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Abstract

Central banks can operate with negative equity, and many have done so in history without undermining trust in fiat money. However, there are limits. How negative can central bank equity be before fiat money loses credibility? We address this question using a global game approach motivated by the fall of the Bank of Amsterdam (1609–1820). We solve for the unique break point where negative equity and asset illiquidity render fiat money worthless. We draw lessons on the role of fiscal support and central bank capital in sustaining trust in fiat money.

Key words: central banks, negative equity, fiat money, trust

JEL Codes: E42, E58, N13

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I. Introduction

Money is a social convention. One party accepts it as payment in the expectation that others will also do so in the future. But at what point do these expectations break down, causing money to lose its value? In this paper, we identify the limits beyond which trust in fiat money falls away. Our analysis is motivated by the fall of the Bank of Amsterdam (1609-1820), perhaps the best known of the public deposit banks in Europe. The Bank of Amsterdam was arguably an early precursor to a modern central bank in that it issued fiat money and conducted monetary policy to stabilise its value. By examining what it takes for an issuer of fiat money to fail, we draw lessons on the central bank underpinnings of the institution of money. These lessons have resonance and relevance even today.

How then does a central bank fail? Indeed, how *could* a central bank fail when it can always "print" more money? It is well-known that central banks can operate with negative equity, and many have done so throughout history without undermining trust in fiat money (eg Stella 1997; Archer and Moser-Boehm 2013). However, this is not to say that there are no limits. We can pose the question in the following, more precise way: how negative can central bank equity be before fiat money loses credibility?

In our model, failure refers to the central bank's inability to achieve its public policy objective. Ours is not a bank run model; we are not looking at the default of a private institution faced with a liquidity shock (Diamond and Dybvig 1983). Neither is it a currency crisis model in which speculators decide whether they should attack a currency (Krugman 1979; Morris and Shin 1998). Nor is it about a lack of central bank independence and commitment to its policy objective (Obstfeld, 1995). Instead, our model is crucially about money demand, and the central bank's ability to adjust the money supply in response to changes in money demand. If it lacks the room for manoeuvre to do so, its policy objective could be at risk. The failure to achieve its policy objective may imply that it will ultimately be replaced by another institution, as happened in 1820, when the Bank of Amsterdam was shut down and succeeded by today's central bank of the Netherlands, De Nederlandsche Bank.

We address the limits of fiat money using a global game approach motivated by the features that eventually led to the failure of the Bank of Amsterdam.[1] In our model, the value to users of bank money (ie the deposits issued by the Bank of Amsterdam) depends on its value in settlement of wholesale trade transactions, and therefore on the volume of trade and the general buoyancy of the economy. In the face of a negative shock to the economy that reduces the value to users, there is reduced demand for bank money. The network effects in the use of bank money further amplify the decline in the value to users of holding bank money. Other things equal, the excess supply of bank money would put downward pressure on the exchange rate (the "agio").

In principle, the Bank of Amsterdam could respond to the negative shock by reducing the money supply to restore the agio to the desired level. It could do so by selling local coins in the daily open market and debiting the accounts of the buyers, thereby reducing the money supply. However, in the presence of illiquid loans on the balance sheet, there is a hard limit to the reduction in the money stock. The hard limit binds more as losses mount and negative equity eats into the asset value of the Bank. Once the Bank has sold all the liquid assets (the coins), it only has illiquid assets (the loans). The sales needed to stabilise the agio cannot go further, as there is no more capacity for a reduction in the deposits. We solve for the unique "break point" of the global game where negative equity and asset illiquidity renders fiat money worthless.

Three key features stand out from our model, which resonate even for debates of today.

First, while the network effects of fiat money allow monetary regimes to persist for

[1]The model in this paper complements empirical estimations of the long-term negative effect of greater lending by the Bank on confidence in bank money; see Frost, Shin and Wierts (2020).

quite some time, there are limits to how resilient such arrangements can be. Being able to issue fiat money gives the central bank considerable latitude to leverage up its balance sheet without loss of confidence in the value of money. Yet the Bank of Amsterdam's failure is a vivid lesson in how a central bank that loses public trust can push its luck too far. When it goes beyond the threshold – or break point – it fails.

Second, this break point binds harder when central bank equity becomes more negative and when economic fundamentals are weaker. Crucially, the Bank of Amsterdam did not receive fiscal support from a sovereign with (adequate) power to tax. The ultimate backing for the value of money is the fiscal sustainability of the consolidated public sector, consisting of the central bank and fiscal authorities (Sims 1994; Cúrdia and Woodford 2011; Reis 2015). In this sense, fiat currencies need backing, and modern central banks need the credible fiscal support from the government that flows from the sustainability of public finances. The credibility of fiat money may be at risk if holders of fiat money doubt the willingness of the fiscal authority to recapitalise the central bank in times of need. The loss of confidence manifests itself in a switch in the portfolio of monetary instruments used in the economy. In our model, the loss of trust results in a coordinated switch away from Bank money to metal coins. In a modern context such a shock could occur through dollarisation, as we have seen in the case of many emerging market economies, or potentially through "cryptoisation", where the portfolio decision in the money system tilts towards modern digital assets such as cryptocurrencies or private stablecoins.

Third, our global game model delivers sharp predictions on the nature of the break point. As the fundamental uncertainty among investors dissipates, the relationship between the state of the economy and trust in fiat money approaches a "step function": there is a discrete jump at the break point below which fiat money becomes worthless.

By drawing on this historical episode about the Bank of Amsterdam, and the relatively simple period of competition between fiat money and alternatives, we can shed light on the economics underlying fiat currency.[2] Our main contribution is the modelling exercise on how trust in fiat money can be lost. For this, we specify a portfolio choice problem between competing forms of money in the presence of network effects. We argue that our results have a bearing on much broader issues of central bank solvency and the governance of fiat money.[3]

The rest of this paper is organised as follows. Section II briefly introduces the Bank of Amsterdam and its demise. In section III, we present our global game model of the Bank's downfall and the collapse of bank money as a unique equilibrium outcome. Section IV draws implications for modern central banks, highlighting the conditions under which a loss of trust may or may not occur. Section V concludes.

II. The Bank of Amsterdam and its Downfall

The Bank of Amsterdam (*Wisselbank*, or "Exchange Bank") was a public giro or payments bank owned by the municipality of Amsterdam. The Bank was founded in the context of a large number of circulating metal coins in the early 17th century, and the debasement of those coins by the deliberate mixing of base metals into gold and silver coins (Kindleberger and Aliber 2005; Schnabel and Shin 2004, 2018). In the Bank's founding decree, it was given a mandate to 'check all agio (of the current

[2] There are other historical examples of public deposit banks that failed, such as Stockholms Banco (see Edvinsson et al 2018). Yet the international role of the Bank of Amsterdam, and its substantial institutional evolution over time, make it a much more relevant test case for broader issues of trust in fiat money.

[3] Our paper bears similarities to Schilling et al (2020). They consider a run on a central bank that issues a central bank digital currency (CBDC) in a Diamond and Dybvig framework, with an additional price stability objective for the central bank. In their model there are no alternatives to CBDC as a means of payment. The central bank can always deliver on its obligation by printing more money, but at the cost of sacrificing its inflation target. Instead, we focus on the case where alternative means of payment do exist, and the central bank needs to shrink its balance sheet when facing a decline in the demand for fiat money.

money) and confusion of coin, and to be of use to all persons who are in need of any kind of coin in business'.[4]

The early Bank of Amsterdam resembled what we now know as a "stablecoin" – where account-based money is backed by assets of stable value. Indeed, customers would physically deposit metal coins with the Bank and account balances were recorded in a central ledger. These deposit balances could be transferred to other account holders without cost, or withdrawn for a small fee.[5]

A. Proto-Central Bank

Over time, the Bank departed from the strict application of full backing – without initially undermining its credibility. A key date is 1683, when the Bank ended the policy of redeemability of deposits into coin (Uittenbogaard 2009, Quinn and Roberds 2014). In this sense, the Bank started issuing fiat money. This change was crucial for the role the Bank would play at the heart of the international payment system.

However, the shift from a "rigid" to an "elastic" structure was not a complete shift. At the same time as removing the redeemability of bank deposits into coin, the Bank introduced a separate "receipt" system that allowed coin holders to sell their high-quality (metal) trade coins to the Bank. They then had the option to repurchase the same coins after a fixed period – typically six months – for a small fee ($\frac{1}{4}$ percent for silver coins and $\frac{1}{2}$ percent for gold coins). During this period, the coin sellers would have a deposit claim at the Bank, and the trade coins under receipt would be earmarked –

[4]The agio referred at the time to the premium on different types of currency, ie the difference between the rate of exchange in the market and the nominal value. As will be shown later, the agio or premium on Bank guilders relative to current guilders came to be an important indicator of confidence in the stability of the Bank of Amsterdam.

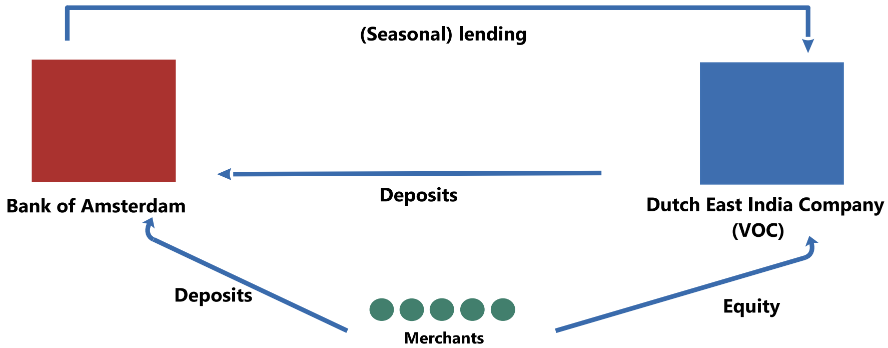
[5]The term "stablecoin" first entered the lexicon of monetary economics through Facebook's Libra proposal in 2019. Stablecoins are private cryptocurrencies (or "crypto-assets") that seek to maintain a stable value against assets or fiat currencies. See eg Arner et al (2020), FSB (2020), and Gorton and Zhang (2023).

"encumbered" – for potential withdrawal. Instead, (metal) local coins such as current guilders were not eligible for these swaps – they were "unencumbered". These local coins were used for day-to-day payments and could be bought and sold for Bank money in the daily open market.

With this key policy change, the bank effectively moved from a rigid to an elastic stablecoin, combining redeemability with fiat money. As part of its monetary operations, the Bank of Amsterdam engaged in asset purchases and sales in the daily open market to stabilise the value of the agio. The Bank expanded the money stock by purchasing coins – and thus crediting the seller's account – when the agio rose. It contracted the money stock by selling coins – and debiting the buyer's account – when the agio fell. These market operations resemble a modern central bank that changes base money through an asset purchase programme by quantitative easing (QE) or tightening (QT). Specifically, the Bank of Amsterdam sought to keep the agio of Bank guilders to current guilders in a target range between 4 and 5%, and thus to ensure Bank guilders could serve as a stable unit of account (van Dillen 1925, pp 433-4; Quinn and Roberds 2019, p 751).

The Bank of Amsterdam maintained a close relationship with another key institution of the time – the Dutch East India Company (*Verenigde Oostindische Compagnie*, VOC). The VOC, founded in Amsterdam in 1602, is often considered the world's first joint-stock company, and played a crucial role in European trade with Asia for nearly two centuries. Shareholders of the VOC, which included the largest merchants of the day, were also among the largest depositors at the Bank. In ports in the Netherlands and elsewhere in Europe, the VOC would load ships with precious metal coins from the mines of the New World, and exchange these for goods from Asia (de Vries 2003). Given the seasonal patterns of trade by the VOC, the Bank regularly lent to the VOC to provide settlement liquidity for the wholesale payment system. Finally, the VOC itself

Figure 1: Selected relationships between the Bank and the VOC



Source: authors' elaboration.

held deposits at the Bank, and made wholesale transactions in bank money. Figure 1 shows selected relationships between the two institutions.

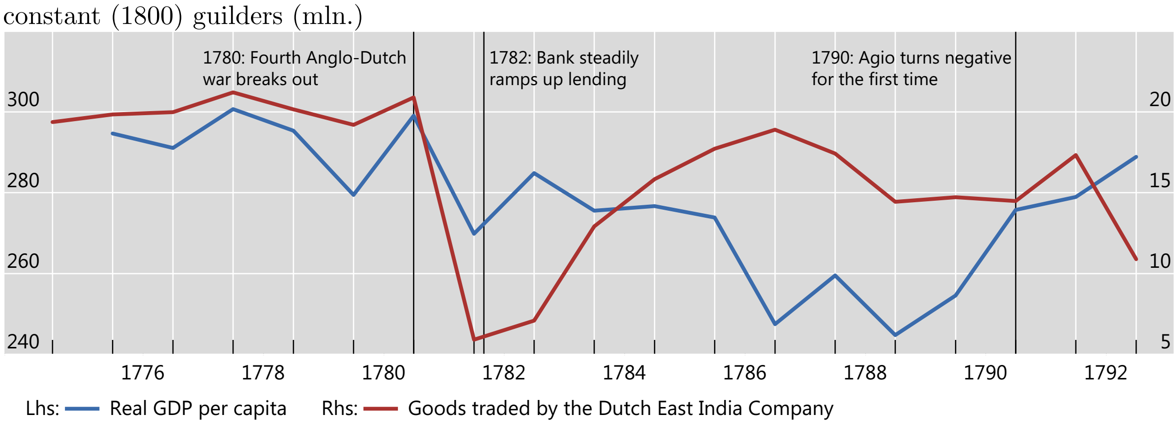
B. Downfall of the Bank of Amsterdam

The resilience of the Bank of Amsterdam and its success over many decades came under increasing strain in the late 1770s. Under the economic pressures generated by war with the English, the Bank departed more seriously from sound practice by lending on a more substantial scale to the VOC, in a sustained and non-transparent way.

Crucially, the Bank of Amsterdam lacked fiscal support. While the Bank's public sector ownership by the city of Amsterdam gave it some degree of financial backing from the city tax authorities (and also the ability to mutualise losses across segments of Amsterdam society), this was not sufficient for the large scale of activities of the Bank, given the large volume of international trade through Amsterdam. The Bank operated with slightly negative equity for most of its existence, but as a result of weak governance it did not have proper safeguards for when equity turned more deeply negative.[6]

[6]The weak governance of the Bank meant that the safeguards and operational structure needed to support a durable fiat currency were sorely lacking. Janssen (2015) and van 't Hart (2009) relate how the close relationships between the Amsterdam municipal authorities, the VOC and the commissioners

Figure 2: Output and trade in the Netherlands during the Bank's downfall, 1775-92



Source: van Zanden and van Leeuwen 2012; KNAW Huygens; authors' calculations.

The pivotal event was the Fourth Anglo-Dutch war (1780-84). This military conflict posed a grave economic shock and strained the VOC, which had become the main borrower of the Bank of Amsterdam. Shipping volumes by the VOC fell dramatically; sales of trade goods in the Netherlands dropped from 20.9 million guilders in 1780 to only 5.9 million in 1781 (Figure 2).

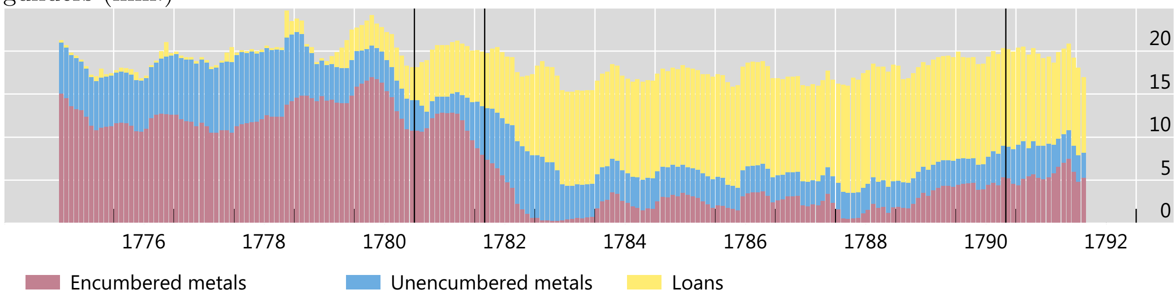
The drop in revenues and loss of many ships imposed catastrophic financial and operational losses for the VOC. Loans that were already extended could no longer be repaid. Yet throughout 1782, the Bank steadily ramped up its lending to the VOC; outstanding loans rose to a peak of 7.8 million guilders in February 1783. As loans grew (to a full 71% of the Bank's assets), the metal stock fell, from 17.6 million guilders in 1776 to 7.8 million in 1783 (Figure 3). This was because Bank account holders with receipts withdrew their (encumbered) trade coins by exercising their right to redeem (Figure 4).

During the first half of 1783, the Bank responded to downward pressure on the agio by selling 3.5 million worth of guilder coins into the market (Quinn and Roberds 2016). By the summer of 1783, guilders were now only backed by metal coins for 28% of their

of the Bank made the latter susceptible to pressure to act in disregard of its charter.

Figure 3: Asset backing of the Bank during its downfall, 1775-92

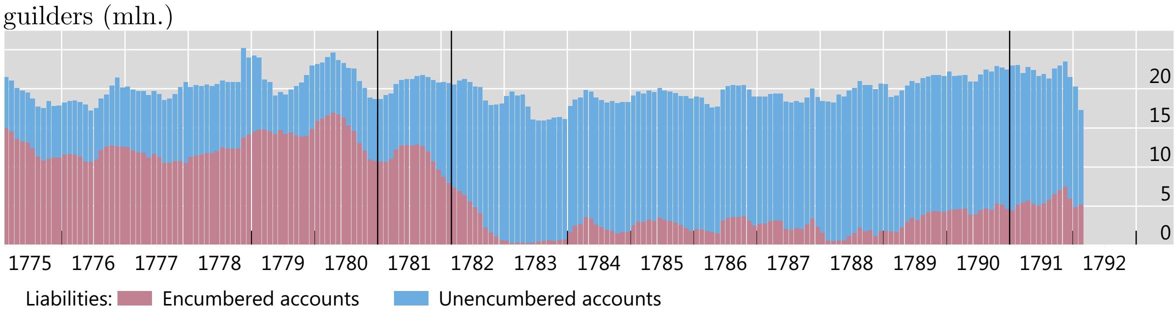
guilders (mln.)



Source: van Dillen 1934; Quinn and Roberts 2016; authors' calculations.

Figure 4: Bank liabilities: encumbered vs unencumbered accounts, 1775-92

guilders (mln.)

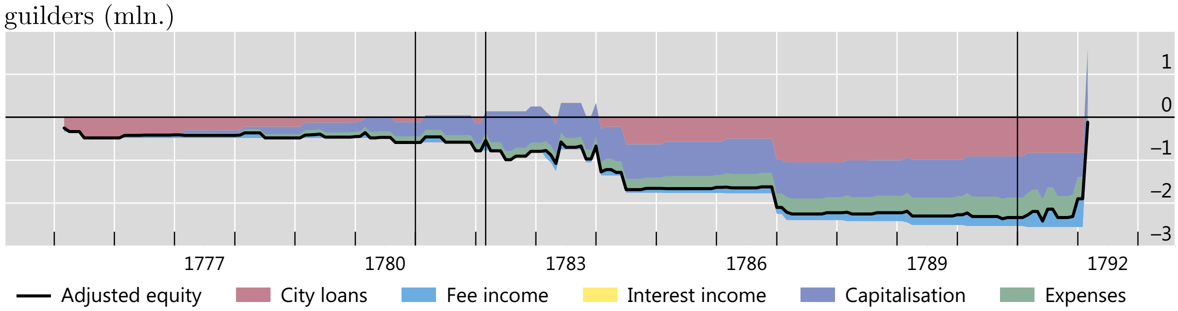


Note: Encumbered accounts refer to those where account holders held a receipt entitling them to redeem their coin after a fixed period (typically six months). Unencumbered accounts were those without a redeemability option. The Bank typically created balances in unencumbered accounts through purchases of coins in open market operations, and by granting loans. Source: Quinn and Roberts 2016; authors' calculations.

value, from 97% just four years earlier. With the conclusion of the war in May 1784, the Bank had accumulated a large credit exposure which soon became non-performing.

The Bank's insolvency – and the inability of the city authorities to recapitalise it – are important elements in its downfall. The Bank's income sources comprised mainly fees from the receipt system, gains on sales of coin, and interest margins on loans. However, while the loans to the VOC became non-performing, the bank had not been rebuilding capital to cover these losses, as profits were regularly distributed to the city. Moreover, it did not have seigniorage income of modern central banks, nor

Figure 5: Adjusted equity of the Bank, 1775-1792



Source: Quinn and Roberds 2016; authors' calculations.

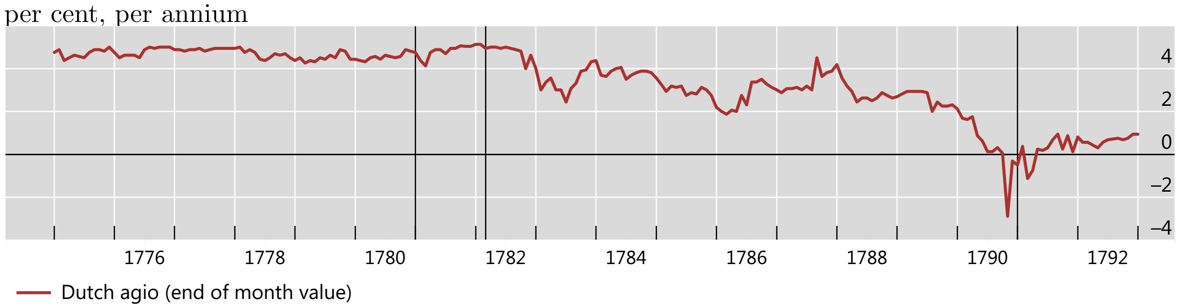
an adequate fiscal backstop.[7] The city of Amsterdam did make limited attempts to recapitalise the Bank, but the funds were quickly diverted back to city coffers (Quinn and Roberds 2016). From the perspective of modern central banking theory, the City of Amsterdam's fiscal capacity was insufficient to provide the sovereign backing of an institution that had become a proto-central bank.

The extent of lending exposures remained opaque for a further decade. Market developments, as indicated by the agio, suggest that market participants were sceptical about the full solvency of the Bank of Amsterdam and started to question the value of Bank money. In July 1789, as the Bastille was stormed in Paris and uncertainty spread across Europe, the agio on the Bank guilder dropped to 2%, and eventually turned negative in October 1790–February 1791 (Figure 6).

The final chapter came in 1795, after the invasion of the Netherlands by French revolutionary armies. It was then that the true extent of the Bank's insolvency came to light. The new authorities decreed that the Bank's accounts would be made public, revealing the low metal stock. The agio on Bank guilders dropped to nearly -30% on the revelation. From 1795 to 1820, the Bank lived on as a severely weakened institution.

[7] As the Bank of Amsterdam did not hold a portfolio of government securities, nor did it issue circulating bank notes, this meant that the Bank did not have access to the most common source of revenue for modern central banks, which is seigniorage income from notes backed by government bonds.

Figure 6: Bank agio during the period of analysis, 1775-92



Source: van Dillen (1934); Quinn and Roberds (2016); authors' calculations.

After William, Prince of Orange-Nassau, proclaimed himself King William I in 1813, he founded the De Nederlandsche Bank in 1814, today's central bank of the Netherlands (Vanthoor 2006; Uittenbogaard 2015). The Bank of Amsterdam was finally closed in 1820.

The economic fallout from the war, compounded by the downfall of the VOC and the failure of the Bank of Amsterdam were severe. Income per capita fell by 17% between 1794 and 1807 (van Zanden and van Leeuwen 2012). The Bank guilder lost its role in international finance, and the centre of gravity in European finance shifted definitively to London (Carlos and Neal 2011).

III. Model

To better understand the Bank's downfall, we build a global game model. At the heart of our model is the portfolio decision of merchants who decide on their holdings of bank money and coins. "Bank money" refers to deposit balances maintained at the Bank of Amsterdam. A payment using bank money is settled when the Bank of Amsterdam debits the account of the payer and credits the account of the receiver, much like the modern monetary system based on high-powered money issued by the central bank. Merchants are analogous to modern-day commercial banks in this regard; merchants held accounts at the Bank of Amsterdam just as modern commercial banks hold accounts at the central bank. In our model, merchants derive value from bank money because it gives them access to the wholesale payment system. Specifically, it allowed clearing and settlement of financial instruments such as bills of exchange that underpinned supply chain finance for international trade (Schnabel and Shin 2004, 2018).

We assume that the value of the flow of services from bank money is increasing in the strength of overall economic activity – the "buoyancy" or "fundamentals". Moreover, we assume that the value derived from holding bank money is subject to network effects, ie the value derived by a particular merchant is increasing in the total stock of bank money held by other merchants. The rationale is that the network effects of money will enhance the coordination value of using a common means of settlement for transactions.[8] The merchant's alternative to (account-based) bank money is circulating local (metal) coins. These can be used to make daily payments, but they are more cumbersome for settling trade transactions. Key to our model is that the network effects are stronger for bank money as compared with coins.

The monetary policy objective of the Bank of Amsterdam is to maintain a stable premium (or agio) of bank money over coins. Equivalently, we may view the monetary policy objective as maintaining a target exchange rate between Amsterdam bank money and coins. This objective is met through market operations by selling or buying assets in the daily open market so that bank money supply is set equal to bank money demand at the target agio.

Concretely, the balance sheet of the Bank of Amsterdam has coins C and (illiquid)

[8]Early discussions on the network properties of money date back to Jevons (1875) and Menger (1892). In a seminal paper, Jones (1976) focuses on the cost externality of searching for complementary trading partners to explain the emergence of media of exchange. See also eg Lewis (1969), Ostroy and Starr (1974) and Kiyotaki and Wright (1989).

loans L as assets, and bank money M and equity E as liabilities. The balance sheet identity is:

$$C + L = M + E. \tag{1}$$

The policy objective of the Bank of Amsterdam is to maintain the agio at the target level by adjusting M through the purchase and sale of coins C. The purchase of an asset is paid for by creating a deposit for the seller of the asset, and conversely, the sale of an asset entails the debit of the buyer's account. In this way, asset purchases and sales are mirrored by the change in the stock of bank money, and the Bank can adjust the money stock so that the market clearing price of bank money aligns with the target agio.

Naturally, the details of the monetary policy operations pursued by the Bank of Amsterdam were more complex than depicted in our stylized account above. In our model, for simplicity, we concentrate on the buying and selling of local coins by the Bank in the daily market, abstracting from other institutional details (see Quinn and Roberds 2014, 2016).[9]

A. Money Demand

There are three dates $\{0, 1, 2\}$, two types of money (bank money and coins), and a continuum [0,1] of risk-neutral merchants. Coins are the numeraire in our model with a unit price of 1, and bank money has price p.

We assume that economic fundamentals Θ (the "state of the economy") are log-normally distributed, and $\theta = log\,\Theta$ is normally distributed with mean y and standard deviation $1/\sqrt{\alpha}$. These features are common knowledge. We may interpret the dy-

[9] We abstract from the so-called receipt system which operated as a (quasi-)repurchase standing facility whereby large merchants could swap trade coins for bank money. Our model applies to the case where all encumbered trade coins that carried a receipt had been repurchased so that the Bank of Amsterdam was left with only illiquid loans and local coins on the asset side of its balance sheet. We thank one of the referees for this point.

namic process of economic fundamentals $\{\theta_t\}$ as following a random walk with Gaussian increments, where $E(\theta_t) = \theta_{t-1} = y$.

At date 0, merchants are born, and each is endowed with one unit of wealth. The Bank of Amsterdam sets the target price of bank money in terms of coins at $\bar{p} = 1 + \bar{\gamma}$, where $\bar{\gamma} > 0$ denotes the target agio.

At the beginning of date 1, before merchants make their portfolio decisions, the state of the economy θ is drawn. The Bank of Amsterdam observes the state θ, but merchants do not. Instead, each merchant i bases his portfolio decision on his own type v_i:

$$v_i = \theta + \varepsilon_i, \tag{2}$$

where θ is the state of the economy and $\{\varepsilon_i\}$ are independent and identically distributed normal random variables with mean 0 and standard deviation $1/\sqrt{\beta}$. Merchant i's type v_i consists of a common component θ and an idiosyncratic component ε_i. Each merchant knows his own type, but he does not observe θ and ε_i separately.

After the merchants learn their own type, they choose their portfolio between coins and bank money. Merchant i's gross utility of holding one unit of bank money is given by:

$$u_i(m) = v_i \cdot f(m), \tag{3}$$

where $f(m)$ is a bounded and increasing function of aggregate bank money holding $m \in [0, 1]$. We assume that $f(0) > 0$. Merchant i's utility of holding a unit of coins is normalised to 1. Since merchants are risk-neutral, it is without loss of generality to consider the binary action game where a merchant either holds coins only or bank money only. Merchant i holds bank money if

$$\mathbb{E}\left(\frac{u_i(m)}{1 + \bar{\gamma}}\right) \geq 1, \tag{4}$$

and otherwise holds coins. The aggregate bank money holding m is given by the *proportion* of merchants who hold bank money.

Finally, in period 2, the state of the economy is revealed to all. The agio is determined as the market-clearing price of bank money when the demand for bank money equals supply. Moreover, loans mature and merchants derive their utility from either bank money or coins.

The network effects associated with holding bank money introduce strategic complementarities whereby the attractiveness of holding bank money is increasing in the extent to which others hold bank money. With complete information, multiple equilibria may arise in the manner of Diamond and Dybvig (1983). Instead, we employ global game methods to derive a unique equilibrium (Morris and Shin 1998, 2003; Goldstein and Pauzner 2005).

B. Unique Equilibrium

Our model defines a global game in *private values*, implying that each player conditions his strategy on his own valuation of bank money and infers the strategies of others from the joint distribution over valuations (Morris and Shin 2003).

We solve for the unique equilibrium, which turns out also to be dominance solvable. This means that the unique equilibrium can be obtained from the iterated deletion of strictly dominated strategies. The equilibrium is characterised by two equations in two unknowns: a break point θ^* for economic fundamentals, below which the agio fails to meet the target, and a marginal type of merchant v^* who separates the population into those who hold bank money and those who hold coins. Merchants with types above v^* hold bank money while merchants below v^* hold coins.

To solve for the unique equilibrium we proceed in several steps.

As a first step, we confine attention to *switching strategies* whereby merchant i

holds bank money if and only if $v_i \geq v^*$ for some threshold v^*. We show that there is a unique threshold value v^* such that the marginal type of merchant v^* is indifferent between holding bank money and coins. This marginal type v^* satisfies the indifference condition:

$$\frac{v^*}{1+\bar{\gamma}} \cdot F(v^*, y) = 1, \tag{5}$$

where $F(v^*, y)$ is the expected value of $f(m)$ conditional on type v^* and ex ante mean y. In other words,

$$F(v^*, y) = \int_0^1 f(m)\, g(m \,|\, v^*, y) dm, \tag{6}$$

where $g(\cdot \,|\, v^*, y)$ denotes the posterior density over aggregate money holdings m conditional on v^* and y. Observe that the left-hand side of (5) is the expected utility of holding bank money, while the right-hand side is the utility of holding coins.

Let us denote by $G(\cdot \,|\, v^*, y)$ the cumulative distribution function corresponding to the density $g(\cdot \,|\, v^*, y)$. We can now derive the following closed-form expression for G, which is a key second step in our solution.

Lemma 1 *Suppose that all merchants use switching strategies around v^*. Then the cumulative distribution function over aggregate money holdings conditional on v^* and y is given by $G(m \,|\, v^*, y)$, where*

$$G(m \,|\, v^*, y) = \Phi\left(\frac{\alpha}{\sqrt{\alpha+\beta}}(v^* - y) + \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(m)\right). \tag{7}$$

The proof of Lemma 1 is presented in the online appendix.

The closed-form expression for G enables us to solve for the marginal type v^* in the limiting case where the idiosyncratic component of v_i becomes negligible relative to the common component. Specifically, given α, in the limiting case where $\beta \to \infty$, observe that $G(m \,|\, v^*, y)$ converges to the identity function in (7) so that the probability density function over aggregate bank money holdings goes to the uniform density. Hence, in the limit where $\beta \to \infty$, equation (5) becomes

$$\frac{v^*}{1+\bar{\gamma}} \cdot F = 1, \tag{8}$$

where $F = \int_0^1 f(m)dm$, which is invariant to v^* or y. Hence,

$$v^* = (1+\bar{\gamma})\,/F. \tag{9}$$

By continuity, for any α, there is a β sufficiently large such that there exists a unique solution for v^* that solves (5).

As a third step, we now turn to finding the break point θ^* for economic fundamentals below which the agio fails to meet the target. Conditional on the realisation of θ, merchant types $\{v_i\}$ are independently and normally distributed with mean θ and standard deviation $1/\sqrt{\beta}$. Therefore, given the state of the economy θ, if all merchants follow the switching strategy around v^*, then money demand $D(\theta)$ is given by:

$$D(\theta) = \Pr(v_i \geq v^* \,|\, \theta) = \Phi\left(\sqrt{\beta}(\theta - v^*)\right), \tag{10}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Clearly, the demand for bank money $D(\theta)$ is increasing in θ.

The monetary policy rule of the Bank of Amsterdam sets money supply equal to money demand so as to maintain the agio at its target $\bar{\gamma}$. Recall that the Bank of Amsterdam observes θ, and hence can condition on the realised value of θ.

Note, however, an important asymmetry in the conduct of monetary operations. The Bank of Amsterdam can always lower the agio through asset purchases, as there is no upper bound to the quantity of money that can be created by funding the purchase of assets (other than the availability of coins to buy). However, in order for the Bank of Amsterdam to *raise* the agio, it must have sufficient liquid assets to sell (ie coins in its own vaults) so as to reduce the stock of bank money through quantitative tightening. If there are illiquid loans on the balance sheet or equity turns sufficiently negative, there is a hard limit on the sale of assets to raise the agio. This restricts the Bank's scope to reduce its money supply. This hard limit determines the break point θ^*, which is defined as the value of θ below which the agio falls below the target $\bar{\gamma}$.

Using (10) and (5), the market clearing condition for bank money at state θ is:

$$\begin{aligned} M(\theta) &= D(\theta) \\ &= \Phi\left(\sqrt{\beta}\left(\theta - v^*\right)\right) \\ &= \Phi\left(\sqrt{\beta}\left(\theta - \left(1+\bar{\gamma}\right)/F(v^*, y)\right)\right). \end{aligned} \tag{11}$$

The break point is given by the value of θ where bank money demand D is equal to the total illiquid asset holdings L minus equity E. From the balance sheet identity (1), we must have $M \geq L - E$ since the holdings of coins are always non-negative, ie $C \geq 0$. In other words, once the Bank of Amsterdam runs out of coins ($C = 0$), the break point θ^* is the solution to:

$$\Phi\left(\sqrt{\beta}\left(\theta^* - v^*\right)\right) = L - E, \tag{12}$$

or

$$\theta^* = v^* + \frac{\Phi^{-1}\left(L - E\right)}{\sqrt{\beta}}. \tag{13}$$

From (13), the break point is increasing in L and decreasing in E, indicating that higher levels of illiquid assets or lower levels of equity can be counterbalanced only by stronger economic fundamentals. For the Bank of Amsterdam, which did not have explicit fiscal support, L was large and E was deeply negative. It thus could not respond in an effective way to the deterioration of economic fundamentals.

We are now ready to draw the strands together and state the first of two main propositions. Although a fairly standard result in global games, Proposition 2 underscores our simple "network idea" of money as a force of coordination among merchants. Indeed, joint modelling of the positive network effects of money, combined with the agents' portfolio choice problem, is a key innovation in our model of money demand.

Proposition 2 (Unique equilibrium) *For any α, there is a β sufficiently large such that there exists a unique equilibrium which is characterised by the joint solution (v^*, θ^*) to equations (5) and (13). This equilibrium is in switching strategies and is dominance solvable.*

The uniqueness of the threshold point v^* for sufficiently large β has already been demonstrated, and the solution for the break point θ^* follows from (13). What remains to complete the proof of Proposition 2 is to show that merchants with valuations above v^* strictly prefer to hold bank money while merchants with valuations below v^* strictly prefer to hold coins. These steps in the proof of Proposition 2 are given in the online appendix.

Finally, we need to complete the argument by showing that the equilibrium characterised by (5) and (13) is dominance solvable. Morris and Shin (2003) show that in a global game with *i)* strategic complementarities in payoffs and *ii)* the existence of "dominance" regions, the equilibrium in switching strategies also proves to be the only outcome that survives the iterated deletion of strictly dominated strategies. First, regarding strategic complementarity, the payoffs in our model are such that the payoff advantage to a merchant to holding bank money (relative to holding coins) is strictly increasing in the share of other merchants that holds bank money. Second, regarding dominance regions, we can show that for sufficiently low (high) merchant types it is strictly dominant to hold coins (bank money) even if all other merchants hold bank money (coins). Therefore, by appealing to the result of Morris and Shin (2003) on dominance solvability, we can complete our proof. The details are in the online appendix.

C. Deteriorating Fundamentals and Money Collapse

We now turn our attention to how public information of deteriorating economic fundamentals can hasten the break of the agio from the target level. The role of public information in influencing the feasibility of coordination was first explored in Morris and Shin (2002, 2004). The punchline here is that when the economy is in the doldrums, the agio may fail even though it would have survived had public information about the economic fundamentals been different. In other words, the bar for maintaining the agio is set higher when the public perception has already deteriorated.

Our focus is on the ex ante mean y of fundamentals θ. We draw lessons on the role of public information on fundamentals by examining the limiting case where $\alpha \to \infty$ and $\beta \to \infty$ but such that $\sqrt{\beta}/\alpha \to k$, for some constant $k > 0$. Observe from (13) that in this limit, the break point θ^* converges to the switching point v^*. The distribution function $G(m \,|\, v^*, y)$ converges to $G_k(m \,|\, v^*, y)$, with:

$$G_k(m \,|\, v^*, y) = \Phi\left(\frac{(v^* - y)}{k} + \Phi^{-1}(m)\right). \tag{14}$$

Our next proposition characterises the equilibrium in this limiting case with a focus on the comparative statics of the break point θ^*, with respect to the ex ante mean y. In this limit, the money demand function D becomes a step function, collapsing to zero below the break point θ^*. This implies that the market clearing price of bank money (relative to coins) also collapses to zero.

The comparative statics with respect to ex ante mean y are key. When y falls, fundamentals are commonly known to deteriorate. In this adverse economic environment, the break point for the monetary system, where the demand for bank money collapses, shifts upward. Hence, bad public information coming from a low value of y results in a more fragile monetary system. The same fundamentals θ that would have otherwise been consistent with the maintenance of the agio are now not strong enough to sustain the agio at the target level. The value of bank money then collapses to zero.

Proposition 3 (Break point) *There is a constant $k_0 > 0$ such that in the limit, as $\alpha \to \infty$ and $\beta \to \infty$ but such that $\sqrt{\beta}/\alpha \to k$, with $k > k_0$, there exists a unique equilibrium (v^*, θ^*). In this limit, the break point θ^* is a strictly decreasing function of y. Around this break point θ^*, the money demand function is a step function given by:*

$$D(\theta) = \begin{cases} 0, & \text{if } \theta < \theta^*, \\ 1, & \text{if } \theta \geq \theta^*, \end{cases} \tag{15}$$

and the market clearing price of bank money is a step function

$$p = \begin{cases} 0, & \text{if } \theta < \theta^*, \\ 1 + \bar{\gamma}, & \text{if } \theta \geq \theta^*. \end{cases} \tag{16}$$

The proof of Proposition 3 is given in the online appendix.

Even though $\beta \to \infty$, the information value of the ex ante mean y is preserved by the fact that α is also becoming large at the same time. The restriction that the ratio $\sqrt{\beta}/\alpha$ is finite in the limit ensures that the ex ante mean y retains a powerful force in shaping the equilibrium outcome, even when all merchants' valuations converge to the same point θ^*.

Finally, the restriction that $k > k_0$ ensures that there is a unique threshold point v^* that defines the marginal type. Hence, there is a unique equilibrium in switching strategies. We show in the online appendix that the threshold k_0 for k is given by:

$$k_0 = \frac{(1+\bar{\gamma})(f(1)-f(0))}{\sqrt{2\pi}f(0)^2}. \tag{17}$$

Proposition 3 highlights the important role played by public perceptions of fundamentals. The fact that others are fearful sets a more pessimistic tone to the coordination problem. As such, the same fundamentals that would otherwise have been consistent with the maintenance of the agio are now associated with the collapse of trust in fiat money.

IV. Implications for Modern Central Banks

Our model determines a unique break point where negative equity and asset illiquidity make fiat money worthless. However, we do not explicitly consider the possibility of recapitalisation of the fiat money issuer. In the presence of fiscal support, the granting of resources by the government would have the effect of easing the equity constraint. This would increase the scope for asset sales to reduce the money supply and to defend the exchange rate (the agio).

Banks are the modern day equivalent of the merchants in our model. Banks hold fiat money in the form of reserves at the central bank. Like the merchants, they face a portfolio decision with respect to fiat currency and its alternatives.[10] In our model, the agio is the relative price of fiat money versus its alternative, ie coins. The modern day equivalent of the relative price of fiat money differs across countries. For small open

[10] A distinction can be made between private digital alternatives and central bank digital currencies (CBDC) that would be a direct liability of the central bank and thus use the existing numeraire. A growing body of literature assesses the design of CBDCs and their potential effects on the monetary system and the structure of financial intermediation; see eg Schilling et al (2020) and Fernández-Villaverde et al (2021). Drawing on examples from monetary history, Bordo and Roberds (2023) argue that successful CBDCs need to combine microeconomic efficiency with macroeconomic credibility which may require some politically uncomfortable compromises.

and emerging economies, it would often be the US dollar,[11] while new alternatives may arise eg due to "cryptoisation" or the emergence of stablecoins.[12] A shift from fiat money would be visible in quantities and relative prices – especially exchange rates – which become a barometer for trust in fiat money. In line with our model predictions, episodes of dollarisation in past decades show that when this happens, it happens fast (Agur 2023). Such a rapid move could be very damaging to the overall economy.

Today, modern central banks face a challenging environment. After the great financial crisis of 2008, during the euro area crisis of 2010-2 and yet again in the wake of the Covid-19 pandemic of 2020, central banks loosened policy for an extended period – buying up assets and issuing new liabilities in the form of modern fiat money. These QE policies helped to stabilise financial markets, but they also had side effects on the financial positions of central banks. Many central banks have experienced a reduction in profits, and in some cases even losses and negative equity, due to the combination of large balance sheets and rising interest rates.[13]

Key insights of the model, related eg to the demand for fiat money, still apply to this situation. Fluctuations in demand for fiat money are reflected in the liabilities of central banks. Negative equity and illiquid assets constrain the ability to shrink balance

[11]Dollarisation and the possibility of central bank money being substituted by electronic money have been discussed eg by Santomero and Seater (1996), Friedman (2000) and Goodhart (2000). In a theoretical model of currency dominance, Coppola et al. (2023) rationalise features of the current dominance of the U.S. dollar in international finance and relate their theory to historical experiences. They apply this to the prominence of the Bank of Amsterdam guilder in the 18th century, as well as the ongoing debate about the potential rise of the Chinese renminbi at the expense of the dollar.

[12]Cryptocurrencies form a very poor substitute to fiat money, given their high volatility and primary use as a speculative asset. Experience from El Salvador, where bitcoin was adopted as legal tender, suggests that there are substantial costs of a move toward cryptocurrencies (Alvarez et al 2022). For a theoretical discussion of monetary competition in a model with two currencies – the dollar and bitcoin – see Schilling and Uhlig (2020).

[13]As a recent example, the Riksbank reported a loss of just over SEK 80 billion (ca. USD 8 billion) in 2022, and as a result the Riksbank's equity turned negative. As the Swedish Governor Erik Thedéen explained, "to maintain confidence in an independent monetary policy in the long term, it is necessary that the Riksbank is financially independent, that is, has sufficient equity and earnings to cover its costs," (Riksbank 2023). By law, the Swedish government needs to restore the Riksbank's equity to a basic level of SEK 40 billion.

sheets. As underscored in our model, tightening policy can be much more difficult than easing. These considerations make some recent proposals to write off government bonds held on centralbank balance sheets particularly dangerous.[14] However, there are also large differences with the past. First, the magnitude of the balance sheet constraints is much smaller relative to shifts in fiat money than it was in the days of the Bank of Amsterdam. In those times, these constraints depended heavily on loans to a single borrower. The Bank was vulnerable to outflows in encumbered accounts in addition to fluctuations in fiat money. Second, the modern institutional position of central banks as part of the general government is much stronger, especially as regards the fiscal backstop. As a result, such losses do not compromise the ability of central banks to fulfil their mandates (Bell et al 2023).[15]

The demise of the Bank of Amsterdam shows how a loss of trust in fiat money becomes a real possibility if a credible fiscal backstop is absent. As equity turned more deeply negative and the proportion of illiquid loans increased on its balance sheet, the Bank passed the point of policy insolvency (Stella and Lönnberg 2008). At this point it was no longer able to achieve its policy objectives. To regain policy solvency, recapitalisation by the fiscal authorities is needed. Such recapitalisations can take different forms, including the issuance of government bonds that may substitute for illiquid assets and negative equity, or the distribution of negative dividends by a central bank to the government (Archer and Moser-Boehm 2013). These fresh resources can help the central bank to regain its footing and continue to use its balance sheet for policy purposes.

[14]Indeed, some prominent economists have argued that central banks can simply write off government bonds, or convert them to 0% perpetuals, to provide debt relief (Various authors 2021). Notably, these proposals were made prior to the recent bout of higher inflation around the world.

[15]See also del Negro and Sims (2015): in extreme cases the fiscal authority will need to provide support and recapitalise the central bank. Moreover, see Wessels and Broeders (2022) as to why capital adequacy may be important for a credible, independent central bank over a medium-term horizon, even if central banks cannot default formally.

Yet what if the fiscal authorities are unable to recapitalise? In theoretical models, it is well-known that fiscal policy has an important impact on the overall price level. For certain configurations of fiscal and monetary policy, the price level may be indeterminate, resulting in stochastically fluctuating, explosive inflation (Sims 1994; del Negro and Sims 2015). In cases where fiscal positions deteriorate and the solvency of the consolidated public sector is in doubt, there is the risk that trust in both government debt and the fiat money in which it is denominated could break down simultaneously.

In practice, there is great uncertainty around where exactly the break point for trust in fiat money would be. Yet given the stakes for the economy as a whole, central banks and governments would do well to stay far clear of this critical threshold. In order to ensure the public good nature of fiat money in the future, sound central bank finances and fiscal backing are key.

V. Conclusion

Money is a social convention. Yet it is also the key yardstick for the value of disparate goods, services, claims and assets. Sound money allows individuals, firms (including financial firms) and governments to transact, and to record their obligations to one another in a way that binds the economy together. The governance of money is about ensuring a flexible system that meets the needs of the economy and yet is robust enough to ensure confidence. Experience with monetary institutions through the ages has given rise to central banks as the key institution at the heart of the monetary system. This is not to say that central banks always get it right; the Bank of Amsterdam is the poster child of what can go wrong when governance goes awry. Yet the solution to date has been to bolster the mandate and solvency of central banks, whose governance arrangements have continued to evolve with the economic challenges of their time.

In this paper, we have drawn lessons from the experience of this proto-central bank which issued a highly successful form of fiat money for nearly two centuries. The move of the Bank of Amsterdam from a rigid to an elastic structure allowed its success in supporting wholesale payments and international trade. Yet after a severe economic shock, and in the absence of fiscal backing, the same network effects that had initially sustained trust in the Bank guilder worked to expedite its ultimate downfall. Our global game model formalises the conditions under which trust in fiat money can evaporate. It shows that there is a unique break point for trust in fiat money, which is more likely to be reached in a severe shock when central bank equity turns deeply negative and fiscal support is lacking. When uncertainty declines, the move from one regime (high trust) to another (breakdown of trust) becomes a step function, and the downfall can be swift and precipitous.

Overall, our analysis demonstrates the value in reviewing historical precedents for understanding the monetary systems of today. In a context of high inflation, high global uncertainty and competition between both sovereign currencies and now cryptocurrencies, it is particularly relevant to understand monetary competition and the factors that could lead to shifts between different monetary regimes. Finding and analysing the incentives and governance underlying these structures may be a fruitful avenue for further research.

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Online Appendix

Proof of Lemma 1.

Under the assumption that all merchants follow the switching around v^*, first observe that the posterior about θ, conditional on marginal type v^*, is normally distributed with mean $(\alpha\, y + \beta\, v^*)/(\alpha + \beta)$ and standard deviation $1/\sqrt{\alpha + \beta}$. Then, let θ_0 be such that if $\theta = \theta_0$, the proportion of merchants with type higher than v^* is exactly m. This yields:

$$Pr(v \geq v^* \,|\, \theta_0) = m \quad \Rightarrow \quad \theta_0 = v^* - \frac{\Phi^{-1}(1-m)}{\sqrt{\beta}}.$$

For the cumulative distribution function, $G(m \,|\, v^*, y)$, we derive:

$$\begin{aligned}
G(m \,|\, v^*, y) &= Pr(\theta < \theta_0 \,|\, v^*) = \Phi\left(\sqrt{\alpha+\beta}\left(\theta_0 - \frac{\alpha y + \beta v^*}{\alpha+\beta}\right)\right) \\
&= 1 - \Phi\left(\sqrt{\alpha+\beta}\left(\frac{\alpha y + \beta v^*}{\alpha+\beta} - \theta_0\right)\right) \\
&= 1 - \Phi\left(\sqrt{\alpha+\beta}\left(\frac{\alpha y + \beta v^*}{\alpha+\beta} - v^* + \frac{\Phi^{-1}(1-m)}{\sqrt{\beta}}\right)\right) \\
&= 1 - \Phi\left(\frac{\alpha}{\sqrt{\alpha+\beta}}(y - v^*) + \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(1-m)\right) \\
&= \Phi\left(\frac{\alpha}{\sqrt{\alpha+\beta}}(v^* - y) + \sqrt{\frac{\alpha+\beta}{\beta}}\Phi^{-1}(m)\right).
\end{aligned}$$

■

Proof of Proposition 2.

First, we have already shown in subsection III.B that for a given α, the joint solution (v^*, θ^*) uniquely solves equations (5) and (13) if β is sufficiently large.

Second, to verify that the switching strategies constitutes an equilibrium, we have to show that, given that all other merchants adhere to their switching strategies, a merchant with type $v_i < v^*$ has no incentive to hold bank money, and similarly, a merchant with type $v_i > v^*$ has no incentive to hold coins. That is, we have to show that $v_i\, F(v_i,\, y)/(1+\bar{\gamma}) < 1$ if $v_i < v^*$, and $v_i\, F(v_i,\, y)/(1+\bar{\gamma}) > 1$ if $v_i > v^*$. Note that $v^*\, F(v^*,\, y)/(1+\bar{\gamma}) = 1$ for the marginal type of merchant v^*. Define $V(v_i) = v_i\, F(v_i,\, y)$ and look at $dV/dv_i = F(v_i,\, y) + v_i \cdot dF(v_i,\, y)/dv_i$. Using (7), in the limit as $\beta \to \infty$ for given α, $F(v_i,\, y)$ converges to $F = \int_0^1 f(m)\, dm$, which is invariant to v_i, so that $dF(v_i,\, y)/dv_i$ converges to 0. Therefore, in the limit, we get $dV/dv_i = F > 0$, since $f(m) > 0$ for all m in $[0, 1]$. Hence, by continuity, for sufficiently large β, we have $V(v_i)$ is increasing in v_i. This means that $v_i\, F(v_i,\, y)/(1+\bar{\gamma}) < v^*\, F(v^*,\, y)/(1+\bar{\gamma}) = 1$ if $v_i < v^*$, and $v_i\, F(v_i,\, y)/(1+\bar{\gamma}) > v^*\, F(v^*,\, y)/(1+\bar{\gamma}) = 1$ if $v_i > v^*$.

Third, note that for sufficiently low merchant types it is strictly dominant ("low dominance" region) to hold coins even if all other merchants hold bank money (namely for $v_i < (1+\bar{\gamma})/f(1)$) and strictly dominant ("high dominance" region) to hold bank money for sufficiently high types even if all other merchants hold coins (namely for $v_i > (1+\bar{\gamma})/f(0)$). Starting from a dominance region, we can apply iterative elimination of dominated strategies – from above and below – and stop at the proposed switching strategy profile. For the proof that only this equilibrium survives iterated deletion of strictly dominated strategies, see Morris and Shin (2003). ■

Proof of Proposition 3.

First, we may write indifference condition (5) as $z(v^*) = 0$, with $z(v^*) = v^* - h(v^*)$ and $h(v^*) = (1+\bar{\gamma})/F(v^*)$, where $F(v^*) = \int_0^1 f\,(m)\, dG_k(m \,|\, v^*, y)$ and $G_k(\cdot)$ as specified in (14). Since $0 < f(0) \leq F(v^*) \leq f(1) < \infty$ for all v^*, note that $z(v^*)$ is continuous and differentiable in v^* with $z(0) < 0$ and $z(v^*) > 0$ for $v^* > (1+\bar{\gamma})/f(0)$. Hence, $z(v^*)$ crosses the zero-line at least once. For uniqueness of v^* we require that $z'(v^*) = 1 - h'(v^*) > 0$, or equivalently $h'(v^*) < 1$.

To derive $h'(v^*) = -(1+\bar{\gamma})F'(v^*)/F(v^*)^2$ we first calculate $F'(v)$. Using integration by parts, $F(v^*)$ can be written as:

$$\begin{aligned} F(v^*) &= [f(m)G_k(z \,|\, v^*, y)]_0^1 - \int_0^1 f'(m)G_k(m \,|\, v^*, y)dm \\ &= f(1)G_k(1 \,|\, v^*, y) - f(0)G_k(0 \,|\, v^*, y) - \int_0^1 f'(m)G_k(m \,|\, v^*, y)dm \\ &= f(1) - \int_0^1 f'(m)G_k(m \,|\, v^*, y)dm, \end{aligned}$$

since $G_k(1 \,|\, v^*, y) = 1$ and $G_k(0 \,|\, v^*, y) = 0$. Observe that

$$\partial G_k(m| \, v^*, y)/\partial v^* = \frac{1}{k}\phi\left(\frac{v^* - y}{k} + \Phi^{-1}(m)\right),$$

with $\phi(\cdot) = \Phi'(\cdot)$ the standard normal density function. For the derivative $F'(v^*) = dF(v^*)/dv^*$ it now follows (recall $f' > 0$):

$$F'(v^*) = -\frac{1}{k}\int_0^1 f'(m)\phi\left(\frac{(v^* - y)}{k} + \Phi^{-1}(m)\right)dm < 0.$$

Substituting $F'(v^*)$ in $h'(v^*)$ yields:

$$h'(v^*) = \frac{(1+\bar{\gamma})\left(\int_0^1 f'(m)\phi\left(\frac{(v^*-y)}{k} + \Phi^{-1}(m)\right)dm\right)}{k\,F(v^*)^2} > 0.$$

Since $\phi(\cdot) \leq 1/\sqrt{2\pi}$ and $\int_0^1 f'(m)dm = f(1) - f(0) > 0$, some algebraic manipulations yield:

$$0 < h'(v^*) \leq \frac{(1+\bar{\gamma})(f(1) - f(0))}{k\sqrt{2\pi}f(0)^2} = k_0/k.$$

Hence, for $k > k_0$ we have that $h'(v^*) < 1$ and therefore uniqueness of v^*. As a result, given a unique v^* for $k > k_0$, there is also a unique solution for θ^* that solves (13).

Second, in the limit as $\beta \to \infty$, from (10), the money demand function converges to a step function with the jump at $\theta^* = v^*$. Below the break point θ^*, the demand for bank money is zero while the supply of bank money is at least $L - E$. Therefore, the market clearing price of bank money is zero. Above the break point θ^*, with the agio at the target $\bar{\gamma}$, the supply of bank money is equal to the demand of bank money. As such, the market clearing price p equals the target price $\bar{p} = 1 + \bar{\gamma}$.

Third, it remains to be shown that the break point θ^* is a decreasing function of y. To derive this result, first note that $G_k(m \,|\, v^*, y)$ in (14) is decreasing in y. Therefore, a larger y gives rise to a first-order stochastic shift of the aggregate bank money holding distribution to the right. Since $f(\cdot)$ is an increasing function, we have that the first-order stochastic shift puts more weight on higher utility of holding bank money. Hence,

$$\int_0^1 f(m) dG_k(m \,|\, v^*, y') > \int_0^1 f(m) dG_k(m \,|\, v^*, y) \quad \text{if} \quad y' > y.$$

Therefore, from the indifference condition (5), the marginal type v^* is a decreasing function of y. Finally, from (13), we have that $d\theta^*/dy = d\theta^*/dv^* \cdot dv^*/dy < 0$, because $d\theta^*/dv^* = 1$. Hence, the break point θ^* is decreasing in y as well. ■

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