Reducing Carbon using Regulatory and Financial Market Tools *

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Abstract

We study the conditions under which debt securities that make the cost of debt contingent on the issuer's carbon emissions, similar to sustainability-linked loans and bonds, can be equivalent to a carbon tax. The paper proposes a model in which standard and environmentally-oriented agents can adopt polluting and nonpolluting technologies, with the latter being less profitable than the former. A carbon tax can correct the laissez-faire economy in which the polluting technology is adopted by standard agents, but requires sufficient political support. Carbon-contingent securities provide an alternative price incentive for standard agents to adopt the nonpolluting technology, but require sufficient funds to fully substitute the regulatory tool. Absent political support for the tax, carbon-contingent securities can only improve welfare, but the same is not true when some support for a carbon tax exists. Understanding the conditions under which the regulatory and capital market tools are substitutes or complements within one economy is an important steppingstone in thinking about carbon pricing globally. It sheds light, for instance, on how developed economies can deploy finance to curb carbon emissions in developing economies where support for a carbon tax does not exist.

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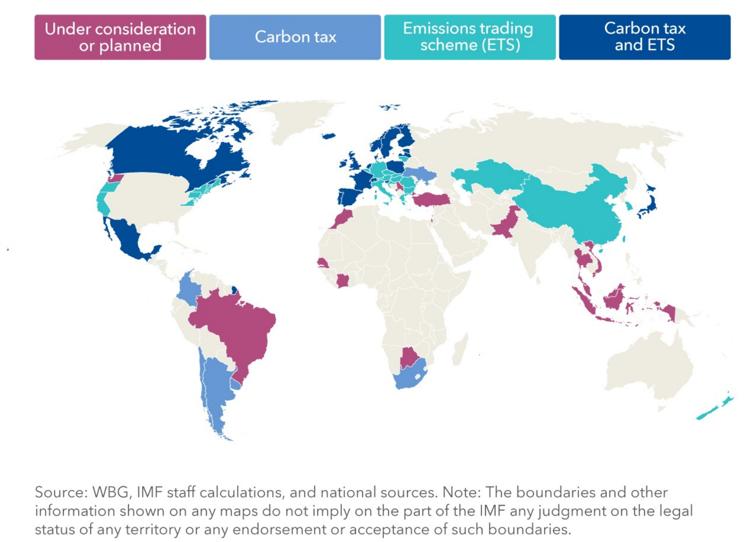
1 Introduction

There is widespread scientific consensus that the Earth's climate has undergone significant warming since the late 1800s, and human activities, particularly the release of greenhouse gases like carbon dioxide and methane, are the primary drivers of this warming. Consequently, the issue of reducing and pricing carbon dioxide emissions has risen on the agenda of policymakers and has been the subject of numerous debates. As illustrated in Figure 1, there is considerable heterogeneity across countries with respect to whether or not a carbon pricing regulation has been implemented and the form that it takes, with some countries adopting a carbon tax, others a cap-and-trade system, and a few others having adopted both.[1] There are many reasons behind this fragmented regulation. At the international level, there are complex considerations around what would constitute an equitable climate transition that takes into account the fact that the countries most exposed to climate damages are the ones that have contributed the least to global emissions and are also the ones least equipped with the resources to finance the climate transition.[2] At the domestic level, the implementation of carbon pricing schemes is subject to considerable political frictions, with skepticism coming both from the public and politicians alike.[3]

Figure 1. Carbon Pricing Regulation
The figure captures the current state of carbon pricing regulation worldwide as downloaded from the up-to-date carbon pricing dashboard developed by the World Bank Group. Source *https://carbonpricingdashboard.worldbank.org*, accessed November 2022.



Even when regulation has been implemented, the carbon prices implied by the adopted regulatory tools are largely below the consensus level needed to incentivize the achievement of the Paris Agreement goal to remain below the 1.5°C degree rise in global temperature. Furthermore, the investment estimates needed to achieve this goal are significant and range from \$5 trillion per year by 2030 (World Resources Institute, 2021) to \$6.9 trillion per year (OECD, 2018). Many developing countries such as India, argue that developed countries that have been responsible for large emissions during their industrialization over many

[1] A carbon tax (also known as Pigouvian tax) involves charging a tax on each unit of pollution. A cap-and-trade system involves capping the total quantity of emissions allowed, distributing rights to emitters within this total, and allowing them to trade the permits among themselves.

[2] A comprehensive discussion around these issues can be found in Nordhaus (2020).

[3] The lack of public support has been an impediment to achieving environmental targets through carbon pricing regulation. Prominent examples are the Washington State's two failed carbon tax referendums from 2016 and 2018 (see Anderson et al. (2019)), and the suspension of fuel taxes in France (which were planned to be raised as part of the government's decarbonization plan) in response to the Yellow Vest Movement (see Douenne and Fabre (2022)).

years should be responsible for bearing most of the costs of the transition. Indeed, in 2009 developed countries committed to jointly mobilize \$100 billion a year by 2020 to help developing countries adapt to climate change, but these funds have been slow to come by and as of 2020 were still about \$17 billion short.[4]

The amount of financial resources that needs to be mobilized in order to support the climate transition is significant, and well beyond the scope of what governments can provide. Financial markets are now playing an increasingly important role by providing a platform through which investors can channel funds towards projects with environmental, social and sustainability-related outcomes. A prominent example is the market for sustainable debt securities, which has grown exponentially in recent years from a total issuance volume of \$109 billion pre-2012, to \$5,910 billion as of 2022 (see Figures 2 and 3 below).[5] Of these, \$1,611 billion consist of sustainability-linked debt, a new class of instruments introduced in 2018 which have an interest rate that is contingent on the issuer's performance against a sustainability-related target, which in most cases is represented by greenhouse gas emissions.[6].

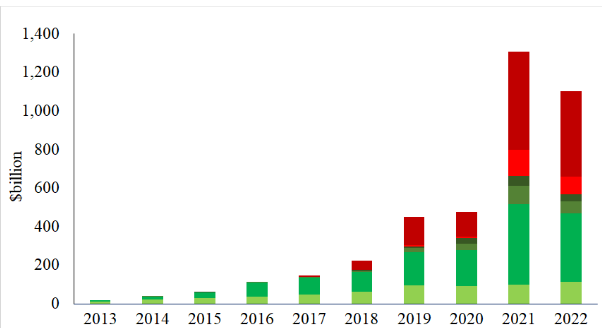


Figure 2. Sustainable Debt Issuance per Year

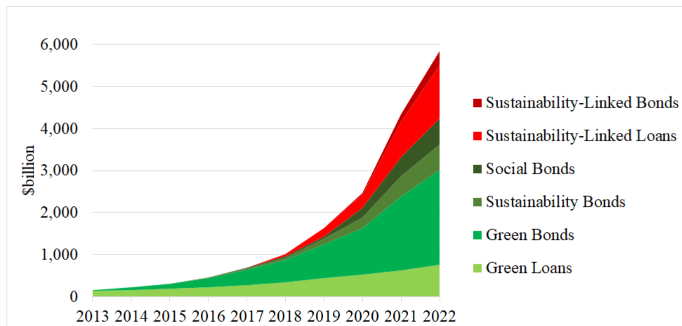


Figure 3. Cumulative Sustainable Debt Issuance

Importantly, the capital mobilized through sustainability-linked debt is orders of magnitude larger than the \$100 billion pledge to developing countries, and this form of carbon-contingent financing has a wider reach, being implemented in countries where support for regulation has been insufficient (see Figure 4 below). By combining the global nature of capital markets with the carbon-pricing incentives of regulation, these securities have the potential to be an important tool for reducing carbon.

Motivated by this stylized evidence, in this paper we study the interaction between regulatory and financial market tools for pricing carbon within one economy, focusing on the role of political support for environmental regulation. The regulatory tool we focus on is a carbon tax that can only be implemented by the regulator if at least half of the voters are better off with the tax. In other words, the implementation of the tax is subject to a median-voter constraint. The financial market tool is represented by carbon-contingent securities which have a payoff that increases (decreases) if the issuer's carbon emissions are in excess (deficit) of a predetermined target, in a manner that resembles the one observed in sustainability-linked debt instruments. The focus on a single economy is a necessary first step to study how regulation and financial markets jointly shape incentives to reduce emissions while abstracting from

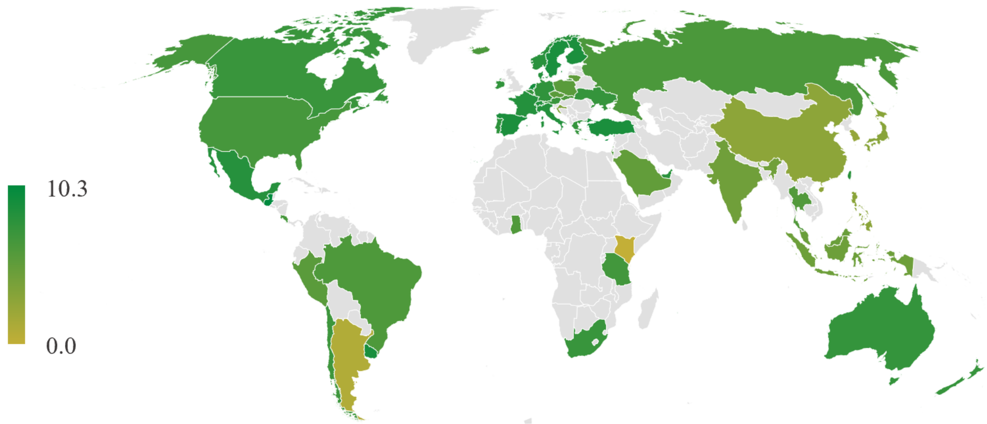
[4]Details can be found in the OECD report `https://www.oecd.org/climate-change/finance-usd-100-billion-goal/`

[5]This market comprises project-based securities such as green, social and sustainable bonds and loans, as well as outcome-based securities such as sustainability-linked loans and bonds which make the cost of debt contingent on outcomes such as the issuers' reduction in carbon emissions. Barbalau and Zeni (2022) provide an overview of these markets and analyse firms' incentives to issuing each class of securities.

[6]For example, Uruguay issued a \$1.5bn sustainability-linked bond in 2022, with an initial coupon of %5.75 which can increase by 15bps if Uruguay fails to reduce a specific carbon emission metric by more than 50%, and it can decrease by 15bps if the reduction is higher than 52%. Market-wide statistics and descriptions of the targets underlying sustainability-linked debt instruments can be found in, for instance, Barbalau and Zeni (2022) and Kölbel and Lambillon (2022)

Figure 4. Percentage of Sustainability-Linked Debt Issuance

This figure shows the geographical distribution of sustainability-linked debt (which includes corporate and government issued sustainability-linked loans and bonds) relative to all debt (corporate and government issued loans and bonds) issued since 2013. Data are collected from Bloomberg. A more intense shade of green indicates a higher proportion of sustainability-linked debt relative to total debt.



cross-country considerations such as international agreements and carbon leakage effects.

We start by proposing a baseline model which features standard and environmentally-oriented agents that are risk-neutral and behave atomistically with respect to global emissions. Environmental agents internalize the negative impact of the emissions associated with their actions. Specifically, they derive non-pecuniary utility (dis-utility) from taking actions that reduce (increase) emissions, and this is irrespective of the impact on aggregate emissions.[7] Each agent has endowments which she can either invest in polluting and non-polluting production technologies, with the latter being less profitable than the former, or lend to other agents through carbon-contingent debt securities. There is a regulator that internalizes aggregate emissions and aims to set a carbon tax to maximize social welfare, but is subject to a median voter constraint. To focus on real effects in our welfare analysis, we treat the agents' non-pecuniary utility as a purely decisional utility, and do not include it in our notion of welfare.[8]

The model predicts that in a laissez-faire economy without taxes nor financial markets, standard agents will invest in the polluting technology and environmental ones in the green, non-polluting technology. If exposure to climate shocks is higher than the profitability loss brought about by investing in the less profitable green technology, the regulator will find it optimal to implement a carbon tax and by doing so correct the laissez-faire economy, improve welfare and reduce emissions. However, the regulator can only implement a tax if it is supported by the median voter, where agents support a given tax if it increases their utility relative to the counterfactual without the tax. Furthermore, environmental agents derive utility from supporting a carbon tax, in line with the assumption that they internalize the emissions associated with their actions. We use group-based ethical voter models (Feddersen and Sandroni (2006))

[7] These agents are similar to the so-called warm-glow agents in Andreoni (1990), the warm-glow agents in Huang and Kopytov (2022) and Goldstein et al. (2022), the narrow-mandate investors in Oehmke and Opp (2022b), the values-aligned investors in Green and Roth (2021) and Landier and Lovo (2020). Empirical evidence on such preferences for sustainable investing is provided by Riedl and Smeets (2017), Bonnefon et al. (2022), Humphrey et al. (2021), Heeb et al. (2023a). We discuss green preferences in more detail in Section 3.

[8] This is in line with Broccardo et al. (2022) and Inderst and Opp (2022). We discuss this choice and its implications in greater detail in Section 3.

to quantify the emissions associated with the voting action.

Carbon-contingent financing arises only in the absence of a carbon tax, with environmental agents acting as lenders and standard agents as borrowers. Carbon-contingent securities offer an alternative carbon-pricing incentive for standard agents to switch to the green technology, but the extent to which these securities can fully substitute regulation depends on the funds of environmental agents, who are effectively financing the transition. When the funds deployed through carbon-contingent markets are sufficiently large, the financial market solution can fully substitute regulation and achieves the same level of welfare and emissions reduction as the carbon tax. Pricing emissions through financial markets also creates welfare gains when environmental funds are small, provided that the political constraint is binding. Therefore, when there is no support for a carbon tax, the introduction of financial markets circumvents the political constraint and is welfare improving.

In the baseline model, the introduction of financial markets for pricing carbon does not change voting outcomes, with environmental agents always supporting the tax and standard agents always voting against it. However, on the intensive margin, support for the tax decreases when markets exist. Standard agents are less willing to vote for the tax when the market solution for pricing carbon exists because the latter provides them with a platform though which they can monetize environmental agents' preferences and thus get financially rewarded for reducing emissions. For environmental agents, who act as lenders in carbon contingent-markets and are effectively subsidizing the transition of standard agents, tax support is lower because they get to reform others through markets.[9]

To understand the intensive margin interaction between market-based and regulatory tools for pricing carbon, we extend the model to a continuum of production technologies with emissions that can be reduced at a convex cost and a continuum of agents with heterogeneous environmental preferences. In the continuous model, the regulator can implement a revenue-neutral tax which involves redistributing the tax revenues equally across agents.[10] Again, the regulator can only implement a tax that is at most as high as that supported by the median voter. The model predicts that the issuance of carbon-contingent securities, the market-implied price of carbon, and the resulting emission abatement generated by financial markets are a decreasing function of the tax, suggesting again that the two tools can be used as substitutes. In line with the linear model, we show that the presence of financial markets reduces the median voter's willingness to support any given tax proposed by the regulator. Furthermore, for admissible ranges of model parameters, the model predicts that the median voter is a borrower, so the counterproductive effect of introducing financial markets in terms of crowding-out political support for the tax is driven by two pecuniary motives. First, in the presence of the tax, the median voter stands to profit less from issuing the carbon-contingent security. Second, relative to the economy without financial markets, the voter receives a lower tax redistribution because the contribution of financial markets to reduce emissions reduces the tax proceeds collected by the regulator.

When solving for the regulator's optimal tax rate in the presence of financial markets, we find that the unconstrained optimal carbon tax in the presence of financial markets is strictly lower than the optimal tax if financial markets did not exist. However, as discussed, the median voter's support for any given

[9]Note that this does not mean they prefer markets to the tax, but since the cost of reforming others is lower of equal than the benefit they get from doing so, due to their green preferences, their support for the tax will be weaker.

[10]The choice of revenue-neutral taxation is to preserve consistency with the linear model framework, in which the tax simply serves as an incentive to modify standard agents' investment choices from polluting to non-polluting technologies, and is never actually collected. Furthermore, this assumption is in lien with large scale revenue-neutral carbon taxes implemented in countries such as the UK and Canada.

tax is also reduced. So although financial markets reduce the need for regulatory intervention, they also increase the stringency of the political constraint, with the implication that they may no longer be able to efficiently circumvent the political constraint but might actually increase the stringency of the political constraint. The implications in terms of welfare and emissions reduction depend on which force dominates.

Relative to a counterfactual economy without financial markets which is unconstrained and can implement the optimal tax, we find that the introduction of financial markets typically results in a lower welfare and an inefficiently higher or lower level of emissions reduction. Similar levels of welfare and emissions reduction can only be achieved if the decrease in support that is brought about by markets is not high enough to make the implementation of the (optimally lower) tax unfeasible, i.e. if the political constraint does not bind. But otherwise, if markets make the implementation of the tax redundant (negative optimal tax) or unfeasible (negative median voter constraint), such that only a tax equal to zero can be implemented, then the introduction of financial markets is detrimental.

Relative to a politically constrained counterfactual economy without financial markets which cannot implement the optimal tax, the introduction of financial markets typically increases welfare and reduces emissions. Even when markets decrease political support to the point of making the implementation of the tax unfeasible, i.e. negative median voter constraint, welfare can still be improved if the carbon abatement achieved through markets is higher than that achieved through the constrained tax implemented in the counterfactual economy. However, when the economy with financial markets implements the politically constrained tax, i.e. median voter constraint binds, it will lead to a lower welfare and higher emissions relative to the counterfactual economy without markets, which implements a (lower) constrained tax.

The extended model is able to generate the observed co-existence of a carbon tax policy and carbon contingent finance, thereby rationalizing why countries with environmentally-oriented voters have both high carbon taxes and active sustainable finance markets. However, our model suggests that this co-existence is likely to be inefficient if the reduction in political support that markets bring about is higher than the carbon abatement they enable and the consequent reduction in the need for regulatory intervention. An important implication of our model is that introducing financial markets for pricing carbon in economies where some support for regulation exists can be dangerous[11], and carbon-contingent funds are best directed to markets without carbon taxes, that is, markets where support for taxes and voters' environmental concerns are low. Not only there is more abatement potential in such economies which cannot implement taxes, but the risk that markets would crowd out regulation is also lower.

The rest of the paper is organized as follows: in Section 2 we provide a review of the related literature, underlying the original contribution of our work; in Section 3 we present and solve the baseline model; in Section 4 we present and solve the extended model; in Section 5 we conclude.

2 Literature

This paper contributes to understanding how security design can enable financial markets to complement government regulation in reducing negative externalities. First, it contributes to the literature studying the interaction between financial markets and corporate behavior, which has largely focused on understanding the conditions under which and channels through which investments by agents with pro-social and/or pro-environmental preferences can have an impact by reforming the firms. The channel most studied is the cost of capital channel. Notable papers in this literature stream include Heinkel et al. (2001) who study

[11] This is in line with debates regarding markets being a potentially dangerous placebo (see Heeb et al. (2023b)).

how exclusionary ethical investing impacts corporate behavior, Pastor et al. (2020) who study how shifts in customers' tastes for green products and investors' tastes for green holdings produce positive social impact, Edmans et al. (2022) who study the effectiveness of exclusion versus titling investment strategies in terms of reducing externalities, Oehmke and Opp (2022b) who study the conditions for impact in a context in which investors can relax firms' financial constraints for responsible production, and Landier and Lovo (2020) who study how ESG funds should invest to maximize social welfare in a setup in which financing markets are subject to a search friction. Chowdhry et al. (2019) also study the conditions under which impact investments improve social outcomes, but they focus on the role of contracting and security design when firms that cannot commit to social goals are jointly financed by profit and socially-motivated investors. Goldstein et al. (2022) show how the presence of investors with heterogeneous ESG preferences influences information aggregation by prices and affects firms' cost of capital, which may indirectly affect their production. Finally, Hart and Zingales (2017) and Broccardo et al. (2022) emphasize a governance rather than a cost of capital channel. We are also effectively studying a cost of capital channel, but our channel is contractual and embedded in security design, and is not driven by the equilibrium interaction between agents and the ensuing price or governance pressure exerted, often through equity investments. We abstract from corporate governance and a firm's decision to reform by taking the technologies as given and only look at which will be financed in equilibrium. Our focus is instead on the interaction between security design and regulation, which is absent in all the works cited above.

The literature stream that our paper is most related to is the one at the intersection of finance and corporate behavior, but which also brings regulation into the picture. Heider and Inderst (2021) examine optimal environmental policy through emission caps and pricing when firms need costly external financing, Döttling and Rola-Janicka (2022) study environmental and financial regulation in a setup with financially constrained firms and endogenous climate-related transition and physical risks, and Oehmke and Opp (2022a) study the role of green capital requirements as a tool to incentivize bank lending to green firms when taxes on emissions are not available. Hong et al. (2021) study the welfare implications of investment mandates which involve restricting a fixed fraction of the representative investor's portfolio to hold firms that meet sustainability guidelines. Inderst and Opp (2022) study the interaction between financial regulation taking the form of a taxonomy for sustainable investment products, and traditional tools for environmental regulation such as taxes on externalities or production standards. Biais and Landier (2022) study the complementarity between firms, which can invest in green technologies, and the government, which can impose emission caps but has limited commitment power. Ramadorai and Zeni (2021) find that firms' abatement actions depend greatly on their beliefs about climate regulation, and that both informational frictions and reputational concerns can amplify responses to climate regulation, increasing its effectiveness. Huang and Kopytov (2022) show that in the presence of socially responsible investors, pollution can increase with regulation stringency because regulation reshapes firms' shareholders composition and makes polluting firms' shareholders less averse to holding polluting shares. While most of the literature has focused on market responses to (the anticipation) of regulation, our focus is on how markets affect the implementation of regulation, placing emphasis on political constraints and voting. We are, to the best of our knowledge, the first ones to note that a specific security design can substitute regulation. In doing so we build on the work of Barbalau and Zeni (2022), who study firms' incentives to issue securities that make the cost of debt contingent on sustainability-related outcomes such as carbon emissions. We show that a carbon-contingent security design can fully substitute a carbon tax for which there is insufficient political support if the capital deployed through such markets is sufficiently high.

While our paper delivers the insight that financial market solution for pricing carbon can substitute regulation when political support is missing, we also find that it weakens support for regulation and, under certain conditions, can lead to less efficient outcomes. Thus, our paper also relates to the literature showing that socially responsible investments can have counterproductive implications. Green and Roth (2021) show that ESG investing strategies that focus on the social value of the companies included in their portfolio, with no regard for the implications of these investments on total welfare, allocate their capital inefficiently from the perspective of generating impact and financial returns. Gupta et al. (2022) highlight that socially responsible investors who value acquiring firms with high negative production externalities that they can reform, create trading gains that can actually cause a potential delay in reform. Bisceglia et al. (2022) point to the failure of socially responsible investors to internalize the impact of their investment on product market competition, resulting in concentration of green capital to few firms and increased market power. Hartzmark and Shue (2023) provide empirical evidence that common strategies which direct capital away from brown firms and towards green firms may be counterproductive by making brown firms browner without making green firms greener.

The environmental agents in our model are effectively implementing a Coasian solution by subsidizing the technology shift of agents that would otherwise pollute. Adrian et al. (2022) estimate the gains that could be realized from phasing out coal, and make a case for a Coasean bargain whereby we would be better off by paying the polluter to stop polluting. They discuss the role of international agreements that feature compensation conditional on phasing out coal, as well as blended finance which would leverage public funds to de-risk investments in renewable energy and catalyze investments from capital markets. In contrast to this paper, we propose a model that features a purely decentralized market solution that does not rely on international agreements or collaborations, which are subject to political frictions. We show that if agents with environmental preferences are given an option to personally subsidize the decarbonization efforts of other agents through private financial solutions, they will optimally do so and this has the advantage of circumventing the political constraints faced by a regulator. Our paper is therefore also related to the literature on the private provision of public goods (Besley and Ghatak, 2007), which studies the incentives and advantages that private actors might have in dealing with public goods provision, and it also contributes to the political economy literature on private versus public regulation (Maxwell et al. (2000), Egorov and Harstad (2017)). We demonstrate the emergence of private tool for pricing emissions which can be equivalent to and substitute a regulatory tool. Importantly, we outline the conditions under which this private solution presents a comparative advantage relative to the public one by overcoming political frictions that prevent the implementation of regulation, as well those under which its introduction may backfire by weakening support for public regulation. Finally, we also contribute to the political economy literature on the drivers of individual support for climate policies (Drews and Van den Bergh (2016), Dechezleprêtre et al. (2022),Heeb et al. (2023b)) by showing that, in line with empirical evidence, support for regulation depends importantly on self-interest considerations preferences and tax redistributions.

3 Simple Model

We start with a simple linear model featuring two technologies, two time periods $t = 0, 1$, two types of agents (standard and environmentally-oriented), and a regulator which sets a carbon tax to maximize social welfare subject to a median voter constraint.

There are two technologies, which take as input capital I to produce output y and carbon emissions e. The polluting technology, indexed by π, yields output y_π and emissions e_π given by

$$y_\pi = \pi I \qquad \text{and} \qquad e_\pi = I,$$

where $\pi > 1$ is a production parameter. The non-polluting or green technology, indexed by g, yields output y_g and zero emissions

$$y_g = gI \qquad \text{and} \qquad e_g = 0,$$

with g a green production parameter which satisfies $1 < g < \pi$.

There are two types of risk-neutral agents indexed by $i = 1, 2$, namely:

(i) standard agents, indexed by $i = 1$, who form a proportion θ of the population, are each endowed with capital h_1, and have utility that is increasing in consumption

$$U_1 = C_1, \tag{1}$$

where consumption is given by the output of their investment.

(ii) environmentally-oriented or green agents, indexed by $i = 2$, who form a proportion $1 - \theta$ of the population, have capital h_2 and utility

$$U_2 = C_2 - \eta e_2, \tag{2}$$

where e_2 are the emissions associated with their actions, and η is a green preference parameter which is assumed to satisfy $\eta > \pi - g$. This preference specification captures the idea that agents dislike the emissions associated with their actions and which they feel responsible for.

While there is no consensus on the terminology or modelling of environmental preferences, a broad distinction can be made between warm-glow agents that derive personal (dis-)utility from their actions, and consequentialists who internalize the consequences or impact of their actions on others. This goes back to Andreoni (1990) who distinguishes between acts that enter utility as private goods (warm-glow altruism) or public goods (pure altruism). The literature has thus labelled as consequentialist those agents that take into account the impact of their actions on aggregate social welfare, which typically requires that they are big enough and are not atomistic (as in Broccardo et al. (2022), Gupta et al. (2022) or Oehmke and Opp (2022b)). On the other hand, warm-glow refers to agents that derive purely personal utility from their actions, which is what motivates atomistic agents to engage in prosocial behavior even if this is inconsequential for aggregate outcomes. However, leaving welfare considerations apart, a distinction can be made between actions that grant personal satisfaction independently of the outcome of those actions, and actions that grant personal satisfaction which is proportional to the actions' outcome. With this in mind, we can make a distinction between agents who: 1) refuse to take certain actions that do not satisfy given moral or ethical criteria e.g. refusal to hold dirty shares (such agents include the exclusionary investors in Heinkel et al. (2001) and Hong et al. (2021), the deontological agents in Dangl et al. (2023) and Broccardo et al. (2022)); 2) derive utility from taking actions that can be labelled as right or wrong independently of the outcome of those actions e.g. derive (dis-)utility from holding what can be categorized as green (brown) shares (such agents include the the warm-glow non-consequentialist agents in Inderst and Opp (2022), the non-consequentialist agents in Dangl et al. (2023), or the taste investor in baseline model of Pastor et al. (2020)); 3) derive personal utility that depends on the impact or consequences of their actions, irrespective of action labelling e.g. holding a brown rather than a green shares if doing so reduces externalitities (such agents include those in Edmans et al. (2022), the narrow-mandate investors in Oehmke and Opp (2022b), the values-aligned investors in Green and Roth (2021) and Landier and Lovo (2020), the warm-glow agents in Huang and Kopytov (2022), Goldstein et al. (2022), Hart and Zingales (2017) and Broccardo et al. (2022)); 4) derive utility from taking actions that have an impact on others, or in other words, actions that have consequences for aggregate outcomes e.g. reducing aggregate negative externalities (such agents include the broad-mandate investors in Oehmke and Opp (2022b); Gupta et al. (2022); the impact-aligned investors in Green and Roth (2021); impact-aligned agents in Landier and Lovo

(2020); the consequentialist agents Broccardo et al. (2022) and Dangl et al. (2023)).

The environmental agents in our model can be best placed in third category listed above. Their preferences can be labelled as warm-glow preferences in the sense that they derive personal non-pecuniary (dis-)utility from the negative externalities associated with their actions. However, insofar as they correctly internalize the consequences of their actions they can also be thought of as narrow consequentialists in the sense that the negative externalities they internalize are evaluated relative to a counterfactual in which the action would not have been taken.[12]

The regulator maximizes utilitarian social welfare given by

$$W = \theta C_1 + (1 - \theta)C_2 - \lambda E. \tag{3}$$

where $E = \theta e_1 + (1-\theta)e_2$ denotes aggregate emissions given by a weighted sum of the emissions produced by standard and environmental agents, e_1 and e_2 respectively, and λ is a climate parameter which captures the economic damage associated with one additional unit of emissions in the atmosphere, also referred to as the social cost of carbon. Note that environmental agents' green preference parameter η does not enter the regulator's welfare function, which allows us to capture only real effects while excluding obvious, mechanical effects resulting from agents' green preferences.[13] This is akin to treating η as merely decisional utility, since it remains relatively unaffected by the aggregate outcomes that are actually achieved, and is in line with Broccardo et al. (2022) and Inderst and Opp (2022).

In Appendix A.1 we consider a specification in which agents too internalize exposure to global emissions, and their preferences are $U_1 = C_1 + \lambda E$ and $U_2 = C_2 + \eta e_2 + \lambda E$. This can be conceptualized as capturing a global climate shocks that affects them irrespective of their preferences, such as a natural disaster or the negative effects of pollution on health which affect the entire population. Under this specification our standard and environmental agents are similar to the pure altruists and impure altruists, specifically, in Andreoni (1990) or the warm-glow atomistic agents in Huang and Kopytov (2022) who care about the externalities associated with their actions beyond their contribution to aggregate outcomes. The key implication of considering this preference variation is that agents now have a motive to vote for a carbon tax. The key results on equivalence and the weakening support for tax remain unchanged, but this variation delivers starker voting predictions in terms of the detrimental effect that the introduction of markets has for the support for regulation.

3.1 Laissez-Faire Benchmark

In the decentralized economy, agents choose to produce output using the polluting or non-polluting technologies. Denote the capital investment in the polluting and non-polluting technology by I_π and I_g, respectively, and denote agent i's green preference using η_i, which for the standard agent $i = 1$ takes the value $\eta_1 = 0$ and for the environmental agent $i = 2$ takes the value $\eta_2 = \eta$. Recall that emissions are only produced by the investment in the polluting technology, that is, $e_\pi = I_\pi$ and $e_g = 0$. Thus, agent i's problem of allocating its endowment to the polluting and the green technology, is

$$U_i^* = \max_{I_\pi, I_g} \; \pi I_\pi + g I_g - \eta_i I_\pi \;\text{ such that }\; I_\pi + I_g \leq h_i. \tag{4}$$

[12]In line with the discussion in Hart and Zingales (2017), who note that consequentialism is defined as "the doctrine that the morality of an action is to be judged solely by its consequences".

[13]Including the preference parameter η in the regulator's welfare function is inconsequential for setting the tax and has the main implications that financial markets alone can achieve a higher welfare than the carbon tax even when the latter is not subject to political constraints. We deal with this case in Appendix A.2.

Given that we assumed $\pi > g > 1$ and $\eta > \pi - g$, the standard agent $i = 1$ will invest all available capital in the polluting technology, $I_\pi^* = h_1$, whereas the environmental agent will invest all capital in the non-polluting technology, $I_g^* = h_2$.

Taking account of such choices, aggregate emissions are $E^* = \theta e_1^* + (1-\theta)e_2^* = \theta h_1$, the utility of the standard agent is $U_1^* = \pi h_1$, that of the environmental agent is $U_2^* = g h_2$ and social welfare is

$$W^* = \theta \pi h_1 + (1-\theta) g h_2 - \lambda \theta h_1. \tag{5}$$

3.2 Carbon Tax

Suppose that the regulator can alter the laissez-faire economy by imposing a tax τ on the emissions produced by the polluting technology π, and by doing so alter the investment decisions of the agents. Denoting $E^\tau = \theta e_1^\tau + (1-\theta)e_2^\tau$ the sum of the optimal emissions of the standard and environmental agents given the tax, the utilitarian social welfare is

$$W^\tau = \theta C_1^\tau + (1-\theta) C_2^\tau - \lambda E^\tau \tag{6}$$

with C_1^τ and C_2^τ the consumption of the standard and environmental agents, respectively, evaluated at their optimal investment choices given the tax τ.

It is straightforward to show that any tax $\tau \geq 0$ will not change the actions of the environmental agent relative to the benchmark laissez-faire economy in which the green technology is adopted. It is therefore sufficient to focus on the standard agent's problem, which in the presence of the tax becomes

$$U_1^\tau = \max_{I_\pi, I_g} \; g I_g + (\pi - \tau) I_\pi \quad \text{such that} \quad I_\pi + I_g = h_1. \tag{7}$$

Optimal investment choices given the tax τ are

$$\begin{array}{llll} I_g^\tau = h_1 & \text{and} & I_\pi^\tau = 0 & \text{if} \quad \tau \geq \pi - g \\ I_g^\tau = 0 & \text{and} & I_\pi^\tau = h_1 & \text{otherwise,} \end{array} \tag{8}$$

and the emissions associated with the standard agent's choices are $e_1^\tau = 0$ if $\tau \geq \pi - g$, and $e_1^\tau = h_1$ otherwise. Substituting agents' optimal choices into the utilitarian social welfare in (6), we have

$$W = \begin{cases} W^\tau = \theta g h_1 + (1-\theta) g h_2 & \text{if} \quad \tau \geq \pi - g \\ W^* = \theta \pi h_1 + (1-\theta) g h_2 - \lambda \theta h_1 & \text{otherwise.} \end{cases} \tag{9}$$

Thus, implementing a carbon tax that is sufficiently high to incentivize the transition to the green technology, i.e. $\tau \geq \pi - g$, yields a higher welfare if $\lambda > \pi - g$. Therefore, the optimal tax is $\tau = \pi - g$ if $\lambda > \pi - g$, and $\tau = 0$ otherwise.

We focus henceforth on the case in which $\lambda > \pi - g$, such that the tax should be optimally implemented. In this case, aggregate emissions are zero, $E^\tau = 0 < E^*$, and utilitarian social welfare is higher relative to the laissez-faire economy

$$W^\tau = \theta g h_1 + (1-\theta) g h_2 > W^*. \tag{10}$$

3.3 Constrained Carbon Tax

As discussed in the introduction, political constraints are an important friction to the implementation of carbon pricing schemes, which requires sufficient political support from the population. The regulator is subject to a political constraint in the sense that it can only implement a tax τ that makes at least half of the population better off. Formally, the regulator must solve a constrained maximization problem of the type

$$\max_{\tau} \; W^{\tau} \;\; \text{such that } \tau \leq \tau_{0.5}, \tag{11}$$

which states that the optimal tax should be at most equal to that supported by the median voter, denoted as $\tau_{0.5}$. We now outline the voting problem and derive an explicit expression for $\tau_{0.5}$.

The voting problem. We consider sincere, non-strategic voters that vote for (against) the tax based on whether their utility with the tax, U_i^{τ}, is higher (lower) than their utility is the laissez-faire economy, U_i^*. We first consider the case of the standard agent, $i = 1$. The standard agent's utility in an economy with a carbon tax is $U_1^{\tau} = gh_1$ whereas in the laissez-faire economy, it is $U_1^* = \pi h_1$. Therefore, the standard agent is always going to vote against the tax, since $\pi > g$.

Let us now turn to the voting problem of the environmental agent, $i = 2$, who internalizes the emissions associated with her actions. To the extent that the voting action contributes to shaping emissions, the carbon emission implications of her vote should also be internalized. We capture this intuition by allowing the environmental agent to suffer disutility from voting against the tax or, alternatively stated, to derive utility from supporting the tax. However, allowing the environmental agent to internalize consequences of voting against the tax raises the important questions of what is the contribution of one's voting action to shaping emissions. This is related to a long standing (political science) literature on how voters internalize the decision to vote and the consequent voting turnout. Because voters are small, several theories aim to explain the paradox of voting (see Feddersen (2004) and Geys (2006) for excellent reviews). The theories that lend themselves best to our setup are "group-based" model of turnout, in which group members vote because they believe themselves to be ethically obliged to act in a manner that is consistent with the group's interest as in "ethical agent" models.[14]

We use the group-based ethical voter approach to allow environmental agents to determine and internalize the implications of their voting action. The environmental agents in our model are interpreted as ethical voters, who as the rule utilitarians in Harsanyi (1977) receive an additional payoff for acting according to an ethical rule with the property that if everyone acts according to this rule, social welfare will be maximized. Following Feddersen and Sandroni (2006), the ethical rule is that which produces the best outcome from that agent's perspective if all voters of the same type act according to that rule, while taking as given the behavior of agents with different preference types. In other words, from the point of view of these voters, the benefit from voting for a certain policy is the best outcome that is achieved if all other voters sharing that type vote for that policy. Applied to our model, and in line with group-based ethical voter models, the benefit from voting in favour is the best outcome (i.e., a reduction in aggregate emissions $\theta h_1 = E^* - E^{\tau}$) assuming all other voters of the same type (i.e., $1 - \theta$) do the same. Recalling that our green voter derives disutility η from the emissions associated with its actions, the emissions associated with an environmental agent's voting action can be defined as

$$\eta \frac{(E^* - E^{\tau})}{1 - \theta}. \tag{12}$$

[14] An alternative group-based theory of turnout involves agents voting because they are directly coordinated and rewarded by leaders as in "mobilization" models (Uhlaner, 1989).

When voting, the agent trades off her utility in the laissez-faire economy with her utility in an economy with a tax plus the (green) benefit of supporting the tax, which is given by the possible emission reduction associated with voting for the tax. Thus, she votes in favour of the tax if

$$U_2^\tau + \underbrace{\eta\frac{(E^* - E^\tau)}{1-\theta}}_{\text{voting benefit}} > U_2^* \tag{13}$$

Since $U_2^* = U_2^\tau = gh_2$, the environmental agent always supports the tax.

The median voter threshold, denoted $\tau_{0.5}$, which is the maximum tax that makes the median voter indifferent between supporting or not the carbon tax, is

$$\tau_{0.5} = \begin{cases} \pi - g & \text{if} \quad \theta < 0.5 \\ 0 & \text{if} \quad \theta > 0.5. \end{cases} \tag{14}$$

PROPOSITION 1. *Suppose that $\lambda > \pi - g$ such that the implementation of the carbon tax is desirable. Then if the median voter is an environmentally-oriented type $\theta < 0.5$, then the tax $\tau^c = \pi - g$ achieves the unconstrained optimum in* (10)*, aggregate emissions are zero $E^c = 0$ and welfare is*

$$W^c = \theta gh_1 + (1-\theta)gh_2 > W^*.$$

If the median voter is a standard type $\theta > 0.5$, then a tax cannot be implemented $\tau^c = 0$, aggregate emissions are $E^c = \theta h_1$ and welfare is

$$W^c = \theta\pi h_1 + (1-\theta)gh_2 - \lambda\theta h_1 = W^*.$$

We have thus derived the optimal carbon tax policy in the absence of financial markets. In the next subsection we will consider the regulator's problem when financial markets for pricing carbon exists, and agents can lend or borrow through carbon-contingent debt securities.

3.4 Carbon-Contingent Financing

So far, we have studied each agent's decisions assuming access to own capital only, represented by their endowments $h_i, i = 1, 2$. In what follows, we allow for external financing. Specifically, we introduce carbon-contingent debt securities similar to those observed in the market for sustainable finance and allow agents to borrow and lend by lending and borrowing through these securities. In this setup, we assume that agent i can issue, at time $t = 0$, a debt security with principal value d_i and time $t = 1$ payoff given by

$$\bar{r}d_i - \rho(\bar{e}_i - e_i^\rho), \tag{15}$$

where $\bar{r}$ is a fixed interest rate, $\bar{e}_i$ denotes agent i's benchmark emissions, set at time $t = 0$, which are essentially the counterfactual of what emissions would be in the absence of external financing, and e_i^ρ denotes emissions realized at time $t = 1$ with external financing through a carbon-contingent contract. This return specification is analogous to that underlying sustainability-linked loans and bonds, which feature a fixed interest rate component and a variable component that is contingent on the deviation of realized emissions from a benchmark that is agreed at contract issuance. If realized emissions are higher than the benchmark, i.e. $e_i^\rho > \bar{e}_i$, then the interest rate in (15) increases and vice versa.

Although the specification in (15) is in line with empirically observed security designs, exchanging the principal plays no role in incentivizing the reduction of carbon emissions and the key feature of this security is the contingent component. Therefore, to ease exposition we solve for the market equilibrium by setting the fixed component to zero and we only focus on the equilibrium pricing of the contingent component.[15]

The Borrower's Problem. A borrower entering contract (15) can get rewarded upon reducing emissions below the benchmark. Consider first the case of the standard agent $i = 1$. If there is no carbon tax $\tau = 0$, then benchmark emissions in (15) are $\bar{e}_1 = h_1$ and the standard agent can monetize a reduction in emissions relative to this benchmark, i.e. $e_1^\rho < \bar{e}_1$. The problem solved by the standard agent if it were to issue the carbon-contingent contract is

$$\mathcal{B}_1 = \max_{I_\pi, I_g} \; \pi I_\pi + g I_g + \rho(h_1 - I_\pi) \;\; \text{such that } I_g + I_\pi \leq h_1, \tag{16}$$

which yields solution $I_g = h_1$ if $\rho \geq \pi - g$, and $I_g = 0$ otherwise.[16] When determining whether or not to issue a carbon-contingent security, the agent compares the utility from borrowing with the utility in the laissez-faire economy $U_1^* = \pi h_1$. If the price of carbon implied by the carbon-contingent contract is sufficiently high to incentivize the transition to the green technology, i.e. $\rho \geq \pi - g$, the utility of the standard agent is

$$\mathcal{B}_1 = g h_1 + \rho h_1, \tag{17}$$

which is higher than the utility from using internal finance only, $U_1^* = \pi h_1$, with the implication that the agent is willing to borrow at this rate. However, if the contingent rate is not sufficiently high to incentivize switching to the green technology, i.e. $\rho < \pi - g$, the standard agents' utility from borrowing is $\mathcal{B}_1 = \pi h_1 = U_1^*$ and the agent is indifferent between issuing the security or not. Given that entering the contract will involve neither a technology shift nor a contingent cash flow, since $\bar{e}_1 - e_1^\rho = 0$, we will abstract from this trivial case when considering the carbon-contingent financing equilibrium.

If there is a carbon tax, then the standard agent's benchmark emissions are $\bar{e}_1 = 0$, and her utility from issuing the carbon-contingent security is

$$\mathcal{B}_1^\tau = \max_{I_\pi, I_g} \; \pi I_\pi + g I_g - \tau I_\pi - \rho I_\pi \;\; \text{such that } I_g + I_\pi \leq h_1, \tag{18}$$

which yields $I_g = h_1$ if $\tau + \rho \geq \pi - g$. This holds at the optimal tax $\tau = \pi - g$, with the implication that the agent will optimally invest in the green technology irrespective of the rate ρ. Therefore, the agent is indifferent between entering the contract or not and, as before, we abstract from considering this trivial case in equilibrium as it does not involve a technology shift or a contingent payoff.

Consider now the case of the environmentally-oriented agent $i = 2$, who internalizes the emissions associated with her actions. If through carbon-contingent financing the agent reduces her emissions she will register a utility gain proportional to this reduction and the green preference parameter, i.e. $\eta(\bar{e}_2 - e_2^\rho)$. However, independently of whether a tax exists or not, this green agent prefers to invest in the non-polluting technology so her benchmark emissions in the absence of external financing are $\bar{e}_2 = 0$ and she cannot physically reduce emissions further. Because of the technology constraint which does not allow producing negative emissions, the best she can do is enable other agents to reduce their emissions by participating in carbon-contingent financing markets. Therefore, the carbon-contingent financing tool allows

[15] This is without loss of generality. In Appendix A.3 we consider specification (15) derive the equilibrium fixed rate $\bar{r}$ and the contingent rate ρ .

[16] Here we implicitly assume that when indifferent on the intensive margin, that is, when $\rho = \pi - g$ the agent always prefers to implement the green technology. Relaxing the assumption does not change the equilibrium outcome.

the environmental agent to circumvent her technology constraint and contribute to reducing the emissions of other agents in the economy. Formally, we can see that upon issuing the carbon-contingent security, the environmental agent faces the following investment problem

$$\mathcal{B}_2^\tau = \max_{I_\pi, I_g} \ \pi I_\pi + g I_g - \tau I_\pi - \rho I_\pi - \eta I_\pi \quad \text{such that } I_g + I_\pi \leq h_2. \tag{19}$$

Investing in the green technology is optimal if $\tau + \rho + \eta \geq \pi - g$, which holds for any tax $\tau \geq 0$ or contingent rate $\rho \geq 0$ since $\eta > \pi - g$. The agent is thus indifferent between borrowing or not, as she is investing in the green technology anyway and has no emission reductions to monetize.

The Lender's Problem The lenders in carbon-contingent markets are financing the adoption of non-polluting technologies and receive a return that decreases with the carbon emissions reduction that is financed through the security. An individual lender i is responsible for a share q_i of the carbon emission reductions generated through carbon-contingent markets, which is determined in equilibrium. If the standard agent were to act as a lender, then her problem would be

$$\mathcal{L}_1 = \max \ (\pi - \tau) h_1 - \rho q_1. \tag{20}$$

Recalling the borrowing problem of the environmental and standard agents, the emission reduction is $q_1 = 0$ when the borrower (i) is an environmental agent, or (ii) is a standard agent subject to a carbon tax. In both cases, the lender realized the same utility with or without entering the contract. On the other hand, the utility from lending to another standard agent which can deliver a positive emission reduction $q_1 > 0$ (i.e., in absence of the tax) is strictly lower than that from not entering the contract for any $\rho > 0$. Thus, it is not optimal for the standard agent to be a lender or, in other words, to reward emission reductions.

Moving on to the problem faced by an environmental agent who acts like a lender in carbon-contingent markets we note that she receives a variable payoff, $-\rho q_2$, which depends on the carbon emissions reduction enabled through carbon-contingent markets in equilibrium. Given that the environmental agent generates no emissions by investing in her preferred green technology, she will only internalize the emissions associated with lending, which are captured by the equilibrium emissions associated with the carbon-contingent security, q_2. The environmental agent's utility from acting as a lender is

$$\mathcal{L}_2 = \max g h_2 - \rho q_2 + \eta q_2 \quad \text{such that} \quad g h_2 - \rho q_2 \geq 0, \tag{21}$$

where the first term is the return from investing in the green technology, the subsequent term is the cash flow associated with the contingent security, and the last term captures green preferences related to the emissions associated with the agent's lending action. The constraint captures the idea that although this class of investors may be willing to reward emission reductions, they will only do so up to the point that they deplete their wealth.

Note that if lending is associated with an emission reduction (increase) $q_2 > 0$ ($q_2 < 0$), the utility of the agent increases (decreases) via the green preference channel, but it decreases (increases) via the financial channel, i.e., the variable part of the contingent-security payoff. Therefore, if lending through the contingent security increases emissions (i.e., $q_2 < 0$), then environmental agents would require compensation at a minimum rate $\rho \geq \eta$. Since $\eta > \pi - g$, and recalling the standard agent's investment problem in (85), implies an equilibrium in which the standard agent borrows from the environmental one at the contingent rate $\rho \geq \pi - g$ and switches to the non-polluting technology. If, on the other hand, lending generates emissions reduction (i.e., $q_2 > 0$), then environmental agents will be willing to forgo $\rho \leq \eta$ for each unit of emissions reduction achieved, provided the financing constraint is verified. Since $\eta > \pi - g$, then an equilibrium in which the standard agent $i = 1$ is willing to borrow through the contingent security and implement the green technology can arise for any contingent rate $\rho \in [\pi - g, \eta]$.

Market clearing. In equilibrium, the total carbon emission reduction enabled through carbon-contingent lending by environmental agents must meet the emissions reductions supplied by the standard agents who are borrowing, that is

$$(1-\theta)q_2 = \theta(\bar{e}_1 - e_1^\rho). \tag{22}$$

This implies that each environmental agent's lending activity is responsible for an equilibrium emissions reduction $q_2 = \frac{\theta}{1-\theta}(\bar{e}_1 - e_1^\rho) = \frac{\theta}{1-\theta}h_1$.[17]

In such an equilibrium, the financing constraint is thus $gh_2 - \rho\frac{\theta}{1-\theta}h_1$ and is non-negative provided

$$\rho \leq \bar{\rho} = g\frac{h_2}{h_1}\frac{1-\theta}{\theta}. \tag{23}$$

It follows that if the endowments of environmental agents satisfy

$$h_2 \geq \frac{\pi - g}{g}\frac{\theta}{1-\theta}h_1, \tag{24}$$

then $\bar{\rho} \geq \pi - g$ and an equilibrium with a constraint-admissible rate $\rho \in [\pi - g, min(\bar{\rho}, \eta)]$ always exists. However, if lenders' endowments are such that the budget constraint in (24) is violated, then the carbon-contingent financing solution is not enough to incentivize the technology switch of the entire population of standard agents. In such a case, a smaller share $\theta_d \in [0, \theta)$ of standard agents, given by $\theta_d = \frac{g(1-\theta)h_2}{(\pi-g)h_1}$,[18] could still borrow at the limit rate $\rho = \pi - g$, and switch to the green technology g, whereas the remainder of standard agents would continue to invest in the polluting technology π using internal finance only. We formalize these results in the following proposition:

PROPOSITION 2. *If there is no carbon tax, then a market for carbon-contingent financing arises in which environmental agents act as lenders and standard agents as borrowers. In such case*

- *if environmental agents' endowments h_2 are sufficiently large to satisfy the inequality in* (24)*, emissions are priced at $\rho \in [\pi - g, \eta]$ and carbon-contingent debt financing enables all standard agents in the economy to adopt the green technology;*
- *otherwise, emissions are priced at $\rho = \pi - g$ and only a share $\theta_d = \frac{g(1-\theta)h_2}{(\pi-g)h_1} < \theta$ of standard agents can access carbon-contingent debt financing and switch to the green technology, whereas the remainder $\theta - \theta_d$ continue to adopt the polluting technology.*

The existence of a market for carbon-contingent securities depends on whether the tax is implemented. If the carbon tax is implemented, then all emissions are priced at the tax rate $\tau = \pi - g$ and all agents adopt the green technology, so there is no scope for pricing carbon via the financial market solution. On the other hand, if there is no tax, then carbon contingent finance arises and the extent to which it enables a complete technology switch depends on the share of environmental agents' endowments.

[17] This comes from the fact that $\bar{e}_1 = h_1$ since the counterfactual economy is a laissez-faire with investment in polluting technologies whereas $e_1^\rho = 0$ with contingent securities since it is never strictly optimal to borrow and not switch to the non-polluting technology.

[18] This is determined such that the constraint binds at $\rho = \bar{\rho} = \pi - g$.

3.5 Carbon-Contingent Financing and Political Constraints

The previous section has shown that carbon-contingent financing emerges only in the absence of the carbon tax. Borrowing through the issuance of carbon contingent securities is optimal for standard agents, whereas lending via these securities is optimal for environmental agents. We now take a step back and show how the possibility of being a lender or borrower in carbon-contingent debt markets affects agents' willingness to vote in favor of a carbon tax $\tau = \pi - g$, derive the constrained optimal tax and assess the implications for welfare and emissions relative to the benchmark results outlined in Proposition 1.

Consider first the case in which there are sufficient funds to finance the transition of all standard agents. The utility of standard borrowers that switch to the green technology is

$$\mathcal{B}_1 = gh_1 + \rho h_1, \tag{25}$$

which is higher than their utility with the tax, $U_1^\tau = gh_1$, since $\rho \geq \pi - g > 0$. So these agents are better off with securities rather than the tax because they are financially rewarded for reducing their emissions. Note also that they are strictly better off with these securities than in the laissez-faire, i.e. $\mathcal{B}_1 > U_1^* = \pi h_1$, since $\rho > \pi - g$. In fact, it holds that that $\mathcal{B}_1 > U_1^* > U_1^\tau$. The utility of standard agents is reduced when a carbon tax is implemented, and this is even more so relative to the counterfactual in which emissions are priced through markets. Although the presence of financial markets does not change the voting outcome, the existence of financial markets for pricing carbon reduces standard agents' support for the tax.

Let us define support for tax as difference in utility with the tax relative to the counterfactual economy without and with carbon-contingent markets. The standard agent's support for the tax when markets do not exist is

$$\mathit{Support}_1\ \tau = U_1^\tau - U_1^* = -(\pi - g)h_1, \tag{26}$$

while support for the tax in the presence of financial markets is given by

$$\mathit{Support}_1\ \tau|\rho = U_1^\tau - \mathcal{B}_1 = -\rho h_1. \tag{27}$$

Given that $\rho \geq \pi - g$ it is clear that support is weaker when markets exist, which is also illustrated in Figure 5.

Moving on to the voting problem of environmental agents, let us first note that their utility in an economy with financial markets, where they would optimally act as lenders, is

$$\mathcal{L}_2 = gh_2 + (\eta - \rho)q_2 = gh_2 + (\eta - \rho)\frac{\theta}{1-\theta}h_1. \tag{28}$$

We can thus define the environmental agent's support for the tax in the presence of financial markets as

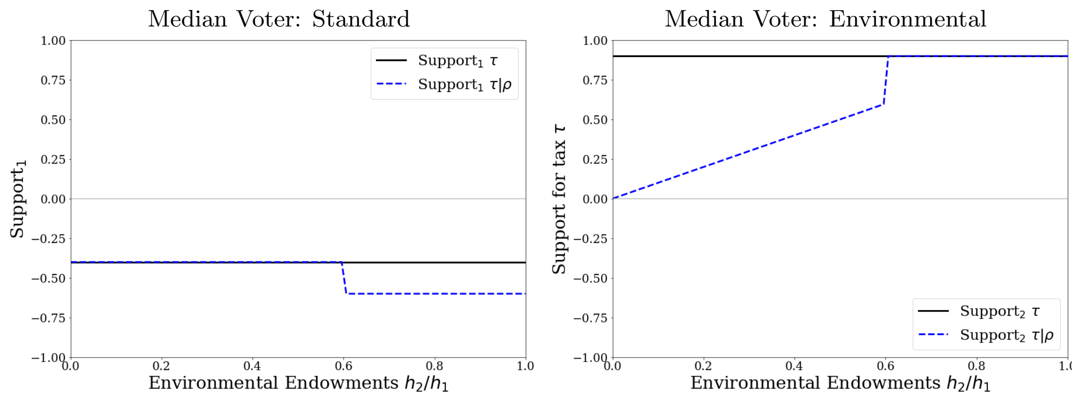
$$\mathit{Support}_2\ \tau|\rho = U_2^\tau + \underbrace{\eta\frac{\theta}{1-\theta}h_1}_{\text{voting benefit}} - \mathcal{L}_2 = \rho\frac{\theta}{1-\theta}h_1, \tag{29}$$

with $\rho \in [\pi - g, \eta]$ depending on the size of the market, as outlined in Proposition 2. Thus, support is always positive and the green agents have a preference for voting for the tax since the tax allows them to achieve the exact same environmental benefit while paying no financial cost. On the other hand, support for the tax when markets do not exist is

$$\mathit{Support}_2\ \tau = U_2^\tau + \eta\frac{\theta}{1-\theta}h_1 - U_2^* = \eta\frac{\theta}{1-\theta}h_1, \tag{30}$$

and since $\rho \leq \eta$, for this agent too, support for the tax is lower when markets exist.

Figure 5. Voting Problem
The left plot shows the support function in (26) (black line) and the support function in (26) (blue line) when the median voter is a standard type (i.e. $\theta > 0.5$) as a function of the endowments of environmental agents h_2. The right plot shows the support function in (29) (black line) and the support function in (30) (blue line) when the median voter is an environmental type (i.e. $\theta < 0.5$) as a function of the endowments of environmental agents h_2. In the regions of model parameters where $\rho \in [\pi - g, \eta]$, we set $\rho = \eta$. Other model parameters are $\pi = 1.4$, $g = 1$, $h_1 = 1$, $\eta = 0.6$.



Let us now consider the case in which the funds deployed through carbon-contingent markets are insufficient to fund the transition of all standard agents. For standard agents, support for the tax is higher relative to the case when markets can fund the transition of everyone because when markets are small the price of carbon is set at the minimum admissible rate $\pi - g$ and so they stand to benefit less from reducing emissions. So for this class of agents support for the tax decreases with size of carbon-contingent markets. On the other hand, environmental agents' tax support when markets are small

$$Support_2 \ \tau|\rho = U_2^{\tau} + \eta\frac{\theta}{1-\theta}h_1 - (gh_2 + (\eta - \rho)\frac{\theta_d}{1-\theta}h_1) = \eta\frac{\theta - \theta_d}{1-\theta}h_1 + \rho\frac{\theta_d}{1-\theta}h_1, \tag{31}$$

is lower relative to the case when markets are big enough to enable the transition of all agents.[19] Environmental agents' support for the tax increases with the size of carbon-contingent markets because markets provide a carbon emission outcome that is increasingly more similar to that achieved by the tax, but this comes at an increasingly higher cost. So for this class of agents support for the tax increases because the market alternative provides at best the same environmental outcome at a higher cost. Note that we made the simplifying (and conservative) assumption that the benefit of supporting the tax is evaluated relative to the laissez-faire counterfactual and not relative to a counterfactual in which at least some emissions would have already been reduced through financial markets. We consider this latter case in Appendix A.4 and show that doing so would only strengthen our result that the introduction of markets weakens support for regulation. These results are plotted in Figure 5.

In sum, relative to the case in which financial markets do not exist, introducing the market solution for pricing carbon reduces environmental agents' support for the tax because markets give them the opportunity to reform others,[20] whereas standard agents weaker support is driven by their preference for the market solution which allows them to monetize green preferences and get rewarded if they reduce emissions. Furthermore, whereas standard agents' support for the taxes decreases with the size of carbon-contingent

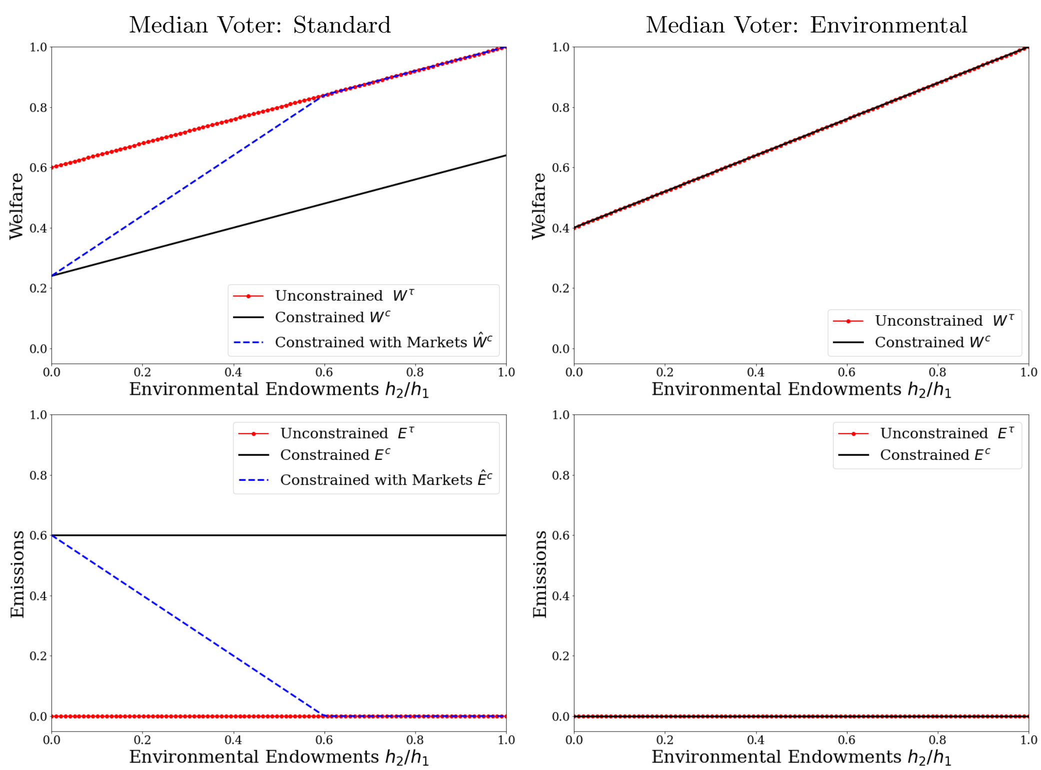
[19] To see this, substitute $\rho = \pi - g$ in (31) and $\rho = \eta$ in (29).

[20] This becomes apparent when comparing (31) and (29) with (30), since the perceived benefit from doing so, captured by the green preference parameter η, is higher or equal than the cost they incur to reform others, captured by ρ.

markets, and is driven by higher market-implied carbon price, environmental agents' support for the tax increases because the market alternative provides at best the same environmental outcome at a higher cost.

The implications for aggregate welfare and emissions relative to welfare and emissions in the politically constrained economy, W^c and E^c respectively, and these same quantities in an unconstrained economy where the optimal tax can be implemented, W^τ and E^τ, are summarized in the following proposition and illustrated in Figure 6.

Figure 6. Reducing Carbon: Regulatory vs Financial Market Tool
The plots show the equilibrium welfare (left top and bottom plots) and emissions (right top and bottom plots) as a function of the endowments of environmental agents h_2. The top plots refer to the case of $\theta < 0.5$, when the median voter is an environmental type. The bottom plots refer to the case $\theta > 0.5$, when the median voter is a standard type. The blue line is welfare in presence of financial markets. The black thick line is welfare with the politically constrained tax. The black dotted line is the reference welfare achieved from an unconstrained tax. Other parameters are $h_1 = 1$, $\lambda = 1$, $\pi = 1.4$, $g = 1$.



PROPOSITION 3. *Suppose that $\lambda > \pi - g$ such that the implementation of the carbon tax is desirable.*

- *If the median voter is an environmental type $\theta < 0.5$, then there is voting in support of the carbon tax $\hat{\tau}^c = \pi - g$ and no carbon-contingent financing. Emissions are zero and welfare*

$$\widehat{W}^c = \theta g h_1 + (1-\theta) g h_2 = W^\tau = W^c > W^*$$

- *If the median voter is a standard type $\theta > 0.5$, then there is no carbon tax $\hat{\tau}^c = 0$ and carbon-contingent financing arises at the market rate ρ which depends on environmental agents' endowments as outlined in Proposition 2.*
  - *If there are sufficient funds, emissions are reduced to zero $\hat{E}^c = 0$ and welfare is*

$$\widehat{W}^c = \theta g h_1 + (1-\theta) g h_2 = W^\tau > W^c = W^*$$

– *If funds are insufficient, emissions are* $\hat{E}^c = (\theta - \theta_d)h_1$ *and welfare is*

$$W^\tau > \widehat{W}^c = \theta\pi h_1 + (1-\theta)gh_2 - (\pi - g)\theta_d h_1 - \lambda(\theta - \theta_d)h_1 > W^c = W^*$$

When the median voter is an environmental type, the tax can be implemented and carbon-contingent financing does not arise. Welfare and emissions reduction are the same as those achieved in the unconstrained economy where the optimal tax can be implemented. If the median voter is a standard type then the tax cannot be implemented. Relative to a politically constrained economy without markets, which cannot implement the tax, welfare is higher and increases in the relative endowments of environmental agents, who are financing the transition. When the capital mobilized through carbon-contingent markets is high enough to finance the transition of all standard agents, welfare is the same as in the unconstrained economy. Therefore, when political support for tax does not exist, introducing markets is welfare improving and is equivalent to the unconstrained carbon tax if markets are sufficiently large to finance the transition of all agents. Similarly, introducing markets reduces emissions relative to the politically constrained tax, and can equal the level achieved under the unconstrained tax.

4 Extended Model

The simple model, in light of being linear delivers either-or type of predictions and cannot rationalize the empirical evidence showing that contingent finance co-exists with carbon pricing regulation. To understand the interaction between market-based and regulatory tools on the intensive margin, we extend the model to allow for a continuum of agents with heterogeneous environmental preferences, as well as a continuum of production technologies with a convex carbon abatement cost. Specifically, instead of assuming either a polluting or a non-polluting production technology, we allow agents to adjust the degree of pollution of their production technology by investing in abatement, which is parameterized by $\delta \in [0, 1]$, such that an investment of \$1 delivers output and emissions

$$y(\delta) = \pi - \phi(\delta) \quad \text{and} \quad e(\delta) = 1 - \delta,$$

where $\phi(\delta)$ represents the cost of emissions abatement, assumed to be convex $\phi(\delta) = \frac{1}{2}\phi\delta^2$ with $\phi \leq 2\pi$.[21]

There is a mass one of agents $i \in [0, 1]$, with equal endowments $h_i = \$1$, environmental preferences η_i increasing monotonically in i, verifying $\eta_i < \phi$,[22] and utility

$$U_i = C_i - \eta_i e_i, \tag{32}$$

with C_i consumption and e_i emissions associated with agent i's investment choices.

The regulator maximizes utilitarian social welfare, is given by

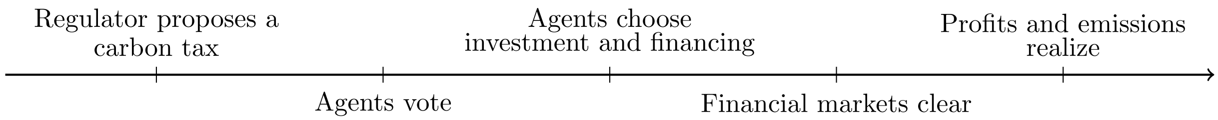
$$\mathcal{W} = \int_0^1 C_i di - \lambda E, \tag{33}$$

with λ the social cost associated with carbon emissions.

[21] This is to ensure that any abatement technology $\delta \in [0, 1]$ will produce positive output $y(\delta)$ and it is never optimal to completely shut down production. Note that the assumed range for abatement $\delta \in [0, 1]$ implies that emissions can at most be reduced to zero, and an agent cannot produce negative emissions.

[22] This is a necessary condition to ensure that the optimal technologies lie in the admissible range $\delta \in [0, 1]$, as clarified below.

As in the case of the simple model, our aim is to determine the conditions under which financial markets for pricing carbon can substitute the regulatory tool and the implications for welfare. We solve the model following a backward induction approach according to the timeline below



We first determine the agents' optimal investment and financing choices in the joint presence of a given carbon tax and a market for carbon-contingent securities. We then input these choices into the agents' utilities at the timing of voting, and derive the maximum admissible tax that each agent can support assuming each fully internalizes the behavior of others and the adjustment of financial markets to the tax. Solving for the maximum tax as a function of the agent's type will allow us to determine the median-voter constraint, which we then input into the regulator problem of finding the constrained-optimal tax which maximizes the utilitarian social welfare.

4.1 Laissez-Faire Benchmark

As a benchmark, we outline the investment choices, utilitarian social welfare, and aggregate emissions in a laissez-faire economy with no financial markets and no carbon tax. In a decentralized economy without financial markets nor taxes, each agent chooses abatement (i.e. the production technology) δ_i to maximize the utility in (32) and solves

$$U_i^* = \max_{\delta_i} C_i(\delta_i) - \eta_i e(\delta_i), \tag{34}$$

with consumption given by investment output $C_i(\delta_i) = y(\delta_i) = \pi - \phi(\delta_i)$ and emissions $e(\delta_i) = (1 - \delta_i)$. The optimal abatement choice is given by the individual environmental preference scaled by the cost of abatement

$$\delta_i^* = \frac{\eta_i}{\phi}. \tag{35}$$

Note that the parameter restriction $\eta_i < \phi$ is necessary to ensure that we obtain an interior solution for abatement $\delta \in [0, 1]$. The utility of the agent i becomes

$$U_i^* = C_i(\delta_i^*) - \eta_i e_i(\delta_i^*) = \pi - \frac{1}{2}\frac{\eta_i^2}{\phi} - \eta_i(1 - \frac{\eta_i}{\phi}), \tag{36}$$

and denoting aggregate emissions $E^* = \int_0^1 e_i(\delta_i^*)di$, welfare in the laissez-faire economy is

$$\mathcal{W}^* = \int_0^1 C_i(\delta_i^*)di - \lambda E^* = \int_0^1 \Big(\pi - \frac{1}{2}\frac{\eta_i^2}{\phi}\Big)di - \lambda(1 - \frac{\bar{\eta}}{\phi}), \tag{37}$$

with $\bar{\eta} = \int_0^1 \eta_i di$ the average green preferences in the economy.

4.2 Carbon tax

The regulator considers imposing a tax τ on the emissions e_i produced by each agent i so as to maximize the utilitarian social welfare. To preserve consistency with the previous framework in which the regulator never effectively collects tax revenues, and also motivated by actual large-scale carbon tax implementations, we assume that the carbon tax is revenue-neutral.[23] Specifically, we assume that the total proceeds

[23]Examples of revenue-neutral carbon taxes have been applied to both firms and consumers. The largest revenue-neutral carbon tax applied to firms is UK's Climate Change Levy active since 2001. As for carbon taxes applied to both firms and consumers, the one with greatest coverage has been implemented in the Canadian province of British Columbia since 2001.

from the tax $\mathcal{R}^\tau = \int_0^1 \tau e_i di$ are redistributed to agents $i \in [0,1]$ in the form of tax rebates.

The use of carbon tax rebates has been shown to be an important lever used by regulators to address various frictions and improve welfare. When regulation is unilateral and firms can shift production to deregulated countries, tax rebates can limit leakage if they are designed to reflect firms' exit propensities (Martin et al. (2014); Fowlie and Reguant (2022)). In the context of political constraints, a review by Carattini et al. (2018) suggests that public acceptance for a carbon tax is higher if the use of the tax proceeds is clearly specified. Similarly, empirical work by Fremstad et al. (2022) focusing on the USA and Switzerland has shown that having tax rebates increases political support for a given tax.

In line with this literature, our model shows that the choice of tax rebates can affect welfare in the presence of political constraints by shaping agent i's willingness to vote for the tax τ. Given that our primary focus is to understand the equilibrium interaction between regulatory and financial tools for reducing carbon, we consider a simple specification of the rebates which involves distributing an equal sum κ^τ to each agent i so as to equate total tax revenues

$$\int_0^1 \kappa^\tau di = \mathcal{R}^\tau. \tag{38}$$

In principle, it could be possible to design rebates in such a way as to increase political support for a tax. We show in Appendix B.1 that such rebates would have to penalize agents i with higher environmental preferences η_i and compensate those with lower ones. In practice though, the design of agent-level tax rebates, especially those that involve contracting on preferences, are difficult to implement.

We first determine the benchmark optimal carbon tax in the absence of political constraints. Then, we determine the median-voter threshold for a given tax rebate choice and we study the problem a regulator that is constrained to implement a tax that is at most as high as that supported by the median voter.

Agent problem. In the presence of a carbon tax τ, agent i's utility is

$$U_i^\tau = \max_\delta C_i(\delta_i) - \tau e_i(\delta_i) + \kappa^\tau - \eta_i e_i(\delta), \tag{39}$$

and optimal abatement increases with the tax and is given by

$$\delta_i^\tau = \delta_i^* + \frac{\tau}{\phi}, \tag{40}$$

with $\delta_i^* = \frac{\eta_i}{\phi}$ and $\phi > \eta_i + \tau$ so as to abstract from corner solutions where the technology constraint would be violated. Substituting (40) into the utility in (39) we get

$$U_i^\tau = U_i^* + \frac{1}{2}\frac{\tau^2}{\phi} - \tau + \frac{\tau\eta_i}{\phi} + \kappa^\tau. \tag{41}$$

Regulator problem. Welfare in the presence of the tax becomes

$$\mathcal{W}^\tau = \int_0^1 (C_i(\delta_i^\tau) - \tau e_i(\delta_i^\tau) + \kappa^\tau)di - \lambda E^\tau \tag{42}$$

with $E^\tau = \int_0^1 e_i(\delta_i^\tau)di$. Note that since the carbon tax revenues $\mathcal{R}^\tau = \int_0^1 \tau e_i(\delta_i^\tau)di$ are equal to the total rebates $\int_0^1 \kappa^\tau di$, the second and third terms in the integral cancel out and welfare becomes, after some rearrangements

$$\mathcal{W}^\tau = \mathcal{W}^* - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\bar{\eta}\tau}{\phi} + \lambda\frac{\tau}{\phi}. \tag{43}$$

Welfare in (43) is therefore independent of the specification of the rebates and has a unique optimum in $\tau = \lambda - \bar{\eta}$. This is a standard result whereby in the absence of frictions, the redistribution scheme for the tax revenues does not affect the optimal tax rate (Pigou (1920)). We show in Appendix B.1 the following

PROPOSITION 4. *The tax that maximizes the utilitarian social welfare*

$$\mathcal{W}^o = \max_{\tau \geq 0} \mathcal{W}^\tau \tag{44}$$

with W^τ *as in* (43) *is* $\tau^o = \lambda - \bar{\eta}$.

Importantly, the proposition states that the optimal tax is below the Pigouvian benchmark (i.e., the social cost of emissions λ) by an amount that reflects the average green preference $\bar{\eta}$. This is because the regulator internalizes that the existence of green preferences affect agents' investment choices and results in a certain abatement even in the absence of the tax.[24] If such preferences were already very high so that $\bar{\eta} > \lambda$, then the regulator would optimally choose $\tau^o = 0$. We avoid this extreme case and work in a more realistic range of model parameters in which $\lambda > \bar{\eta}$.

4.3 Constrained Carbon Tax

We now introduce the political constraint which means that the tax that the regulator can implement can be at most as high as that supported by the median voter. We outline below the voting problem and derive an explicit expression for the median-voter threshold.

The voting problem. As with the simple model, agents $i \in [0, 1]$ decide to vote for (against) the tax τ by comparing their utility with the tax, U_i^τ, with their utility in the laissez-faire economy, U_i^*. Voting is also an action (in addition to investing) so each agent i internalizing the emissions associated with their actions will also internalize the benefit of voting for the tax, and this will be proportional to the green preference η_i. As before, such benefit is defined as the total emissions reduction achieved by the tax relative to the laissez-faire, $E^* - E^\tau$, divided by the number of voters in favour of the tax, which we denote as green voters $\mathcal{G}$ and which is determined in equilibrium. Let us define support for the tax as

$$Support_i\ \tau = U_i^\tau + \eta_i \underbrace{\frac{(E^* - E^\tau)}{\mathcal{G}}}_{\text{voting benefit}} - U_i^*. \tag{45}$$

So agent $i \in \mathcal{G}$ votes in favour of the tax if $Support_i\ \tau \geq 0$. Substituting the expression for utility in the presence of the tax U_i^τ, from (41), and noticing that $E^* - E^\tau = \frac{\tau}{\phi}$, agent i votes in favour of the tax if

$$Support_i\ \tau = \eta_i \frac{\tau}{\phi \mathcal{G}} + \frac{1}{2}\frac{\tau^2}{\phi} - \tau + \frac{\tau \eta_i}{\phi} + \kappa^\tau \geq 0. \tag{46}$$

The inequality is unambiguously more likely to be satisfied the higher the green preference of the agent $i \in [0, 1]$. Therefore, the equilibrium group of voters supporting the tax can be defined as $\mathcal{G} = 1 - i$, where i is the indifference voter whose support (45) that is exactly zero. In order for the tax to be implemented, the indifference type needs to be at most the median voter $i = 0.5$. To understand how the support varies with the tax, let us substitute in (45) the explicit expression for the rebates, $\kappa^\tau = \tau(1 - \frac{\bar{\eta}}{\phi} - \frac{\tau}{\phi})$, which

[24] We must note here that the abatement level reflects the simple average $\bar{\eta}$ as a consequence of the fact that endowments are normalized to $h_i = \$1$ for each $i \in [0, 1]$. If endowments h_i were heterogeneous across agents, then there would be an endowment-weighted average green preference $\int_0^1 w_i \eta_i di$ with $w_i = \frac{h_i}{\int_0^1 h_i di}$. In such a case, the higher (lower) the correlation between wealth and environmental preferences, the higher (lower) the baseline abatement level achieved in the laissez-faire.

can be backed from from $\int_0^1 \kappa^\tau di = \int_0^1 \tau e_i(\delta^\tau)$. We can thus express median voter support as[25]

$$Support_{0.5}\ \tau = \eta_{0.5}\frac{2\tau}{\phi} - \frac{\tau\bar{\eta}}{\phi} - \frac{1}{2}\frac{\tau^2}{\phi} + \frac{\tau\eta_{0.5}}{\phi}, \tag{47}$$

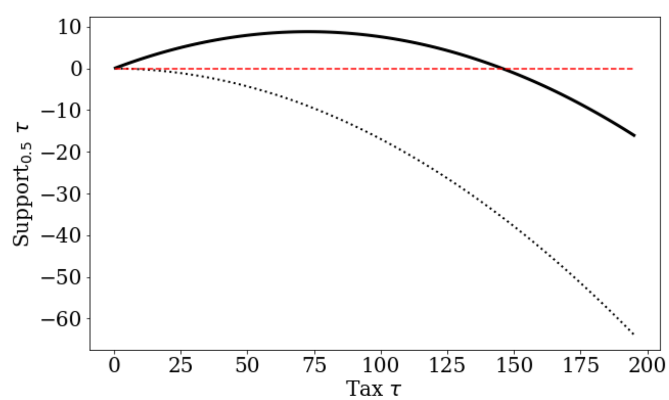
from which we can back out the median voter threshold tax $\tau_{0.5}$ by solving for $Support_{0.5}\ \tau_{0.5} = 0$

$$\tau_{0.5} = 2(3\eta_{0.5} - \bar{\eta}). \tag{48}$$

The median voter tax threshold increases with the green preference of the median voter $\eta_{0.5}$ and decreases with the average green preferences in the economy $\bar{\eta}$. Figure 7 plots the median voter support as a function of the tax. The thick black line refers to concave preferences $\eta_i = \eta\sqrt{i}$, whereas the dotted black line refers to convex preferences $\eta_i = \eta i^4$. The red dots mark the zero support threshold below which support cannot be obtained from the majority of the population. Note that everything else equal, convex (concave) preferences imply lower (higher) support for the tax. The intuition is that when green preferences increase at an increasing rate (convex case) the median voter preference $\eta_{0.5}$ is further away from the mean $\bar{\eta}$, and the median voter's preferences are less representative of average preferences. In the case plotted in Figure 7, that is, when preferences η_i are strongly convex in i, support from the median voter is roughly zero and the regulator cannot implement a tax.

Figure 7. Voting Problem

The plot shows the median voter support function in (47) as a function of τ in \$ per ton of carbon. The thick black line refers to concave preferences $\eta_i = \eta\sqrt{i}$. The dotted black line refers to convex preferences $\eta_i = \eta i^4$. The red line is the threshold tax beyond which there is not enough support from the majority. The preference parameter $\eta = \$50$/ton of carbon. Other model parameters are $\pi = 150$ and $\phi = 300$.



We solve in Appendix B.1 the problem of a politically constrained regulator who can only implement a tax that is at most as high as that supported by the median voter. The ensuing constrained optimal tax is summarized in the following proposition

[25]In our model, rebates take a specific functional form which is determined by the revenue-neutrality constraint in (38). Specifically, rebates increase in a concave manner as a function of τ. Such concavity is important for generating the realistic prediction that the type i which is indifferent between supporting or not the tax is decreasing as a function of the tax τ. For example, setting the rebates $\kappa^\tau = 0$ in (45) would flip this prediction implying that a higher tax τ receives more support from the population. This prediction is a consequence of the fact that, abstracting from tax rebates, marginal benefits from the tax τ are linear in τ, whereas costs are concave in τ, since each agent can optimally adjust to the tax by choosing a different abatement technology. This is merely an artifact of our functional form assumptions. Assuming equal rebates is a fairly straighforward and innocuous way of recovering the intuition that support decreases with the tax but the same would be obtained if the regulator were to spend the tax revenues on a public good with diminishing returns to scale.

PROPOSITION 5. *The tax that maximizes the constrained utilitarian social welfare*

$$\mathcal{W}^c = \max_{\tau \geq 0} \mathcal{W}^\tau \quad \textit{such that} \;\; \tau \leq \tau_{0.5}, \tag{49}$$

with W^τ as in (43) *and $\tau_{0.5}$ as in* (48) *is*

$$\tau^c = \min\{\lambda - \bar{\eta}, \tau_{0.5}\} \tag{50}$$

if $\tau_{0.5} > 0$, otherwise is $\tau^c = 0$.

Note that unlike the unconstrained result in Proposition 4, the optimal carbon tax policy now depends on the tax rebates through the constraint $\tau_{0.5}$. When the constraint binds, the optimal tax cannot be implemented. In such a case, we want to understand whether financial markets, that is, lending and borrowing through carbon-contingent securities can improve welfare by better exploiting heterogeneity in the distribution of green preferences across agents.

4.4 Carbon-Contingent Financing

Given a certain carbon tax τ, we now derive the conditions under which a market for carbon-contingent financing exists and the equilibrium price of carbon implied by this market. We introduce carbon-contingent securities along the lines of those studied in the simple model. Specifically, we assume that each agent i can issue a carbon-contingent security which effectively rewards the issuer for reducing emissions and imposes a monetary penalty for increasing emissions relative to a benchmark agreed at security issuance. The payoff to the security issuer is given by

$$\rho(\bar{e}_i - e_i(\delta_i)), \tag{51}$$

where $e_i(\delta_i)$ denotes the issuer i's actual emissions at time $t = 1$ and $\bar{e}_i = e_i(\delta_i^\tau)$ the benchmark emissions in the counterfactual scenario where the security is not issued and a given carbon tax τ exists.

For what follows, it is useful to use a compact notation for agent i's consumption for a given tax τ, security price ρ, and abatement technology choice δ_i as

$$C_i^{\tau,\rho}(\delta_i) = \pi - \phi(\delta_i) - \tau e_i(\delta_i) + \kappa^{\tau,\rho}, \tag{52}$$

where $\kappa^{\tau,\rho}$ denotes the tax rebate in an economy with financial markets, which will be determined in equilibrium.

Borrower's problem. Denote $\mathcal{B} \subset [0, 1]$ the set of agents that issue the carbon-contingent contract and stand to receive a financial compensation if they reduce emissions relative to their benchmark. These correspond to the borrowers in sustainability-linked debt markets. Denote $\mathcal{B}_i$ issuer i's utility for a given tax τ and carbon price ρ, which is given by

$$\mathcal{B}_i = \max_{\delta_i} C_i^{\tau,\rho}(\delta_i) - \eta_i e_i(\delta_i) + \rho(\bar{e}_i - e_i(\delta_i)) \tag{53}$$

The optimal carbon abatement choice for a given tax τ and security-implied carbon price ρ is

$$\delta_i^{\tau,\rho} = \delta_i^\tau + \frac{\rho}{\phi} \tag{54}$$

where $\delta_i^\tau = \frac{\eta_i+\tau}{\phi}$ is the optimal abatement technology choice in the counterfactual economy where a tax exists and the security is not issued, derived in (40).[26] Substituting the optimal technology choice back into the utility in (53), we show in Appendix B.2 that the borrower's utility can be expressed as

$$\mathcal{B}_i = U_i^\tau - (\kappa^\tau - \kappa^{\tau,\rho}) + \frac{1}{2}\frac{\rho^2}{\phi}. \tag{55}$$

Given that the existence of financial markets for pricing carbon reduces tax revenues, the term $\kappa^\tau - \kappa^{\tau,\rho}$ capturing the difference in the tax rebates due to financial markets, will be positive. So although borrowers can monetize their emission reductions through markets, captured by the term $\frac{1}{2}\frac{\rho^2}{\phi}$, the introduction of financial markets also has the effect of reducing their utility through the loss in rebates.

Lender's problem. Denote now the set of lenders, which act as buyers of carbon-contingent contracts, as $\mathcal{L} \subset [0,1]-\mathcal{B}$. Denote q_i the emissions reduction that agent i is financing through the carbon-contingent contract at price ρ, which can also be thought of as the quantity of carbon emissions purchased by agent $i \in \mathcal{L}$. Agent i continues to invest in the abatement technology δ_i^τ in (40) so her utility from consumption is obtained by evaluating consumption in (52) at δ_i^τ, which is the abatement rate that would be optimal in an economy with a tax only. Define for expositional simplicity $C_i^{\tau,\rho} = C_i^{\tau,\rho}(\delta_i^\tau)$ as the consumption from abating at the tax-only rate, whose expression is derived in Appendix B.2. The lender's problem is to decide the optimal abatement q_i to finance, by solving the problem

$$\mathcal{L}_i = \max_{q_i} C_i^{\tau,\rho} - \eta_i e_i(\delta_i^\tau) + \eta_i q_i - \rho q_i \text{ such that } C_i^{\tau,\rho} - \rho q_i \geq 0. \tag{56}$$

The constraint is the equivalent of the budget constraint introduced in (91). From the linearity of the problem, it follows that

$$\begin{aligned} q_i^{\tau,\rho} &= \frac{C_i^{\tau,\rho}}{\rho} \quad \text{if} \quad \rho \leq \eta_i \\ q_i^{\tau,\rho} &= 0 \quad \text{otherwise.} \end{aligned} \tag{57}$$

Substituting the optimal quantity back into the utility in (56), the utility of lender i given the tax τ and the security-implied carbon price ρ can be expressed as

$$\mathcal{L}_i = U_i^\tau - (\kappa^\tau - \kappa^{\tau,\rho}) + (\eta_i - \rho)\frac{C_i^{\tau,\rho}}{\rho}1\{\eta_i \geq \rho\}, \tag{58}$$

meaning that agent i has a utility gain from purchasing the security if its green preference η_i is stronger than the market-implied price of carbon ρ.

Market clearing. Market clearing requires that the total amount of carbon emissions reduction financed by the lenders $i \in \mathcal{L}$, given by $\int_{\mathcal{L}} q_i^{\tau,\rho} di$ with $q_i^{\tau,\rho}$ as in (57) equates the total emissions reduction supplied by the borrowers $i \in \mathcal{B}$, given by $\int_{\mathcal{B}} (\bar{e}_i - e_i(\delta_i^{\tau,\rho}))di$. Optimal abatement in (54) is such that for each issuer $i \in \mathcal{B}$, emissions reduction is $\bar{e}_i - e_i(\delta_i^{\tau,\rho}) = \frac{\rho}{\phi}$. Therefore, substituting optimal demand and supply as a function of the market price ρ, market clearing requires that

$$\int_{\mathcal{B}} \frac{\rho}{\phi} di = \int_{\mathcal{L}} \frac{C_i^{\tau,\rho}}{\rho} di. \tag{59}$$

[26] Here we continue to abstract from corner solutions in which optimal abatement would be above the technology constraint $\delta \leq 1$. The condition for an interior solution is $\eta_i < \phi - \tau - \rho$, and we already assumed that ϕ is large enough so that each agent i's preferences verify $\eta_i < \phi - \tau$. In Appendix B.2, we outline the equilibrium in which the technology constraint is violated such that $\delta = 1$.

Equilibrium. In order to pin down the equilibrium price of carbon for a given tax τ, we have to specify the equilibrium set of lenders $\mathcal{L}$ and borrowers $\mathcal{B}$ in the economy. Define the net gains from issuing the security as the difference between the agent i's utility associated with issuing a carbon-contingent security in (55), i.e. being a borrower, and the utility associated with purchasing the security in (58), i.e. being a lender, as

$$\Pi_i = \mathcal{B}_i - \mathcal{L}_i = \frac{1}{2}\frac{\rho^2}{\phi} - (\eta_i - \rho)\frac{C_i^{\tau,\rho}}{\rho}1\{\eta_i \geq \rho\}. \tag{60}$$

If the profits Π_i are monotonically decreasing in the type i, that is if

$$\frac{\partial}{\partial i}\Pi_i < 0 \quad \forall i \in [0,1], \tag{61}$$

then the single-crossing property allows us to solve for a cutoff type x verifying $\Pi_x^\tau = 0$ such that the equilibrium set of borrowers is given by $\mathcal{B} = [0, x)$ and the lenders are $\mathcal{L} = [x, 1]$. In Appendix **??**, we provide a sufficient condition for the profits in (60) to be monotonically decreasing in the type i, which amounts to assuming that the baseline profits π are large enough.[27] Assume that model parameter are such that the issuance profits in (60) satisfy the monotonicity condition in Appendix B.2. We introduce the following carbon-contingent market equilibrium

PROPOSITION 6. *For a given tax τ, the pair (ρ^τ, x^τ) consisting of the market price of carbon and cutoff type constitutes an equilibrium if these jointly verify the market clearing condition and the indifference condition, that is*

$$\rho^\tau = \sqrt{\frac{\phi}{x^\tau}\int_{x^\tau}^{1} C_i^{\tau,\rho^\tau}\,di} \quad \text{and} \quad \frac{1}{2}\frac{(\rho^\tau)^2}{\phi} = (\eta_{x^\tau} - \rho^\tau)\frac{C_{x^\tau}^{\tau,\rho^\tau}}{\rho^\tau}. \tag{62}$$

Figure 8. Equilibrium carbon-contingent financing as a function of the tax
The plots show the equilibrium rate ρ^τ in \$/CO2 (left plot) and the cutoff type x^τ (right plot) as a function of the tax τ when preferences are convex $\eta_i = \eta i^4$ (dashed line) and concave $\eta_i = \eta\sqrt{i}$ (thick line) in the type $i \in [0,1]$. Other model parameters are $\eta = 75$\$/CO2, $\phi = 300$, and $\pi = 150$.

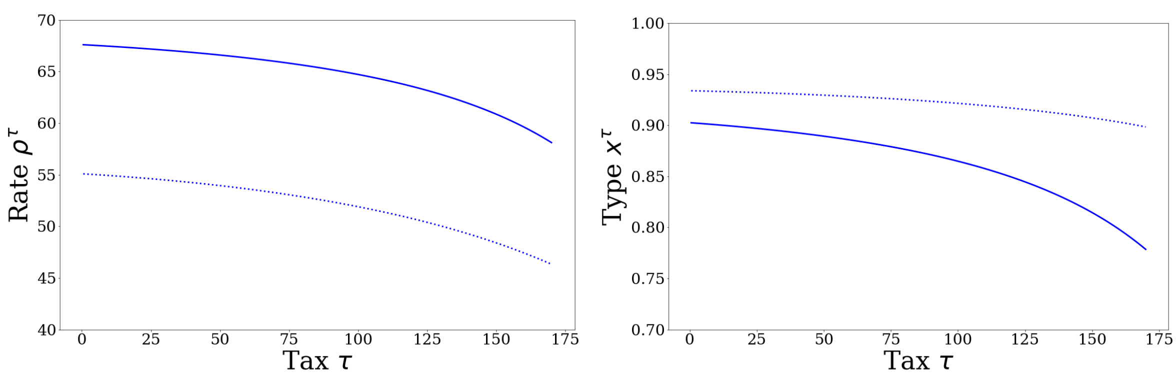


Figure 8 shows the equilibrium price of carbon and cutoff type as a function of the tax τ. Dashed and thick lines refer to the case of convex preferences ($\eta_i = \eta i^4$) and concave preferences ($\eta_i = \eta\sqrt{i}$), respectively. The left plot shows that independently of the functional form of the green preferences, the market price of carbon decreases as a function of the tax.[28] As the market price of carbon decreases with the tax, the financial profits in (55) that the borrower can make from issuing the security decrease with the tax.

[27] This assumption was already imposed earlier to ensure that each agent i optimally produces at least some emissions e_i (i.e., it is never optimal to shut down operations).

[28] This is because a higher tax decreases the aggregate demand for carbon-contingent securities by reducing the budget available to lenders (see the left equation in (62)).

Consequently, and as shown in the right plot of Figure 8, the cutoff indifference type, i.e. the fraction of agents acting as borrowers, decreases as a function of the tax. Taken together, these results suggest that total abatement generated by financial markets, which is quantified by $\frac{\rho^\tau x^\tau}{\phi}$, also decreases as a function of the tax. This is an important result showing that financial markets are less effective in those economies where there is regulation in place.

Finally, for what follows, it is important to note that for the range of model parameters which admit interior solutions as assumed in this model, the equilibrium cutoff type x^τ is always above the median voter $i = 0.5$. This implies that the median voter is typically a borrower of carbon contingent securities.

4.5 Carbon-Contingent Financing and Political Constraints

We now solve for agent i's voting problem in the presence of financial markets. Tax support depends on the utility of being a carbon-contingent lender or borrower in an economy with a carbon tax versus one without, as well as the voting benefit. As in the simple model, the voting benefit is evaluated conservatively, and it depends on emission reductions brought about by tax relative to the laissez-faire, which is given by $\frac{\tau}{\phi}$.[29]

Denote $\mathcal{L}_i^\tau$ and $\mathcal{B}_i^\tau$ the lender and borrower's utility in (58) and (55), respectively, evaluated at the equilibrium market price of carbon ρ^τ for a given tax τ, while $\mathcal{L}_i^*$ and $\mathcal{B}_i^*$ denote the equilibrium utilities when $\tau = 0$ and ρ^* is the equilibrium market price of carbon when $\tau = 0$. We can define support for a carbon tax conditional on the existence of financial markets as

$$Support_i \ \tau|\rho = \max\{\mathcal{L}_i^\tau, \mathcal{B}_i^\tau\} + \underbrace{\eta_i \frac{\tau}{\phi(1-i)}}_{\text{voting benefit}} - \max\{\mathcal{L}_i^*, \mathcal{B}_i^*\}, \tag{63}$$

where the max operator captures the idea that agent i can optimally be a lender or borrower depending on the utility of doing so. Showing that financial markets reduce support for regulation boils down to evaluating whether tax support when markets exist, $Support_i \ \tau|\rho$ in (63), is lower than tax support without markets, $Support_i \ \tau$ in (45), which in turn amount to showing that

$$\max\{\mathcal{L}_i^*, \mathcal{B}_i^*\} - \max\{\mathcal{L}_i^\tau, \mathcal{B}_i^\tau\} > U_i^* - U_i^\tau. \tag{64}$$

If inequality (64) is verified, the presence of financial markets is said to decrease support for the carbon tax. In an economy with financial markets, we distinguish between three types of agents i. First, we note that absent a carbon tax the financial market equilibrium is such that a fraction $i \in [0, x^*)$ of agents optimally act as borrowers whereas agents $i \in [x^*, 1]$ act as lenders. However, introducing a tax will change this equilibrium and, as shown in the right plot of Figure 8, will shift the indifferent type down to $x^\tau < x^*$, meaning that the tax can induce a fraction of agents $[x^\tau, x^*)$ to switch from being borrowers to being lenders. Therefore, agents $i \in [0, x^\tau)$ act as borrowers in the carbon-contingent market independently of whether a tax exists; agents $i \in (x^\tau, x^*]$ act as borrowers in the carbon-contingent market when there is no tax, but become lenders if the tax τ is implemented; agents $i \in [x^*, 1]$, act as lenders independently of the tax implementation.

Borrower. Consider first the case in which agent i is a borrower irrespective of the tax implementation. Given the utility specification in (55), the left term of inequality (64) becomes

[29] A more precise emission attribution would include equilibrium adjustments in the abatement generated by financial markets as a result of the tax. As discussed in the simple model, this would only strengthen our key results, while considerably complicating derivations, so we stick to the more conservative simplification.

$$\mathcal{B}_i^* - \mathcal{B}_i^\tau = U_i^* - U_i^\tau + \kappa^\tau - \kappa^{\tau,\rho^\tau} + \frac{1}{2}\frac{(\rho^*)^2 - (\rho^\tau)^2}{\phi}, \tag{65}$$

with the implication that support for the tax decreases in the presence of financial markets for two reasons. First, because financial markets reduce emissions, they will lower carbon tax revenues in equilibrium, so each agent i receives lower rebates with markets $\kappa^{\tau,\rho^\tau} < \kappa^\tau$. Second, because the market price of carbon decreases with the tax, $\rho^* > \rho^\tau$ the profits from issuing the carbon-contingent security also decrease, such that the $\frac{1}{2}\frac{(\rho^*)^2-(\rho^\tau)^2}{\phi}$ is positive. Consequently, the inequality (64) is verified and support for the tax decreases in the presence of financial markets.

Lender-Borrower. There is also the case in which the agent i is a borrower in carbon-contingent market absent the tax, but becomes a lender when the tax is implemented. In such a case, the left term of inequality (64) becomes

$$\mathcal{B}_i^* - \mathcal{L}_i^\tau = U_i^* - U_i^\tau + \kappa^\tau - \kappa^{\tau,\rho^\tau} + \big(\frac{1}{2}\frac{(\rho^*)^2}{\phi} - (\eta_i - \rho^\tau)\frac{C_i^{\tau,\rho^\tau}}{\rho^\tau}\big). \tag{66}$$

The term in parenthesis is the difference between the net profits from being a borrower in the market without the tax and the utility gain from being a lender in the market with the tax. We can prove that this term is weakly positive for any type $i \in [x^\tau, x]$. Recalling the carbon-contingent market equilibrium (62) in the presence of the tax, at the equilibrium cutoff type $i = x^\tau$ we have that $(\eta_i - \rho^\tau)\frac{C_i^{\tau,\rho^\tau}}{\rho^\tau} = \frac{1}{2}\frac{(\rho^\tau)^2}{\phi} < \frac{1}{2}\frac{(\rho^*)^2}{\phi}$ and the term in parenthesis is strictly positive. On the other hand, for $i = x^*$ the term in parenthesis is exactly zero. By continuity and monotonicity, the last term is strictly positive for any interior type $i \in (x^\tau, x)$. Given that the tax rebates term $\kappa^\tau - \kappa^{\tau,\rho^\tau}$ is also positive, inequality (64) is verified and financial markets decrease support for the tax in this case too.

Lender. Finally, when agent i is a lender irrespective of the tax implementation, we can write the left term of (64) as

$$\mathcal{L}_i^* - \mathcal{L}_i^\tau = U_i^* - U_i^\tau + \kappa^\tau - \kappa^{\tau,\rho^\tau} + \Big((\eta_i - \rho^*)\frac{C_i^*}{\rho^*} - (\eta_i - \rho^\tau)\frac{C_i^{\tau,\rho^\tau}}{\rho^\tau}\Big), \tag{67}$$

where C_i^* is the utility from consumption obtained by evaluating $C_i^{\tau,\rho}(\delta_i)$ in (52) at $\tau = 0$, $\delta_i = \delta_i^*$ and $\rho = \rho^*$. The term in parenthesis is the difference between the lender i's utility gain in an economy without a tax and one in which a tax exists. Recalling that each lender i purchases an optimal equilibrium quantity of emission reductions $q_i^* = \frac{C_i^*}{\rho^*}$ without the tax and $q_i^{\tau,\rho^\tau} = \frac{C_i^{\tau,\rho^\tau}}{\rho^\tau}$ with the tax, this term can be re-arranged as

$$\eta_i(q_i^* - q_i^{\tau,\rho^\tau}) - (\rho^* q_i^* - \rho^\tau q_i^{\tau,\rho^\tau}). \tag{68}$$

The first term in (68) is the difference in the lender i's utility from financing emissions reduction without and with the tax. We show in Appendix B.3 that the optimal emissions reduction financed by the lender decreases with the tax, such that $q_i^* > q_i^{\tau,\rho^\tau}$.[30] The second term in (68) is the difference in the lending cost without and with the tax. Since $q_i^* > q_i^{\tau,\rho^\tau}$ and $\rho^* > \rho^\tau$, this term is also positive and progressively larger than the first term, the higher the tax τ. Therefore, the expression in (68) is negative and increasing in magnitude the higher the tax τ. Put differently, as the level of the tax becomes higher, the reduction in the cost of lending brought about by the tax (second term in (68)) outweights the utility loss

[30] Recalling that $q_i^{\tau,\rho^\tau} = \frac{C_i^{\tau,\rho^\tau}}{\rho^\tau}$, note that a higher tax decreases both the numerator as well as the denominator. However, the consumption budget C_i^{τ,ρ^τ} available to the lender decreases in τ at a rate which is higher than the rate of decrease in the carbon price ρ^τ.

caused by being able to finance a lower quantity of emissions reduction (first term in (68)). Therefore, the term in parenthesis in (67) is negative, which increases lender's preference for the tax. However, the term in (67) representing the difference in tax revenues, $\kappa^{\tau} - \kappa^{\tau,\rho^{\tau}}$, is positive and therefore drives the lender's preferences away from the tax. The relative magnitude of the second and third terms in (67) will determine whether the lender's support for the tax in (63) is eventually higher or lower than its support in the baseline economy without financial markets. Either way, for the high types $i \in [x^*, 1]$, the voting benefit in (63) is high enough to make the tax τ always desirable.

In sum, our model suggests that agents with strong environmental preferences, who act as lenders in carbon-contingent markets, will not experience a decrease in support for the tax. Empirically, Heeb et al. (2023b) study the relationship between market-based solutions for combating climate change and political support for environmental regulation by conducting a survey on the Swiss population, which is one with strong environmental preferences. The authors find that giving people the option to carry out green investments (i.e., in our model terms, be lenders in the carbon-contingent market) does not reduce their support for a carbon tax, which is in line with our model predictions. However, our model also shows that agents with weaker environmental preferences, which optimally act as borrowers in the carbon-finance market will decrease their support for the tax, as they have the possibility of monetizing emission reductions in financial markets. In most markets, such agents with lower environmental preferences and are arguably more representative of the median voter. Indeed, our quantitative model suggests that the marginal voter is typically a borrower of carbon-contingent securities. This is because in the region of admissible model parameters which admit interior solutions for (ρ^{τ}, x^{τ}), the cutoff type typically satisfies $x^{\tau} > 0.5$ for any τ sufficiently large, implying that the median voter support function in (63) reflects that one of the borrower in (65). Therefore, the median voter support function is

$$Support_{0.5}\ \tau|\rho = Support_{0.5}\ \tau - (\kappa^{\tau} - \kappa^{\tau,\rho^{\tau}}) - \frac{1}{2}\frac{(\rho^*)^2 - (\rho^{\tau})^2}{\phi}. \quad (69)$$

Recalling the expression for $Support_{0.5}\ \tau$ in (45) and substituting the tax rebate term $\kappa^{\tau,\rho^{\tau}} = \kappa^{\tau} - \tau \int_0^{x^{\tau}} \frac{\rho^{\tau}}{\phi} di$ for a revenue-neutral tax, we can solve for the median voter threshold tax $\hat{\tau}_{0.5}$ beyond which support from the majority is lost (i.e. the tax verifying $Support_{0.5}\tau|\rho = 0$). The latter satisfies the implicit expression

$$\hat{\tau}_{0.5} = \tau_{0.5} - 2\rho^{\hat{\tau}_{0.5}} x^{\hat{\tau}_{0.5}} - \frac{(\rho^*)^2 - (\rho^{\hat{\tau}_{0.5}})^2}{\hat{\tau}_{0.5}}. \quad (70)$$

The median voter threshold $\hat{\tau}_{0.5}$ is lower than that the threshold without markets, $\tau_{0.5}$. This can be observed by noticing that when $\hat{\tau}_{0.5} > \tau_{0.5}$, the second and third term in (70) are both negative hence the right-hand side is lower than $\tau_{0.5}$ and the equality cannot be satisfied.

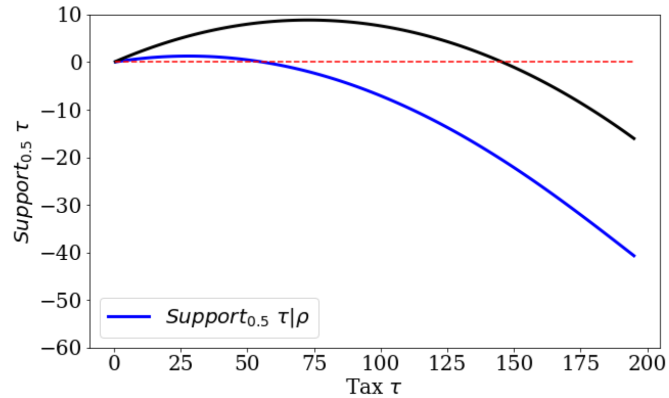
Figure 9 shows the new support function of the median voter in (69) (blue line) against the baseline support function in (45) (black line) for the case of concave preferences.[31] As discussed, when the median voter is a borrower the reduction in political support for the tax is unambigously reduced by the introduction of financial markets.

4.6 Constrained Carbon Tax with Carbon-Contingent Financing

As in the baseline model, the regulator is subject to a political constraint in that it must propose a tax which is supported by at least half of the population. The regulator's welfare function in the presence of

[31] The case of convex preferences is excluded since, as illustrated in Figure 7, tax support was zero even in the absence of financial markets.

Figure 9. Voting Problem
The plot shows the median voter support function in (47) (black line) and the median voter support function in (69) (blue line) as a function of τ in \$ per ton of carbon. The blue thin line is the support function in (69) where the contribution of the rebates component $\kappa^{\tau} - \kappa^{\tau,\rho^{\tau}}$ is set to zero. The red line is the threshold tax beyond which there is not enough support from the majority. Preferences are assumed concave $\eta_i = \eta\sqrt{i}$ with $\eta = \$50$/ton of carbon. Other model parameters are $\pi = 150$ and $\phi = 300$.



carbon-contingent financing can be expressed as follows

$$\widehat{W}^{\tau} = \int_0^{x^{\tau}} \Big(C_i(\delta_i^{\tau,\rho^{\tau}}) - \tau e_i(\delta_i^{\tau,\rho^{\tau}}) + \kappa^{\tau,\rho^{\tau}} + \rho^{\tau}(\bar{e}_i - e_i(\delta_i^{\tau,\rho^{\tau}}))\Big) di + \int_{x^{\tau}}^{1} \Big(C_i(\delta_i^{\tau}) - \tau e_i(\delta_i^{\tau}) + \kappa^{\tau,\rho^{\tau}} - \rho^{\tau} q_i^{\tau,\rho^{\tau}}\Big) di - \lambda E^{\tau,\rho^{\tau}} \tag{71}$$

with $C_i(\cdot) = \pi - \phi(\cdot)$, optimal abatement $\delta_i^{\tau,\rho} = \delta_i^{\tau} + \frac{\rho^{\tau}}{\phi}$ and total emissions $E^{\tau,\rho^{\tau}} = E^{\tau} - \frac{\rho^{\tau} x^{\tau}}{\phi}$. The first integral captures the total consumption of borrowers $i \in [0, x^{\tau}]$, which consists of output from investing in the abatement technology $\delta_i^{\tau,\rho^{\tau}}$, the tax rebates, and the financial reward from reducing emissions beyond the counterfactual benchmark, net of the carbon tax. The second integral captures the total consumption of lenders $i \in (x^{\tau}, 1]$, consisting of the output from investing the technology δ_i^{τ}, the tax rebate, net the tax payment and the amount invested in carbon-contingent securities. Recalling that lenders optimally purchase carbon-contingent securities until they meet their budget constraint, this second integral is zero.

Substituting the market clearing condition (59) in the financial rewards from issuing carbon-contingent securities, $\rho^{\tau}(\bar{e}_i - e_i(\delta_i^{\tau,\rho^{\tau}}))$, and recalling that the tax rebates $\kappa^{\tau,\rho}$ equate the total proceeds from the tax, the expression in (71) can be simplified, as we show in Appendix B.4, to

$$\widehat{\mathcal{W}}^{\tau} = \mathcal{W}^{\tau} - \int_0^{x^{\tau}} \Big(\frac{\eta_i \rho^{\tau}}{\phi} + \frac{\tau\rho^{\tau}}{\phi} + \frac{1}{2}\frac{(\rho^{\tau})^2}{\phi}\Big) di + \lambda \frac{\rho^{\tau} x^{\tau}}{\phi}. \tag{72}$$

The first term is the regulator's welfare function in the absence of financial markets, as outlined in (43). The second term represents the increase in total abatement costs due to the presence of financial markets.[32] The third term is the social benefit achieved from the additional emissions reduction. When deciding the optimal tax, the regulator weights this benefit against the increase in the total abatement costs, which increase as a function of the tax because of the interaction term $\frac{\tau\rho^{\tau}}{\phi}$. We prove in Appendix B.4 the following

[32]This term exists because of convex abatement costs, which imply that it is progressively more expensive to reduce an additional unit of emissions. Convex abatement cost curves have been empirically estimated in the literature (see Du et al. (2015) for the case of China) and are necessary in our setting to provide interior solutions. A linear specification would collapse the problem to the simple setting outlined in the previous section.

PROPOSITION 7. *The optimal carbon tax maximizing the regulator problem*

$$\widehat{\mathcal{W}}^o = \max_{\tau \geq 0} \widehat{\mathcal{W}}^\tau \tag{73}$$

with $\widehat{W}^\tau$ *as in* (72) *is given by*

$$\hat{\tau}^o = \lambda - \frac{\bar{\eta} + \rho^o x^o (1 + \frac{1}{2}\rho^o_\tau + \bar{\eta}^o_\tau) + (\rho^o_\tau x^o + \rho^o x^o_\tau)(\frac{1}{2}\rho^o + \bar{\eta}^o)}{1 + (\rho^o_\tau x^\tau + \rho^o x^o_\tau)} \tag{74}$$

with ρ^o *and* x^o *the equilibrium prices and cutoff type evaluated at* $\tau = \hat{\tau}^o$, ρ^o_τ *and* x^o_τ *the derivatives evaluated at* $\tau = \hat{\tau}^o$, *the term* $\bar{\eta}^o = \frac{1}{x^o}\int_0^{x^o} \eta_i di$ *and* $\bar{\eta}^o_\tau$ *its derivative evaluated at* $\tau = \hat{\tau}^o$.
The constrained carbon tax maximizing the regulator problem

$$\widehat{\mathcal{W}}^c = \max_{\tau \geq 0} \widehat{\mathcal{W}}^\tau \quad \textit{such that} \quad \tau \leq \hat{\tau}_{0.5} \tag{75}$$

with $\hat{\tau}_{0.5}$ *the median voter threshold in* (70), *is given by*

$$\hat{\tau}^c = \min\{\hat{\tau}^o, \hat{\tau}_{0.5}\} \tag{76}$$

if $\hat{\tau}_{0.5} > 0$, *otherwise is* $\hat{\tau}^c = 0$.

Appendix B.4 shows that the unconstrained optimal tax $\hat{\tau}^o$ in the presence of financial markets is lower than the counterfactual tax without financial markets, $\tau^o = \lambda - \bar{\eta}$. In other words, financial markets reduce the need of regulatory intervention because a fraction of emissions will already be reduced through carbon-contingent financing. In fact, it is possible that this reduction is sufficiently large to make the unconstrained tax $\hat{\tau}^o$ in (74) negative, implying that the regulator imposes no tax, that is, $\hat{\tau}^c = 0$.[33] However, we need to recall that markets also reduce political support for a given tax, as the median voter threshold $\hat{\tau}_{0.5}$ in (70) is strictly lower than $\tau_{0.5}$. In sum, on the one hand, financial markets increase the stringency of the political constraint by reducing the median voter threshold, $\hat{\tau}_{0.5} < \tau_{0.5}$. On the other hand, markets reduce the need for regulatory intervention in the first place, $\hat{\tau}^o < \tau^o$. The equilibrium implications of introducing financial markets on welfare and carbon emissions thus depend on the relative magnitude of these two effects.

The following proposition summarizes the possible scenarios relative to a counterfactual economy without financial markets.

PROPOSITION 8. *Suppose that* $\lambda - \bar{\eta} > 0$ *so that the implementation of a carbon tax in the baseline economy is always desirable. Then the following scenarios are possible:*

1. *If* $\tau^c = \tau^o$, *then the economy without financial markets is not politically constrained and* $\mathcal{W}^c = \mathcal{W}^o$. *In such a case:*
   (a) *if* $\hat{\tau}^o < 0$, *then the economy with financial markets cannot implement the tax, i.e.* $\hat{\tau}^c = 0$. *Relative to the counterfactual economy without markets, which can implement the unconstrained optimal tax, welfare is lower* $\widehat{\mathcal{W}}^c < \mathcal{W}^c$ *and emissions are also lower* $\hat{E}^c < E^c$.
   (b) *if* $\hat{\tau}_{0.5} < 0 < \hat{\tau}^o$, *then the economy with financial markets cannot implement the tax, i.e.* $\hat{\tau}^c = 0$. *Relative to the counterfactual economy without markets, which can implement the unconstrained optimal tax, welfare is lower* $\widehat{\mathcal{W}}^c < \mathcal{W}^c$, *whereas emissions are higher* $\hat{E}^c > E^c$.
   (c) *if* $0 < \hat{\tau}^o < \hat{\tau}_{0.5}$, *then the economy with financial markets can implement the optimal tax, i.e.* $\hat{\tau}^c = \hat{\tau}^o$. *Welfare* $\widehat{\mathcal{W}}^c$ *is marginally higher (lower) than that in the counterfactual economy with-*

[33] We exclude the possibility of negative taxes as unrealistic in practice.

out markets $\mathcal{W}^c$ if environmental preferences η_i are concave or linear (convex) in i. Emissions $\hat{E}^c$ are approximately equal to those in the counterfactual economy without markets, E^c.

(d) *otherwise, the economy with financial markets can only implement the politically constrained tax, i.e. $\hat{\tau}^c = \hat{\tau}_{0.5}$. Relative to the counterfactual economy without markets, which can implement the unconstrained optimal tax, welfare is lower $\widehat{\mathcal{W}}^c < \mathcal{W}^c$, whereas emissions are higher $\hat{E}^c > E^c$.*

2. *If $\tau^c = \tau_{0.5}$, then the economy without financial markets is politically constrained and $\mathcal{W}^c < \mathcal{W}^o$. In such a case:*

(e) *if $\hat{\tau}^o < 0$, then the economy with financial markets cannot implement the tax, i.e. $\hat{\tau}^c = 0$. Relative to the counterfactual economy without markets, which can only implement the constrained tax, welfare is higher $\widehat{\mathcal{W}}^c > \mathcal{W}^c$ and emissions are lower $\hat{E}^c < E^c$.*

(f) *if $\hat{\tau}_{0.5} < 0 < \hat{\tau}^o$, then the economy with financial markets cannot implement the tax, i.e. $\hat{\tau}^c = 0$. If the abatement achieved through financial markets satisfies $\rho^* x^* > \tau_{0.5}$ ($\rho^* x^* < \tau_{0.5}$), then welfare $\widehat{\mathcal{W}}^c$ is higher (lower) relative to the counterfactual economy without markets $\mathcal{W}^c$, whereas emissions $\hat{E}^c$ are lower (higher) than those in counterfactual economy E^c.*

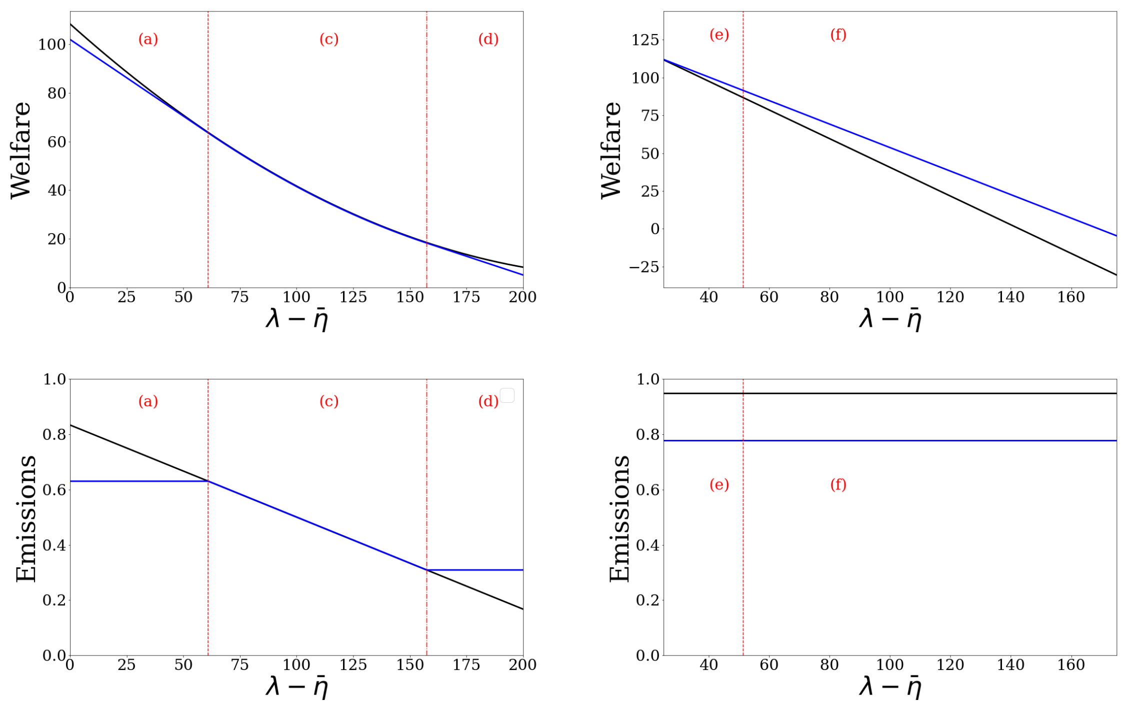
(g) *otherwise, the economy with financial markets can only implement the politically constrained tax, i.e. $\hat{\tau}^c = \hat{\tau}_{0.5}$. Relative to the counterfactual economy without markets, which can only implement the constrained tax, welfare is lower $\widehat{\mathcal{W}}^c < \mathcal{W}^c$, whereas emissions are higher $\hat{E}^c > E^c$.*

In Appendix B.4, we provide a proof of Proposition 8 based on an approximation of the equilibrium tax rate in (74). When an optimal tax can be implemented, introducing financial markets can provide at best a marginally higher welfare than the counterfactual economy without markets. This occurs in scenario 1.(c), which is when the abatement provided through financial markets does not significantly alter equilibrium support for the tax nor does it make the optimal tax negative. The marginal difference in welfare achieved by this economy relative to the counterfactual depends on the distribution of environmental preferences which are reflected in different costs of abatement associated with financial markets (mid term in equation (72)). When preferences are linear or concave in i, abatement costs are typically lower, resulting in a marginally higher welfare.[34]

If the abatement provided through financial markets is sufficiently large to make the tax redundant (that is, scenario 1.(a)), then the regulator imposes no tax, i.e. $\hat{\tau}^o = 0$. In such a case, we show that the emissions reduction is higher than the benchmark achieved with the tax only, resulting in lower aggregate emissions. However, such extra abatement is inefficient from the point of view of the regulator, as it comes at higher costs than the social cost of carbon λ. Consequently, welfare in such a scenario is lower than welfare in the unconstrained scenario without financial markets. However, if the abatement provided through financial markets is such that the tax is positive $\hat{\tau}^o > 0$ and greater than the median voter threshold $\hat{\tau}_{0.5} < \hat{\tau}^o$, we are in the worse case scenarios in which financial markets have the primary effect of reducing support for the tax while not supplying significant emissions reduction relative to the counterfactual (these are scenarios 1.(b) and 1.(d) in Proposition 8). In other words, the emissions reduction provided through markets is not sufficiently high to make the tax redundant but support for regulation is reduced to make the economy politically constrained. In such scenarios, equilibrium emissions reduction as well as welfare are lower than in the counterfactual economy without markets, which can implement the unconstrained optimal tax. This result illustrates that the introduction of markets can have a backfiring effect which emerges only when we take into account the equilibrium implications on the voting decision.

[34]We show this formally in the Appendix. Intuitively, the total abatement costs introduced by financial markets are the sum of the additional abatement costs faced by each borrower i of carbon-contingent securities. The latter increase in $i \in [0, x^\tau]$ as a function of η_i. Because of the Jensen's inequality, the sum of convex η_i is higher than the sum of concave η_i.

Figure 10. Reducing Carbon: Carbon Tax vs Carbon-Contingent Financing
The y-axis shows the equilibrium welfare (top plots) and emissions (bottom plots) achieved by an economy with the carbon tax only (black line) and by an economy with the combined presence of financial markets and the tax (blue line). The x-axis shows the unconstrained optimal benchmark tax $\tau^o = \lambda - \bar{\eta}$ in \$ per ton of carbon for varying values of the social cost λ. The left top and bottom plots refer to the case of concave preferences $\eta_i = \eta\sqrt{i}$ for $i \in [0,1]$, for which the optimal tax $\tau^o = \lambda - \bar{\eta}$ can always be implemented (case 1. in Proposition 8). The right top and bottom plots refer to the case of convex preferences $\eta_i = \eta i^4$ for $i \in [0,1]$, for which the optimal tax $\tau^o = \lambda - \bar{\eta}$ can never be implemented (case 2. in Proposition 8). Other parameters are $\eta = \$75$ per ton of carbon, $\phi = 300$, $\pi = 150$.



These cases are illustrated in the left top and bottom plots in Figure 10, which show the equilibrium welfare and carbon emissions with financial markets (blue line) and without financial markets (black line). The x-axis is the reference unconstrained optimal tax $\tau^o = \lambda - \bar{\eta}$, which increases as a function of the social cost of carbon parameter λ for a given distribution of preferences. As observed, when the social cost λ is low so that the abatement provided through financial markets is more than what is efficient for the regulator (scenario 1.(a)), welfare is lower but financial markets achieve a higher emissions reduction than the counterfactual economy. On the other hand, when the social cost λ is very high, the abatement provided through financial markets does not exceed what is optimal for the regulator, yet the political support for the tax decreases to the point of making the implementation of the unconstrained tax unfeasible (scenario 1.(d)). In the intermediate region, the introduction of financial markets provides equivalent outcomes in terms of welfare and emissions relative to the counterfactual without markets.

Finally, when the baseline economy does not support the implementation of a carbon tax, then there is scope for financial markets to provide a strictly better alternative to the tax only. This occurs in scenario 2.(e) and 2.(f) and is illustrated in the right top and bottom plots in Figure 10. When the abatement achieved through financial markets alone, captured by $\rho^* x^*$, is higher than that achieved through the constrained carbon tax implemented in the counterfactual economy, captured by $\tau_{0.5}$, then both emissions reduction as well as welfare are strictly higher. However, if the abatement provided through financial markets is low, we are back in the negative scenario whereby financial markets tighten the political constraint without supplying sufficient emissions reduction, which results in lower welfare and higher emissions relative to the politically constrained counterfactual without financial markets (scenario 2.(g)).

5 Conclusions

We start by proposing a baseline model in which financially- and environmentally-motivated agents can invest their endowments in polluting or non-polluting technologies, with the latter being less profitable than the former. We show that a carbon tax corrects the laissez-faire allocation in which the polluting technology is adopted by standard agents, and has the effect of increasing welfare and decreasing emissions. If there is no political support for a carbon tax, carbon-contingent financing provided by environmentally-motivated agents can effectively substitute the carbon tax. Whether the financial market solution partially or fully substitutes regulation depends importantly on the endowments of environmental agents who, by lending to financially-motivated agents via carbon-contingent contracts, are essentially subsidizing their transition to the non-polluting technology. We show that when environmental agents are endowed with sufficiently large funds, financial markets circumvent the political constraint achieving the same welfare and the same emissions reduction as the unconstrained optimal tax. Pricing emissions through financial markets also creates welfare gains when environmental funds are small, provided that the political constraint is binding. Importantly though, although financial markets do not affect voting outcomes in this linear model where regulation or financial market tools for pricing carbon cannot co-exist, we show that support for the tax decreases on the intensive margin. This suggests that financial markets may have possible negative consequences on welfare and carbon emissions in a model that supports the intensive margin interaction between regulatory and markets tools for pricing carbon.

We therefore extend the model to a continuum of agents with heterogeneous environmental preferences and production technologies with emissions that can be reduced at a convex abatement cost. We study the case of a regulator which is politically constrained in implementing a revenue-neutral carbon tax which involves redistributing the revenues from the tax equally across voters. Solving for the agents' financing and investment decisions while taking into account the financial markets' response to the tax, we show that taxation and carbon-contingent financing can co-exist and derive the implications in terms of welfare and aggregate emissions. They are still characterized by a substitution relationship, and the share of emission reduction enabled through carbon-contingent financing is smaller the higher the tax, suggesting that such capital flows are best directed to unregulated markets where they can have more impact. However, as implied by the linear model, we show that financial markets reduce agents' willingness to vote for the tax. In equilibrium, such effect can result in actual welfare losses if the baseline economy without financial markets can support the implementation of an optimal tax, or if the abatement provided by financial markets is not large enough. In addition to providing a quantitative analysis of the channels introduced in the simple model, the extended model highlight the role of tax rebates on the political support for the tax and the interaction with financial markets in equilibrium.

A Simple Model Robustness

A.1 Agent Preference Variations

Let us condier the case when agents internalize global emissions, such that standard agents' utility is

$$U_1 = C_1 - \lambda E$$

and environmentally-oriented or green agents' utility is

$$U_2 = C_2 - \lambda E - \eta e_2.$$

Note that whereas the environmental entrepreneurs dislike the emissions associated with their actions and which they feel responsible for, both types of entrepreneurs are affected by total carbon emissions. The latter can be conceptualized as capturing a global climate shock that affects them irrespective of their preferences and over which they have no control. Thus, entrepreneurs are atomistic with respect to the global climate shock, which can be thought as a natural disaster or the negative effects of pollution on health which affect the entire population.
Regulator maximizes utilitarian social welfare given by

$$W = \theta C_1 + (1-\theta)C_2 - \lambda E. \tag{77}$$

where λ is exposure to global emissions, interpreted as climate shocks, and $E = \theta e_1 + (1-\theta)e_2$.

The Voting Problem. The agents' utilities in an economy with a carbon tax are $U_i^\tau = gh_i - \lambda E^\tau$ for $i = 1, 2$. On the other hand, in the laissez-faire economy the standard agent's utility is $U_1^* = \pi h_1 - \lambda E^*$ and for the environmental agent is $U_2^* = gh_2 - \lambda E^*$. The standard agent votes for the tax if $U_1^\tau > U_1^*$.

$$U_1^\tau - U_1^* = -(\pi - g)h_1 + \lambda(E^* - E^\tau) = -(\pi - g)h_1 + \lambda\theta h_1 > 0 \ \text{ if } \ \lambda\theta > \pi - g.$$

So, the standard agent supports the tax if $\lambda\theta > \pi - g$ but does not support it otherwise.
On the other hand, the environmental agent internalizes the possible emission reduction associated with its action to support the tax and votes in favour of the tax if

$$U_2^\tau + \underbrace{\eta\frac{(E^* - E^\tau)}{1-\theta}}_{\text{voting benefit}} > U_2^*$$

which can be rewritten as

$$\lambda(E^* - E^\tau) + \eta\frac{(E^* - E^\tau)}{1-\theta} = \lambda\theta h_1(1 + \frac{\eta}{1-\theta}) > 0$$

So the environmental agent always votes in favour of the tax. The median voter threshold is therefore

$$\tau_{0.5} = \begin{cases} \pi - g & \text{if} \ \ \theta < 0.5 \\ \pi - g & \text{if} \ \ \theta > 0.5 \quad \text{and} \quad \pi - g < \lambda\theta \\ 0 & \text{otherwise.} \end{cases} \tag{78}$$

The Voting Problem with Markets. Let us consider first the case in which there are sufficient funds to finance the transition of all standard agents. Standard agent, who act as borrowers, prefer markets since

$$\mathcal{B}_1 = gh_1 + \rho h_1 > U_1^\tau = gh_1 \tag{79}$$

The environmental agent's utility, who acts as lender, is

$$\mathcal{L}_2 = gh_2 + (\eta - \rho)q_2 - \lambda E^\rho = gh_2 + (\eta - \rho)\frac{\theta}{1-\theta}h_1 \tag{80}$$

and the agent votes for tax if

$$U_2^\tau + \underbrace{\eta\frac{\Delta E}{1-\theta}}_{\text{voting benefit}} > \mathcal{L}_2$$

- If the voting benefit depends on $\Delta E = E^* - E^\tau = \theta h_1$, then the green agent votes in favour of the tax

$$gh_2 + \eta\frac{\theta}{1-\theta}h_1 > gh_2 + \eta\frac{\theta}{1-\theta}h_1 - \rho\frac{\theta}{1-\theta}h_1 \iff$$
$$0 > -\rho\frac{\theta}{1-\theta}h_1 \tag{81}$$

- If the voting benefit depends on $\Delta E = E^\rho - E^\tau = 0$, then the green agent preferes markets

$$gh_2 < gh_2 + (\eta - \rho)\frac{\theta}{1-\theta}h_1$$

  so if there is no voting benefit both agents prefer markets and there is no support for the tax.

In sum, if carbon-contingent markets are big, we can have a situation when the standard agents does not vote for the tax while the green one does. This occurs when the benefit of voting for the tax is evaluated relative to a laissez faire economy. However, if the voting benefit is evaluated relative to the economy with markets, which in this case can reduce all emissions, then there is no voting benefit and both agents prefer markets.

Let us now consider the case when there are insufficient funds to finance the transition of all standard agents. The standard agent's problem, who acts as borrower, is

$$\mathcal{B}_1 = gh_1 + \rho h_1 - \lambda(\theta - \theta_d)h_1 > U_1^\tau = gh_1 \tag{82}$$

so the agent prefers market if $\rho = \pi - g > \lambda(\theta - \theta_d)$.
The green agent's utility, who acts as lender, is

$$\mathcal{L}_2 = gh_2 + (\eta - \rho)\frac{\theta_d}{1-\theta}h_1 - \lambda(\theta - \theta_d)h_1 \tag{83}$$

- If the voting benefit depends on $\Delta E = E^* - E^\tau = \theta h_1$, then the green agent votes in favour of the tax if

$$gh_2 + \eta\frac{\theta}{1-\theta}h_1 > gh_2 + (\eta - \rho)\frac{\theta_d}{1-\theta}h_1 - \lambda(\theta - \theta_d)h_1$$
$$0 > -\eta\frac{\theta - \theta_d}{1-\theta}h_1 - \rho\frac{\theta_d}{1-\theta}h_1 - \lambda(\theta - \theta_d)h_1 \tag{84}$$

  so the green agent votes in favour of the tax. However, by comparing (81) with (84) we can see that tax support is even stronger when carbon-contingent markets are small, which is in line with the baseline model.

- If the voting benefit depends on $\Delta E = E^\rho - E^\tau = (\theta - \theta_d)h_1$, then the green agent prefers tax if

$$gh_2 + \eta\frac{(\theta-\theta_d)h_1}{1-\theta} > gh_2 + (\eta-\rho)\frac{\theta_d}{1-\theta}h_1 - \lambda(\theta-\theta_d)h_1$$

$$0 > -\eta\frac{\theta-2\theta_d}{1-\theta}h_1 + \rho\frac{\theta_d}{1-\theta}h_1 - \lambda(\theta-\theta_d)h_1$$

In sum, if markets are small, agents' preference for the market solution decreases.

A.2 Welfare Variation

$$W = \theta U_1 + (1-\theta)U_2 - \lambda E.$$

Welfare with a tax is

$$W = \theta g h_1 + (1-\theta)gh_2$$

Welfare with financial markets and sufficient funds

$$W = \theta g h_1 + (1-\theta)gh_2 + \eta\theta h_1$$

is higher than with tax and higher than laissez-faire.
Welfare with financial markets and insufficient funds

$$\begin{aligned} W &= (\theta-\theta_d)\pi h_1 + \theta_d(gh_1+\rho h_1) - (1-\theta)(gh_2 + (\eta-\rho)\frac{\theta_d h_1}{1-\theta}) - \lambda(\theta-\theta_d)h_1 \\ &= \theta\pi h_1 + (1-\theta)gh_2 - (\pi-g)\theta_d h_1 + \eta\theta_d h_1 - \lambda(\theta-\theta_d)h_1 \\ &= \theta\pi h_1 + (1-\theta)gh_2 + [\lambda-(\pi-g)]\theta_d h_1 + \eta\theta_d h_1 - \lambda\theta h_1 \end{aligned}$$

is higher than in laissez-faire and higher than economy with tax if $(\eta-\rho)\theta_d > \lambda(\theta-\theta_d)$.

A.3 Security Payoff Specification

Consider security payoff with exchange of principal and fixed rate

$$\bar{r}d_1 - \rho(\bar{e}_i - e_i^\rho).$$

The Borrower's Problem. A borrower entering contract (15) can get rewarded upon reducing emissions below the benchmark. Consider first the case of the standard agent $i=1$. If there is no carbon tax $\tau = 0$, then benchmark emissions in (15) are $\bar{e}_1 = h_1$ and the standard agent can profit if she produces less emissions relative to this benchmark, that is, if $e_1^\rho < \bar{e}_1$. The problem solved by the standard agent if it were to borrow d_1 through the issuance of carbon-contingent debt is

$$\mathcal{B}_1 = \max_{I_\pi, I_g} \ \pi I_\pi + gI_g - \bar{r}d_1 + \rho(h_1 - I_\pi) \ \text{ such that } I_g + I_\pi \le h_1 + d_1, \tag{85}$$

which yields solution $I_g = h_1 + d_1$ if $\rho \ge \pi - g$, and $I_g = 0$ otherwise.[35] When determining whether or not to issue a carbon-contingent security, the agent compares the utility from borrowing with the utility in the laissez-faire economy with no carbon tax and no markets $U_1^* = \pi h_1$. If the price of carbon implied by the carbon-contingent debt contract is sufficiently high to incentivize the transition to the green technology,

[35] Here we implicitly assume that when indifferent on the intensive margin, that is, when $\rho = \pi - g$ the agent always prefers to implement the green technology. Relaxing the assumption does not change the equilibrium outcome.

i.e. $\rho \geq \pi - g$, the utility of the standard agent who acts as a borrower is

$$\mathcal{B}_1 = g(h_1 + d_1) - \bar{r}d_1 + \rho h_1 = (g + \rho)h_1 + (g - \bar{r})d_1 \geq \pi h_1 + (g - \bar{r})d_1, \tag{86}$$

which is higher than the utility from using internal finance only, $U_1^* = \pi h_1$, if $\bar{r} \leq g$. However, if the contingent rate is not sufficiently high to incentivize switching to the green technology, i.e. $\rho < \pi - g$, the standard agents' utility from borrowing is

$$\mathcal{B}_1 = \pi(h_1 + d_1) - \bar{r}d_1 - \rho d_1 = \pi h_1 - \bar{r}d_1 + (\pi - \rho)d_1 > \pi h_1 - (\bar{r} - g)d_1, \tag{87}$$

which is higher than the utility from using internal finance only, $U_1^* = \pi h_1$, if $\bar{r} \leq g$.

If there is a carbon tax, then the standard agent's benchmark emissions are $\bar{e}_1 = 0$, and her utility from borrowing is

$$\mathcal{B}_1^\tau = \max_{I_\pi, I_g} \; \pi I_\pi + g I_g - \tau I_\pi - \bar{r}d_1 - \rho I_\pi \;\; \text{such that } I_g + I_\pi \leq h_1 + d_1, \tag{88}$$

which yields $I_g = h_1 + d_1$ if $\tau + \rho \geq \pi - g$. This holds at the optimal tax $\tau = \pi - g$, with the implication that the agent will optimally invest in the non-polluting technology irrespective of the rate ρ. Thus, there is no contingent component associated with the payoff in (15), which will simply degenerate into a fixed payoff $\bar{r}d_1$, with the agent being willing to borrow if $\bar{r} \leq g$, i.e. $\mathcal{B}_1^\tau = g(h_1 + d_1) - \bar{r}d_1 \geq U_1^\tau = gh_1$. In other words, the debt contract is reduced to a plain vanilla contract with a fixed rate and serves no role in incentivizing emission reduction.[36] We abstract from such equilibria that do not entail a technology shift or a contingent payment.

Consider now the case of the environmentally-oriented agent $i = 2$, who internalizes the emissions associated with her actions. If through borrowing the agent reduces her emissions she will register a utility gain proportional to this reduction and the green preference parameter, i.e. $\eta(\bar{e}_2 - e_2^\rho)$. However, independently of whether a tax exists or not, this green agent prefer to invest in the non-polluting technology so her benchmark emissions in the absence of external financing are $\bar{e}_2 = 0$ and she cannot reduce emissions further. Because of the technology constraint which does not allow producing negative emissions, the best she can do is keep investing in her preferred green technology and enable other agents to reduce their emissions by participating in carbon-contingent financing markets.Upon issuing the carbon-contingent security, she faces the following investment problem

$$\mathcal{B}_2^\tau = \max_{I_\pi, I_g} \; \pi I_\pi + g I_g - \tau I_\pi - \bar{r}d_2 - \rho I_\pi - \eta I_\pi \;\; \text{such that } I_g + I_\pi \leq h_2 + d_2. \tag{89}$$

Investing in the green technology is optimal if $\tau + \rho + \eta \geq \pi - g$, which holds for any tax $\tau \geq 0$ or contingent rate $\rho \geq 0$ since $\eta \geq \pi - g$. So there will be no contingent component associated with the payoff in (15), which will simply degenerate into a fixed payoff $\bar{r}d_2$. For the environmental agents to be willing to borrow, it must be that the fixed rate satisfies $\bar{r} \leq g$ i.e. that is when $\mathcal{B}_2 = \mathcal{B}_2^\tau = g(h_2 + d_2) - \bar{r}d_2 \geq U_2^* = U_2^\tau = gh_2$.

We now determine the equilibrium market price of carbon implied by the lending rate ρ, the baseline return $\bar{r}$, and the supply of credit to the standard agent $i = 1$ by solving the lender's problem.

The Lender's Problem. Standard agents $i = 1$ decide the optimal amount of lending d_1, and invest the remainder $h_1 - d_1$ in their preferred technology. An individual lender i is responsible for a share q_i of the carbon emission reductions generated through carbon-contingent markets, which is determined in

[36] We assume a technology constraint whereby emissions can be at most reduced to zero and cannot be negative.

equilibrium. If the standard agents were to act as lenders, then their problem would be

$$\mathcal{L}_1 = \max_{d_1 \leq h_1} (\pi - \tau)(h_1 - d_1) + \bar{r}d_1 - \rho q_1, \tag{90}$$

If the borrower were an environmental agent or a standard agent subject to a carbon tax, the emission reduction financed was $q_1 = 0$. In such scenario, the standard agent is willing to lend at the fixed interest rate $\bar{r} \geq g$ in the presence of the tax $\tau = \pi - g$, or at the rate $\bar{r} \geq \pi$ if there is no tax $\tau = 0$.[37] The utility from lending to another standard agent which can deliver a positive emission reduction $q_1 > 0$ (i.e., in absence of the tax) is strictly lower than that from not entering the contract for any $\rho > 0$ and any fixed rate that would be acceptable by the borrower (which must satisfy $\bar{r} \leq g$). Therefore, plain vanilla lending at the rate $\bar{r} = g$ (to environmental agents or standard agents subject to a tax) could occur in equilibrium but we abstract from such equilibria which do not involve a technology switch nor a contingent payoff.

Environmental agents $i = 2$ decide the optimal amount of lending d_2, and invest the remainder $h_2 - d_2$ in the green technology. They receive a variable payoff, $\bar{r}d_2 - \rho q_2$, which depends on the carbon emissions reduction enabled through carbon-contingent markets in equilibrium. Given that the environmental agent generates no emissions by investing in her preferred green technology, she will only internalize the emissions associated with lending, which are captured by the equilibrium emissions associated with the carbon-contingent security, q_2. The environmental agent's utility from acting as a lender is

$$\mathcal{L}_2 = \max_{d_2 \leq h_2} g(h_2 - d_2) + \bar{r}d_2 - \rho q_2 + \eta q_2, \tag{91}$$

where the first term is the return from investing in the green technology, the subsequent two terms are the cash flows associated with the contingent security, and the last term captures green preferences related to the emissions associated with the agent's lending action. This lending problem is subject to the financing constraint

$$g(h_2 - d_2) + \bar{r}d_2 - \rho q_2 \geq 0, \tag{92}$$

which captures the idea that although this class of investors may be willing to reward emission reductions, they will only do so up to the point that they deplete their wealth.

From (91), it follows that the fixed indifference rate at which the environmental agent is willing to lend any amount $d_2 \in [0, h_2]$ must satisfy $\bar{r} \geq g$, i.e. $\mathcal{L}_2 \geq U_\tau^* = U_2^* = gh_2$. The rest of the analysis is the same as in the main body of the paper.

A.4 Voting Benefit Variation

When markets are sufficiently large to fund transition of all agents, emissions are zero and relative to this counterfactual markets provide no additional emissions reduction benefit and the voting benefit in (29) is zero. Consequently, the support of environmental agents becomes

$$Support_2\ \tau|\rho = U_2^\tau - (gh_2 + (\eta - \rho)\frac{\theta}{1-\theta}h_1) = -(\eta - \rho)\frac{\theta}{1-\theta}h_1 \tag{93}$$

which is even lower than that in (29).

When carbon-contingent markets are small and can only fund the transition of a fraction of standard

[37]That is because with a tax the rate must satisfy $\mathcal{L}_1^\tau = g(h_1 - d_1) + \bar{r}d \geq U_1^\tau = gh_1$ and without a tax it must satisfy $\mathcal{L}_1 = \pi(h_1 - d_1) + \bar{r}d_1 \geq U_1^* = \pi h_1$.

agents and thus partially reduce aggregate emissions, support is

$$Support_2\ \tau|\rho = U_2^\tau + \eta\frac{\theta-\theta_d}{1-\theta}h_1 - (gh_2 + (\eta-\rho)\frac{\theta_d}{1-\theta}h_1) = \eta\frac{\theta-\theta_d}{1-\theta}h_1 + \rho\frac{\theta_d}{1-\theta}h_1,$$

meaning that support for the tax is even lower relative to the case in (31).

B Extended Model

B.1 Welfare without Financial Markets

Proof [Proposition 5]. In the absence of financial markets, welfare for a given tax τ reads

$$\mathcal{W}^\tau = \int_0^1 (C_i(\delta_i^\tau) - \tau e_i(\delta_i^\tau) + \kappa^\tau)di - \lambda E^\tau \tag{94}$$

with $C_i(\delta_i^\tau) = \pi - \frac{1}{2}\phi(\delta_i^\tau)^2$, total emissions $E^\tau = \int_0^1 e_i(\delta_i^\tau)di$ and $\delta_i^\tau = \frac{\eta_i+\tau}{\phi}$. Substituting these terms and recalling the revenue-neutrality condition $\int_0^1 \kappa^\tau di = \mathcal{R}^\tau = \int_0^1 \tau e_i(\delta_i^\tau)$ we get

$$\begin{aligned}
\mathcal{W}^\tau &= \int_0^1 (\pi - \frac{1}{2}\frac{(\eta_i^2+\tau^2+2\eta_i\tau)}{\phi})di - \lambda\int_0^1(1-\frac{\eta_i+\tau}{\phi})di \\
&= \int_0^1 (\pi - \frac{1}{2}\frac{\eta_i^2}{\phi})di - \lambda\int_0^1(1-\frac{\eta_i}{\phi})di - \int_0^1(\frac{1}{2}\frac{\tau^2}{\phi} + \frac{\eta_i\tau}{\phi})di + \lambda\int_0^1\frac{\tau}{\phi}di \\
&= \mathcal{W}^* - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\bar\eta\tau}{\phi} + \lambda\frac{\tau}{\phi}.
\end{aligned} \tag{95}$$

Therefore, the welfare gain $\mathcal{W}^\tau - \mathcal{W}^*$ is zero for $\tau = 0$, increasing in $\tau \in [0, \lambda-\bar\eta]$, reaching a global maximum in $\lambda - \bar\eta$, and decreasing for $\tau \in (\lambda-\bar\eta, 2(\lambda-\bar\eta)]$, reaching again zero in $2(\lambda-\bar\eta)$. Given the choice of constant tax rebates κ^τ across agents $i \in [0,1]$, we showed in the main text that each agent i's willingness to support the tax increases monotonically in i. Therefore, the political constraint is expressed in terms of the maximum tax $\tau_{0.5}$ that makes the median voter $i = 0.5$ indifferent between supporting or not the regulation. Any tax τ strictly higher that $\tau_{0.5}$ violates the political constraint. It is therefore immediate to see that if $\tau_{0.5} \in (0, \lambda-\bar\eta)$, the highest welfare gain that the regulator can achieve is when $\tau = \tau_{0.5}$. If $\tau_{0.5} < 0$ then the regulator can only impose $\tau = 0$, whereas if $\tau_{0.5} > \lambda - \bar\eta$ then the regulator can maximize welfare by imposing a tax $\tau = \lambda - \bar\eta$.

Rebates Design

For a generic distribution of rebates κ_i^τ, the regulator's problem can be expressed as

$$\max_{\tau\geq 0} \mathcal{W}^\tau \quad \text{such that} \quad \int_0^1 \big(1\{Support_i\ \tau \geq 0\}\big)di \geq 0.5. \tag{96}$$

with $\mathcal{W}^\tau$ as in (95) and support for the tax (45)

$$Support_i\ \tau = \eta_i\frac{\tau}{\phi\mathcal{G}} + \frac{1}{2}\frac{\tau^2}{\phi} - \tau + \frac{\tau\eta_i}{\phi} + \kappa_i^\tau. \tag{97}$$

Suppose that the regulator decides to allocate the tax proceeds heterogeneously across agents in such a way that each agent i is indifferent between voting or not for the tax. For all voters to support the tax, i.e. $\mathcal{G} = 1$, the tax rebates κ_i^τ should verify

$$Support_i\ \tau = \eta_i \frac{\tau}{\phi} + \frac{1}{2}\frac{\tau^2}{\phi} - \tau + \frac{\tau\eta_i}{\phi} + \kappa_i^\tau = 0. \tag{98}$$

Solving for κ_i^τ we get

$$\tag{99}$$

Note that the first term τe_i^τ is the payment that the regulator receives from agent i whereas agent's green preference η_i enters the rebate negatively, with the implication that agents with lower green preferences will receive higher rebates and viceversa. The regulator's budget constraint imposes that the total rebates do not exceed the tax proceeds, that is

$$\int_0^1 \kappa_i^\tau di \le \mathcal{R}^\tau \quad \longleftrightarrow \quad \int_0^1 \tau(e_i^\tau - \frac{\eta_i}{\phi} + \frac{1}{2}\frac{\tau}{\phi})di \le \int_0^1 \tau e_i^\tau di. \tag{100}$$

The inequality is verified provided that the average green preference satisfies $\frac{1}{2}\lambda \le \bar{\eta}$. Furthermore, as discussed around Proposition 4, the average green preference must also satisfy $\bar{\eta} \le \lambda$, so the admissible range of values for average green preferences is $\bar{\eta} \in [\frac{1}{2}\lambda, \lambda]$. In such a case, an agent-specific tax rebate can circumvent the political constraint.

B.2 The Financing Problem

The borrower's problem. From the point of view of an issuer of the carbon-contingent security, for a given carbon tax τ, the abatement problem reads

$$\mathcal{B}_i = \max_{\delta_i} \pi - \phi(\delta_i) - \tau e_i(\delta_i) + \kappa^{\tau,\rho} - \eta_i e_i(\delta_i) + \rho(\bar{e}_i - e_i(\delta_i)) \ \text{ such that } \delta_i \le 1, \tag{101}$$

with $\delta_i \le 1$ a technology constraint which captures the idea that abatement can at most reduce emissions to zero (but not negative), $\kappa^{\tau,\rho}$ the lump-sum payment tax rebate when markets exist, and $\bar{e}_i$ the benchmark emissions that agent i produces if not issuing the security. As for the simple model, the implicit assumption is that when determining whether or not to issue the security, the agent i acts as atomistic and takes the lump-sum payment $\kappa^{\tau,\rho}$ as given. This is because a change in agent i's financing strategy has a negligible impact on total emissions and thus on the tax proceeds in equilibrium. Solving for the optimal abatement we get

$$\delta_i^{\tau,\rho} = \delta_i^\tau + \frac{\rho}{\phi} \text{ if } \frac{\rho}{\phi} + \delta_i^\tau < 1 \tag{102}$$

and $\delta_i^{\tau,\rho} = 1$ otherwise.

We first consider the case in which the technology constraint is not violated and the problem admits an internal solution, i.e. $\delta_i^{\tau,\rho} < 1$, which is the case we focus on in the paper. Substituting the optimal abatement choice in the utility problem (101), and noting that $e_i(\delta_i^{\tau,\rho}) = e_i(\delta_i^\tau) - \frac{\rho}{\phi} = \bar{e}_i - \frac{\rho}{\phi}$, we get

$$\begin{aligned}
\mathcal{B}_i &= \pi - \phi(\delta_i^{\tau,\rho}) - (\eta_i + \tau)e_i(\delta_i^{\tau,\rho}) + \rho\frac{\rho}{\phi} + \kappa^{\tau,\rho} \\
&= \pi - \phi(\delta_i^\tau) - \frac{1}{2}\frac{\rho^2}{\phi} - \frac{\rho(\eta_i + \tau)}{\phi} - (\eta_i + \tau)(e_i(\delta_i^\tau) - \frac{\rho}{\phi}) + \rho\frac{\rho}{\phi} + \kappa^{\tau,\rho} \\
&= \pi - \phi(\delta_i^\tau) - (\eta_i + \tau)e_i(\delta_i^\tau) + \frac{1}{2}\frac{\rho^2}{\phi} + \kappa^{\tau,\rho} \pm \kappa^\tau \\
&= U_i^\tau - (\kappa^\tau - \kappa^{\tau,\rho}) + \frac{1}{2}\frac{\rho^2}{\phi}.
\end{aligned} \tag{103}$$

On the other hand, if the technology constraint is violated and a corner solution is obtained, i.e. $\delta_i^{\tau,\rho} = 1$ such that $e(\delta_i^{\tau,\rho}) = 0$, the utility is

$$\begin{aligned}\mathcal{B}_i &= \pi - \phi(\delta_i^{\tau,\rho}) - (\eta_i + \tau)e_i(\delta_i^{\tau,\rho}) + \rho(1-\delta_i^{\tau}) + \kappa^{\tau,\rho} \\ &= \pi - \frac{1}{2}\phi + \rho(1-\delta_i^{\tau}) + \kappa^{\tau,\rho}.\end{aligned} \tag{104}$$

The lender's problem. From the point of view of the lender, abatement strategies are kept fixed and the investment problem simply reads

$$\mathcal{L}_i = \max_{q_i} C_i^{\tau,\rho} - \eta_i e_i(\delta_i^{\tau}) + \eta_i q_i - \rho q_i \quad \text{such that } C_i^{\tau,\rho} - \rho q_i \geq 0, \tag{105}$$

with $C_i^{\tau,\rho}$ the consumption evaluated at the abatement rate which would be optimal in a tax-only economy δ_i^{τ}, namely $C_i^{\tau,\rho} = C_i^{\tau,\rho}(\delta_i^{\tau}) = \pi - \phi(\delta_i^{\tau}) - \tau e_i(\delta_i^{\tau}) + \kappa^{\tau,\rho}$. Solving for the optimal quantity of emissions financed

$$q_i^{\tau,\rho} = \begin{cases} \frac{C_i^{\tau,\rho}}{\rho} & \text{if } \eta_i > \rho, \\ [0, \frac{C_i^{\tau,\rho}}{\rho}] & \text{if } \eta_i = \rho \\ 0 & \text{otherwise.} \end{cases} \tag{106}$$

which yields

$$\begin{aligned}\mathcal{L}_i &= C_i^{\tau,\rho} - \eta_i e_i(\delta^{\tau}) + (\eta_i - \rho)q_i^{\tau,\rho} \\ &= \pi - \phi(\delta_i^{\tau}) - \tau e_i(\delta_i^{\tau}) + \kappa^{\tau,\rho} - \eta_i e_i(\delta^{\tau}) + (\eta_i - \rho)q_i^{\tau,\rho} \pm \kappa^{\tau} \\ &= U_i^{\tau} - (\kappa^{\tau} - \kappa^{\tau,\rho}) + (\eta_i - \rho)q_i^{\tau,\rho}.\end{aligned} \tag{107}$$

Single-Crossing Condition. Let us focus on the case in which the abatement problem of the issuer admits an internal solution (i.e., the technology constraint is not violated). The net gains from issuance of the carbon-contingent security are

$$\Pi_i = \mathcal{B}_i - \mathcal{L}_i = \frac{\rho^2}{2\phi} - (\eta_i - \rho)q_i^{\tau,\rho} \tag{108}$$

We want to prove that

$$\frac{\partial}{\partial i}\Pi_i < 0 \tag{109}$$

for each $i \in [0,1]$. The first term in (108) is a constant function of the type i. On the other hand, the second term in (108) is equal to zero if $\eta_i \leq \rho$ and equal to $(\eta_i - \rho)\frac{C_i^{\tau,\rho}}{\rho}$ for $\eta_i > \rho$. To prove (109), it is therefore sufficient to prove that

$$\frac{d}{di}\left((\eta_i - \rho)C_i^{\tau,\rho}\right) > 0 \text{ for } \eta_i > \rho. \tag{110}$$

Recalling the expression for consumption

$$\begin{aligned}C_i^{\tau,\rho} &= \pi - \phi(\delta_i^{\tau}) - \tau e_i(\delta_i^{\tau}) + \kappa^{\tau,\rho} = \pi - \frac{1}{2}\frac{\eta_i^2 + \tau^2 + 2\eta_i\tau}{\phi} - \tau\left(1 - \frac{\eta_i + \tau}{\phi}\right) + \kappa^{\tau,\rho} \\ &= \pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\eta_i^2}{\phi} - \tau + \frac{\tau^2}{\phi} + \kappa^{\tau,\rho}\end{aligned} \tag{111}$$

Tax rebates in the presence of markets are $\kappa^{\tau,\rho} = \kappa^{\tau} - \tau\int_{\mathcal{B}} \frac{\rho}{\phi} di$. Since we are looking for the monotonicity condition, we substitute $\mathcal{B} = [0,i]$ which gives $\kappa^{\tau,\rho} = \kappa^{\tau} - \tau\frac{\rho i}{\phi} = \tau\left(1 - \frac{\bar{\eta}}{\phi} - \frac{\tau}{\phi}\right) - \frac{\tau\rho i}{\phi}$. Therefore the condition becomes

$$\frac{d}{di}\big((\eta_i - \rho)\big(\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\eta_i^2}{\phi} - \frac{\tau\bar{\eta}}{\phi} - \frac{\tau\rho i}{\phi}\big)\big) > 0. \tag{112}$$

Expanding the derivative, we get

$$\eta_i^{'}\big(\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\eta_i^2}{\phi} - \frac{\tau\bar{\eta}}{\phi} - \frac{\tau\rho i}{\phi}\big) - (\eta_i - \rho)(\frac{\eta_i}{\phi}\eta_i^{'} + \frac{\tau\rho}{\phi}) > 0. \tag{113}$$

Dividing everything by $\eta_i^{'}$, with $\eta_i^{'} > 0$, we have

$$\pi - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{1}{2}\frac{\eta_i^2}{\phi} - \frac{\tau\bar{\eta}}{\phi} - \frac{\tau\rho i}{\phi} - (\eta_i - \rho)(\frac{\eta_i}{\phi} + \frac{\tau\rho}{\phi\eta_i^{'}}) > 0 \tag{114}$$

Since the inequality must hold for any i such that $\eta_i > \rho$, a sufficient condition is

$$\pi > \frac{1}{2}\frac{\tau^2}{\phi} + \frac{1}{2}\frac{\eta_i^2}{\phi} + \frac{\tau\bar{\eta}}{\phi} + \frac{\tau i\rho}{\phi} \tag{115}$$

which evaluated in the highest type $i = 1$ becomes

$$\pi > \frac{1}{2}\frac{\tau^2}{\phi} + \frac{1}{2}\frac{\eta_1^2}{\phi} + \frac{\tau\bar{\eta}}{\phi} + \frac{\tau\rho}{\phi}. \tag{116}$$

Such condition is always verified in our admissible range of model parameters in that, in order to justify a certain amount of emissions e_i deriving from each agent i's investment choices, we assumed high profitability $\pi > \frac{1}{2}\phi$, with $\phi >> 1$ to justify abatement choices below the technology threshold $\delta_i^* = \frac{\eta_i}{\phi} < 1$ for each $i \in [0, 1]$.
A similar argument applies to the case in which the technology constraint is violated. After rearrangement of the utility in (104), net profits read

$$\Pi_i = \mathcal{B}_i - \mathcal{L}_i = (1 - \delta_i^{\tau})\rho - \frac{1}{2}\phi(1 - \delta_i^{\tau})^2 - (\eta_i - \rho)q_i^{\tau,\rho} \tag{117}$$

for $\eta_i \leq \rho$, profits from issuance of the security are strictly smaller than $\frac{\rho^2}{2\phi}$ and decreasing in i, while for $\eta_i > \rho$, the lender's profits are left unaltered and are decreasing in i provided the sufficient condition in (116) is verified.

Proof. **[Proposition 6]** Provided that π verifies the condition in (116), we look for a cutoff type $x \in (0, 1)$ and a rate ρ such that the market clearing condition and the indifference condition are jointly verified

$$\int_0^x (\bar{e}_i - e_i(\delta_i^{\tau,\rho}))di = \int_x^1 q_i^{\tau,\rho} di \quad \text{and} \quad \Pi_x = 0. \tag{118}$$

For any $i < x$, the borrower's net profits from issuance of the security in (108) are strictly positive. Therefore, it must be that the indifference type x is such that $\eta_x > \rho$. The conditions become

$$\int_0^x \frac{\rho}{\phi} di = \int_x^1 \frac{C_i^{\tau,\rho}}{\rho} di, \quad \frac{\rho^2}{2\phi} - (\eta_x - \rho)\frac{C_i^{\tau,\rho}}{\rho} = 0 \tag{119}$$

which rearranged become

$$\rho = \sqrt{\frac{1}{x}\phi \int_x^1 C_i^{\tau,\rho} di}, \quad \frac{\rho^2}{2\phi} = (\eta_x - \rho)\frac{C_x^{\tau,\rho}}{\rho}. \tag{120}$$

from which the equilibrium pair ρ and x is determined numerically.
Now if $\rho + \eta_x > \phi - \tau$, then an equilibrium of this type cannot exist in that we have reached a corner

solution in which the rate ρ is so high that the optimal abatement technologies of some issuers $0 < i < x$ go beyond the available cleanest technology, $\delta = 1$. In such a corner solution, profits from issuing the contingent security are defined as in (117). Provided that π verifies (116), the indifference condition becomes

$$(1-\delta_x^\tau)\rho - \frac{1}{2}\phi(1-\delta_x^\tau)^2 = (\eta_x - \rho)\frac{C_x^{\tau,\rho}}{\rho}, \tag{121}$$

whereas the market clearing condition becomes

$$\int_0^{\underline{x}} \frac{\rho}{\phi} di + \int_{\underline{x}}^{x} (1-\delta_i^\tau) di = \int_x^1 \frac{C_i^{\tau,\rho}}{\rho} di \tag{122}$$

with $\underline{x} < x$ the type whose optimal abatement verifies $\delta_{\underline{x}}^\tau = 1 - \frac{\rho}{\phi}$.

B.3 Equilibrium Emissions Reduction Financed

Let us show that the equilibrium emissions reduction purchased by each agent $i \in [x^\tau, 1]$ decreases with the tax τ. This amounts to proving that

$$\frac{\partial}{\partial \tau} q_i^{\tau,\rho^\tau} = \frac{\partial}{\partial \tau}\frac{C_i^{\tau,\rho^\tau}}{\rho^\tau} < 0. \tag{123}$$

Recalling that both the equilibrium price of carbon ρ^τ as well as the consumption C_i^{τ,ρ^τ} are decreasing in τ, we can express the derivative as

$$\frac{\partial}{\partial \tau}\frac{C_i^{\tau,\rho^\tau}}{\rho^\tau} = \frac{1}{\rho^\tau}\frac{C_i^{\tau,\rho^\tau}}{\rho^\tau}|\frac{\partial \rho^\tau}{\partial \tau}| - \frac{1}{\rho^\tau}|\frac{\partial C_i^{\tau,\rho^\tau}}{\partial \tau}|. \tag{124}$$

From the equilibrium expression for ρ^τ in (120), we notice that the price ρ^τ increases when x^τ decreases. Since the equilibrium cutoff type x^τ decreases when the tax become lower, then an upperbound on the magnitude of the derivative $|\frac{\partial \rho^\tau}{\partial \tau}|$ is determined by taking x^τ as a constant. That is, we have

$$|\frac{\partial \rho^\tau}{\partial \tau}| \geq \frac{1}{2}\frac{1}{\rho^\tau}\frac{\phi}{x^\tau}\int_{x^\tau}^1 |\frac{\partial C_i^{\tau,\rho^\tau}}{\partial \tau}| di \tag{125}$$

Noticing that the derivative $|\frac{\partial C_i^{\tau,\rho^\tau}}{\partial \tau}|$ is independent of i, we can take the term out of the integral and get

$$|\frac{\partial \rho^\tau}{\partial \tau}| \geq \frac{1}{2}\frac{1}{\rho^\tau}\frac{\phi(1-x^\tau)}{x^\tau}|\frac{\partial C_i^{\tau,\rho^\tau}}{\partial \tau}| \tag{126}$$

Therefore, we have that

$$\frac{\partial}{\partial \tau}\frac{C_i^{\tau,\rho^\tau}}{\rho^\tau} \geq \frac{1}{\rho^\tau}\frac{C_i^{\tau,\rho^\tau}}{\rho^\tau}\frac{1}{2}\frac{1}{\rho^\tau}\frac{\phi(1-x^\tau)}{x^\tau}|\frac{\partial C_i^{\tau,\rho^\tau}}{\partial \tau}| - \frac{1}{\rho^\tau}|\frac{\partial C_i^{\tau,\rho^\tau}}{\partial \tau}| \tag{127}$$

from which it follows that the condition (123) in satisfied if

$$(\frac{C_i^{\tau,\rho^\tau}}{\rho^\tau}\frac{1}{2}\frac{1}{\rho^\tau}\frac{\phi(1-x^\tau)}{x^\tau} - 1) < 0. \tag{128}$$

Recalling that $q_i^{\tau,\rho^\tau} = \frac{C_i^{\tau,\rho^\tau}}{\rho^\tau}$ for each $i \in [x^\tau, 1]$, we have, rearranging

$$q_i^{\tau,\rho^\tau} < 2\big(\frac{\rho^\tau x^\tau}{\phi}\frac{1}{1-x^\tau}\big) \text{ for } i \in [x^\tau, 1]. \tag{129}$$

The left-hand side is the emissions reduction financed by lender $i \in [x^\tau, 1]$, while the right-hand side is twice the total emissions reduction supplied by the borrowers, $2\frac{\rho^\tau x^\tau}{\phi}$, divided by the share of lenders in the economy, $\frac{1}{1-x^\tau}$. We note that due to the market clearing condition, the right-hand side is twice the amount of emissions reduction that each agent $i \in [x^\tau, 1]$ would finance if all agents had the same budget constraint. In practice then, the inequality is stating that each agent i cannot hold more than twice of what she would be entitled if consumption constraints were homogenous across agents. Since we assume equal endowments $h_i = 1\$$ and since the marginal effect of η_i on q_i^{τ,ρ^τ} is limited by the technology constraint $\eta_i << \phi$, this inequality is satisfied in our framework.

B.4 Welfare with Financial Markets

Welfare with financial markets reads

$$\begin{aligned}
\widehat{\mathcal{W}}^\tau &= \int_0^{x^\tau} \Big(C_i(\delta_i^{\tau,\rho^\tau}) - \tau e_i(\delta_i^{\tau,\rho^\tau}) + \kappa^{\tau,\rho^\tau} + \rho^\tau(\bar{e}_i - e_i(\delta_i^{\tau,\rho^\tau}))\Big)di \\
&+ \int_{x^\tau}^1 \Big(C_i(\delta_i^\tau) - \tau e_i(\delta_i^\tau) + \kappa^{\tau,\rho^\tau} - \rho^\tau q_i^{\tau,\rho^\tau}\Big)di - \lambda E^{\tau,\rho^\tau} \\
&= \int_0^{x^\tau} \big((\pi - \phi(\frac{\eta_i + \rho^\tau + \tau}{\phi})) - \tau e_i(\delta_i^{\tau,\rho^\tau}) + \kappa^{\tau,\rho} + \frac{(\rho^\tau)^2}{\phi}\big)di - \lambda E^{\tau,\rho^\tau}
\end{aligned} \tag{130}$$

and substituting for the market clearing condition (59) and recalling that the tax revenues are fully redistributed

$$\widehat{\mathcal{W}}^\tau = \int_0^{x^\tau} \big((\pi - \phi(\frac{\eta_i + \tau}{\phi}) - \frac{1}{2}\frac{(\rho^\tau)^2}{\phi} - \frac{\eta_i\rho^\tau}{\phi} - \frac{\tau\rho^\tau}{\phi})) - \tau e_i(\delta_i^{\tau,\rho^\tau}) + \kappa^{\tau,\rho}\big)di + \int_{x^\tau}^1 C_i(\delta_i^\tau)di - \lambda E^{\tau,\rho^\tau} \tag{131}$$

$$= \int_0^1 (\pi - \phi(\frac{\eta_i + \tau}{\phi}))di - \int_0^{x^\tau} (\frac{1}{2}\frac{(\rho^\tau)^2}{\phi} + \frac{\eta_i\rho^\tau}{\phi} + \frac{\tau\rho^\tau}{\phi})di - \lambda E^{\tau,\rho^\tau} \tag{132}$$

$$= \int_0^1 (\pi - \phi(\frac{\eta_i}{\phi}))di - \lambda E^* - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\bar{\eta}\tau}{\phi} + \lambda\frac{\tau}{\phi} - \int_0^{x^\tau} (\frac{1}{2}\frac{(\rho^\tau)^2}{\phi} + \frac{\eta_i\rho^\tau}{\phi} + \frac{\tau\rho^\tau}{\phi})di + \lambda\frac{x^\tau\rho^\tau}{\phi} \tag{133}$$

$$= \mathcal{W}^* - \frac{1}{2}\frac{\tau^2}{\phi} - \frac{\bar{\eta}\tau}{\phi} + \lambda\frac{\tau}{\phi} - x^\tau\rho^\tau(\frac{1}{2}\frac{\rho^\tau}{\phi} + \frac{\bar{\eta}^\tau}{\phi} + \frac{\tau}{\phi}) + \lambda\frac{x^\tau\rho^\tau}{\phi} \tag{134}$$

$$= \mathcal{W}^\tau - x^\tau\rho^\tau(\frac{1}{2}\frac{\rho^\tau}{\phi} + \frac{\bar{\eta}^\tau}{\phi} + \frac{\tau}{\phi} - \frac{\lambda}{\phi}) \tag{135}$$

with $\bar{\eta}^\tau = 1/(x^\tau)\int_0^{x^\tau} \eta_i di$.

Proof. [Proposition 7]. Figure 11 shows the welfare function in (131) (blue line) against the baseline welfare in (95) in absence of financial markets (black line). The welfare function in (131) is concave in τ with a unique maximum verifying

$$\frac{d}{d\tau}\widehat{\mathcal{W}}^\tau\Big|_{\tau=\hat{\tau}^o} = 0. \tag{136}$$

Expanding the derivative, one gets

$$\begin{aligned}
\frac{d}{d\tau}\widehat{\mathcal{W}}^\tau &= \frac{d}{d\tau}\mathcal{W}^\tau - (\rho_\tau^\tau x^\tau + \rho^\tau x_\tau^\tau)(\frac{1}{2}\frac{\rho^\tau}{\phi} + \frac{\bar{\eta}^\tau}{\phi} + \frac{\tau}{\phi} - \frac{\lambda}{\phi}) - \rho^\tau x^\tau(\frac{1}{2}\frac{\rho_\tau^\tau}{\phi} + \frac{1}{\phi} + \frac{\bar{\eta}_\tau^\tau}{\phi}) \\
&= -\frac{\tau}{\phi} + \frac{\lambda - \bar{\eta}}{\phi} - (\rho_\tau^\tau x^\tau + \rho^\tau x_\tau^\tau)(\frac{1}{2}\frac{\rho^\tau}{\phi} + \frac{\bar{\eta}^\tau}{\phi} + \frac{\tau}{\phi} - \frac{\lambda}{\phi}) - \rho^\tau x^\tau(\frac{1}{2}\frac{\rho_\tau^\tau}{\phi} + \frac{1}{\phi} + \frac{\bar{\eta}_\tau^\tau}{\phi})
\end{aligned} \tag{137}$$

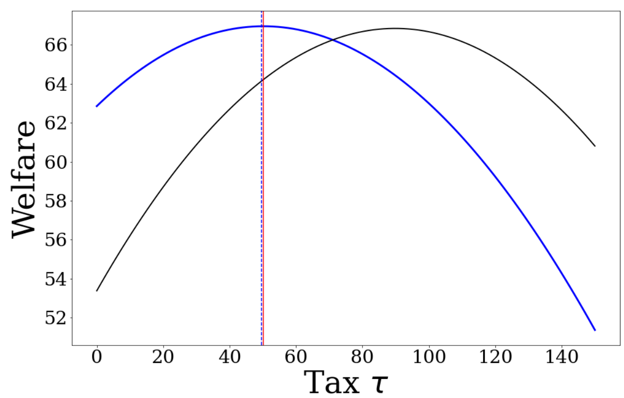
therefore solving for τ this gives the implicit expression

$$\hat{\tau}^o = \lambda - \frac{\bar{\eta} + \rho^o x^o(1 + \frac{1}{2}\rho_\tau^o + \bar{\eta}_\tau^o) + (\rho_\tau^o x^o + \rho^o x_\tau^o)(\frac{1}{2}\rho^o + \bar{\eta}^o)}{1 + (\rho_\tau^o x^\tau + \rho^o x_\tau^o)} \tag{138}$$

with ρ^o and x^o the equilibrium prices and cutoff type evaluated at the optimal tax in the presence of financial markets $\tau = \hat{\tau}^o$, ρ^o_τ and x^o_τ the derivatives evaluated in $\tau = \hat{\tau}^o$, $\bar{\eta}^o = \frac{1}{x^o}\int_0^{x^o} \eta_i di$ and $\bar{\eta}^o_\tau$ the derivative evaluated in $\tau = \hat{\tau}^o$.

Figure 11. Welfare

The blue line shows the welfare function in (131) against the tax rate τ. The black line is the reference welfare function in (95). The vertical red line is the optimal tax rate $\hat{\tau}^o$ verifying (138). The dashed blue line is the approximate rate in (139). Preferences are assumed convex $\eta_i = \eta i^4$ with $\eta =$ \$50/ton of carbon. Other model parameters are $\pi = 150$ and $\phi = 300$.



The red vertical line in Figure 11 indicates the optimal $\hat{\tau}^o$ solution in (138), the blue dashed line is an approximate solution which assumes that the change in the equilibrium price and cutoff type with respect to the tax is approximately zero and, in particular, they are the levels obtained in the absence of financial markets $\rho^\tau = \rho^*$ and $x^\tau = x^*$. As observed, given our model specification, changes in financial markets outcomes as a function of the tax have a negligible effect on the optimal tax rate. Consequently, we work with the approximate solution

$$\hat{\tau}^o \approx \lambda - \bar{\eta} - \rho^* x^*. \tag{139}$$

Note that the optimal tax rate with markets is below the optimal tax rate without markets $\tau^o = \lambda - \bar{\eta}$, by an amount that is proportional to the abatement provided by financial markets.

Let us now solve for the constrained optimal tax obtained by imposing the median voter constraint. As for the case without financial markets, the welfare function is increasing between $[0, \hat{\tau}^o]$ and decreasing afterwards. Therefore, if $0 < \hat{\tau}_{0.5} < \hat{\tau}^o$, the best outcome that the regulator can get is achieved by imposing a constrained tax $\hat{\tau}^c = \hat{\tau}_{0.5}$. If $\hat{\tau}_{0.5} < 0 < \hat{\tau}^o$, then the regulator's choice is bounded by the positivity constraint , $\hat{\tau}^c = 0$. Note also from (139) that the unconstrained tax can be negative $\hat{\tau}^o < 0$ if abatement provided by financial markets is large. In such a case, the regulator will also impose a tax $\hat{\tau}^c = 0$.

Proof. [Proposition 8]. Based on the result in Proposition 5, we can derive an explicit expression for welfare $\mathcal{W}^c$ in an economy without financial markets

$$\mathcal{W}^c = \begin{cases} \mathcal{W}^o = \mathcal{W}^* + \frac{1}{2}\frac{(\lambda - \bar{\eta})^2}{\phi} & \text{if } \tau^o = \lambda - \bar{\eta} < \tau_{0.5}, \\ \mathcal{W}^* + \tau_{0.5}\frac{\lambda - \bar{\eta} - \frac{1}{2}\tau_{0.5}}{\phi} & otherwise. \end{cases} \tag{140}$$

whereas emissions absent markets are

$$E^c = \begin{cases} E^o = E^* - \frac{\lambda - \bar{\eta}}{\phi} & \text{if } \lambda - \bar{\eta} < \tau_{0.5}, \\ E^* - \frac{\tau_{0.5}}{\phi} & otherwise. \end{cases} \tag{141}$$

The following scenario are possible:

1. if $\tau^c = \tau^o$, meaning if $\lambda - \bar{\eta} < \tau_{0.5}$, then $\mathcal{W}^c = \mathcal{W}^o$ and $E^c = E^o$. Then
   (a) If $\hat{\tau}^o < 0$, the admissible tax $\hat{\tau}^c$ is bounded by the positivity constraint, meaning $\hat{\tau}^c = 0$. Substituting this into (131) we have that welfare with financial markets takes the value

$$\begin{aligned} \widehat{\mathcal{W}^c} &= \mathcal{W}^* + \frac{\rho^* x^*}{\phi}(\lambda - \bar{\eta}^* - \frac{1}{2}\rho^*) \\ &\approx \mathcal{W}^* + \frac{\rho^* x^*}{\phi}(\lambda - \bar{\eta} - \frac{1}{2}\rho^* x^*) \end{aligned} \tag{142}$$

   where the last approximation followed by noticing that the cutoff type without the tax takes high values $x^* \approx 1$. Therefore, we have

$$\widehat{\mathcal{W}^c} - \mathcal{W}^c = \frac{\rho^* x^*}{\phi}(\lambda - \bar{\eta} - \frac{1}{2}\rho^* x^*) - \frac{1}{2}\frac{(\lambda - \bar{\eta})^2}{\phi} \tag{143}$$

   the first term in (143) reaches the maximum in $\rho^* x^* = \lambda - \bar{\eta}$, meaning that equation (143) is zero if abatement is exactly $\rho^* x^* = \lambda - \bar{\eta}$ and negative otherwise. Since $\hat{\tau}^o < 0$, we know from (139) that $\rho^* x^* > \lambda - \bar{\eta}$ implying that equation (143) is negative. As far as emissions are concerned, we have that

$$\hat{E}^c = E^* - \frac{\rho^* x^*}{\phi} < E^c = E^o = E^* - \frac{(\lambda - \bar{\eta})}{\phi}. \tag{144}$$

   (b) If $\hat{\tau}_{0.5} < 0 < \hat{\tau}^o$, then the admissible tax $\hat{\tau}_{0.5}$ is again set at $\hat{\tau}^c = 0$, however, the abatement provided by financial markets is now $\rho^* x^* < \lambda - \bar{\eta}$. This implies that the welfare difference (143) is again negative but emissions $\hat{E}^c$ are higher than the benchmark without financial markets.

   (c) If $0 < \hat{\tau}^o < \hat{\tau}_{0.5}$, then the regulator can implement the unconstrained optimal tax and welfare with financial markets $\widehat{\mathcal{W}^c}$ achieves the unconstrained optimum. Substituting the optimal tax (139) into (131), we have

$$\widehat{\mathcal{W}^c} = \mathcal{W}^* + \frac{1}{2}\frac{(\lambda - \bar{\eta})^2}{\phi} + \frac{1}{2}\frac{(x^o \rho^o)^2}{\phi} + \frac{\bar{\eta} x^o \rho^o}{\phi} - (\frac{1}{2}\frac{(x^o (\rho^o)^2)}{\phi} + \frac{x^o \bar{\eta}^o \rho^o}{\phi}) \tag{145}$$

   and since $\mathcal{W}^c = \mathcal{W}^* + \frac{1}{2}\frac{(\lambda - \bar{\eta})^2}{\phi}$ we have

$$\widehat{\mathcal{W}^c} - \mathcal{W}^c = \frac{\rho^o x^o}{\phi}(\bar{\eta} - \bar{\eta}^o - \frac{1}{2}(1 - x^o)\rho^o). \tag{146}$$

   Denote the preference distribution as $\eta_i = \eta i^\beta$ for a certain $\beta > 0$. Then it is simple to show that

$$\bar{\eta} - \bar{\eta}^o = \frac{\eta}{\beta + 1}(1 - (x^o)^\beta), \tag{147}$$

   which implies that

$$\widehat{\mathcal{W}^c} - \mathcal{W}^c = \frac{\rho^o x^o}{\phi}(\frac{\eta}{\beta + 1}(1 - (x^o)^\beta) - \frac{1}{2}(1 - x^o)\rho^o), \tag{148}$$

which is strictly positive if

$$\frac{\eta}{\beta+1} > \frac{\rho^o}{2}\frac{(1-x^o)}{(1-(x^o)^\beta)}. \tag{149}$$

The higher β, the less likely it is that this inequality is verified (left hand side is lower whereas right hand side becomes larger). Viceversa, the lower β, the more likely it is that this inequality is verified. For $\beta = 1$, that is when preferences are linear, the inequality is verified since $\rho < \eta$ necessarily from the equilibrium condition, proving the result.

(d) When the tax $\hat{\tau}^c = \hat{\tau}_{0.5}$, welfare $\widehat{\mathcal{W}^c}$ is given by (131) evaluated in $\tau = \hat{\tau}_{0.5}$, that is

$$\begin{aligned}
\widehat{\mathcal{W}}^c &= \mathcal{W}^* - \frac{1}{2}\frac{\hat{\tau}_{0.5}^2}{\phi} + \frac{\hat{\tau}_{0.5}(\lambda-\bar{\eta})}{\phi} - x^{\hat{\tau}_{0.5}}\rho^{\hat{\tau}_{0.5}}(\frac{1}{2}\frac{\rho^{\hat{\tau}_{0.5}}}{\phi} + \frac{\bar{\eta}^{\hat{\tau}_{0.5}}}{\phi} + \frac{\hat{\tau}_{0.5}}{\phi} - \frac{\lambda}{\phi}) \\
&= \mathcal{W}^* + \frac{\hat{\tau}_{0.5}}{\phi}(\lambda - \bar{\eta} - \frac{1}{2}\hat{\tau}_{0.5}) - x^{\hat{\tau}_{0.5}}\rho^{\hat{\tau}_{0.5}}(\frac{1}{2}\frac{\rho^{\hat{\tau}_{0.5}}}{\phi} + \frac{\bar{\eta}^{\hat{\tau}_{0.5}}}{\phi} + \frac{\hat{\tau}_{0.5}}{\phi} - \frac{\lambda}{\phi}) \\
&< \mathcal{W}^* + \frac{(\lambda-\bar{\eta}-\rho^*x^*)}{\phi}(\lambda-\bar{\eta}-\frac{1}{2}(\lambda-\bar{\eta}-\rho^*x^*)) - x^{\hat{\tau}_{0.5}}\rho^{\hat{\tau}_{0.5}}(\frac{1}{2}\frac{\rho^{\hat{\tau}_{0.5}}}{\phi} + \frac{\bar{\eta}^{\hat{\tau}_{0.5}}}{\phi} + \frac{\hat{\tau}_{0.5}}{\phi} - \frac{\lambda}{\phi}) \\
&= \mathcal{W}^* + \frac{(\lambda-\bar{\eta}-\rho^*x^*)}{\phi}(\frac{\lambda-\bar{\eta}}{2} + \frac{1}{2}\rho^*x^*) - x^{\hat{\tau}_{0.5}}\rho^{\hat{\tau}_{0.5}}(\frac{1}{2}\frac{\rho^{\hat{\tau}_{0.5}}}{\phi} + \frac{\bar{\eta}^{\hat{\tau}_{0.5}}}{\phi} + \frac{\hat{\tau}_{0.5}}{\phi} - \frac{\lambda}{\phi}) \\
&= \mathcal{W}^* + \frac{1}{2}\frac{(\lambda-\bar{\eta})^2}{\phi} - \frac{1}{2}\rho^*x^*\frac{(\lambda-\bar{\eta})}{\phi} + \frac{\rho^*x^*}{\phi}(\frac{\lambda-\bar{\eta}}{2} - \frac{1}{2}\rho^*x^*) - x^{\hat{\tau}_{0.5}}\rho^{\hat{\tau}_{0.5}}(\frac{1}{2}\frac{\rho^{\hat{\tau}_{0.5}}}{\phi} + \frac{\bar{\eta}^{\hat{\tau}_{0.5}}}{\phi} + \frac{\hat{\tau}_{0.5}}{\phi} - \frac{\lambda}{\phi}) \\
&= \mathcal{W}^* + \frac{1}{2}\frac{(\lambda-\bar{\eta})^2}{\phi} - \frac{1}{2}\frac{(\rho^*x^*)^2}{\phi} - x^{\hat{\tau}_{0.5}}\rho^{\hat{\tau}_{0.5}}(\frac{1}{2}\frac{\rho^{\hat{\tau}_{0.5}}}{\phi} + \frac{\bar{\eta}^{\hat{\tau}_{0.5}}}{\phi} + \frac{\hat{\tau}_{0.5}}{\phi} - \frac{\lambda}{\phi})
\end{aligned} \tag{150}$$

where the inequality follows from noticing that, necessarily for $\hat{\tau}^c = \hat{\tau}_{0.5}$, it must be that $0 < \hat{\tau}_{0.5} < \hat{\tau}^o \approx \lambda - \bar{\eta} - \rho^*x^*$. Subtracting the unconstrained welfare without financial markets, we get

$$\widehat{\mathcal{W}}^c - \mathcal{W}^c < -\frac{1}{2}\frac{(\rho^*x^*)^2}{\phi} - x^{\hat{\tau}_{0.5}}\rho^{\hat{\tau}_{0.5}}(\frac{1}{2}\frac{\rho^{\hat{\tau}_{0.5}}}{\phi} + \frac{\bar{\eta}^{\hat{\tau}_{0.5}}}{\phi} + \frac{\hat{\tau}_{0.5}}{\phi} - \frac{\lambda}{\phi}) \tag{151}$$

which is negative in our range of model parameters. To compute emissions, we notice that when the marginal change in the equilibrium price and cutoff types $\rho^\tau_\tau \approx 0$ and $x^\tau_\tau \approx 0$, the threshold takes the explicit expression

$$\hat{\tau}_{0.5} \approx \tau_{0.5} - 2\rho^*x^* \tag{152}$$

Emissions with financial markets are therefore

$$\begin{aligned}
\hat{E}^c &= E^* - \frac{\hat{\tau}_{0.5}}{\phi} - \frac{x^{\hat{\tau}_{0.5}}\rho^{\hat{\tau}_{0.5}}}{\phi} \\
&\approx E^* - \frac{\tau_{0.5}}{\phi} + \frac{2\rho^*x^*}{\phi} - \frac{x^{\hat{\tau}_{0.5}}\rho^{\hat{\tau}_{0.5}}}{\phi} \\
&> E^* - \frac{\tau_{0.5}}{\phi} + \frac{\rho^*x^*}{\phi}
\end{aligned} \tag{153}$$

these are larger than E^c provided that $\tau_{0.5} - \rho^*x^* < \lambda - \bar{\eta}$. Since necessarily $0 < \hat{\tau}_{0.5} < \hat{\tau}^o$, substituting the expressions in (152) and (139), we have that $\tau_{0.5} - \rho^*x^* < \lambda - \bar{\eta}$, which gives the result.

2. If $\tau^c = \tau_{0.5}$, meaning if $\lambda - \bar{\eta} > \tau_{0.5}$, then $\mathcal{W}^c < \mathcal{W}^o$ and $E^c > E^o$. Before proceeds with the possible scenario, we note that when the political constraint is binding for the economy without financial markets, it can never be that the economy with financial markets implements an unconstrained optimal tax $0 < \hat{\tau}^o < \hat{\tau}_{0.5}$. This can be derived from (152) and (139), noticing that for $\hat{\tau}^o < \hat{\tau}_{0.5}$, it must be that $\rho^*x^* < \tau_{0.5} - (\lambda - \bar{\eta}) < 0$. Then we have

(e) if $\hat{\tau}^o < 0$, then the admissible tax $\hat{\tau}^c = 0$. In such a case, since we showed that $\rho^* x^* > \lambda - \bar{\eta} > \tau_{0.5}$, emissions in the economy with financial markets $\hat{E}^c$ are always lower than the benchmark emissions with the tax only, E^c. Welfare takes the value

$$\widehat{\mathcal{W}}^c = \mathcal{W}^* + \frac{\rho^* x^*}{\phi}(\lambda - \bar{\eta}^* - \frac{1}{2}\rho^*) \approx \mathcal{W}^* + \frac{\rho^* x^*}{\phi}(\lambda - \bar{\eta} - \frac{1}{2}\rho^* x^*) \tag{154}$$

where again the approximation comes from the fact that in absence of the tax, the equilibrium cutoff type $x^* \approx 1$ and $\bar{\eta}^* \approx \bar{\eta}$. This is compared against the constrained welfare

$$\mathcal{W}^c = \mathcal{W}^* + \frac{\tau_{0.5}}{\phi}(\lambda - \bar{\eta} - \frac{1}{2}\tau_{0.5}). \tag{155}$$

from which derives immediately that $\widehat{\mathcal{W}}^c > \mathcal{W}^c$ if $\rho^* x^* > \tau_{0.5}$, and $\widehat{\mathcal{W}}^c < \mathcal{W}^c$ otherwise. We recall that since $\hat{\tau}^o < 0$, we have that $\rho^* x^* > \lambda - \bar{\eta} > \tau_{0.5}$, which proves the result.

(f) if $\hat{\tau}_{0.5} < 0 < \hat{\tau}^o$, then again the admissible tax $\hat{\tau}^c = 0$. However here $\rho^* x^* < \lambda - \bar{\eta}$ since $\hat{\tau}^o > 0$. Following the previous arguments, we have that if $\rho^* x^* < \tau_{0.5} < \lambda - \bar{\eta}$, then the economy with financial markets generates higher emissions $\hat{E}^c > E^c$ and lower welfare $\widehat{\mathcal{W}}^c < \mathcal{W}^c$. Otherwise, financial markets generate lower emissions $\hat{E}^c < E^c$ and higher welfare $\widehat{\mathcal{W}}^c > \mathcal{W}^c$.

(g) the remaining case is when $0 < \hat{\tau}_{0.5} < \hat{\tau}^o$. In such a case the regulator implements the constrained tax $\hat{\tau}^c = \hat{\tau}_{0.5}$. Using the approximation $\hat{\tau}_{0.5} \approx \tau_{0.5} - 2\rho^* x^*$, welfare reads

$$\begin{aligned}
\widehat{\mathcal{W}}^c &\approx \mathcal{W}^* + \frac{\tau_{0.5} - 2\rho^* x^*}{\phi}(\lambda - \bar{\eta} - \rho^* x^* - \frac{1}{2}\tau_{0.5} + \rho^* x^*) + \frac{\rho^* x^*}{\phi}(\lambda - \frac{1}{2}\rho^* - \bar{\eta}_*) \\
&= \mathcal{W}^* + \frac{\tau_{0.5}}{\phi}(\lambda - \bar{\eta} - \frac{1}{2}\tau_{0.5}) - \frac{2\rho^* x^*}{\phi}(\lambda - \bar{\eta} - \frac{1}{2}\tau_{0.5}) + \frac{\rho^* x^*}{\phi}(\lambda - \frac{1}{2}\rho^* - \bar{\eta}^*) \\
&= \mathcal{W}^c - \frac{\rho^* x^*}{\phi}(\lambda - 2\bar{\eta} - \tau_{0.5} + \frac{1}{2}\rho^* + \bar{\eta}^*)
\end{aligned} \tag{156}$$

this is less than $\mathcal{W}^c$ if

$$\frac{1}{2}\rho^* > \bar{\eta} - \bar{\eta}^* + \tau_{0.5} - (\lambda - \bar{\eta}) \tag{157}$$

since $\lambda - \bar{\eta} > \tau_{0.5}$, a sufficient condition for this to be satisfied is that

$$\frac{1}{2}\rho > \bar{\eta} - \bar{\eta}_x \tag{158}$$

which is always satisfied in our range of model parameters since $x^* \approx 1$ and $\bar{\eta} \approx \bar{\eta}^*$. As for the emissions, we have that

$$\begin{aligned}
\hat{E}^c &= E^* - \frac{\hat{\tau}_{0.5}}{\phi} - \frac{x^{\hat{\tau}_{0.5}} \rho^{\hat{\tau}_{0.5}}}{\phi} \\
&\approx E^* - \frac{\tau_{0.5}}{\phi} + \frac{2\rho^* x^*}{\phi} - \frac{x^{\hat{\tau}_{0.5}} \rho^{\hat{\tau}_{0.5}}}{\phi} \\
&> E^* - \frac{\tau_{0.5}}{\phi} + \frac{\rho^* x^*}{\phi} > E^* - \frac{\tau_{0.5}}{\phi} = E^c.
\end{aligned} \tag{159}$$

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