A Behavioral New Keynesian Model of a Small Open Economy under Limited Foresight*

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Abstract

This paper investigates exchange rate dynamics in open economies by incorporating bounded rationality. We develop a small open-economy New Keynesian model with an incomplete asset market, wherein decision-makers possess limited foresight and can plan for only a finite distance into the future. The equilibrium dynamics depend on the degree of foresight and the decision-makers' belief-updating behaviors that approximate continuation values at the end of their planning horizons. Limited foresight leads to dynamic overshooting of forecast errors in the real exchange rate across different time horizons, while also differentiating the term structure of expectations. This framework hence provides a micro-foundation for understanding time and forecast horizon variability in uncovered interest parity (UIP) puzzles.

Keywords: Finite planning horizon; value function learning; small open economy; exchange rate; UIP violations

JEL codes: E43; E70; F31; F41

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1 Introduction

A burgeoning body of research studies and survey evidence shows that economic decision-makers often exhibit strong biases in forming expectations, deviating from the standard assumption of rational expectations (RE). In particular, recent empirical evidence concerning open economies suggests that subjective expectations, rather than alternative forces, could be the main drivers of many RE-based violations.[1] Prompted by the "forward guidance puzzle" (Del Negro, Giannoni and Patterson, 2015), some studies focusing on theoretical models in closed economies have found significant policy consequences by relaxing the RE assumption.[2] However, relatively little attention has been given to open-economy macro models. In this paper, we aim to bridge this gap between theory and empirical evidence by introducing bounded rationality within an open-economy context.

Our goals are twofold. First, we develop a small open-economy New Keynesian (SOE-NK) model, integrating the concept of bounded rationality. We consider a particular approach of modeling bounded rationality developed by Woodford (2019)—limited foresight. The model is broad enough for applications in typical open-economy studies, while also encapsulating RE analyses when decision-makers' foresight extends infinitely into the future. We further investigate how limited foresight affects the equilibrium dynamics and forecast errors of the real exchange rate, in comparison to the standard RE case. Second, we demonstrate that our model provides an intrinsic micro-foundation for several well-known puzzles related to the uncovered interest parity (UIP) under RE, particularly those concerning discrepancies across different time and forecast horizons.

To facilitate comparison, we use the standard SOE-NK model, outlined by Galí and Monacelli (2005), as our reference model, and introduce two departures from this standard model. First, we assume that decision-makers form expectations subject to limited foresight; that is, they consider only a finite planning horizon (henceforth, FH) into the future. To evaluate potential situations that may be reached at the end of their planning horizons, they employ a *coarse* continuation value function learned from past experiences. This signifies that decision-makers incorporate all relevant information and policy alterations within the FH. However, when approximating the more distant future beyond their planning horizons, their value function becomes less accurate compared to the RE scenario. Second, we assume an incomplete asset market. This assumption stems from the idea that provided decision-makers

[1] Among others, Kalemli-Özcan and Varela (2022) document that several uncovered interest parity (UIP) violations, such as the Fama (1984) puzzle and the predictability reversal puzzles of the real exchange rate (e.g., Engel, 2016; Valchev, 2020), disappear in advanced economies when one uses the actual expectations of exchange rates from survey data instead of ex-post realized exchange rates.

[2] For closed-economy studies, see, among others, Angeletos and Lian (2018), Gabaix (2020), Woodford and Xie (2019, 2022), and Dupraz, Bihan and Matheron (2022).

cannot make state-contingent plans as they can in the RE benchmark, assuming Arrow-Debreu markets would be logically inconsistent. From a practical viewpoint, this incomplete market structure also allows us to examine the impact of external shocks originating from the international financial market on the domestic economy.

The value function in our FH model significantly differs from the RE benchmark in two key aspects. First, unlike the RE model, the value function in the FH model does not encapsulate all state variables of the economy. Specifically, we assume that the value function only includes individual state variables, excluding aggregate state variables such as aggregate stochastic shocks affecting the economy. As a consequence, when envisioning the future beyond their planning horizons, decision-makers do not factor in the evolution of these aggregate state variables.

Second, the construction of the coarse value function is backward-looking, in contrast to the forward-looking nature of dynamic programming in the RE benchmark. Decision-makers with limited foresight extend the value function over time by updating it based on past experiences, which means extrapolating the value function. This reflects the concept that obtaining a value function as precise as in the RE benchmark is computationally implausible in a complex world. For the sake of simplicity, we model this value function update as a constant-gain learning process. Inherently, our FH model nests the RE reference model as a limiting case. As decision-makers' planning horizons extend towards infinity, or when they adopt the precise value function aligned with RE, their planning problem yields the same outcomes for variables as found in the unique RE equilibrium of the reference model.

Our economy is buffeted by one domestic shock and one external shock: the domestic productivity shock and the foreign interest rate shock. We compare the equilibrium dynamics between FH and RE models by examining impulse response functions. We find that the FH model generates more persistent, hump-shaped movements of aggregate variables. Furthermore, the FH model inherently produces systematic forecast errors of the real exchange rate, a feature absent in the RE benchmark. The role of value function learning should be noted. Decision-makers' value function learning in the FH model results in hump-shaped dynamics for their value functions over time, influencing the dynamic patterns of the macro aggregates. Furthermore, the value function learning significantly impacts the dynamic behavior of the forecast errors, by causing dynamic overshooting of the forecast errors and leading to a sign reversal over the time horizon. We also remark that due to the wealth effect and the valuation effect acting during the process of value function learning, the value function exhibits dynamic movements in opposite directions in response to the two different shocks.

Our FH model offers explanations for various puzzling violations of the RE-based UIP condition, especially those related to time and forecast horizon variability. The RE-UIP condition, a renowned theoretical asset pricing condition from the RE-based open-economy macro model, dictates that an increase in the interest rate differential between domestic and foreign currency bonds should match the RE-based expected future domestic currency depreciation. This results in *unpredictable* excess returns on currency bonds for any time horizon and predicts the *horizon invariance* on the exchange rate reaction to the term structure of forecasted future interest rate differentials.

However, comprehensive empirical studies have refuted the predictions from the RE-UIP condition, particularly in advanced countries. Observations reveal predictable short-run excess returns for higher interest rate currency bonds (Fama, 1984; Eichenbaum and Evans, 1995), with excess return predictability reversing its sign across time horizons (e.g., Bacchetta and van Wincoop, 2010; Engel, 2016; Valchev, 2020). Additionally, the horizon invariance of the real exchange rate to the forecast of future interest rate differential is challenged, with a documented overreaction and underreaction of the exchange rate to expected short-term and long-term interest rate differentials, respectively. It is also referred to as the "forward guidance exchange rate puzzle" (Galí, 2020). Whereas our model could also speak to other forms of UIP violations in the literature, we focus particularly on these two, because the FH model offers a compelling micro-founded explanation for both the time and forecast horizon aspects of the UIP violations.

In our FH model, the dynamic patterns of predictable excess returns are mirrored by the systematic forecast errors of the real exchange rate, which exhibit dynamic overshooting over the time horizon caused by value function learning. We perform a regression analysis on the predictability of excess returns using simulated data from the calibrated FH model. The results demonstrate that the FH model produces an unconditional profile of excess return predictability and a reversal of predictability that are qualitatively consistent with the empirical literature. Furthermore, we find that while the FH model predicts opposite profiles of excess return predictability conditioned on the two types of shocks, the foreign interest rate shock predominantly drives the unconditional profile of excess return predictability.

In addition, our FH model suggests that it can also explain the empirical evidence of the breakdown of the forecast horizon invariance via the value function learning process in the model and resulting breakdown of the Law of Iterated Expectations (LIE). In the FH model, decision-makers form expectations at time t for the future period $t + k$ based on the assumption that the counterfactual endogenous variables are constructed on aggregate conditions with a remaining planning horizon $h - k$. Thus, as term of the forecasts increases, the expectations become more tightly linked to decision-makers' value functions at the end of planning horizon. When decision-makers learn and update their value functions, expectations of future endogenous variables differ from the expectations of expected future endogenous variables. This disrupts the LIE. Consequently, the FH model results in an asymmetric reaction of the real exchange rate to short-term and long-term forecasts of interest rate differentials. We perform a regression analysis using the simulated data and find that the FH model generates empirically consistent overreaction and underreaction of the real exchange rate to forecasts of future short-term and long-term interest rate differentials, respectively.

Related Literature. This paper contributes to recent studies in the literature that scrutinize the RE assumption by considering behavioral biases or alternatives to assess their implications on policy analyses and empirical relevance.[3] In this context, we explore the consequences of limited foresight in an open-economy setting. Compared with other popular behavioral variants, the FH approach naturally differentiates between expectation biases influenced by the length of planning horizons and those guided by the approximating behavior of the value function. In consequence, our FH model provides micro-founded explanations for the RE-UIP violations, particularly those over time and forecast horizon.

Our paper builds on and expands the scope of FH models. In terms of modeling bounded rationality, Gust, Herbst and López-Salio (2022) show that Woodford (2019)'s approach is particularly relevant for aligning with aggregate data. Xie (2020) posits that this method facilitates the examination of equilibrium dynamics without having to tackle equilibrium selection issues. Woodford and Xie (2022) emphasize the importance of the bounded rationality approach for policy analysis and its welfare consequences, particularly under the zero lower bound conditions.[4] Notably, these studies are conducted within a closed-economy setting. Our paper broadens this framework to an open economy, underlining its implications for foreign shock transmissions and the dynamics of exchange rate and interest rate differential. Moreover, from a methodological perspective, we augment the FH model by introducing a method to incorporate additional endogenous state variables—the real exchange rate and net foreign asset position—into our analysis.

The open-economy environment of this paper begins with the literature on the macro models of monetary transmission mechanisms with open capital account from back in the 1960s. In these analyses, the equilibrium analyses are based on the UIP condition with no excess returns. Subsequent advancements in dynamic stochastic general equilibrium (DSGE) open-economy macro models, under the full-information RE hypothesis, continue to rely on

[3]The literature has developed several approaches to model bounded rationality that address the forward guidance puzzle, such as cognitive discounting (Gabaix, 2020), level-k thinking (e.g., García-Schmidt and Woodford, 2019; Farhi and Werning, 2019), lack of common knowledge (Angeletos and Lian, 2018), and finite planning horizons (Woodford, 2019).

[4]See Woodford and Xie (2022) for empirical evidence that supports the assumption of FHs.

the UIP condition.[5] However, numerous empirical studies report frequent rejections of the RE-UIP condition. A strand of works previously mentioned documents predictable excess returns of the real exchange rate and its reversal of the sign over the time horizon. It also reports the breakdown of the forecast horizon invariance as implied by the RE-UIP condition.

In particular, several studies utilizing expectation survey data indicate that in advanced economies, systematic forecast errors from the subjective expectations of economic agents are the primary source of RE-UIP violations (Froot and Frankel, 1989; Chinn and Frankel, 2019; Kalemli-Özcan and Varela, 2022; Candian and De Leo, 2023), rather than alternative explanations such as risk premia or financial frictions (e.g., Gabaix and Maggiori, 2015). Inspired by these findings, our paper applies a subjective expectation formation featuring agents with limited foresight to the standard open-economy New Keynesian model. Our work thus offers a new theoretical framework capable of explaining the time and forecast horizon aspects of the UIP violations.[6] Furthermore, by focusing on the expectation channels of the behavioral agents, our model stresses a distinct perspective on the source of UIP wedges compared to the recent work by Itskhoki and Mukhin (2021). They consider a segmented financial market with noise traders and risk-averse intermediaries, where limits-to-arbitrage results in a wedge in the RE-UIP condition. In contrast, our model attributes endogenous deviations from the RE-UIP condition to decision-makers' behavioral responses to aggregate shocks.

This paper aligns with the literature on open-economy macro models employing bounded-rational agents to address exchange rate puzzles. Gourinchas and Tornell (2004) model investors with distorted beliefs, resulting in misperceptions about the relative weight of persistent versus transitory interest rate shocks. Their model can predict positive excess returns for bonds bearing higher interest rates, but fails to predict the reversal of excess return across different time horizons. Candian and De Leo (2023) extend this work by incorporating investors' extrapolation of underlying shocks, showing that the model can predict the excess return reversal if investors' perception of shocks over-extrapolates the actual shocks. Valente, Vasudevan and Wu (2021) also model decision-makers receiving noisy signals and extrapolating an exogenous interest-rate process to address various exchange rate puzzles. These analyses focus on decision-makers' misperception and extrapolation of underlying shocks. Kolasa, Ravgotra and Zabczyk (2022) tackle UIP puzzles by developing

[5] For examples of the earliest contributions, see Mundell (1963), Fleming (1962), and Dornbusch (1976). Among many others, see Clarida, Gali and Gertler (2001) and Galí and Monacelli (2005) for DSGE open-economy macro models.

[6] The literature has also developed various approaches in the RE framework to address some parts of these UIP puzzles, such as by considering time-varying risk premia (Verdelhan, 2010), infrequent portfolio adjustments (Bacchetta and van Wincoop, 2010), or the convenience yield (Valchev, 2020).

an open-economy macro model with cognitive discounting, as in Gabaix (2020).[7]

Unlike the explanations mentioned above, decision-makers do not misperceive an exogenous process in our model. Instead, the behavioral biases stem from their limited foresight in planning, along with the coarse value functions they use and update, all of which affect their expectations for endogenous variables. Our approach is also related to Molavi, Tahbaz-Salehi and Vedolin (2023), which employs a model with constraints on the complexity of agents' beliefs when addressing the UIP puzzle across the time horizon. However, their approach does not provide an explanation for the breakdown of forecast horizon invariance. The micro-foundation for both dynamic overshooting of the forecast error over the time horizon and contrasting short-term and long-term forecasts across the forecast horizon is a distinctive feature of our FH approach.

This paper proceeds as follows. Section 2 illustrates an SOE-NK model in which decision-makers are subject to limited foresight. Section 3 summarizes the full equilibrium conditions when decision-makers share a homogeneous planning horizon and discusses the solution method of the model. Section 4 analyzes the equilibrium dynamics of the FH model. Section 5 applies the FH model to address the UIP puzzles. Section 6 discusses the robustness of the main results by considering alternative setups of the model, including extending to heterogeneous planning horizons. Section 7 concludes the paper.

2 A Small Open-Economy New Keynesian Model under Limited Foresight

We develop an SOE-NK model in which decision-makers have bounded rationality. The model setup is similar to the standard SOE-NK DSGE model developed in Galí and Monacelli (2005). The world consists of a continuum of small open economies that lie on the unit interval. The consumption basket of households includes domestically produced goods and imported foreign goods. Producers of domestic goods have market power and can set prices in the domestic currency (i.e., producer currency pricing (PCP)). We assume nominal rigidity in goods prices whereby firms have a limited ability to reset their prices following Calvo pricing.

Our model differs from the benchmark SOE-NK model in the following aspects. First, we replace the assumption of infinite-horizon dynamic planning with planning under limited foresight à la Woodford (2019). Second, we assume incomplete asset markets such that only

[7] Crucini, Shintani and Tsuruga (2020) argue that the high persistence in purchasing power parity (PPP) deviation and the law of one price (LOP) deviation cannot be explained solely by nominal rigidities; they also provide a rationale through cognitive discounting.

state noncontingent claims are available to trade. We consider two types of bonds. One is a domestic currency bond that is only traded domestically, and the other is an international bond (denominated in foreign currency) that is traded with the rest of the world. In addition, we also introduce the random foreign interest rate shock from the rest of the world to study the effects of aggregate disruptions stemming from the international financial market. These disruptions include instances of country premium shocks and foreign monetary policy shocks. In this section, we show that the assumption of FH yields behavioral analogs of the structural equations of open-economy macro models, such as the open-economy Euler equations for domestic and foreign bonds and the open-economy New Keynesian supply curve.

2.1 Households

Let us begin with a description of the households' forward planning problem. The small open economy consists of infinitely many identical households indexed in the unit interval $[0,1]$. At any time t, household i seeks to maximize

$$\hat{\mathbb{E}}_t^i \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left(u\left(C_\tau^i\right) - \varpi\left(N_\tau^i\right)\right), \tag{2.1}$$

where C_τ^i is the consumption composite at date τ and N_τ^i is the labor supply of household i. Function $u(\cdot)$ denotes a periodic utility, which is strictly increasing and concave, while function $\varpi(\cdot)$ denotes a periodic disutility of labor supply, which is strictly increasing and convex. Parameter $0 < \beta < 1$ is the subjective discount factor. Operator $\hat{\mathbb{E}}_t^i$ describes the subjective expectation operator of household i at time t; and we will specify this expectation operator later such that it features the assumption of limited foresight.

The consumption basket is a composite index of home goods, $C_{H,\tau}^i$, and imported foreign goods, $C_{F,\tau}^i$. The home good is a constant elasticity of substitution (CES) aggregation of different varieties of home-made goods, $C_{H,\tau}^i(j)$, where $j \in [0,1]$. The imported goods are purchased from a variety of small open economies; thus, they are an aggregate of goods from each such country, $l \in [0,1]$, denoted as $C_{l,\tau}^i$. Finally, a good imported from country l is an aggregator of different varieties of goods made in country l, $C_{l,\tau}(j)$. These aggregates are given by

$$C_\tau^i = \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,\tau}^i)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,\tau}^i)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where $1-\alpha$ governs the degree of home bias, and

$$C_{H,\tau}^i = \left(\int_0^1 C_{H,\tau}^i(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad C_{F,\tau}^i = \left(\int_0^1 C_{l,\tau}^{i\;\frac{\gamma-1}{\gamma}} dl \right)^{\frac{\gamma}{\gamma-1}}, \quad C_{l,\tau}^i = \left(\int_0^1 C_{l,\tau}^i(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where η, ϵ, and γ represent the elasticities of substitution between home and foreign goods, within-country varieties, and across-country varieties, respectively.

The household faces the following sequential budget constraint:

$$\int_0^1 P_{H,\tau}(j)C^i_{H,\tau}(j)dj+\int_0^1\int_0^1 P_{l,\tau}(j)C^i_{l,\tau}(j)dj\,dl+\frac{B^i_{\tau+1}}{1+i_\tau}+\frac{\mathcal{E}_\tau B^{*,i}_{\tau+1}}{1+i^*_\tau}=B^i_\tau+\mathcal{E}_\tau B^{*,i}_\tau+W_\tau N^i_\tau+\Phi_\tau,$$

where $P_{H,\tau}$ is the nominal price index for domestically produced final goods, $P_{l,\tau}(j)$ is the nominal price for final good j produced in country l, $B^i_{\tau+1}$ is the nominal payoff in period $\tau+1$ of the domestic bond portfolio that household i holds at the end of period τ, and $1+i_\tau$ is the (one-period ahead) riskless gross domestic nominal interest rate of the domestic bond. Variable $\mathcal{E}_\tau$ is the effective nominal exchange rate between the home country and the rest of the world in units of domestic currency. Variable $B^{*,i}_{\tau+1}$ is the nominal payoff of a foreign bond in foreign currency, and $1+i^*_\tau$ is the nominal interest rate for the international bond. Variable W_τ is the nominal wage, and Φ_τ is the nominal profit of firms transferred to individual household i (households own the domestic firms, but the dividends transfer is beyond household i's control).

The household's static cost minimization problem for consumption expenditure yields the following demand functions for consumption goods:

$$C^i_{H,\tau}(j)=\left(\frac{P_{H,\tau}(j)}{P_{H,\tau}}\right)^{-\epsilon}C^i_{H,\tau},\qquad C^i_{l,\tau}(j)=\left(\frac{P_{l,\tau}(j)}{P_{l,\tau}}\right)^{-\epsilon}C^i_{l,\tau},\qquad C^i_{l,\tau}=\left(\frac{P_{l,\tau}}{P_{F,\tau}}\right)^{-\gamma}C^i_{F,\tau},$$

$$C^i_{H,\tau}=(1-\alpha)\left(\frac{P_{H,\tau}}{P_\tau}\right)^{-\eta}C^i_\tau,\qquad C^i_{F,\tau}=\alpha\left(\frac{P_{F,\tau}}{P_\tau}\right)^{-\eta}C^i_\tau,$$

where the price indices for domestically produced final goods and foreign-produced goods are given as

$$P_{H,\tau}=\left(\int_0^1 P_{H,\tau}(j)^{1-\epsilon}dj\right)^{\frac{1}{1-\epsilon}},P_{l,\tau}=\left(\int_0^1 P_{l,\tau}(j)^{1-\epsilon}dj\right)^{\frac{1}{1-\epsilon}},P_{F,\tau}=\left(\int_0^1 {P_{l,\tau}}^{1-\gamma}dl\right)^{\frac{1}{1-\gamma}}.$$

The aggregate consumer price index (CPI) is given as

$$P_\tau=\left[(1-\alpha)P^{1-\eta}_{H,\tau}+\alpha P^{1-\eta}_{F,\tau}\right]^{\frac{1}{1-\eta}},$$

which also implies

$$P_\tau C_\tau^i = \int_0^1 P_{H,\tau}(j) C_{H,\tau}^i(j)dj + \int_0^1 \int_0^1 P_{l,\tau}(j) C_{l,\tau}^i(j)djdl.$$

Then, the sequential budget constraint can be rewritten as

$$P_\tau C_\tau^i + \frac{B_{\tau+1}^i}{1+i_\tau} + \frac{\mathcal{E}_\tau B_{\tau+1}^{*,i}}{1+i_\tau^*} = B_\tau^i + \mathcal{E}_\tau B_\tau^{*,i} + W_\tau N_\tau^i + \Phi_\tau. \tag{2.2}$$

For the sake of parsimony, we follow the assumption on the labor market in Woodford (2019). That is, the country has a labor organization in which each household is asked by firms to supply its share of the aggregate domestic labor demand N_τ. Thus, the expected path of $N_\tau^i = N_\tau$ is beyond household i's control. This implies that the expected path of equilibrium income (in domestic currency); that is,

$$W_\tau N_\tau + \Phi_\tau = P_{H,\tau} Y_\tau, \tag{2.3}$$

is exogenous to individual household i.

Now, we characterize household i's intertemporal consumption-saving decisions under limited foresight. Suppose that in each period, household i engages in explicit forward planning for finite h periods ahead; that is, household i has a planning horizon h. For simplicity, we assume that h is exogenous and time-invariant. At time t with the vector of aggregate state variables $\boldsymbol{z}_t$, the household chooses state-contingent plans $\{C_\tau^i(\boldsymbol{z}_\tau)\}$ only for the possible states $\boldsymbol{z}_\tau$ within periods $t \leq \tau \leq t+h$. The household chooses the finite-horizon plans to maximize the objective

$$\mathbb{E}_t^h \left[\sum_{\tau=t}^{t+h} \beta^{\tau-t} u(C_\tau^i) + \beta^{h+1} v\left(\mathcal{B}_{t+h+1}^i, \mathcal{B}_{t+h+1}^{*,i}; \boldsymbol{z}_{t+h}\right) \right], \tag{2.4}$$

where $v(\cdot; \boldsymbol{z}_{t+h})$ is the value function that the household uses to approximate continuation values for each possible state $\boldsymbol{z}_{t+h}$ at the end of planning horizon. It is a *coarse* value function such that it is contingent only on individual states instead of the complete state-contingent structure as under the RE case. Here $\mathcal{B}_\tau \equiv B_\tau / P_{\tau-1}$ and $\mathcal{B}_\tau^* \equiv B_\tau^* / P_{\tau-1}^*$ denote the nominal value of bonds maturing in period τ deflated by last-period price indices, and thus $\mathcal{B}_\tau$ and $\mathcal{B}_\tau^*$ are real variables that are purely predetermined in period $\tau - 1$.

By combining (2.2) and (2.3), household i's budget constraint can be expressed in real terms as follows:

$$C_\tau^i + \frac{\mathcal{B}_{\tau+1}^i}{1+i_\tau} + \frac{Q_\tau \mathcal{B}_{\tau+1}^{*,i}}{1+i_\tau^*} = \frac{\mathcal{B}_\tau^i}{\Pi_\tau} + \frac{Q_\tau \mathcal{B}_\tau^{*,i}}{\Pi_\tau^*} + \mathcal{S}_\tau Y_\tau, \tag{2.5}$$

where $\Pi_{\tau+1} \equiv P_{\tau+1}/P_\tau$ and $\Pi_{\tau+1}^* \equiv P_{\tau+1}^*/P_\tau^*$ are the domestic and foreign inflation rates, $Q_\tau \equiv P_\tau^* \mathcal{E}_\tau / P_\tau$ is the real exchange rate, and $\mathcal{S}_\tau Y_\tau$ is the household income from wages and dividends (beyond the household's control). Here, we have used the condition that firms' total income is equal to their wage and dividend payment to households; that is,

$$\frac{W_\tau N_\tau + \Phi_\tau}{P_\tau} = \frac{P_{H,\tau} Y_\tau}{P_\tau} = \mathcal{S}_\tau Y_\tau,$$

where $\mathcal{S}_\tau \equiv P_{H,\tau}/P_\tau$ is the ratio between the price index for domestically produced goods and the price index for the aggregate consumption basket.

In the forward planning exercise with horizon $h \geq 1$ at any date $t \leq \tau \leq t+h-1$, the first-order conditions to maximize lifetime utility with respect to C_τ^i, $\mathcal{B}_{\tau+1}^i$, and $\mathcal{B}_{\tau+1}^{*,i}$ yield

$$u'\left(C_\tau^i\right) = \beta \mathbb{E}_t^h \left[(1+i_\tau)\, u'(C_{\tau+1}^i)/\Pi_{\tau+1} | \boldsymbol{z}_\tau \right], \tag{2.6}$$

$$u'\left(C_\tau^i\right) = \beta \mathbb{E}_t^h \left[(1+i_\tau^*)\, u'(C_{\tau+1}^i)(Q_{\tau+1}/Q_\tau)(1/\Pi_{\tau+1}^*) | \boldsymbol{z}_\tau \right], \tag{2.7}$$

for each possible state $\boldsymbol{z}_\tau$, given state $\boldsymbol{z}_t$ at the time of the planning exercise. In terminal period $\tau = t+h$ where the forward planning is truncated, or in the case of $h=0$, the first-order conditions related to the value function are

$$u'(C_{t+h}^i) = \beta(1+i_{t+h}) v_1(\mathcal{B}_{t+h+1}^i, \mathcal{B}_{t+h+1}^{*,i}; \boldsymbol{z}_{t+h}), \tag{2.8}$$

$$u'(C_{t+h}^i) = \beta(1+i_{t+h}^*) v_2(\mathcal{B}_{t+h+1}^i, \mathcal{B}_{t+h+1}^{*,i}; \boldsymbol{z}_{t+h})/Q_{t+h}, \tag{2.9}$$

for each possible state $\boldsymbol{z}_{t+k}$.

Note that in the household's finite-horizon forward planning problem, if the household's subjective expectation operator $\mathbb{E}_t^h[\cdot]$ is the model-consistent expectation and the value function $v(\cdot; \boldsymbol{z}_{t+h})$ is the accurate model-consistent value function with a complete state-contingent structure (as in standard dynamic programming under infinite planning horizons), the household's optimization problem replicates the conventional intertemporal optimization problem. That is, in such a case, the household makes the optimal infinite-horizon contingent plans under RE.

However, the decision-making under limited foresight features optimal plans and expectation formations that deviate from the infinite-horizon RE benchmark. At date t, the household constructs a contingent plan for the subsequent h forward dates but implements the plan only for the current date t. When the following date $t+1$ arrives, the household reconstructs the contingent plans for future h dates, which are not necessarily identical to those made at the *previous* date t. The household implements the new plans only for the current date $t+1$. In terms of expectation formation, at each date t, the h-horizon household makes a contingent plan up to date $t+h$. At each date τ within the planning horizon $t \leq \tau \leq t+h$, the household is assumed to plan forward for the remaining $t+h-\tau$ dates. In addition, the household assumes that spending and pricing decisions made by other households and firms at any date τ within its planning horizon are made with the same remaining planning horizon $t+h-\tau$.

The expectation operator for the h-horizon household can be converted with the model-consistent expectation. For any endogenous variable X_τ determined at date τ ($t \leq \tau \leq t+h$), the household's expectation conditional on state $\boldsymbol{z}_t$ at date t is assumed to satisfy

$$\mathbb{E}_t^h[X_\tau|\boldsymbol{z}_t] = \mathbb{E}_t\left[X_\tau^{t+h-\tau}\right], \tag{2.10}$$

where operator $\mathbb{E}[\cdot]$ is the standard model-consistent expectation operator and $t+h-\tau$ represents the remaining planning horizon at date τ. The household's expectation for X_τ conditional on future state $\boldsymbol{z}_\tau$ in its period-t planning exercise is given by

$$\mathbb{E}_t^h[X_\tau|\boldsymbol{z}_\tau] = \mathbb{E}_\tau\left[X_\tau^{t+h-\tau}\right]. \tag{2.11}$$

Finally, the household's expectation for $X_{\tau+1}$ conditional on the same information structure above is given by

$$\mathbb{E}_t^h[X_{\tau+1}|\boldsymbol{z}_\tau] = \mathbb{E}_\tau\left[X_{\tau+1}^{t+h-\tau-1}\right]. \tag{2.12}$$

Under the expectation formation, the first-order conditions (2.6)-(2.9) can be rewritten with the model-consistent operator as follows:

$$u'(C_\tau^{t+h-\tau}) = \beta\mathbb{E}_\tau\left[(1+i_\tau^{t+h-\tau})\frac{u'(C_{\tau+1}^{t+h-\tau-1})}{\Pi_{\tau+1}^{t+h-\tau-1}}\right], \tag{2.13}$$

$$u'(C_\tau^{t+h-\tau}) = \beta\mathbb{E}_\tau\left[(1+i_\tau^{*,t+h-\tau})\frac{u'(C_{\tau+1}^{t+h-\tau-1})}{\Pi_{\tau+1}^{*,t+h-\tau-1}}\frac{Q_{\tau+1}^{t+h-\tau-1}}{Q_\tau^{t+h-\tau}}\right], \tag{2.14}$$

$$u'(C_{t+h}^0) = \beta(1+i_{t+h}^0)v_1(\mathcal{B}_{t+h+1}^0, \mathcal{B}_{t+h+1}^{*,0}; \boldsymbol{z}_{t+h}), \tag{2.15}$$

$$u'(C_{t+h}^0) = \beta(1+i_{t+h}^{*,0})v_2(\mathcal{B}_{t+h+1}^0, \mathcal{B}_{t+h+1}^{*,0}; \boldsymbol{z}_{t+h})/Q_{t+h}^0. \tag{2.16}$$

In the nonstochastic steady state, we have that $1+\bar{i} = \beta^{-1}\bar{\Pi}$ and $1+\bar{i}^* = \beta^{-1}\bar{\Pi}^*$, and the value function in the steady state is given by

$$v(\mathcal{B},\mathcal{B}^*) = (1-\beta)^{-1} u\left(\frac{(1-\beta)\mathcal{B}}{\bar{\Pi}} + \frac{(1-\beta)\bar{Q}\mathcal{B}^*}{\bar{\Pi}^*} + \bar{\mathcal{S}}\bar{Y}\right).$$

Details of deriving the steady state value function can be found in Appendix A.

We define the domestic variables after log-linear approximation as

$$\hat{c}_t \equiv \log\left(\frac{C_t}{\bar{C}}\right), \qquad \hat{y}_t \equiv \log\left(\frac{Y_t}{\bar{Y}}\right), \qquad \hat{\imath}_t \equiv \log\left(\frac{1+i_t}{1+\bar{i}}\right),$$
$$\hat{b}_t \equiv \frac{\mathcal{B}_t - \bar{\mathcal{B}}}{\bar{\Pi}\bar{C}}, \qquad \hat{q}_t \equiv \log\left(\frac{Q_t}{\bar{Q}}\right), \qquad \pi_t \equiv \log\left(\frac{\Pi_t}{\bar{\Pi}}\right),$$

and for the foreign variables,

$$\hat{\imath}_t^* \equiv \log\left(\frac{1+i_t^*}{1+\bar{i}^*}\right), \qquad \hat{b}_t^* \equiv \frac{\bar{Q}(\mathcal{B}_t^* - \bar{\mathcal{B}}^*)}{\bar{\Pi}^*\bar{C}}, \qquad \pi_t^* \equiv \log\left(\frac{\Pi_t^*}{\bar{\Pi}^*}\right),$$

Throughout the paper, we use lowercase to denote variables after taking logs unless otherwise stated, and further use hats to denote log-deviation from the steady state.

Log-linearizing equations (2.13) and (2.14) yield

$$\hat{c}_\tau^{t+h-\tau} = \mathbb{E}_\tau[\hat{c}_{\tau+1}^{t+h-\tau-1}] - \sigma^{-1}\left[\hat{\imath}_\tau^{t+h-\tau} - \mathbb{E}_\tau \pi_{\tau+1}^{t+h-\tau-1}\right], \tag{2.17}$$
$$\hat{c}_\tau^{t+h-\tau} = \mathbb{E}_\tau[\hat{c}_{\tau+1}^{t+h-\tau-1}] - \sigma^{-1}\left[\hat{\imath}_\tau^{*,t+h-\tau} + \mathbb{E}_\tau(\hat{q}_{\tau+1}^{t+h-\tau-1} - \hat{q}_\tau^{t+h-\tau} - \pi_{\tau+1}^{*,t+h-\tau-1})\right], \tag{2.18}$$

for any date $t \le \tau \le t+h-1$ with horizon $h \ge 1$, where $\sigma^{-1} \equiv -u'(\bar{C})/(u''(\bar{C})\bar{C})$ is the elasticity of intertemporal substitution of households.

Under the assumption that households adopt the steady-state value function in their forward-planning exercise and that there is no learning in the value function, log-linearizing equations (2.15) and (2.16) yields

$$\hat{c}_\tau^0 = -\sigma^{-1}\hat{\imath}_\tau^0 + (1-\beta)\hat{b}_{\tau+1}^0 + (1-\beta)\hat{b}_{\tau+1}^{*,0}, \tag{2.19}$$
$$\hat{c}_\tau^0 = -\sigma^{-1}\hat{\imath}_\tau^{*,0} + (1-\beta)\hat{b}_{\tau+1}^0 + (1-\beta)\hat{b}_{\tau+1}^{*,0} + \sigma^{-1}\hat{q}_\tau^0. \tag{2.20}$$

Details of the derivation can be found in Appendix B. In Section 2.5, we also introduce a learning process in the value function.

2.2 Firms

A set of continuum producers $f \in [0,1]$ in the economy produce a variety of differentiated intermediate goods as inputs for the domestically produced final goods. The intermediate goods market is monopolistically competitive, and the producers of each intermediate good can be price-setters in domestic currency (that is, PCP) but face staggered pricing, as in the style of Calvo (1983) and Yun (1996). Specifically, we assume that at each time, fraction $1-\theta$ of firms are randomly selected to be able to reoptimize their prices. A producer j that belongs to the remaining fraction θ cannot reset its price, and we assume that its price satisfies $P_{H,t}(j) = P_{H,t-1}(j)\bar{\Pi}_H$, where $\bar{\Pi}_H$ is the inflation rate of the domestic goods in the nonstochastic steady state. This implies that the prices are automatically revised by considering the long-run inflation rate for domestically produced goods. This assumption is an open-economy variation of Woodford (2019), which implies that all equilibrium relative prices among the varieties of domestic goods are the same as those under flexible prices in the steady state.

At time t, similar to the objective function of households, firm f with a k-period planning horizon that can reset its price chooses $P^f_{H,t}$ to maximize

$$\max_{P^f_{H,t}} \mathbb{E}^k_t \left[\sum_{\tau=t}^{t+k} (\beta\theta)^{\tau-t} \lambda_\tau H\left(r^f_\tau; \mathcal{S}_\tau, \boldsymbol{z}_\tau\right) + \tilde{v}\left(r^f_{t+k}; \mathcal{S}_{t+k}, \boldsymbol{s}_{t+k}\right) \right], \tag{2.21}$$

where $\lambda_\tau \equiv \int u_c(C^i_\tau)di$ is the average marginal utility of household consumption and $r^f_\tau \equiv \frac{P^f_{H,t}}{P_{H,\tau}} \bar{\Pi}^{\tau-t}_H$ denotes the relative price between firm f's goods price and domestically produced final goods. $H\left(r^f_\tau; \mathcal{S}_\tau, \boldsymbol{z}_\tau\right)$ represents the real profits of the firm at date τ, where $\boldsymbol{z}_\tau$ is the vector of real state variables that are beyond firm f's control. A detailed expression of the functional form $H(\cdot)$ can be found in Appendix C.

The last term $\tilde{v}(r^f_{t+k}; \mathcal{S}_{t+k}, \boldsymbol{s}_{t+k})$ in (2.21) is the firm's value function at the end of the planning horizon that is used to approximate the value of discounted future real profits from date $t+k+1$ onward. Here $\boldsymbol{s}_{t+k}$ is a coarse vector for the real state variables, which is a subset of $\boldsymbol{z}_{t+k}$.

We begin with the assumption that the firm's value function $\tilde{v}(\cdot)$ is the one learned from the nonstochastic steady-state equilibrium; that is, we now consider the following steady-state firm value function:

$$\tilde{v}\left(r^f\right) = (1-\theta\beta)^{-1} \bar{\lambda} H\left(r^f; \bar{\mathcal{S}}, \bar{\boldsymbol{z}}\right), \tag{2.22}$$

where $\bar{\lambda} = u_c(\bar{C})$ is the constant value of λ_τ in the steady state. In Section 2.5, we relax this assumption by incorporating a learning process into $\tilde{v}(\cdot)$.

The firm's expectation formation through $\mathbb{E}_t^k[\cdot]$ is isomorphic to that of the household. That is, the firm with planning horizon k assumes that the endogenous variables determined at any date τ in $t \leq \tau \leq t+k$ are based on the decisions of all agents in the economy with the remaining planning horizon $t+k-\tau$. Therefore, the firm's subjective expectation operator for endogenous variables is represented by the model-consistent expectation in the same fashion as in equations (2.10)-(2.12), where h is now replaced with k.

Then, with the notation $p_{H,t}^f \equiv \log[P_{H,t}^f/(P_{H,t-1}\bar{\Pi}_H)]$, any firm f that reoptimizes its price at time t with a k-period planning horizon sets $p_{H,t}^f = p_{H,t}^k$, which is given by

$$p_{H,t}^k = \mathbb{E}_t \sum_{\tau=t}^{t+k} (\beta\theta)^{\tau-t} \left[\pi_{H,\tau}^{t+k-\tau} + (1-\beta\theta)\widehat{mc}_\tau^{t+k-\tau} \right], \tag{2.23}$$

where $\widehat{mc}_\tau \equiv \log\left(\mathcal{MC}_\tau/\overline{\mathcal{MC}}\right)$ is the log-deviation of the real marginal cost, $\mathcal{MC}_\tau \equiv \frac{MC_\tau}{P_{H,\tau}}$, around its steady state at date τ. In particular, it satisfies $\widehat{mc}_t = -H'(1;1,\boldsymbol{z}_t)/H''(1;1,\bar{\boldsymbol{z}})$. The details of deriving (2.23) and $\widehat{mc}_t$ can be found in Appendix C.

The evolution of the aggregate domestic price index $P_{H,t}$ satisfies

$$P_{H,t}^{1-\epsilon} = \theta(P_{H,t-1}\bar{\Pi}_H)^{1-\epsilon} + (1-\theta)(P_{H,t}^f)^{1-\epsilon},$$

and its log-linear approximation around the steady state with constant domestic inflation rate $\bar{\Pi}_H$ yields

$$\pi_{H,t}^k = (1-\theta)p_{H,t}^k.$$

Thus, equation (2.23) becomes

$$\pi_{H,t}^k = (1-\theta)\mathbb{E}_t \sum_{\tau=t}^{t+k} (\beta\theta)^{\tau-t} \left[\pi_{H,\tau}^{t+k-\tau} + (1-\beta\theta)\widehat{mc}_\tau^{t+k-\tau} \right]. \tag{2.24}$$

The isomorphic form of the equation holds if we replace k with any horizon $j \geq 0$. That is, $\{\pi_{H,t}^j\}$ for any horizon $j \geq 1$ satisfies the following recursive form:

$$\pi_{H,t}^j = \kappa\widehat{mc}_t^j + \beta\mathbb{E}_t\pi_{H,t+1}^{j-1}, \tag{2.25}$$

where $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta$, and when $j=0$, we have

$$\pi_{H,t}^0 = \kappa\widehat{mc}_t^0. \tag{2.26}$$

2.3 Labor Market and Real Marginal Cost

The wage is determined following the approach of Woodford (2019), which abstracts labor supply decision-making from any individual household while maintaining the aggregate labor-supply curve as in the canonical New Keynesian models. As mentioned in Section 2.1, the labor market organization has representatives who bargain for wages on behalf of households. Henceforth, we drop the superscripts on the planning horizon for the sake of parsimony when they are redundant for explicitly understanding the equilibrium relationships. We formally state the full equilibrium conditions with the FH in Section 3.

A representative determines the number of working hours provided by households for any given wage, and households must supply that number of hours and receive the same wage. There are many such representatives, and no representative has any market power. Then, the representatives choose the number of hours N_t to maximize the average utility of the households in the economy, which yields the following labor supply:

$$\varpi_N(N_t) = \lambda_t \frac{W_t}{P_t}.$$

As in Galí and Monacelli (2005), we assume the standard disutility function of the labor supply in the form of

$$\varpi(N_t) = \frac{N_t^{1+\varphi}}{1+\varphi},$$

where φ is the inverse of the Frisch elasticity of labor supply. Then, the labor supply equation after taking log becomes

$$\varphi n_t = -\sigma c_t + w_t - p_t. \tag{2.27}$$

Each firm $j \in [0,1]$ has a linear technology represented by the production function

$$Y_t(j) = A_t N_t(j),$$

hence the marginal cost is common across domestic firms. The real marginal cost in terms of domestic prices is then given by

$$\mathcal{MC}_t = (1-\tau)\frac{W_t}{P_{H,t} A_t},$$

where τ is an employment subsidy,[8] and the log of the real marginal cost becomes:

$$mc_t = \log(1-\tau) + w_t - p_{H,t} - a_t. \tag{2.28}$$

2.4 Closing the Economy

Exchange Rate and the Terms of Trade. From the definition of the real exchange rate, we have the following accounting relationship between logs of the nominal exchange rate, real exchange rate, and domestic and foreign price indices:

$$q_t = e_t + p_t^* - p_t, \tag{2.29}$$

which yields the following log-linearized equation

$$\hat{\varepsilon}_t = \hat{q}_t - \hat{q}_{t-1} + \pi_t - \pi_t^*. \tag{2.30}$$

Here, $\hat{\varepsilon}_t \equiv \log(\frac{\mathcal{E}_t}{\bar{\varepsilon}\mathcal{E}_{t-1}})$ is the log-deviation of the nominal depreciation rate from its steady-state value $\bar{\varepsilon}$.

Without loss of generality, the price of foreign composite goods in the foreign currency is normalized to one; that is, $p_t^* = 1$. We further assume that the law of one price always holds, hence $e_t = p_{F,t}$. Then, taking into account the linearized CPI index

$$p_t = (1-\alpha)p_{H,t} + \alpha p_{F,t}, \tag{2.31}$$

we have

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\hat{\varepsilon}_t. \tag{2.32}$$

The terms of trade between the domestic country and the rest of the world are defined as $S_t \equiv P_{F,t}/P_{H,t}$. Taking the log of this expression yields

$$s_t = p_{F,t} - p_{H,t}. \tag{2.33}$$

By further combining (2.29) and (2.31), the terms of trade in the first-order approximation satisfies

$$\hat{s}_t = \frac{\hat{q}_t}{(1-\alpha)}. \tag{2.34}$$

[8] As discussed in Galí and Monacelli (2005), the constant employment subsidy τ is set to correct the distortion associated with firms' market power, which requires $1-\tau = 1-1/\epsilon$. It is financed via contemporaneous lump-sum taxes on households. Thus, Φ_t in (2.3) is the net income of the dividend transfer less taxes.

International Goods Market Clearing. The international market-clearing condition for domestically produced goods j is

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C^{*,i}_{H,t}(j)di, \tag{2.35}$$

for all $j \in [0,1]$ and any t, where $C^{*,i}_{H,t}(j)$ is the demand from foreign country i for good j produced in the domestic country.

With the assumption of identical preferences across countries and the law of one price for any goods j, aggregating (2.35) across j and utilizing (2.34) yield

$$\hat{y}_t = \vartheta_{yc}\hat{c}_t + \vartheta_{ys}\hat{s}_t, \tag{2.36}$$

where $\vartheta_{yc} \equiv (1-\alpha)\bar{\mathcal{S}}^{-\eta}\bar{C}/\bar{Y}$ and $\vartheta_{ys} \equiv \alpha[\gamma + \eta(1-\alpha)]\bar{\mathcal{S}}^{-\eta}\bar{C}/\bar{Y}$. Details of the derivations can be found in Appendix D.

Utilizing the log-linearized production function $\hat{y}_t = \hat{a}_t + \hat{n}_t$ and the labor supply function (2.27), along with (2.31), (2.33), and (2.36), the log-linearized real marginal cost (2.28) becomes:

$$\widehat{mc}_t = (\sigma + \varphi\vartheta_{yc})\hat{c}_t + (\alpha + \varphi\vartheta_{ys})\hat{s}_t - (1+\varphi)\hat{a}_t. \tag{2.37}$$

Without loss of generality, we assume that the steady-state level of domestic productivity is $\bar{A} = 1$, which implies $\bar{a} = 0$. Consequently, this leads to $\hat{a}_t = a_t$.

Domestic Monetary Policy. For the domestic interest rate, we consider a monetary policy rule intended to stabilize the domestic CPI inflation rate following a standard Taylor-type form; that is,[9]

$$\hat{\imath}_t = \phi_\pi \pi_t, \tag{2.38}$$

where $\phi_\pi > 1$ is a constant parameter.

Evolution of the Foreign Bond. Without loss of generality, we assume that the foreign inflation rate is always one; that is, $\Pi^*_t = \bar{\Pi}^* = 1$ and $\pi^*_t = 0$ for any t. Then, the foreign nominal interest rate $\hat{\imath}^*_t$ is equal to $\hat{r}^*_t$ and the steady-state relationship satisfies $1 + \bar{r}^* = \beta^{-1}$. In addition, given that the domestic bonds are not internationally tradable and are cleared domestically in equilibrium, we assume the net supply of domestic bonds in this small open economy is always zero ($\mathcal{B}_t = 0$ for any t).

[9] Our numerical results and conclusions are also robust to targeting domestic inflation measured by producer price index (PPI) $\pi_{H,t}$.

In equilibrium, the resource constraint (2.5) of the domestic economy now becomes:

$$\frac{Q_t\mathcal{B}^*_{t+1}}{1+i^*_t} = \frac{Q_t\mathcal{B}^*_t}{\Pi^*_t} + \mathcal{S}_t Y_t - C_t.$$

After log-linearization around the steady state and by noticing $\hat{\mathcal{S}}_t = -\alpha\hat{s}_t$, we have

$$\hat{b}^*_{t+1} = \beta^{-1}(\hat{b}^*_t + \vartheta_1\hat{q}_t - \vartheta_2\alpha\hat{s}_t + \vartheta_2\hat{y}_t - \hat{c}_t) - \vartheta_1\hat{q}_t + \vartheta_1\hat{r}^*_t, \tag{2.39}$$

where $\vartheta_1 \equiv \frac{\bar{\mathcal{B}}^*\bar{Q}}{\bar{C}}$ and $\vartheta_2 \equiv \frac{\bar{\mathcal{S}}\bar{Y}}{\bar{C}}$. This equation governs the evolution of the foreign bond (net foreign asset position) $\hat{b}^*_t$ in the equilibrium.

Foreign Interest Rate Function. A small open-economy feature of our model is that the foreign real interest rate, denoted by $\hat{r}^*_t$, is exogenously set by the rest of the world. Schmitt-Grohé and Uribe (2003) show that linearized small open-economy models under RE with an incomplete asset market yield equilibrium dynamics with random walk components for open-economy related variables, if the rate of return of the internationally traded assets is exogenously determined abroad. Thus, one needs stationarity-inducing devices to solve small open-economy models under RE with an incomplete asset market.

We assume that the foreign interest rate faced by the domestic country endogenously responds to its level of net foreign asset position:

$$\hat{r}^*_t = \phi_b\tilde{b}^*_{t+1} + \mu_t, \tag{2.40}$$

where $\tilde{b}^*_{t+1}$ is the cross-sectional average of the foreign bond holdings across households (which is beyond household i's control), $\phi_b < 0$ is a constant, and μ_t is the random foreign interest rate shock.[10] In particular, $\phi_b < 0$ indicates that as the domestic country hold more net foreign asset position from the rest of the world, it is charged with a lower interest rate by the international financial market. Similarly, if the country borrows ($\tilde{b}^*_{t+1} < 0$), it is charged a higher interest rate when it borrows more. This specification follows the standard practice of the external debt-elastic interest rate function, as proposed by Schmitt-Grohé and Uribe (2003).

Importantly, as discussed in Woodford (2019) and Xie (2020), a model with limited foresight by design always guarantees a unique equilibrium solution in the closed-economy setting, regardless of the restrictions on the monetary/fiscal policy reaction function. This feature of equilibrium determinacy also holds in our small open-economy model under limited

[10] In equilibrium, we have $\tilde{b}^*_{t+1} = \hat{b}^*_{t+1}$.

foresight.[11] However, we still impose the same assumption (2.40) in the model of limited foresight to facilitate its comparison with the counterpart model under RE, so that we can isolate the role of limited foresight on the different equilibrium performances.

2.5 Decision-makers' Learning in Value Functions

Thus far, we have assumed that the value functions of households $v\left(\mathcal{B},\mathcal{B}^*\right)$ and of firms $\tilde{v}\left(r^f\right)$ are the fixed ones learned from the nonstochastic stationary environment. Starting from this section, we relax this assumption such that decision-makers update their value functions over time based on their past experiences. Similar to Woodford (2019), we assume that the learning behaviors of households and firms follow a constant-gain process:

$$v_{t+1}(\mathcal{B},\mathcal{B}^*) = \gamma_v v_t^{est}(\mathcal{B},\mathcal{B}^*) + (1-\gamma_v)v_t(\mathcal{B},\mathcal{B}^*),$$

$$\tilde{v}_{t+1}\left(r^f\right) = \gamma_{\tilde{v}}\tilde{v}_t^{est}\left(r^f\right) + (1-\gamma_{\tilde{v}})\tilde{v}_t\left(r^f\right),$$

where $\gamma_v, \gamma_{\tilde{v}} \in [0,1]$ are learning gain parameters. That is, decision-makers *extrapolate* future value functions (which will be used in the planning exercise at time $t+1$) using their beliefs about the value functions at time t and estimates of the value function obtained as a result of their planning exercises at time t. Therefore, the value functions that describe decision-makers' perceptions of the future beyond their planning horizons reflect past and estimated value functions. This extrapolation can lead to slow-moving components of economic indicators.

We now consider a local approximation of the dynamics implied by the constant-gain learning rule through a perturbation of the steady-state solution. We parameterize a log-linear approximation of $v_1(\mathcal{B},\mathcal{B}^*)$ in the household's optimal finite-horizon plan with respect to the domestic bond as

$$\log(v_{1,t}(\mathcal{B},\mathcal{B}^*)/v_1^*(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)) = -\sigma\left[\nu_t + \chi_t\hat{b} + \zeta_t\hat{b}^*\right].$$

Here we use $v^*(\cdot)$ to denote the steady-state value function. Using this approximation, we can compute a log-linear approximation of the solution to the household's optimal finite-horizon plan in period t.

Let $C_t^i(\mathcal{B},\mathcal{B}^*)$ denote the optimal expenditure plan of household i under the counterfactual assumption $\mathcal{B}^i = \mathcal{B}$. Then, the derivative of the estimated value function is equal

[11] When there is no learning in agents' value function, the equilibrium under limited foresight is stationary even with an exogenous path of $\hat{r}_t^*$. However, when agents learn and update their value function, a device similar to (2.40) is necessary to guarantee the stationarity of equilibrium.

to

$$v^{est}_{1,t}(\mathcal{B},\mathcal{B}^*) = \mathbb{E}^i_t[u_C(C^i_t(\mathcal{B},\mathcal{B}^*)/\Pi_t].$$

By log-linearization, we then have

$$\log(v^{est}_{1,t}(\mathcal{B},\mathcal{B}^*)/v^*_1(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)) = -\sigma\hat{c}^h_t(\hat{b},\hat{b}^*) - \pi^h_t, \tag{2.41}$$

where h is the planning horizon of household i. Our log-linear approximation of the optimal household plan satisfies $\hat{c}^h_t(\hat{b},\hat{b}^*) = \hat{c}^h_t(\bar{b},\bar{b}^*) + \hat{c}^h_{1,t}\hat{b} + \hat{c}^h_{2,t}\hat{b}^*$. Approximating the left-hand side as $-\sigma\left[\nu^{est}_t + \chi^{est}_t\hat{b} + \zeta^{est}_t\hat{b}^*\right]$ of equation (2.41) directly implies

$$\nu^{est}_t = \hat{c}^h_t + \sigma^{-1}\pi^h_t - \zeta^{est}_t\hat{b}^*_t,$$

where we have used the condition $\hat{b}_t = 0$ for all t. Equating the coefficients on both sides by substituting the expression of $\hat{c}^h_t(\hat{b},\hat{b}^*)$ yields

$$\chi^{est}_t = \hat{c}^h_{1,t},$$

$$\zeta^{est}_t = \hat{c}^h_{2,t}.$$

The intercept term of the estimated marginal value of the domestic bond, ν^{est}_t, depends on current consumption, the CPI inflation, and the foreign bond.

Together with the constant-gain learning rule, we have

$$\begin{aligned}
\nu_{t+1} &= \gamma_v\nu^{est}_t + (1-\gamma_v)\nu_t,\\
\chi_{t+1} &= \gamma_v\chi^{est}_t + (1-\gamma_v)\chi_t,\\
\zeta_{t+1} &= \gamma_v\zeta^{est}_t + (1-\gamma_v)\zeta_t.
\end{aligned}$$

We can show that χ_t and ζ_t are univariately mean-reverting to a constant $1-\beta$, and thus we assume that these two variables have converged. Details can be found in Appendix E.

Similarly, we parameterize a log-linear approximation of $v_2(\mathcal{B},\mathcal{B}^*)$ in the households' optimal finite-horizon plan with respect to the foreign bond as

$$\log(v_{2,t}(\mathcal{B},\mathcal{B}^*)/v^*_2(\bar{\mathcal{B}},\bar{\mathcal{B}}^*)) = -\sigma\left[\nu^*_t + \chi'_t\hat{b} + \zeta'_t\hat{b}^*\right].$$

Using this approximation, we can compute a log-linear approximation of the solution to the household's optimal finite-horizon plan in period t.

Let $C^i_t(\mathcal{B},\mathcal{B}^*)$ denote the optimal expenditure plan of household i under the counterfactual assumption $\mathcal{B}^{*i} = \mathcal{B}^*$. Then, the derivative of the estimated value function will be equal to

$$v_{2,t}^{est}(\mathcal{B}, \mathcal{B}^*) = \hat{\mathbb{E}}_t^i[u_C(C_t^i(\mathcal{B}, \mathcal{B}^*))Q_t/\Pi_t^*].$$

Note we have assumed that $\Pi_t^* = 1$ for any t. By log-linearization, we then have

$$\log(v_{2,t}^{est}(\mathcal{B}, \mathcal{B}^*)/v_2^*(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)) = -\sigma \hat{c}_t^h(\hat{b}, \hat{b}^*) + \hat{q}_t^h. \tag{2.42}$$

Our log-linear approximation of the optimal household plan satisfies $\hat{c}_t^h(\hat{b}, \hat{b}^*) = \hat{c}_t^h(\bar{b}, \bar{b}^*) + \hat{c}_{1,t}^h\hat{b} + \hat{c}_{2,t}^h\hat{b}^*$. Approximating the left-hand side as $-\sigma[\nu_t^{*,est} + \chi_t^{\prime,est}\hat{b} + \zeta_t^{\prime,est}\hat{b}^*]$ and equating coefficients yield

$$\nu_t^{*,est} = \hat{c}_t^h - \sigma^{-1}\hat{q}_t^h - (1-\beta)\hat{b}_t^* = \nu_t^{est} - \sigma^{-1}(\hat{q}_t^h + \pi_t^h),$$

$$\chi_t^{\prime,est} = \hat{c}_{1,t}^h = \chi_t^{est},$$

$$\zeta_t^{\prime,est} = \hat{c}_{2,t}^h = \zeta_t^{est},$$

where we have utilized the fact that $\zeta_t^{est} = 1-\beta$ as shown in Appendix E. Thus, the estimated marginal value of the foreign bond depends on the estimated marginal value of the domestic bond, the current real exchange rate, and CPI inflation.

Note further that the constant-gain learning rule yields

$$\nu_{t+1}^* = \gamma_v \nu_t^{*,est} + (1-\gamma_v)\nu_t^*.$$

Therefore, we have characterized the learning process of the household value function.

For the firm (with planning horizon k), we similarly have

$$\tilde{\nu}_t^{est} = (1-\theta)^{-1}\pi_{H,t}^k,$$

and

$$\tilde{\nu}_{t+1} = \gamma_{\tilde{v}}\tilde{\nu}_t^{est} + (1-\gamma_{\tilde{v}})\tilde{\nu}_t.$$

3 Equilibrium Characterization with Homogeneous Planning Horizons across Agents

In this section, we focus on the steps to pin down the equilibrium path with the assumption that all the agents share the same planning horizon h. Therefore, the equilibrium dynamics of aggregate variables satisfy $\hat{y}_t = \hat{y}_t^h$, $\pi_t = \pi_t^h$, etc., and also $\hat{b}^*_{t+1} = \hat{b}^{*,h}_{t+1} = \tilde{b}^*_{t+1}$. Focusing on the case of homogeneous agents allows us to abstract from the aggregation problem across the population, while we can still analyze how the equilibrium dynamics change with respect to the degree of foresight (i.e., the common planning horizon h). In Section 6, we extend the analyses to heterogeneous agents with different planning horizons and discuss the robustness of the main results.[12]

First, given state variables $\{\hat{q}_{t-1}, \hat{b}^*_t, \nu_t, \nu^*_t, \tilde{\nu}_t\}$ and exogenous shocks $\{a_t, \mu_t\}$ that follow AR(1) processes, we solve the problem of the finite planning exercise in period t. Let $\hat{y}^j_{\tau|t}$ be the value of $\hat{y}_\tau$ that is predicted at date τ as a result of aggregation of decisions made by agents with (counterfactual) planning horizon $j = h + t - \tau$, which is calculated at date t by agents with planning horizon h. It is a function of the state $\{\hat{q}_{t-1}, \hat{b}^*_t, \nu_t, \nu^*_t, \tilde{\nu}_t\}$ and $\{a_t, \mu_t\}$ in period t. Then, we have the actual aggregate output in period t given by $\hat{y}_t = \hat{y}^h_{t|t}$. Similarly, we can define other variables in the finite planning exercise with the same notation. The additional subscript $|t$ matters because different value functions are used in finite planning in different periods.

The equilibrium conditions for the finite planning exercise in period t are given in Appendix F. The system consists of a finite number of equations as a function of state variables $\{\hat{q}_{t-1}, \hat{b}^*_t, \nu_t, \nu^*_t, \tilde{\nu}_t\}$ and exogenous shocks $\{a_t, \mu_t\}$. Thus, we can solve for all endogenous variables $\{\hat{c}^{h+t-\tau}_{\tau|t}, \hat{y}^{h+t-\tau}_{\tau|t}, \hat{\imath}^{h+t-\tau}_{\tau|t}, \hat{r}^{*,h+t-\tau}_{\tau|t}, \pi^{h+t-\tau}_{H,\tau|t}, \pi^{h+t-\tau}_{\tau|t}, \hat{q}^{h+t-\tau}_{\tau|t}, \hat{s}^{h+t-\tau}_{\tau|t}, \hat{\varepsilon}^{h+t-\tau}_{\tau|t}, \hat{b}^{*,h+t-\tau}_{\tau+1|t}\}_{\tau=t}^{t+h}$ with a unique solution. The actual aggregate variables in period t are then given by

$$
\begin{aligned}
&\hat{c}_t = \hat{c}^h_{t|t}, && \hat{y}_t = \hat{y}^h_{t|t}, && \hat{\imath}_t = \hat{\imath}^h_{t|t}, && \hat{r}^*_t = \hat{r}^{*,h}_{t|t}, && \pi_{H,t} = \pi^h_{H,t|t}, \\
&\pi_t = \pi^h_{t|t}, && \hat{q}_t = \hat{q}^h_{t|t}, && \hat{s}_t = \hat{s}^h_{t|t}, && \hat{\varepsilon}_t = \hat{\varepsilon}^h_{t|t}, && \hat{b}^*_{t+1} = \hat{b}^{*,h}_{t+1|t}.
\end{aligned} \tag{3.1}
$$

From period t to period $t+1$, the value functions evolve over time; that is,

$$\nu_{t+1} = \gamma_v \nu^{est}_t + (1-\gamma_v)\nu_t, \tag{3.2}$$

$$\nu^*_{t+1} = \gamma_v \nu^{*,est}_t + (1-\gamma_v)\nu^*_t, \tag{3.3}$$

$$\tilde{\nu}_{t+1} = \gamma_{\tilde{v}} \tilde{\nu}^{est}_t + (1-\gamma_{\tilde{v}})\tilde{\nu}_t, \tag{3.4}$$

[12] Our conclusions are also robust when extending the set of exogenous shocks to include domestic demand shock and domestic interest rate shock, in addition to the productivity shock and foreign interest rate shock.

where

$$\nu_t^{est} = \hat{c}_t + \sigma^{-1}\pi_t - (1-\beta)\hat{b}_t^*, \tag{3.5}$$

$$\nu_t^{*,est} = \nu_t^{est} - \sigma^{-1}(\hat{q}_t + \pi_t), \tag{3.6}$$

$$\tilde{\nu}_t^{est} = (1-\theta)^{-1}\pi_{H,t}. \tag{3.7}$$

Now, we describe how to solve the planned solution at date τ calculated in period t, which is characterized by the equilibrium conditions of the finite planning exercise as shown in Appendix F. We can write the solution to any endogenous variable $x_{\tau|t}^j$ except $\hat{b}_{\tau+1|t}^{*j}$ in forward planning as a function of the state variables and exogenous shocks; that is,

$$x_{\tau|t}^j = \psi_{x,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{x,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{x,a}^j a_\tau + \psi_{x,\mu}^j \mu_\tau + \psi_{x,\nu}^j \nu_t + \psi_{x,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{x,\nu^*}^j \nu_t^*, \tag{3.8}$$

for any (counterfactual) $j \geq 0$, and similarly,

$$\hat{b}_{\tau+1|t}^{*j} = \psi_{b,q}^j \hat{q}_{\tau-1|t}^{j+1} + \psi_{b,b}^j \hat{b}_{\tau|t}^{*,j+1} + \psi_{b,a}^j a_\tau + \psi_{b,\mu}^j \mu_\tau + \psi_{b,\nu}^j \nu_t + \psi_{b,\tilde{\nu}}^j \tilde{\nu}_t + \psi_{b,\nu^*}^j \nu_t^*. \tag{3.9}$$

Then, one can solve the undetermined coefficients via the equilibrium conditions of the finite planning exercise for any j in the following steps: (i) utilizing the equilibrium conditions for $j = 0$ and equating the coefficients yields the coefficients for $j = 0$; (ii) solving the undetermined coefficients for any (counterfactual) j by forward induction. That is, given the coefficients for $j - 1$, the undetermined coefficients for j are uniquely given by the equilibrium conditions for j. See Appendix G for the detailed procedure.

Thus far, we have derived the solution of the entire forward planning calculated in period t. Then, one can easily solve for the equilibrium path (3.1) with the evolution of the state variables (3.2)-(3.4) together with exogenous shocks.

4 Equilibrium Analyses

In this section, we investigate the equilibrium features of the FH model. We first calibrate the model parameters to match key moments of the aggregate dynamics of Canada with some structural parameters standard in the literature. Next, we analyze the equilibrium dynamics by examining impulse response functions of the FH model, comparing it with its RE counterpart and the case of the FH model without learning in the value function. Finally, we discuss how the models with limited foresight generate systematic forecast errors in the real exchange rate.

Table 1: Calibrated Parameters

Parameter	Value	Description
h	8	Length of Planning Horizon (quarter)
β	0.99	Subjective Discount Factor
σ	2	Inverse of Intertemporal Elasticity of Substitution
α	0.15	1 - Home Bias
φ	1	Inverse of the Frisch Elasticity of Labor Supply
θ	0.75	Calvo-Yun Sticky Price Parameter
γ	1.5	Elasticity of Substitution between Goods of Foreign Countries
η	1.5	Elasticity of Substitution between Goods of Home and Rest-of-the-World
ϕ_π	2.15	Monetary Policy Reaction Coefficient to the CPI Inflation Rate
ϕ_b	-0.01	External Bond Sensitivity of the Foreign Interest Rate
ρ_x	0.9	Persistence of Shocks
γ_v	0.165	Household's Learning Gain
σ_a	0.0114	Std. Dev. of TFP Shock
σ_μ	0.0050	Std. Dev. of Foreign Interest Rate Shock

4.1 Calibrated Parameters

We calibrate the FH model to a quarterly frequency; see Table 1. Following the common practice in the open-economy macro literature, we assume a symmetric steady state across the domestic country and the rest of the world, with $\bar{\mathcal{B}}^* = 0$, $1 + \bar{r}^* = 1/\beta$, and $\bar{Q} = 1$. It directly implies $\bar{\mathcal{S}} = \bar{S} = 1$ and $\bar{C} = \bar{Y}$.

The following parameters are standard in the literature. We set the subjective discount factor $\beta = 0.99$, the inverse of intertemporal elasticity of substitution $\sigma = 2$, and the inverse of the Frisch elasticity of labor supply $\varphi = 1$. Parameter α, which governs the home bias $(1 - \alpha)$, is set to 0.15. The Calvo-Yun price stickiness parameter is $\theta = 0.75$, implying an average duration of four quarters between two consecutive price adjustments. We set the parameter of policy reaction in the Taylor rule $\phi_\pi = 2.15$, following Clarida, Gali and Gertler (1999), and the parameters of trade elasticity γ and η to 1.5, following the values used in Backus, Kehoe and Kydland (1994) and Chari, Kehoe and McGrattan (2002). We fix the sensitivity of the foreign interest rate to foreign bond holdings ϕ_b to -0.01, following Benigno (2009) and Justiniano and Preston (2010). Together with the steady-state values, we calculate the coefficients ϑ_1, ϑ_2, ϑ_{yc}, and ϑ_{ys} by their definitions.

Following Woodford and Xie (2022), we set the length of the planning horizon $h = 8$ (that is, eight quarters). This is a *conservative* value because empirical findings suggest an even shorter planning horizon; for instance, Gust, Herbst and López-Salio (2022) estimate an average planning horizon as being one-quarter of the U.S. economy.[13] If the planning horizon

[13] The survey evidence in Coibion et al. (2023) suggests that household planning horizons in the U.S. are no more than two years and they may well be capable of planning forward about three or four quarters.

Table 2: Data and Model-Implied Second-Order Moments

	Data	Model-FH	Model-RE
Targeted Moments			
$\sigma(\Delta y) \times 100$	0.89	0.89	0.84
$\sigma(\Delta c)/\sigma(\Delta y)$	0.93	0.93	1.14
$\sigma(\Delta q)/\sigma(\Delta y)$	2.19	2.03	2.30
$\rho(q)$	0.98	0.96	0.90
$\rho(\Delta y, \Delta c)$	0.49	0.41	0.35
Non-Targeted Moments			
$\rho(\varepsilon)$	0.34	0.29	0.15
$\sigma(\varepsilon)/\sigma(\Delta y)$	2.16	2.46	2.66
$\rho(\varepsilon, \Delta q)$	0.94	0.99	0.99
$\rho(\Delta c, \Delta q)$	-0.07	-0.39	-0.55

Notes: Among the targeted moments, $\rho(q)$ presents the persistence of the real exchange rate and $\rho(\Delta y, \Delta c)$ represents the cross-correlation between output growth and consumption growth. Similar notations apply to the non-targeted moments. For model-implied second-order moments, each entry is the median of the moments derived from 10,000 simulations, each spanning 188 quarters. The length in quarters matches that of the Canadian data we utilized.

is shorter, our conclusions are strengthened as they deviate more from the case under RE and the numerical results become quantitatively much stronger.

We assume that both exogenous shocks exhibit a persistence of 0.9, following the assumption of shock persistence as posited by Candian and De Leo (2023). We then calibrate the three remaining parameters: the household's learning gain parameter γ_v and the standard deviations of the two exogenous shocks σ_a and σ_μ.[14] The three parameters are jointly calibrated to match the following five moments, utilizing Canadian data from 1961:2Q to 2007:4Q:[15] (i) the standard deviation of output growth, $\sigma(\Delta y)$; (ii) the relative standard deviation of consumption growth to output growth, $\sigma(\Delta c)/\sigma(\Delta y)$; (iii) the relative standard deviation of real exchange rate growth to output growth, $\sigma(\Delta q)/\sigma(\Delta y)$; (iv) the persistence of the real exchange rate, $\rho(q)$; and (v) the cross-correlation between output growth and consumption growth, $\rho(\Delta y, \Delta c)$. We obtain a learning gain of $\gamma_v = 0.165$,[16] and standard deviations of the shocks $\sigma_a = 0.0114$ and $\sigma_\mu = 0.0050$. As a consequence, the relative

[14] In our benchmark numerical analysis, we nullify the firm's learning behavior on its value function by setting $\gamma_{\tilde{v}} = 0$ for parsimony. Incorporating the firm's learning behavior has negligible effects on our findings in Sections 4 and 5. Thus, we leave the case of incorporating the firm's learning behavior in the robustness checks; see Appendix J.

[15] The data source and variable construction can be found in Appendix H. We stop the sample at 2007 to exclude the period of the Great Recession.

[16] The calibrated value of the households' learning gain parameter is also close to the benchmark estimate in Gust, Herbst and López-Salio (2022), which stands around 0.14.

standard deviation of domestic productivity and foreign interest shocks stands at 2.28.

Table 2 reports a number of moments for the macroeconomic aggregates, comparing both the data and the model-generated moments with the same calibrated parameters. The FH model matches the overall moments reasonably well, including the five targeted moments and other non-targeted ones. In addition, the FH model fits the data overall better than the RE counterpart does, as the calibration is based on executing the FH model. Compared to the data, the calibrated FH model tends to overpredict the negative correlation between the consumption and real exchange rate growths, $\rho(\Delta c, \Delta q)$. Nevertheless, the FH model still performs significantly better than the RE model along this dimension.

4.2 Impulse Responses

We now analyze the equilibrium dynamics by comparing the impulse response functions between the FH and RE models. To isolate the role of value function learning, we also compare with the FH model in which the learning gain in value function is zero (i.e., $\gamma_v = 0$, labeled as FH-NG). We consider the actual transition dynamics of variables under perfect foresight shocks as impulse responses, where decision-makers are aware of the shock's complete future path upon its impact on the economy. The actual transition dynamics help study how value-function learning influences actual (not expected) aggregate dynamics, as decision-makers update their value function over time.

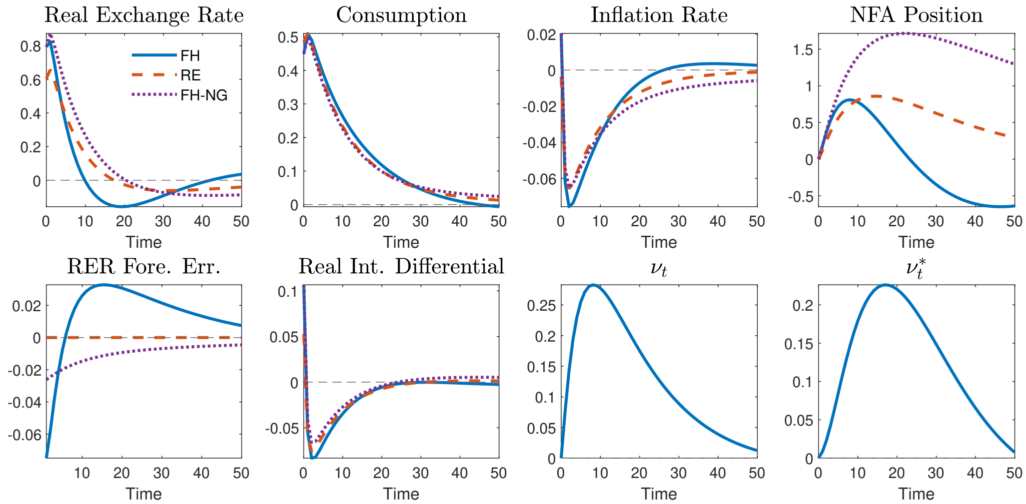
Figure 1 illustrates the impulse responses of the variables of interest to the two structural shocks, respectively, with a size of one standard deviation. In essence, across all three models, the domestic productivity and foreign interest rate shocks (shown in panels (a) and (b), respectively) conform to their roles in price and quantity determination. The former causes real exchange rate depreciation and consumption increase, triggering a temporary rise in CPI inflation due to the pass-through of exchange rate depreciation, followed by a supply shock-induced inflation decline. The latter shock depreciates the real exchange rate, curbs consumption, and elevates CPI inflation via exchange rate pass-through. Both shocks prompt households to boost savings, resulting in a gradual rise in net foreign asset positions $\hat{b}_t^*$ in subsequent periods. Also, represented as $\hat{r}_t - \hat{r}_t^*$ (with $\hat{r}_t$ being the nominal interest rate minus expected one-period ahead inflation), the real interest rate differential between domestic and foreign bonds initially surges due to shocks but later adjusts downwards, indicating higher subsequent real returns from foreign bonds.[17]

Despite those common patterns, notable differences emerge across the models. Specif-

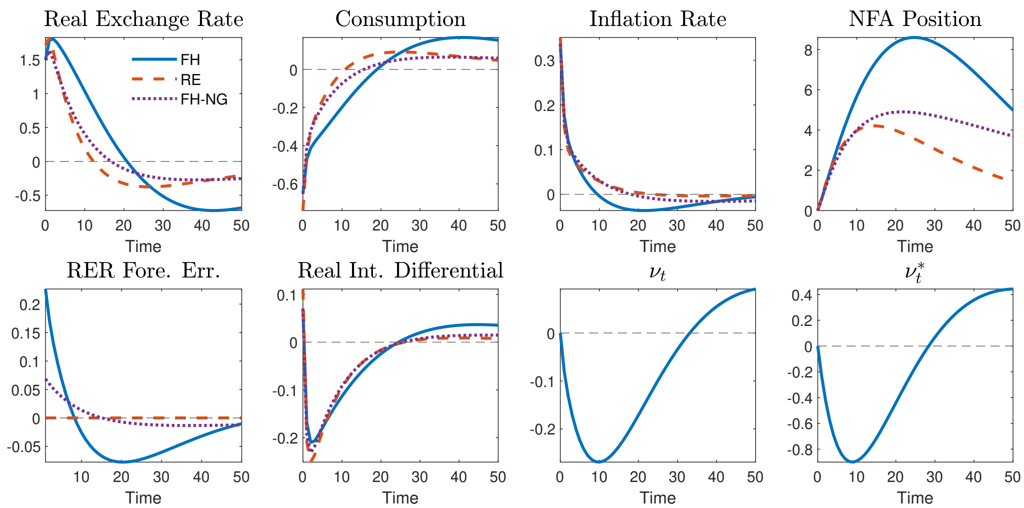
[17] The figure shows initial real interest rate differential surges in both the FH and RE models, indicating that the surge is not specific to the FH model. Instead, the root cause of the initial surges is attributed to the substantial persistence of the shocks, which is set to 0.9.

Figure 1: Impulse Responses of Selected Variables to Structural Shocks

(a) To Domestic Productivity Shock



(b) To Foreign Interest Rate Shock



Notes: This figure shows the impulse responses of selected variables in three models, subject to one standard deviation domestic productivity shock (panel (a)) and to one standard deviation foreign interest rate shock (panel (b)). "FH" refers to the benchmark limited foresight model, "RE" refers to the rational expectation model, and "FH-NG" refers to the limited foresight model with no learning gain ($\gamma_v = 0$).

ically, in comparison to the RE model, the FH model generally exhibits more persistent, hump-shaped movements of the aggregate variables. Furthermore, the FH model generates non-trivial dynamics of the forecast errors of the real exchange rate, which is a feature absent in the RE model. We now examine these differences in detail.

Dynamics of the Value Functions. To understand the responses of the aggregate dynamics in the FH model, we first examine the dynamics of ν_t and ν_t^*. These are related to the log-linear approximation of marginal values of domestic and foreign bonds, $v_{1,t}(\mathcal{B}, \mathcal{B}^*)$ and $v_{2,t}(\mathcal{B}, \mathcal{B}^*)$, respectively, as detailed in Section 2.5. In the FH model, when shocks occur at $t = 0$, households use the steady-state evaluated priors ν_0 and ν_0^* for planning. Subsequently, they observe the realized macro outcomes and estimate ν_0^{est} and $\nu_0^{*,est}$, using (3.5) and (3.6). Households then update ν_1 and ν_1^* by averaging the estimates with the previous values of ν_0 and ν_0^*, as outlined in (3.2) and (3.3). These ν_1 and ν_1^* are used for planning in period 1. After observing the realized macroeconomic outcome at the end of period 1, they obtain estimates ν_1^{est} and $\nu_1^{*,est}$. These are used to update ν_2 and ν_2^* for planning in period 2. This process is repeated thereafter.

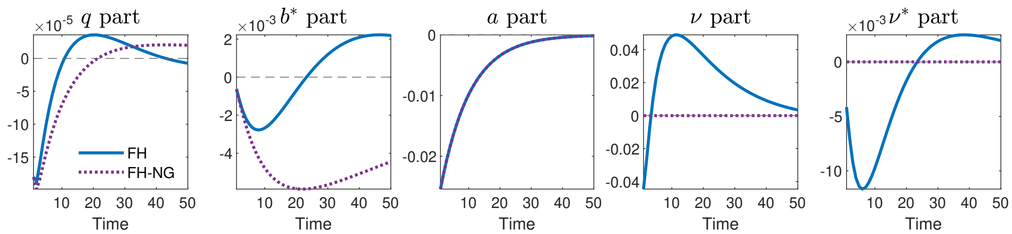
Households' learning behavior results in hump-shaped dynamics for ν_t and ν_t^*. With a learning gain of $\gamma_v = 0.165$, households replace 16.5% of the prior values with values estimated from previous planning exercises, which emphasizes the prior values and creates persistent dynamics. When exogenous shocks occur, households slowly update their values using the estimated values, which then gradually revert to steady-state as the shocks fade, creating hump-shaped dynamics. Panel (a) suggests ν_t and ν_t^* follow inverse U-shaped dynamics with the productivity shock, while panel (b) suggests they display U-shaped dynamics with the foreign interest rate shock.

To understand why the hump-shaped dynamics of ν_t and ν_t^* exhibit opposite directions based on two different shocks, we now need to investigate the estimates ν_t^{est} and $\nu_t^{*,est}$ more precisely. As detailed in Section 2.5, they are related to the log-linear approximations of the estimated marginal values of domestic and foreign bonds, $v_{1,t}^{est}(\mathcal{B}, \mathcal{B}^*)$ and $v_{2,t}^{est}(\mathcal{B}, \mathcal{B}^*)$, respectively. For the domestic bond, the approximation is $-\sigma\left[\nu_t^{est} + (1-\beta)\hat{b}_t^*\right]$, positioning ν_t^{est} as the opposite of this value. Per (3.5), given the predetermined net foreign asset position $\hat{b}_t^*$, the bond's estimated marginal value increases with a decline in both consumption and inflation. Intuitively, an increase in the marginal utility of consumption raises the marginal value of the real domestic bond via the standard wealth effect, while inflation reduces the domestic bond's real value. Similarly, $\nu_t^{*,est}$ serves as the opposite of the estimated marginal value of the foreign bond. Combining (3.5) and (3.6) yields $\nu_t^{*,est} = \hat{c}_t - \sigma^{-1}\hat{q}_t - (1-\beta)\hat{b}_t^*$. Therefore, given the predetermined net foreign asset position, $\nu_t^{*,est}$ decreases as consumption falls and the real exchange rate depreciates. The intuition is similar to the domestic bond case; an increase in the marginal utility of consumption raises the marginal value of the real foreign bond via the standard wealth effect, and a depreciation of the real exchange rate increases the foreign bond's real value (in domestic currency).

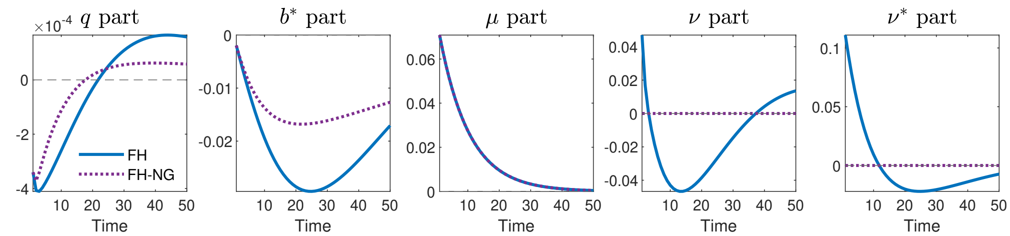
With σ and β calibrated in Table 1, the equations for estimated value function (3.5) and

Figure 2: Decomposition of the Forecast Error Dynamics for the Real Exchange Rate

(a) To Domestic Productivity Shock



(b) To Foreign Interest Rate Shock



Notes: This figure shows the impulse response functions of the components of the one-period ahead forecast errors of the real exchange rate in the two models, subject to one standard deviation domestic productivity shock (panel (a)) and one standard deviation foreign interest rate shock (panel (b)). "FH" refers to the benchmark limited foresight model and "FH-NG" refers to the limited foresight model with $\gamma_v = 0$.

(3.6) become $\nu_t^{est} = \hat{c}_t + 0.5\pi_t - 0.01\hat{b}_t^*$ and $\nu_t^{*,est} = \hat{c}_t - 0.5\hat{q}_t - 0.01\hat{b}_t^*$. Figure 1 suggests that the two exogenous shocks affect the left-hand-side variables differently, both in direction and magnitude. Concerning the coefficients on the variables, the responses of consumption, inflation, and the real exchange rate tend to be the primary drivers of ν_t^{est} and $\nu_t^{*,est}$. As a consequence, combined with the households' learning behavior regarding value functions, ν_t and ν_t^* follow inverse U-shaped dynamics in response to the productivity shock and U-shaped dynamics in response to the foreign interest rate shock.

Dynamics of RER Forecast Errors. We now turn our attention to the dynamics of the one-period ahead forecast errors of the real exchange rate, $\hat{q}_{t+1} - \hat{\mathbb{E}}_t\hat{q}_{t+1}$. Figure 1 shows that the FH model inherently produces systematic forecast errors due to a discrepancy between expectation and realization.

The dynamics of the exchange rate forecast errors significantly depend on the learning behavior in the value function and display contrasting patterns based on the two shocks. When subjected to the domestic productivity shock, as shown in panel (a), the FH model initially presents a short-run negative forecast error that reverses to positive in subsequent periods. When responding to the foreign interest rate shock (panel (b)), the FH model initially shows a positive forecast error in the short-run, which later reverses to negative in the ensuing periods.[18]

These findings suggest that in response to a domestic productivity shock, households overestimate the real depreciation in the short-run, but underestimate it in the long-run. Conversely, when responding to a foreign interest rate shock, households underestimate the depreciation in the short-run, but overestimate it in the long-run. On the other hand, although the FH-NG model also displays contrasting patterns of forecast errors after the two shocks, the magnitudes and dynamic reversals are significantly less pronounced.

Figure 2 displays the decomposition of the dynamics associated with the one-period ahead forecast errors of the real exchange rate, as shown in Figure 1. In the FH model, the forecast error of the one-period ahead real exchange rate can be written as

$$\begin{aligned}\hat{q}_{t+1} - \hat{\mathbb{E}}_t \hat{q}_{t+1} = & \underbrace{\underbrace{\left(\psi^8_{q,q} - \psi^7_{q,q}\right)\hat{q}_t}_{q \text{ part}} + \underbrace{\left(\psi^8_{q,b} - \psi^7_{q,b}\right)\hat{b}^*_{t+1}}_{b^* \text{ part}} + \underbrace{\left(\psi^8_{q,a} - \psi^7_{q,a}\right)\rho_a a_t}_{a \text{ part}} + \underbrace{\left(\psi^8_{q,\mu} - \psi^7_{q,\mu}\right)\rho_\mu \mu_t}_{\mu \text{ part}}}_{\text{forecast errors from finite forward planning}} \\ & + \underbrace{\underbrace{\left(\psi^8_{q,\nu}\nu_{t+1} - \psi^7_{q,\nu}\nu_t\right)}_{\nu \text{ part}} + \underbrace{\left(\psi^8_{q,\nu^*}\nu^*_{t+1} - \psi^7_{q,\nu^*}\nu^*_t\right)}_{\nu^* \text{ part}}}_{\text{forecast errors from value function extrapolation (learning)}} . \end{aligned} \tag{4.1}$$

That is, the forecast error of the real exchange rate in the FH model can be decomposed into six components $(q, b^*, a, \mu, \nu, \nu^*)$ categorized by forecast errors stemming from (i) finite forward planning and (ii) value function extrapolation (learning). In contrast, the forecast error in the FH-NG model only includes the first four components, given that $\nu_t = \nu^*_t = 0$ for all t. Figure 2 suggests that quantitatively, the forecast errors caused by value function extrapolation in the FH model, absent in the FH-NG model, play a crucial role due to the non-monotonic, hump-shaped behavior in value functions. This results in dynamic overshooting of the forecast error, leading to a sign reversal over the time horizon.

5 Addressing the UIP Puzzles across Horizons

In this section, we show that the FH model can address some puzzling features of the time and forecast horizon aspects of the UIP condition predicted by the RE framework. In the quantitative exercise, recognizing that the calibrated parameters in Table 1 do not target any empirical moments related to the UIP violations, our numerical analyses thus provide external validity for how the calibrated model is apt in explaining the qualitative features of

[18]This result evokes the empirical findings of Angeletos, Huo and Sastry (2021), wherein a dynamic overshooting of forecast errors of inflation and unemployment is observed from the U.S. survey data, notwithstanding the distinctions in the underlying environment and the variables under analysis.

expectation formations related to RE-UIP violations.

We begin by formally describing the RE-UIP condition and its implications.

5.1 RE-UIP Condition and Its Implications

The asset pricing equations of the real domestic currency bond and the real foreign currency bond in the RE framework (corresponding to (2.6)-(2.9) in the FH framework) are stated as follows:

$$u'(C_t) = \beta \mathbb{E}_t \left[(1+i_t)\, u'(C_{t+1})/\Pi_{t+1} \right], \tag{5.1}$$

$$u'(C_t) = \beta \mathbb{E}_t \left[(1+i_t^*)\, u'(C_{t+1})(Q_{t+1}/Q_t)(1/\Pi_{t+1}^*) \right]. \tag{5.2}$$

Combining (5.1) and (5.2) yields

$$\mathbb{E}_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \left(\frac{1+i_t}{\Pi_{t+1}} - \frac{1+i_t^*}{\Pi_{t+1}^*} \frac{Q_{t+1}}{Q_t} \right) \right] = 0. \tag{5.3}$$

The log-linear approximation of equation (5.3) around the nonstochastic steady state gives

$$\mathbb{E}_t \hat{q}_{t+1} - \hat{q}_t = \hat{r}_t - \hat{r}_t^*, \tag{5.4}$$

where $\hat{r}_t \equiv \hat{\imath} - \mathbb{E}_t \pi_{t+1}$ and $\hat{r}_t^* \equiv \hat{\imath}^* - \mathbb{E}_t \pi_{t+1}^*$. Equation (5.4) is the RE-UIP condition in real form. This implies that when the real interest rate differential between domestic and foreign currency bonds is positive (that is, $\hat{r}_t - \hat{r}_t^* > 0$), future real depreciation $\mathbb{E}_t \hat{q}_{t+1} - \hat{q}_t > 0$ should result of the same magnitude.[19]

We define the ex-post real excess return on foreign currency bonds from period t to $t+1$ as follows:

$$\Delta_{t+1} \equiv \hat{q}_{t+1} - \hat{q}_t + \hat{r}_t^* - \hat{r}_t. \tag{5.5}$$

Then, the UIP condition (5.4) under RE implies

$$\mathbb{E}_t \Delta_{t+1} = 0, \tag{5.6}$$

indicating that the ex-post excess return Δ_{t+1} should be unpredictable with the information

[19] The literature also often uses the nominal version of the RE-UIP condition. Choosing either nominal or real version is not consequential for our results. We use the real version just to facilitate the discussion with Section 4. Also, note that the RE-UIP condition (5.4) is a simple specification and it can be extended to more complex forms, for example, by considering a stationary trend in the real exchange rate dynamics. For the sake of parsimony, we use the simplest specification to emphasize the role of behavioral biases from the FH models in addressing the puzzles that we focus on in an essential modeling environment.

set at time t. A corollary of the unpredictability result is that when we extend equation 5.6 to time $t+k$ and apply the law of iterated expectation (LIE), we obtain,

$$\mathbb{E}_t \Delta_{t+k} = 0, \tag{5.7}$$

where $\Delta_{t+k} \equiv \hat{q}_{t+k} - \hat{q}_{t+k-1} + \hat{r}^*_{t+k-1} - \hat{r}_{t+k-1}$ is the ex-post one-period excess return between time $t+k-1$ and $t+k$. Thus, the RE-UIP condition implies that the ex-post excess return for any future time horizon $t+k$ is unpredictable based on the information set at time t. We will refer to this unpredictability result as the time horizon aspect of the RE-UIP condition.

In addition, there is another implication of the RE-UIP condition from the perspective of term structure. Iterating the expectation term of real exchange rate $\mathbb{E}_t \hat{q}_{t+1}$ forward in (5.4) by the LIE yields

$$\hat{q}_t = \sum_{k=0}^{\infty} \mathbb{E}_t[\hat{r}^*_{t+k} - \hat{r}_{t+k}] + \lim_{T\to\infty} \mathbb{E}_t \hat{q}_{t+T}.$$

Following Galí (2020), we decompose the sum of expectations into the short-term and long-term:

$$\hat{q}_t = D^S_t(M) + D^L_t(M) + \lim_{T\to\infty} \mathbb{E}_t \hat{q}_{t+T}, \tag{5.8}$$

where M is the threshold period for the short-term and the long-term, and

$$D^S_t(M) \equiv \sum_{k=0}^{M-1} \mathbb{E}_t[\hat{r}^*_{t+k} - \hat{r}_{t+k}], \qquad D^L_t(M) \equiv \sum_{k=M}^{\infty} \mathbb{E}_t[\hat{r}^*_{t+k} - \hat{r}_{t+k}],$$

are defined as the sum of expectations on the short- and long-term real interest rate differential, respectively.[20] Since the real exchange rate is a stationary variable, one can assume that $\lim_{T\to\infty} \mathbb{E}_t \hat{q}_{t+T} = 0$. Then, (5.8) indicates that the RE-UIP condition predicts the horizon invariance for the impact of the forecast of the real interest rate differential on the real exchange rate; the forecast of the short-term interest rate differential $D^S_t(M)$ and the forecast of the long-term interest rate differential $D^L_t(M)$ have identical effects on the current real exchange rate with the same weight. We will refer to this horizon invariance result as the forecast horizon aspect of the RE-UIP condition.

[20] The definition of the real interest rate differential in Galí (2020) is represented as $\hat{r}^* - \hat{r}$. Thus, in the discussion regarding the forecast horizon invariance, we follow Galí (2020)'s definition to maintain consistency, whereas in the rest of the paper, we adopt the common practice by defining the real interest rate differential as $\hat{r} - \hat{r}^*$.

5.2 Excess Return Predictability and the Predictability Reversal

A challenge to the RE-UIP condition (5.4) is a predictable excess return observed in the data. Early studies, such as Fama (1984) and Eichenbaum and Evans (1995), show a short-run positive predictable excess return of currency bonds that bear higher interest rates. Furthermore, recent studies (e.g., Bacchetta and van Wincoop, 2010; Engel, 2016; Valchev, 2020) document that the movements of the predictable excess return are more complicated over the time horizon; the excess return is positive in the short run, whereas it reverses to negative in the long run. Thus, the UIP violations have time horizon variability.

In the FH model, the ex-post one-period excess return on foreign currency bonds between time t and $t+k$ is, by construction, equivalent to the one-period ahead forecast error of the real exchange rate.

$$\Delta_{t+1} \equiv \hat{q}_{t+1} - \hat{q}_t + \hat{r}_t^* - \hat{r}_t = \hat{q}_{t+1} - \hat{\mathbb{E}}_t q_{t+1}.$$

This formulation intentionally disregards other potential factors in excess return predictability driven by time-varying risk or liquidity premia. Thus, our model isolates the expectation channels from other factors and is also consistent with the empirical finding that the major source of the UIP deviations in advanced countries is subjective forecast error (e.g., Froot and Frankel, 1989; Chinn and Frankel, 2019; Kalemli-Özcan and Varela, 2022; Candian and De Leo, 2023).

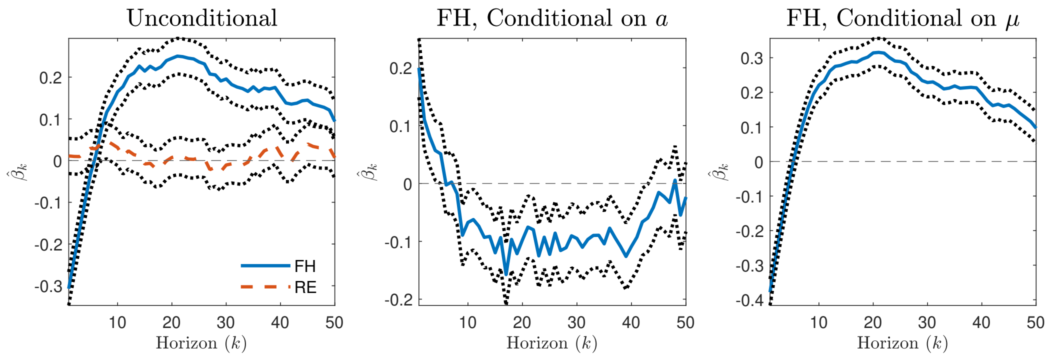
Following the empirical specification in the literature, we run the regression model using the simulated data from RE and FH models as follows:

$$\Delta_{t+k} = \beta_0 + \beta_k(\hat{r}_t - \hat{r}_t^*) + \xi_t, \tag{5.9}$$

where the coefficient β_k captures the predictable excess return at time t for future horizon k. Since $\hat{r}_t - \hat{r}_t^*$ is an explanatory variable, a negative β_k suggests that the foreign currency bonds yield a positive real excess return when the foreign bonds carry a higher real interest rate ($\hat{r}_t^* > \hat{r}_t$). Conversely, a positive β_k implies the opposite.

The left panel in Figure 3 illustrates the estimates of excess return coefficients, $\hat{\beta}_k$, across time horizon k, where the dotted lines indicate 95% confidence intervals. In the RE model, the confidence intervals for the estimates nearly always contain zero, highlighting the unpredictability of excess returns as implied by the RE-UIP condition. In contrast, the FH model demonstrates the predictability of excess returns. It initially exhibits a negative $\hat{\beta}_k$ for the first five time horizons, which then reverses to positive, peaks at time horizon $k = 21$, and then diminishes. Thus, the FH model predicts that the foreign currency bond yields a short-run positive real excess return when the foreign bond bears a higher real interest rate.

Figure 3: Excess Return Predictability across Time Horizons: Regression Coefficients



Notes: This figure presents the estimates of excess return coefficients, $\hat{\beta}_k$, across time horizon k. The solid lines represent point estimates, while the dotted lines represent 95% confidence intervals. These estimates are obtained using samples from 100 simulations, each spanning 188 quarters, of the models. The left panel uses series generated by the two shocks, the middle panel uses data series from the domestic productivity shock only, and the right panel uses series from the foreign interest rate shock only. In the left panel, "FH" refers to the benchmark limited foresight model, and "RE" refers to the rational expectation model.

Meanwhile, it predicts a negative excess return in the medium- and long-run time horizons, echoing empirical findings in the referenced literature.[21]

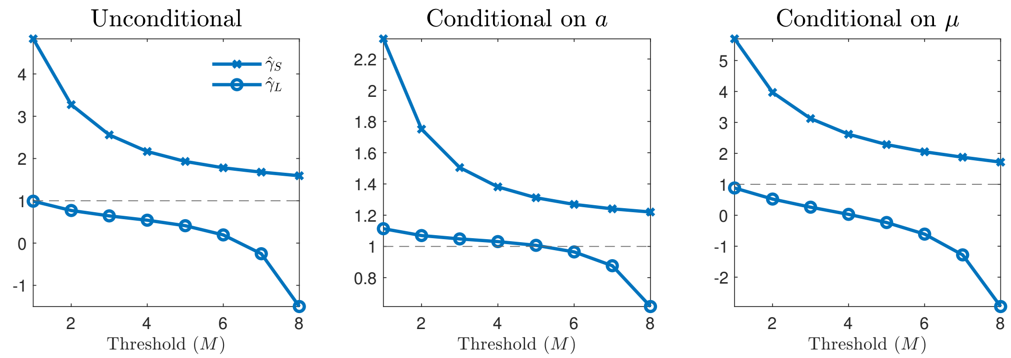
Notably, the left panel of Figure 3 displays the *unconditional* profile of excess return predictability, which incorporates both shocks in the model. As shown in Figure 1, the exchange rate forecast errors in the FH model respond in qualitatively opposite ways to the two shocks. This is mirrored in the middle and right panels of Figure 3, which display the $\hat{\beta}_k$ estimates, each conditioned on a distinct shock. Although these two panels indicate the prediction of opposite excess return profiles in the FH model, conditional on the two shocks, the foreign interest rate shock drives the unconditional profile of excess return predictability.[22]

The sign reversal of forecast errors in the FH model, induced by the finite planning horizon and value function learning, introduces a novel explanation for the reversal of predictability. This addition enriches existing theoretical explanations in the literature that attribute this phenomenon to infrequent portfolio decisions (Bacchetta and van Wincoop, 2010), convenience yields (Valchev, 2020), and over-extrapolation on misperceived shocks (Candian and De Leo, 2023).

[21]The model-predicted excess return coefficients $\hat{\beta}_k$ are quantitatively larger (smaller) if the planning horizon decreases (increases) than the benchmark planning horizon in the FH model. See Figure J.8 in Appendix J for the regression coefficients under planning horizons $h = 2, 4$, and 40, respectively.

[22]The RE model has no forecast errors as displayed in Figure 1, implying no excess returns, regardless of conditioning on the two shocks. This suggests that although a financial shock (e.g., a risk premium shock) can be a source of the foreign interest rate shock in our model, this shock cannot create a wedge from our RE-UIP condition. This is because the shock is already incorporated into the foreign interest rate $\hat{r}_t^*$ via (2.40) in our model. Thus, in our model, the expectation channel in the FH model, which characterizes how agents respond to the shocks, is the fundamental source of the RE-UIP violation.

Figure 4: Reaction Coefficients of Real Exchange Rate to the Term Structure of the Forecast of the Interest Rate Differentials



Notes: This figure shows the estimates of $\hat{\gamma}_S$ and $\hat{\gamma}_L$, representing the reaction of the real exchange rate to the forecasts of short- and long-term interest rate differentials in the benchmark FH model. The estimates are obtained using samples from 100 simulations, each spanning 188 quarters, of the models. The left panel uses series generated by the two shocks, the middle panel uses series from the domestic productivity shock only, and the right panel uses series from the foreign interest rate shock only.

5.3 Breakdown of the Forecast Horizon Invariance

Another challenge to the RE-UIP condition is the empirical breakdown of the forecast horizon invariance in condition (5.8). Galí (2020) builds the following regression specification based on (5.8):

$$\hat{q}_t = \gamma_0 + \gamma_S D_t^s(M) + \gamma_L D_t^L(M) + \zeta_t, \tag{5.10}$$

where γ_0, γ_S, and γ_L are regression coefficients and ζ_t is an orthogonal error term. If the forecast horizon invariance result from the RE-UIP holds, then the regression coefficients should be $\gamma_0 = 0$ and $\gamma_S = \gamma_L = 1$.

Galí (2020) tests specification (5.10) using the data on government zero-coupon bond yields and inflation swaps at different maturities from the U.S., the U.K., and Germany, and finds robust results on $\hat{\gamma}_S > 1$ and $\hat{\gamma}_L < 1$ for those countries. The empirical findings imply that the current real exchange rate *overreacts* to the forecast of the short-term interest rate differential but *underreacts* to the forecast of the long-term interest rate differential, rejecting the prediction of the forecast horizon invariance. Thus, the UIP violations exhibit forecast horizon variability, and Galí (2020) refers to this phenomenon as the "forward guidance exchange rate puzzle."

We conduct the same exercise using the series generated by the FH model and show that the FH model can address the short-term overreaction and the long-term underreaction of the real exchange rate. For a given threshold horizon M between the short-term and the long-term, together with the given planning horizon h, we construct the expected real interest rate differentials under limited foresight as

$$D_t^S(M) = \sum_{k=0}^{M-1} \mathbb{E}_t^h[\hat{r}^*_{t+k} - \hat{r}_{t+k}], \qquad D_t^L(M) = \sum_{k=M}^{h} \mathbb{E}_t^h[\hat{r}^*_{t+k} - \hat{r}_{t+k}], \tag{5.11}$$

where $0 < M \leq h$.[23]

We run the regression for the empirical specification (5.10), using the constructed variables from the FH model. Figure 4 displays the estimates of the reaction coefficients $\hat{\gamma}_S$ and $\hat{\gamma}_L$ under the threshold $1 \leq M \leq 8$. The left panel shows that, across all the threshold M, $\hat{\gamma}_S > 1$ and $\hat{\gamma}_L < 1$. Furthermore, as the threshold M increases, the reaction coefficients decrease monotonically. This behavior indicates that the FH model qualitatively predicts the breakdown of the forecast horizon invariance, aligning with the findings of Galí (2020). The foreign interest rate shock (as shown in the right panel) primarily drives the breakdown of the forecast horizon invariance in this unconditional profile. The domestic productivity shock (shown in the middle panel) plays a lesser role.

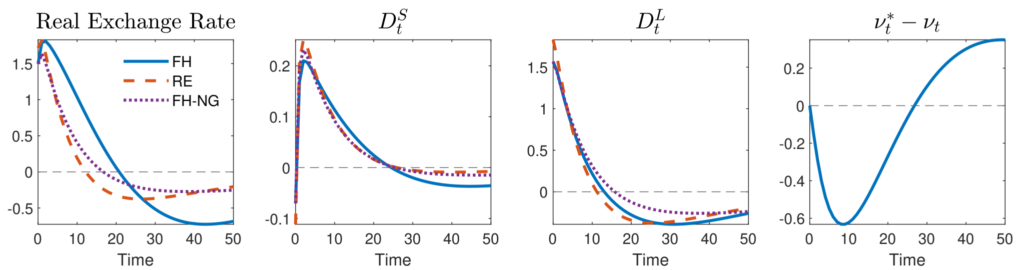
In contrast to the FH model, the simulated series consistently yield $\hat{\gamma}_S = \hat{\gamma}_L = 1$ in both the RE and FH-NG models. Hence, the value function learning in the FH model play a pivotal role in the breakdown of the forecast horizon invariance in Figure 4. What are the mechanisms at play here? First, the LIE does not apply in the FH model with value function learning. This is because expectations formed at any time t are contingent on the value functions at that same time t. When decision-makers learn and update their value functions, expectations of future endogenous variables differ from the *expectations of expected* future endogenous variables. For example, $\hat{\mathbb{E}}_t x_{t+2}$ is the subjective expectation for any endogenous variable x_{t+2}, which is based on time-t value functions, whereas $\hat{\mathbb{E}}_{t+1} x_{t+2}$ is the subjective expectation conditional on the value functions at time $t+1$. As the value functions at times t and $t+1$ are typically different (and their evolution follows a constant-gain learning process), in general $\hat{\mathbb{E}}_t x_{t+2} \neq \hat{\mathbb{E}}_t[\hat{\mathbb{E}}_{t+1} x_{t+2}]$. This variation disrupts the forecast horizon invariance (5.8), a result derived from forward iteration and the LIE. Thus, our FH model suggests that the empirical evidence of Galí (2020) also implies the rejection of the LIE.

Next, we aim to understand the causes of the specific asymmetrical directions of the reaction coefficients $\hat{\gamma}_S$ and $\hat{\gamma}_L$ in Figure 4. Notably, decision-makers form expectations at time t for future periods $t+k$ based on the assumption that the counterfactual endogenous variables are constructed on aggregate conditions with a *remaining* planning horizon of $h-k$. Therefore, as k increases, the expectations become more dependent on the decision-

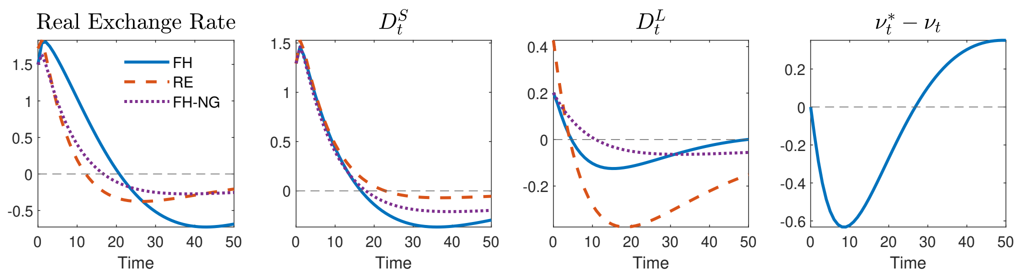
[23] The detailed procedure to construct the variables in (5.11) can be found in Appendix I.

Figure 5: Impulse Responses of the Real Exchange Rate, D_t^S, D_t^L, and $\nu_t^* - \nu_t$ to Foreign Interest Rate Shock

(a) $M = 1$



(b) $M = 8$



Notes: This figure shows the impulse responses to foreign interest rate shock in the three models for the following variables: the real exchange rate, the cumulative forecasts of short-term real interest rate differentials (D_t^S), the cumulative forecasts of long-term real interest rate differentials (D_t^S), and the difference between variables regarding the marginal values of holding foreign and domestic bonds ($\nu_t^* - \nu_t$). Panel (a) presents the case in which the threshold horizon for the short- and long-term (M) is set to 1, while panel (b) presents the case in which M is set to 8. "FH" refers to the benchmark limited foresight model, "RE" refers to the rational expectation model, and "FH-NG" refers to the limited foresight model with no learning gain ($\gamma_v = 0$). The planning horizon (h) is set to 8.

makers' value functions. For example, with the planning horizon $h = 8$, the decision-makers' perception of the counterfactual interest rate differential at eight periods ahead is based on the following equilibrium condition:[24]

$$\hat{r}^{*,0}_{t+8|t} - \hat{r}^{0}_{t+8|t} = \hat{q}^{0}_{t+8|t} + \sigma(\nu_t^* - \nu_t). \tag{5.12}$$

It shows that, given $\hat{q}_{t+8|t}$, the term $\nu_t^* - \nu_t$ directly influences the counterfactual real interest rate differential at time $t+8$ with a substantial effect (by noting the calibrated value $\sigma = 2$). Intuitively, given $\hat{q}^0_{t+8|t}$, when decision-makers discern a higher relative marginal value of holding foreign bonds over domestic bonds (signaled by a drop in $\nu_t^* - \nu_t$), they project these values onto their long-term expectations. This leads to an increase in relative foreign bond prices and a corresponding decrease in the counterfactual interest rate ($\hat{r}^{*,0}_{t+8|t} - \hat{r}^{0}_{t+8|t}$) at time $t+8$. Such an extrapolation is absent in both the RE and FH-NG models.

[24]See equilibrium condition (F.13) in Appendix F.

Thus, in the FH model, $D_t^L(M)$, which encompasses the expected counterfactual interest rate differential at $t+8$, is strongly linked with $\nu_t^* - \nu_t$. The model solutions (3.8) and (3.9) show that the value functions affect both counterfactual variables across remaining horizons $h-k$ and current equilibrium variables at time t. However, their impact diminishes as future horizons draw nearer, corresponding to an increase in (remaining) planning horizon and a convergence of policy coefficients tied to the value functions towards zero, as seen in the RE case. Therefore, the impact of $\nu_t^* - \nu_t$ on $D_t^S(M)$ is weaker compared to $D_t^L(M)$ in the FH model.

Figure 5 illustrates these dynamics, showing the impulse responses to a foreign interest rate shock by varying threshold horizons $M = 1$ (panel (a)) and $M = 8$ (panel (b)). In the FH model, the real exchange rate $\hat{q}_t$ deviates from the sum of the forecasts of short-term and long-term interest rate differentials, $D_t^S(M)$ and $D_t^L(M)$. The dynamics in the FH model show a weaker link between $D_t^S(M)$ and $\nu_t^* - \nu_t$, which is more pronounced when $M = 1$, whereas the relationship between $D_t^L(M)$ and $\nu_t^* - \nu_t$ strengthens when $M = 8$. In contrast, the RE and FH-NG models align $\hat{q}_t$ with the sum of $D_t^S(M)$ and $D_t^L(M)$, per forecast horizon invariance.

In addition, the dynamics of $\hat{q}_t$ in the FH model diverge more significantly from the expected interest rate differential for the more distant future. As illustrated in Figure 5, $\hat{q}_t$ in the blue solid line bears a strong resemblance to $D_t^S(M)$ in their dynamics for both cases of $M = 1$ and $M = 8$. Moreover, the magnitude of the responses of $D_t^S(M)$ increases as M increases. This implies that in the regression exercise of (5.10), the response coefficient of $\hat{q}_t$ on $D_t^S(M)$, $\hat{\gamma}_S$, reaches its peak when M is at its smallest. In contrast, $\hat{q}_t$ demonstrates a relatively weaker resemblance to $D_t^L(M)$ and a greater divergence when $M = 8$. Therefore, in the regression exercise, $D_t^L(M)$ would play a less significant role in explaining the variation of $\hat{q}_t$ compared to $D_t^S(M)$, suggesting $\hat{\gamma}_L < \hat{\gamma}_S$, and its role would become less positive as M increases. This observation provides a rationale for the regression results shown in Figure 4.

6 Robustness

We evaluate the robustness of the main results in Section 5 by taking into account alternative lengths of planning horizon h, by varying the firm's learning gain parameter $\gamma_{\tilde{v}}$, and by extending the model to incorporate agents with heterogeneous planning horizons. In these robustness checks, we set all structural parameters as in Section 4.2 except for the parameter of interest. The qualitative aspects of the main results remain broadly robust, with the principal takeaways unchanged by those considerations. To save space, we leave the details to Appendix J.

7 Concluding Remarks

In this paper, we reconsider the conclusions from the RE assumption in a standard SOE-NK model by assuming that decision-makers are subject to limited foresight when making decisions. Our analysis indicates that the dynamics of the model's equilibrium are significantly influenced by both the degree of decision-makers' foresight and the manner in which they update their value functions. The FH model generates dynamic overshooting of the forecast errors of the real exchange rate across time horizons, along with inherent differences in the formation of short-term and long-term expectations, none of which can be observed in the RE model.

Our model provides an intrinsic micro-foundation for those renowned puzzles of the RE-UIP condition that feature time and forecast horizon variability. We show that our model can explain (i) the time-varying excess return predictability and its reversal of sign over the time horizon and (ii) the breakdown of the forecast horizon invariance, marked by the diverse responses of the real exchange rate to the term structure of the forecast of the real interest rate differential.

The model environment presented in our paper is small-scale, which indicates the necessity for future research to enlarge the model with an increased emphasis on quantitative aspects. Natural extensions could involve the incorporation of additional frictions and wedges, along with endogenous variables such as capital. It may also be beneficial to estimate the quantitative model in line with dynamic moments from a more comprehensive data set, including financial and expectation-related elements. We discuss these issues in another companion paper.

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Online Appendix: Not for Publication

"A Behavioral New Keynesian Model of a Small Open Economy under Limited Foresight"

Seunghoon Na and Yinxi Xie

A Steady-State Value Function of Households

The steady-state value function solves the following Bellman equation:

$$v(\mathcal{B}, \mathcal{B}^*) = \max_{C, \mathcal{B}', \mathcal{B}^{*\prime}} \{u(C) + v(\mathcal{B}', \mathcal{B}^{*\prime})\}$$

s.t.

$$\beta\mathcal{B}' + \beta\frac{\bar{Q}\mathcal{B}^{*\prime}\bar{\Pi}}{\bar{\Pi}^*} = \mathcal{B} + \frac{\bar{Q}\mathcal{B}^*\bar{\Pi}}{\bar{\Pi}^*} + (\bar{\mathcal{S}}\bar{Y} - C)\bar{\Pi}.$$

The first-order conditions yield

$$v_1(\mathcal{B}', \mathcal{B}^{*\prime}) = \frac{\beta u'(C)}{\bar{\Pi}}, \qquad v_2(\mathcal{B}', \mathcal{B}^{*'}) = \frac{\beta\bar{Q}u'(C)}{\bar{\Pi}^*},$$

$$v_1(\mathcal{B}, \mathcal{B}^*) = \frac{u'(C)}{\bar{\Pi}}, \qquad v_2(\mathcal{B}, \mathcal{B}^*) = \frac{\bar{Q}u'(C)}{\bar{\Pi}^*},$$

where the last two equations come from the envelope theorem.

It can be easily verified that the following solution satisfies the above system of first-order conditions, which is given by

$$v(\mathcal{B}, \mathcal{B}^*) = (1-\beta)^{-1}u\left(\frac{(1-\beta)\mathcal{B}}{\bar{\Pi}} + \frac{(1-\beta)\bar{Q}\mathcal{B}^*}{\bar{\Pi}^*} + \bar{\mathcal{S}}\bar{Y}\right).$$

B Log-Linearization of the F.O.C.s of Households in the Ending Period of Forward Planning

We now show the steps of log-linearizing equations (2.15) and (2.16) under the assumption that households use a steady-state value function in their forward planning. First, taking logs of (2.15) and conducting first-order Taylor expansion at the steady state with notation $\tau = t + h$ yields

$$\ln u'(C_\tau^0) = \ln\beta + \ln(1 + i_\tau^0) + \ln v_1(\mathcal{B}_{\tau+1}^0, \mathcal{B}_{\tau+1}^{*;0})$$

$$\Rightarrow \frac{u''(\bar{C})}{u'(\bar{C})}(C_\tau^0 - \bar{C}) = \frac{1+i_\tau^0 - (1+\bar{i})}{1+\bar{i}} + \frac{v_{1,1}(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}{v_1(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}(\mathcal{B}_{\tau+1}^0 - \bar{\mathcal{B}}) + \frac{v_{1,2}(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}{v_1(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}(\mathcal{B}_{\tau+1}^{*,0} - \bar{\mathcal{B}}^*)$$

$$\Rightarrow \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\hat{c}_\tau^0 = \hat{\imath}_\tau^0 + \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\frac{(1-\beta)}{\bar{\Pi}}\frac{\mathcal{B}_{\tau+1}^0 - \bar{\mathcal{B}}}{\bar{C}} + \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\frac{(1-\beta)\bar{Q}}{\bar{\Pi}^*}\frac{\mathcal{B}_{\tau+1}^{*,0} - \bar{\mathcal{B}}^*}{\bar{C}}. \tag{B.1}$$

Given the definition of $\sigma^{-1} \equiv -\frac{u'(\bar{C})}{u''(\bar{C})\bar{C}}$, (B.1) can be rewritten as

$$\hat{c}_\tau^0 = -\sigma^{-1}\hat{\imath}_\tau^0 + (1-\beta)\hat{b}_{\tau+1}^0 + (1-\beta)\hat{b}_{\tau+1}^{*,0},$$

which gives (2.19).

Similarly, log-linearizing equation (2.16) yields

$$\ln u'(C_\tau^0) = \ln\beta + \ln(1+i_\tau^{*,0}) + \ln v_2(\mathcal{B}_{\tau+1}^0, \mathcal{B}_{\tau+1}^{*,0}) - \ln Q_\tau^0$$

$$\Rightarrow \frac{u''(\bar{C})}{u'(\bar{C})}(C_\tau^0 - \bar{C}) = \frac{1+i_\tau^{*,0} - (1+\bar{i}^*)}{1+\bar{i}^*} + \frac{v_{2,1}(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}{v_2(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}(\mathcal{B}_{\tau+1}^0 - \bar{\mathcal{B}}) + \frac{v_{2,2}(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}{v_2(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)}(\mathcal{B}_{\tau+1}^{*,0} - \bar{\mathcal{B}}^*) - \frac{Q_\tau^0 - \bar{Q}}{\bar{Q}}$$

$$\Rightarrow \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\hat{c}_\tau^0 = \hat{\imath}_\tau^0 + \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\frac{(1-\beta)}{\bar{\Pi}}\frac{\mathcal{B}_{\tau+1}^0 - \bar{\mathcal{B}}}{\bar{C}} + \frac{u''(\bar{C})\bar{C}}{u'(\bar{C})}\frac{(1-\beta)\bar{Q}}{\bar{\Pi}^*}\frac{\mathcal{B}_{\tau+1}^{*,0} - \bar{\mathcal{B}}^*}{\bar{C}} - q_\tau^0.$$

After plugging the definition of σ, we have

$$\hat{c}_\tau^0 = -\sigma^{-1}\hat{\imath}_\tau^{*,0} + (1-\beta)\hat{b}_{\tau+1}^0 + (1-\beta)\hat{b}_{\tau+1}^{*,0} + \sigma^{-1}\hat{q}_\tau^0,$$

which gives (2.20).

C Firm Profit Function and Optimal Pricing Solution

In this section, we first show the explicit expression for profit function $H(\cdot)$ of the firms in each period and then derive the firms' optimal pricing decision given by (2.23). The period profit function of firms is the same regardless of whether they are infinitely forward-looking or have limited foresight. We therefore derive the profit function by considering the case of the standard RE framework with infinite planning horizons.

In the RE framework, a firm f that is able to reoptimize its goods price sets $P_{H,t}^f$ to maximize

$$\max_{P_{H,t}^f} \mathbb{E}_t \sum_{k=0}^{\infty}(\beta\theta)^k \left[\left(\frac{C_{t+k}}{C_t}\right)^{-\sigma}\left(\frac{P_t}{P_{t+k}}\right) Y_{t+k}(j)\left(P_{H,t}^f \bar{\Pi}_H^k - MC_{t+k}\right)\right], \tag{C.1}$$

subject to the demand constraint

$$Y_{t+k}(j) \leq \left(\frac{P^f_{H,t}\bar{\Pi}^k_H}{P_{H,t+k}}\right)^{-\epsilon} \underbrace{\left(C_{H,t+k} + \int_0^1 C^l_{H,t+k} dl\right)}_{\equiv Y_{t+k}}.$$

We rewrite the firm's problem (C.1) as follows:

$$\max_{P^f_{H,t}} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \left[C^{-\sigma}_{t+k} \left(\frac{P^f_{H,t}\bar{\Pi}^k_H}{P_{H,t+k}}\right)^{-\epsilon} Y_{t+k} \frac{P_{H,t+k}}{P_{t+k}} \left(\frac{P^f_{H,t}\bar{\Pi}^k_H}{P_{H,t+k}} - \frac{MC_{t+k}}{P_{H,t+k}}\right)\right], \tag{C.2}$$

where we have applied the demand constraint and dropped C_t and P_t (note that they are taken as given by firm f at time t, and dropping them does not change the solution to the optimality problem).

Now, let us define

$$\lambda_{t+k} \equiv C^{-\sigma}_{t+k}, \qquad r^f_{H,t+k} \equiv \frac{P^f_{H,t}\bar{\Pi}^k_H}{P_{H,t+k}},$$

$$\mathcal{S}_{t+k} \equiv \frac{P_{H,t+k}}{P_{t+k}}, \qquad \mathcal{MC}_{t+k} \equiv \frac{MC_{t+k}}{P_{H,t+k}}.$$

Then, the optimality problem (C.2) can be summarized as follows:

$$\max_{P^f_{H,t}} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \left[\lambda_{t+k} H(r^f_{H,t+k}; \mathcal{S}_{t+k}, \boldsymbol{z}_{t+k})\right], \tag{C.3}$$

where $H(r^f_{H,t+k}; \mathcal{S}_{t+k}, Z_{t+k})$ is the function of real profit in period $t+k$; that is,

$$H(r^f_{H,t+k}; \mathcal{S}_{t+k}, \boldsymbol{z}_{t+k}) = \left(r^f_{H,t+k}\right)^{-\epsilon} Y_{t+k}\mathcal{S}_{t+k} \left(r^f_{H,t+k} - \mathcal{MC}_{t+k}\right), \tag{C.4}$$

and $\boldsymbol{z}_{t+k}$ is the vector of all real state variables at time $t+k$. In the steady state with $r^f_H = \mathcal{S} = 1$, the derivative of function $H(\cdot)$ becomes

$$H'(1; 1, \bar{\boldsymbol{z}}) = \bar{Y}(1 - \epsilon + \epsilon \cdot \overline{\mathcal{MC}}) = 0 \tag{C.5}$$

by noting that the real marginal cost in the steady state is $\overline{\mathcal{MC}} = (\epsilon - 1)/\epsilon$.

Now, we show that the firms' optimal pricing decision is given by (2.23). We assume that the firms use a value function to approximate discounted future profits beyond its planning horizon that is learned from the nonstochastic steady state given by (2.22). Then, the first-order condition of maximizing the firm's objective function (2.21) is

$$\mathbb{E}_t^f\left[\sum_{\tau=t}^{t+k}(\beta\theta)^{\tau-t}\lambda_\tau H_1\left(\frac{P_{H,t}^f\bar{\Pi}_H^{\tau-t}}{P_{H,\tau}};\mathcal{S}_\tau,\boldsymbol{z}_\tau\right)\frac{P_{H,t}\bar{\Pi}_H^{\tau-t}}{P_{H,\tau}}+\frac{(\beta\theta)^{k+1}}{1-\beta\theta}\bar{\lambda}H_1\left(\frac{P_{H,t}^f\bar{\Pi}_H^k}{P_{H,t+k}};\bar{\mathcal{S}},\bar{\boldsymbol{z}}\right)\frac{P_{H,t}\bar{\Pi}_H^k}{P_{H,t+k}}\right]=0. \tag{C.6}$$

Log-linearizing (C.6) around the steady state yields

$$\mathbb{E}_t^f\left\{\sum_{\tau=t}^{t+k}(\beta\theta)^{\tau-t}\left[p_{H,t}^f-\sum_{s=t}^{\tau}\pi_{H,s}-m_\tau\right]+\frac{(\beta\theta)^{k+1}}{1-\beta\theta}\left[p_{H,t}^f-\sum_{s=t}^{t+k}\pi_{H,s}\right]\right\}=0, \tag{C.7}$$

where

$$p_{H,t}^f\equiv\log\left(\frac{P_{H,t}(f)}{P_{H,t-1}\bar{\Pi}_H}\right),\qquad \pi_{H,t}\equiv\log\left(\frac{\Pi_{H,t}}{\bar{\Pi}_H}\right),$$

and

$$m_t\equiv-\frac{H'(1;1,\boldsymbol{z}_t)}{H''(1;1,\bar{\boldsymbol{z}})}=\frac{Y_t}{\bar{Y}}\left(\frac{\epsilon}{\epsilon-1}mc_t-1\right). \tag{C.8}$$

We define

$$\hat{m}_t\equiv m_t-\bar{m},\qquad \widehat{mc}_t\equiv\log\left(\frac{\mathcal{MC}_t}{\overline{\mathcal{MC}}}\right),$$

where $\bar{m}$ is the value of m_t in the nonstochastic steady state.

By noting that $\bar{m}=\frac{\bar{Y}}{\bar{Y}}\left(\frac{\epsilon}{\epsilon-1}\overline{\mathcal{MC}}-1\right)=0$, we have $\hat{m}_t=m_t$. Then, the log-linear approximation of (C.8) yields

$$m_t=\widehat{mc}_t.$$

Thus, by replacing m_t with $\widehat{mc}_t$ in (C.6) and reorganizing its expression, we have the firms' optimal pricing $p_{H,t}^f$ characterized by (2.23).

D International Goods-Market Clearing Condition

The international market clearing condition for domestically produced goods j is

$$Y_t(j)=C_{H,t}(j)+\int_0^1 C_{H,t}^{*,i}(j)di \tag{D.1}$$

for all $j\in[0,1]$ and all t, where $C_{H,t}^{*,i}(j)$ is the demand from foreign country i for good j produced in the domestic country. With the assumption of identical preferences across countries and the law of one price for all goods j, we can rewrite condition (D.1) as follows:

$$Y_t(j)=\left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon}\left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t+\alpha\int_0^1\left(\frac{P_{H,t}}{\mathcal{E}_{i,t}P_{F,t}^{*,i}}\right)^{-\gamma}\left(\frac{P_{F,t}^{*,i}}{P_t^{*,i}}\right)^{-\eta}C_t^{*,i}di\right],$$

where $\mathcal{E}_{i,t}$ is the bilateral nominal exchange rate between the domestic country and the foreign country i (i.e., the price of country i's currency in units of domestic country currency), $P_{F,t}^{*,i}$ and $P_t^{*,i}$ are price indices of imported goods and consumption goods in foreign country i in units of its own currency, respectively, and $C_t^{*,i}$ is foreign country i's consumption.

By the definition of the aggregate output of the domestic economy, $Y_t \equiv \left[\int_0^1 Y_t(j)^{1-\frac{1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$, we can further rewrite the above equation as follows:

$$
\begin{aligned}
Y_t &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\mathcal{E}_{i,t} P_{F,t}^{*,i}}\right)^{-\gamma} \left(\frac{P_{F,t}^{*,i}}{P_t^{*,i}}\right)^{-\eta} C_t^{*,i} di \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha)C_t + \alpha \int_0^1 \left(\frac{\mathcal{E}_{i,t} P_{F,t}^{*,i}}{P_{H,t}}\right)^{\gamma-\eta} Q_{i,t}^{\eta} C_t^{*,i} di\right] \\
&= \mathcal{S}_t^{-\eta}\left[(1-\alpha)C_t + \alpha \int_0^1 \left(S_t^i S_{i,t}\right)^{\gamma-\eta} Q_{i,t}^{\eta} C_t^{*,i} di\right],
\end{aligned}
\tag{D.2}
$$

where $\mathcal{E}_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_t^{*,i}}{P_t}$ is the bilateral real exchange rate between the domestic country and country i, $S_t^i \equiv \frac{P_{F,t}^{*,i}}{P_{H,t}^{*,i}}$ is the effective terms of trade for country i, and $S_{i,t} \equiv \frac{\mathcal{E}_{i,t} P_{H,t}^{*,i}}{P_{H,t}}$ is the bilateral terms of trade between the home country and country i.

To facilitate tractability, we further assume that in the steady state,

$$
\bar{C} = (\bar{S}^i \bar{S}_i)^{\gamma-\eta} \bar{Q}_i^{\eta} \bar{C}^{*,i} \tag{D.3}
$$

for any country i.[25] Then, the log-linear approximation of equation (D.2) is given by

$$
\begin{aligned}
\bar{Y}\hat{y} &= (1-\alpha)\bar{\mathcal{S}}^{-\eta}\bar{C}(\hat{c}_t + \alpha\eta\hat{s}_t) + \alpha\bar{\mathcal{S}}^{-\eta}\bar{C}\left[\int_0^1 (\gamma-\eta)(\hat{s}_t^i + \hat{s}_{it}) + (\eta\hat{q}_{it} + \alpha\hat{s}_t + \hat{c}_t^i) di\right] \\
&= \bar{\mathcal{S}}^{-\eta}\bar{C}\left[(1-\alpha)\hat{c}_t + \alpha\gamma\hat{s}_t + \alpha\eta\hat{q}_t + \alpha\hat{c}_t^*\right],
\end{aligned}
\tag{D.4}
$$

where $\hat{c}_t^* \equiv \int_0^1 \hat{c}_t^i di$. Here we have utilized the conditions $\hat{\mathcal{S}}_t = -\alpha\hat{s}_t$, $\int_0^1 \hat{s}_t^i di = 0$, and $\int_0^1 \hat{s}_{i,t} di = \hat{s}_t$.

The goods market clearing condition (D.4) nests the scenario analyzed in Galí and Monacelli (2005), in which all countries are assumed to be symmetric in the steady state and households feature perfect risk sharing. The symmetry assumption implies $\bar{Y} = \bar{C}$ and $\bar{\mathcal{S}} = 1$, while the perfect risk sharing condition implies $\hat{c}_t = \hat{c}_t^* + \frac{1}{\sigma}\hat{q}_t$. Plugging these two

[25] The assumption of symmetry at the steady state necessarily ensures (D.3) to hold.

conditions into (D.4), we have

$$\hat{y} = \hat{c}_t + \alpha\gamma\hat{s}_t + \alpha(\eta - \frac{1}{\sigma})\hat{q}_t,$$

which is exactly the same goods market clearing condition derived in Galí and Monacelli (2005) without imposing the Cole-Obstfeld parameterization.

By assuming no foreign demand shocks, the world income is a constant, hence $\hat{c}_t^* = \hat{y}_t^* = 0$. Then, using the relation $\hat{s}_t = \frac{\hat{q}_t}{1-\alpha}$, we can rewrite equation (D.4) as

$$\hat{y}_t = \vartheta_{yc}\hat{c}_t + \vartheta_{ys}\hat{s}_t,$$

where $\vartheta_{yc} \equiv (1-\alpha)\bar{\mathcal{S}}^{-\eta}\bar{C}/\bar{Y}$ and $\vartheta_{ys} \equiv \alpha[\gamma + \eta(1-\alpha)]\bar{\mathcal{S}}^{-\eta}\bar{C}/\bar{Y}$. This gives equation (2.36).

E Proof of the Mean-Reverting Processes of χ_t and ζ_t

First, we log-linearize the resource constraint (2.5) as follows:

$$\hat{c}_t + \beta(\hat{b}_{t+1} + \hat{b}_{t+1}^*) - \beta\vartheta_1'\hat{\imath}_t + \beta\vartheta_1(\hat{q}_t - \hat{r}_t^*) = \hat{b}_t - \vartheta_1'\pi_t + \hat{b}_t^* + \vartheta_1\hat{q}_t + \vartheta_2(\hat{y}_t - \alpha\hat{s}_t),$$

where $\vartheta_1' \equiv \frac{\bar{\mathcal{B}}}{\bar{\Pi}\bar{C}}$. Here we have used the relation $\hat{\mathcal{S}}_t = -\alpha\hat{s}_t$, $\pi_t^* = 0$, hence $\hat{\imath}_t^* = \hat{r}_t^*$, together with the steady-state relationship $\bar{\Pi}^* = 1$ and $\beta^{-1} = (1+\bar{\imath})/\bar{\Pi} = (1+\bar{\imath}^*)/\bar{\Pi}^*$. It can be rewritten as

$$\hat{b}_t + \hat{b}_t^* = \beta(\hat{b}_{t+1} + \hat{b}_{t+1}^*) + [\hat{c}_t - \beta\vartheta_1'\hat{\imath}_t + \beta\vartheta_1(\hat{q}_t - \hat{r}_t^*) + \vartheta_1'\pi_t - \vartheta_1\hat{q}_t - \vartheta_2(\hat{y}_t - \alpha\hat{s}_t)]\,. \tag{E.1}$$

In the FH model, we can rewrite equation (E.1) at any date τ as the version of interest:

$$\hat{b}_\tau^{j+1} + \hat{b}_\tau^{*j+1} = \beta(\hat{b}_{\tau+1}^j + \hat{b}_{\tau+1}^{*,j}) + \left[\hat{c}_\tau^j - \beta\vartheta_1'\hat{\imath}_\tau^j + \beta\vartheta_1(\hat{q}_\tau^j - \hat{r}_\tau^{*,j}) + \vartheta_1'\pi_\tau^j - \vartheta_1\hat{q}_\tau^j - \vartheta_2(\hat{y}_\tau^j - \alpha\hat{s}_\tau^j)\right], \tag{E.2}$$

where j is the (counterfactual) planning horizon at date τ.

Let time t be the point at which forward planning occurs. Then, iterating (E.2) forward to the end of the planning horizon yields

$$\begin{aligned}
&\hat{b}_t^{h+1} + \hat{b}_t^{*,h+1} \\
&= \mathbb{E}_t \sum_{j=0}^{h} \beta^j \left[\hat{c}_{t+j}^{h-j} - \beta\vartheta_1'\hat{\imath}_{t+j}^{h-j} + \beta\vartheta_1(\hat{q}_{t+j}^{h-j} - \hat{r}_{t+j}^{*,h-j}) + \vartheta_1'\pi_{t+j}^{h-j} - \vartheta_1\hat{q}_{t+j}^{h-j} - \vartheta_2(\hat{y}_{t+j}^{h-j} - \alpha\hat{s}_{t+j}^{h-j})\right] \\
&+ \beta^{h+1}\mathbb{E}_t(\hat{b}_{t+h+1}^0 + \hat{b}_{t+h+1}^{*,0}),
\end{aligned} \tag{E.3}$$

where $\hat{b}_t^{h+1}$ and $\hat{b}_t^{*,h+1}$ are the household's initial financial position in period t.

We parameterize the log-linear approximations of $v_1(\mathcal{B}, \mathcal{B}^*)$ and $v_2(\mathcal{B}, \mathcal{B}^*)$ by

$$\begin{aligned}
\log(v_{1,t}(\mathcal{B}, \mathcal{B}^*)/v_1^*(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)) &= -\sigma(\nu_t + \chi_t \hat{b} + \xi_t \hat{b}^*), \\
\log(v_{2,t}(\mathcal{B}, \mathcal{B}^*)/v_2^*(\bar{\mathcal{B}}, \bar{\mathcal{B}}^*)) &= -\sigma(\nu_t^* + \chi_t' \hat{b} + \xi_t' \hat{b}^*).
\end{aligned}$$

Then the first-order conditions of a household's optimality problem at the end of its planning horizon (2.15) and (2.16) can be log-linearized as

$$\begin{aligned}
\hat{c}_{t+k}^0 &= -\sigma^{-1}\hat{\imath}_{t+k}^0 + \nu_t + \chi_t \hat{b}_{t+k+1}^0 + \xi_t \hat{b}_{t+k+1}^{*,0}, \\
\hat{c}_{t+k}^0 &= -\sigma^{-1}\hat{\imath}_{t+k}^{*,0} + \nu_t^* + \chi_t' \hat{b}_{t+k+1}^0 + \xi_t' \hat{b}_{t+k+1}^{*,0} + \sigma^{-1}\hat{q}_{t+k}^0,
\end{aligned}$$

which implies $\chi_t = \chi_t'$ and $\xi_t = \xi_t'$.

In the standard dynamic programming problem under the RE assumption in the benchmark model, the household's holdings of domestic bonds $\hat{b}$ and foreign bonds $\hat{b}^*$ (in terms of the domestic currency) are perfect substitutes in their value functions. Since the bond holdings $\hat{b}$ and $\hat{b}^*$ are also perfect substitutes in the budget constraint of the household in the finite planning problem, we have $\chi_t = \xi_t$. Thus, the Euler equation at the end of the planning horizon reduces to

$$\tilde{b}_{t+h+1}^0 \equiv \hat{b}_{t+h+1}^0 + \hat{b}_{t+h+1}^{*,0} = \chi_t^{-1}(\hat{c}_{t+h}^0 - \nu_t + \sigma^{-1}\hat{\imath}_{t+h}^0), \tag{E.4}$$

where $\tilde{b}$ represents the total holdings of bond positions.

Similar to Woodford (2019), by the household's optimal expenditure conditions (2.17) and (2.19), together with (E.3), we have

$$\hat{c}_t^h = g_k(\chi_t)\tilde{b}_t^{h+1} + rest,$$

where "rest" indicates the terms not including total asset position $\tilde{b}_t^{h+1}$ and

$$g_k(\chi_t) \equiv \frac{\chi_t}{\beta^{k+1} + \left(\frac{1-\beta^{h+1}}{1-\beta}\right)\chi_t}.$$

Thus, we have $\chi_t^{est} = g_k(\chi_t)$. Because the evolution process of χ_t follows a constant-gain learning rule; that is,

$$\chi_{t+1} = \gamma g_k(\chi_t) + (1-\gamma)\chi_t,$$

χ_t monotonically converges to the fixed point $1-\beta$. Similarly, since $\chi_t = \xi_t$ for any t, the same is true for the evolution process of ξ_t.

F Equilibrium Conditions of the Forward Planning

This section summarizes the equilibrium conditions in the finite planning exercise calculated in period t. Let $y^{j}_{\tau|t}$ be the value of $\hat{y}_\tau$ that is predicted at date τ as a result of aggregation of decisions made by agents with (counterfactual) planning horizon $j = h + t - \tau$, which is calculated at date t by agents with planning horizon h. It is a function of the state $\{\hat{q}_{t-1}, \hat{b}^*_t, \nu_t, \nu^*_t, \tilde{\nu}_t\}$ and exogenous shocks $\{a_t, \mu_t\}$ in period t. Then, the actual aggregate output in period t is given by $\hat{y}_t = \hat{y}^h_{t|t}$. Similarly, we define other variables in the finite planning exercise with the same notation. The additional subscript $|t$ matters because different value functions are used in finite planning in different periods. All the exogenous shocks are assumed to follow an AR(1) process.

In the forward planning exercise by the agents in period t with planning horizon h, at any date $t \leq \tau < t + h - 1$, we have

$$
\begin{aligned}
\hat{c}^{h+t-\tau}_{\tau|t} &= \mathbb{E}_\tau \hat{c}^{h+t-\tau-1}_{\tau+1|t} - \frac{1}{\sigma}(\hat{\imath}^{h+t-\tau}_{\tau|t} - \mathbb{E}_\tau \pi^{h+t-\tau-1}_{\tau+1|t}), && \text{(F.1)}\\
\hat{q}^{h+t-\tau}_{\tau|t} &= \mathbb{E}_\tau \hat{q}^{h+t-\tau-1}_{\tau+1|t} + \hat{r}^{*,h+t-\tau}_{\tau|t} - (\hat{\imath}^{h+t-\tau}_{\tau|t} - \mathbb{E}_\tau \pi^{h+t-\tau-1}_{\tau+1|t}), && \text{(F.2)}\\
\hat{\varepsilon}^{h+t-\tau}_{\tau|t} &= \hat{q}^{h+t-\tau}_{\tau|t} - \hat{q}^{h+t-\tau+1}_{\tau-1|t} + \pi^{h+t-\tau}_{\tau|t}, && \text{(F.3)}\\
\hat{s}^{h+t-\tau}_{\tau|t} &= \frac{\hat{q}^{h+t-\tau}_{\tau|t}}{1-\alpha}, && \text{(F.4)}\\
\pi^{h+t-\tau}_{H,\tau|t} &= \kappa \widehat{mc}^{h+t-\tau}_{\tau|t} + \beta \mathbb{E}_\tau \pi^{h+t-\tau-1}_{H,\tau+1|t}, && \text{(F.5)}\\
\widehat{mc}^{h+t-\tau}_{\tau|t} &= (\sigma + \varphi\vartheta_{yc})\hat{c}^{h+t-\tau}_{\tau|t} + (\alpha + \varphi\vartheta_{ys})\hat{s}^{h+t-\tau}_{\tau|t} - (1+\varphi)a_\tau, && \text{(F.6)}\\
\hat{\imath}^{h+t-\tau}_{\tau|t} &= \phi_\pi \pi^{h+t-\tau}_{\tau|t}, && \text{(F.7)}\\
\hat{y}^{h+t-\tau}_{\tau|t} &= \vartheta_{ys}\hat{s}^{h+t-\tau}_{\tau|t} + \vartheta_{yc}\hat{c}^{h+t-\tau}_{\tau|t}, && \text{(F.8)}\\
\pi^{h+t-\tau}_{\tau|t} &= (1-\alpha)\pi^{h+t-\tau}_{H,\tau|t} + \alpha \hat{\varepsilon}^{h+t-\tau}_{\tau|t}, && \text{(F.9)}\\
\hat{r}^{*,h+t-\tau}_{\tau|t} &= \phi_b \hat{b}^{h+t-\tau}_{\tau+1|t} + \mu_\tau, && \text{(F.10)}\\
\hat{b}^{*,h+t-\tau}_{\tau+1|t} &= \beta^{-1}(\hat{b}^{*,h+t+1-\tau}_{\tau|t} + \vartheta_1 \hat{q}^{h+t-\tau}_{\tau|t} - \vartheta_2 \alpha \hat{s}^{h+t-\tau}_{\tau|t} + \vartheta_2 \hat{y}^{h+t-\tau}_{\tau|t} - \hat{c}^{h+t-\tau}_{\tau|t})\\
&\quad - \vartheta_1 \hat{q}^{h+t-\tau}_{\tau|t} + \vartheta_1 \hat{r}^{*,h+t-\tau}_{\tau|t}, && \text{(F.11)}
\end{aligned}
$$

where $\hat{q}^{h+1}_{t-1|t}$ is simply a notational simplification defined by $\hat{q}^{h+1}_{t-1|t} \equiv \hat{q}_{t-1}$ and similarly $\hat{b}^{h+1}_{t|t} \equiv \hat{b}^*_t$. Here $\vartheta_{yc} = (1-\alpha)\frac{\bar{S}^{-\eta}\bar{C}}{\bar{Y}}$, $\vartheta_{ys} = \alpha[\gamma + \eta(1-\alpha)]\frac{\bar{S}^{-\eta}\bar{C}}{\bar{Y}}$, $\vartheta_1 = \frac{\bar{\mathcal{B}}^*\bar{Q}}{\bar{C}}$, and $\vartheta_2 = \frac{\bar{S}\bar{Y}}{\bar{C}}$.

At the end of finite planning date $\tau = t + h$, we have

$$\hat{c}^0_{\tau|t} = -\frac{1}{\sigma}\hat{\imath}^0_{\tau|t} + (1-\beta)\hat{b}^{*,0}_{\tau+1|t} + \nu_t, \tag{F.12}$$

$$\hat{q}^0_{\tau|t} = \hat{r}^{*,0}_{\tau|t} - \hat{\imath}^0_{\tau|t} + \sigma(\nu_t - \nu^*_t), \tag{F.13}$$

$$\hat{\varepsilon}^0_{\tau|t} = \hat{q}^0_{\tau|t} - \hat{q}^1_{\tau-1|t} + \pi^0_{\tau|t}, \tag{F.14}$$

$$\hat{s}^0_{\tau|t} = \frac{\hat{q}^0_{\tau|t}}{1-\alpha}, \tag{F.15}$$

$$\pi^0_{H,\tau|t} = \kappa\widehat{mc}^0_{\tau|t} + (1-\theta)\beta\tilde{\nu}_t, \tag{F.16}$$

$$\widehat{mc}^0_{\tau|t} = (\sigma + \varphi\vartheta_{yc})\hat{c}^0_{\tau|t} + (\alpha + \varphi\vartheta_{ys})\hat{s}^0_{\tau|t} - (1+\varphi)a_\tau, \tag{F.17}$$

$$\hat{\imath}^0_{\tau|t} = \phi_\pi\pi^0_{\tau|t}, \tag{F.18}$$

$$\hat{y}^0_{\tau|t} = \vartheta_{ys}\hat{s}^0_{\tau|t} + \vartheta_{yc}\hat{c}^0_{\tau|t}, \tag{F.19}$$

$$\pi^0_{\tau|t} = (1-\alpha)\pi^0_{H,\tau|t} + \alpha\hat{\varepsilon}^0_{\tau|t}, \tag{F.20}$$

$$\hat{r}^{*,0}_{\tau|t} = \phi_b\hat{b}^0_{\tau+1|t} + \mu_\tau, \tag{F.21}$$

$$\hat{b}^{*,0}_{\tau+1|t} = \beta^{-1}(\hat{b}^{*,1}_{\tau|t} + \vartheta_1\hat{q}^0_{\tau|t} - \vartheta_2\alpha\hat{s}^0_{\tau|t} + \vartheta_2\hat{y}^0_{\tau|t} - \hat{c}^0_{\tau|t}) - \vartheta_1\hat{q}^0_{\tau|t} + \vartheta_1\hat{r}^{*,0}_{\tau|t}. \tag{F.22}$$

The above system of equations consists of a finite number of equations as a function of state variables $\{\hat{q}_{t-1}, \hat{b}^*_t, \nu_t, \nu^*_t, \tilde{\nu}_t\}$ and exogenous shocks $\{a_t, \mu_t\}$. Thus, we can solve for all endogenous variables $\{\hat{c}^{h+t-\tau}_{\tau|t}, \hat{y}^{h+t-\tau}_{\tau|t}, \hat{\imath}^{h+t-\tau}_{\tau|t}, \hat{r}^{*,h+t-\tau}_{\tau|t}, \pi^{h+t-\tau}_{H,\tau|t}, \pi^{h+t-\tau}_{\tau|t}, \hat{q}^{h+t-\tau}_{\tau|t}, \hat{s}^{h+t-\tau}_{\tau|t}, \hat{\varepsilon}^{h+t-\tau}_{\tau|t}, \hat{b}^{*,h+t-\tau}_{\tau+1|t}\}^{t+h}_{\tau=t}$ with a unique solution. See Appendix G for the detailed solution method.

G Solution to Policy Functions

We now show the solution to the policy functions for the equilibrium characterized in Section 3 and Appendix F. Similar to expressions (3.8)-(3.9), we can write the solution to any endogenous variable $x^j_{\tau|t}$ except for $\hat{b}^{*j}_{\tau+1|t}$ in forward planning as a function of the state variables with exogenous shocks; that is,

$$x^j_{\tau|t} = \psi^j_{x,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{x,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{x,a}a_\tau + \psi^j_{x,\mu}\mu_\tau + \psi^j_{x,\nu}\nu_t + \psi^j_{x,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{x,\nu^*}\nu^*_t, \tag{G.1}$$

for any (counterfactual) $j \geq 0$, and

$$\hat{b}^{*j}_{\tau+1|t} = \psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{b,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t. \tag{G.2}$$

First, we aim to pin down the coefficients for $j = 0$. From (F.12)-(F.22), one can easily eliminate $\{\hat{\varepsilon}^0_{\tau|t}, \hat{s}^0_{\tau|t}, \widehat{mc}^0_{\tau|t}, \hat{\imath}^0_{\tau|t}, \hat{r}^{*,0}_{\tau|t}\}$. Additionally, note that since $y^0_{\tau|t}$ only enters (F.19), we only need to solve $\{\hat{c}^0_{\tau|t}, \hat{q}^0_{\tau|t}, \pi^0_{\tau|t}, \pi^0_{H,\tau|t}, \hat{b}^{*,0}_{\tau+1|t}\}$, and then $\hat{y}^0_{\tau|t}$ is uniquely pinned down by (F.19).

We solve $\{\hat{c}^0_{\tau|t}, \hat{q}^0_{\tau|t}, \pi^0_{\tau|t}, \pi^0_{H,\tau|t}\}$ by equating coefficients. Note that from (F.12),

$$\hat{c}^0_{\tau|t} = -\frac{\phi_\pi}{\sigma}\pi^0_{\tau|t} + (1-\beta)\hat{b}^{*,0}_{\tau+1|t} + \nu_t,$$

and equating the coefficients yields

$$\begin{aligned}
\psi^0_{c,q} &= -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,q} + (1-\beta)\psi^0_{b,q}, & \psi^0_{c,b} &= -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,b} + (1-\beta)\psi^0_{b,b},\\
\psi^0_{c,a} &= -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,a} + (1-\beta)\psi^0_{b,a}, & \psi^0_{c,\mu} &= -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,\mu} + (1-\beta)\psi^0_{b,\mu},\\
\psi^0_{c,\nu} &= -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,\nu} + (1-\beta)\psi^0_{b,\nu} + 1, & \psi^0_{c,\tilde{\nu}} &= -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,\tilde{\nu}} + (1-\beta)\psi^0_{b,\tilde{\nu}},\\
\psi^0_{c,\nu^*} &= -\frac{\phi_\pi}{\sigma}\psi^0_{\pi,\nu^*} + (1-\beta)\psi^0_{b,\nu^*}.
\end{aligned}$$

Similarly, from (F.13),

$$\hat{q}^0_{\tau|t} = \phi_b\hat{b}^0_{\tau+1|t} + \mu_\tau - \phi_\pi\pi^0_{\tau|t} + \sigma(\nu_t - \nu^*_t),$$

which yields

$$\begin{aligned}
\psi^0_{q,q} &= \phi_b\psi^0_{b,q} - \phi_\pi\psi^0_{\pi,q}, & \psi^0_{q,b} &= \phi_b\psi^0_{b,b} - \phi_\pi\psi^0_{\pi,b}\\
\psi^0_{q,a} &= \phi_b\psi^0_{b,a} - \phi_\pi\psi^0_{\pi,a}, & \psi^0_{q,\mu} &= \phi_b\psi^0_{b,\mu} - \phi_\pi\psi^0_{\pi,\mu} + 1,\\
\psi^0_{q,\nu} &= \phi_b\psi^0_{b,\nu} - \phi_\pi\psi^0_{\pi,\nu} + \sigma, & \psi^0_{q,\tilde{\nu}} &= \phi_b\psi^0_{b,\tilde{\nu}} - \phi_\pi\psi^0_{\pi,\tilde{\nu}},\\
\psi^0_{q,\nu^*} &= \phi_b\psi^0_{b,\nu^*} - \phi_\pi\psi^0_{\pi,\nu^*} - \sigma.
\end{aligned}$$

Similarly, from (F.16)-(F.17),

$$\pi^0_{H,\tau|t} = \kappa(\sigma + \varphi\vartheta_{yc})\hat{c}^0_{\tau|t} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\hat{q}^0_{\tau|t} - \kappa(1+\varphi)a_\tau + (1-\theta)\beta\tilde{\nu}_t,$$

which yields

$$\begin{aligned}
\psi^0_{\pi_H,q} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^0_{c,q} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,q}, & \psi^0_{\pi_H,b} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^0_{c,b} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,b},\\
\psi^0_{\pi_H,a} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^0_{c,a} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,a} - \kappa(1+\varphi), & \psi^0_{\pi_H,\mu} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^0_{c,\mu} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,\mu},\\
\psi^0_{\pi_H,\nu} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^0_{c,\nu} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,\nu}, & \psi^0_{\pi_H,\tilde{\nu}} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^0_{c,\tilde{\nu}} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,\tilde{\nu}} + (1-\theta)\beta,\\
\psi^0_{\pi_H,\nu^*} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^0_{c,\nu^*} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^0_{q,\nu^*}.
\end{aligned}$$

Similarly, from (F.14) and (F.20),

$$\pi^0_{\tau|t} = (1-\alpha)\pi^0_{H,\tau|t} + \alpha(\hat{q}^0_{\tau|t} - \hat{q}^1_{\tau-1|t} + \pi^0_{\tau|t}),$$

which yields

$$\begin{aligned}
(1-\alpha)\psi^0_{\pi,q} &= (1-\alpha)\psi^0_{\pi_H,q} + \alpha\psi^0_{q,q} - \alpha, & (1-\alpha)\psi^0_{\pi,b} &= (1-\alpha)\psi^0_{\pi_H,b} + \alpha\psi^0_{q,b},\\
(1-\alpha)\psi^0_{\pi,a} &= (1-\alpha)\psi^0_{\pi_H,a} + \alpha\psi^0_{q,a}, & (1-\alpha)\psi^0_{\pi,\mu} &= (1-\alpha)\psi^0_{\pi_H,\mu} + \alpha\psi^0_{q,\mu},\\
(1-\alpha)\psi^0_{\pi,\nu} &= (1-\alpha)\psi^0_{\pi_H,\nu} + \alpha\psi^0_{q,\nu}, & (1-\alpha)\psi^0_{\pi,\tilde{\nu}} &= (1-\alpha)\psi^0_{\pi_H,\tilde{\nu}} + \alpha\psi^0_{q,\tilde{\nu}},\\
(1-\alpha)\psi^0_{\pi,\nu^*} &= (1-\alpha)\psi^0_{\pi_H,\nu^*} + \alpha\psi^0_{q,\nu^*}.
\end{aligned}$$

Similarly, from (F.19) and (F.22),

$$\begin{aligned}
\hat{b}^{*,0}_{\tau+1|t} &= \beta^{-1}(\hat{b}^{*,1}_{\tau|t} + \vartheta_1\hat{q}^0_{\tau|t} - \vartheta_2\alpha\hat{s}^0_{\tau|t} + \vartheta_2\hat{y}^0_{\tau|t} - \hat{c}^0_{\tau|t}) - \vartheta_1\hat{q}^0_{\tau|t} + \vartheta_1\hat{r}^{*,0}_{\tau|t}\\
&= \beta^{-1}\left\{\hat{b}^{*,1}_{\tau|t} + \left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\hat{q}^0_{\tau|t} - (1-\vartheta_2\vartheta_{yc})\hat{c}^0_{\tau|t}\right\} - \vartheta_1\hat{q}^0_{\tau|t} + \vartheta_1\hat{r}^{*,0}_{\tau|t},
\end{aligned}$$

and it implies

$$(1-\vartheta_1\phi_b)\hat{b}^{*,0}_{\tau+1|t} = \beta^{-1}\left\{\hat{b}^{*,1}_{\tau|t} + \left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\hat{q}^0_{\tau|t} - (1-\vartheta_2\vartheta_{yc})\hat{c}^0_{\tau|t}\right\} - \vartheta_1\hat{q}^0_{\tau|t} + \vartheta_1\mu_\tau,$$

which yields

$$\begin{aligned}
(1-\vartheta_1\phi_b)\psi^0_{b,q} &= \beta^{-1}\left\{\left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,q} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,q}\right\} - \vartheta_1\psi^0_{q,q},\\
(1-\vartheta_1\phi_b)\psi^0_{b,b} &= \beta^{-1}\left\{1 + \left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,b} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,b}\right\} - \vartheta_1\psi^0_{q,b},\\
(1-\vartheta_1\phi_b)\psi^0_{b,a} &= \beta^{-1}\left\{\left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,a} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,a}\right\} - \vartheta_1\psi^0_{q,a},\\
(1-\vartheta_1\phi_b)\psi^0_{b,\mu} &= \beta^{-1}\left\{\left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,\mu} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,\mu}\right\} - \vartheta_1\psi^0_{q,\mu} + \vartheta_1,\\
(1-\vartheta_1\phi_b)\psi^0_{b,\nu} &= \beta^{-1}\left\{\left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,\nu} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,\nu}\right\} - \vartheta_1\psi^0_{q,\nu},\\
(1-\vartheta_1\phi_b)\psi^0_{b,\tilde{\nu}} &= \beta^{-1}\left\{\left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,\tilde{\nu}} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,\tilde{\nu}}\right\} - \vartheta_1\psi^0_{q,\tilde{\nu}},\\
(1-\vartheta_1\phi_b)\psi^0_{b,\nu^*} &= \beta^{-1}\left\{\left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^0_{q,\nu^*} - (1-\vartheta_2\vartheta_{yc})\psi^0_{c,\nu^*}\right\} - \vartheta_1\psi^0_{q,\nu^*}.
\end{aligned}$$

Thus, in the case of $j=0$, we have 35 simple linear equations for 35 undetermined coefficients. By solving these linear equations, we obtain the solution of $\{\hat{c}^0_{\tau|t}, \hat{q}^0_{\tau|t}, \pi^0_{\tau|t}, \pi^0_{H,\tau|t}, \hat{b}^{*,0}_{\tau+1|t}\}$ at date τ calculated in period t. Then one can easily obtain all other endogenous variables at date τ calculated in period t.

Next, we solve the undermined coefficients for any (counterfactual) j as a function of the coefficients for $j-1$ through (F.1)-(F.11). Similarly, we can eliminate $\{\hat{\varepsilon}^j_{\tau|t}, \hat{s}^j_{\tau|t}, \widehat{mc}^j_{\tau|t}, \hat{i}^j_{\tau|t}, \hat{i}^{*,j}_{\tau|t}\}$ in (F.1)-(F.11). Additionally, since $\hat{y}^j_{\tau|t}$ only enters (F.8), we only need to solve $\{\hat{c}^j_{\tau|t}, \hat{q}^j_{\tau|t}, \pi^j_{\tau|t}, \pi^j_{H,\tau|t}, \hat{b}^{*,j}_{\tau+1|t}\}$, and $\hat{y}^j_{\tau|t}$ is pinned down by (F.8).

To solve $\{\hat{c}^j_{\tau|t}, \hat{q}^j_{\tau|t}, \pi^j_{\tau|t}, \pi^j_{H,\tau|t}, \hat{b}^{*,j}_{\tau+1|t}\}$, from (F.1), we have

$$\hat{c}^j_{\tau|t} = \mathbb{E}_\tau \hat{c}^{j-1}_{\tau+1|t} - \frac{1}{\sigma}(\phi_\pi \pi^j_{\tau|t} - \mathbb{E}_\tau \pi^{j-1}_{\tau+1|t}),$$

and substituting with the policy functions yields

$$\begin{aligned}
&\psi^j_{c,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{c,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{c,a}a_\tau + \psi^j_{c,\mu}\mu_\tau + \psi^j_{c,\nu}\nu_t + \psi^j_{c,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{c,\nu^*}\nu^*_t \\
&= \mathbb{E}_\tau\{\psi^{j-1}_{c,q}[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{c,b}[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{b,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{c,a}a_{\tau+1} + \psi^{j-1}_{c,\mu}\mu_{\tau+1} + \psi^{j-1}_{c,\nu}\nu_t + \psi^{j-1}_{c,\tilde{\nu}}\tilde{\nu}_t + \psi^{j-1}_{c,\nu^*}\nu^*_t\} \\
&- \frac{\phi_\pi}{\sigma}[\psi^j_{\pi,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{\pi,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{\pi,a}a_\tau + \psi^j_{\pi,\mu}\mu_\tau + \psi^j_{\pi,\nu}\nu_t + \psi^j_{\pi,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{\pi,\nu^*}\nu^*_t] \\
&+ \frac{1}{\sigma}\mathbb{E}_\tau\{\psi^{j-1}_{\pi,q}[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{\pi,b}[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{b,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{\pi,a}a_{\tau+1} + \psi^{j-1}_{\pi,\mu}\mu_{\tau+1} + \psi^{j-1}_{\pi,\nu}\nu_t + \psi^{j-1}_{\pi,\tilde{\nu}}\tilde{\nu}_t + \psi^{j-1}_{\pi,\nu^*}\nu^*_t\},
\end{aligned}$$

where we have substituted $\hat{q}^j_{\tau|t} = \psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t$ and the similar expression of $\hat{b}^{*,j}_{\tau+1|t}$.

By equating the coefficients, we obtain

$$\begin{aligned}
\psi^j_{c,q} &= \psi^{j-1}_{c,q}\psi^j_{q,q} + \psi^{j-1}_{c,b}\psi^j_{b,q} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,q} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,q} + \psi^{j-1}_{\pi,b}\psi^j_{b,q}], \\
\psi^j_{c,b} &= \psi^{j-1}_{c,q}\psi^j_{q,b} + \psi^{j-1}_{c,b}\psi^j_{b,b} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,b} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,b} + \psi^{j-1}_{\pi,b}\psi^j_{b,b}], \\
\psi^j_{c,a} &= \psi^{j-1}_{c,q}\psi^j_{q,a} + \psi^{j-1}_{c,b}\psi^j_{b,a} + \rho_a\psi^{j-1}_{c,a} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,a} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,a} + \psi^{j-1}_{\pi,b}\psi^j_{b,a} + \rho_a\psi^{j-1}_{\pi,a}], \\
\psi^j_{c,\mu} &= \psi^{j-1}_{c,q}\psi^j_{q,\mu} + \psi^{j-1}_{c,b}\psi^j_{b,\mu} + \rho_\mu\psi^{j-1}_{c,\mu} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,\mu} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,\mu} + \psi^{j-1}_{\pi,b}\psi^j_{b,\mu} + \rho_\mu\psi^{j-1}_{\pi,\mu}], \\
\psi^j_{c,\nu} &= \psi^{j-1}_{c,q}\psi^j_{q,\nu} + \psi^{j-1}_{c,b}\psi^j_{b,\nu} + \psi^{j-1}_{c,\nu} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,\nu} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,\nu} + \psi^{j-1}_{\pi,b}\psi^j_{b,\nu} + \psi^{j-1}_{\pi,\nu}], \\
\psi^j_{c,\tilde{\nu}} &= \psi^{j-1}_{c,q}\psi^j_{q,\tilde{\nu}} + \psi^{j-1}_{c,b}\psi^j_{b,\tilde{\nu}} + \psi^{j-1}_{c,\tilde{\nu}} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,\tilde{\nu}} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,\tilde{\nu}} + \psi^{j-1}_{\pi,b}\psi^j_{b,\tilde{\nu}} + \psi^{j-1}_{\pi,\tilde{\nu}}], \\
\psi^j_{c,\nu^*} &= \psi^{j-1}_{c,q}\psi^j_{q,\nu^*} + \psi^{j-1}_{c,b}\psi^j_{q,\nu^*} + \psi^{j-1}_{c,\nu^*} - \frac{\phi_\pi}{\sigma}\psi^j_{\pi,\nu^*} + \frac{1}{\sigma}[\psi^{j-1}_{\pi,q}\psi^j_{q,\nu^*} + \psi^{j-1}_{\pi,b}\psi^j_{b,\nu^*} + \psi^{j-1}_{\pi,\nu^*}].
\end{aligned}$$

Similarly, from (F.2), we have

$$\hat{q}^j_{\tau|t} = \mathbb{E}_\tau \hat{q}^{j-1}_{\tau+1|t} + \phi_b \hat{b}^j_{\tau+1|t} + \mu_\tau - (\phi_\pi \pi^j_{\tau|t} - \mathbb{E}_\tau \pi^{j-1}_{\tau+1|t}),$$

and substituting with the policy functions yields

$$\begin{aligned}
&\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t \\
&= \mathbb{E}_\tau\{\psi^{j-1}_{q,q}[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{q,b}[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{b,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{q,a}a_{\tau+1} + \psi^{j-1}_{q,\mu}\mu_{\tau+1} + \psi^{j-1}_{q,\nu}\nu_t + \psi^{j-1}_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^{j-1}_{q,\nu^*}\nu^*_t\} \\
&+ \phi_b[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&+ \mu_\tau - \phi_\pi[\psi^j_{\pi,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{\pi,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{\pi,a}a_\tau + \psi^j_{\pi,\mu}\mu_\tau + \psi^j_{\pi,\nu}\nu_t + \psi^j_{\pi,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{\pi,\nu^*}\nu^*_t] \\
&+ \mathbb{E}_\tau\{\psi^{j-1}_{\pi,q}[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{\pi,b}[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{b,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{\pi,a}a_{\tau+1} + \psi^{j-1}_{\pi,\mu}\mu_{\tau+1} + \psi^{j-1}_{\pi,\nu}\nu_t + \psi^{j-1}_{\pi,\tilde{\nu}}\tilde{\nu}_t + \psi^{j-1}_{\pi,\nu^*}\nu^*_t\}.
\end{aligned}$$

By equating the coefficients, we obtain

$$\begin{aligned}
\psi^j_{q,q} &= \psi^{j-1}_{q,q}\psi^j_{q,q} + \psi^{j-1}_{q,b}\psi^j_{b,q} + \phi_b\psi^j_{b,q} - \phi_\pi\psi^j_{\pi,q} + \psi^{j-1}_{\pi,q}\psi^j_{q,q} + \psi^{j-1}_{\pi,b}\psi^j_{b,q}, \\
\psi^j_{q,b} &= \psi^{j-1}_{q,q}\psi^j_{q,b} + \psi^{j-1}_{q,b}\psi^j_{b,b} + \phi_b\psi^j_{b,b} - \phi_\pi\psi^j_{\pi,b} + \psi^{j-1}_{\pi,q}\psi^j_{q,b} + \psi^{j-1}_{\pi,b}\psi^j_{b,b}, \\
\psi^j_{q,a} &= \psi^{j-1}_{q,q}\psi^j_{q,a} + \psi^{j-1}_{q,b}\psi^j_{b,a} + \rho_a\psi^{j-1}_{q,a} + \phi_b\psi^j_{b,a} - \phi_\pi\psi^j_{\pi,a} + \psi^{j-1}_{\pi,q}\psi^j_{q,a} + \psi^{j-1}_{\pi,b}\psi^j_{b,a} + \rho_a\psi^{j-1}_{\pi,a}, \\
\psi^j_{q,\mu} &= \psi^{j-1}_{q,q}\psi^j_{q,\mu} + \psi^{j-1}_{q,b}\psi^j_{b,\mu} + \rho_\mu\psi^{j-1}_{q,\mu} + \phi_b\psi^j_{b,\mu} + 1 - \phi_\pi\psi^j_{\pi,\mu} + \psi^{j-1}_{\pi,q}\psi^j_{q,\mu} + \psi^{j-1}_{\pi,b}\psi^j_{b,\mu} + \rho_\mu\psi^{j-1}_{\pi,\mu}, \\
\psi^j_{q,\nu} &= \psi^{j-1}_{q,q}\psi^j_{q,\nu} + \psi^{j-1}_{q,b}\psi^j_{b,\nu} + \psi^{j-1}_{q,\nu} + \phi_b\psi^j_{b,\nu} - \phi_\pi\psi^j_{\pi,\nu} + \psi^{j-1}_{\pi,q}\psi^j_{q,\nu} + \psi^{j-1}_{\pi,b}\psi^j_{b,\nu}, \\
\psi^j_{q,\tilde{\nu}} &= \psi^{j-1}_{q,q}\psi^j_{q,\tilde{\nu}} + \psi^{j-1}_{q,b}\psi^j_{b,\tilde{\nu}} + \psi^{j-1}_{q,\tilde{\nu}} + \phi_b\psi^j_{b,\tilde{\nu}} - \phi_\pi\psi^j_{\pi,\tilde{\nu}} + \psi^{j-1}_{\pi,q}\psi^j_{q,\tilde{\nu}} + \psi^{j-1}_{\pi,b}\psi^j_{b,\tilde{\nu}}, \\
\psi^j_{q,\nu^*} &= \psi^{j-1}_{q,q}\psi^j_{q,\nu^*} + \psi^{j-1}_{q,b}\psi^j_{b,\nu^*} + \psi^{j-1}_{q,\nu^*} + \phi_b\psi^j_{b,\nu^*} - \phi_\pi\psi^j_{\pi,\nu^*} + \psi^{j-1}_{\pi,q}\psi^j_{q,\nu^*} + \psi^{j-1}_{\pi,b}\psi^j_{b,\nu^*}.
\end{aligned}$$

Similarly, from (F.5)-(F.6), we have

$$\pi^j_{H,\tau|t} = \kappa(\sigma + \varphi\vartheta_{yc})\hat{c}^j_{\tau|t} + \frac{\kappa(\alpha + \varphi\vartheta_{ys})}{1-\alpha}\hat{q}^j_{\tau|t} - \kappa(1+\varphi)a_\tau + \beta\mathbb{E}_\tau\pi^{j-1}_{H,\tau+1|t},$$

and substituting with the policy functions yields

$$
\begin{aligned}
&\psi^j_{\pi_H,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{\pi_H,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{\pi_H,a}a_\tau + \psi^j_{\pi_H,\mu}\mu_\tau + \psi^j_{\pi_H,\nu}\nu_t + \psi^j_{\pi_H,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{\pi_H,\nu^*}\nu^*_t \\
&= \kappa(\sigma+\varphi\vartheta_{yc})[\psi^j_{c,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{c,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{c,a}a_\tau + \psi^j_{c,\mu}\mu_\tau + \psi^j_{c,\nu}\nu_t + \psi^j_{c,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{c,\nu^*}\nu^*_t] \\
&+ \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&- \kappa(1+\varphi)a_\tau \\
&+ \beta\mathbb{E}_\tau\{\psi^{j-1}_{\pi_H,q}[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{\pi_H,b}[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{b,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&+ \psi^{j-1}_{\pi_H,a}a_{\tau+1} + \psi^{j-1}_{\pi_H,\mu}\mu_{\tau+1} + \psi^{j-1}_{\pi_H,\nu}\nu_t + \psi^{j-1}_{\pi_H,\tilde{\nu}}\tilde{\nu}_t + \psi^{j-1}_{\pi_H,\nu^*}\nu^*_t\}.
\end{aligned}
$$

By equating the coefficients, we obtain

$$
\begin{aligned}
\psi^j_{\pi_H,q} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,q} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,q} + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,q} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,q}], \\
\psi^j_{\pi_H,b} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,b} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,b} + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,b} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,b}], \\
\psi^j_{\pi_H,a} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,a} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,a} - \kappa(1+\varphi) + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,a} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,a} + \rho_a\psi^{j-1}_{\pi_H,a}], \\
\psi^j_{\pi_H,\mu} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,\mu} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,\mu} + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,\mu} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,\mu} + \rho_\mu\psi^{j-1}_{\pi_H,\mu}], \\
\psi^j_{\pi_H,\nu} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,\nu} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,\nu} + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,\nu} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,\nu} + \psi^{j-1}_{\pi_H,\nu}], \\
\psi^j_{\pi_H,\tilde{\nu}} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,\tilde{\nu}} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,\tilde{\nu}} + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,\tilde{\nu}} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,\tilde{\nu}} + \psi^{j-1}_{\pi_H,\tilde{\nu}}], \\
\psi^j_{\pi_H,\nu^*} &= \kappa(\sigma+\varphi\vartheta_{yc})\psi^j_{c,\nu^*} + \frac{\kappa(\alpha+\varphi\vartheta_{ys})}{1-\alpha}\psi^j_{q,\nu^*} + \beta[\psi^{j-1}_{\pi_H,q}\psi^j_{q,\nu^*} + \psi^{j-1}_{\pi_H,b}\psi^j_{b,\nu^*} + \psi^{j-1}_{\pi_H,\nu^*}].
\end{aligned}
$$

Similarly, from (F.3) and (F.9), we have

$$
\pi^j_{\tau|t} = (1-\alpha)\pi^j_{H,\tau|t} + \alpha(\hat{q}^j_{\tau|t} - \hat{q}^{j+1}_{\tau-1|t} + \pi^j_{\tau|t}),
$$

and substituting with the policy functions yields

$$
\begin{aligned}
&(1-\alpha)[\psi^j_{\pi,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{\pi,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{\pi,a}a_\tau + \psi^j_{\pi,\mu}\mu_\tau + \psi^j_{\pi,\nu}\nu_t + \psi^j_{\pi,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{\pi,\nu^*}\nu^*_t] \\
&= (1-\alpha)[\psi^j_{\pi_H,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{\pi_H,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{\pi_H,a}a_\tau + \psi^j_{\pi_H,\mu}\mu_\tau + \psi^j_{\pi_H,\nu}\nu_t + \psi^j_{\pi_H,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{\pi_H,\nu^*}\nu^*_t] \\
&+ \alpha[\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] - \alpha\hat{q}^{j+1}_{\tau-1|t}.
\end{aligned}
$$

By equating the coefficients, we obtain

$$(1-\alpha)\psi^j_{\pi,q} = (1-\alpha)\psi^j_{\pi_H,q} + \alpha\psi^j_{q,q} - \alpha, \qquad (1-\alpha)\psi^j_{\pi,b} = (1-\alpha)\psi^j_{\pi_H,b} + \alpha\psi^j_{q,b},$$
$$(1-\alpha)\psi^j_{\pi,a} = (1-\alpha)\psi^j_{\pi_H,a} + \alpha\psi^j_{q,a}, \qquad (1-\alpha)\psi^j_{\pi,\mu} = (1-\alpha)\psi^j_{\pi_H,\mu} + \alpha\psi^j_{q,\mu},$$
$$(1-\alpha)\psi^j_{\pi,\nu} = (1-\alpha)\psi^j_{\pi_H,\nu} + \alpha\psi^j_{q,\nu}, \qquad (1-\alpha)\psi^j_{\pi,\tilde{\nu}} = (1-\alpha)\psi^j_{\pi_H,\tilde{\nu}} + \alpha\psi^j_{q,\tilde{\nu}},$$
$$(1-\alpha)\psi^j_{\pi,\nu^*} = (1-\alpha)\psi^j_{\pi_H,\nu^*} + \alpha\psi^j_{q,\nu^*}.$$

Finally, from (F.8) and (F.11), we have

$$(1-\vartheta_1\phi_b)\hat{b}^{*,j}_{\tau+1|t} = \beta^{-1}\left\{\hat{b}^{*,j+1}_{\tau|t} + \left[\vartheta_1 - \vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\hat{q}^j_{\tau|t} - (1-\vartheta_2\vartheta_{yc})\hat{c}^j_{\tau|t}\right\} - \vartheta_1\hat{q}^j_{\tau|t} + \vartheta_1\mu_\tau,$$

and substituting with the policy functions yields

$$\begin{aligned}
&(1-\vartheta_1\phi_b)[\psi^j_{b,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{b,b}\hat{b}^{*,j+1}_{\tau|t} + \psi^j_{b,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{b,\nu}\nu_t + \psi^j_{b,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{b,\nu^*}\nu^*_t] \\
&= \beta^{-1}\hat{b}^{*,j+1}_{\tau|t} + \vartheta_1\mu_\tau \\
&+ \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right][\psi^j_{q,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{q,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{q,a}a_\tau + \psi^j_{q,\mu}\mu_\tau + \psi^j_{q,\nu}\nu_t + \psi^j_{q,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{q,\nu^*}\nu^*_t] \\
&- \beta^{-1}(1-\vartheta_2\vartheta_{yc})[\psi^j_{c,q}\hat{q}^{j+1}_{\tau-1|t} + \psi^j_{c,b}\hat{b}^{j+1}_{\tau|t} + \psi^j_{c,a}a_\tau + \psi^j_{c,\mu}\mu_\tau + \psi^j_{c,\nu}\nu_t + \psi^j_{c,\tilde{\nu}}\tilde{\nu}_t + \psi^j_{c,\nu^*}\nu^*_t]
\end{aligned}$$

By equating the coefficients, we obtain

$$\begin{aligned}
(1-\vartheta_1\phi_b)\psi^j_{b,q} &= \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,q} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,q} \\
(1-\vartheta_1\phi_b)\psi^j_{b,b} &= 1 + (\beta^{-1}-1) + \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,b} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,b} \\
(1-\vartheta_1\phi_b)\psi^j_{b,a} &= \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,a} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,a} \\
(1-\vartheta_1\phi_b)\psi^j_{b,\mu} &= \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,\mu} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,\mu} + \vartheta_1 \\
(1-\vartheta_1\phi_b)\psi^j_{b,\nu} &= \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,\nu} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,\nu} \\
(1-\vartheta_1\phi_b)\psi^j_{b,\tilde{\nu}} &= \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,\tilde{\nu}} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,\tilde{\nu}} \\
(1-\vartheta_1\phi_b)\psi^j_{b,\nu^*} &= \left[(\beta^{-1}-1)\vartheta_1 - \beta^{-1}\vartheta_2\frac{\alpha-\vartheta_{ys}}{(1-\alpha)}\right]\psi^j_{q,\nu^*} - \beta^{-1}(1-\vartheta_2\vartheta_{yc})\psi^j_{c,\nu^*}.
\end{aligned}$$

Thus, given the undetermined coefficients for $j-1$, we have 35 simple linear equations for the 35 undetermined coefficients for j, which yields a unique solution. Since we have derived the undetermined coefficients for the case of $j=0$, we can solve the expressions for $\{\hat{c}^j_{\tau|t}, \hat{q}^j_{\tau|t}, \pi^j_{\tau|t}, \pi^j_{H,\tau|t}, b^{*,j+1}_{\tau|t}\}$ by forward induction. All the other endogenous variables can be easily derived with the solutions of these four endogenous variables.

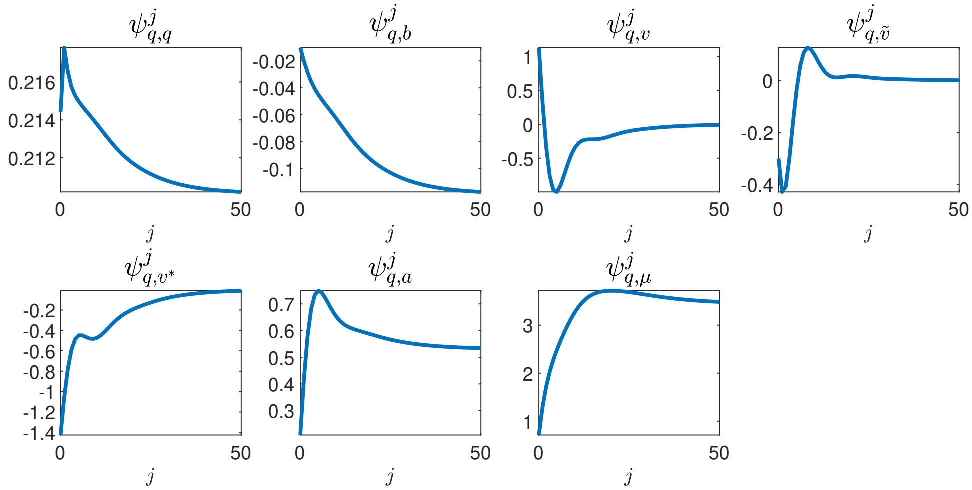
Thus far, we have derived the solution of the entire forward planning calculated in period t. Then, one can easily solve the equilibrium path solution (3.1) with the evolution of the state variables (3.2)-(3.4).

Policy Function Coefficients. With the calibrated parameters in Table 1, Figures G.6 and G.7 report the policy coefficients of the five variables $x \in \{q, b, c, \pi, \pi_H\}$ for different planning horizons $j \in [0, 50]$ quarters. As j increases, all coefficients converge to the unique RE equilibrium values, with household value functions $\{\psi^j_{x,\nu}, \psi^j_{x,\tilde{\nu}}, \psi^j_{x,\nu^*}\}$ becoming zero. As planning horizon j decreases, policy coefficients deviate from RE values, exhibiting non-monotonic, bumpy movements. Some coefficients, such as $\psi^j_{q,\nu}$, may even change signs with shorter horizons.

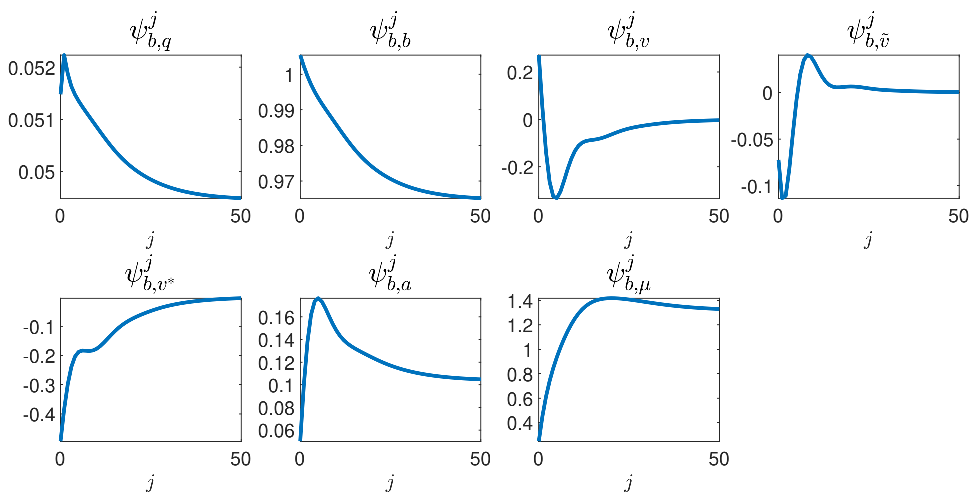
These policy coefficients entail several behavioral implications. Shorter planning horizons give rise to deviations from the RE equilibrium, attributed by limited foresight. Moreover, these biases do not consistently follow the same direction, as indicated by the non-monotonic policy coefficients. As a consequence, when decision-makers operate within a relatively short planning horizon, the resulting equilibrium can diverge in various ways from the RE equilibrium. Conversely, even with an extended planning horizon, expectations regarding distant future variables—assessed using policy coefficients for a truncated remaining planning horizon—may substantially deviate from RE.

Figure G.6: Policy Coefficients of the Five Variables with Planning Horizon $j \in [0, 50]$

(a) Real Exchange Rate



(b) Net Foreign Asset Position



(c) Consumption

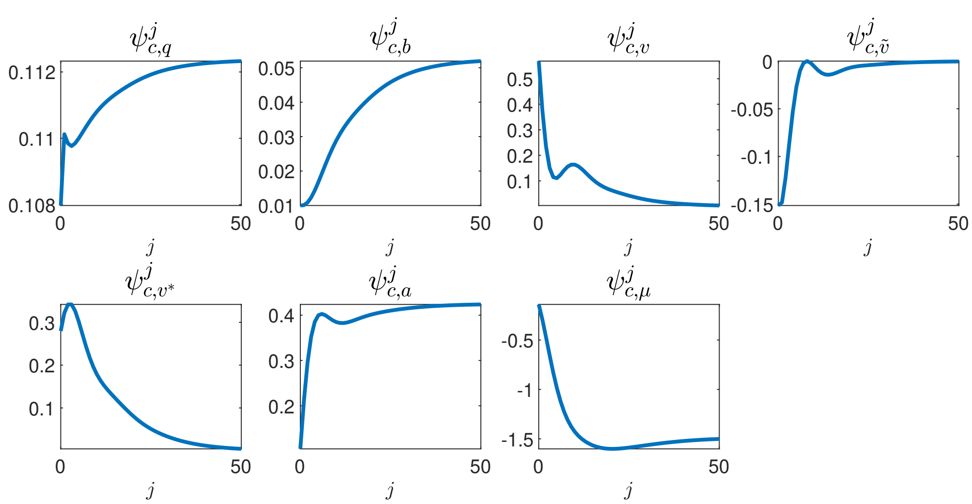
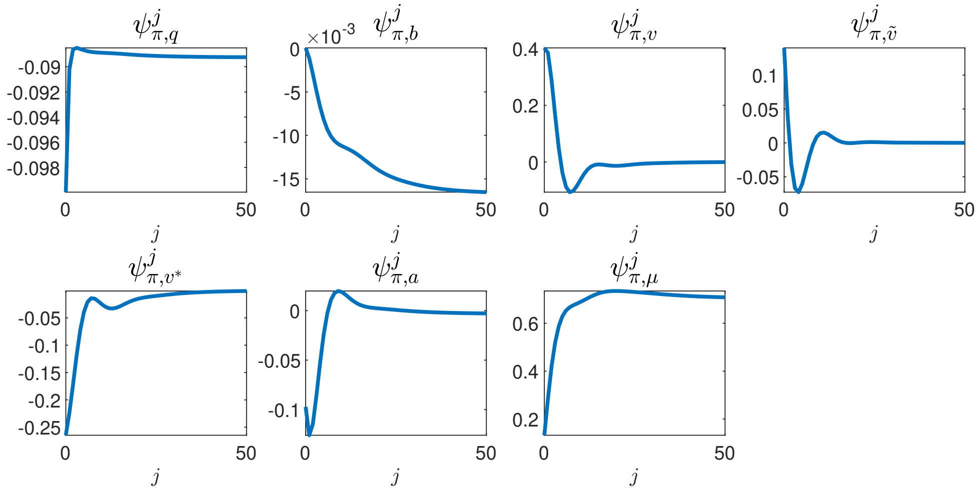
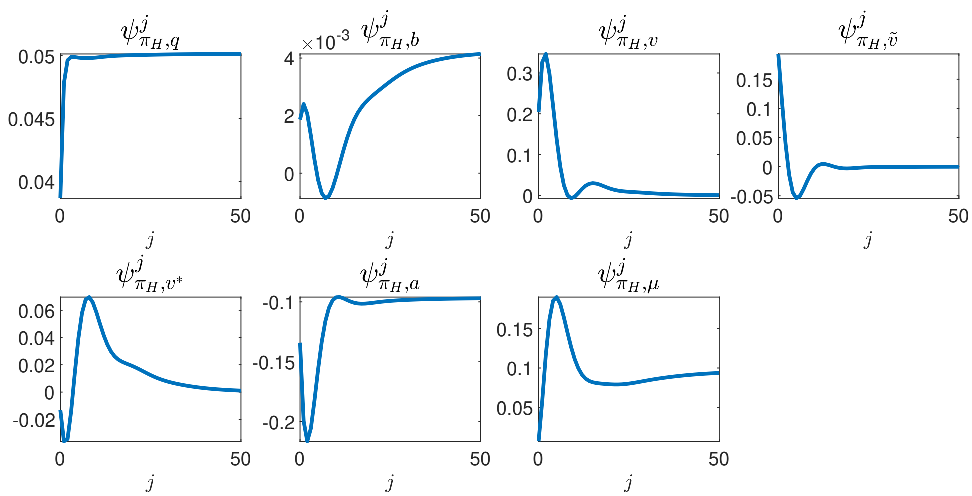


Figure G.7: Policy Coefficients of the Five Variables with Planning Horizon $j \in [0, 50]$ (continued)

(d) CPI Inflation



(e) Domestic Goods Inflation



H Data Sources and Variable Construction

The data used in the calibration of Section 4.1 are constructed as follows, all at a quarterly frequency:

- *Canadian per capita real output growth and consumption growth.* We obtain the Canadian data of real GDP [V6E06896], household real final consumption expenditure [V6A89012], and population [V1] from Statistics Canada (StatsCan). The code in brackets represents StatsCan mnemonic. We divide the two data series by population to calculate the per capita real output and consumption and take the log difference over time to get the corresponding growth rates.

- *U.S. and Canada price levels.* We obtain the headline consumer price index (CPI) of the U.S. and Canada from Federal Reserve Economic Data [FRED: CPIAUCSL and CPALCY01CAQ661N]. The U.S. CPI data is seasonally adjusted, whereas the latter is not. Thus, we follow the U.S. census bureau model X-13ARIMA-SEATS to seasonally adjust the Canadian CPI data series. We further normalize the data series such that the average price level for both the U.S. and Canada in 2015 is 100.

- *U.S.-Canada real exchange rate.* We obtain the average of the daily nominal exchange rate between U.S. and Canadian dollars from Federal Reserve Economic Data [FRED: CCUSMA02CAM618N]. The data is in units of Canadian dollars. We calculate the real exchange rate by multiplying the nominal exchange rate with the U.S. price level and dividing it by the Canadian price level.

I Construction of Short-Term and Long-Term Interest Rate Differentials

We construct the corresponding model-generated series via the following steps. First, to construct the simulated expected real interest rate differential under limited foresight, for each time t and for any horizon k, we need to pin down $\{\hat{q}^{h-k}_{t+k|t}, \hat{b}^{h-k}_{t+k+1|t}\}^{h}_{k=0}$ to back up $\{\hat{\pi}^{h-k}_{t+k|t}, \hat{\imath}^{h-k}_{t+k|t}, \hat{r}^{*,h-k}_{t+k|t}\}^{h}_{k=0}$. For each k, the solution to the forward planning exercise (3.8) yields

$$\mathbb{E}_t\hat{q}^{h-k}_{t+k|t} = \psi^{h-k}_{q,q}\mathbb{E}_t\hat{q}^{h-k+1}_{t+k-1|t} + \psi^{h-k}_{q,b}\mathbb{E}_t\hat{b}^{h-k+1}_{t+k|t} + \psi^{h-k}_{q,a}\rho^k_a a_t + \psi^{h-k}_{q,\mu}\rho^k_\mu\mu_t + \psi^{h-k}_{q,\nu}\nu_t + \psi^{h-k}_{q,\nu^*}\nu^*_t,$$

where $\mathbb{E}_t\hat{q}^{h+1}_{t-1|t} = \hat{q}_{t-1}$ and $\mathbb{E}_t\hat{b}^{h+1}_{t|t} = \hat{b}_t$ (that is, the equilibrium pre-determined real exchange rate and the net foreign asset position at time t). Then, the expected inflation under limited foresight satisfies

$$\mathbb{E}^h_t\pi_{t+k} \equiv \mathbb{E}_t\pi^{h-k}_{t+k|t} = \psi^{h-k}_{\pi,q}\mathbb{E}_t\hat{q}^{h-k+1}_{t+k-1|t} + \psi^{h-k}_{\pi,b}\mathbb{E}_t\hat{b}^{h-k+1}_{t+k|t} + \psi^{h-k}_{\pi,a}\rho^k_a a_t + \psi^{h-k}_{\pi,\mu}\rho^k_\mu\mu_t + \psi^{h-k}_{\pi,\nu}\nu_t + \psi^{h-k}_{\pi,\nu^*}\nu^*_t.$$

Since $\hat{\imath}^{h+t-\tau}_{\tau|t} = \phi_\pi\hat{\pi}^{h+t-\tau}_{\tau|t}$, we have the expected nominal interest rate given by

$$\mathbb{E}^h_t\hat{\imath}_{t+k} \equiv \mathbb{E}_t\hat{\imath}^{h-k}_{t+k|t} = \phi_\pi\mathbb{E}_t\pi^{h-k}_{t+k|t},$$

and then we construct the expected real interest rate under limited foresight as

$$\mathbb{E}_t^h \hat{r}_{t+k} \equiv \mathbb{E}_t \hat{\imath}_{t+k|t}^{h-k} - \mathbb{E}_t \pi_{t+k+1|t}^{h-k-1}, \tag{I.1}$$

for any $0 \leq k < h$, and for the case of $k = h$, we construct $\mathbb{E}_t^h \hat{r}_{t+h} \equiv \mathbb{E}_t \hat{\imath}_{t+h|t}^0$.

Second, for the expected foreign real interest rate under limited foresight, since (2.40) yields $\hat{r}_{\tau|t}^{*,h+t-\tau} = \phi_b \hat{b}_{\tau+1|t}^{h+t-\tau} + \mu_\tau$ for any $t \leq \tau \leq t+h$, we construct

$$\mathbb{E}_t^h \hat{r}_{t+k}^* \equiv \mathbb{E}_t \hat{r}_{t+k|t}^{h-k} = \phi_b \mathbb{E}_t \hat{b}_{t+k+1|t}^{h-k} + \rho_\mu^k \mu_t. \tag{I.2}$$

Therefore, for a given the threshold horizon for the short-term and the long-term M and the given planning horizon h, we can construct the cumulative forecasts of real interest rate differentials under limited foresight as

$$D_t^S(M) = \sum_{k=0}^{M-1} \mathbb{E}_t^h [\hat{r}_{t+k}^* - \hat{r}_{t+k}], \qquad D_t^L(M) = \sum_{k=M}^{h} \mathbb{E}_t^h [\hat{r}_{t+k}^* - \hat{r}_{t+k}], \tag{I.3}$$

where $0 < M \leq h$.

J Robustness Checks

Planning Horizon h. First, we consider planning horizon $h \in \{2, 4, 40\}$ in the benchmark model with homogeneous agents. Subject to the two exogenous shocks, Figure J.8 illustrates the regression coefficients for both excess return predictability and the real exchange rate's response to short- and long-term real interest rate differentials with each planning horizon h. In summary, the attributes of the primary findings in Section 5 become stronger when planning horizons shorten and weaker as they lengthen, in comparison to the benchmark planning horizon $h = 8$.

Firm's Learning Gain $\gamma_{\tilde{v}}$. Next, we consider firm's learning behavior on its value function with various learning gain $\gamma_{\tilde{v}} \in \{0.1, 0.5, 0.99\}$. Figure J.9 illustrates the related regression coefficients based on each learning gain $\gamma_{\tilde{v}}$. Clearly, the outcomes for excess return predictability are not significantly impacted by variations in $\gamma_{\tilde{v}}$. Moreover, the results of the breakdown of the forecast horizon invariance are found to be unaffected by $\gamma_{\tilde{v}}$.

Heterogeneous Planning Horizons across Agents. Lastly, we extend the model to heterogeneous planning horizons across agents and examine its consequences for the aggregate variables. We assume that fraction $\omega_h \in (0,1)$ of households and fraction $\tilde{\omega}_h \in (0,1)$ of firms have planning horizon h. Following Woodford (2019), we further assume that those agents with horizon h make their decisions by assuming that all other agents have the same planning horizon of h. After the planning exercises in each period, the estimated value functions across households are aggregated by

$$\nu_t^{est} = \sum_h \omega_h \left[\hat{c}_t^h + \sigma^{-1}\pi_t^h - (1-\beta)\hat{b}_t^{*,h}\right], \tag{J.1}$$

$$\nu_t^{*,est} = \sum_h \omega_h \left[\nu_t^{est} - \sigma^{-1}(\hat{q}_t^h + \pi_t^h)\right], \tag{J.2}$$

and the estimated value functions across firms are aggregated by

$$\tilde{\nu}_t^{est} = \sum_h \tilde{\omega}_h (1-\theta)^{-1}\pi_{H,t}^h. \tag{J.3}$$

We consider a special case in which fractions ω_h and $\tilde{\omega}_h$ follow the same geometric distributions:

$$\omega_h = \tilde{\omega}_h = \rho^h(1-\rho)$$

for any $h \geq 0$, where $\rho \in (0,1)$. Then, the aggregate variables become the weighted average among the population. The current endogenous variables are aggregated as follows:

$$\hat{c}_t = \sum_h \omega_h \hat{c}_t^h, \quad \hat{y}_t = \sum_h \omega_h \hat{y}_t^h, \quad \hat{\imath}_t = \sum_h \omega_h \hat{\imath}_t^h, \quad \hat{r}_t^* = \sum_h \omega_h \hat{r}_t^{*,h}, \quad \pi_{H,t} = \sum_h \omega_h \pi_t^h,$$
$$\pi_t = \sum_h \omega_h \pi_t^h, \quad \hat{q}_t = \sum_h \omega_h \hat{q}_t^h, \quad \hat{s}_t = \sum_h \omega_h \hat{s}_t^h, \quad \hat{\varepsilon}_t = \sum_h \omega_h \hat{\varepsilon}_t^h, \quad \hat{b}_{t+1}^* = \sum_h \omega_h \hat{b}_{t+1}^h. \tag{J.4}$$

The k-period ahead expectations for CPI inflation are aggregated as follows:

$$\begin{aligned}
\mathbb{E}_t^{agg}\pi_{t+k} &\equiv \sum_h w_h \mathbb{E}_t^h \pi_{t+k} = \sum_h w_h \mathbb{E}_t \pi_{t+k|t}^{h-k} \\
&= \mathbb{E}_t\left[\rho^k(1-\rho)\pi_{t+k|t}^0 + \rho^{k+1}(1-\rho)\pi_{t+k|t}^1 + \rho^{k+2}(1-\rho)\pi_{t+k|t}^2 + \cdots\right] \\
&= \rho^k \mathbb{E}_t\left[\pi_{t+k|t}\right].
\end{aligned}$$

Similarly, we obtain

$$\mathbb{E}_t^{agg}\hat{q}_{t+k} = \rho^k\mathbb{E}_t\left[\hat{q}_{t+k|t}\right], \qquad \mathbb{E}_t^{agg}\hat{\imath}_{t+k} = \rho^k\mathbb{E}_t\left[\hat{\imath}_{t+k|t}\right], \qquad \mathbb{E}_t^{agg}r_{t+k}^* = \rho^k\mathbb{E}_t\left[r_{t+k|t}^*\right].$$

Thus, we have

$$\mathbb{E}_t\left[\pi_{t+k|t}\right] = \rho^{-k}\mathbb{E}_t^{agg}\pi_{t+k}, \qquad \mathbb{E}_t\left[\hat{q}_{t+k|t}\right] = \rho^{-k}\mathbb{E}_t^{agg}\hat{q}_{t+k},$$
$$\mathbb{E}_t\left[\hat{\imath}_{t+k|t}\right] = \rho^{-k}\mathbb{E}_t^{agg}\hat{\imath}_{t+k}, \qquad \mathbb{E}_t\left[\hat{r}^*_{t+k|t}\right] = \rho^{-k}\mathbb{E}_t^{agg}\hat{r}^*_{t+k},$$

and the k-step ahead domestic real interest rate becomes

$$\mathbb{E}_t[\hat{r}_{t+k|t}] = \mathbb{E}_t\left[\hat{\imath}_{t+k|t}\right] - \mathbb{E}_t\left[\pi_{t+k+1|t}\right]. \tag{J.5}$$

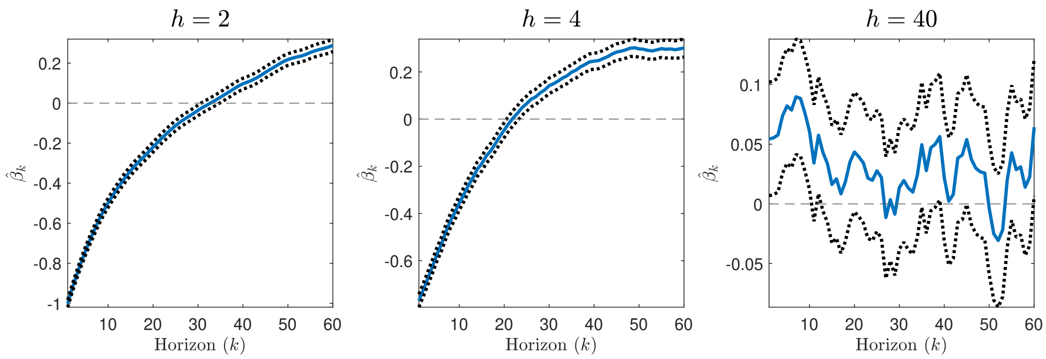
We can then construct $D_t^S(M) = \sum_{k=0}^{M-1}\mathbb{E}_t[\hat{r}^*_{t+k|t} - \hat{r}_{t+k|t}]$ and $D_t^L(M) = \sum_{k=M}^{\infty}\mathbb{E}_t[\hat{r}^*_{t+k|t} - \hat{r}_{t+k|t}]$.

For the numerical exercise, we set $\rho = 8/9$ so that the average planning horizon in the economy is $\rho/(1-\rho) = 8$ quarters, the same as the planning horizon of the homogeneous agents in Section 4.[26] Figure J.10 illustrates the related regression coefficients; it shows that the coefficients for excess return predictability across various time horizons display a pattern similar to our benchmark results, as shown in Section 5. Additionally, there is a breakdown of the forecast horizon invariance, as shown in the middle and right panels. One observation is that the long-term reaction coefficient of the real exchange rate, $\hat{\gamma}_S$, exhibits quantitatively small variations across different thresholds M. This is because the long-term expected real interest rate differential $D_t^L(M)$ also shows relatively small variations across these thresholds, due to the aggregation effect of the value functions across heterogeneous planning horizons.

[26] For numerical aggregation, we consider planning horizons ranging from 0 to 200. Also, we nullify the firms' learning in their value function.

Figure J.8: Robustness Check with Alternative Planning Horizons $h \in \{2, 4, 40\}$

(a) Excess Return Predictability across Time Horizons



(b) Reaction Coefficients of Real Exchange Rate to the Term Structure of the Forecasted Interest Rate Differentials

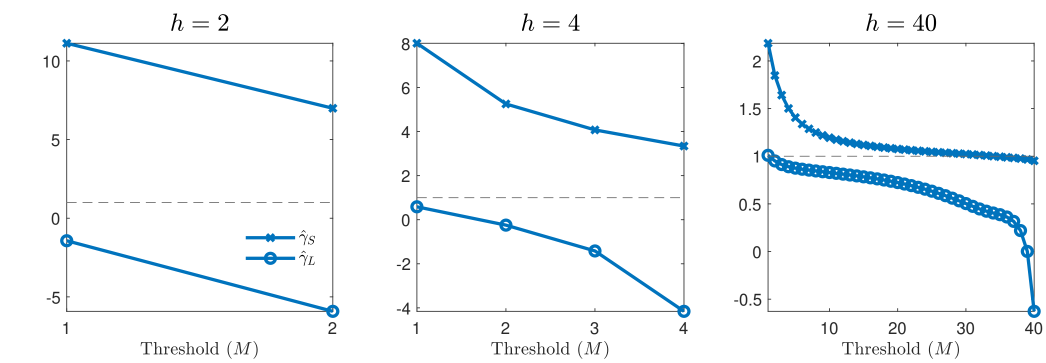
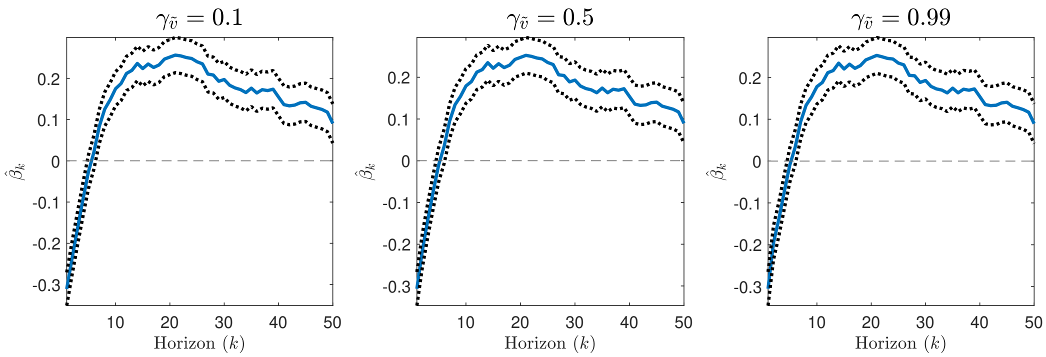


Figure J.9: Robustness Check with Different Firm's Learning Gains $\gamma_{\tilde{v}} \in \{0.1, 0.5, 0.99\}$

(a) Excess Return Predictability across Time Horizons



(b) Reaction Coefficients of Real Exchange Rate to the Term Structure of the Forecasted Interest Rate Differentials

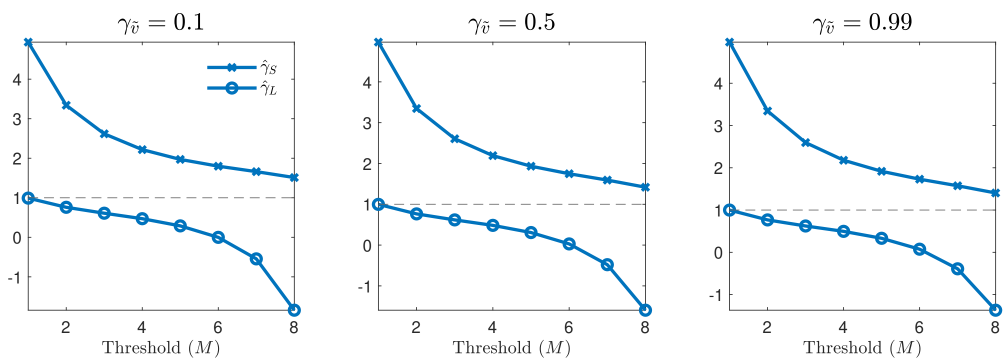
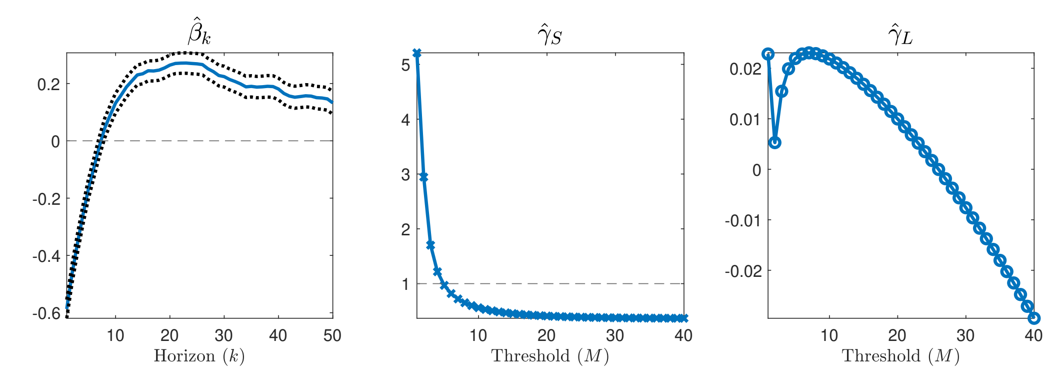


Figure J.10: Heterogeneous Planning Horizon across Agents: Excess Return Predictability and Reactions Coefficients of Real Exchange Rate to the Term Structure of the Forecast



Notes: The average planning horizon of the population is set to be eight quarters ($\rho = 8/9$).