. That is, they induce all firms to (1) choose the efficient precision of private information π^{x*}, (2) follow the efficient rule $\hat{n}(x; \pi^{x*})$ to determine whether or not to upgrade their technology, and (3) set prices (under dispersed information) according to rules $\hat{p}_0(x; \pi^{x*})$ and $\hat{p}_1(x; \pi^{x*})$ that, when followed by all firms, induce, in each state θ, demands for the intermediate products equal to the efficient levels $\hat{y}_0(\theta; \pi^{x*})$ and $\hat{y}_1(\theta; \pi^{x*})$, and hence result in firms employing labor according to the efficient rules $\hat{l}_0(\theta; \pi^{x*})$ and $\hat{l}_1(\theta; \pi^{x*})$.

For any precision of private information π^x (possibly different from π^{x*}), and any θ, let $\hat{M}(\theta; \pi^x)$ denote the amount of money supplied to the representative household in state θ when all firms are expected to acquire information of precision π^x. The policy $\hat{M}(\cdot; \pi^x)$ is designed so that, when all firms choose their technology according to the efficient rule $\hat{n}(x; \pi^x)$ and set prices according to $\hat{p}_0(x; \pi^x)$ and $\hat{p}_1(x; \pi^x)$, the resulting employment decisions coincide with the efficient ones $\hat{l}_0(\theta; \pi^x)$ and $\hat{l}_1(\theta; \pi^x)$ for an economy with private information of precision π^x.

The following lemma characterizes the monetary policy $\hat{M}(\cdot; \pi^x)$.

Lemma 4. *Assume that the precision of private information is exogenously fixed at π^x for all firms. Any monetary policy $\hat{M}(\cdot; \pi^x)$ that, together with some fiscal policy $\hat{T}(\cdot; \pi^x)$, implements the efficient use of information (for precision π^x) as an equilibrium is of the form*

$$\hat{M}(\theta; \pi^x) = m\hat{l}_0(\theta; \pi^x)^{1+\varepsilon} \left((\gamma^{\varphi} - 1) \hat{N}(\theta; \pi^x) + 1 \right)^{\frac{(1+\varepsilon)(v-1)-1}{v-1}},$$

for all θ, where m is an arbitrary positive constant. The monetary policy $\hat{M}(\cdot;\pi^x)$ induces all firms with the same technology to set the same price, irrespective of their information about θ.

As in other economies with nominal rigidities, the monetary policy $\hat{M}(\cdot;\pi^x)$ implements the efficient allocation by inducing firms to disregard their private information about the fundamental θ when setting their prices, and condition the latter only on the type of technology adopted. That prices do not respond to firms' information about θ, conditional on the selected technology, is necessary to avoid allocative distortions in the induced employment and production decisions. In fact, given the selected technology, relative prices must not vary with firms' signals about θ when the latter are imprecise. The monetary policy in Lemma 4 is designed so that, even if firms could condition their prices on θ, thus bypassing the nominal rigidity, they would not find it optimal to do so. Under the proposed policy, variations in employment and production decisions in response to changes in fundamentals are sustained by adjusting the money supply in a way that replicates the same allocations sustained when money is constant and prices are flexible.

The result in Lemma 4 may suggest that the monetary authority needs to know the cost of information to compute the optimal money supply in each state θ. However, as anticipated above, this is not the case. In fact, it suffices that the authority observes the cross-sectional distribution of employment and technology choices for it to be able to compute the amount of money that needs to be supplied.

Lemma 4 in turn permits us to establish the following:

Proposition 3. *All the results about the structure of the optimal fiscal policy in the previous section for the case of flexible prices carry over to the economy with price rigidities under consideration.*

The proof in the Appendix first shows that, when information is exogenous and of precision π^x, any fiscal policy that induces efficiency in information usage must induce firms to set prices that, given the technology choice, are invariant in the firms' signals. The only policies that satisfy this property take the form $T_0(r) = r/(v-1)$ and $T_1(r,\theta;\pi^x) = r/(v-1) + s(\theta;\pi^x)$, as in Lemma 2. It then shows that, under any such a fiscal policy, when the monetary policy is the one in Lemma 4, all firms have incentives to set prices that induce them to hire the efficient amount of labor in each state. Building on these observations, the proof then shows that, when the monetary policy takes the form in Lemma 4, the net private benefit that each firm with signal x expects from adopting the new technology continues to be given by $\mathbb{E}[\mathcal{R}(\theta;\pi^x)|x,\pi^x]$, as in the case of flexible prices. This property, in turn, implies that the extra subsidy $s(\theta;\pi^x)$ to the innovating firms must satisfy the conditions in Lemma 2 and, when information is endogenous, the additional condition (12) in Lemma 3.

The above result in turn implies implies that the Pigouvian fiscal policy of Proposition 1, in which the extra subsidy to the innovating firms takes the form

$$s(\theta;\pi^{x*}) = \frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})},$$

when paired with the monetary policy of Lemma 4 (specialized to $\pi^x = \pi^{x*}$), continues to realign the private value from adopting the new technology with its social counterpart, state by state. Once this realignment is established, the value that firms assign to information acquisition coincides with the social value, inducing all firms to acquire the efficient amount of private information when expecting other firms to do the same, as in the economy with flexible prices. Similar arguments imply that, when the fiscal or monetary authorities do not know the cost of information acquisition, it remains possible to implement the efficient acquisition and usage of information but it becomes necessary to expand the contingencies in the policies, for example by conditioning the policies on the cross-sectional distribution of employment and technology choices.

5 Conclusions

We investigate optimal fiscal and monetary policy in economies in which firms face uncertainty about the profitability of new investment opportunities such as a new technology or production process, and the profitability of such opportunities is affected by investment spillovers. We show that firms can be induced to acquire information efficiency and then use it in society's best interest through a fiscal policy that, in addition to correcting for firms' market power, provides those firms investing in the new technology with a subsidy that makes them internalize the effects of their investments on the production of intermediate and final goods. This result shows how the power of Pigouvian corrections extends to economies in which neither the collection nor the usage of information is verifiable. The same fiscal policy induces efficiency in information acquisition and usage when firms set prices under dispersed information (nominal rigidities), provided that it is accompanied by a monetary policy that makes firms disregard their endogenous private information when setting prices and only use information for technology adoption. Similar results obtain in markets in which externalities originate in pollution, and/or spillovers from investments in human capital.

Our analysis can be extended in several directions. We assume a static general-equilibrium economy in which investment, employment, and pricing decisions occur simultaneously. Technology adoption, however, is often a dynamic process. In future work, it would be interesting to extend the analysis to incorporate information externalities that naturally arise when firms can choose when to invest and learn from the observation of other firms' investment choices, as, e.g., in Dasgupta (2007), but in a setting with endogenous private information. It would also be interesting to enrich the model to allow for partial information aggregation in financial markets and study how inefficiencies in investment and production interact with those in the trading of financial assets (see also Angeletos, Lorenzoni, and Pavan (2023), and Pavan, Sundareas and Vives (2022) for models with some of these ingredients).

Finally, it would be interesting to extend the analysis to economies in which firms, in addition to acquiring information about the profitability of new technologies, expand the set of available products over time and strategically choose when to replace existing products with new ones, thus contributing to the understanding of how governments can increase the efficiency of the innovation diffusion process.

References

[1] Akcigit, U., S. Caicedo, E. Miguelez, V. Sterzi, and S. Stantcheva, 2018, Dancing with the Stars: Innovation through Interactions, *NBER Working Paper* 24466.

[2] Akcigit, U., J.R. Grigsby, T. Nicholas, and S. Stantcheva, 2022, Taxation and Innovation in the 20th Century, *Quarterly Journal of Economics*, 137(1), 329-385.

[3] Akcigit, U., D. Hanley, and S. Stantcheva, 2022, Optimal Taxation and R&D Policies, *Econometrica*, 90(2), 645-684.

[4] Alvarez F., D. Argente, F. Lippi, E. Mendez, and D. Van Patten, 2022, Strategic Complementarities in a Dynamic Model of Fintech Adoption, *mimeo*, New York University.

[5] Angeletos, G.M., L. Iovino and J. La'O, 2016, Real Rigidity, Nominal Rigidity, and the Social Value of Information, *American Economic Review*, 106, 200-227.

[6] Angeletos, G.M., and J. La'O, 2020, Optimal Monetary Policy with Informational Frictions, *Journal of Political Economy* 128(3), 1027-1064.

[7] Angeletos, G.M. and A. Pavan, 2007, Efficient Use of Information and Social Value of Information, *Econometrica*, 75, 1103-1142.

[8] Angeletos, G.M., G. Lorenzoni, and A. Pavan, 2023, Wall Street and Silicon Valley: A Delicate Interaction, *The Review of Economic Studies*, 90(3), 1041-1088.

[9] Bergemann, D., and J. Välimäki, 2002, Information Acquisition and Efficient Mechanism Design, *Econometrica*, 70, 3, 2002, 1007–33.

[10] Bloom, N., R. Griffith, and J. Van Reenen, 2002, Do R&D Tax Credits Work? Evidence From a Panel of Countries 1979–1997, *Journal of Public Economics*, 85 (1), 1-31.

[11] Colombo, L., G. Femminis and A. Pavan, 2014, Information Acquisition and Welfare, *The Review of Economic Studies*, 81, 1438-1483.

[12] Correia, I., J.P. Nicolini and P. Teles, 2008, Optimal Fiscal and Monetary Policy: Equivalence Results, *Journal of Political Economy*, 116, 141-170.

[13] Dasgupta, A., 2007, Coordination and Delay in Global Games, *Journal of Economic Theory*, 134, 195-225.

[14] Grossman, S. J., and J. E. Stiglitz, 1980), On the impossibility of Informationally Efficient Markets, *American Economic Review*, 70(3), 393-408.

[15] Llosa, G. and V. Venkatesvaran, 2022, Efficiency under Endogenous Information Choice: a Price-Setting Application, *Journal of the European Economic Association*, 20, 1619-1646.

[16] Pavan, A., S. Sundareasan, and X. Vives, 2022, (In)efficiency in Information Acquisition and Aggregation through Prices, *mimeo*, Imperial College, IESE, and Northwestern University.

[17] Vives, X., 1988 Aggregation of Information in Large Cournot Markets, *Econometrica*, 56, 851-76

Appendix

Proof of Lemma 1. Fix π^x and drop it from all expressions to ease the notation. Efficiency requires that any two firms with the same technology employ the same amount of labor. Letting $n(x)$ denote the probability that a firm receiving signal x adopts the new technology, and $l_1(\theta)$ and $l_0(\theta)$ the amount of labor employed by the firms adopting the new technology and by those retaining the old one, respectively, we have that the planner's problem can be written as

$$\max_{n(x),l_1(\theta),l_0(\theta)} \int_\theta C(\theta)d\Omega(\theta) - k\int_\theta N(\theta)d\Omega(\theta) +$$
$$-\frac{1}{1+\varepsilon}\int_\theta \left[l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta))\right]^{1+\varepsilon} d\Omega(\theta) +$$
$$-\int_\theta \mathcal{Q}(\theta)\left(N(\theta)-\int_x n(x)\,\Phi(x|\theta)\right)d\Omega(\theta),$$

where $\Omega(\theta)$ is the cumulative distribution function of θ (with density $\omega(\theta)$), $\Phi(x|\theta)$ is the cumulative distribution function of x given θ (with density $\phi(x|\theta)$), $\mathcal{Q}(\theta)$ is the multiplier associated with the constraint $N(\theta) = \int_x n(x)\,d\Phi(x|\theta)$, and

$$C(\theta) = \left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{v}{v-1}}, \tag{A.1}$$

with

$$y_1(\theta) = \gamma\Theta\left(1+\beta N(\theta)\right)^\alpha l_1(\theta)^\psi, \tag{A.2}$$

and

$$y_0(\theta) = \Theta\left(1+\beta N(\theta)\right)^\alpha l_0(\theta)^\psi. \tag{A.3}$$

The first-order condition with respect to $l_1(\theta)$ is thus equal to

$$\psi\left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{1}{v-1}} \left(\gamma\Theta\left(1+\beta N(\theta)\right)^\alpha\right)^{\frac{v-1}{v}} l_1(\theta)^{\psi\frac{v-1}{v}-1}$$
$$-\left(l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta))\right)^\varepsilon = 0.$$

Letting

$$L(\theta) \equiv l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta)), \tag{A.4}$$

and using (A.1) and (A.2), we have that the first order condition for $l_1(\theta)$ above can be expressed as

$$\psi C(\theta)^{\frac{1}{v}} y_1(\theta)^{\frac{v-1}{v}} = l_1(\theta)L(\theta)^\varepsilon. \tag{A.5}$$

Following similar steps, the first-order condition for $l_0(\theta)$ yields

$$\psi C(\theta)^{\frac{1}{v}} y_0(\theta)^{\frac{v-1}{v}} = l_0(\theta)L(\theta)^\varepsilon. \tag{A.6}$$

Jointly, the above first-order conditions – together with (A.3)-(A.4) – yield

$$l_0(\theta) = \psi^{\frac{1}{1+\varepsilon-\psi}} \left(\Theta \left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{\frac{1}{1+\varepsilon-\psi}} \left(\left(\gamma^{\varphi}-1\right) N\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v\varepsilon}{(v-1)(1+\varepsilon-\psi)}}, \tag{A.7}$$

and

$$l_1(\theta) = \gamma^{\varphi} l_0(\theta). \tag{A.8}$$

Notice that (A.8) implies that, at the efficient allocation, the total labor demand, as defined in (A.4), is equal to

$$L\left(\theta\right) = l_0\left(\theta\right)\left[\left(\gamma^{\varphi}-1\right) N\left(\theta\right)+1\right]. \tag{A.9}$$

The above conditions are both necessary and sufficient given that the planner's problem has a unique stationary point in (l_0, l_1) for any θ.
Differentiating the government's objective with respect to $N(\theta)$, we have that

$$\mathcal{Q}(\theta) = \frac{v}{v-1} C(\theta)^{\frac{1}{v}} \left[y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right] + \frac{\alpha\beta}{1+\beta N(\theta)} C(\theta) - k - L(\theta)^{\varepsilon}\left(l_1(\theta) - l_0(\theta)\right). \tag{A.10}$$

Lastly, consider the effect on welfare of changing $n(x)$ from 0 to 1, which is equal to

$$\Delta(x) \equiv \int_{\theta} \mathcal{Q}(\theta)\phi\left(x|\theta\right)\omega\left(\theta\right) d\theta.$$

Using the fact that $\phi\left(x|\theta\right)\omega\left(\theta\right) = f\left(\theta|x\right) g\left(x\right)$, where $f\left(\theta|x\right)$ is the conditional density of θ given x, and $g(x)$ is the marginal density of x, we have that

$$\Delta(x) \overset{sgn}{=} \int_{\theta} \mathcal{Q}(\theta) f\left(\theta|x\right) d\theta = \mathbb{E}[\mathcal{Q}(\theta)|x].$$

Hence, efficiency requires that $n(x) = 1$ if $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ and $n(x) = 0$ if $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$.
Now use (A.5) and (A.6) to observe that

$$L(\theta)^{\varepsilon}\left(l_1(\theta) - l_0(\theta)\right) = \psi C(\theta)^{\frac{1}{v}} \left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right).$$

Replacing the above expression into (A.10), we have that

$$\mathcal{Q}(\theta) = \left(\frac{v-\psi\left(v-1\right)}{v-1}\right) C(\theta)^{\frac{1}{v}} \left[y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right] + \frac{\alpha\beta}{1+\beta N\left(\theta\right)} C(\theta) - k.$$

Using (A.1), (A.2), (A.3), and (A.8), after some manipulations, we have that

$$C(\theta)^{\frac{1}{v}}\left(y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right)=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1}{v-1}}\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}l_0(\theta)^{\psi}\left(\gamma^{\varphi}-1\right), \tag{A.11}$$

and

$$C(\theta)=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{v}{v-1}}\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}l_0(\theta)^{\psi}.$$

It follows that

$$\mathcal{Q}(\theta)=\psi^{\frac{\psi}{1+\varepsilon-\psi}}\Theta^{\frac{1+\varepsilon}{1+\varepsilon-\psi}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{1+\varepsilon}{\varphi(1+\varepsilon-\psi)}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1+\varepsilon)}{1+\varepsilon-\psi}}\times$$
$$\times\left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)}{1+\beta N(\theta)}\right)-k. \tag{A.12}$$

When the parameters satisfy the conditions in the lemma, $\mathcal{Q}$ is increasing in both N (for given θ) and in θ (for given N). That, for any θ, $\mathcal{Q}$ is increasing in N implies that welfare is convex in N under the first best, i.e., when θ is observable by the firms (and hence by the planner) at the time the technology choices are made. Such a property implies that the first-best choice of N is either $N=0$ or $N=1$, for all θ. This last property, along with the fact that $\mathcal{Q}$ is increasing in θ for any N, implies that the first-best level of N is increasing in θ. This property, in turn, implies that the efficient strategy $\hat{n}\left(x\right)$ is monotone. For any θ and $\hat{x}$, let $\bar{\mathcal{Q}}(\theta|\hat{x})$ denote the function defined in (A.12) when $N(\theta)=1-\Phi(\hat{x}|\theta)$, that is, when firms adopt the new technology if and only if $x>\hat{x}$. Under the parameters' restrictions in the lemma, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$ is continuous, strictly increasing in $\hat{x}$, and such that

$$\lim_{\hat{x}\to-\infty}\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]<0<\lim_{\hat{x}\to+\infty}\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}].$$

Hence, the equation $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]=0$ admits one and only one solution. Let $\hat{x}$ denote the solution to this equation. Then note that $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x]<0$ for $x<\hat{x}$ and $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x]>0$ for $x>\hat{x}$. We conclude that, under the assumptions in the lemma, there exists a threshold $\hat{x}$ such that the technology-adoption rule $\hat{n}\left(x\right)=\mathbb{I}(x\geq\hat{x})$ along with the employment strategies $\hat{l}_1(\theta)$ and $\hat{l}_0(\theta)$ satisfying the first-order conditions above, constitute a solution to the planner's problem. Q.E.D.

Proof of Lemma 2. As in the proof of Lemma 1, we drop π^x from all formulas to ease the notation. We also drop θ when there is no risk of confusion.

Each firm with the new technology chooses p_1 to maximize

$$\frac{p_1 y_1 - W l_1}{P} + T_1\left(\frac{p_1 y_1}{P}\right), \tag{A.13}$$

taking W and P as given, accounting for the fact that y_1 is given by (5), with C exogenous to the firm's problem, and with l_1 given by (7). The first-order condition with respect to p_1 is given by

$$(1-v)\,CP^{v-1}p_1^{-v} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{1}{P}\frac{dT_1\left(p_1 y_1/P\right)}{dr}\frac{d\left(p_1 y_1\right)}{dp_1} = 0. \tag{A.14}$$

Using (5) and (7), we have that

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi}\frac{l_1}{p_1}, \tag{A.15}$$

and

$$\frac{d\left(p_1 y_1\right)}{dp_1} = (1-v)\,CP^{v}p_1^{-v}. \tag{A.16}$$

Replacing (A.15) and (A.16) into (A.14), using (5), and rearranging terms, we obtain that

$$\frac{1-v}{v}\frac{y_1 p_1}{P} + \frac{1}{\psi}\frac{W}{P}l_1 + \frac{1-v}{v}\frac{dT_1\left(p_1 y_1/P\right)}{dr}\frac{y_1 p_1}{P} = 0. \tag{A.17}$$

Next use (1) and (5), along with (A.8), to observe that, in any equilibrium implementing the efficient allocation, firms must set prices equal to (hereafter we use "hats" to denote variables under the rules inducing the efficient allocation)

$$\hat{p}_1 = \left(\left(\gamma^{\varphi}-1\right)\hat{N}+1\right)^{\frac{1}{v-1}}\gamma^{\frac{\varphi}{1-v}}\hat{P}, \tag{A.18}$$

and

$$\hat{p}_0 = \left(\left(\gamma^{\varphi}-1\right)\hat{N}+1\right)^{\frac{1}{v-1}}\hat{P}, \tag{A.19}$$

with

$$\hat{P} = \left(\hat{p}_1^{1-v}\hat{N} + \hat{p}_0^{1-v}\left(1-\hat{N}\right)\right)^{\frac{1}{1-v}}. \tag{A.20}$$

Market-clearing in the labor market requires that

$$\frac{\hat{W}}{\hat{P}} = \hat{L}^{\varepsilon}.$$

Then use (A.4) and (A.8) to note that $\hat{L} = \hat{l}_0\left[(\gamma^{\varphi}-1)\hat{N}+1\right]$. Next, use (A.5) to observe that efficiency requires that

$$-\psi\hat{C}^{\frac{1}{v}}\hat{y}_1^{\frac{v-1}{v}} + \hat{L}^{\varepsilon}\hat{l}_1 = 0. \tag{A.21}$$

Condition (A.17) then implies that T implements the efficient allocation only if

$$\frac{1}{v} = \frac{v-1}{v} \frac{dT_1\left(\hat{p}_1\hat{y}_1/\hat{P}\right)}{dr}.$$

Because $\hat{p}_1\hat{y}_1/\hat{P}$ is state dependent, we thus have that T_1 must be affine and satisfy

$$T_1(r) = \frac{1}{v-1}r + s, \tag{A.22}$$

with s invariant in r. Furthermore, one can show that, under the policy (A.22), the payoff of each firm adopting the new technology is quasi-concave in its price, which implies that the above first-order condition is also sufficient for the firm to choose $p_1 = \hat{p}_1$.

Similar arguments imply that the transfer to those firms retaining the old technology must be equal to

$$T_0(r) = \frac{1}{v-1}r \tag{A.23}$$

for these firms to find it optimal to set $p_0 = \hat{p}_0$.

Next, consider technology adoption. When the policy satisfies (A.22) and (A.23), with $s(\theta)$ possibly depending on θ, each firm finds it optimal to adopt the new technology if $\mathbb{E}[\mathcal{R}(\theta)|x] > 0$ and retain the old one if $\mathbb{E}[\mathcal{R}(\theta)|x] < 0$, where

$$\mathcal{R}(\theta) \equiv \left(\frac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + s(\theta) - k \tag{A.24}$$

is the extra profit (net of the subsidy) from adopting the new technology relative to retaining the old one. Now use the proof of Lemma 1 to note that efficiency requires that each firm invests if $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ and does not invest if $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$, where $\mathcal{Q}(\theta)$ can be conveniently rewritten as

$$\mathcal{Q}(\theta) = \left(\tfrac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \tfrac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}(\theta)} - k.$$

When the economy satisfies the conditions of Lemma 1, $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ turns from negative to positive at $x = \hat{x}$. Hence, for the policy defined by (A.22) and (A.23) to induce efficiency in technology adoption it is both necessary and sufficient that $\mathbb{E}[\mathcal{R}(\theta)|x]$ turns from negative to positive at $x = \hat{x}$. Q.E.D.

Proof of Corollary 1. Use the derivations in the proof of Lemma 2 to observe that

$$\mathcal{R}(\theta) = \mathcal{Q}(\theta) - \frac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}(\theta)} + s(\theta).$$

Next observe that the function

$$\mathcal{Q}(\theta) - \frac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}(\theta)}$$

is non-decreasing in θ under the conditions in Lemma 1. We thus have that, when $s(\theta) = \bar{s}_{\pi^x}$ for all θ, $\mathbb{E}\left[\mathcal{R}(\theta)|x\right]$ turns from negative to positive at $x = \hat{x}$, implying that the fiscal policy T satisfies all the conditions in Lemma 2 and hence is optimal. Q.E.D.

Proof of Lemma 3. The proof is in two parts. Part 1 characterizes the efficient precision of information π^{x*}. Part 2 uses the characterization in part 1 to establish the claim in the lemma.

Part 1. Using the results in Lemma 1, we have that, for any π^x, irrespective of whether the economy satisfies the restrictions in Lemma 1, ex-ante welfare under the efficient allocation is equal to

$$\begin{aligned}\mathcal{W} = &\int_\theta \Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^\alpha \hat{l}_0(\theta;\pi^x)^\psi \left(\left(\gamma^\varphi-1\right)\hat{N}(\theta;\pi^x)+1\right)^{\frac{v}{v-1}} d\Omega(\theta) + \\ &- k\int_\theta \hat{N}(\theta;\pi^x)\, d\Omega(\theta) - \int_\theta \frac{\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}}{1+\varepsilon}\left(\left(\gamma^\varphi-1\right)\hat{N}(\theta;\pi^x)+1\right)^{1+\varepsilon} d\Omega(\theta) - \mathcal{I}(\pi^x).\end{aligned}$$

Using the envelope theorem, we then have that π^{x*} solves

$$\begin{aligned}&\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})\left(\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^{x*})} + \frac{v\left(\gamma^\varphi-1\right)}{(v-1)\left(\left(\gamma^\varphi-1\right)\hat{N}(\theta;\pi^{x*})+1\right)}\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right] \\ &-k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right] + \mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left(\left(\gamma^\varphi-1\right)\hat{N}(\theta;\pi^{x*})+1\right)^\varepsilon\left(\gamma^\varphi-1\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right] = \frac{d\mathcal{I}(\pi^{x*})}{d\pi_x}.\end{aligned} \tag{A.25}$$

The above condition identifies the efficient precision of private information π^{x*}.

Part 2. Suppose that all firms other than i acquire information of precision π^{x*} and consider firm i's problem. Under the policy in the lemma, in each state θ, the price that maximizes firm i's profit coincides with the one that induces the efficient allocation for precision π^{x*}, irrespective of firm i's choice of π^x_i. This price is equal to $\hat{p}^*_1$ if the firm adopts the new technology and $\hat{p}^*_0$ if the firm retains the old technology, where $\hat{p}^*_1$ and $\hat{p}^*_0$ are given by the functions in (A.18) and (A.19), respectively, evaluated at $\pi^x = \pi^{x*}$. Note that we use the combination between "^" and "*" to denote variables under the efficient allocation for precision π^{x*} (this notation applies not only to $\hat{p}_1^*$ and $\hat{p}_0^*$ but to all expressions below).

Dropping θ from the argument of each function to ease the notation, we have that firm i's value function is equal to

$$\bar{\Pi}_i(\pi_i^x) \equiv \sup_{\varsigma:\mathbb{R}\to[0,1]} \Pi_i(\varsigma;\pi_i^x),$$

where

$$\Pi_i(\varsigma;\pi_i^x) \equiv \mathbb{E}\left[\hat{r}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{r}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right] - \mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]$$
$$+\,\mathbb{E}\left[\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x),$$

with $\bar{n}(\pi_i^x;\varsigma) \equiv \int \varsigma(x)d\Phi(x|\theta,\pi_i^x)$ denoting the probability that firm i adopts the new technology when using the strategy $\varsigma:\mathbb{R}\to[0,1]$, and $\hat{T}_1^*$ and $\hat{T}_0^*$ denoting the transfers received when generating (real) revenues $\hat{r}_1^* = \hat{p}_1^*\hat{y}_1^*/\hat{P}^*$ and $\hat{r}_0^* = \hat{p}_0^*\hat{y}_0^*/\hat{P}^*$ under the new and the old technology, respectively.

Substituting $\hat{r}_f^* = \hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}$, $f=0,1$, into $\Pi_i(\varsigma;\pi_i^x)$ and using (1), we have that

$$\Pi_i(\varsigma;\pi_i^x) = \mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right]$$
$$-\,\mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^*\right]$$
$$+\,\mathbb{E}\left[\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).$$

Accordingly,

$$\frac{\partial\Pi_i(\varsigma;\pi_i^x)}{\partial\pi_i^x} = \mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right]$$
$$-\,\mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\right]$$
$$+\,\mathbb{E}\left[\left(\frac{\hat{T}_1^*-\hat{T}_0^*}{\hat{P}^*}\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right] - k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right] - \frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x}. \quad \text{(A.26)}$$

Replacing

$$\hat{T}_1^* - \hat{T}_0^* = s + \frac{1}{v-1}\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}$$

into (A.26), we obtain that

$$\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}(\gamma^{\varphi}-1)\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x} l_0^{*\psi\frac{v-1}{v}}\right]$$
$$-\mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left((\gamma^{\varphi}-1)\,\hat{l}_0^*\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right)\right]+\mathbb{E}\left[s\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}. \quad \text{(A.27)}$$

Recall that, when $\pi_i^x = \pi^{x*}$, the optimal investment strategy is the efficient one: $\varsigma = \hat{n}^*$. Using the envelope theorem, we thus have that

$$\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial \Pi_i(\hat{n}^*;\pi^{x*})}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}(\gamma^{\varphi}-1)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]$$
$$-\mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left((\gamma^{\varphi}-1)\,\hat{l}_0^*\frac{\partial \hat{N}^*}{\partial \pi^x}\right)\right]+\mathbb{E}\left[s\frac{\partial \hat{N}^*}{\partial \pi^x}\right]-k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x},$$

where $\partial \hat{N}^*/\partial \pi^x$ is the marginal change in the measure of firms adopting the new technology that obtains when one changes π^x at $\pi^x = \pi^{x*}$, holding $\hat{n}^*$ fixed. For the proposed policy to induce efficiency in information acquisition, it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. This requires that

$$\mathbb{E}\left[\frac{v\,(\gamma^{\varphi}-1)\,\hat{C}\,(\theta;\pi^{x*})}{(v-1)\left((\gamma^{\varphi}-1)\,\hat{N}\,(\theta;\pi^{x*})+1\right)}\frac{\partial \hat{N}(\theta;\pi^{x*})}{\partial \pi^x}\right]$$
$$-\mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left((\gamma^{\varphi}-1)\,\hat{N}\,(\theta;\pi^{x*})+1\right)^{\varepsilon}(\gamma^{\varphi}-1)\frac{\partial \hat{N}(\theta;\pi^{x*})}{\partial \pi^x}\right]$$
$$+\mathbb{E}\left[s(\theta)\frac{\partial \hat{N}(\theta;\pi^{x*})}{\partial \pi^x}\right]-k\mathbb{E}\left[\frac{\partial \hat{N}(\theta;\pi^{x*})}{\partial \pi^x}\right]=\frac{\partial \mathcal{I}(\pi^{x*})}{\partial \pi^x}, \quad \text{(A.28)}$$

where we reintroduce all the arguments of the various functions to make the result consistent with the claim in the main text.

Comparing (A.28) with Condition (A.25) in part 1, we thus have that the policy in Lemma 3 induces the firms to acquire the efficient precision of private information only if, in addition to $s(\theta)$ satisfying the property in Lemma 2, it also satisfies Condition (12). Q.E.D.

Proof of Lemma 4. We drop π^x from all formulas to ease the notation. Using (A.5) and (A.6), we have that

$$\hat{l}_1(\theta)\hat{L}\,(\theta)^{\varepsilon} = \psi\hat{C}(\theta)^{\frac{1}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}},$$

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi \hat{C}(\theta)^{\frac{1}{v}} \hat{y}_0(\theta)^{\frac{v-1}{v}},$$

with $\hat{L}(\theta)$ defined by (A.4). The Dixit and Stiglitz demand system implies that $y_i = C(P/p_i)^v$. Hence, efficiency requires that the prices set by any two firms adopting the same technology coincide, which means that they must be independent of the signal x, conditional on the choice of technology. Let $\hat{p}_1$ be the (state-invariant) price set by firms adopting the new technology and $\hat{p}_0$ the price set by firms retaining the old technology. Let $\hat{P}(\theta)$ denote the price of the final good in state θ when all firms follow the efficient rules. Efficiency requires that such prices satisfy

$$\hat{l}_1(\theta)\hat{L}(\theta)^{\varepsilon} = \psi \hat{C}(\theta)\left(\hat{P}(\theta)/\hat{p}_1\right)^{v-1}, \tag{A.29}$$

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi \hat{C}(\theta)\left(\hat{P}(\theta)/\hat{p}_0\right)^{v-1}, \tag{A.30}$$

from which we obtain that

$$\frac{\hat{p}_0}{\hat{p}_1} = \left(\frac{\hat{l}_1(\theta)}{\hat{l}_0(\theta)}\right)^{\frac{1}{v-1}},$$

which, using (A.8), implies that

$$\hat{p}_1 = \gamma^{\frac{\varphi}{1-v}} \hat{p}_0.$$

The price of the final good is then equal to

$$\hat{P}(\theta) = \left((\gamma^{\varphi} - 1)\hat{N}(\theta) + 1\right)^{\frac{1}{1-v}} \hat{p}_0. \tag{A.31}$$

Combining (A.30) with the cash-in-advance constraint $M = PC$, we have that, in each state θ,

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi \hat{M}(\theta)\hat{P}(\theta)^{v-2} \hat{p}_0^{1-v},$$

and therefore

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi \hat{M}(\theta)\left((\gamma^{\varphi} - 1)\hat{N}(\theta) + 1\right)^{\frac{v-2}{1-v}} \hat{p}_0^{-1},$$

where we also used (A.31) to express $\hat{P}(\theta)$ as a function of $\hat{N}(\theta)$ and $\hat{p}_0$. Finally, using Condition (A.9), we obtain that, in each state θ, the money supply must be given by

$$\hat{M}(\theta) = \frac{1}{\psi}\hat{l}_0(\theta)^{1+\varepsilon}\left((\gamma^{\varphi} - 1)\hat{N}(\theta) + 1\right)^{\frac{(1+\varepsilon)(v-1)-1}{v-1}} \hat{p}_0.$$

It is immediate to verify that the same conclusion can be obtained starting from (A.29). Because $\hat{p}_0$ can be taken to be arbitrary, the result in the lemma obtains by setting $m = \frac{1}{\psi}\hat{p}_0$. Q.E.D.

Proof of Proposition 3. The proof is in two parts. Part 1 shows that, when information is exogenous and the monetary policy is the one in Lemma 4 (which, by virtue of the lemma, is the only one that can induce efficiency in information usage), any optimal fiscal policy must take the form $T_0(r) = r/(v-1)$ and $T_1(r) = r/(v-1) + s$, for some s that is invariant in r. The reason why this result is not implied by Lemma 2 and requires a separate proof is that the information upon which the firms set their prices is different from the one considered in Lemma 2; this implies that, in principle, the way the government provides incentives to the firms may be different from what established for flexible prices. Part 2 then uses the result in Part 1 to establish the conclusions in the proposition.

Part 1. Fix the precision of private information π^x and drop it to ease the notation. We also drop θ from the arguments of the various functions below when there is no risk of confusion. Consider first the pricing decision of a firm that adopts the new technology. The firm sets p_1 to maximize

$$\mathbb{E}\left[\frac{p_1 y_1 - W l_1}{P} + T_1(r_1) \middle| x\right], \tag{A.32}$$

where $r_1 = p_1 y_1 / P$, taking C, W, and P as given, and accounting for the fact that the demand for its product is given by

$$y_1 = C\left(\frac{P}{p_1}\right)^v, \tag{A.33}$$

and that the amount of labor that the firm will need to procure is given by

$$l_1 = \left(\frac{y_1}{\gamma\Theta(1+\beta N)^\alpha}\right)^{\frac{1}{\psi}}.$$

The first-order condition for the maximization of (A.32) with respect to p_1 is given by

$$\mathbb{E}\left[(1-v)\, C P^{v-1} p_1^{-v} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{1}{P}\frac{dT_1(r_1)}{dr}\frac{d(p_1 y_1)}{dp_1} \middle| x\right] = 0. \tag{A.34}$$

Using

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi}\frac{l_1}{p_1}, \tag{A.35}$$

$$\frac{d(p_1 y_1)}{dp_1} = (1-v)\, C P^v p_1^{-v},$$

and (A.33), we have that (A.34) can be rewritten as

$$\mathbb{E}\left[(1-v)\frac{y_1}{P} + \frac{W}{P}\frac{v}{\psi}\frac{l_1}{p_1} + \frac{dT_1(r_1)}{dr}\frac{(1-v)\, y_1}{P} \middle| x\right] = 0.$$

Multiplying all the addenda by p_1/v, we have that

$$\mathbb{E}\left[\frac{1-v}{v}\frac{y_1p_1}{P}+\frac{1}{\psi}\frac{W}{P}l_1+\frac{1-v}{v}\frac{dT_1\left(r_1\right)}{dr}\frac{y_1p_1}{P}\right|x\bigg]=0. \tag{A.36}$$

Suppose that all other firms follow policies that induce the efficient allocations, meaning that they follow the rule $\hat{n}(x)$ to determine which technology to use and then set prices $\hat{p}_0$ and $\hat{p}_1$ that depend only on the technology adopted but not on the signal x, as in the proof of Lemma 4. Consistently with the notation used above, we add "hats" to all relevant variables to highlight that these are computed under the efficient rules.
Observe that market clearing in the labor market requires that

$$\frac{\hat{W}}{\hat{P}}=\hat{L}^{\varepsilon}, \tag{A.37}$$

and recall that, as established in the Proof of Lemma 1,

$$\hat{L}=\hat{l}_0\left[\left(\gamma^{\varphi}-1\right)\hat{N}+1\right].$$

Also, observe that efficiency requires that

$$-\psi\hat{C}^{\frac{1}{v}}\hat{y}_1^{\frac{v-1}{v}}+\hat{L}^{\varepsilon}\hat{l}_1=0.$$

Accordingly, using Condition (A.36), we have that each firm adopting the new technology finds it optimal to set the price $\hat{p}_1$ that sustains the efficient allocation only if

$$\mathbb{E}\left[\frac{1-v}{v}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\hat{C}^{\frac{1}{v}}\hat{y}_1^{\frac{v-1}{v}}+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\right|x\bigg]=0, \tag{A.38}$$

where $\hat{r}_1=\hat{p}_1\hat{y}_1/\hat{P}$. Using again (A.33), we have that $\hat{y}_1^{-\frac{1}{v}}=\hat{C}^{-\frac{1}{v}}\frac{\hat{p}_1}{\hat{P}}$, which allows us to rewrite Condition (A.38) as

$$\mathbb{E}\left[\frac{1-v}{v}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\right|x\bigg]=0,$$

or, equivalently, as

$$\mathbb{E}\left[\frac{\hat{y}_1\hat{p}_1}{\hat{P}}\left(\frac{1}{v}+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\right)\right|x\bigg]=0.$$

It follows that, to induce the firm to set the efficient price $\hat{p}_1$ irrespective of his signal x, the fiscal policy must satisfy $dT_1\left(r_1\right)/dr=1/(v-1)$ for all r_1. Furthermore, one can verify that, when $dT_1(r_1)/dr = 1/(v-1)$ for all r_1, the firm's payoff is quasi-concave in p_1, which implies that setting the price $p_1 = \hat{p}_1$ is indeed optimal for the firm, for all x. To see that the firm's payoff is quasi-concave in p_1 note that, when all other firms follow the efficient rules and

$$T_1(r) = \frac{r}{v-1} + s = \frac{1}{v-1}\left(\frac{p_1 y_1}{P}\right) + s,$$

where s is invariant in r, the firm's objective (A.32) is equal to

$$\mathbb{E}\left[\frac{v}{v-1}\frac{p_1 y_1}{\hat{P}} - \frac{\hat{W}}{\hat{P}} l_1 + s \middle| x\right].$$

Using (A.33) and (A.35), we have that the first derivative of the firm's objective with respect to p_1 is

$$\mathbb{E}\left[-v\frac{y_1}{\hat{P}} + \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{l_1}{p_1} \middle| x\right],$$

whereas the second derivative is

$$\mathbb{E}\left[\frac{1}{p_1}\left(v^2\frac{y_1}{\hat{P}} - \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\left(\frac{v}{\psi}+1\right)\frac{l_1}{p_1}\right) \middle| x\right].$$

From the analysis above, we have that, when $p_1 = \hat{p}_1$, $y_1 = \hat{y}_1$ and $l_1 = \hat{l}_1$ in each state θ. Furthermore, irrespective of x, the derivative of the firm's objective function with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is

$$\mathbb{E}\left[-v\frac{\hat{y}_1}{\hat{P}} + \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{\hat{l}_1}{\hat{p}_1} \middle| x\right] = 0. \tag{A.39}$$

Using (A.39), we then have that the second derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is negative. Because the firm's objective function has a unique stationary point at $p_1 = \hat{p}_1$, we conclude that the firm's payoff is quasi-concave in p_1. Applying similar arguments to the firms retaining the old technology, we have that any fiscal policy that induces efficiency in information usage must pay to each firm retaining the old technology a transfer equal to $T_0(r_0)$, such that $dT_0(r_0)/dr = 1/(v-1)$, and that any such policy indeed induces these firms to set a price equal to $\hat{p}_0$ irrespective of the signal x. Thus, we conclude that any policy inducing efficiency in information usage must have the structure

$$T_0(r) = \frac{1}{v-1}r, \tag{A.40}$$

and

$$T_1(\theta, r) = \frac{1}{v-1} r + s(\theta), \tag{A.41}$$

where we reintroduce the dependence of s on θ in light of the analysis below.

Part 2. Observe that, under any monetary and fiscal policy that implement the efficient allocation, the "real revenues," i.e., the revenues expressed in terms of the consumption of the final good, must be the same as under flexible prices. This follows from the fact that equilibrium in the market for intermediate goods implies that

$$\hat{y}_f = \hat{C} \left(\frac{\hat{P}}{\hat{p}_f} \right)^v,$$

for $f = 0, 1$, which means that $\hat{p}_f / \hat{P}$ – and hence $\hat{r}_f = (\hat{p}_f \hat{y}_f)/\hat{P}$ – is uniquely pinned down by the efficient allocation. Because the transfers to the firms are in terms of "real revenues," and because "real wages" are also uniquely pinned down by the efficient allocation (as one case see from (A.37)), the value of adopting the new technology and of acquiring information must coincide with their counterparts under flexible prices. In turn, this implies that the subsidy to the innovating firms $s(\theta)$ must satisfy the same conditions as in Lemma 2 when information is exogenous, and those in Lemma 3 when information is endogenous. Finally, that the conclusions in Propositions 1 and 2 hold follows directly from the same arguments as in the proofs of these propositions. Q.E.D.

Optimal Fiscal and Monetary Policy with Investment Spillovers and Endogenous Private Information

Online Supplement

Luca Colombo[*] Gianluca Femminis[†] Alessandro Pavan[‡]

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Abstract

This document contains an extension to a family of economies in which the firms' managers, and hence the representative household, are risk averse; that is, they have diminishing marginal utility for the consumption of the final good. All numbered items in this document contain the prefix "S". Any numbered reference without the prefix "S" refers to an item in the main text.

[*]Università Cattolica del Sacro Cuore (lucava.colombo@unicatt.it).
[†]Università Cattolica del Sacro Cuore (gianluca.femminis@unicatt.it).
[‡]Northwestern University and CEPR (alepavan@northwestern.edu).

S.1 Richer Economies with Risk-Averse Managers

Consider the following economy in which the firms' managers are risk averse and set prices under imperfect information about the underlying fundamentals. Consistently with the rest of the pertinent literature, we assume that each manager is a member of a representative household, whose utility function is given by

$$U = \frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon} - \int \mathcal{I}(\pi_i^x)di,$$

where $R \geq 0$ is the coefficient of relative risk aversion in the consumption of the final good (the case $R = 0$ corresponds to what examined in Section 4 in the main text). The assumption that all managers are members of the same representative household is meant to capture the existence of a rich set of financial instruments that make the market complete in the sense of allowing the managers to fully insure against idiosyncratic consumption risk. The latter property, in turn, isolates the frictions (and associated inefficiencies) that originate in the presence of investment spillovers and endogenous dispersion of information at the time of technology adoption from the more familiar inefficiencies that originate in the lack of insurance possibilities.

As in the baseline model, each agent provides the same amount of labor (i.e., $l_i = l$ for all i), which is a consequence of the assumption that labor is homogenous and exchanged in a competitive market. Being a member of the representative household, each manager maximizes her firm's market valuation taking into account that the profits the firm generates will be used for the purchase of the final good. This means that each manager maximizes

$$\mathbb{E}\left[C^{-R}\left(\frac{p_i y_i - Wl_i}{P} + T\right)\middle| x_i; \pi_i^x\right] - kn_i - \mathcal{I}(\pi_i^x),$$

where C^{-R} is the representative household's marginal utility of consumption of the final good.

The representative household is endowed with an amount M of money provided by the government as a function of θ before the markets open (but after firms make their technology and price decisions). The household faces a cash-in-advance constraint according to which the maximal expenditure on the purchase of the final good cannot exceed M, that is,

$$PY \leq M.$$

The representative household collects profits from all firms and wages from all workers and uses them to repay M to the government at the end of the period. The government maximizes the ex-ante utility of the representative household, which is given by

$$\mathcal{W} = \mathbb{E}\left[\frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon}\right] - \mathcal{I}(\pi^x),$$

by means of a monetary policy $M(\cdot)$ and a fiscal policy $T(\cdot)$, subject to the constraint that the tax deficit be non-positive in each state.

The timing of events is the same as in Section 4 in the main text (note, in particular, that prices are set under dispersed information about θ, that is, each p_i is based on x_i instead of θ). This richer economy is consistent with most of the assumptions typically made in the pertinent Macroeconomics literature.

S.1.1 Efficient Allocation

The following proposition characterizes the efficient allocation in this economy.

Proposition S.1. *(1) Let* $\varphi \equiv \frac{v-1}{v-\psi(v-1)}$ *and* $\bar{R} \equiv 1 - \frac{(v-1)(1+\varepsilon)}{(1+\varepsilon)v+\varepsilon\psi(1-v)}$. *Assume that* $\gamma^{\varphi} \geq 1+\beta$, $\psi < \min\left\{1, \frac{1+\varepsilon}{\varepsilon(v-1)}\right\}$, *and* $0 \leq R \leq \bar{R}$. *For any precision of private information* π^x, *there exists a threshold* $\hat{x}(\pi^x)$ *such that efficiency requires that* $\hat{n}(x;\pi^x) = \mathbb{I}(x \geq \hat{x}(\pi^x))$. *The threshold* $\hat{x}(\pi^x)$, *along with the functions* $\hat{N}(\theta;\pi^x)$, $\hat{l}_1(\theta;\pi^x)$, *and* $\hat{l}_0(\theta;\pi^x)$, *satisfy the following properties:*

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^x)+1\right)^{\frac{1}{\varphi}}\right)^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times\right.$$

$$\left.\times\left(\frac{\gamma^{\varphi}-1}{\varphi\left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^x)+1\right)}+\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^x)}\right)\right|\hat{x}(\pi^x),\pi^x\right]=k,$$

$$\hat{N}(\theta;\pi^x) = 1-\Phi\left(\hat{x}(\pi^x)|\theta;\pi^x\right),$$

$$\hat{l}_0(\theta;\pi^x) = \psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}}\times$$

$$\times\left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^{x*})+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}, \quad \text{(S.1)}$$

and

$$\hat{l}_1(\theta;\pi^x) = \gamma^{\varphi}\hat{l}_0(\theta;\pi^x), \quad \text{(S.2)}$$

where $\Theta \equiv \exp(\theta)$.
(2) The efficient acquisition of private information is implicitly defined by the solution to

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^{x*})}+\frac{v}{v-1}\frac{(\gamma^{\varphi}-1)}{\left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^{x*})+1\right)}\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]+$$

$$+\mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left[(\gamma^{\varphi}-1)\hat{N}(\theta;\pi^{x*})+1\right]^{\varepsilon}(\gamma^{\varphi}-1)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi^x}.$$

The restriction $0 \leq R \leq \bar{R}$ guarantees that the marginal utility of consuming the final good does not decrease 'too quickly' with C. Along with the other restrictions in the proposition, which are the same as in Lemma 1 in the main text, this property implies that the efficient investment strategy is monotone. When, instead, $R > \bar{R}$, a higher value of θ may entail a low enough marginal utility of consumption to induce the planner to ask some firms receiving a high signal to refrain from investing in the new technology. As we clarify below, our key results extend to this case, but the exposition is less transparent.

S.1.2 Equilibrium Allocation

Firms choose both their technology and the price for their intermediate goods under dispersed information about θ. Given these choices, they acquire labor l to meet their demands, after observing θ and the total investment N in the new technology. In this richer economy, the equilibrium price of the final good and the demands for the intermediate products continue to be given by the same conditions as in the main text. Likewise for the labor demands. Because labor is undifferentiated and the labor market is competitive, the supply of labor is then given by

$$\frac{W}{P}C^{-R} = l^{\varepsilon},$$

where the right-hand side is the marginal disutility of labor, whereas the left-hand side is the marginal utility of expanding the consumption of the final good by W/P units, starting from a level of consumption equal to C. Market clearing in the labor market then requires that

$$\frac{W}{P}C^{-R} = \left(\int l_i di\right)^{\varepsilon}.$$

Let $p_1(x;\pi^x)$ and $l_1(x,\theta;\pi^x)$ denote the equilibrium price and labor demand, respectively, of each firm that invests in the new technology. The corresponding functions for the firms that retain the old technology are $p_0(x;\pi^x)$ and $l_0(x,\theta;\pi^x)$.[1]

The above equilibrium conditions are standard. The following definition identifies the components of the equilibrium allocation that are most relevant for our analysis.

Definition S.1. Given the monetary policy $M(\cdot)$ and the fiscal policy $T(\cdot)$, an **equilibrium** is a precision π^x of private information, along with an investment strategy $n(x;\pi^x)$ and a pair of price functions $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$ such that, when each firm $j \neq i$ chooses a precision of information equal to π^x and then chooses its technology according to $n(x;\pi^x)$ and sets its price according to $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$, each firm i maximizes its market valuation by doing the same.

The following definition clarifies what it means that $M(\cdot)$ and $T(\cdot)$ are optimal.

Definition S.2. The monetary policy $M^*(\cdot)$ along with the fiscal policy $T^*(\cdot)$ are **optimal** if they implement the efficient acquisition and usage of information as an equilibrium. That is, if they induce all firms to choose the efficient precision of information π^{x*}, follow the efficient rule $\hat{n}(x;\pi^{x*})$ to determine whether or not to upgrade their technology, and set prices according to rules $\hat{p}_0(x;\pi^{x*})$

[1] As in the baseline model, the dependence of these functions on π^x reflects the fact that, in each state θ, the measure of firms N adopting the new technology depends on the precision π^x of firms' information.

and $\hat{p}_1(x;\pi^{x*})$ that, when followed by all firms, induce in each state θ demands for the intermediate products equal to $\hat{y}_0(\theta;\pi^{x*})$ and $\hat{y}_1(\theta;\pi^{x*})$ and result in firms employing labor according to the efficient schedules $\hat{l}_0(\theta;\pi^{x*})$ and $\hat{l}_1(\theta;\pi^{x*})$.

For any precision of private information π^x (possibly different from π^{x*}), and any θ, let $\hat{M}(\theta;\pi^x)$ denote the optimal money supply in state θ. The following lemma characterizes the monetary policy $\hat{M}(\cdot;\pi^x)$.

Lemma S.1. *Suppose that the precision of private information is exogenously fixed at π^x for all firms. Any monetary policy $\hat{M}(\cdot;\pi^x)$ that, together with some fiscal policy $\hat{T}(\cdot)$, implements the efficient use of information (for precision π^x) as an equilibrium is of the form*

$$\hat{M}(\theta;\pi^x) = m\hat{l}_0(\theta;\pi^x)^{\frac{1+\varepsilon}{1-R}} \left(\left(\gamma^{\varphi}-1\right) \hat{N}\left(\theta;\pi^x\right)+1 \right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

for all θ, where m is an arbitrary positive constant. The monetary policy $\hat{M}(\cdot;\pi^x)$ induces all firms with the same technology to set the same price, irrespective of their information about θ.

As in other economies with nominal rigidities, the monetary policy $\hat{M}(\cdot;\pi^x)$ induces firms to disregard their private information about the fundamentals, and set prices based only on the selected technology. That prices do not respond to firms' information about θ is necessary to avoid allocative distortions in the induced employment and productions decisions. Relative prices must not vary with firms' signals about θ when the latter signals are imprecise. The monetary policy in Lemma S.1 is designed so that, even if firms could condition their prices on θ, they would not find it optimal to do so. Under the proposed policy, variations in employment and production decisions in response to changes in fundamentals are sustained by adjusting the money supplied in a way that replicates the same allocations sustained when the supply of money is constant and prices are flexible.

Lemma S.1, in turn, permits us to establish the following result.

Proposition S.2. *Irrespective of whether the economy satisfies the conditions in Proposition S.1, the fiscal policy*

$$T_0^*\left(r\right) = \frac{1}{v-1}r,$$

and

$$T_1^*\left(\theta,r\right) = \frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})} + \frac{1}{v-1}r,$$

along with the monetary policy

$$M^*(\theta) = m\hat{l}_0(\theta;\pi^{x*})^{\frac{1+\varepsilon}{1-R}} \left(\left(\gamma^{\varphi}-1\right) \hat{N}\left(\theta;\pi^{x*}\right)+1 \right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

are optimal.

The monetary policy in the proposition (which belongs to the family in Lemma S.1, specialized to $\pi^x = \pi^{x*}$) neutralizes the effects of price rigidity by replicating the same allocations as under flexible prices. When paired with the fiscal policy in the proposition, it guarantees that, if firms were constrained to acquire information of precision π^{x*}, they would follow the efficient rule $\hat{n}(x;\pi^{x*})$ to choose which technology to operate and then set prices $\hat{p}_0(x;\pi^x)$ and $\hat{p}_1(x;\pi^x)$ that induce the efficient labor demands, and hence the efficient production of the intermediate and final goods. This is accomplished through a fiscal policy that, in addition to offsetting firms' market power with a familiar revenue subsidy $r/(v-1)$, realigns the private value of upgrading the technology with the social value through an additional subsidy to the innovating firms that operates as a Pigouvian correction. As in the baseline economy, the subsidy

$$s(\theta) = \frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})}$$

makes each firm internalize the marginal effect of the investment in the new technology on the production of the final good, in each state θ. Once this realignment is established, the value that firms assign to acquiring information coincides with its social counterpart, inducing all firms to acquire the efficient amount of private information when expecting other firms to do the same.

S.2 Proofs

Proof of Proposition S.1. The proof is in two parts, each corresponding to the two claims in the proposition.

Part 1. Fix the precision of private information π^x and then drop it from all expressions to ease the notation. Let $n(x)$ denote the probability that a firm receiving signal x adopts the new technology, and $l_1(\theta)$ and $l_0(\theta)$ the amount of labor employed by the firms adopting the new technology and by those retaining the old one, respectively. The planner's problem can be written as

$$\max_{n(x),l_1(\theta),l_0(\theta)} \int_\theta \frac{C(\theta)^{1-R}}{1-R} d\Omega(\theta) - k\int_\theta N(\theta)d\Omega(\theta) + \\ -\frac{1}{1+\varepsilon}\int_\theta \left[l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta))\right]^{1+\varepsilon} d\Omega(\theta) + \\ -\int_\theta \mathcal{Q}(\theta)\left(N(\theta)-\int_x n(x)\Phi(x|\theta)\right)d\Omega(\theta),$$

where $\Omega(\theta)$ denotes the cumulative distribution function of θ (with density $\omega(\theta)$), $\Phi(x|\theta)$ the cumulative distribution function of x given θ (with density $\phi(x|\theta)$), $\mathcal{Q}(\theta)$ the multiplier associated with the constraint $N(\theta) = \int_x n(x)\,d\Phi(x|\theta)$, and

$$C(\theta) = \left(y_1(\theta)^{\frac{v-1}{v}}N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{v}{v-1}}, \tag{S.3}$$

with

$$y_1(\theta) = \gamma\Theta\left(1+\beta N(\theta)\right)^\alpha l_1(\theta)^\psi, \tag{S.4}$$

and

$$y_0(\theta) = \Theta\left(1 + \beta N(\theta)\right)^{\alpha} l_0(\theta)^{\psi}. \tag{S.5}$$

Using (S.3) and (S.4), the first-order condition of the planner's problem with respect to $l_1(\theta)$ can be written as

$$\psi C(\theta)^{-R}\left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1 - N(\theta))\right)^{\frac{1}{v-1}} \left(\gamma\Theta\left(1 + \beta N(\theta)\right)^{\alpha}\right)^{\frac{v-1}{v}} l_1(\theta)^{\psi\frac{v-1}{v}-1} - \left(l_1(\theta)N(\theta) + l_0(\theta)(1 - N(\theta))\right)^{\varepsilon} = 0.$$

Letting

$$L(\theta) \equiv l_1(\theta)N(\theta) + l_0(\theta)(1 - N(\theta)), \tag{S.6}$$

and using (S.3) and (S.4), we have that the above first-order condition reduces to

$$\psi C(\theta)^{\frac{1-vR}{v}} y_1(\theta)^{\frac{v-1}{v}} = l_1(\theta)L(\theta)^{\varepsilon}. \tag{S.7}$$

Following similar steps, the first-order condition with respect to $l_0(\theta)$ yields

$$\psi C(\theta)^{\frac{1-vR}{v}} y_0(\theta)^{\frac{v-1}{v}} = l_0(\theta)L(\theta)^{\varepsilon}. \tag{S.8}$$

Using (S.4) and (S.5), the ratio between (S.7) and (S.8) can be written as

$$\gamma^{\frac{v-1}{v}}\left(\frac{l_1(\theta)}{l_0(\theta)}\right)^{\psi\frac{v-1}{v}} = \frac{l_1(\theta)}{l_0(\theta)},$$

which implies that

$$l_1(\theta) = \gamma^{\varphi} l_0(\theta). \tag{S.9}$$

Notice that (S.9) entails that, at the efficient allocation, the total labor demand, as defined in (S.6), is equal to

$$L(\theta) = l_0(\theta)\left[\left(\gamma^{\varphi} - 1\right)N(\theta) + 1\right]. \tag{S.10}$$

Using (S.4) and (S.5), we can also write aggregate consumption as

$$C(\theta) = \Theta\left(1 + \beta N(\theta)\right)^{\alpha}\left(\gamma^{\frac{v-1}{v}} l_1(\theta)^{\psi\frac{v-1}{v}} N(\theta) + l_0(\theta)^{\psi\frac{v-1}{v}}(1 - N(\theta))\right)^{\frac{v}{v-1}}.$$

Using (S.9), we can rewrite the latter expression as

$$C(\theta) = \Theta\left(1 + \beta N(\theta)\right)^{\alpha} l_0(\theta)^{\psi}\left(\left(\gamma^{\varphi} - 1\right)N(\theta) + 1\right)^{\frac{v}{v-1}}. \tag{S.11}$$

Next, use (S.9) and (S.11) to rewrite (S.8) as

$$\psi\left(\Theta\left(1 + \beta N(\theta)\right)^{\alpha}\right)^{\frac{1-vR}{v}} l_0(\theta)^{\psi\frac{1-vR}{v}}\left(\left(\gamma^{\varphi} - 1\right)N(\theta) + 1\right)^{\frac{1-vR}{v-1}} \times \left(\Theta\left(1 + \beta N(\theta)\right)^{\alpha}\right)^{\frac{v-1}{v}} l_0(\theta)^{\psi\frac{v-1}{v}} = l_0(\theta)L(\theta)^{\varepsilon},$$

which, using (S.10), can be expressed as

$$\psi\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}}$$
$$=l_0(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\varepsilon}.$$

From the derivations above, we have that the efficient labor demands are given by

$$l_0(\theta)=\psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}},\qquad\text{(S.12)}$$

and by (S.9).

Note that $l_0(\theta)>0$ for all θ. Also note that the above conditions are both necessary and sufficient given that the planner's problem has a unique stationary point in (l_0,l_1) for each θ.

Next, consider the derivative of the planner's problem with respect to $N(\theta)$. Ignoring that $N(\theta)$ must be restricted to be in $[0,1]$, we have that

$$\mathcal{Q}(\theta)\equiv C(\theta)^{-R}\frac{dC(\theta)}{dN\left(\theta\right)}-k-L(\theta)^{\varepsilon}\left(l_1(\theta)-l_0(\theta)\right).$$

The derivative $dC(\theta)/dN(\theta)$ is computed holding the functions $l_1(\theta)$ and $l_0(\theta)$ fixed, and varying the proportion of firms investing into the new technology and the amounts that each firm produces for given technology choice when N changes.

Lastly, consider the effect on welfare of changing $n(x)$ from 0 to 1, which is equal to

$$\Delta(x)\equiv\int_{\theta}\mathcal{Q}(\theta)\phi\left(x|\theta\right)\omega\left(\theta\right)d\theta.$$

Using the fact that $\phi\left(x|\theta\right)\omega\left(\theta\right)=f\left(\theta|x\right)g\left(x\right)$, where $f\left(\theta|x\right)$ is the conditional density of θ given x and $g(x)$ is the marginal density of x, we have that

$$\Delta(x)\overset{sgn}{=}\int_{\theta}\mathcal{Q}(\theta)f\left(\theta|x\right)d\theta=\mathbb{E}[\mathcal{Q}(\theta)|x].$$

Hence, efficiency requires that all managers receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x]>0$ adopt the new technology, whereas all those receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x]<0$ retain the old one.

Next, use (S.3) to observe that

$$C(\theta)^{-R}\tfrac{dC(\theta)}{dN(\theta)}=\tfrac{v}{v-1}C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right]+$$

$$+C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{-\frac{1}{v}}\tfrac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta)+y_0(\theta)^{-\frac{1}{v}}\tfrac{\partial y_0(\theta)}{\partial N(\theta)}(1-N(\theta))\right],$$

and (S.4) and (S.5) to note that

$$y_1(\theta)^{-\frac{1}{v}}\tfrac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta)+y_0(\theta)^{-\frac{1}{v}}\tfrac{\partial y_0(\theta)}{\partial N(\theta)}(1-N(\theta))$$

$$=\tfrac{\alpha\beta}{1+\beta N(\theta)}\left(y_1(\theta)^{\frac{v-1}{v}}N\left(\theta\right)+y_0(\theta)^{\frac{v-1}{v}}\left(1-N\left(\theta\right)\right)\right)=\tfrac{\alpha\beta}{1+\beta N(\theta)}C(\theta)^{\frac{v-1}{v}},$$

where the last equality uses again (S.3).

Finally, using (S.7) and (S.8), we have that

$$\psi C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right)=L(\theta)^{\varepsilon}\left(l_1(\theta)-l_0(\theta)\right).$$

We conclude that

$$\mathcal{Q}(\theta)=\left(\frac{v-\psi\left(v-1\right)}{v-1}\right)C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right]+C(\theta)^{1-R}\frac{\alpha\beta}{1+\beta N\left(\theta\right)}-k.$$

Using (S.4), (S.5), (S.9), and (S.11), after some manipulations, we have that

$$C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right)=$$
$$=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}}\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}\left(\gamma^{\varphi}-1\right). \quad \text{(S.13)}$$

Using (S.11), we also have that

$$C(\theta)^{1-R}=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{v(1-R)}{v-1}}\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}.$$

It follows that

$$\mathcal{Q}(\theta)=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{v(1-R)}{v-1}}\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}\times$$
$$\times\left(\frac{\gamma^{\varphi}-1}{\varphi[\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1]}+\frac{\alpha\beta}{1+\beta N\left(\theta\right)}\right)-k.$$

Next, recall that the optimal labor demand for the firms retaining the old technology is given by (S.12). Replacing the expression for $l_0(\theta)$ into that for $\mathcal{Q}(\theta)$, we obtain that

$$\mathcal{Q}(\theta)=\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times$$
$$\times\left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)}{1+\beta N(\theta)}\right)-k.$$

Note that, when the parameters satisfy the conditions in the proposition, $\mathcal{Q}$ is increasing in both N (for given θ) and in θ (for given N). That, for any θ, $\mathcal{Q}(\theta)$ is increasing in N implies that welfare is convex in N under the first best, i.e., when θ is observable by the planner at the time the investment decisions are made. In turn, such a property implies that the first-best choice of N is either $N=0$ or $N=1$, for all θ. This observation, along with the fact that $\mathcal{Q}(\theta)$ is increasing in θ for any N then implies that the first-best level of N is increasing in θ. These properties, in turn, imply that the optimal investment policy is monotone. For any $\hat{x}$, let

$$\bar{N}(\theta|\hat{x})\equiv 1-\Phi(\hat{x}|\theta)$$

denote the measure of firms investing in the new technology at θ when firms follow the monotone rule $n(x)=\mathbb{I}(x>\hat{x})$. Then let

$$\bar{\mathcal{Q}}(\theta|\hat{x}) \equiv \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right)\bar{N}(\theta|\hat{x})+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \left(1+\beta\bar{N}(\theta|\hat{x})\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times \times \left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)\bar{N}(\theta|\hat{x})+1\right)}{1+\beta\bar{N}(\theta|\hat{x})}\right)-k$$

denote the function $\mathcal{Q}(\theta)$ characterized above, specialized to $N(\theta)=\bar{N}(\theta|\hat{x})$.

Observe that, under the parameters' restrictions in the proposition, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$ is continuous, strictly increasing in $\hat{x}$, and such that

$$\lim_{\hat{x}\to-\infty}\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]<0<\lim_{\hat{x}\to+\infty}\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}].$$

Hence, the equation $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]=0$ admits exactly one solution. Letting $\hat{x}$ denote the solution to this equation, we have that $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x]<0$ for $x<\hat{x}$, and $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x]>0$ for $x>\hat{x}$. We conclude that, under the assumptions in the proposition, there exists a threshold $\hat{x}(\pi^x)$ such that the investment strategy $\hat{n}\left(x;\pi^x\right)=\mathbb{I}(x\geq\hat{x}(\pi^x))$ along with the employment strategies $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$ in the proposition satisfy all the first-order conditions of the planner's problem. The threshold $\hat{x}(\pi^x)$ solves

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \left(1+\beta\hat{N}\left(\theta;\pi^x\right)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times \times \left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)}{1+\beta\hat{N}\left(\theta;\pi^x\right)}\right)\middle|\hat{x}(\pi^x),\pi^x\right]=k,$$

with $\hat{N}\left(\theta;\pi^x\right)=1-\Phi(\hat{x}(\pi^x)|\theta;\pi^x)$.

Finally note that, irrespective of whether the parameters satisfy the conditions in the proposition (recall that these conditions guarantee that $\hat{n}\left(x;\pi^x\right)$ is monotone), any solution to the planner's problem must be such that the functions $\hat{l}_0(\theta;\pi^x)$ and $\hat{l}_1(\theta;\pi^x)$ satisfy Conditions (S.1) and (S.2) in the proposition and $\hat{n}\left(x;\pi^x\right)=\mathbb{I}(\mathbb{E}[\hat{\mathcal{Q}}(\theta;\pi^x)|x,\pi^x]>0)$, where

$$\hat{\mathcal{Q}}(\theta;\pi^x) \equiv \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times \times \left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)}{1+\beta\hat{N}(\theta;\pi^x)}\right)-k,$$

with $\hat{N}(\theta;\pi^x)=\int_{\theta}\hat{n}\left(x;\pi^x\right)d\Phi\left(x|\theta,\pi^x\right)$.

Part 2. For any precision of private information π^x, use Conditions (S.10) and (S.11) in part (1) to write ex-ante welfare as

$$\begin{aligned}&\mathbb{E}\left[\mathcal{W}|\pi^x\right]= \\ &=\frac{1}{1-R}\int_{\theta}\Theta^{1-R}\left(1+\beta\hat{N}\left(\theta;\pi^x\right)\right)^{\alpha(1-R)}\hat{l}_0\left(\theta;\pi^x\right)^{\psi(1-R)}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{v}{v-1}(1-R)}d\Omega\left(\theta\right)+ \\ &\quad-k\int_{\theta}\hat{N}\left(\theta;\pi^x\right)d\Omega\left(\theta\right)-\int_{\theta}\frac{\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}}{1+\varepsilon}\left[\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right]^{1+\varepsilon}d\Omega\left(\theta\right)-\mathcal{I}(\pi^x).\end{aligned}$$

Using the envelope theorem, we have that the marginal effect of a variation in the precision of private information on welfare is given by

$$\frac{d\mathbb{E}\left[\mathcal{W}|\pi^x\right]}{d\pi^x} =$$
$$= \mathbb{E}\left[\hat{C}\left(\theta;\pi^x\right)^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^x\right)}+\frac{v\left(\gamma^{\varphi}-1\right)}{\left(v-1\right)\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]+$$
$$-k\mathbb{E}\left[\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]+\mathbb{E}\left[\hat{l}_0\left(\theta;\pi^x\right)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]-\frac{d\mathcal{I}(\pi_x)}{d\pi_x}.$$

The result in part 2 then follows from the fact that, at the optimum, the above derivative must be equal to zero. Q.E.D.

Proof of Lemma S.1. We drop π^x from all formulas to ease the notation. Using (S.7) and (S.8), we have that

$$\hat{l}_1(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}},$$
$$\hat{l}_0(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}}.$$

The Dixit and Stiglitz demand system implies that $y_i = C\left(P/p_i\right)^v$. Hence, the prices set by any two firms adopting the same technology coincide, implying that they are independent of the signal x. Let $\hat{p}_1$ be the (state-invariant) price set by the firms investing in the new technology, and $\hat{p}_0$ that set by firms retaining the old technology. Let $\hat{P}(\theta)$ denote the price of the final good when all firms follow the efficient policies. Efficiency requires that such prices satisfy

$$\hat{l}_1(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{C}(\theta)^{1-R}\left(\hat{P}\left(\theta\right)/\hat{p}_1\right)^{v-1}, \tag{S.14}$$

$$\hat{l}_0(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{C}(\theta)^{1-R}\left(\hat{P}\left(\theta\right)/\hat{p}_0\right)^{v-1}, \tag{S.15}$$

from which we obtain that

$$\frac{\hat{p}_0}{\hat{p}_1} = \left(\frac{\hat{l}_1(\theta)}{\hat{l}_0(\theta)}\right)^{\frac{1}{v-1}},$$

which, using (S.9), implies that

$$\hat{p}_1 = \gamma^{\frac{\varphi}{1-v}}\hat{p}_0.$$

The price of the final good is then equal to

$$\hat{P}\left(\theta\right) = \left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1}{1-v}}\hat{p}_0. \tag{S.16}$$

Combining the cash-in-advance constraint $M = PC$ with (S.15), we then have that

$$\hat{l}_0(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{M}(\theta)^{1-R}\hat{P}\left(\theta\right)^{v+R-2}\hat{p}_0^{1-v},$$

and therefore

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi \hat{M}(\theta)^{1-R}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta)+1\right)^{\frac{v+R-2}{1-v}}\hat{p}_0^{R-1},$$

where we also used (S.16). Finally, using Condition (S.10), we obtain that

$$\hat{M}(\theta)^{1-R} = \frac{1}{\psi}\hat{l}_0(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{v-1}}\hat{p}_0^{1-R}.$$

It is immediate to verify that the same conclusion can be obtained starting from (S.14). Because $\hat{p}_0^{1-R}$ can be taken to be arbitrary, the result in the lemma obtains by setting $m^{1-R} = \frac{1}{\psi}\hat{p}_0^{1-R}$. Q.E.D.

Proof of Proposition S.2. The proof is in two parts and establishes a more general result than the one in the proposition. Part 1 fixes the precision of information and identifies a condition on the fiscal policy $T(\cdot)$ that guarantees that, when $T(\cdot)$ is paired with the monetary policy of Lemma S.1, and the economy satisfies the parameters' restrictions of Proposition S.1, firms have incentives to use information efficiently when the latter is exogenous. Part 2 identifies an additional restriction on the fiscal policy that, when combined with the condition in part 1, guarantees that, when the economy satisfies the parameters' restrictions of Proposition S.1, agents have also incentives to acquire information efficiently. The arguments in parts 1 and 2 also allow us to establish that, irrespective of whether or not the economy satisfies the parameters' restrictions of Proposition S.1, when $M(\cdot)$ and $T(\cdot)$ are the specific policies of Proposition S.2, any firm that expects all other firms to acquire and use information efficiently has incentives to do the same.

Part 1. We fix the precision of information π^x and drop it to ease the notation. We also drop θ from the arguments of the various functions when there is no risk of confusion.

Consider first the pricing decision of a firm that adopts the new technology. The firm sets p_1 to maximize

$$\mathbb{E}\left[C^{-R}\left(\frac{p_1 y_1 - W l_1}{P} + T_1(r_1)\right)\bigg| x\right], \tag{S.17}$$

where $r_1 = p_1 y_1 / P$, taking C, W, and P as given, and accounting for the fact that the demand for its product is given by

$$y_1 = C\left(\frac{P}{p_1}\right)^{v}, \tag{S.18}$$

and that the amount of labor that it will need to procure is given by

$$l_1 = \left(\frac{y_1}{\gamma\Theta\left(1+\beta N\right)^{\alpha}}\right)^{\frac{1}{\psi}}.$$

The first-order condition for the maximization of (S.17) with respect to p_1 is given by

$$\mathbb{E}\left[C^{-R}\left(\left(1-v\right)CP^{v-1}p_1^{-v} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{1}{P}\frac{dT_1(r_1)}{dr}\frac{d(p_1 y_1)}{dp_1}\right)\bigg| x\right] = 0. \tag{S.19}$$

Using

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi}\frac{l_1}{p_1}, \tag{S.20}$$

$$\frac{d\left(p_1 y_1\right)}{dp_1} = (1-v)\, C P^v p_1^{-v},$$

and (S.18), we have that (S.19) can be rewritten as

$$\mathbb{E}\left[C^{-R}\left((1-v)\frac{y_1}{P} + \frac{W}{P}\frac{v}{\psi}\frac{l_1}{p_1} + \frac{dT_1\left(r_1\right)}{dr}\frac{(1-v)\, y_1}{P}\right)\middle| x\right] = 0.$$

Multiplying all the addenda by p_1/v, we have that

$$\mathbb{E}\left[\frac{1-v}{v}C^{-R}\frac{y_1 p_1}{P} + \frac{1}{\psi}C^{-R}\frac{W}{P}l_1 + \frac{1-v}{v}C^{-R}\frac{dT_1\left(r_1\right)}{dr}\frac{y_1 p_1}{P}\middle| x\right] = 0. \tag{S.21}$$

Suppose that all other firms follow policies that induce the efficient allocations, meaning that they follow the rule $\hat{n}(x)$ to determine which technology to use and then set prices $\hat{p}_0$ and $\hat{p}_1$ that depend only on the technology they adopted but not on the signal x, as in the proof of Lemma S.1. Hereafter, we add 'hats' to all relevant variables to highlight that these are computed under the efficient policies. Observe that market clearing in the labor market requires that

$$\hat{C}^{-R}\frac{\hat{W}}{\hat{P}} = \hat{L}^{\varepsilon}, \tag{S.22}$$

and recall that, as established in the Proof of Proposition S.1,

$$\hat{L} = \hat{l}_0\left[\left(\gamma^{\varphi}-1\right)\hat{N}+1\right].$$

Also, consider that efficiency requires that

$$-\psi \hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}} + \hat{L}^{\varepsilon}\hat{l}_1 = 0.$$

Accordingly, using Condition (S.21), we have that each firm adopting the new technology finds it optimal to set the price $\hat{p}_1$ only if

$$\mathbb{E}\left[\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}} + \hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}} + \frac{1-v}{v}C^{-R}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\middle| x\right] = 0, \tag{S.23}$$

where $\hat{r}_1 = \hat{p}_1\hat{y}_1/\hat{P}$. Using again (S.18), we have that $\hat{y}_1^{-\frac{1}{v}} = \hat{C}^{-\frac{1}{v}}\frac{\hat{p}_1}{\hat{P}}$, which allows us to rewrite Condition (S.23) as

$$\mathbb{E}\left[\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}} + \hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}} + \frac{1-v}{v}\hat{C}^{-R}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\middle| x\right] = 0,$$

or, equivalently,

$$\mathbb{E}\left[\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}\left(\frac{1}{v} + \frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\right)\middle| x\right] = 0.$$

It follows that, when $dT_1\left(\hat{r}_1\right)/dr = 1/(v-1)$, the first-order condition of the firm's optimization problem with respect to its price is satisfied. Furthermore, one can verify that, under the proposed fiscal policy, the firm's payoff is quasi-concave in p_1, which implies that setting a price $p_1 = \hat{p}_1$ is indeed optimal for the firm. To see that the firm's payoff is quasi-concave in p_1 note that, when all other firms follow the efficient policies and

$$T_1(r) = \frac{r}{v-1} + s = \frac{1}{v-1}\left(\frac{p_1 y_1}{P}\right) + s,$$

where s may depend on θ but is invariant in r, the firm's objective (S.17) is equal to

$$\mathbb{E}\left[\hat{C}^{-R}\left(\frac{v}{v-1}\frac{p_1 y_1}{\hat{P}} - \frac{\hat{W}}{\hat{P}}l_1 + s(\theta)\right)\middle| x\right].$$

Using (S.18) and (S.20), we have that the first derivative of the firm's objective with respect to p_1 is

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{y_1}{\hat{P}} + \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{l_1}{p_1}\right)\middle| x\right],$$

whereas the second derivative is

$$\mathbb{E}\left[\frac{\hat{C}^{-R}}{p_1}\left(v^2\frac{y_1}{\hat{P}} - \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\left(\frac{v}{\psi}+1\right)\frac{l_1}{p_1}\right)\middle| x\right].$$

From the analysis above, we have that $y_1 = \hat{y}_1$ and $l_1 = \hat{l}_1$ in each state θ when $p_1 = \hat{p}_1$. Furthermore, irrespective of x, the derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{\hat{y}_1}{\hat{P}} + \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{\hat{l}_1}{\hat{p}_1}\right)\middle| x\right] = 0. \tag{S.24}$$

Using (S.24), we then have that the second derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is negative. Because the firm's objective function has a unique stationary point at $p_1 = \hat{p}_1$, we conclude that the firm's payoff is quasi-concave in p_1. Applying similar arguments to the firms retaining the old technology, we have that a fiscal policy that pays each firm retaining the old technology a policy equal to $T_0(r) = r/(v-1)$ induces these firms to set the price $\hat{p}_0$ irrespective of the signal x.

Next, consider the firms' technology choice. Hereafter, we reintroduce θ in the notation. When

$$T_0(r) = \frac{1}{v-1}r, \tag{S.25}$$

and

$$T_1(\theta, r) = s(\theta) + \frac{1}{v-1}r, \tag{S.26}$$

no matter the shape of the function $s(\theta)$, each firm anticipates that, by innovating, it will set a price $\hat{p}_1$, hire $\hat{l}_1(\theta)$, and produce $\hat{y}_1(\theta)$ in each state θ, whereas, by retaining the old technology, it will set a price $\hat{p}_0$, hire $\hat{l}_0(\theta)$, and produce $\hat{y}_0(\theta)$. Let

$$\hat{\mathcal{R}}(\theta) \equiv \hat{C}(\theta)^{-R}\left(\hat{r}_1(\theta) - \hat{r}_0(\theta) - \frac{\hat{W}(\theta)}{\hat{P}(\theta)}\left(\hat{l}_1(\theta) - \hat{l}_0(\theta)\right) + T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) - k,$$

where $\hat{r}_1(\theta)$ and $\hat{r}_0(\theta)$ are the firm's (real) revenues when the firm follows the efficient policies, after adopting the new technology and retaining the old one, respectively. Each firm receiving signal x finds it optimal to adopt the new technology if

$$\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \geq 0,$$

and retain the old technology if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \leq 0$. Recall from (S.18) that the Dixit and Stiglitz demand system implies that $\hat{p}_f = \hat{P}(\theta)\,\hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{-\frac{1}{v}}$, so that $\hat{r}_f(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{\frac{v-1}{v}}$, for $f = 0, 1$. Also, recall that market clearing in the labor market implies that

$$\frac{\hat{W}(\theta)}{\hat{P}(\theta)}\hat{C}(\theta)^{-R} = \hat{L}(\theta)^{\varepsilon}.$$

Hence, $\hat{\mathcal{R}}(\theta)$ can be rewritten as

$$\hat{\mathcal{R}}(\theta) = \hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) - \hat{L}(\theta)^{\varepsilon}\left(\hat{l}_1(\theta) - \hat{l}_0(\theta)\right) + \\ + \hat{C}(\theta)^{-R}\left(T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) - k.$$

Using the fact that the efficient allocation satisfies the following two conditions (see the proof of Proposition S.1)

$$\psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}} = \hat{l}_1(\theta)\hat{L}(\theta)^{\varepsilon},$$

and

$$\psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}} = \hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon},$$

we have that $\hat{\mathcal{R}}(\theta)$ can be further simplified as follows:

$$\hat{\mathcal{R}}(\theta) \;=\; (1-\psi)\,\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) \;+\; \hat{C}(\theta)^{-R}\left(T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) \;-\; k.$$

Next, use (S.18) to note that

$$\hat{r}_f(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\hat{y}_f(\theta)^{\frac{v-1}{v}},$$

for $f = 0, 1$. It follows that

$$T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta)) = s(\theta) + \frac{1}{v-1}\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right).$$

Accordingly, $\hat{\mathcal{R}}(\theta)$ can be written as

$$\hat{\mathcal{R}}(\theta) = \left(\frac{v - \psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \hat{C}(\theta)^{-R}s(\theta) - k. \tag{S.27}$$

Recall from the proof of Proposition S.1 that efficiency requires that each firm adopts the new technology if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] > 0$ and retains the old one if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] < 0$, where $\hat{\mathcal{Q}}(\theta)$ is given by

$$\hat{\mathcal{Q}}(\theta) \equiv \left(\frac{v - \psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left[\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right] + \hat{C}(\theta)^{1-R}\frac{\alpha\beta}{1+\beta\hat{N}(\theta)} - k.$$

Hence, we conclude that the proposed policy induces all firms to follow the efficient technology-adoption rule $\hat{n}(x)$ if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \geq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \geq 0$, and $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \leq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \leq 0$.

As shown in the proof of Proposition S.1 (see Equations (S.13) and (S.12), respectively),

$$\hat{C}(\theta)^{\frac{1-vR}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) = \\ = \left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}} \left(\Theta\left(1+\beta\hat{N}\left(\theta\right)\right)^{\alpha}\right)^{1-R} \hat{l}_0(\theta)^{\psi(1-R)}\left(\gamma^{\varphi}-1\right),$$

and

$$\hat{l}_0(\theta) = \psi^{\frac{1}{1+\varepsilon+\psi(R-1)}} \left(\Theta\left(1+\beta\hat{N}\left(\theta\right)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}.$$

Using the last two expressions, we have that the first addendum in (S.27) can be rewritten as

$$\left(\frac{v-\psi\left(v-1\right)}{v-1}\right)\hat{C}\left(\theta\right)^{\frac{1-vR}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) = \\ = \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\frac{\gamma^{\varphi}-1}{\varphi}\right).$$

When the economy satisfies the conditions in Proposition S.1, the above expression is increasing in N (for given θ) and in θ (for given N). In this case, when the second addendum $\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)$ in (S.27) is non-decreasing in θ, then $\hat{\mathcal{R}}(\theta)$ is non-decreasing in θ, implying that $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right]$ is non-decreasing in x. As in the baseline model, we thus have that, when the economy satisfies the parameters' restrictions in Proposition S.1, a subsidy $s(\theta)$ to the innovating firms satisfying conditions (a) and (b) below guarantees that firms find it optimal to follow the efficient rule $\hat{n}(x)$:

(a) $\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)$ non-decreasing in θ;

(b)

$$\mathbb{E}\left[\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)\middle|\hat{x}\right] = \mathbb{E}\left[\frac{\alpha\beta\hat{C}(\theta)^{1-R}}{1+\beta\hat{N}\left(\theta\right)}\middle|\hat{x}\right].$$

The analysis above also reveals that, when the fiscal policy takes the form in (S.25) and (S.26) with

$$s(\theta) = \frac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}\left(\theta\right)},$$

for all θ, and the monetary policy takes the form in Lemma S.1, then irrespective of whether or not the economy satisfies the conditions in Proposition S.1, each firm expecting all other firms to follow the efficient technology adoption rule $\hat{n}(x)$, and setting prices according to $\hat{p}_0$ and $\hat{p}_1$ (thus inducing the efficient employment decisions), finds it optimal to do the same.

Part 2. We now show that, when the economy satisfies the conditions in Proposition S.1, the fiscal policy in (S.25) and (S.26), when paired with the monetary policy

$$M^*(\theta) = m\hat{l}_0\left(\theta;\pi^{x*}\right)^{\frac{1+\varepsilon}{1-R}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

implement the efficient acquisition and usage of information if and only if the subsidy $s(\theta)$ to the innovating firms, in addition to properties (a) and (b) in part 1, is such that

$$\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right].$$

To see this, suppose that all firms other than i acquire information of precision π^{x*} and follow the efficient technology and pricing rules. Consider firm i's problem. As shown above, irrespective of the information acquired by the firm, under the proposed fiscal and monetary policies, the firm finds it optimal to set a price equal to $\hat{p}_1^*$ after adopting the new technology and equal to $\hat{p}_0^*$ after retaining the old one, where $\hat{p}_1^*$ and $\hat{p}_0^*$ are given by the values of $\hat{p}_1$ and $\hat{p}_0$, respectively, when the precision of private information is π^{x*}.

Let

$$\hat{N}^*(\theta)\equiv\hat{N}\left(\theta;\pi^{x*}\right),$$

$$\hat{l}_0^*(\theta)\equiv\hat{l}_0(\theta;\pi^{x*}),$$

$$\hat{l}_1^*(\theta)\equiv\hat{l}_1(\theta;\pi^{x*}),$$

$$\hat{y}_1^*(\theta)\equiv\gamma\Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\hat{l}_1^*(\theta)^{\psi},$$

$$\hat{y}_0^*(\theta)\equiv\Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\hat{l}_0^*(\theta)^{\psi},$$

$$\hat{C}^*(\theta)=\hat{Y}^*(\theta)\equiv\left(\hat{y}_1^*(\theta)^{\frac{v-1}{v}}\hat{N}^*(\theta)+\hat{y}_0^*(\theta)^{\frac{v-1}{v}}\left(1-\hat{N}^*(\theta)\right)\right)^{\frac{v}{v-1}},$$

$$\hat{W}^*(\theta)\equiv\hat{W}(\theta;\pi^{x*}),$$

and

$$\hat{P}^*\left(\theta\right)\equiv\left(\hat{p}_1^{*1-v}\hat{N}^*\left(\theta\right)+\hat{p}_0^{*1-v}(1-\hat{N}^*\left(\theta\right)\right)^{\frac{1}{1-v}}.$$

Dropping the state θ from the argument of each function, as well as all the arguments of the fiscal policy, so as to ease the exposition, we have that firm i's market valuation (i.e., its payoff) is equal to

$$\bar{\Pi}_i(\pi_i^x)\equiv\sup_{\varsigma:\mathbb{R}\to[0,1]}\Pi_i(\varsigma;\pi_i^x),$$

where

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x)&\equiv\mathbb{E}\left[\hat{C}^{*-R}\left(\hat{r}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{r}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]\\&-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]\\&+\mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]-k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right]-\mathcal{I}(\pi_i^x),\end{aligned}$$

with $\bar{n}(\pi_i^x;\varsigma) \equiv \int \varsigma(x) d\Phi(x|\theta,\pi_i^x)$ denoting the probability that firm i adopts the new technology when using the strategy $\varsigma : \mathbb{R} \to [0,1]$, and $\hat{T}_1^*$ and $\hat{T}_0^*$ denoting the transfers received when generating (real) revenues $\hat{r}_1^* = \hat{p}_1^*\hat{y}_1^*/\hat{P}^*$ and $\hat{r}_0^* = \hat{p}_0^*\hat{y}_0^*/\hat{P}^*$ under the new and the old technology, respectively. Using (S.18), we have that $\hat{r}_f^* = \hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}$ for $f = 0,1$. Hence,

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) =& \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\hat{y}_1^{*\frac{v-1}{v}}\bar{n}(\pi_i^x;\varsigma) + \hat{y}_0^{*\frac{v-1}{v}}(1-\bar{n}(\pi_i^x;\varsigma))\right)\right]\\ &- \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{l}_0^*(1-\bar{n}(\pi_i^x;\varsigma))\right)\right]\\ &+ \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{T}_0^*(1-\bar{n}(\pi_i^x;\varsigma))\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).\end{aligned}$$

Using

$$\hat{y}_1^* = \gamma\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_1^{*\psi}, \tag{S.28}$$

$$\hat{y}_0^* = \Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_0^{*\psi}, \tag{S.29}$$

and

$$\hat{l}_1^* = \gamma^{\varphi}\hat{l}_0^*, \tag{S.30}$$

we have that

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) =& \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right] +\\ &- \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^*\right] +\\ &+ \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{T}_0^*(1-\bar{n}(\pi_i^x;\varsigma))\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).\end{aligned}$$

Accordingly, the marginal effect of a change in π_i^x on firm i's objective is given by

$$\begin{aligned}\frac{\partial\Pi_i(\varsigma;\pi_i^x)}{\partial\pi_i^x} =& \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right] +\\ &- \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\right] +\\ &+ \mathbb{E}\left[\hat{C}^{*-R}\left(\frac{\hat{T}_1^*-\hat{T}_0^*}{\hat{P}^*}\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right] - k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right] - \frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x},\end{aligned} \tag{S.31}$$

where $\partial\bar{n}(\pi_i^x;\varsigma)/\partial\pi_i^x$ is the marginal effect of varying π_i^x on the probability that the firm adopts the new technology at θ, holding fixed the rule ς.

Next, recall again that, for $f = 0,1$,

$$\hat{r}_f^* \equiv \frac{\hat{p}_f^*\hat{y}_f^*}{\hat{P}^*} = \hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}.$$

Using (S.28) and (S.29), we have that

$$\hat{r}_1^* - \hat{r}_0^* = \hat{C}^{*\frac{1}{v}} \Theta^{\frac{v-1}{v}} \left(1+\beta\hat{N}^*\right)^{\alpha\frac{v-1}{v}} \left(\gamma^{\frac{v-1}{v}} \hat{l}_1^{*\psi\frac{v-1}{v}} - \hat{l}_0^{*\psi\frac{v-1}{v}}\right).$$

Therefore, using (S.30) and the structure of the proposed fiscal policy, we have that

$$\hat{T}_1^* - \hat{T}_0^* = s + \frac{1}{v-1}\hat{C}^{*\frac{1}{v}} \left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} (\gamma^{\varphi}-1)\, \hat{l}_0^{*\psi\frac{v-1}{v}}.$$

Substituting this expression in (S.31), we obtain that

$$\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}} \left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} (\gamma^{\varphi}-1)\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right] +$$
$$- \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left((\gamma^{\varphi}-1)\,\hat{l}_0^*\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right)\right] + \mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right] - k\mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right] - \frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}.$$

Next recall that, when $\pi_i^x = \pi^{x*}$, the optimal investment strategy is the efficient one, i.e., $\varsigma = \hat{n}^*$, where $\hat{n}^*(x) \equiv \hat{n}(x;\pi^{x*})$ is the efficient technology choice for a firm receiving signal x after acquiring information of precision π^{x*}. Using the envelope theorem, we thus have that

$$\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial \Pi_i(\hat{n}^*;\pi^{x*})}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}} \left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} (\gamma^{\varphi}-1)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right] +$$
$$- \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left((\gamma^{\varphi}-1)\,\hat{l}_0^*\frac{\partial \hat{N}^*}{\partial \pi^x}\right)\right] + \mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial \hat{N}^*}{\partial \pi^x}\right] - k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right] - \frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x},$$

where $\partial \hat{N}^*/\partial \pi^x$ is the marginal change in the measure of firms adopting the new technology that obtains when one changes π^x at $\pi^x = \pi^{x*}$, holding the strategy $\hat{n}^*$ fixed. Note that, in writing the expression above, we use the fact that, when $\varsigma = \hat{n}^*$, $\bar{n}(\pi_i^x;\varsigma) = \hat{N}^*$, which implies that

$$\frac{\partial \bar{n}(\pi_i^{x*};\hat{n}^*)}{\partial \pi_i^x} = \frac{\partial \hat{N}^*}{\partial \pi^x}.$$

For the fiscal policy to induce efficiency in information acquisition (when paired with the monetary policy in the proposition), it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. Given the derivations above, this requires that

$$\frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}} \left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} (\gamma^{\varphi}-1)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right] +$$
$$- \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left((\gamma^{\varphi}-1)\,\hat{l}_0^*\frac{\partial \hat{N}^*}{\partial \pi^x}\right)\right] +$$
$$+ \mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial \hat{N}^*}{\partial \pi^x}\right] - k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right] = \frac{\partial \mathcal{I}(\pi^{x*})}{\partial \pi^x}. \quad \text{(S.32)}$$

Next, use (S.22) and (S.30) to note that

$$\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}=\left(\hat{l}_1^*\hat{N}^*+\hat{l}_0^*\left(1-\hat{N}^*\right)\right)^{\varepsilon}=\hat{l}_0^{*\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)^{\varepsilon}.$$

Hence, using the fact that $\hat{C}^{*\frac{1-vR}{v}}=\hat{C}^{*1-R}\hat{C}^{*\frac{1-v}{v}}$, along with the fact that, as shown in the proof of Proposition S.1,

$$\hat{C}^*=\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_0^{*\psi}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)^{\frac{v}{v-1}},$$

we have that

$$\hat{C}^{*\frac{1-vR}{v}}=\hat{C}^{*1-R}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{1-v}{v}}\hat{l}_0^{*\psi\frac{1-v}{v}}\frac{1}{\left(\gamma^{\varphi}-1\right)\hat{N}^*+1}.$$

It follows that (S.32) is equivalent to

$$\mathbb{E}\left[\frac{v\left(\gamma^{\varphi}-1\right)\hat{C}^{*1-R}}{(v-1)\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)}\frac{\partial\hat{N}^*}{\partial\pi^x}\right]+$$
$$-\mathbb{E}\left[\hat{l}_0^{*1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\right]+$$
$$+\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial\hat{N}^*}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^*}{\partial\pi^x}\right]=\frac{\partial\mathcal{I}(\pi^{x*})}{\partial\pi^x}. \quad \text{(S.33)}$$

Recall that the efficient precision of private information π^{x*} solves

$$\mathbb{E}\left[\hat{C}^{*1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}^*}+\frac{v\left(\gamma^{\varphi}-1\right)}{(v-1)\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)}\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\right]$$
$$+\mathbb{E}\left[\hat{l}_0^{*1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N^*+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^*}{\partial\pi^x}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi_x}. \quad \text{(S.34)}$$

Comparing (S.33) with (S.34), we have that, for the policy T to implement the efficient acquisition and usage of information (when paired with the monetary policy in the proposition, which, by virtue of Lemma S.1, is the only monetary policy that can induce efficiency in both information usage and information acquisition), the subsidy s to the innovating firms must satisfy the following condition

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right]=\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^{x*})}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right],$$

where we reintroduce the arguments of the various functions.

Finally, note that, independently of whether the economy satisfies the conditions in Proposition S.1, when the subsidy to the innovating firms is equal to

$$s(\theta)=\frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})}$$

in each state, then, as shown in part 1, the private value $\mathcal{R}$ that each firm assigns to adopting the new technology coincides with the social value $\mathcal{Q}$ in each state, implying that the firm finds it optimal to acquire the efficient amount of private information and then uses it efficiently when expecting all other firms to do the same. This establishes the claim in the proposition. Q.E.D.