Market Risk Premium Expectation: Combining Option Theory with Traditional Predictors*

Hong Liu† Yueliang (Jacques) Lu‡ Weike Xu§ Guofu Zhou¶

First draft: December 2022

Current version: December 2023

*We greatly thank Yakov Amihud, Tyler Beason, Hui Chen, Kevin Crotty, Sina Ehsani, Bing Han, Yufeng Han, Cheng-Few Lee, Zhongling (Danny) Qin, Weidong Tian, Robert Van Ness, Paul Whelan, Yangru Wu, Chuanhai Zhang, conference and seminar participants at the First CUHK-RAPS Conference on Asset Pricing and Investment, the 33rd Annual Conference on Financial Economics and Accounting, 2023 China International Risk Forum, 2023 Financial Management Association Annual Meeting, 2023 Financial Markets and Corporate Governance Conference, 2023 Hong Kong Conference for Fintech, AI, and Big Data in Business, 2023 PKU-NUS Annual International Conference on Quantitative Finance and Economics, 2023 Wolfe Research Options in Investment Management Conference, Fudan University, Hunan University, Hunan Normal University, Jiangxi University of Finance and Economics, Kyung Hee University, Renmin University of China, Shanghai Advanced Institute of Finance (SAIF), Tsinghua University, Washington University in St. Louis, and Xi'an Jiaotong University for their insightful comments and suggestions.

†Washington University in St. Louis. Email: liuh@wustl.edu

‡Clemson University. Email: yuelial@clemson.edu

§Clemson University. Email: weikex@clemson.edu

¶Washington University in St. Louis. Email: zhou@wustl.edu. Corresponding author.

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Abstract

Recently there is a growing literature on predicting the market risk premium from the option market, shedding new insights on the traditional voluminous literature of market predictability that relies on economic state variables. This paper provides a novel link between these literatures. Theoretically, we derive a generalized lower bound on the expected market risk premium that combines both options and state variables. Empirically, we find that the new bound significantly enhances the out-of-sample market predictability compared to using either type of information alone, with gains more pronounced in the short horizons such as one to three months.

Keywords: Out-of-sample predictability, equity risk premium, lower bound, option-implied information, recovery

JEL Classification: G1, G11, G12, G17

1 Introduction

The expected equity market excess return, or market risk premium, is one of the central quantities in finance and macroeconomics. Going as far back as Dow (1920), the literature on market predictability attempts to gain insights into the economic and financial variables that drive the market risk premium. For example, Fama and French (1988, 1989), Campbell and Shiller (1988a,b, 1998), and Huang, Jiang, Tu, and Zhou (2015) find that variables such as dividend-price ratio, earnings-price ratio, and investor sentiment can predict market returns.[1] Breaking a new ground, Martin (2017) shows that option prices prove useful on the future market return by bounding the market risk premium from below. Subsequently, Martin and Wagner (2019); Chabi-Yo and Loudis (2020); Kadan and Tang (2020); Chabi-Yo, Dim, and Vilkov (2022), among others, propose additional bounds. Back, Crotty, and Kazempour (2022) provide formal tests of the bounds and find that they are inadequate for the data. The open question is then how to improve the bounds.

In this paper, we provide a novel study that combines the insights of two literatures together, the time-series predictability and the option recovery theory, to predict the market risk premium. Specifically, we derive a generalized bound that incorporates both the risk-neutral volatility computed from option prices and the economic state variables. We show that the generalized bound performs well in forecasting the market risk premium out-of-sample and generates substantial economic gains consistently over time. In particular, it outperforms substantially the results when the method of each of the literatures is used alone.

Back, Crotty, and Kazempour (2022) provide the first link between the two literatures. Using a forecasting combination approach (Rapach, Strauss, and Zhou, 2010), they combine the bound forecasts with those forecasts from Welch and Goyal (2008) macro variables in a regression framework. While their approach is inspirational, it differs from ours. Instead of a statistical motivation, we focus on how to incorporate any related economic variables into the theoretical

[1] Rapach and Zhou (2022) provide a recent survey of the literature.

bound, so that a state-dependent bound can be used to bound the market risk premium directly. Our setup appears to connect economic state variables more explicitly to the market risk premium in a structural manner.

Empirically, we construct the generalized bound using a set of commonly studied time-series predictors, including the 14 macro variables by Welch and Goyal (2008), the five sentiment indices by Baker and Wurgler (2006); Huang et al. (2015); Shapiro, Sudhof, and Wilson (2022), and the short interest index (SII) by Rapach, Ringgenberg, and Zhou (2016). For comparison, we compare the performance of the generalized bounds with other established option bounds, including Martin (2017) bound, the slackness-adjusted bound (BCK) introduced by Back, Crotty, and Kazempour (2022), and the risk-neutral moments bound (CYL) proposed by Chabi-Yo and Loudis (2020). We confirm the finding in Back, Crotty, and Kazempour (2022) that the mean bounds of Martin and CYL are substantially smaller than the mean market excess return (around 8.26%) by roughly 2.5% to 5% per year, suggesting a substantial slackness. The median bounds are even smaller by roughly 10% to 12.5%. Although adding past slackness adjustment of Back, Crotty, and Kazempour (2022) increases the bound, the mean and median of BCK bound are still much lower compared to those of market excess return. In contrast, the generalized bounds are much higher than the existing bounds in terms of both mean and median, and the magnitudes become comparable to the targeted realized return. For instance, the mean value of generalized bounds by macro variables, sentiment, and short interest index are around 6.44%, 7.66%, and 13.63%, respectively. Notably, the median of SII-based bound, around 14.10%, is also comparable to that of market excess return, around 15.10%, whereas the medians of BCK and CYL bounds are both lower than 5%.

To analyze the out-of-sample predictive power of the bounds, we calculate the out-of-sample R^2_{OS} statistic relative to the historical average benchmark and assess its statistical significance with Clark and West (2007) test. We find some evidence of outperformance over adjustment by the existing bounds (especially for BCK and CYL bounds), but the predictive power mainly comes from the period after 2008/09. For instance, the R^2_{OS} statistics from Martin, BCK and CYL (restricted) bounds are -0.59%, -0.54%, and -0.76% during the evaluation period 2006–2022, but jump to 2.14%, 2.80%, and 3.25% during 2011–2022, with the latter two turning marginally significant at the 10% level. The weak or mixed evidence suggests that it is important to improve the bounds further by incorporating economic state variables, the focus of our paper.

Indeed, we find that the generalized bounds demonstrate much improved out-of-sample predictive power over different out-of-sample evaluation periods, and also over of 2006–2022 where those established bounds underperform, even in the presence of the financial crisis. Specifically, both sentiment- and SII-based bounds produce positive R^2_{OS} larger than 3%, suggesting a meaningful degree of return predictability in terms of investment value.[2] Moreover, the generalized bounds continue to outperform the existing bounds in the post-crisis era by generating much larger R^2_{OS} statistics, with 8 of them significant at the 10% level and 6 significant at the 5% level. Among three category of traditional variables, the bound incorporating SII or sentiments outperforms the bound with macro variables, with the former exhibiting the most robust performance. For example, the SII-based bound consistently generates significant R^2_{OS} values of 7.88%, 17.47%, and 20.00% across the three out-of-sample periods. One plausible explanation for the underperformance of WG macro-based bounds may be attributed to the lack of consistent out-of-sample evidence for these macro variables, a critique famously articulated by Welch and Goyal (2008).

The statistically strong performance by combining option theory with traditional variables is also economically valuable. We find that neither of those existing bounds in the literature can consistently outperform the historical average benchmark in terms of Sharpe ratio and certainty equivalent gain (CER). The underperformance is amplified during the 2008/09 financial crisis period. Overall, they tend to generate higher average return than the benchmark but at the cost of even higher volatility, leading to smaller Sharpe ratios and negative CERs. For example, the Sharpe ratios for Martin, BCK, and CYL (restricted) bounds are 0.18, 0.22, and 0.24, respectively, compared to the Sharpe ratio of 0.26 for the benchmark. In contrast, the generalized bounds excel in managing downside risk during the market downturn: they produce comparable standard deviations to Martin or CYL bounds but much larger average returns. As a result, we find that the majority of generalized bounds outperform the benchmark with substantial investment values.

[2] As highlighted by Campbell and Thompson (2008), an R^2_{OS} of 0.5% for monthly return forecast indicates a meaningful degree of return predictability in terms of increased annual portfolio returns for a mean-variance investor.

For instance, the SII-based bound yields Sharpe ratios of 0.63 and 1.04 during the out-of-sample periods of 2006–2022 and 2011–2022, respectively. The difference in two CERs, referred to as the utility gain (Pástor and Stambaugh, 2000; Campbell and Thompson, 2008), indicates that investors would be willing to pay 10.09%, 8.88%, 7.68%, or 7.73% *per annum* for access to the information in the SII-based generalized bound compared to the historical average, Martin bound, BCK bound, or CYL restricted bound, respectively.

We offer several explanations for the superb performance of generalized bounds. Firstly, when examining the cumulative squared error, we observe results similar to those in Back, Crotty, and Kazempour (2022), indicating substantial underperformance of bound-based forecasts during the financial crisis and the COVID-19 pandemic, followed by partial recovery. Remarkably, only the generalized bound fully recovers, delivering a smaller forecasting error than the benchmark. Secondly, Martin, BCK, and CYL bounds exhibit much higher forecast variance compared to the benchmark, resulting in negative R^2_{OS} statistics. In contrast, the generalized bound better regularizes forecast variability, achieving lower forecast variance and higher precision. Lastly, we introduce a new approach to investigate bound forecasts by comparing the distributions of the bounds and realized market returns. The distribution of realized market returns is highly dispersed, ranging from extreme negative to extreme positive values, which can not be effectively captured by the concentrated and right-skewed Martin, BCK, and CYL bounds. In contrast, the generalized bound demonstrates greater dispersion and a less pronounced right skew compared to the Martin and others. Despite the uncertain and evolving nature of the data-generating process for expected market return, the generalized option bound, combining risk-neutral moments and state variables, manages to mimic the distribution of expected market return, significantly contributing to its out-of-sample predictive power.

We observe in particular that, when Martin (2017) bound is employed as a lower threshold, it may not adequately capture market dynamics during normal periods, leading to slackness. Conversely, it might be disproportionately large during extreme market events, such as financial crises, resulting in the violation of NCC. Moreover, we observe a reasonable correlation between the adjustment factor in the generalized bound and the economic sentiment. Specifically, the

Martin bound is scaled up when economic sentiment is on the rise, and conversely, it is scaled down when economic sentiment is on the fall. Since the adjustments made by the generalized bounds each month vary depending on the chosen state variable and the macroeconomic condition, the overall trend indicates that the generalized bounds consistently surpass the Martin bound most of the time. This pattern suggests a partial alleviation of slackness by using state variables as a real-time adjustment.

We conduct a series of robustness checks. We find that the predictive power of option bounds is primarily concentrated in subperiods characterized by high sentiment or low risk aversion. Furthermore, our findings remain robust across alternative constructions of generalized bounds, extended data samples, and longer-horizon forecasts. In summary, Martin, BCK, and CYL bounds exhibit reasonable predictability when used for forecasting longer-term market returns. However, our generalized bounds, incorporating both option theory and traditional variables, consistently demonstrate substantial forecasting gains across short-term (1-month), medium-term (3- or 6-month), and long-term (12-month) horizons.

Our paper contributes to two key lines of literature concerning the market risk premium. The first line, with Martin (2017) as a notable example, investigates how elusive it is to estimate the (conditional) expected return, which dates back to Merton (1980), Black (1993), and Elton (1999). The line of research, including Martin and Wagner (2019); Chabi-Yo and Loudis (2020); Kadan and Tang (2020); Heston (2021); Martin (2021); Back, Crotty, and Kazempour (2022); Bakshi, Gao, and Xue (2022); Chabi-Yo, Dim, and Vilkov (2022), provides substantial insights on the market risk premium. The second line, as emphasized by Welch and Goyal (2008); Campbell and Thompson (2008); Spiegel (2008), challenges researchers on whether any model forecasts of the equity risk premium can be any better than the historical mean out-of-sample. While studies, including Jagannathan and Liu (2019); Farmer, Schmidt, and Timmermann (2023); Kelly, Malamud, and Zhou (2023), suggest the possibility of beating the historical mean, our study, which combines insights from both lines, demonstrates an even greater predictive power. This provides a deeper understanding of the underlying drivers.

The rest of this paper is structured as follows. Section 2 presents the theoretical framework, followed by the out-of-sample test in Section 3. Robustness checks are provided in Section 4. Finally, we conclude in Section 5.

2 Theory

In this section, based on Martin (2017), we provide the theoretical underpinnings for the important role of state variables.

For a discrete time subscript t, let S_t denote the time-t price of a stock market index (inclusive of dividends). $R_{t,t+1} = \frac{S_{t+1}}{S_t}$ denotes the gross market return over the time period from t to the next time $t+1$, and $R_{f,t,t+1}$ is the gross risk-free return over the same time period. We denote the real-world probability measure by $\mathbb{P}$, and the information set at time t by $\mathscr{F}_t$. Let $M_{t,t+1}$ be a stochastic discount factor (SDF) over the period from t to $t+1$. Consequently, the risk-neutral probability measure $\mathbb{Q}$ satisfies

$$\frac{d\mathbb{Q}}{d\mathbb{P}}|_{\mathscr{F}_{t+1}} = R_{f,t,t+1}M_{t,t+1}. \tag{1}$$

Similarly, we define $R_{t,T} = \frac{S_T}{S_t}$ and $R_{f,t,T}$ for any $T = 1,2,\cdots$ in a dynamic setting, with $\mathbb{E}_t^{\mathbb{P}}(M_{t,T}R_{t,T}) = 1$ holding. The length of the period can be arbitrary. For simplicity, we omit the subscript t and use the notations R_T and M_T to denote the gross market return and the SDF over the period $[t,T]$, respectively.

2.1 Martin (2017) bound

Martin (2017) decomposes the market risk premium into two components

$$\begin{aligned} \mathbb{E}_t^{\mathbb{P}}(R_T) - R_{f,t,T} &= \left[\mathbb{E}_t^{\mathbb{P}}(M_T R_T^2) - R_{f,t,T}\right] - \left[\mathbb{E}_t^{\mathbb{P}}(M_T R_T^2) - \mathbb{E}_t^{\mathbb{P}}(R_T)\right], \\ &= \frac{1}{R_{f,t,T}} Var_t^{\mathbb{Q}}(R_T) - Cov_t^{\mathbb{P}}(M_T R_T, R_T), \end{aligned} \tag{2}$$

where the first component, the risk-neutral variance, can be computed directly from time-t prices of index options, as known from the work of Breeden and Litzenberger (1978). The second component is a covariance term. We use the superscript $\mathbb{P}$ to highlight the fact that those quantities are under the real-world probability measure. Henceforth, we drop the superscript and use $\mathbb{E}_t(\cdot)$ to represent the mean conditional expectation under the $\mathbb{P}$-measure.

Martin (2017) imposes a restriction that is termed *negative correlation condition* (NCC). He further shows that NCC holds theoretically under mild conditions in a variety of asset pricing settings, and it also holds empirically when a typical factor structure for the SDF is assumed.

Definition 1. *The negative correlation condition (NCC) holds if*

$$Cov_t\left(M_T R_T, R_T\right) \leq 0,$$

for all M_T under the real-world probability measure.

By NCC, the risk-neutral variance can be viewed as a lower bound of the equity risk premium (the expected market excess return), which is

$$\mathbb{E}_t\left(R_T\right) - R_{f,t,T} \geq \frac{1}{R_{f,t,T}} Var_t^{\mathbb{Q}}\left(R_T\right), \tag{3}$$

Martin (2017) provides the first test on whether the implied risk-neutral volatility bounds could be related directly to the equity premium, going beyond early related studies by Merton (1980), Black (1993), and Elton (1999). However, recent studies on the lower bound of the equity premium remain controversial due to the fact that the NCC is pivotal to obtaining the lower bound and yet there is no direct quantification of it. For instance, Bakshi, Crosby, Gao, and Zhou (2021) exploits theoretical and empirical constructions that challenge the NCC. They use options on the S&P 500 index and STOXX 50 equity index and conclude that the overall tests favor the rejection.

Back, Crotty, and Kazempour (2022) test Martin bounds along with others at different horizons conditionally and reject the hypothesis that they are tight for market risk premium. Therefore, using the lower bounds as forecasts of market risk premium appears insufficient in many cases due to their high slackness. Goyal, Welch, and Zafirov (2021) also examine those option bounds and demonstrate that the out-of-sample performance is never statistically significant. As a result, Back, Crotty, and Kazempour (2022) propose to add past mean slackness to Martin's (2017) option bounds as a potential solution. But the solution is impeded by the lack of enough data to estimate mean slackness, as they stress that 150 years of data is necessary for the 'bound + mean slackness' strategy to achieve a substantial improvement in out-of-sample performance.

2.2 A Generalization of Martin (2017) bound

We generalize Martin (2017) bound for an economy where asset prices depend on a vector x_t of state variables (such as sentiment (e.g. Asriyan, Fuchs, and Green, 2019; Hore, 2015)). In such an economy, for any security with a return process R_t (not just the market portfolio), in equilibrium we have $\mathbb{E}[R_T] = f(x)$, for some function $f(\cdot)$ and that $\mathbb{E}[M_T R_T^2] = g(x)$, for some function $g(\cdot) > 0$.

Therefore, we have

$$\mathbb{E}[R_T] = k(x)\mathbb{E}(M_T R_T^2), \tag{4}$$

where $k(x) \equiv f(x)/g(x)$. It is straightforward to show that the NCC assumption in Martin (2017) (and thus Martin's lower bound) is equivalent to assuming $k(x) \geq 1$ for all realizations of x.

It follows from Equations (2) and (4) that the market risk premium satisfies:

$$\mathbb{E}_t\left(R_T\right) - R_{f,t,T} = k(x)\left(\underbrace{\frac{1}{R_{f,t,T}} Var_t^{\mathbb{Q}}\left(R_T\right)}_{\text{Martin (2017) bound}} + R_{f,t,T}\right) - R_{f,t,T}, \tag{5}$$

which links the risk-neutral option bound to the state variable vector. When $k(x) \geq 1$, Equation (5) reduces to Martin (2017) bound. Following Martin (2017), we express the risk-neutral variance in terms of a SVIX index over the period $[t,T]$ and compute the Martin bound, $b_{t,M}$ as

$$\begin{aligned} b_{t,M} &= \frac{1}{R_{f,t,T}} Var_t^{\mathbb{Q}}(R_T) \\ &= (T-t) R_{f,t,T} \mathrm{SVIX}^2_{t \to T}, \end{aligned} \tag{6}$$

where $\mathrm{SVIX}^2_{t \to T}$ is defined via the formula,

$$\mathrm{SVIX}^2_{t \to T} = \frac{2}{(T-t) R_{f,t,T} S_t^2} \left[\int_0^{F_{t,T}} \mathrm{put}_{t,T}(K) dK + \int_{F_{t,T}}^{\infty} \mathrm{call}_{t,T}(K) dK \right], \tag{7}$$

where $\mathrm{put}_{t,T}(K)$ ($\mathrm{call}_{t,T}(K)$) denotes the market price of a put (call) option with strike K and maturity $T-t$, and $F_{t,T}$ is the forward price of the underlying.

Equation (5) essentially combines the forward-looking feature (option prices) and backward-looking feature (state variables). The function form for $k(x_t)$ can be either linear or non-linear. Compared with Back, Crotty, and Kazempour (2022), instead of adding the past mean values as a correction for slackness or relying on statistical combinations to include macro variables' forecasts, we provide a theoretical motivation to use the state variables as a real-time correction to the option bound.

In the next section, we will empirically test the efficacy of Equation (5) in forecasting market risk premium out-of-sample. We also compare both statistical and economic performances of our generalized bound with other known bounds in the literature, including the slackness-adjusted bound by Back, Crotty, and Kazempour (2022), and the bound by Chabi-Yo and Loudis (2020) that is based on the first four risk-neutral moments of the market excess return.

3 Out-of-Sample Tests

In this section, we first discuss the data, followed by the forecast construction and the criteria used to evaluate the out-of-sample performance. We then present our main out-of-sample results.

3.1 Bound and traditional variables

We use S&P 500 index options obtained from OptionMetrics to compute Martin (2017) bound, running from January 1996 to December 2022. We apply the same filters as in Martin (2017) to clean the data.We then construct time series of option bounds at time horizons $T - t = 1, 3, 6$, and 12 months. We interpolate the bound linearly to match maturities of 30, 90, 180, and 360 days. To compute the slackness-adjusted bounds, we follow Back, Crotty, and Kazempour (2022) by matching these option bounds with realized market excess returns on the S&P 500 index compounded over the 21, 63, 126, and 252 trading days. The daily return data for the S&P 500 index are obtained from CRSP. For the Chabi-Yo and Loudis (2020) bound, we consider both unrestricted and restricted bounds with data from Fousseni Chabi-Yo's website.

For state variables, we collect data for the following return predictors that are commonly used in the time-series return predictability literature. The traditional variables can be broadly grouped into three categories. The Internet Appendix contains the detailed description and the data source.

- Financial/macro variables (14 variables by Welch and Goyal, 2008, short for WG)
- Sentiment (five variables by Baker and Wurgler, 2006; Huang et al., 2015; Shapiro, Sudhof, and Wilson, 2022)[3]
- Short interest index (Rapach, Ringgenberg, and Zhou, 2016)[4]

It is worth noting that option bounds with maturities of 30, 90, 180, and 360 days are computed at a daily frequency, whereas the traditional state variables are at a monthly frequency. We merge the state variables with the option bounds computed at the last trading day of each month, resulting in a time series of combined predictors at a monthly frequency. Throughout the paper, we focus on the out-of-sample predictability of 1-month return (thus the 30-day option bound), which is

[3]The data for Baker and Wurgler (2006) sentiment is updated to June 2022. For Baker and Wurgler (2006) and Huang et al. (2015), we use both orthogonal and non-orthogonal measures.

[4]We also consider alternative short interest measure–the short selling efficiency in Chen, Da, and Huang (2022). The results are similar.

the primary horizon as in the influential study by Welch and Goyal (2008). But we also examine longer-horizon forecasts in Section 4.4.

3.2 Forecast construction

Stock market excess return predictability is typically analyzed in the context of a predictive regression model (see, for instance, Welch and Goyal, 2008; Campbell and Thompson, 2008). In contrast, by option theory, the bound is already a meaningful expected return (Martin, 2017). Therefore, we use the bound directly as the forecast of the expected market return.

Let r_t denote the market excess return at time t. The forecast of subsequent market excess return, r_{t+1} is given by

$$\hat{r}_{t+1} = b_{t,i}, \quad i \in (M, BCK, CYL, LLXZ), \tag{8}$$

where $b_{t,M}$, $b_{t,BCK}$, $b_{t,CYL}$, and $b_{t,LLXZ}$ denote the Martin (2017) bound, Back, Crotty, and Kazempour (2022) slackness-adjusted bound, Chabi-Yo and Loudis (2020) bound, and our generalized bound by incorporating state variables computed at time t, respectively.

Our theoretical framework, defined in Equation (5), provides a flexible approach to include state variables into option bounds. For 14 WG macro variables, we use a number of dimension reduction techniques in a high-dimensional setting to extract a representative component and use it for x in Equation (5). The shrinkage methods include predictor average that takes the cross-sectional average of (normalized) variables, principal components (PCA), partial least squares (PLS) (Wold, 1966, 1975; Kelly and Pruitt, 2013, 2015), and scaled-PCA (Huang, Jiang, Li, Tong, and Zhou, 2022). For five sentiment measures, we apply the same shrinkage methods. For short interest index, we use it directly. In the Internet Appendix, we also consider using each variable individually to construct the generalized bound and repeat the analyses in the main paper.

To empirically construct the generalized bound, we choose $k(x) = \exp(\alpha + \beta x)$ in Equation (5) as the adjustment factor, where α and β are constants. Alternative function forms are discussed in

Section 4.2. At time t, the generalized bound is computed as

$$b_{t,LLXZ} = \exp(\hat{\alpha}_t + \hat{\beta}_t x_t) \times \left(b_{t,M} + R_{f,t,T}\right) - R_{f,t,T}, \tag{9}$$

where $\{\hat{\alpha}_t, \hat{\beta}_t\}$ are estimated *recursively* by solving the following optimization problem:

$$\min_{\alpha,\beta} \sum_{s=1}^{t-1} \left\{ r_{s+1} - \left[\exp(\alpha + \beta x_s) \times \left(b_{s,M} + R_{f,s}\right) - R_{f,s} \right] \right\}^2. \tag{10}$$

We carefully use information up to time t only to avoid any forward-looking bias.

To ensure we have enough observations to estimate $\{\alpha, \beta\}$, we use the first 10 years (from 1996:01 to 2005:12) of the full sample period as the initial in-sample estimation period, thus 120 observations. The subsequent 17 years, from 2006:01 to 2022:12 (204 observations), constitutes the out-of-sample period for forecast evaluation. This relatively long out-of-sample period covers the 2008/09 global financial crisis and COVID-19 pandemic, which allows us to analyze the efficacy of option bounds under a variety of economic conditions. We also consider using longer initial windows such as 15 and 20 years, so our out-of-sample periods cover 2011:01–2022:12 and 2016:01-2022:12. These two short periods allow us to evaluate the bound performance in the post-financial crisis era.

Table 1 presents summary statistics of the 1-month market excess return and the bounds over the period from 2006:01 to 2022:12. We follow Back, Crotty, and Kazempour (2022) to compute market excess returns from S&P 500 index returns using the risk-free rate from Ken French's website. Panel A confirms the message of Back, Crotty, and Kazempour (2022) that the mean market excess return (around 8.26%) is substantially larger than the mean bounds, by roughly 2.5% to 5% per year. Further, the slackness-adjusted bound (BCK) by Back, Crotty, and Kazempour (2022) and the risk-neutral moments bound (CYL) by Chabi-Yo and Loudis (2020) are generally higher than Martin bound, so realized slackness is lower. However, compared with the mean and median of market realized return, those bounds are still much lower, suggesting substantial slackness.

Panel B of Table 1 reports the summary statistics for the generalized bounds. WG Average, WG PCA, WG PLS, and WG S-PCA are the generalized bounds based on the cross-sectional average, first principal component, partial least square, and scaled first principal component extracted from the 14 macro variables in Welch and Goyal (2008), respectively. Sent Average, Sent PCA, Sent PLS, and Sent S-PCA represent those resulting from applying different shrinkage methods to the five sentiment measures (Baker and Wurgler, 2006; Huang et al., 2015; Shapiro, Sudhof, and Wilson, 2022). SII denotes the generalized bound based on short interest index (Rapach, Ringgenberg, and Zhou, 2016). In contrast to Panel A, we observe that the generalized bounds, except for WG Avg and WG PCA, are mostly higher than BCK or CYL bounds, and their magnitudes are similar to the target market return. For instance, the mean value of the generalized bounds after applying PCA, PLS, and scaled-PCA to sentiment variables are 8.50%, 7.53%, and 7.66%, respectively. Notably, when using the short interest index as the state variable, the mean and median of the generalized bound, around 13.63% and 14.10%, become much comparable to those of the market excess return, around 8.26% and 15.10%. In contrast, the mean and median of BCK or CYL bounds are lower than 6% and 5%, respectively.

We next test whether the option bounds can successfully predict the market excess return out of sample.

3.3 Out-of-sample statistical gains

We assess the forecast accuracy with out-of-sample R^2_{OS} statistic relative to the prevailing historical average benchmark.[5] Given T forecasts in the out-of-sample evaluation period, R^2_{OS} statistic essentially measures the relative reduction in mean square prediction error (MSPE),

$$R^2_{OS} = 1 - \frac{\sum_{t=0}^{T-1}(r_{t+1} - b_{t,i})^2}{\sum_{t=0}^{T-1}(r_{t+1} - \bar{r}_t)^2}, \tag{11}$$

[5] The historical average of market risk premium is constructed from the daily return series of the S&P 500 index, starting from July 3, 1962 on CRSP.

where r_{t+1} is the realized return at time $t+1$, $b_{t,i}$ is the option bound i computed at time t, and $\bar{r}_t$ is the historical average forecast made at time t. A positive $R^2_{OS} > 0$ indicates that the bound generates a lower MSPE than the benchmark, thereby the out-of-sample evidence of return predictability of option bound. To assess the statistical significance of R^2_{OS}, we use the Clark and West's (2007) *MSPE-adjusted* statistic.

Before we report the main out-of-sample results, we first plot the time series of the Martin, BCK, CYL bounds, as well as the SII-based generalized bound in Figure 1. Consistent with the plots in Back, Crotty, and Kazempour (2022), the bounds are very volatile and occasionally quite high. The peaks occur in periods when measures of market uncertainty, such as the 08/09 financial crisis or the COVID-19 pandemic, are also very high. While the option bounds in the four panels appear to exhibit high correlation, our generalized bound consistently surpasses other bounds, staying above the historical average benchmark.

Table 2 reports out-of-sample R^2_{OS} statistics for 1-month market excess return forecasts based on option bounds for three out-of-sample periods. Martin Bound denotes the Martin (2017) bound, BCK Bound denotes the Back, Crotty, and Kazempour (2022) slackness-adjusted bound, and CYL Bounds denote the unrestricted and restricted (R) bounds from Chabi-Yo and Loudis (2020). Column (1) of Table 2 reports the results for the "long" out-of-sample period 2006:01–2022:12, which includes both 2008/09 financial crisis and 2020 COVID-19 pandemic. In Panel A, we find that option bounds in the existing literature perform poorly out of sample, all generating negative R^2_{OS} statistics. For example, Martin, BCK, and CYL (restricted) produce R^2_{OS} statistic of -0.59%, -0.54%, and -0.76%, respectively, suggesting weak or no predictive power of option bounds. By contrast, Panel B demonstrates much improved out-of-sample predictive power of the generalized bounds, except for the ones based on WG variables. Specifically, sentiment-based bounds produce economically sizable R^2_{OS} statistics that are larger than 3% (see for instance, Campbell and Thompson, 2008). Moreover, the SII-based bound generates an even larger R^2_{OS}, around 7.88% and significant at the 5% level.

Columns (3) and (4) of Table 2 report the results for two shorter periods, 2011:01–2022:12

and 2016:01–2022:12, after the 2008/09 global financial crisis. Compared to column (1), we find that option bounds, both existing and generalized ones, perform much better in the post-crisis era. For example, the R^2_{OS} statistics for Martin, BCK, and CYL (restricted) bounds jump to 2.14%, 2.80%, and 3.25%, respectively, during the 2011:01–2022:12 out-of-sample period, and the latter two become marginally significant at the 10% level. Nevertheless, the generalized bounds still outperform the existing ones: all generalized bounds in columns (3) and (4) become positive with 8 of them significant at the 10% level and 6 significant at the 5% level. Furthermore, we observe that both sentiment- and SII-based bounds consistently produce R^2_{OS} statistics larger than 10% in magnitude.

Among three category of traditional variables, the bound incorporating SII or sentiments outperforms the bound with macro variables, with the former exhibiting the most robust performance. To illustrate, the SII-based bound consistently generates significant R^2_{OS} values of 7.88%, 17.47%, and 20.00% across the three out-of-sample periods. One possible explanation for the underperformance of WG macro-based bounds may be attributed to the absence of consistent out-of-sample support for these macro variables, a criticism notably expressed by Welch and Goyal (2008). As detailed in the Internet Appendix, when constructing generalized bounds using 14 WG variables individually, merely 6 out of them yield positive R^2_{OS} statistics over the period 2006–2022, with only one of them achieving (marginally) significant results at the 10% level.[6]

We provide several statistical explanations for the superb performance of generalized bounds in Section 3.5. Before that, we first explore the economic values of generalized bounds, apart from the statistical gains in Table 2.

3.4 Out-of-sample economic values

In this subsection, we assess the economic values of different option bounds. We compare the benchmark and bound-based forecasts in terms of their economic values to a mean-variance

[6] Neely, Rapach, Tu, and Zhou (2014) show that applying principal component regression on 14 WG variables cannot outperform the historical average benchmark, either (see Table 3, page 1786, in their paper).

investor. Specifically, consider an investor who allocates across equities and a risk-free asset (the Treasury bill) each month. At the end of month t, the investor faces the objective function

$$\arg_{\hat{\omega}_{t+1}} \hat{\omega}_{t+1}\hat{r}_{t+1} - \frac{\gamma}{2}\hat{\omega}_{t+1}^2\hat{\sigma}_{t+1}^2, \tag{12}$$

where γ denotes the coefficient of relative risk aversion, $\{\hat{\omega}_{t+1}, 1-\hat{\omega}_{t+1}\}$ are allocation weights to the market portfolio and the risk-free asset at month $t+1$, $\hat{r}_{t+1}$ is the investor's forecast of market excess return, and $\hat{\sigma}_{t+1}^2$ is the forecast of the variance of the market excess return. The optimal mean-variance portfolio weight on the market can be computed as[7]

$$\hat{\omega}_{t+1}^{\star} = \left(\frac{1}{\gamma}\right)\left(\frac{\hat{r}_{t+1}}{\hat{\sigma}_{t+1}^2}\right), \tag{13}$$

Figure 2 plots the log cumulative excess returns for portfolios using market excess return forecasts based on Martin, BCK, CYL, and SII-based generalized bound over the out-of-sample period 2006:01–2022:12. The figure also depicts the log cumulative excess return for the portfolio based on the historical average benchmark forecast. Figure 2 reveals that the portfolio that incorporates the information from both option market and stock market generally exhibits superior performance compared to the portfolio based on the historical average benchmark forecast. In sharp contrast, the portfolio that solely relies on the option market information, for instance, Martin or CYL bound, underperforms the benchmark. Consistent with the patterns of statistical gains in Table 2, we find that the main reason that option bounds do not perform well relative to the benchmark is due to the 2008/09 financial crisis period. In contrast to the Martin, BCK and CYL bounds in Panels A, B and C of Figure 2, we find that the generalized bound excels in managing downside risk during the market downturn.

Over the out-of-sample period, we compute four performance measures: (1) average portfolio returns, μ_p, (2) portfolio return standard deviation, σ_p, (3) Sharpe ratio, $SR_p = \frac{\mu_p}{\sigma_p}$, and (4) certainty equivalent return (CER), $CER_p = \mu_p - \frac{\gamma}{2}\sigma_p$. The results are presented in Table 3. In Panel A,

[7] We follow Campbell and Thompson (2008) to set $\gamma = 3$ and to constrain the portfolio weight on the market portfolio to lie between $[0, 1.5]$ in Equation (13).

we find that although portfolios based on existing option bounds mostly achieve higher average returns than that based on historical average benchmark, they also exhibit much higher standard deviations, leading to smaller Sharpe ratios and negative CERs. For instance, the Sharpe ratios for Martin, BCK, and CYL (restricted) bounds are 0.18, 0.22, and 0.24, respectively, compared to that of 0.26 for the benchmark. In contrast, the generalized bounds produce comparable standard deviations to Martin or CYL bound but much larger average returns. As a result, we find that 6 out of 8 the generalized bounds produce Sharpe ratios larger than the benchmark, with 6 of them larger than 0.30, and SII-based bound larger than 0.6. We also consider adjusting for a proportional transaction cost of 50 basis points per transaction. The results are robust and are available in the Internet Appendix.

In light of the finding in Table 2 that option bounds may underperform during the financial crisis period, we also assess the out-of-sample economic gains during the post-crisis period 2011:01–2022:12 in Table 4. Indeed, we observe a much improved economic values for Martin, BCK and CYL bounds, with Sharpe ratios ranging from 0.67 to 0.73 and CERs ranging from 6.04% to 7.80%, slightly above those of the benchmark (around 0.64 and 4.83%). Notwithstanding, the generalized bounds continue to outperform the historical average benchmark with both higher Sharpe ratio and larger CERs. Using WG macro variables, the best performed generalized bound (WG PLS) achieves a Sharpe ratio of 0.83 and a CER of 9.89%. Using sentiment variables, most of generalized bounds produce a Sharpe ratio larger than 0.80 and a CER larger than 10%. Notably, the SII-based bound remains the best-performing in terms of economic value: it achieves an impressive Sharpe ratio of 1.04 and a CER of 14.92% over the period 2011:01–2022:12. The difference between two CERs, known as the utility gain (Pástor and Stambaugh, 2000; Campbell and Thompson, 2008), suggests that the investor would be willing to pay 10.09%, 8.88%, 7.68%, and 7.73% *per annum* to have access to the information in the SII-based generalized bound relative to the historical average, Martin bound, BCK bound, and CYL restricted bound, respectively.

Overall, we demonstrate consistent out-of-sample evidence of market return predictability by combining option theory with economic state variables. The results are both statistically and economically significant. To further understand the superb performance of the generalized bound, we next investigate several plausible explanations from a statistical point of view.

3.5 Statistical explanations

In this subsection, we provide three possible ways to explain the exceptional out-of-sample performance of the generalized bound. We first follow Welch and Goyal (2008) to plot the cumulative squared errors over time and then follow Theil (1966) to conduct a bias-variance decomposition. Finally, we provide a novel approach to evaluate the efficacy of option bounds by comparing their time-series distributions.

3.5.1 Cumulative squared error

To gain a better understanding of those forecasts, we first employ the methodology outlined by Welch and Goyal (2008) to calculate the cumulative differences in squared forecast errors between the historical average benchmark and the bound-based forecast,

$$\text{squared error difference} = (r_{t+1} - \bar{r}_t)^2 - (r_{t+1} - b_{t,i})^2, \tag{14}$$

where r_{t+1} is the realized return at time $t+1$, $b_{t,i}$ is the option bound i computed at time t, and $\bar{r}_t$ is the historical average forecast computed at time t.

Figure 3 plots the cumulative squared error difference over the out-of-sample period 2006:01–2022:12. This tool provides a straightforward way to assess whether a competing forecast can consistently outperform the benchmark over time. An increasing (decreasing) curve indicates that the competing forecast consistently exhibits a lower (higher) MSPE than the benchmark. Moreover, a curve that concludes higher (lower) at the end of the out-of-sample period than at the beginning is equivalent to a positive R^2_{OS} in Equation (11). Similar to the finding in Back, Crotty, and Kazempour (2022), we find that the bound-based forecasts substantially underperform and then partially recover in the financial crisis and in the COVID-19 pandemic. However, only the generalized bound fully recover by achieving a higher point at the end of the period than at the beginning. The outperformance during those extreme market downturn periods results in the overall significantly positive R^2_{OS} in Table 2 for the generalized bound.

3.5.2 Bias-variance trade-off

The R^2_{OS} statistic in Equation (11) essentially compares two MSPEs between two forecasts. We next adopt the decomposition of MSPE proposed by Theil (1966):

$$MSPE = \left(\bar{\hat{e}}\right)^2 + Var(\hat{e}), \tag{15}$$

where $\hat{e}$ signifies the forecast error, $\left(\bar{\hat{e}}\right)^2$ is the squared forecast bias, and $Var(\hat{e})$ is the forecast variance.

Figure 4 presents four scatterplots on the variance-bias plane. Each panel compares the historical average benchmark with an option bound. Within Panels A, B, and C, we observe that historical average forecasts lie in the top left, while Martin, BCK, and CYL bounds are situated in the bottom right. This suggests that option bounds exhibit much higher forecast variance compared to the benchmark, leading to negative R^2_{OS} statistics. In contrast, in Panel D, we note that the historical average is located in the top right of the panel, while the generalized bound is in the bottom left, suggesting that the generalized bound achieves both much lower forecast variance and higher forecast precision. In summary, the generalized bound better regularizes forecast variability, leading to substantially improved out-of-sample forecast accuracy.

3.5.3 Bound distribution

Apart from the time-series plot of the cumulative squared error difference and the classic bias-variance analysis, we next propose an alternative way to explain the superb performance of the generalized bound achieved by combing option theory with economic state variables.

In Figure 5, we plot the distribution of the realized 1-month market excess return (compounded over the 21 trading days) versus the 1-month option bound during the period 2006:01–2022:12. As depicted in each panel, the distribution of realized market excess return is very dispersed, ranging from extreme negative values to extreme positive values. In contrast, Martin, BCK, and CYL bounds in Panels A, B, and C are much concentrated and right skewed. Moreover, the modes of the those three distributions are just slightly above zero, which also suggests that those existing lower bounds are too loose. This observation is consistent with the summary statistics in Table 1 where we find that the mean bounds (less than 5%) are substantially smaller than the market excess return (around 8.26%) per year, and the median bounds (less than 5%) are even smaller compared to the median of market excess return (around 15.10%).

In Panel D of Figure 5, we observe that the distribution of the SII-based generalized bound demonstrates greater dispersion and a less pronounced right skew compared to the Martin and other bounds, though it remains less dispersed than the market excess return. More importantly, the mode and the median of the generalized bound distribution become comparable to those of the market excess return. This is also consistent with the summary statistics in Table 1 where the median of SII-based bound is 14.10%, which is similar to the median of market excess return. In other words, despite the uncertain and evolving nature of the data-generating process for expected market return, the generalized option bound, combining risk-neutral moments from option prices and economic state variables, manages to mimic the distribution of expected market return, significantly contributing to its out-of-sample predictive power.

3.6 Dynamics of bound adjustment, $k(x)$

We generalize Martin (2017) bound by theoretically incorporating the economic state variables into the bound construction. In this subsection, we compare the Martin and the generalized bounds.

Figure 6 comprises four panels, with the first three presenting bar plots that depict three generalized bounds derived from sentiment using three different shrinkage techniques, and the last one presenting the bound derived from the short interest index. Additionally, within each panel, a bar plot illustrating the Martin bound is also included. While the adjustment in each month varies depending on the chosen state variable, the overarching trend reveals that the generalized bounds consistently surpass the Martin bound most of the time. This pattern suggests a partial alleviation of slackness. It is also noteworthy that the adjustments during the 2008/09 financial crisis and the 2020 COVID-19 pandemic exhibit opposing trends: during the former period, we generally observe a *downward* adjustment of the Martin bound, whereas in the latter, an *upward* adjustment prevails.

In the context of Equation (5), the adjustment by the generalized bound essentially depends on the $k(x)$. An upward (downward) adjustment is equivalent to $k(x) > 1$ ($k(x) < 1$). Figure 7 depicts the estimated $k(x)$ over time based on different state variables. The solid lines depicted in Figure 7 indicate that $k(x)$ experiences a drastic decrease below one in the period spanning 2008-2009, followed by a partial (or complete) recovery to levels above one, contingent on the chosen state variable. The red dotted line represents the economic sentiment index by Shapiro, Sudhof, and Wilson (2022). Overall, we note a reasonable correlation between $k(x)$ and economic sentiment. Specifically, Martin bound is scaled down when economic sentiment is on the fall, and conversely, it is scaled up when economic sentiment is on the rise.

The limited use of Martin bound is contended either because of its significant slackness (Back, Crotty, and Kazempour, 2022), or the breach of the NCC (Bakshi et al., 2021). Theoretically, $k(x) = 1$ is equivalent to $NCC = 0$ (indicating a tight bound); $k(x) > 1$ is equivalent to $NCC < 0$ (which may result in slackness); and $k(x) < 1$ is equivalent to $NCC > 0$ (indicating a violation). In this regard, Figures 7 provides a visual explanation for the efficacy of Martin (2017) bound. We find that $k(x_t)$ consistently exceeds one for the majority of the observed period, notably over the past decade. This observation aligns with the finding of Back, Crotty, and Kazempour (2022), highlighting that the Martin bound, when used as a lower threshold, may not sufficiently capture market dynamics during normal periods. Conversely, during the 2008/09 financial crisis, $k(x)$ dips below one, indicating that the Martin bound, as a lower bound, might be unduly large during extreme market conditions, potentially leading to the violation of the inequality in Equation (3).

In Panel A of Table 5, we present the summary statistics of the time-series of estimated $k(x)$ based on different state variables over the out-of-sample period 2006:01–2022:12. For comparison, we express $k(x)$ in percent. Using WG macro variables, $k(x) > 1$ holds for more than half of the months, with mean values either above or slightly below the threshold of one. This may also help to explain the underperformance of WG-based bounds: the bounds are still loose. Using sentiment or short interest index, we find that $k(x) > 1$ holds for more than 75% cases, and sometimes more than 90%. As a result, the mean values of $k(x)$ range from 1.0031 to 1.0082, and are significant larger than one based on Newey and West (1987) standard errors. In Panel B when we switch to the out-of-sample period 2006:01–2022:12, we find that almost all $k(x)$ estimates become significantly larger than one, suggesting a further correction of slackness in the Martin (2017) bound. For instance, the mean value of $k(x)$ using short interest index becomes 1.0100, significant at the 1% level.

4 Robustness

In this section, we perform a battery of robustness checks, including subperiod analysis with respect to different economic conditions, alternative adjustment form to construct the generalized bound, extended data sample, and the longer-horizon forecast.

4.1 Bounds and economic conditions

As highlighted in Figures 2 and 3, the predictive power of option bounds might be heavily affected by the economic conditions. In light of this, we next compute the out-of-sample R^2_{OS} during periods of "High" or "Low" economic regimes over 2006:01–2022:12, where the regimes are based on sorted values of Shapiro, Sudhof, and Wilson's (2022) news sentiment and Bekaert, Engstrom, and Xu's (2022) risk aversion index (in turn). "High" and "Low" periods are defined using the top and bottom third of sorted values, respectively, in order to have a reasonable number of observations in each regime. The results are reported in Table 6.

We find that the predictive power of option bounds is mainly concentrated in the subperiod of high sentiment or low risk aversion: both BCK slackness-adjusted bound and our generalized bounds produce significantly positive R^2_{OS} statistics, though the R^2_{OS} of the latter remain much larger in magnitude. In contrast, we observe that neither Martin nor CYL bound can outperform the historical average benchmark in subperiods. One possible explanation is that Martin and CYL bound rely sole on the risk-neutral information implied from derivative market, whereas BCK and generalized bounds incorporate both stock market and derivative market information. To examine the robustness of the results, we also analyze the bound forecast during NBER-dated business cycles. The results are reported in the Internet Appendix.

4.2 Alternative adjustment factor $k(x)$

In this subsection, we consider two alternative forms for the adjustment factor, $k(x)$ in Equation (5). We first consider a linear function form such that

$$k(x) = \alpha + \beta x, \tag{16}$$

where α and β are constants. In addition, we also consider incorporating both interaction and individual terms,[8]

$$k(x) = \exp(\alpha + \beta x) + \lambda, \tag{17}$$

where α, β, and λ are constants.

We repeat the out-of-sample test with the new specifications and present the results in Table 7. The results are quite similar to those in Table 2. Combing options with traditional variables greatly improve the out-of-sample forecast accuracy with SII-based generalized bound as the best performing one. Overall, our theory-based generalized bound is robust to the different choices of function form $k(x)$.

[8] We thank Tyler Beason for this valuable suggestion.

4.3 Extended sample period

Our sample begins in 1996 due to the availability of option data. As mentioned in Equation (5), Martin bound is expressed in terms of SVIX index.

Since SVIX and CBOE VIX index are highly correlated (with a correlation coefficient of 0.99), we next use VIX data to construct a similar 'Martin bound',

$$b_{t,M} = \begin{cases} (T-t)R_{f,t,T}\text{SVIX}^2_{t\to T}, & \text{if} \quad t \geq 1996; \\ (T-t)R_{f,t,T}\text{VIX}^2_{t\to T}, & \text{if} \quad t < 1996, \end{cases} \tag{18}$$

and thus extend the full sample period to 1990:01–2022:12. Subsequently, we can construct the generalized bound by Equation (9).

The out-of-sample R^2_{OS} statistics with extended data sample are presented in Table 8. We consider the same three out-of-sample evaluation periods: 2006:01–2022:12, 2011:01–2022:12, and 2016:01–2022:12. We observe similar results as in Table 2. For example, Martin bound cannot outperform the benchmark during the period 2006:01–2022:12, yielding a negative R^2_{OS}, though the predictive power partially recovers after the 2008/09 financial crisis with R^2_{OS} jumping to 2.09% and 2.29%, over the periods 2011:01–2022:12 and 2016:01–2022:12, respectively. The generalized bounds based on sentiment and short interest index continue to outperform both the historical average benchmark and the established option bounds.

Rather than using both VIX and SVIX to construct the bound, we can simply employ VIX to create the entire 'Martin bound' from 1990 to 2022. Figure 8 depicts the 30-day SVIX index alongside the CBOE VIX index. Both indices exhibit nearly identical patterns. As shown in Table 9, both Martin and generalized bounds by VIX exhibit similar results to the ones by SVIX as in Tables 2 and 8. Since VIX and SVIX both capture important aspects of market returns, their disparity can be minimal under certain conditions, such as log-normality (see Martin, 2017).

4.4 Longer-horizon forecast

In this subsection, we conduct out-of-sample forecasting for longer horizons. Following Back, Crotty, and Kazempour (2022), we linearly interpolate the option bounds to correspond with 90, 180, and 360-day maturities. We then align these with realized returns over 63, 126, and 252 trading days. Table 10 presents the out-of-sample R_{OS}^2 statistics for 1-, 3-, 6-, and 12-month market excess return forecasts based on option bounds during the out-of-sample evaluation period 2006:01–2022:12.

We find that option bounds generally perform well relative to the historical mean in long-horizon forecasting, exhibiting mostly positive R_{OS}^2 statistics. Martin and BCK bounds achieve significantly positive R_{OS}^2 statistics when predicting 6- and 12-month market excess returns. CYL bounds perform substantially well in 6- and 12-month market excess return forecasts, displaying large and significantly positive R_{OS}^2 statistics. Additionally, our generalized bounds continue to outperform the historical mean in long-horizon forecasting. For instance, the R_{OS}^2 statistics from the SII-based bound rise to significant values of 11.70%, 18.66%, and 22.83% for 3-, 6-, and 12-month return forecasts, respectively. These findings are expected given the overall upward trend observed in the market over the long term. Similar findings are reported by Campbell and Thompson (2008) in their analysis of forecasting 12-month market excess return through the use of macro variables (refer to Panel B, Table 2, in their paper). They highlight a substantial R_{OS}^2 value, reaching as high as 7.89% for the 12-month return forecast achieved through an unconstrained simple predictive regression.[9] However, it is worth noting that the same regression approach has also faced major criticism since Welch and Goyal (2008) for its inability to outperform the historical average benchmark, particularly when used for 1-month return forecasting.

In summary, we find that existing bounds, including Martin, BCK, and CYL, exhibit strong predictive power when employed to forecast longer-term market returns. Conversely, our generalized bounds, combining option theory and traditional variables, show substantial

[9] The R_{OS}^2 values are not directly comparable as our R_{OS}^2 is derived from using the bound as a meaningful expected return, while the R_{OS}^2 from Campbell and Thompson (2008) is obtained through regression forecasts.

forecasting gains consistently across short-term (1-month), medium-term (3- or 6-month), and long-term (12-month) horizons.

5 Conclusion

Predicting the market risk premium is one of the fundamental challenges in finance, which is of interest because it is a pivotal determinant of the required rate of return for investors to hold assets in asset pricing models. Despite numerous studies on the time-series predictability in the literature, the out-of-sample predictability continues to exhibit limited effectiveness. Recent developments by Martin (2017) and others offer new insights into learning about the expected market return from options prices; however, the empirical performance remains unsatisfactory.

In this paper, we provide a novel approach to combine two lines of literature on market risk premium. We theoretically derive a new bound on the market risk premium by combining risk-neutral variance with traditional economic state variables. We further show that the new bound performs well empirically and the improvement in out-of-sample forecasting is economically substantial. Collectively, our paper provides new insights into the market risk premium by drawing perspectives from both the time-series return predictability literature and the option literature.

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Table 1: Summary statistics

This table reports summary statistics for realized returns and option bounds. The sample comprises monthly observations from 2006:01 to 2022:12. The market excess return is the S&P 500 return in excess of the risk-free rate. Martin Bound denotes the Martin (2017) bound, BCK Bound denotes the Back, Crotty, and Kazempour (2022) slackness-adjusted bound, and CYL Bound denotes the unrestricted and restricted (R) bounds from Chabi-Yo and Loudis (2020). Panel B reports summary statistics of the generalized bounds based on state variables indicated in row headings. WG Average, WG PCA, WG PLS, and WG S-PCA are the generalized bounds based on the cross-sectional average, first principal component, partial least square, and scaled first principal component extracted from the 14 macro variables in Welch and Goyal (2008), respectively. Sent Average, Sent PCA, Sent PLS, and Sent S-PCA represent those resulting from applying different shrinkage methods to the 5 sentiment measures (Baker and Wurgler, 2006; Huang et al., 2015; Shapiro, Sudhof, and Wilson, 2022). SII represents the generalized bound based on short interest index (Rapach, Ringgenberg, and Zhou, 2016). All results are annualized and expressed in percent.

(1)	(2) Mean (%)	(3) S.D. (%)	(4) P10 (%)	(5) P25 (%)	(6) P50 (%)	(7) P75 (%)	(8) P90 (%)
Panel A: Market excess return and established bounds							
Market excess return	8.26	57.53	−72.81	−15.72	15.10	40.82	71.09
Martin Bound	3.72	3.69	1.11	1.59	2.46	4.38	7.16
BCK Bound	5.28	3.52	2.24	3.19	4.30	6.39	9.67
CYL Bound	5.80	5.95	1.79	2.42	3.78	6.95	10.76
CYL Bound (R)	4.90	5.24	1.47	1.97	3.10	5.80	9.40
Panel B: Generalized bounds							
State Variables							
WG Avg	4.44	6.64	0.00	0.91	2.60	5.17	10.40
WG PCA	5.42	4.87	0.00	2.47	4.27	6.91	13.80
WG PLS	12.73	8.57	0.20	6.85	11.76	17.36	23.32
WG S-PCA	6.44	5.78	0.00	2.67	4.68	9.05	16.21
Sent Avg	8.25	5.03	3.75	5.56	7.20	10.04	14.44
Sent PCA	8.50	3.27	5.33	6.45	8.18	9.58	11.69
Sent PLS	7.53	3.61	2.33	5.20	7.63	10.24	11.51
Sent S-PCA	7.66	3.76	3.00	5.15	7.68	10.19	11.97
SII	13.63	6.26	3.13	10.55	14.10	18.00	21.49

Table 2: R^2_{OS} (%) for market risk premium forecast

This table reports out-of-sample R^2_{OS} for 1-month market excess return forecasts by option bounds. Panel A uses the option bounds by Martin (2017), the slackness-adjusted bound by Back, Crotty, and Kazempour (2022), and the four risk-neutral moments-based bounds by Chabi-Yo and Loudis (2020) (both unrestricted and restricted). Panel B uses the generalized bounds based on state variables indicated in row headings. WG Average, WG PCA, WG PLS, and WG S-PCA are the generalized bounds based on the cross-sectional average, first principal component, partial least square, and scaled first principal component extracted from the 14 macro variables, respectively. Sent Average, Sent PCA, Sent PLS, and Sent S-PCA represent those resulting from applying different shrinkage methods to the 5 sentiment measures. SII represents the generalized bound based on short interest index. *, ** and *** indicate significance at the 10%, 5% and 1% levels for the positive R^2_{OS}, respectively, based on Clark and West (2007) test.

(1) Out-of-sample periods	(2) 2006:01–2022:12	(3) 2011:01–2022:12	(4) 2016:01–2022:12
	Panel A: Established bounds		
Martin Bound	−0.59	2.14	2.27
BCK Bound	−0.54	2.80*	2.99
CYL Bound	−0.84	3.89*	3.93
CYL Bound (R)	−0.76	3.25*	3.44
	Panel B: Generalized bounds		
State Variables			
WG Avg	−4.07	1.97	2.30
WG PCA	−0.91	2.68	3.44
WG PLS	−1.60	0.58***	2.49*
WG S-PCA	−1.99	1.95	3.07
Sent Avg	−1.24	3.90*	3.14
Sent PCA	3.55	15.10**	13.63
Sent PLS	3.80	14.30**	12.65
Sent S-PCA	3.37	14.54**	13.29
SII	7.88**	17.47***	20.00**

Table 3: Out-of-sample economic gains, 2006:01–2022:12

This table provides various economic performance measures for a mean-variance investor with a relative risk aversion coefficient of three when using various option bounds as forecasts for market risk premium. The investor reallocates between equities and risk-free bills monthly during the out-of-sample period 2006:01–2022:12. Allocation weights depend on the bound-based return forecasts as indicated by panel headings. WG Average, WG PCA, WG PLS, and WG S-PCA are the generalized bounds based on the cross-sectional average, first principal component, partial least square, and scaled first principal component extracted from the 14 macro variables, respectively. Sent Average, Sent PCA, Sent PLS, and Sent S-PCA represent those resulting from applying different shrinkage methods to the 5 sentiment measures. SII represents the generalized bound based on short interest index. The computed performance measures include out-of-sample average excess return, standard deviation, Sharpe ratio, and certainty equivalent return (CER). All results are annualized.

(1)	(2) Avg. Ret. (%)	(3) S.D. (%)	(4) Sharpe Ratio	(5) CER (%)
Panel A: Benchmark and established bounds				
HA	2.77	10.59	0.26	1.08
Martin Bound	3.09	16.94	0.18	−1.22
BCK Bound	4.23	19.65	0.22	−1.56
CYL Bound	5.54	19.53	0.28	−0.18
CYL Bound (R)	4.41	18.39	0.24	−0.66
Panel B: Generalized bounds				
State Variables				
WG Avg	2.59	17.70	0.15	−2.10
WG PCA	4.82	17.71	0.27	0.11
WG PLS	8.17	18.23	0.45	3.18
WG S-PCA	2.65	17.60	0.15	−1.99
Sent Avg	6.78	21.03	0.32	0.15
Sent PCA	7.91	21.61	0.37	0.91
Sent PLS	6.90	19.16	0.36	1.39
Sent S-PCA	6.54	19.66	0.33	0.74
SII	14.51	23.10	0.63	6.51

Table 4: Out-of-sample economic gains, 2011:01–2022:12

This table provides various economic performance measures for a mean-variance investor characterized by a relative risk aversion coefficient of three when using various option bounds as forecasts for market risk premium. The investor reallocates between equities and risk-free bills on a monthly basis during the out-of-sample period from 2011:01 to 2022:12 (post the 2008/09 financial crisis). The allocation weights depend on the bound-based return forecasts as indicated by the panel headings. WG Average, WG PCA, WG PLS, and WG S-PCA are the generalized bounds based on the cross-sectional average, first principal component, partial least square, and scaled first principal component extracted from the 14 macro variables, respectively. Sent Average, Sent PCA, Sent PLS, and Sent S-PCA represent those resulting from applying different shrinkage methods to the 5 sentiment measures. SII represents the generalized bound based on short interest index. The computed performance measures encompass out-of-sample average excess return, standard deviation, Sharpe ratio, and certainty equivalent return (CER). All results are annualized.

(1)	(2) Avg. Ret. (%)	(3) S.D. (%)	(4) Sharpe Ratio	(5) CER (%)
Panel A: Benchmark and established bounds				
HA	6.25	9.70	0.64	4.83
Martin Bound	8.45	12.67	0.67	6.04
BCK Bound	11.76	17.36	0.68	7.24
CYL Bound	11.45	15.60	0.73	7.80
CYL Bound (R)	10.23	14.22	0.72	7.19
Panel B: Generalized bounds				
State Variables				
WG Avg	7.66	13.11	0.58	5.08
WG PCA	11.25	17.00	0.66	6.91
WG PLS	14.29	17.14	0.83	9.89
WG S-PCA	9.17	15.24	0.60	5.69
Sent Avg	13.39	18.04	0.74	8.51
Sent PCA	15.88	18.30	0.87	10.86
Sent PLS	15.07	17.95	0.84	10.24
Sent S-PCA	15.19	17.85	0.85	10.42
SII	21.03	20.20	1.04	14.92

Table 5: Estimates of $k(x)$, expressed in percent

This table reports summary statistics for the time-series estimates of $k(x_t) = \exp(\alpha_t + \beta_t x_t)$ where $\{\alpha_t, \beta_t\}$ are estimated recursively. WG Average, WG PCA, WG PLS, and WG S-PCA are the generalized bounds based on the cross-sectional average, first principal component, partial least square, and scaled first principal component extracted from the 14 macro variables, respectively. Sent Average, Sent PCA, Sent PLS, and Sent S-PCA represent those resulting from applying different shrinkage methods to the 5 sentiment measures. SII represents the generalized bound based on short interest index. All results expressed in percent. * and *** indicate significance at the 10% and 1% levels, respectively, for the null hypothesis $k(x) = 1$, based on Newey and West (1987) standard errors.

(1)	(2) Mean (%)	(3) P10 (%)	(4) P25 (%)	(5) P50 (%)	(6) P75 (%)	(7) P90 (%)
			Panel A: Out-of-sample period 2006:01–2022:12			
State Variables						
WG Avg	100.04	99.70	99.90	100.00	100.12	100.34
WG PCA	99.97	99.65	99.93	100.21	100.26	100.45
WG PLS	100.44*	99.55	100.23	100.81	101.22	101.81
WG S-PCA	99.98	99.53	99.98	100.19	100.32	101.13
Sent Avg	100.35***	100.01	100.23	100.37	100.52	100.75
Sent PCA	100.40***	100.18	100.27	100.40	100.55	100.68
Sent PLS	100.31***	99.79	100.21	100.39	100.61	100.75
Sent S-PCA	100.33***	99.88	100.18	100.33	100.62	100.76
SII	100.82***	100.10	100.67	100.89	101.12	101.42
			Panel B: Out-of-sample period 2011:01–2022:12			
State Variables						
WG Avg	99.98	99.72	99.91	100.00	100.08	100.17
WG PCA	100.15***	99.86	100.11	100.22	100.25	100.28
WG PLS	100.86***	99.99	100.38	100.86	101.45	102.03
WG S-PCA	100.09*	99.69	100.04	100.18	100.27	100.32
Sent Avg	100.31***	99.68	100.24	100.38	100.49	100.67
Sent PCA	100.42***	100.20	100.31	100.41	100.56	100.68
Sent PLS	100.41***	100.09	100.28	100.39	100.61	100.78
Sent S-PCA	100.38***	100.12	100.20	100.33	100.52	100.74
SII	101.00***	100.73	100.81	100.94	101.14	101.44

Table 6: R^2_{OS} (%) during subperiods of extreme market conditions

This table reports out-of-sample R^2_{OS} for 1-month market excess return forecasts by option bounds over the period from 2006:01 to 2022:12. We compute R^2_{OS} in two subperiods corresponding to the top tercile (High) and the bottom tercile (Low) of values based on economic news sentiment by Shapiro, Sudhof, and Wilson (2022) or the risk aversion index by Bekaert, Engstrom, and Xu (2022). WG Average, WG PCA, WG PLS, and WG S-PCA are the generalized bounds based on the cross-sectional average, first principal component, partial least square, and scaled first principal component extracted from the 14 macro variables, respectively. Sent Average, Sent PCA, Sent PLS, and Sent S-PCA represent those resulting from applying different shrinkage methods to the 5 sentiment measures. SII represents the generalized bound based on short interest index. *, **, and *** indicate significance at the 10%, 5%, and 1% levels for positive R^2_{OS}, respectively, based on the Clark and West (2007) test.

(1)	(2)	(3)	(4)	(5)
	Economic sentiment		Risk aversion	
	High	Low	High	Low
	Panel A: Established bounds			
Martin Bound	−3.12	−1.52	−1.68	−4.41
BCK Bound	1.96**	−1.58	−2.49	4.69***
CYL Bound	−0.61	−3.27	−3.72	−0.28
CYL Bound (R)	−1.84	−2.59	−2.87	−2.33
	Panel B: Generalized bounds			
State Variables				
WG avg	−3.63	−6.78	−7.19	−4.98
WG PCA	−1.93	−0.87	−2.41	2.81
WG PLS	7.14***	−5.66	−7.08	24.55***
WG S-PCA	−3.96	−2.8	−3.88	1.74*
Sent Avg	2.58*	−3.67	−4.99	6.96***
Sent PCA	6.73**	−2.42	−5.88	20.11***
Sent PLS	6.81**	−1.09	−4.84	21.02***
Sent S-PCA	6.11**	−1.84	−5.36	20.49***
SII	12.13***	−1.93	−2.99	26.87***

Table 7: R^2_{OS} (%) by alternative construction forms

This table reports out-of-sample R^2_{OS} statistics for 1-month market excess return forecasts based on the generalized option bounds with alternative construction forms as indicated in panel headings. WG Average, WG PCA, WG PLS, and WG S-PCA are the generalized bounds based on the cross-sectional average, first principal component, partial least square, and scaled first principal component extracted from the 14 macro variables, respectively. Sent Average, Sent PCA, Sent PLS, and Sent S-PCA represent those resulting from applying different shrinkage methods to the 5 sentiment measures. SII represents the generalized bound based on short interest index. *, ** and *** indicate significance at the 10%, 5% and 1% levels for the positive R^2_{OS}, respectively, based on Clark and West (2007) test.

(1) Out-of-sample periods	(2) 2006:01–2022:12	(3) 2011:01–2022:12	(4) 2016:01–2022:12
Panel A: $b_{t,LLXZ} = (\alpha + \beta x_t) \times (b_{t,M} + R_{f,t,T}) - R_{f,t,T}$			
State Variables			
WG Avg	−4.19	2.04	2.40
WG PCA	−0.89	2.66	3.43
WG PLS	−1.62	0.55***	2.47*
WG S-PCA	−3.83	2.31	3.17
Sent Avg	−1.25	3.89*	3.13
Sent PCA	3.58	15.14**	13.65
Sent PLS	3.80	14.30**	12.65
Sent S-PCA	2.71	14.04*	12.78
SII	7.88**	17.47***	19.98**
Panel B: $b_{t,LLXZ} = [\exp(\alpha + \beta x_t) + \lambda] \times (b_{t,M} + R_{f,t,T}) - R_{f,t,T}$			
State Variables			
WG Avg	−4.17	1.95	2.27
WG PCA	−0.88	2.68*	3.45
WG PLS	−1.14	1.47***	3.03**
WG S-PCA	0.58	1.93	1.91
Sent Avg	−1.24	3.90*	3.15
Sent PCA	3.56	15.12**	13.66
Sent PLS	2.29	13.19*	11.61
Sent S-PCA	2.69	13.74*	13.27
SII	7.90**	17.50***	20.04**

Table 8: R^2_{OS} (%) by using both SVIX and VIX to construct option bounds

This table reports out-of-sample R^2_{OS} statistics for 1-month market excess return forecasts based on option bounds using an extended sample. In Panel A, we construct the Martin (2017) bound using VIX during 1990:01–1995:12 and SVIX during 1996:01–2022:12. In Panel B, we construct the generalized bounds based on the Martin bound in Panel A. WG Average, WG PCA, WG PLS, and WG S-PCA are the generalized bounds based on the cross-sectional average, first principal component, partial least square, and scaled first principal component extracted from the 14 macro variables, respectively. Sent Average, Sent PCA, Sent PLS, and Sent S-PCA represent those resulting from applying different shrinkage methods to the 5 sentiment measures. SII represents the generalized bound based on short interest index. *, ** and *** indicate significance at the 10%, 5% and 1% levels for the positive R^2_{OS}, respectively, based on Clark and West (2007) test.

(1) Out-of-sample periods	(2) 2006:01–2022:12	(3) 2011:01–2022:12	(4) 2016:01–2022:12
	Panel A: Martin (2017) bound		
Martin Bound	−0.70	2.09*	2.29
	Panel B: Generalized bounds		
State Variables			
WG Avg	−1.43	2.23	2.78
WG PCA	−1.16	2.89*	3.22
WG PLS	−0.24	0.45	0.95
WG S-PCA	−0.76	0.02	−0.26
Sent Avg	−1.00	4.14**	3.18
Sent PCA	3.75	13.10**	13.83
Sent PLS	3.74	12.21**	13.07
Sent S-PCA	4.23	12.64**	14.03*
SII	8.12**	15.83***	18.87**

Table 9: R^2_{OS} (%) by using VIX only to construct option bounds

This table presents out-of-sample R^2_{OS} statistics for 1-month market excess return forecasts based on option bounds using an extended sample. Panel A reports the results of a modified Martin (2017) bound constructed by VIX over the period 1990:01–2022:12. Panel B reports the results of generalized bounds based on the Martin bound in Panel A. WG Average, WG PCA, WG PLS, and WG S-PCA are the generalized bounds based on the cross-sectional average, first principal component, partial least square, and scaled first principal component extracted from the 14 macro variables, respectively. Sent Average, Sent PCA, Sent PLS, and Sent S-PCA represent those resulting from applying different shrinkage methods to the 5 sentiment measures. SII represents the generalized bound based on short interest index. *, ** and *** indicate significance at the 10%, 5% and 1% levels for the positive R^2_{OS}, respectively, based on Clark and West (2007) test.

(1) Out-of-sample periods	(2) 2006:01–2022:12	(3) 2011:01–2022:12	(4) 2016:01–2022:12
	Panel A: Martin (2017) bound by VIX		
Martin Bound	−0.56	3.36**	3.53*
	Panel B: Generalized bounds		
State Variables			
WG Avg	−1.74	2.78*	3.53
WG PCA	−1.39	3.44*	4.02*
WG PLS	−0.04	0.76	1.29
WG S-PCA	−1.00	2.58*	3.39*
Sent Avg	−1.29	4.73**	3.94
Sent PCA	3.49	13.74**	14.69*
Sent PLS	3.58	12.89**	13.95
Sent S-PCA	2.92	11.99*	13.65
SII	7.99**	16.57***	19.81**

Table 10: R^2_{OS} statistics (%) for longer-horizon forecasts

This table displays out-of-sample R^2_{OS} statistics for 1-, 3-, 6-, and 12-month market excess return forecasts based on option bounds. The out-of-sample period is from 2006:01 to 2022:12, with a 10-year initial estimation window. Panel A uses the option bounds by Martin (2017), the slackness-adjusted bound by Back, Crotty, and Kazempour (2022), and the four risk-neutral moments-based bounds by Chabi-Yo and Loudis (2020) (both unrestricted and restricted). Panel B uses the generalized option bounds based on the state variables indicated in the row heading. WG Average, WG PCA, WG PLS, and WG S-PCA are the generalized bounds based on the cross-sectional average, first principal component, partial least square, and scaled first principal component extracted from the 14 macro variables, respectively. Sent Average, Sent PCA, Sent PLS, and Sent S-PCA represent those resulting from applying different shrinkage methods to the 5 sentiment measures. SII represents the generalized bound based on short interest index. *, ** and *** indicate significance at the 10%, 5% and 1% levels for the positive R^2_{OS}, respectively, based on Clark and West (2007) test.

(1) Horizons	(2) 1-month	(3) 3-month	(4) 6-month	(5) 12-month
		Panel A: Established bounds		
Martin bound	−0.59	2.26	7.14***	9.88***
BCK bound	−0.54	1.99	6.31***	9.64***
CYL bound	−0.84	3.50*	12.73***	25.74***
CYL bound (R)	−0.76	3.19*	12.70***	26.73***
		Panel B: Generalized bounds		
State Variables				
WG avg	−4.07	−10.08	−2.00	−25.42
WG PCA	−0.91	−1.98	−7.65	−17.14
WG PLS	−1.60	6.16***	12.24***	15.22***
WG S-PCA	−1.99	−6.21	−12.66	−22.99
Sent Avg	−1.24	0.43	5.43***	7.47***
Sent PCA	3.55	2.14**	10.29***	12.53***
Sent PLS	3.80	−0.36	6.36***	8.74***
Sent S-PCA	3.37	0.13**	7.14***	8.03***
SII	7.88**	11.70***	18.66***	22.83***

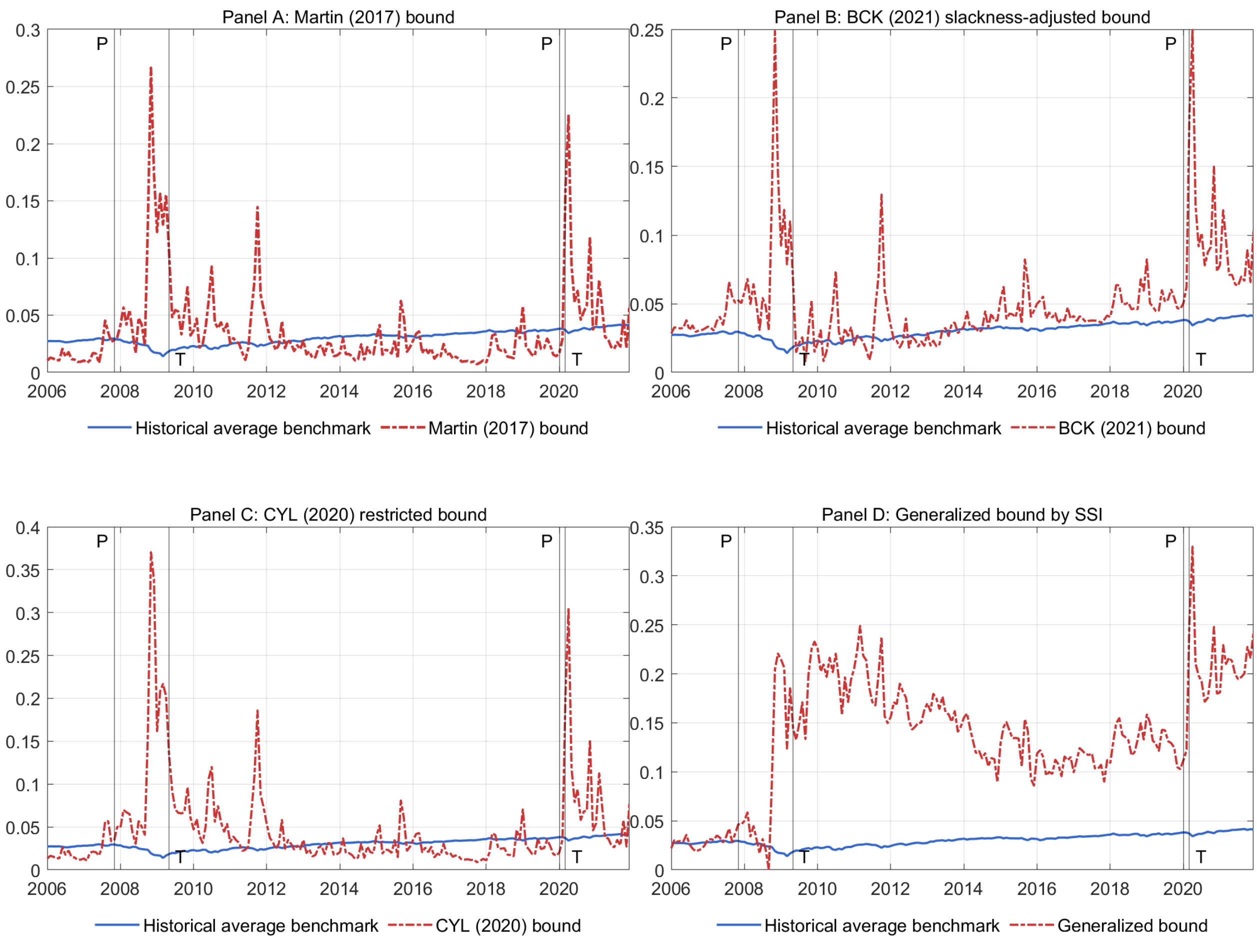

Figure 1: Out-of-sample forecasts

Each panel depicts the 1-month market excess return forecasts by historical average benchmark (blue solid line) and the option bound (red dotted line) as indicated in each panel heading over the out-of-sample period from 2006:01 to 2022:12.

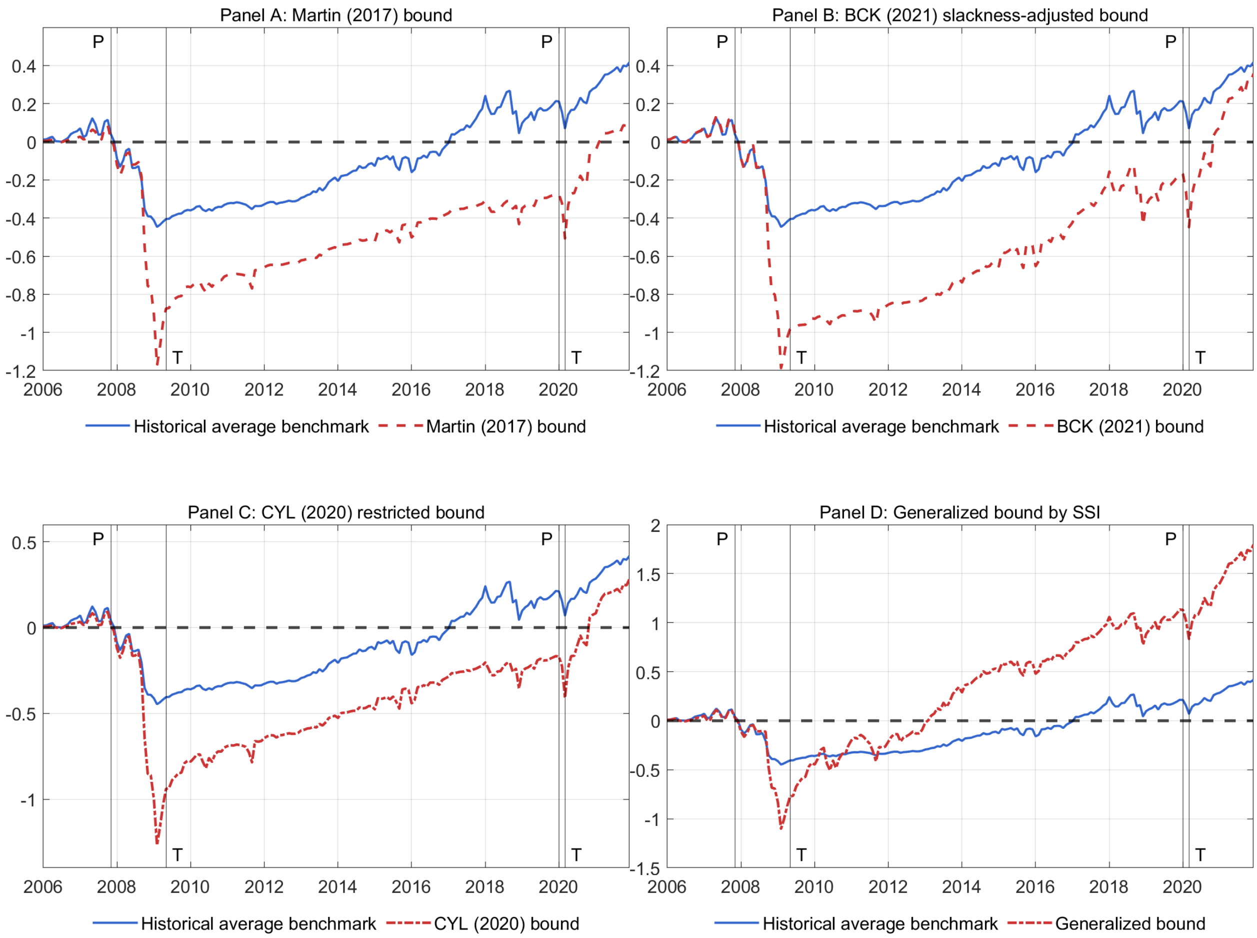

Figure 2: Log cumulative excess returns for out-of-sample bound-based portfolios

Each panel depicts the log cumulative excess return for a portfolio constructed using the historical average benchmark forecast (blue solid line) and the bound-based forecast (red dotted line) indicated in the panel heading for the out-of-sample period from 2006:01 to 2022:12. Vertical lines indicate NBER-dated business-cycle peaks (P) and troughs (T).

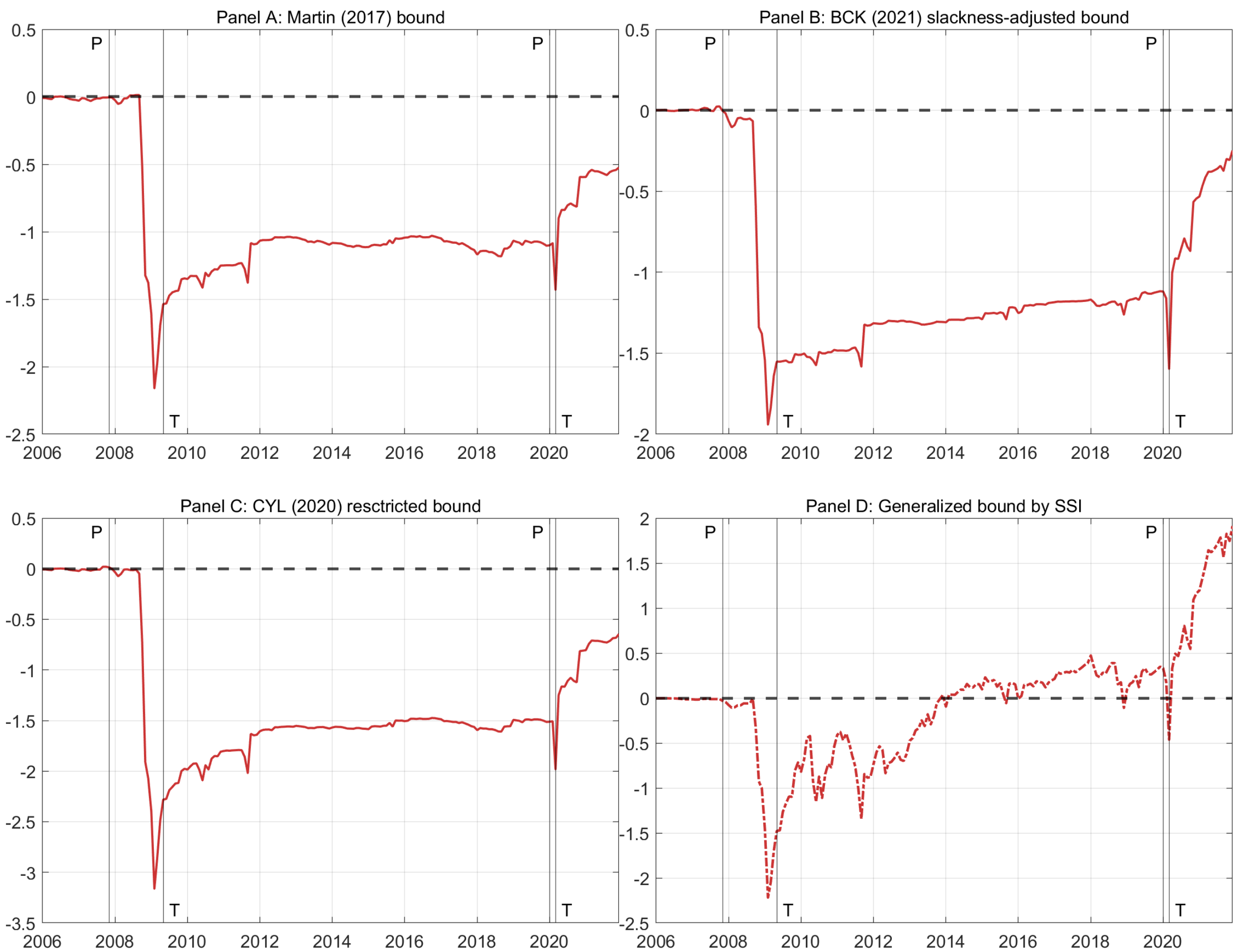

Figure 3: Cumulative squared errors

Each panel depicts the cumulative squared error of the benchmark minus the cumulative squared error of the bound-based forecast over the out-of-sample period from 2006:01 to 2022:12,

$$\text{squared error difference} = (r_{t+1} - \bar{r}_t)^2 - (r_{t+1} - b_{t,i})^2,$$

where r_{t+1} is the realized market excess return, $\bar{r}_t$ is the historical average benchmark, and $b_{t,i}$ is the bound-based forecast indicated in each panel heading. Vertical lines indicate NBER-dated business-cycle peaks (P) and troughs (T).

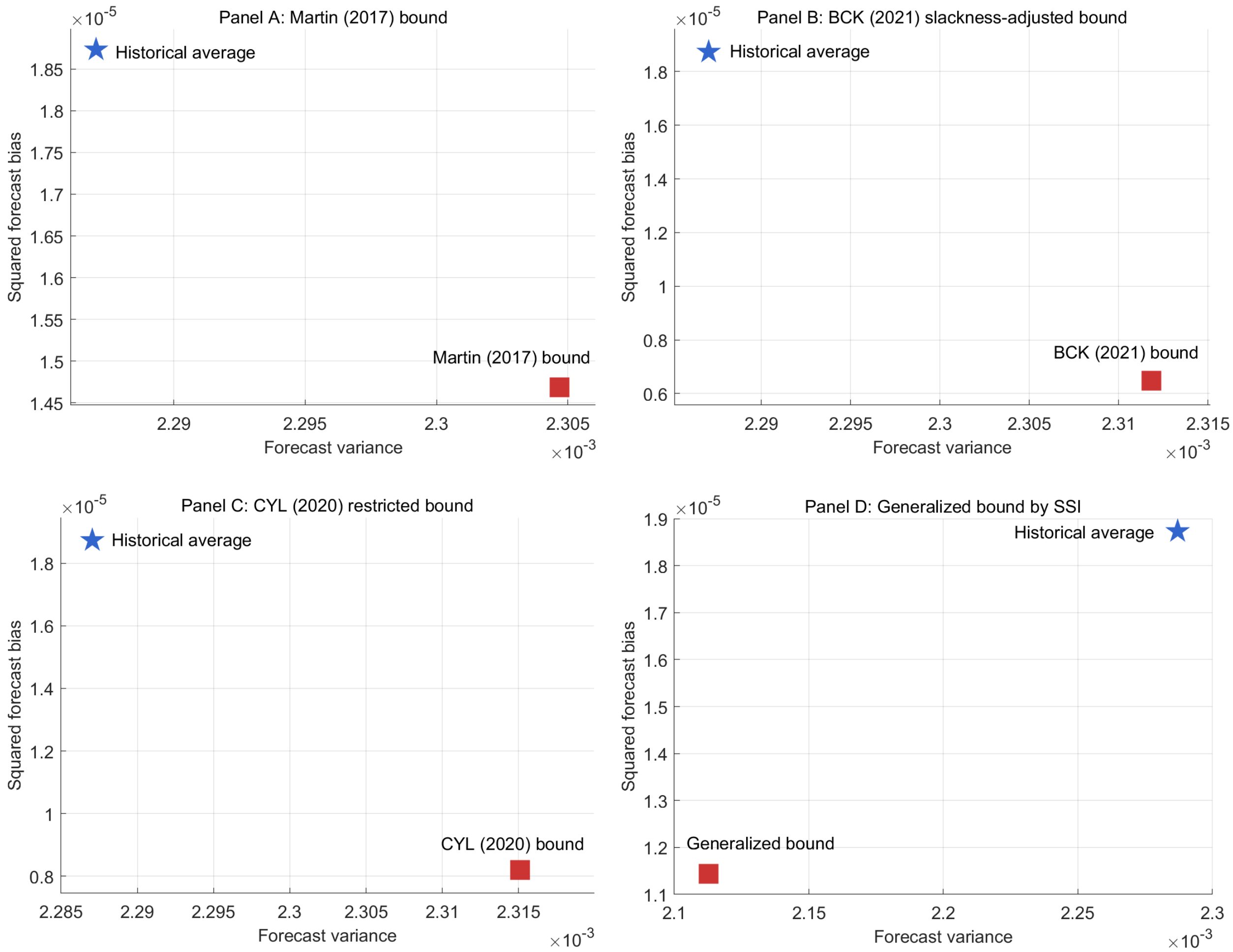

Figure 4: Bias-Variance decomposition

Each panel depicts the bias-variance decomposition of the mean squared prediction error (MSPE) using forecasts derived from option bounds or historical average benchmark during the out-of-sample period from 2006:01 to 2022:12.

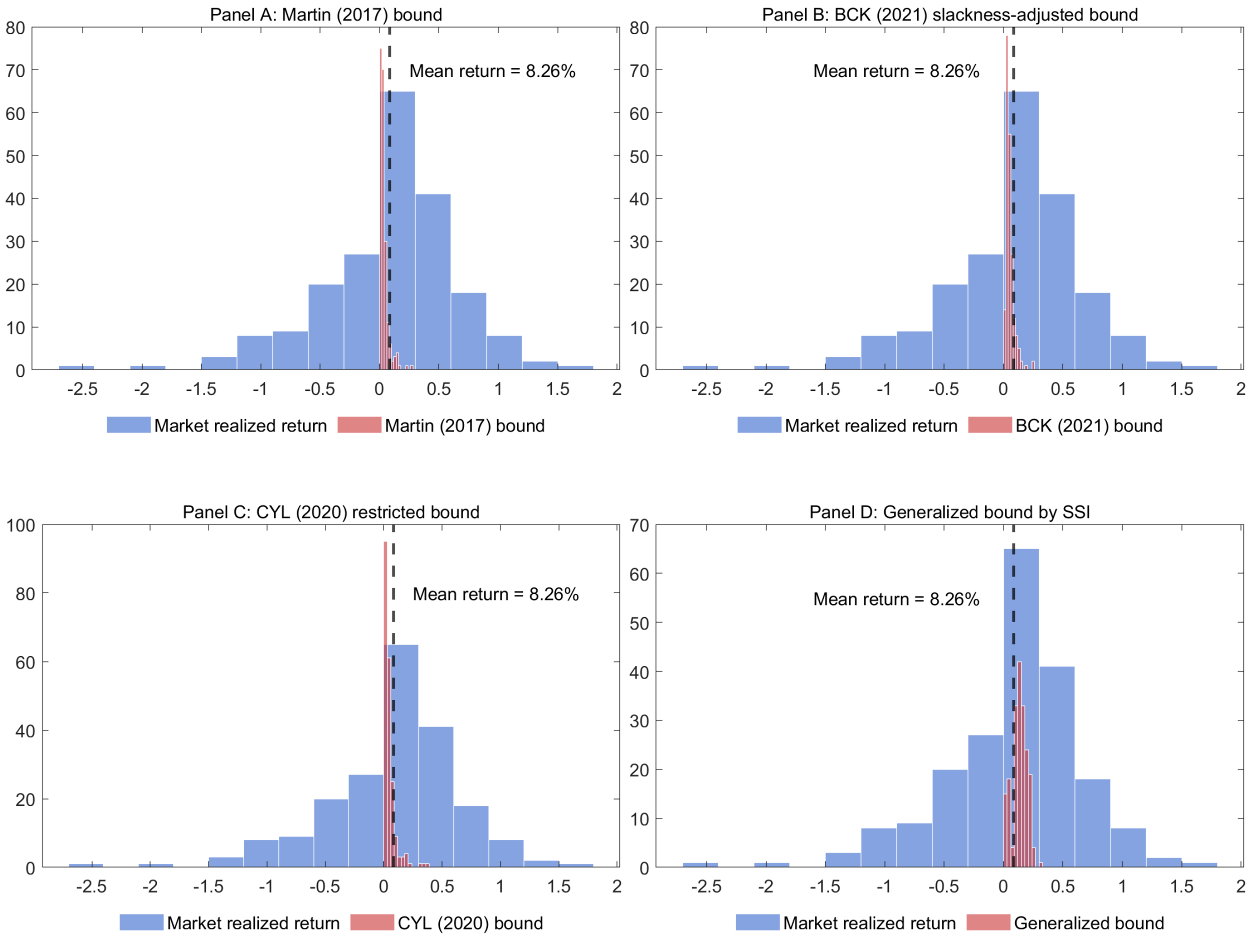

Figure 5: Distribution of option bounds

Each panel depicts the distribution of the realized market excess return and the option bound as indicated in the panel heading over the out-of-sample period from 2006:01 to 2022:12. The vertical dotted line denotes the mean market excess return.

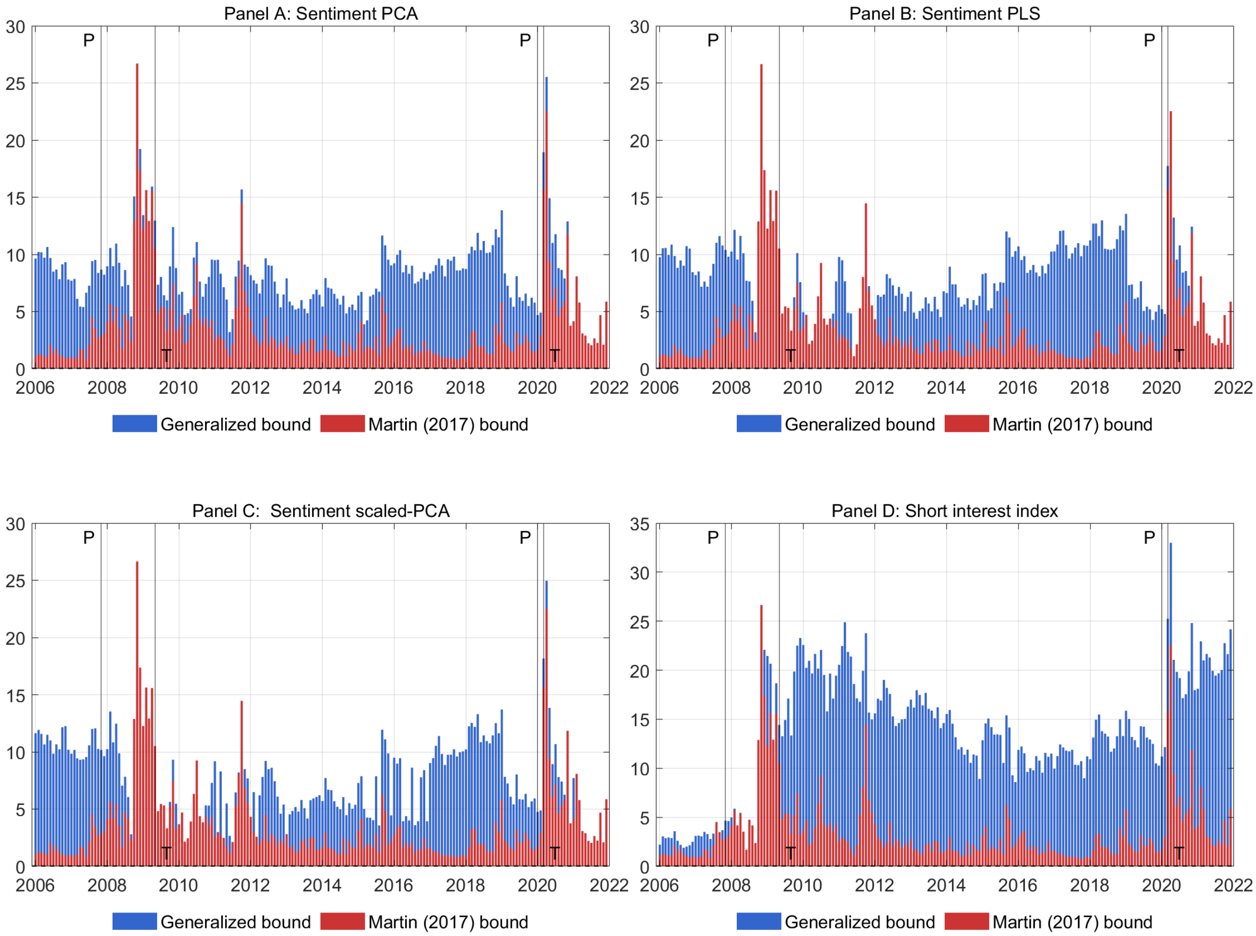

Figure 6: Martin (2017) bound versus generalized bounds

Each panel depicts the bar plot of Martin (2017) bound and the generalized bound based on a state variable indicated in the panel heading over the out-of-sample period from 2006:01 to 2022:12.

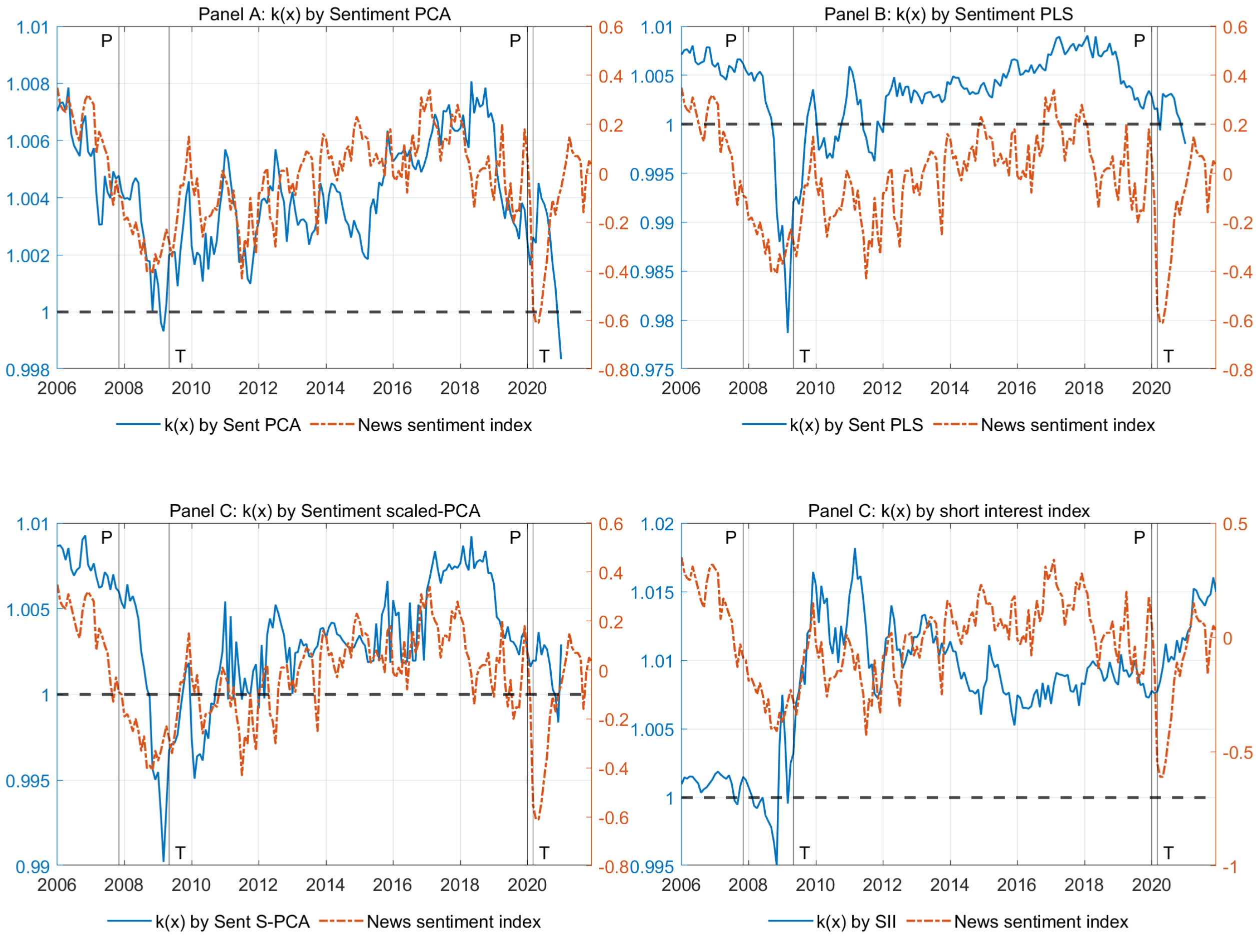

Figure 7: The adjustment factor

Each panel displays the dynamism of the adjustment factor, $k(x_t) = \exp(\alpha + \beta x_t)$ (blue solid line) where x_t is indicated in the panel heading, and $\{\alpha, \beta\}$ are estimated recursively with data up to time t. The red dotted line plots the economic news sentiment index by Shapiro, Sudhof, and Wilson (2022).

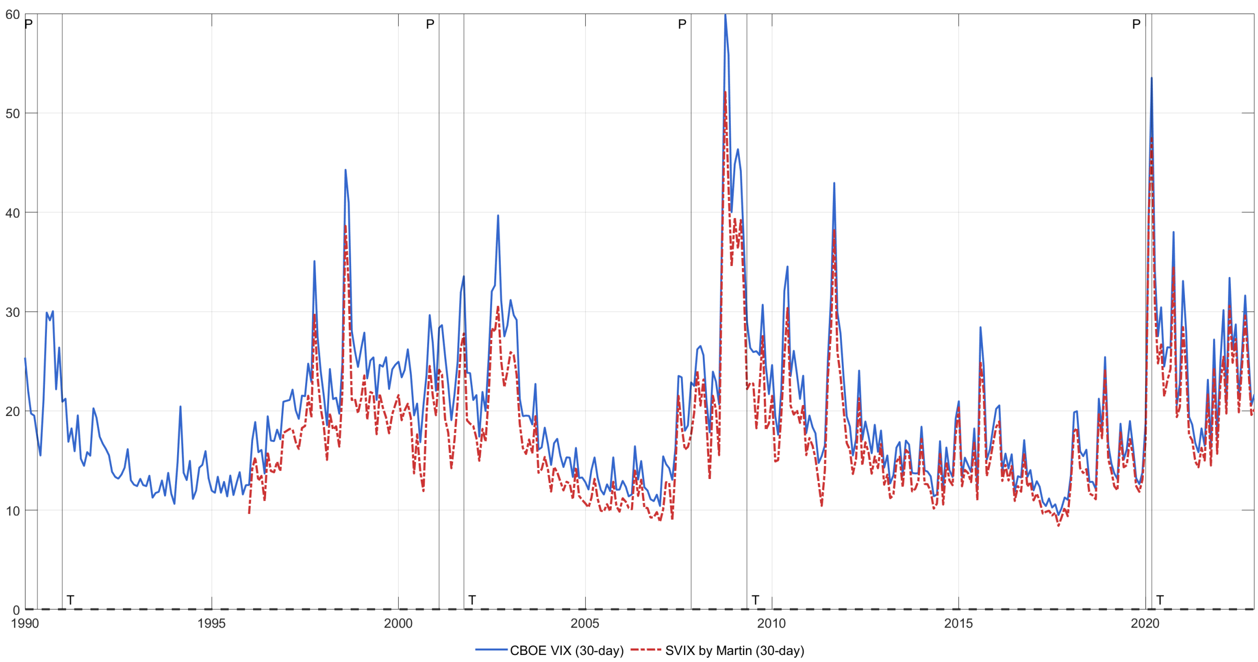

Figure 8: CBOE VIX and SVIX (in percent)

This figure displays the CBOE VIX index and Martin (2017) SVIX index (both expressed as percentages). Vertical lines are used to indicate NBER-dated business-cycle peaks (P) and troughs (T).