The Cross-section of Subjective Expectations: Understanding Prices and Anomalies

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ABSTRACT

We propose a structural model of constant gain learning about future earnings growth that incorporates preferences for the timing of cash flows. As implied by the model, a cross-sectional decomposition using survey forecasts shows that high price-earnings ratios are accounted for by both low expected returns and overly high expected earnings growth. The model quantitatively matches a number of asset pricing moments, as learning about growth interacts strongly with the preference for the timing of cash flows, and provides insights on the roles of risk premia and mispricing in the cross-section of stocks. The magnitudes and timing of the comovement between prices, earnings growth surprises, and anomaly returns are all consistent with a gradual learning process rather than expectations being highly sensitive to the most recent realization. Large earnings growth surprises do not immediately translate into large one-period returns, but instead are gradually reflected in future returns over time.

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It's been known since Basu (1975) and Stattman (1980) that high price ratio stocks (e.g., price-earnings ratios, price-book ratios) earn lower returns than their peers (Rosenberg, Reid, and Lanstein, 1985; Fama and French, 1992; Davis, Fama, and French, 2000). While the one-month difference between Growth and Value stocks has declined over time (Schwert, 2003; Fama and French, 2020), return differences at longer horizons have remained substantial (De la O, Han, and Myers, 2023)[1] and play a large role in accounting for the level of prices (van Binsbergen et al., 2023; Cho and Polk, 2023). Given that a stock's price is the risk-adjusted value of expected future cash flows, these realized return differences imply that high price ratio stocks either have low risk exposure or overly high expected cash flows.

There is a long-standing debate between research advocating either for risk exposure or incorrect cash flow expectations to explain cross-sectional differences in price ratios and subsequent returns.[2] Our innovation is twofold. First, we provide a quantitative decomposition using professional forecasts which measures the relative importance of these two components at varying horizons. Second, we estimate a structural model which relates these decomposition results to duration-based risk premia and learning about earnings growth. Both the empirical decomposition and the structural model emphasize the magnitudes and timing of expected and realized earnings growth and returns, which provides key distinctions from the predictions of existing full-information rational expectations (FIRE), learning, and behavioral models.

To frame our empirical analysis, we propose a model of constant gain learning about firm-level earnings growth with duration-based risk premia. The agent's SDF depends on an aggregate shock. The effect of this shock on firm cash flows is persistent but not permanent, meaning that longer horizon cash flows carry a lower discount rate as they are less exposed to the aggregate shock. Cross-sectional differences in firm earnings depend on each firm's underlying mean earnings growth as well as transitory idiosyncratic shocks. The agent

[1] Table I also confirms that long-term return differences are large even for 1999-2020.

[2] See Fama and French (1995) and Daniel and Titman (1997) for early evidence and Lustig and Nieuwerburgh (2005) and Hou, Karolyi, and Kho (2011) for more recent explanations

attempts to infer the mean growth based on past realizations. Objectively, mean earnings growth is identical across firms. However, the agent's expectation of mean earnings growth will differ across firms based on their realized shocks.

The model gives three key predictions for the dynamics of prices, earnings growth, and returns which distinguish it from other models of expectations. First, a high expected mean earnings growth for a firm will raise the firm's price by increasing the expected future cash flows and lowering the subjective discount rate. In other words, both high subjective expected cash flows and low subjective expected returns will contribute to a high price. This differs from return extrapolation,[3] in which high prices are associated with *high* expected returns.

Second, if the constant gain parameter is small (i.e., learning is slow), then the impact of earnings growth surprises on one-period returns will be small. If the agent largely attributes the surprise to a transitory shock to earnings, rather than updating her expectation of the mean growth rate, then the earnings growth surprise will only lead to a small immediate change in price. Instead, prices will adjust gradually over time as agents update their beliefs. In comparison, diagnostic models and models in which agents overstate the persistence of earnings growth imply that earnings growth surprises would translate more than 1-1 into one-period returns, as current prices are highly sensitive to the most recent realizations.[4]

Third, if the constant gain parameter is small, then positive earnings growth surprises will *decrease* expected next period earnings growth. If the surprise is primarily attributed to a temporary shock to the level of earnings, then the agent believes the surprise will be largely reversed by low earnings growth in the next period. This differs from the diagnostic and exaggerating persistence models mentioned in the second prediction. This also differs from other constant gain learning models, such as Nagel and Xu (2022).[5] These models all imply that a positive surprise increases expected next period earnings growth.

[3]See Jin and Sui (2022) and Barberis et al. (2015).

[4]See the extrapolative expectations model of Hirshleifer, Li, and Yu (2015) or the diagnostic expectations model of Bordalo et al. (2019).

[5]This distinction from Nagel and Xu (2022) comes from the fact that our model incorporates temporary shocks to the level of earnings into the constant gain learning model.

To measure the relative importance of subjective earnings growth expectations and subjective discount rates and test the model predictions, we use a cross-sectional version of the Campbell-Shiller decomposition.[6] Using professional forecasts, we find that 43.3% of dispersion in price-earnings ratios is accounted for by high price ratio firms having higher expected four-year earnings growth and 12.7% of dispersion is accounted for by high price ratio firms having lower expected four-year returns. This confirms the first prediction of the model that both higher expected earnings growth and lower expected returns contribute to a high price. Interestingly, unlike the aggregate time series evidence in Greenwood and Shleifer (2014) and De la O and Myers (2021) that expected returns are positively correlated with price ratios, in the cross-section investors correctly expect lower returns for high price-ratio firms.[7] The remaining dispersion is explained by expectations of future price-earnings ratios, which reflect expectations of earnings growth and returns beyond four years.

For comparison, realized four-year earnings growth and negative returns account for 9.9% and 32.0% of price-earnings ratio dispersion, respectively. Empirically, high price ratio firms are mainly associated with lower returns than their peers, rather than higher future earnings growth. In other words, investors overestimate the earnings growth of high price ratio firms, which leads to consistent disappointment in earnings growth for these firms. While investors do expect lower returns for high price ratio firms, they understate the magnitude of this relationship. Consistent with the fact that investors are disappointed by earnings growth, the realized returns on high price ratios firms are even lower than expected.

In terms of timing, while investors are significantly disappointed by one-year earnings growth for high price ratio firms, this does not immediately lead to large negative one-year returns on these firms. Comparing the decomposition results at the one-year horizon and the four-year horizon, we find that disappointment in future earnings growth for high price ratio firms is largely concentrated at the one-year horizon, while the low returns on these

[6] Because this decomposition is derived from an identity, it holds even if expectations differ from the objective distribution.

[7] Dahlquist and Ibert (2023), Bastianello (2023) and Büsing and Mohrschladt (2023) also find evidence that expected returns are negatively related to price ratios.

firms gradually accumulate over four years. This confirms the second model prediction that, when the constant gain parameter is small, overly high earnings growth expectations will be gradually reflected in future returns.

Why does disappointment in one-year earnings growth not immediately lead to large negative returns? Because, consistent with the third model prediction, a negative earnings growth surprise empirically increases expected next year earnings growth. After observing a bad earnings growth realization, investors appear to largely attribute this to a transitory shock to the level of earnings which will be reversed going forward. This demonstrates the importance of modeling expectations of future earnings as relatively "stubborn" and slow-moving, rather than being highly sensitive to the most recent realizations.

While we mainly focus on decomposing cross-sectional dispersion in price ratios, we can analogously frame these tests in terms of understanding anomaly returns. Focusing on the value, profitability, and investment anomalies, as well as a combination of 22 anomalies from Hou, Xue, and Zhang (2015), we show that each anomaly is associated with large return forecast errors. In other words, the realized one-year returns on each anomaly are much higher than investors expected.[8] Decomposing the unexpected anomaly returns, we find that forecast errors for one-year earnings growth have the correct sign and are large enough to account for these unexpected anomaly returns. In fact, consistent with the second model prediction, we find that one-year earnings growth forecast errors are larger than the one-year unexpected returns, again indicating that earnings growth surprises do not immediately translate into large one-year returns. Instead, we consistently find that positive earnings growth surprises decrease expected next-year earnings growth, in line with the third model prediction.

To demonstrate that our model can quantitatively as well as qualitatively explain our empirical findings, we estimate and test the constant gain learning model. In the words of Brunnermeier et al. (2021), "*we need structural models of belief dynamics that can compete*

[8]A one standard deviation increase in the anomaly variable is associated with a 80 to 330bps increase in return forecast errors.

with [FIRE] models in explaining asset prices and empirically observed beliefs." The six parameters of the model are inferred from realized earnings, average aggregate returns, and the volatility of one-year earnings growth expectations. Despite not using any price information other than average aggregate returns, the model successfully matches a number of untargeted aggregate and cross-sectional asset pricing moments. Importantly, the model also replicates our decomposition results for the price-earnings ratio at the one-year and four-year horizon, both in terms of magnitudes and timing. The model not only matches the dynamics of price-earnings ratios and expectations, but also outperforms standard FIRE models (Berk, Green, and Naik, 1999; Zhang, 2005; Lettau and Wachter, 2007) in matching the dynamics of price-earnings ratios and realized future earnings growth and returns.[9]

The quantified structural model allows us to extend our empirical results in two ways. First, we can go beyond the four-year horizon to estimate that expected earnings growth and expected returns for all horizons account for two-thirds (65.4%) and one-third (34.6%) of price-earnings ratio dispersion, respectively. This is largely due to errors in earnings growth expectations, which account for half (49.4%) of all price-earnings ratio dispersion.

Second, we can examine the underlying mechanisms which drive expected earnings growth and expected returns, namely constant gain learning and duration-based risk sensitivity, to show that the interaction between these two mechanisms is important for generating realistic dispersion in price-earnings ratios. Compared to an economy with no learning and no risk sensitivity, introducing only risk sensitivity has no impact on the dispersion in price-earnings ratios, introducing only learning increases the dispersion by a factor of 2.1, and introducing both increases the dispersion by a factor of 4.4. This highlights the benefit of unifying non-FIRE earnings growth expectations and duration-based risk premia, as the interaction magnifies the sensitivity of prices to changes in beliefs.

Broadly, this paper contributes to the growing literature using subjective expectations

[9]We find that these models struggle to generate risk premia large enough to match the empirical relationship between price-earnings ratios and future returns.

to understand asset prices.[10] In the cross-section, errors in firm-level professional earnings forecasts have been strongly linked to future returns (La Porta, 1996; Frankel and Lee, 1998; Da and Warachka, 2011; So, 2013; van Binsbergen, Han, and Lopez-Lira, 2022) and have been used to study a number of anomalies such as post-earnings announcement drift (Abarbanell and Bernard, 1992), the duration premium (Weber, 2018), and the profitability anomaly (Bouchaud et al., 2019). Moreover, Kozak, Nagel, and Santosh (2018) and Engelberg, Mclean, and Pontiff (2018) find that short legs of multiple long-short anomaly strategies comprise stocks with more optimistic earnings forecasts, whereas Engelberg, McLean, and Pontiff (2020) find that anomaly short legs comprise stocks with more optimistic return forecasts.

We differ from these studies in two important ways. First, to the best of our knowledge, we are the first paper to quantify the relative importance of subjective cash flow and return expectations in accounting for cross-sectional dispersion in price ratios and returns. This decomposition sheds light on the relative importance of risk (discount rates) and mispricing in stock prices. By utilizing expectations of both earnings growth and returns, we are able to measure subjective discount rates and quantitatively link unexpected anomaly returns to errors in earnings growth expectations.[11] Second, we use the expectations data to estimate and test a structural model of expectation formation, preferences, and asset prices which links our decomposition results to underlying "deep" parameters of learning and risk sensitivity.

In terms of the structural model, our work is closely related to the literature on learning about mean consumption or cash flow growth (Bordalo, Gennaioli, Porta, and Shleifer, 2019; Lewellen and Shanken, 2002; Collin-Dufresne, Johannes, and Lochstoer, 2016; Nagel and Xu, 2022) and incorporates duration-based risk premia similar in spirit to Lettau and Wachter (2007). We provide new evidence supporting these types of learning models using the cross-

[10] Amromin and Sharpe (2014); Greenwood and Shleifer (2014); Piazzesi, Salomao, and Schneider (2015); Cassella and Gulen (2018); De la O and Myers (2021); Nagel and Xu (2022); Bordalo et al. (2022) utilize survey expectations for aggregate outcomes such as returns, cash flows and yields.

[11] This differs from the implied cost of capital approach (Chen, Da, and Zhao, 2013; Hommel, Landier, and Thesmar, 2023) in which discount rates are inferred using earnings expectations for observable horizons and assumptions about long-term industry growth or GDP growth.

sectional dynamics of stocks and show that incorporating learning about temporary shocks to the level of earnings creates distinct qualitative predictions for the timing of earnings growth surprises and returns.[12] We also highlight that learning about growth naturally complements duration-based risk premia. Even if the objective timing of cash flows is relatively similar across all firms (Chen et al., 2017), duration-based risk premia can still play an important role in stock prices so long as investors *believe* there is a large difference in the timing of cash flows. In other words, as argued in Jensen (2023), once we depart from FIRE, the compensation for risk that investors require should be disciplined by data on investors' believed risks, not the objective risks.

I. Model of cash flow expectations and discount rates

In this section, we introduce a model with slow-moving biases in cash flow growth expectations and duration-dependent discount rates. We show in Section III that this model quantitatively replicates our empirical findings. Throughout the paper, we use lowercase letters to denote log values, $z \equiv \log(Z)$.

A. *Cash flows and the stochastic discount factor*

For each firm i, the log cash flow $x_{i,t}$ has an aggregate and a firm-level component,

$$x_{i,t} = x_t^{agg} + \tilde{x}_{i,t} \tag{1}$$

$$x_t^{agg} = \phi x_{t-1}^{agg} + u_t \tag{2}$$

$$\tilde{x}_{i,t} = g_i t + v_{i,t}. \tag{3}$$

The aggregate component is an AR(1) process, which can be thought of as business-cycle fluctuations. The firm-level component is a firm-specific trend $g_i t$ plus noise to capture potential cross-sectional differences in growth rates. The shocks $u_t, v_{i,t}$ are uncorrelated and

[12] While Bordalo et al. (2019) incorporate a sluggish form of diagnostic expectations into their calibration, the model still implies that earnings growth surprises translate more than 1-1 into one-period returns.

have variances σ_u^2, σ_v^2.

The agent has a log stochastic discount factor

$$m_{t+1} = -r^f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1} \tag{4}$$

which depends on the aggregate shock u_{t+1}.

B. *Subjective cash flow expectations*

Objectively, the value of g_i is identical across firms, $g_i = \bar{g}$.[13] However, the agent does not know each firm's g_i and forms her subjective expectation $E_t^*[g_i]$ using constant gain learning,

$$E_t^*[g_i] = E_{t-1}^*[g_i] + \beta\left(\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]\right) \tag{5}$$

$$E_t^*[v_{i,t}] = (1-\beta)\left(\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]\right) \tag{6}$$

where β is the constant gain parameter. Specifically, after observing the surprise $\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]$, she attributes portion β to firm-specific growth and portion $(1-\beta)$ to the noisy shock $v_{i,t}$. Her expectation for the future growth of the firm-level component is then

$$E_t^*[\Delta\tilde{x}_{i,t+1}] = E_t^*[g_i] - E_t^*[v_{i,t}]. \tag{7}$$

Her expectation for the future level of the firm-level component is

$$E_t^*[\tilde{x}_{i,t+n}] = \tilde{x}_{i,t} + nE_t^*[g_i] - E_t^*[v_{i,t}]. \tag{8}$$

C. *Prices and subjective risk premia*

Sections I.A and I.B lay out all of the elements and assumptions of the model. In this subsection, we simply combine the agents' beliefs and the stochastic discount factor to calculate the price for various claims. Appendix A gives the details for all of the equations.

To start, let $P_t^{(n)}$ be the price of an n-period aggregate strip, i.e., a claim that pays X_{t+n}^{agg}

[13] Given that our empirical analysis focuses on price-earnings ratios, we normalize $\bar{g}$ to 0 without loss of generality.

in n periods. The aggregate strip price is

$$\begin{aligned} P_t^{(n)} &= E_t^* \left[\left(\prod_{j=1}^{n} M_{t+j} \right) X_{t+n}^{agg} \right] \\ &= \exp\left\{ -nr^f - \gamma\sigma_u^2 \frac{1-\phi^n}{1-\phi} + \frac{1}{2}\sigma_u^2 \frac{1-\phi^{2n}}{1-\phi^2} + \phi^n x_t^{agg} \right\}. \end{aligned} \tag{9}$$

The realized return on the strip is

$$\begin{aligned} R_{t+1}^{(n)} &= \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}} \\ &= \exp\left\{ r^f + \gamma\sigma_u^2\phi^{n-1} - \frac{1}{2}\sigma_u^2\phi^{2(n-1)} + \phi^{n-1}u_{t+1} \right\} \end{aligned} \tag{10}$$

and the subjective expected return on the strip is

$$E_t^*\left[R_{t+1}^{(n)}\right] = \exp\left\{ r^f + \gamma\sigma_u^2\phi^{n-1} \right\}. \tag{11}$$

The first term (r^f) reflects the risk-free rate and the second term ($\gamma\sigma_u^2\phi^{n-1}$) reflects the subjective risk premium, i.e., the compensation agents require for exposure to risk.

Equation (11) shows an important characteristic of this model: longer horizon strips carry a lower subjective risk premium $\gamma\sigma_u^2\phi^{n-1}$. Equation (2) shows that aggregate shocks are persistent but not permanent. This means that longer horizon cash flows are less sensitive to the aggregate shock u_{t+1} and therefore require a lower risk premium. This is similar to the mechanism in Lettau and Wachter (2007).

Each firm i can be viewed as a collection of strips. Specifically, since shocks to the firm-level component $v_{i,t}$ are uncorrelated with the aggregate shock, we can express the firm's price as

$$\begin{aligned} P_{i,t} &= \sum_{n=1}^{\infty} E_t^* \left[\left(\prod_{j=1}^{n} M_{t+j} \right) X_{t+n}^{agg} \tilde{X}_{i,t+n} \right] \\ &= \sum_{n=1}^{\infty} P_t^{(n)} E_t^* \left[\tilde{X}_{i,t+n} \right] \\ &= \sum_{n=1}^{\infty} P_t^{(n)} \exp\left\{ \frac{1}{2}\sigma_v^2 + E_t^*\left[\tilde{x}_{i,t+n}\right] \right\}. \end{aligned} \tag{12}$$

In other words, idiosyncratic firm risk is not priced, so firm prices simply depend on the expected firm-level component $E_t^*\left[\tilde{x}_{i,t+n}\right]$ and the aggregate strip prices $P_t^{(n)}$.

The subjective expected return on firm i is then simply a weighted average of the subjective expected return on the individual strips,

$$\begin{aligned} E_t^*\left[R_{i,t+1}\right] &= E_t^*\left[\frac{X_{t+1}^{agg}\tilde{X}_{i,t+1}+P_{i,t+1}}{P_{i,t}}\right] \\ &= \sum_{n=1}^{\infty} w_{i,t,n} E_t^*\left[R_{t+1}^{(n)}\right] \\ &= \sum_{n=1}^{\infty} w_{i,t,n} \exp\left\{r^f+\gamma\sigma_u^2\phi^{n-1}\right\} \end{aligned} \tag{13}$$

where the weight $w_{i,t,n}=\frac{\exp\{nE_t^*[g_i]\}P_t^{(n)}}{\sum_{n=1}^{\infty}\exp\{nE_t^*[g_i]\}P_t^{(n)}}$ captures how much of the firm's value in equation (12) comes from its horizon n cash flows.

The realized return for firm i is

$$\begin{aligned} R_{i,t+1} &= \frac{X_{t+1}^{agg}\tilde{X}_{i,t+1}+P_{i,t+1}}{P_{i,t}} \\ &= \sum_{n=1}^{\infty} w_{i,t,n} R_{t+1}^{(n)} \frac{E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{E_t^*\left[\tilde{X}_{i,t+n}\right]}. \end{aligned} \tag{14}$$

In addition to depending on a weighted average of realized strip returns $R_{t+1}^{(n)}$, the realized firm return also depends on the change in the expected future firm-level component. From equations (5), (6), and (8), this change in expectations can all be expressed entirely in terms of the surprise about one-period growth $\Delta\tilde{x}_{i,t+1}-E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]$, as for $n\geq 2$ we have that

$$\frac{E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{E_t^*\left[\tilde{X}_{i,t+n}\right]} = \exp\left\{n\beta\left(\Delta\tilde{x}_{i,t+1}-E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]\right)\right\}. \tag{15}$$

D. Model implications

Below, we discuss several implications from the model which will be useful for understanding the empirical findings.

First, increases in $E_t^*\left[g_i\right]$ raise the firm's price in two ways: increasing the expected future cash flows and decreasing the subjective risk premium. From equation (12), a higher expected

g_i naturally increases the value of the firm by increasing the value of future expected cash flows. What is more surprising is that raising $E_t^*[g_i]$ lowers the subjective risk premium. As shown in equation (11), longer horizon cash flows carry a lower risk premium in this model, as they are less sensitive to the aggregate shock u_{t+1}. A higher value for $E_t^*[g_i]$ means that more of the firm's value comes from its longer horizon cash flows and therefore the weights $w_{i,t,n}$ in equation (13) are more concentrated on the low longer horizon $\exp\left\{r^f + \gamma\sigma_u^2\phi^{n-1}\right\}$. The Campbell-Shiller decomposition in Section II will help to quantify the relative importance of these two channels.

Second, if the constant gain parameter β is small, then the impact of cash flow surprises $\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]$ on one-period returns will be small. As shown in equations (14)-(15), a positive surprise $\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]$ will cause the realized return to be higher than agent's expected return ($E_t^*[R_{i,t+1}]$). If β is large, then agents will raise their expectations of g_i substantially after a positive surprise, which will increase the current price and lead to a large positive current return. However, if β is fairly small, then expectations of g_i will only respond slightly in response to surprises, which means we will not observe a large one-period return in response to a cash flow surprise $\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]$. Instead, when β is small, prices will move slowly over time as agents gradually adjust their expectations of g_i. Rephrased, the mapping between cash flow surprises $\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]$ and realized returns $R_{i,t}$ depends on how "stubborn" beliefs are (i.e., how quickly agents change their beliefs).

Third, if the constant gain parameter β is small, then a positive surprise $\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]$ will *decrease* expected next period growth $E_t^*[\Delta\tilde{x}_{i,t+1}]$. Equations (5)-(7) show that this will be true whenever $\beta < 0.5$. This is quite different from models in which an unexpectedly high value for realized growth increases expected future growth. After observing a positive surprise $\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]$, the agent slightly raises her beliefs about g_i but largely attributes the surprise to the noisy shock $v_{i,t}$ which she expects will be reversed going forward.

II. Empirical findings

A. Decomposing the cross-section of price ratios

To understand the contributions of subjective cash flow growth expectations and subjective discount rates to dispersion in stock price ratios, we focus on a cross-sectional version of the Campbell-Shiller decomposition. Note that this decomposition is derived from an identity, meaning that the empirical results do not require any of the assumptions made in the model. In terms of notation, $E_t^*[\cdot]$ denotes subjective expectations. All other operators use the objective probability distribution. For example, $Var(\cdot)$ and $Cov(\cdot,\cdot)$ denote the observable variance or covariance of variables.

For any stock or portfolio of stocks i, the one-year ahead log return $r_{i,t+1}$ can be approximated in terms of the price-earnings ratio $px_{i,t}$, future earnings growth $\Delta x_{i,t+1}$, and the future price-earnings ratio:

$$r_{i,t+1} \approx \kappa + \Delta x_{i,t+1} + \rho px_{i,t+1} - px_{i,t}, \tag{16}$$

where κ and $\rho < 1$ are constants.[14] To understand cross-sectional dispersion in price-earnings ratios, let $\tilde{px}_{i,t}$ be the cross-sectionally demeaned price-earnings ratio of portfolio i and let $\Delta\tilde{x}_{i,t+1}$ and $\tilde{r}_{i,t+1}$ be the cross-sectionally demeaned earnings growth and returns. Rearranging equation (16) and applying subjective expectations $E_t^*[\cdot]$, we see that a higher than average price-earnings ratio must be explained by higher than average expected earnings growth, lower than average expected returns, or a higher than average expected future price-earnings ratio,

$$\tilde{px}_{i,t} \approx \sum_{j=1}^{h} \rho^{j-1} E_t^*\left[\Delta\tilde{x}_{i,t+j}\right] - \sum_{j=1}^{h} \rho^{j-1} E_t^*\left[\tilde{r}_{i,t+j}\right] + \rho^h E_t^*\left[\tilde{px}_{i,t+h}\right]. \tag{17}$$

Importantly, equation (17) does not require that expectations are rational. Because this equation is derived from an identity, it holds under any subjective probability distribution.

[14]Note that this approximation still holds even for non-dividend paying firms. Appendix B discusses the log-linearization in more detail including the role of the payout ratio.

To measure the relative contribution of subjective cash flow growth expectations and subjective discount rates to the dispersion in price-earnings ratios, we decompose the variance of $\tilde{px}_{i,t}$ into three components:

$$1 \approx \underbrace{\frac{Cov\left(\sum_{j=1}^{h}\rho^{j-1}E_t^*\left[\Delta\tilde{x}_{i,t+j}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{CF_h} + \underbrace{\frac{Cov\left(-\sum_{j=1}^{h}\rho^{j-1}E_t^*\left[\tilde{r}_{i,t+j}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{DR_h} + \underbrace{\rho^h\frac{Cov\left(E_t^*\left[\tilde{px}_{i,t+h}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{FPE_h}. \tag{18}$$

Note that $Var\left(\tilde{px}_{i,t}\right)$ is the average squared cross-sectionally demeaned price-earnings ratio, which means it measures the average cross-sectional dispersion in price-earnings ratios. The coefficients CF_h and DR_h give a quantitative measure of how much dispersion in price-earnings ratios is accounted for by dispersion in earnings growth expectations and how much is accounted for by dispersion in discount rates. For example, in the model of Section I, a higher sensitivity to risk (γ) increases DR_h as the duration-dependent risk premia play a larger role in firms' prices. In contrast, when $\gamma = 0$, we have $DR_h = 0$ and all dispersion in price-earnings ratios is accounted for by expected earnings growth and the future price-earnings ratio. Applying the decomposition to multiple horizons h provides information about the timing of expected earnings growth and discount rates. Additionally, the terms in equation (18) can be interpreted as the coefficients from univariate regressions with time fixed effects, e.g., a one unit increase in $px_{i,t}$ is associated with a CF_1 unit increase in expected one-year earnings growth.

When we estimate equation (18) using professional forecasts, we will use expectations of price growth $E_t^*\left[\Delta p_{i,t+j}\right]$ as a proxy for expectations of returns $E_t^*\left[r_{t+j}\right]$. Empirically, realized price growth and returns are closely related with a correlation of 0.997 to 0.999 for the $j = 1, ..., 4$ horizons that we study in our analysis. However, to ensure that the use of this proxy and the approximation error in equation (18) do not impact the results, we also estimate an exact decomposition based on price growth in Appendix C. As shown in Tables

I and AI, the results of this exact decomposition closely match the results from equation (18).

B. Data

The firm-level realized earnings and prices are collected from Compustat and CRSP. The firm-level expected earnings and prices are collected from I/B/E/S (Institutional Brokers' Estimate System) and Value Line. To perform the decomposition from Section II.A, we sort these firms into the classic Value and Growth portfolios. Specifically, for each month t, we construct five value-weighted portfolios sorted by book-to-market.[15] These portfolios capture over 99% of the firm-level cross-sectional variation in price-book ratios.[16] For these portfolios, we measure the expectations at time t for earnings growth, price growth, and the future price-earnings ratio over the next four years. We also track the realized buy-and-hold future earnings growth, returns, and price-earnings ratios over the next four years. The *main sample*, which contains expectations of both earnings growth and price growth, ranges from 1999-2020. For robustness tests, we also use a *long sample* which ranges from 1982-2020 and contains earnings growth expectations. The subsections below provide more detail on the firm-level variable measurements.

B.1. Realized data

The sample of stocks consists of all common stocks (share code 10 and 11) listed on NYSE, AMEX, and NASDAQ. We obtain monthly prices and shares outstanding from the Center for Research in Security Price (CRSP). The firm-level accounting variables are constructed from the quarterly Compustat database. Following Davis et al., 2000 and Cohen et al., 2003, we define book value as stockholders' book equity, plus deferred taxes and investment tax

[15] The book-to-market ratio is defined as the market-cap in the portfolio formation month scaled by total book value from the most recent four quarters. To account for potential data errors, we exclude firms with book-to-market ratios over 100 or below 0.01.

[16] For our sample, the standard deviation across firms in the log price-book ratio is 0.763. For our five portfolios, the standard deviation of log price-book ratios is 0.756, meaning that these portfolios capture the vast majority of the firm-level variation.

credit if available, minus the book value of preferred stock. If stockholders' book equity is not available at Compustat, we define it as the book value of common equity plus the par value of preferred stock, or the book value of assets minus total liabilities in that order. Depending on availability, we use redemption or par value for the book value of preferred stock. To be consistent with the I/B/E/S's definition of earnings, we define earnings as Compustat net income (item NIq) excluding non-I/B/E/S items, which comprise extraordinary items and discontinued operations (item XIDOq), special items (item SPIq), and non-recurring income taxes (item NRTXTq). This aligns with the measure of earnings proposed in Hillenbrand and McCarthy (2022). Monthly dividends are the difference between CRSP monthly returns and CRSP monthly returns excluding dividends multiplied by total market value. At every month, annual earnings at the firm level are defined as the sum of quarterly earnings from the most recent four quarters.[17] The main sample includes all firms which have observable returns $r_{i,t+j}$, earnings growth $\Delta x_{i,t+j}$,, and price-earnings ratios $px_{i,t+j}$ in future years $j = 1, 2, 3, 4$.

B.2. Subjective expectations

The subjective earnings and short-term price expectations are extracted from the I/B/E/S Database. The Summary Statistics of the I/B/E/S Database contains the median forecasts for EPS (earnings per share) since 1976 for shorter horizons and 1982 for longer horizons for U.S. publicly traded companies and the median forecasts for prices at the 12-month horizon since 1999. I/B/E/S gathers their forecasts from hundreds of brokerage and independent analysts who track companies as part of their investment research work. Because the forecasts are not anonymous, analysts have a strong incentive to accurately report their expectations.[18] Furthermore, research on I/B/E/S suggests that financial firms' trades are consistent with their own analysts' forecasts and recommendations, which adds to the evi-

[17] To account for possible data errors or extreme outliers, we winsorize annual earnings cross-sectionally at the 1% level.

[18] See Mikhail, Walther, and Willis (1999); Cooper, Day, and Lewis (2001)

dence that reported forecasts genuinely reflect the beliefs of the firms.[19] More importantly, market participants take seriously these analyst forecasts and trade in line with them, with stock prices increasing (decreasing) shortly after upward (downward) revisions in analyst earnings forecasts (Kothari, So, and Verdi 2016).

The long-term price expectations are obtained from the three-to-five-year price targets from the Value Line Investment Survey. Value Line is an independent investment research and financial publishing firm. The price targets cover approximately 1,700 actively traded U.S. companies every period, approximately 90% of the US publicly listed firms market value.[20] Value Line does not have any investment banking relation with the analyzed firms, nor any other obvious reason for providing biased forecasts. To the best of our knowledge, this is the only widely available survey containing firm-level price forecasts at long horizons.

We construct monthly earnings expectations for every firm in I/B/E/S at different horizons by using the EPS forecasts for up to three Annual Fiscal Periods (FY1-FY3) and the Long-Term Growth measure (LTG) meant to forecast earnings growth over the next "three-to-five years." For each month, we first interpolate across the different horizons in the annual fiscal periods to estimate an expectation over the next twelve months. We repeat this procedure to calculate two-year expectations. To estimate the three-year expectations, we use the two-year expectations and compound them with the long-term growth forecasts. We repeat this procedure to get four-year earnings expectations. We exclude from the main sample the following firms: a) firms without a LTG forecast, b) firms that do not have sufficient forecasts to calculate a 12-month interpolated forecast $E_t^* [\Delta x_{i,t+1}]$, and c) firms that do not have sufficient forecasts in the next year to calculate a 12-month interpolated forecast, $E_{t+1}^* [\Delta x_{i,t+2}]$.[21]

[19] Bradshaw (2004) shows that individual earnings forecasts are correlated to Buy/Sell recommendations, while Chan, Chang, and Wang (2009) show that financial firms' own stock holding changes are significantly positively related to recommendation changes.

[20] Value Line is an industry standard to the extent that it's been documented that a large portion of investment newsletters herds towards Value Line recommendations (Graham, 1999).

[21] This last point ensures that for every firm in the main sample we can calculate revisions $E_{t+1}^* [\Delta x_{i,t+2}] - E_t^* [\Delta x_{i,t+2}]$ to test the third model prediction.

To estimate the price expectations, we obtain the one-year price expectations from the price target in I/B/E/S. We then calculate the four-year price expectation as the three-to-five year price targets from Value Line. We exclude from the main sample those firms missing either a one-year or a three-to-five year price forecast. Since analysts update earnings and price forecasts every month, our expectation data are also in monthly frequency. The main sample covers on average 79.7% of the total market size of firms listed for at least four years in CRSP.

C. Results

Table I shows the results of decomposition (18) applied both in a FIRE (Full Information Rational Expectations) benchmark and using the subjective expectations. The results show the fraction of price-earnings ratio dispersion that is explained by one-year earnings growth expectations and discount rates, as well as the fraction that is explained by four-year earnings growth expectations $\sum_{j=1}^{4}\rho^{j-1}E_t^*[\Delta\tilde{x}_{i,t+j}]$ and discount rates $\sum_{j=1}^{4}\rho^{j-1}E_t^*[\tilde{r}_{i,t+j}]$. We first apply the decomposition under the FIRE benchmark using realized values and compare the results with existing FIRE models of risk premia. Then, we apply the decomposition using subjective expectations and test the implications of our constant gain learning model.

C.1. FIRE Benchmark

Let $E_t^{FIRE}[\cdot]$ denote expectations under FIRE. Because forecast errors $\Delta\tilde{x}_{i,t+j} - E_t^*[\Delta\tilde{x}_{i,t+j}]$ are uncorrelated with time t variables under FIRE, we know that $Cov\left(E_t^{FIRE}[\Delta\tilde{x}_{i,t+j}], \tilde{px}_{i,t}\right) = Cov\left(\Delta\tilde{x}_{i,t+j}, \tilde{px}_{i,t}\right)$. The same logic also applies to FIRE expectations of future returns and future price-earnings ratios. Thus, to evaluate the FIRE benchmark, the first and fourth columns of Table I show the estimates of CF_1, DR_1, FPE_1 and CF_4, DR_4, FPE_4 using the covariance of $\tilde{px}_{i,t}$ with realized future earnings growth, returns, and price-earnings ratios.

Empirically, high price-earnings ratios are associated with lower future returns and slightly higher future earnings growth. The first column of Table I shows that 10.3% of dispersion

Table I

Decomposition of dispersion in price-earnings ratios

This table decomposes the variance of price-earnings ratios using equation (18). The FIRE column report the elements CF_h, DR_h and FPE_h of the decomposition using future earnings growth, future negative returns and future price-earning ratios. The Expected column report the elements of the decomposition using expected earnings growth, expected returns and expected price-earning ratios. The Forecast Errors column reports the contribution of the forecast errors of each element. For instance, $CF_1 = Cov\left(\Delta\tilde{x}_{i,t+1}, \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ is shown in the FIRE column. This component can be split into its expected component $Cov\left(E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ and its error component $Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$. The results for the one-year decomposition are shown on the left and the results for the four year decomposition are shown on the right. Driscoll-Kraay standard errors are calculated and clustered at the year level. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level. The main sample period is 1999 to 2020. The fourth row shows the element CF_h of the decompositions estimated over the longer sample period of 1982-2020.

		One-year horizon ($h=1$)			One-to-four year horizon ($h=4$)		
		FIRE	Expected	Forecast errors	FIRE	Expected	Forecast errors
1999-2020	CF_h	0.103***	0.331***	−0.228***	0.099***	0.433***	−0.335***
		[0.028]	[0.022]	[0.022]	[0.037]	[0.018]	[0.033]
1999-2020	DR_h	0.143***	0.033**	0.110**	0.320***	0.127***	0.192***
		[0.054]	[0.013]	[0.055]	[0.058]	[0.034]	[0.059]
1999-2020	FPE_h	0.746***	0.620***	0.126**	0.550***	0.385***	0.165***
		[0.051]	[0.019]	[0.06]	[0.043]	[0.021]	[0.049]
1982-2020	CF_h	0.137***	0.312***	−0.175***	0.147***	0.462***	−0.316***
		[0.021]	[0.018]	[0.019]	[0.030]	[0.022]	[0.023]

in price-earnings ratios is accounted for by differences in one-year future earnings growth and 14.3% is accounted for by differences in one-year future returns. The remaining 74.6% is accounted for by the future price-earnings ratio.[22] At the fourth year horizon, the difference between CF_h and DR_h widens. As shown in the fourth column of Table I, differences in future earnings growth over the next four years only accounts for 9.9% of dispersion in price-earnings ratios, while differences in future returns account for three times as much of the dispersion (32.0%).[23]

[22] Note that the three coefficients CF_h, DR_h and FPE_h are not mechanically set to equal one. However, the sum of these coefficients is very close to unity, summing 0.992 for the one-year decomposition and 0.969 for the four-year decomposition.

[23] These results are consistent with De la O, Han, and Myers (2023), who use a longer sample (1963-2020) to show that at least 43.6% of dispersion in price-earnings ratio are reflected in differences in returns after ten years.

Table II

Decompositions in Different Asset Pricing Models

This table calculates the variance decomposition for the price-earnings ratio in different asset pricing models and reports the implied cash flow and discount rate components for one year (CF_1, DR_1) and four years (CF_4, DR_4), as well as the infinite-horizon DR_∞. The first, second, and third rows show the results for models of risk premia. These three models are the model of growth options in Berk et al. (1999), the model of costly reversibility of capital in Zhang (2005), and the model of duration risk in Lettau and Wachter (2007). The last row shows the values measured in the data. All models are solved and estimated using the original author calibrations and simulated over a 20-year sample.

Models	CF_1	CF_4	DR_1	DR_4	DR_∞
Berk, Green, & Naik 1999 (Growth Options)	0.61	0.85	0.01	0.03	0.04
Zhang 2005 (Costly Reversibility of Capital)	−0.31	0.69	−0.01	−0.03	−0.03
Lettau & Wachter 2007 (Duration Premium)	0.03	0.24	0.02	0.06	−0.03
Observed Data (Main Sample)	0.10	0.10	0.14	0.32	n.a.

The large role of returns in explaining price dispersion poses a quantitative challenge for traditional FIRE asset pricing models. Even models designed to generate a value premium (i.e., low expected returns for high price ratio stocks) struggle to generate enough dispersion in expected returns to match our findings. In Table II, we simulate three FIRE models for the value premium (Berk et al., 1999; Zhang, 2005; Lettau and Wachter, 2007) using their benchmark specifications and calculate the model CF_h and DR_h. As shown by the value of DR_h, in these models, differences in expected returns only account for a small fraction of the dispersion in price-earnings ratios. Specifically, at the four-year horizon, the three models imply that future returns should account for less than 6% of dispersion in price-earnings ratio, while empirically, we find that they account for 32%. In the data and (to some extent) in the models, DR_h increases as we include more horizons. Thus, we also calculate DR_∞ in the models and find that it is still an order of magnitude smaller than what we observe in the data using just the first four years of realized returns.

These observations highlight the importance of a quantitative framework. While there are certainly FIRE models in which high price ratio stocks have lower exposure to systematic risk,[24] it is difficult to generate a risk premium that is quantitatively large enough to match

[24] In Berk et al., 1999 and Zhang, 2005, existing projects (or capital) cannot be adjusted easily in response to aggregate shocks. Thus, firms whose value mainly comes from future potential projects rather than

the observed relationship between price-earnings ratios and future returns. Appendix D discusses the three models and the simulations in more detail.

C.2. Subjective Expectations

The second and fifth columns of Table I show the results of the decomposition when we use subjective expectations of earnings growth, returns, and future price-earnings ratios rather than assuming FIRE. Comparing the subjective results to the FIRE results, there are three important findings.

First, investors substantially overestimate the extent to which high price-earnings ratio stocks will have high future earnings growth. Differences in expected one-year earnings growth account for nearly a third (33.1%) of all dispersion in price-earnings ratios and differences in expected four-year earnings growth account for 43.3% of all price-earnings ratio dispersion. Given that realized one-year and four-year earnings growth only account for 10.3% and 9.9% of the dispersion, respectively, this means that high price-earnings ratios are consistently associated with disappointment in future earnings growth. Rephrased, more than a third of all dispersion in price-earnings ratios is accounted for by the fact that current price-earnings ratios significantly negatively predict future forecast errors (as shown in the "Forecast errors" columns). The final row of Table I shows that our earnings growth results are qualitatively and quantitatively similar over the longer 1982-2020 sample.

Second, investors understand that expensive stocks will have lower returns (i.e., a high price-earnings ratio is associated with lower future returns), but they underestimate the magnitude of the relationship. As shown in the second row of Table I, differences in expected one-year returns account for 3.3% of dispersion in price-earnings ratios and differences in expected four-year returns account for 12.7%. This contrasts sharply with previous findings for aggregate return expectations, which positively comove with aggregate price ratios (Am-

existing projects carry a lower risk premium. In Lettau and Wachter, 2007, aggregate shocks are partly reversed over time, which makes longer horizon cash flows less exposed to aggregate risk similar to our model in Section I. Firms whose value mostly comes from backloaded cash flows rather than current cash flows therefore carry a lower risk premium.

romin and Sharpe, 2014; Greenwood and Shleifer, 2014; De la O and Myers, 2021). The fact that investors expect lower returns on high price-earnings ratio stocks points against stories in which the cross-section of prices is driven by investors bidding up the price of certain stocks due to *high* expected returns (e.g., return extrapolation). Consistent with the fact that investors overestimate future earnings growth for high $\tilde{px}_{i,t}$, we find that they consistently overestimate the returns for high $\tilde{px}_{i,t}$. In other words, while investors expect lower returns for high $\tilde{px}_{i,t}$ stocks, the realized returns are even worse than expected.

Combined, these first two findings emphasize that the mistakes in investors' expectations are about magnitudes, not directions. Investors understand that high price-earnings ratios are associated with higher future earnings growth and lower future returns, but they overestimate the magnitude of the earnings growth relationship and underestimate the magnitude of the return relationship. This highlights the benefit of using a quantitative decomposition which captures magnitudes as well as correlations to study these expectations. These findings are consistent with the first implication of the constant gain model. In the model, a higher expected g_i raises the firm's price by both raising expected future cash flows and by lowering the expected return. Given that the objective g_i is the same across all firms, a high expected g_i is then followed by disappointing earnings growth and returns that are lower than expected.

The third finding, shown in the third column of Table I, is that the unexpected returns are smaller than the disappointment in earnings growth. While almost one quarter (22.8%) of $\tilde{px}_{i,t}$ dispersion is reflected in one-year earnings growth forecast errors, only 11.0% is reflected in one-year unexpected returns. In other words, the disappointment in earnings growth does not lead to an equally large disappointment in returns. This is consistent with the second implication of the constant gain model. If the gain parameter β is small, then the impact of earnings growth disappointment on one-period returns will be small.

The muted relationship between earnings growth disappointment and unexpected returns has an important implication for expectation formation models. Why is disappointing earnings growth not immediately reflected in unexpected returns? From equation (16), we have that

$$\tilde{r}_{i,t+1} - E_t^* \left[\tilde{r}_{i,t+1}\right] \approx \left(\Delta\tilde{x}_{i,t+1} - E_t^* \left[\Delta\tilde{x}_{i,t+1}\right]\right) + \rho \left(\tilde{px}_{i,t+1} - E_t^* \left[\tilde{px}_{i,t+1}\right]\right). \quad (19)$$

If there is no unexpected change to the price-earnings ratio $\tilde{px}_{i,t+1}$, then disappointment in one-year earnings growth should translate 1-1 into disappointment in one-year returns. For models in which disappointing earnings growth lowers expected future earnings growth (e.g., overstating the persistence, diagnostic expectations of growth, or extrapolating from the most recent realization), then disappointment in earnings growth will also lower $\tilde{px}_{i,t+1} - E_t^* \left[\tilde{px}_{i,t+1}\right]$, as the change in expected future earnings growth lowers the price-earnings ratio. In these models, we should actually expect to see that the disappointment in returns is *larger* in magnitude than the disappointment in earnings growth, as returns capture the disappointment in $t+1$ earnings growth and the downward revision to earnings growth for $t+2$ and beyond.

Thus, the fact that return disappointment is empirically *smaller* in magnitude than earnings growth disappointment is closely tied to the third implication of the constant gain model: earnings growth disappointment *raises* expected future growth $E_{t+1}^* \left[\Delta\tilde{x}_{i,t+2}\right]$. When the gain parameter β is small, earnings growth disappointment is largely attributed to a negative noisy shock, which means that agents expect earnings to revert back to their previous trend. We directly test this implication by regressing earnings growth revisions $E_t^* \left[\Delta x_{i,t+1}\right] - E_{t-1}^* \left[\Delta x_{i,t+1}\right]$ on earnings growth surprises $\Delta x_{i,t} - E_{t-1}^* \left[\Delta x_{i,t}\right]$. In our main sample, this regression gives a significant coefficient of -0.86^{***} (see Table IV). The fact that earnings growth surprises are negatively related to earnings growth revisions and the fact that return disappointment is smaller in magnitude than earnings growth disappointment both provide evidence in support of models in which investors' expectations are relatively "stubborn", i.e., investors expect earnings to largely revert after a shock, rather than models in which expectations of future earnings are highly sensitive to the most recent realization.

D. *Extending to anomaly returns*

The logic in equation (19) can be extended to other forms of cross-sectional return differences. Consider an anomaly variable $\tilde{a}_{i,t}$, such as profitability or investment, which predicts next-period returns. To make comparison across anomalies simple, we normalize $\tilde{a}_{i,t}$ so that it has variance 1 and positively comoves with future returns. From equation (19), we have the identity

$$\underbrace{Cov\left(\tilde{r}_{i,t+1} - E_t^*\left[\tilde{r}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,r}} \approx \underbrace{Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,x}} + \underbrace{\rho Cov\left(\tilde{px}_{i,t+1} - E_t^*\left[\tilde{px}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,px}}. \tag{20}$$

For robustness, Appendix C shows an exact decomposition based on price growth, which gives very similar results.

Under full-information rational expectations, we would have $\sigma_{a,r}, \sigma_{a,x}, \sigma_{a,px} = 0$, i.e., any predictable anomaly returns would be fully anticipated and $\tilde{a}_{i,t}$ would not predict forecast errors. For example, a higher $\tilde{a}_{i,t}$ might be related to higher risk exposure and investors would require higher returns on these stocks as compensation. More broadly, positive values of $\sigma_{a,r}$ indicate that investors understate the relationship between $\tilde{a}_{i,t}$ and future returns. In other words, the high returns on high $\tilde{a}_{i,t}$ stocks are not fully anticipated. Negative values for $\sigma_{a,r}$ indicate that investors not only understand that high $\tilde{a}_{i,t}$ stocks have higher returns, but they exaggerate the magnitude of the relationship.

In comparison, the values for $\sigma_{a,x}$ and $\sigma_{a,px}$ indicate how much the predictable return forecast errors are explained by predictable errors in next-year earnings growth expectations and expectations of the future price-earnings ratio. When high $\tilde{a}_{i,t}$ stocks generate unanticipated high next-period returns, these returns can be explained by unexpectedly high next-period earnings growth. Alternatively, $\tilde{a}_{i,t}$ could positively predict forecast errors for the future price-earnings, which would mean that the unanticipated high returns of high $\tilde{a}_{i,t}$ stocks are due to errors in return expectations and earnings growth expectations at longer

Table III

Unexpected Anomaly Returns

This table measures and decomposes unexpected anomaly returns. For each anomaly $\tilde{a}_{i,t}$, we sort stocks into five equal-value portfolios based on the anomaly variable. The table shows the coefficients of regressing each of the dependent variables on a specific anomaly variable. The three dependent variables are the unexpected return $\tilde{r}_{i,t+1} - E_t^* [\tilde{r}_{i,t+1}]$, the earnings growth forecast errors $\Delta\tilde{x}_{i,t+1} - E_t^* [\Delta\tilde{x}_{i,t+1}]$, and the price-earnings ratio forecast errors $\rho \left(\tilde{px}_{i,t+1} - E_t^* \left[\tilde{px}_{i,t+1}\right]\right)$. For the anomaly variables, Value is measured using the price-earnings ratio. Profitability is measured using gross profitability. Investment is measured using net stock issuance. The Representative Anomaly is the average ranking of each stock across 22 different anomalies. Each anomaly variable is scaled to have a standard deviation of 1 and signed to positively comove with future returns. The sample period is 1999 to 2020. Driscoll-Kraay standard errors are clustered at the portfolio and year level. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.

	Dependent variable		
Anomaly $\tilde{a}_{i,t}$	$\tilde{r}_{i,t+1} - E_t^* [\tilde{r}_{i,t+1}]$	$\Delta\tilde{x}_{i,t+1} - E_t^* [\Delta\tilde{x}_{i,t+1}]$	$\rho \left(\tilde{px}_{i,t+1} - E_t^* \left[\tilde{px}_{i,t+1}\right]\right)$
Value	0.0331^{**}	0.0687^{***}	-0.0380^{**}
	[0.0166]	[0.0066]	[0.0181]
Profitability	0.0081	0.0091	0.0016
	[0.0078]	[0.0113]	[0.0108]
Investment	0.0260^{***}	0.0287^{**}	-0.0043
	[0.0089]	[0.0127]	[0.0069]
Representative anomaly	0.0325^{***}	0.0655^{***}	-0.0353^{***}
	[0.0098]	[0.0165]	[0.0094]

horizons beyond one period, not because of next-period earnings growth.

Table III shows the results for value, profitability, investment, and a representative anomaly comprised of 22 individual anomalies from Hou, Xue, and Zhang (2015).[25] For each anomaly, we sort stocks into five equal-value portfolios based on the anomaly variable.[26] We then measure forecast errors for one-year returns, earnings growth, and price-earnings ratios and regress each of the three variables on the anomaly variable. For Value, we utilize the same portfolios as the previous section and use the current price-earnings ratio as the anomaly variable to align with the tests in the previous section. For the profitability

[25]Because forecasts are provided primarily for large firms, our expectations data is not well suited for studying the size anomaly.

[26]To perform these tests, stocks are required to have one-year expected and realized earnings growth, returns, and price-earnings ratios. We also require that stocks have a future one-year earnings growth expectation $E_{t+1}^* [\Delta\tilde{x}_{i,t+2}]$ for our test of revisions.

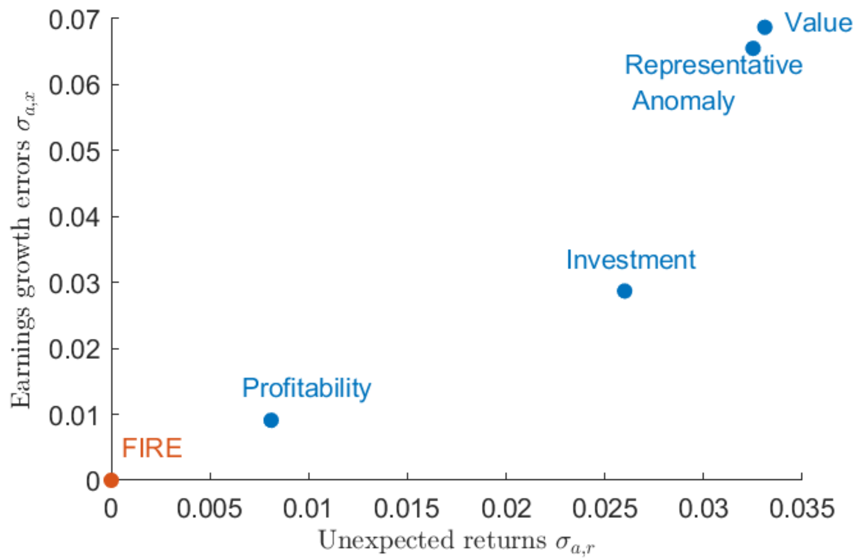

Figure 1. Unexpected anomaly returns. This figure shows the decomposition results $(\sigma_{a,r}, \sigma_{a,x})$ for each anomaly $\tilde{a}_{i,t}$. The x-axis shows $\sigma_{a,r} = Cov\left(\tilde{r}_{i,t+1} - E_t^*\left[\tilde{r}_{i,t+1}\right], \tilde{a}_{i,t}\right)$, which measures how much the anomaly variable predicts unexpected returns. The y-axis shows $\sigma_{a,r} = Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{a}_{i,t}\right)$, which measures how much the anomaly variable predicts one-year earnings growth forecast errors. The anomalies are shown in blue. In red, we show the FIRE benchmark, which is that $\sigma_{a,r}$ and $\sigma_{a,x}$ should equal 0 for all anomalies.

anomaly, we use gross profitability as our measure, specifically revenue minus costs of goods sold relative to total assets. For the investment anomaly, we follow Pontiff and Woodgate (2008) and Fama and French (2008) and use net stock issuance. Finally, the representative anomaly sorts stocks based on 22 different variables and uses the average ranking across these 22 variables in the sorting and in the regressions.

Figure 1 and the first two columns of Table III show the values of $\sigma_{a,r}$ and $\sigma_{a,x}$ for each anomaly. For each anomaly, we estimate a positive value of $\sigma_{a,r}$, meaning that investors do not fully anticipate the high returns on high $\tilde{a}_{i,t}$ stocks. Instead, we find that each anomaly is associated with large positive one-year earnings growth forecast errors, as shown by the estimates of $\sigma_{a,x}$. Further, Figure 1 shows that anomalies with higher $\sigma_{a,r}$ have higher $\sigma_{a,x}$, i.e., larger unanticipated returns are associated with larger one-year earnings growth forecast errors. This demonstrates the benefit of using a quantitative decomposition, as we can state that one-year earnings growth forecast errors not only have the correct sign

Table IV

Revisions in expectations

This table shows the effect of earnings growth surprises on revisions. For each anomaly, we sort stocks into five equal-value portfolios based on the anomaly variable. Each column shows the cofficient from regressing the revision in earnings growth $E^*_{t+1}[\Delta\tilde{x}_{i,t+2}] - E^*_t[\Delta\tilde{x}_{i,t+2}]$ on the earnings growth surprise $\Delta x_{i,t+1} - E^*_t[\Delta x_{i,t+1}]$. Value is measured using the price-earnings ratio. Profitability is measured using gross profitability. Investment is measured using net stock issuance. The Representative Anomaly is the average ranking of each stock across 22 different anomalies. The first row shows the result of the regressions using the main sample period of 1999 to 2020. The second row shows the result of the regressions using the long sample period of 1982 to 2020. Driscoll-Kraay standard errors are clustered at the portfolio and year level. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.

	Value	Profitability	Investment	Representative Anomaly
Main Sample 1999-2020	-0.863^{***} [0.061]	-0.743^{***} [0.0790]	-0.832^{***} [0.027]	-0.825^{***} [0.059]
Long Sample 1982-2020	-0.786^{***} [0.075]	-0.857^{***} [0.027]	-0.777^{***} [0.058]	-0.852^{***} [0.052]

to explain unexpected returns, but also are large enough in magnitude to account for the unexpected anomaly returns.

Interestingly, for all four anomalies, the predictable errors in one-year earnings growth expectations are more than large enough to account for the unexpected one-year returns (i.e., $\sigma_{a,x}$ is greater than $\sigma_{a,r}$). This is analogous to the finding in the price-earnings ratio decomposition that one-year earnings growth surprises are larger in magnitude than the one-year unexpected returns. Once again, we find that this is explained by the fact that positive surprises $\Delta\tilde{x}_{i,t+1} - E^*_t[\Delta\tilde{x}_{i,t+1}]$ *decrease* expected next period growth $E^*_{t+1}[\Delta\tilde{x}_{i,t+2}]$. For each set of anomaly portfolios, Table IV shows the coefficient from regressing the revision $E^*_{t+1}[\Delta\tilde{x}_{i,t+2}] - E^*_t[\Delta\tilde{x}_{i,t+2}]$ on the earnings growth surprise $\Delta x_{i,t} - E^*_{t-1}[\Delta x_{i,t}]$. In each case, we find a significant negative coefficient ranging between -0.74 and -0.86. For robustness, we also repeat the test using earnings growth expectations over the long sample and find similar results.

III. Quantitative model and full decomposition

While the results in Section II support the three qualitative implications of the constant gain learning model, the decomposition in Table I also provides important quantitative implications. As shown in Table II, there may be many models that qualitatively match the decomposition results but struggle to quantitatively match the magnitudes. In this section, we quantify the constant gain learning model using data on realized and expected earnings growth.

The quantified model fulfills three key purposes. First, quoting Brunnermeier et al., 2021, *"Research focus should be on motivating, building, calibrating, and estimating models with non-RE beliefs rather than on merely rejecting RE models. To make further progress, we need structural models of belief dynamics that can compete with RE models in explaining asset prices and empirically observed beliefs."*[27] This model intends to be a step in this direction. It provides a quantitative model that generates realistic asset pricing moments and outperforms the FIRE models of Table II in matching the empirical decomposition results. Notably, the model matches both the magnitudes and timing for the dynamics of price-earnings ratios, realized and expected earnings growth, and realized and expected returns. Second, the quantified model allows us to extend the decomposition in equation (18) beyond the four-year horizon to estimate the full role of expected earnings growth and subjective discount rates in accounting for the dispersion in price-earnings ratios. Third, we can analyze the importance of learning, risk sensitivity, and the interaction between the learning and risk sensitivity using counterfactuals where one or both of these channels are removed (β and/or γ set to 0).

[27] Note that in this paper uses RE as a shorthand for full information rational expectations and specifically highlights learning about parameters as a promising form on non-RE models to explore: *"For example, models of Bayesian learning relax the RE assumption that agents know the model of the world and its parameter values".*

Table V

Model estimation

This table shows the value of the six parameters of the constant-gain learning model and the targeted moments. The parameters for the aggregate cash flow process ϕ, σ_u are derived directly from the autocorrelation and standard deviation of the S&P 500 annual earnings growth. The firm-level volatility σ_v is derived directly from the standard deviation over time of the portfolio-level annual earnings growth (averaged across portfolios). The risk-free rate r^f and risk sensitivity γ are set to match the average one-year Treasury yield and average aggregate equity return during the sample period. The constant-gain parameter β is set to match the standard deviation over time of one-year earnings growth expectations (averaged across portfolios). All aggregate moments are estimated over the full sample period of 1982 to 2020.

Parameter	Value	Moments
	Cash flow process	
ϕ	.828	$AC(\Delta x_{t+1}^{agg})$
σ_u	.337	$\sigma\left(\Delta x_{t+1}^{agg}\right)$
σ_v	.106	$\sigma\left(\Delta x_{i,t+1}\right)$
	SDF	
r_f	4.6%	Risk-free rate
γ	1.63	Average aggregate return
	Learning	
β	1.9%	$\sigma\left(E_t^*\left[\Delta x_{i,t+1}\right]\right)$

A. Estimation

The model only has six parameters, which are all shown in Table V. The parameters for cash flows $(\phi, \sigma_u, \sigma_v)$ are all estimated directly from realized earnings growth for our full sample of 1982-2020. For the aggregate process, the standard deviation and autocorrelation of S&P 500 earnings growth imply a persistence $\phi = 0.828$ and a volatility $\sigma_u = 0.337$. The volatility of individual shocks $\sigma_v = 0.106$ is obtained from the volatility over time of the portfolio-level earnings growth. Appendix E shows the exact formulas mapping these empirical moments to the model parameters. For the agent's stochastic discount factor, the risk-free rate $r^f = 4.6\%$ and the sensitivity to risk $\gamma = 1.63$ are set to match the average one-year Treasury yield and average aggregate stock return of 10.4% for 1982-2020. Finally, the constant gain parameter β is set to match the volatility over time of one-year earnings growth expectations, $\sigma\left(E_t^*\left[\Delta x_{i,t+1}\right]\right) = 13.2\%$.[28]

[28]The model is simulated yearly over 400 periods for 300 firms. To avoid being impacted by the initial value of the expectations $E_0^*\left[g_i\right]$, we calculate all moments after $t = 150$.

Table VI

Model evaluation

This table evaluates the constant-gain learning model by comparing the untargeted aggregate and cross-sectional moments in the model simulations with those observed in that data. Panel A shows the mean, standard deviation and autocorrelation of the aggregate price-earnings ratio as well as the standard deviation of aggregate stock returns. Panel B shows the cross-sectional standard deviations of the price-earnings ratio, future earnings growth and returns, and expected earnings growth and returns. All aggregate moments are estimated over the full sample period of 1982 to 2020. The cross-sectional moments are estimated over the main sample of 1999 to 2020 due to data availability.

	Model	Data
Panel A: Aggregate value		
Mean px_t	2.32	2.98
$\sigma(px_t)$	41.7%	42.5%
$AC(px_t)$	0.79	0.74
$\sigma(r_t)$	11.3%	15.9%
Panel B: Cross-sectional standard deviation		
$\tilde{px}_{i,t}$	22.3%	28.3%
$\Delta\tilde{x}_{i,t+1}$	12.6%	9.1%
$\tilde{r}_{i,t+1}$	6.2%	7.1%
$E_t^*[\Delta\tilde{x}_{i,t+1}]$	15.1%	14.6%
$E_t^*[\tilde{r}_{i,t+1}]$	0.8%	3.3%

B. Model performance

B.1. Aggregate and cross-sectional moments

Even under this straightforward estimation, the model is able to match several relevant moments from the data. Table VI shows a comparison of the untargeted moments in the model and the data. First, despite not using any price information in the estimation other than the average aggregate equity return, the model generates realistic dynamics for the aggregate price-earnings ratio. The unconditional mean, volatility and autocorrelation of the log price-earnings ratio in the model (2.32, 41.7% and 0.79) are consistent with the observed values ($2.98, 42.5\%$ and 0.74) and the model generates volatile returns.

Second, while no information on cross-sectional dispersion was used in the estimation, the model performs well in matching the dispersion in price-earnings ratios, realized and expected earnings growth, and realized and expected returns. The empirical values are based on the standard deviation of the cross-sectionally demeaned values for each variable.

In line with the data, the model generates large cross-sectional differences in price-earnings ratios, realized earnings growth, realized returns, and expected earnings growth. For the sake of parsimony, in the model, subjective discount rates are entirely driven by duration risk premia, see equation (13). Because of this, the model does understate the cross-sectional differences in expected returns. Expanding the model to incorporate other risks into discount rates could help to better match this moment. However, we will show that even without these extra features, the model still succeeds in matching the covariance of price-earnings ratios with expected and realized returns. In other words, while the model does not capture all cross-sectional differences in expected returns, it does capture the portion that is predictable with price-earnings ratios which is what is most important for our purposes.

B.2. Dynamics of prices, cash flows, and returns

A key message of this paper is to emphasize the joint dynamics of price ratios, earnings growth, and returns. Importantly, we want to understand both the expected dynamics and the realized dynamics. The bars in Figure 2 show the one- and four-year price-earnings ratio decomposition results from Table I, along with their 95% confidence intervals. For comparison, the black dots show the values implied by the model. In every case, we cannot reject that the model implied value matches the value measured in the data. Overall, this represents 8 untargeted moments which the model successfully matches, along with the 9 untargeted moments in Table VI.

The left side of Figure 2 shows that high price-earnings ratios are associated with high expected earnings growth. In comparison, the relationship between price-earnings ratios and future realized earnings growth is much smaller, meaning that high price-earnings ratios predict disappointment in future earnings growth. The right side of the figure shows that high price-earnings ratios are also associated with moderately lower expected returns.

The fact that the model matches our decomposition results at multiple horizons highlights that the model is successful both in terms of magnitudes and in terms of timing. While the

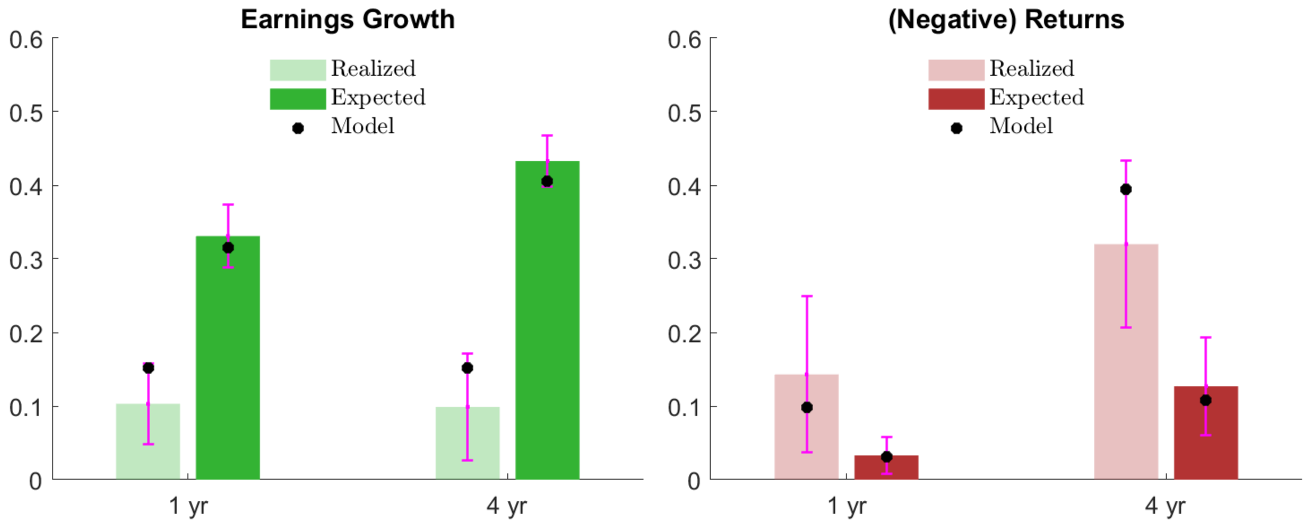

Figure 2. Empirical decomposition and model decomposition. This figure evaluates the one- and four-year decomposition of $\tilde{px}_{i,t}$ dispersion in the model. The light bars show the contribution of realized earnings growth and realized returns to the dispersion of price-earnings ratio obtained in the first and fourth columns of Table I. The dark bars show the contribution of expected earnings growth and expected returns to the dispersion of price-earnings ratio obtained in the second and fifth columns of Table I. Each bar shows Driscoll-Kraay 95% confidence intervals. The black dots show the values of both the realized and expected decomposition implied by the model.

difference in expected and realized one-year earnings growth is large, this does not translate into a large difference between expected and realized one-year negative returns. Instead, agents are slow to adjust their beliefs and the disappointment in earnings growth leads to much lower than expected returns at longer horizons.

C. Full role of objective cash flows, cash flow mistakes, and discount rates

Using the quantified model, we can measure the full role of cash flow growth expectations and subjective discount rates in accounting for price-earnings ratio dispersion. Table VII Panel C shows the decomposition in equation (18) when we extend to the infinite horizon. Specifically, it shows the cross-sectional dispersion $Var\left(\tilde{px}_{i,t}\right)$ and the two components $Cov\left(\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\Delta\tilde{x}_{i,t+j}\right],\tilde{px}_{i,t}\right)$, $Cov\left(-\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\tilde{r}_{i,t+j}\right],\tilde{px}_{i,t}\right)$.

To start, we focus on the final column, which is our main model parameterization. As shown in the second row of Panel C, the model estimates that differences in expected cash flow growth account for two-thirds (65.4%) of all dispersion in price-earnings ratios. Combined with the aggregate time series findings of De la O and Myers (2021), this means that both time series variation in aggregate price ratios and cross-sectional dispersion in price ratios are both primarily explained by expected cash flow growth.[29] However, unlike the aggregate time series findings, we also estimate a non-trivial role for subjective discount rates in accounting for price-earnings ratio dispersion. The fifth row of Panel C shows that low subjective discount rates for high price-earnings ratio firms accounts for roughly one-third (34.6%) of all dispersion in price-earnings ratios.

Looking at the breakdown of the 65.4% contribution from expected earnings growth, we see that this largely comes from forecast errors. The comovement of price-earnings ratios with realized future earnings growth only accounts for 16.0% of the dispersion, meaning that the remaining 49.4% comes from price-earnings ratios predicting forecast errors for earnings growth. As a result, high price-earnings ratios are largely associated with low future returns, with negative realized returns accounting for 84% of all price-earnings ratio dispersion. Note that at the infinite horizon, forecast errors for earnings growth and forecast errors for returns are equal (i.e., the forecast error row for earnings growth and negative returns are exactly opposite). While gradual learning affects how quickly earnings growth surprises are reflected in unexpected returns, eventually all unexpected earnings growth will appear as unexpected returns.

Conveniently, we can summarize the relative importance of realized future earnings growth, errors in earnings growth expectations, and subjective discount rates. The model estimates that realized earnings growth accounts for roughly 1/6 (16.0%) of price-earnings ratio dispersion, errors in earnings growth expectations account for 1/2 (49.4%), and subjective discount rates account for 1/3 (34.6%). Additionally, besides decomposing differences in price-earnings ratios, the model also decomposes the low realized returns earned by expensive stocks. The estimation implies that 41.2%(34.6/84.0) of the difference in returns

[29] This is consistent with the empirical results of Table I, where we find that expected earnings growth over just the first four years already accounts for 43.3% of all price-earnings ratio dispersion.

Table VII

Infinite-horizon decomposition and counterfactual analysis

This table decomposes the cross-sectional dispersion of price-earnings ratios into their infinite-horizon components. Each column shows the decomposition implied by the constant gain learning model using different key parameter choices. Panel A shows the parameters which change for each specification. All other parameters are set to the values in the main specification in Table V. The first column runs a model with no learning or risk sensitivity, $\gamma = 0$ and $\beta = 0$. The second column runs a model with no learning, $\beta = 0$. The third column runs a model with no risk sensitivity, $\gamma = 0$. The main specification is shown in the last column. Models with $\gamma = 0$ are also run with a different risk-free rate $r_f = 10.4\%$ to ensure the average level of equity returns is consistent across all specifications. Panel B shows the magnitudes of the mean aggregate price-earnings ratio and aggregate returns implied by each of the specifications. Panel C shows the decomposition results. The first row shows the implied cross-sectional variance of the price-earnings ratio for each specification. The second and fifth row of Panel C show the amount of cross-sectional variance in $\tilde{px}_{i,t}$ explained by expected earnings growth $\sum_{j=1}^{\infty} E_t^* [\Delta\tilde{x}_{i,t+j}]$ and subjective discount rates $-\sum_{j=1}^{\infty} \rho^{j-1} E_t^* [\tilde{r}_{i,t+j}]$, estimated through the infinite-horizon version of equation (18):

$$Var\left(\tilde{px}_{i,t}\right) = Cov\left(\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\Delta\tilde{x}_{i,t+j}\right], \tilde{px}_{i,t}\right) + Cov\left(-\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\tilde{r}_{i,t+j}\right], \tilde{px}_{i,t}\right).$$

The third and sixth row of Panel C show the amount of cross-sectional variance in $\tilde{px}_{i,t}$ explained by realized earnings growth $\sum_{j=1}^{\infty} \rho^{j-1}\Delta\tilde{x}_{i,t+j}$ and negative realized returns $-\sum_{j=1}^{\infty} \rho^{j-1}\tilde{r}_{i,t+j}$. Finally, the fourth and seventh rows show the amount of cross-sectional variance in $\tilde{px}_{i,t}$ explained by earnings growth forecast errors $\sum_{j=1}^{\infty} \rho^{j-1}(\Delta\tilde{x}_{i,t+j} - E_t^* [\Delta\tilde{x}_{i,t+j}])$ and return forecast errors $-\sum_{j=1}^{\infty} \rho^{j-1}(\tilde{r}_{i,t+j} - E_t^* [\tilde{r}_{i,t+j}])$. The share of the cross-sectional variance of $\tilde{px}_{i,t}$ is shown in parenthesis.

Panel A: Parameter values				
β	0	0	1.9%	1.9%
γ	0	1.63	0	1.63
r^f	10.4%	4.6%	10.4%	4.6%
Panel B: Levels				
Mean px_t	2.32	2.32	2.32	2.32
Mean r_{t+1}	10.4%	10.4%	10.4%	10.4%
Panel C: Decomposing cross-sectional dispersion				
Variance $\tilde{px}_{i,t}$	1.12	1.12	2.33	4.97
Expected earnings growth	1.12	1.12	2.33	3.25
	(100%)	(100%)	(100%)	(65.4%)
Realized earnings growth	1.12	1.12	0.90	0.80
	(100%)	(100%)	(38.4%)	(16.0%)
Forecast errors	0	0	1.44	2.45
	(0%)	(0%)	(61.6%)	(49.4%)
Subjective discount rates	0	0	0	1.72
	(0%)	(0%)	(0%)	(34.6%)
Negative realized returns	0	0	1.44	4.17
	(0%)	(0%)	(61.6%)	(84.0%)
Negative forecast errors	0	0	-1.44	-2.45
	(0%)	(0%)	(-61.6%)	(-49.4%)

between high and low price-earnings ratio stocks reflects subjective discount rates while 58.8% (49.4/84.0) reflects disappointment in earnings growth.

More broadly, by having a structural model, we can investigate the economic role of learning and risk sensitivity in driving the cross-sectional dispersion in price-earnings ratios. The different columns in Table VII show the dispersion in price-earnings ratios and the decomposition results when β and/or γ are set to 0, i.e., learning and/or risk sensitivity are turned off. In all cases, the initial expected g_i is set to 0 for all firms. Thus, the two cases where $\beta = 0$ are equivalent to saying that agents know the objective data-generating process and no longer need to learn the parameters. Given that we are interested in cross-sectional dispersion rather than levels, in the two cases where $\gamma = 0$ we also raise the risk-free rate from 4.6% to 10.4%. As shown in Panel B, this ensures that the aggregate level for price-earnings ratios and equity returns are identical across all four cases and it is only the dispersion that changes. Thus, the two cases where $\gamma = 0$ are equivalent to saying that all firms have the same subjective discount rate of 10.4%.

In the first column, both β and γ are set to 0. In this case, the dispersion in price-earnings ratios is less than 1/4 the value in our main specification (1.12 compared to 4.97). The dispersion in price-earnings ratios comes entirely from differences in expected earnings growth, as there are no differences in subjective discount rates. Price-earnings ratios do not predict earnings growth forecast errors. Instead, all differences in expected earnings growth are simply due to the noise shocks $v_{i,t}$.

In the second column, the model includes risk sensitivity ($\gamma > 0$) but keeps $\beta = 0$. As shown in Panel C, only including risk sensitivity has no effect on the results relative to the first column. This highlights that, in our model, dispersion in expected g_i causes not only dispersion in expected earnings growth but also the dispersion in subjective discount rates. While agents may be sensitive to duration risk, this only matters if firms are expected to differ in the timing of their cash flows.

In comparison, the third column shows that including learning but keeping $\gamma = 0$ does substantially change the results. The dispersion in price-earnings ratios doubles from 1.12 to 2.33. This largely comes from price-earnings ratios now comoving with future earnings growth forecast errors. However, there is also the interesting result that the comovement of price-earnings ratios with realized earnings growth decreases (1.12 to 0.90). The FIRE expectation for future earnings growth is simply $-v_{i,t}$. With learning, expected earnings growth depends on $E_t^*[g_i]$,wich comoves positively with $v_{i,t}$, as a positive shock will tend to increase the guess for g_i. Thus, introducing learning means that the price-earnings ratio, which depends on expected earnings growth, will now be less related to future earnings growth due to the response to shocks $v_{i,t}$.

Finally, the last column shows the interaction from including both risk sensitivity and learning. While risk sensitivity by itself has no effect, once we incorporate learning, increasing γ from 0 to 1.63 more than doubles the dispersion in price-earnings ratios (2.33 to 4.97). Looking at the contribution of subjective discount rates, we clearly see the interaction between risk sensitivity and learning, as dispersion in subjective discount rates now contributes 1.72 (34.6%) to the total dispersion in price-earnings ratios.

More surprisingly, we also find an important interaction between risk sensitivity and learning for the contribution of earnings growth expectations. Given that γ has no impact on equations (5)-(8), changing γ has no effect on expected earnings growth. Thus, the increase in comovement between price-earnings ratios and expected earnings growth (2.33 to 3.25) is entirely due to changes in the price-earnings ratios. Intuitively, incorporating duration-based discount rates increases the sensitivity of price-earnings ratios to expected g_i, and this increased sensitivity is reflected in the larger comovement of price-earnings ratios with expected earnings growth. This logic extends to any model with duration-based discount rates and shows that while discount rates may not affect expected earnings growth, they can be quantitatively important for driving the comovement of price ratios with expected earnings and earnings growth forecast errors.

Overall, the fact that dispersion in price-earnings ratios for $\beta > 0, \gamma > 0$ is more than twice as large as any of the other counterfactuals highlights the natural interaction between preferences for the timing of cash flows and learning about cash flow growth. We find that this interaction is quantitatively important for matching the large empirical dispersion in price-earnings ratios.

IV. Conclusion

We find that subjective expectations have substantial potential to explain the cross-section of stock price ratios and shed light on the relative importance of expected future cash flows and discount rates. Using a variance decomposition, we show that cross-sectional dispersion in price-earnings ratios is primarily explained by predictable errors in subjective expectations of earnings growth. Subjective discount rates play a secondary, but non-trivial role. In a similar vein, we show that investors do not fully anticipate the high returns on anomaly portfolios and that these returns instead largely reflect positive unexpected earnings growth.

To understand this findings, we provide a quantitative model which not only outperforms standard FIRE models in matching the dynamics of prices and realized earnings growth and returns, but also matches the dynamics of prices and expectations. The model features constant gain learning about earnings growth and duration-based risk premia and emphasizes the importance of slow-moving beliefs in order to match the empirical timing of earnings growth expectations and realized returns. In both the model and the data, disappointment in one-period earnings growth does not immediately lead to large negative returns. Instead, returns fall gradually over time as agents adjust their beliefs.

These findings for the cross-section of stock prices are consistent with the aggregate time-series findings of De la O and Myers (2021, 2023), who emphasize that aggregate stock prices are largely driven by subjective earnings growth expectations and that errors in short-term earnings growth expectations play a particularly large role in explaining long-term returns.

This harmony between the aggregate time-series and the cross-section indicates that a single mechanism could potentially explain both dimensions of the data and provides a strong motivation for further research understanding how investors form cash flow expectations and discount rates.

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Appendix

A. Model prices and returns

To derive equation (9), we guess and verify a log-affine form for the strip price, $P_t^{(n)} = \exp\{A(n) + \phi^n x_t^{agg}\}$. The strip price is then pinned down by $P_t^{(0)} = \exp\{x_t^{agg}\}$ (i.e., $A(0) = 0$) and

$$\begin{aligned} P_t^{(n)} &= E_t^*\left[M_{t+1}P_{t+1}^{(n-1)}\right] \\ &= E_t^*\left[\exp\left\{-r^f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1} + A(n-1) + \phi^n x_t^{agg} + \phi^{n-1}u_{t+1}\right\}\right] \\ &= \exp\left\{-r^f - \frac{1}{2}\gamma^2\sigma_u^2 + A(n-1) + \phi^n x_t^{agg} + \frac{1}{2}\left(\phi^{n-1} - \gamma\right)^2\sigma_u^2\right\}. \end{aligned} \tag{A1}$$

This gives that

$$\begin{aligned} A(n) &= A(n-1) - r^f - \gamma\phi^{n-1}\sigma_u^2 + \frac{1}{2}\phi^{2(n-1)}\sigma_u^2 \\ &= -nr^f - \gamma\sigma_u^2\frac{1-\phi^n}{1-\phi} + \frac{1}{2}\sigma_u^2\frac{1-\phi^{2n}}{1-\phi^2}. \end{aligned} \tag{A2}$$

The expected and realized strip returns in equations (10)-(11) then simply utilize the formula for $P_t^{(n)}$. The firm price in equation (12) uses the independence of aggregate and idiosyncratic shocks to simplify $E_t^*\left[\left(\prod_{j=1}^n M_{t+j}\right)X_{t+n}^{agg}\tilde{X}_{i,t+n}\right] = E_t^*\left[\left(\prod_{j=1}^n M_{t+j}\right)X_{t+n}^{agg}\right]E_t^*\left[\tilde{X}_{i,t+n}\right] = P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]$.

Given that the firm price is simply a collection of strip prices, the return for a firm is

$$\begin{aligned} R_{i,t+1} &= \frac{\tilde{X}_{i,t+1}X_{t+1}^{agg} + P_{i,t+1}}{P_{i,t}} = \frac{\sum_{n=1}^\infty P_{t+1}^{(n-1)}E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{\sum_{n=1}^\infty P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]} \\ &= \sum_{n=1}^\infty \frac{P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]}{\sum_{n=1}^\infty P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]}\frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}\frac{E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{E_t^*\left[\tilde{X}_{i,t+n}\right]} \\ &= \sum_{n=1}^\infty w_{i,t,n}R_{t+1}^{(n)}\frac{E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{E_t^*\left[\tilde{X}_{i,t+n}\right]} \end{aligned} \tag{A3}$$

where the weight is $w_{i,t,n} = \frac{P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]}{\sum_{n=1}^\infty P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]} = \frac{\exp\{nE_t^*[g_i]\}P_t^{(n)}}{\sum_{n=1}^\infty \exp\{nE_t^*[g_i]\}P_t^{(n)}}$ from equation (8). Applying expectations, we then get equation (13).

B. *Connecting returns, earnings growth, and price-earnings ratios*

First, we derive the equation for a firm which has zero dividends. For simplicity, we eliminate the index i in this derivation. In this case, the return is equal to the price growth which after log-linearization becomes an exact relationship

$$r_{t+1} = \Delta x_{t+1} - px_t + px_{t+1}. \tag{A4}$$

A high price-earnings ratio px_t must be followed by low future price growth Δp_{t+1} (returns r_{t+1}), high future earnings growth Δx_{t+1}, or a high future price-earnings ratio px_{t+1}.

Now, we consider the case where dividends are non-zero. We start with the one-year return identity of a portfolio

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\left(\frac{P_{t+1}}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}}{\frac{P_t}{D_t}},$$

where P_t and D_t are the current price and dividends. Log-linearizing around $\bar{pd}$, we can represent the price-dividend ratio pd_t in terms of future dividend growth, Δd_{t+1}, future returns, r_{t+1}, and the future price-dividend ratio, pd_{t+1}, all in logs:

$$r_{t+1} \approx \kappa^d + \Delta d_{t+1} - pd_t + \rho pd_{t+1}, \tag{A5}$$

where κ^d is a constant, $\rho = e^{\bar{pd}}/\left(1 + e^{\bar{pd}}\right) < 1$. We can then insert the identity $px_t = pd_t + dx_t$,where $dx_t$is the log payout ratio, into (A5) to obtain

$$r_{t+1} \approx \kappa + \Delta e_{t+1} - px_t + \rho px_{t+1} \tag{A6}$$

where we approximate $(1 - \rho)\, dx_{t+1}$ as 0 given that $(1 - \rho)$ is very close to 0 .[30] Here, $\bar{pd}$ does not need to be the mean price-dividend ratio of this specific stock or portfolio. In order to study cross-sectional variation without resorting to portfolio-specific approximation parameters, we use the average price-dividend ratio of the market for $\bar{pd}$ following Cochrane

[30] The zero dividend relationship in equation (A4) is a special case of equation (A6) as $\bar{pd}$ goes to infinity.

(2011).

While the identity relies on the approximation that $(1-\rho)\,dx_{t+1} \to 0$, empirically this approximation (A6) holds tightly. For horizons of 1 to 4 years, Table I shows that a one unit increase in px_t is associated with almost exactly a one unit increase in $\sum_{j=1}^{h} \rho^{j-1} \Delta x_{t+j} - \sum_{j=1}^{h} \rho^{j-1} r_{t+j} + \rho^h px_{t+h}$.[31] In other words, the approximation error from ignoring the payout ratio and using a single value for ρ accounts for at most 3.1% of variation in price-earnings ratios in the decomposition of equation (18). For robustness, the next section uses an exact relationship to ensure the approximation is not driving our results.

C. *Exact decomposition results*

In this section, we derive all the main results using an exact decomposition of price-earnings ratios based on price growth, rather than the approximate decomposition based on returns. For any stock or portfolio of stocks i, the price-earnings ratio $px_{i,t}$ can be expressed in terms of the one-year ahead log price growth $\Delta p_{i,t+1}$, the future earnings growth $\Delta x_{i,t+j}$, and the future price-earnings ratio:

$$px_{i,t} = \Delta x_{i,t+1} + \Delta p_{i,t+1} + px_{i,t+1}. \tag{A7}$$

This equation is exact and does not contain a log-linearization constant ρ. Applying subjective expectations $E_t^*[\cdot]$, we see that a higher than average price-earnings ratio must be explained by higher than average expected earnings growth, lower than average expected price growth, or a higher than average expected future price-earnings ratio,

$$\tilde{px}_{i,t} = \sum_{j=1}^{h} E_t^*\left[\Delta \tilde{x}_{i,t+j}\right] - \sum_{j=1}^{h} E_t^*\left[\Delta \tilde{p}_{i,t+j}\right] + E_t^*\left[\tilde{px}_{i,t+h}\right]. \tag{A8}$$

Just as the main decomposition, this equation holds under any subjective probability

[31] For example, at the one-year horizon, a one unit increase in px_t is associated with a 0.103 increase in Δx_{t+1}, a 0.143 increase in $-r_{t+1}$, and a 0.746 increase in ρpx_{t+1}. At the four-year horizon, a one unit increase in px_t is associated with a with a 0.099 increase in $\sum_{j=1}^{4} \rho^{j-1} \Delta x_{t+j}$, a 0.320 increase in $-\sum_{j=1}^{4} \rho^{j-1} r_{t+j}$, and a 0.550 increase in $\rho^4 px_{t+4}$.

distribution and we can decompose the variance of $\tilde{px}_{i,t}$ into three components:

$$1 = \underbrace{\frac{Cov\left(\sum_{j=1}^{h} E_t^*\left[\Delta\tilde{x}_{i,t+j}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{CF_h} + \underbrace{\frac{Cov\left(-\sum_{j=1}^{h} E_t^*\left[\Delta\tilde{p}_{i,t+j}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{PG_h} + \underbrace{\frac{Cov\left(E_t^*\left[\tilde{px}_{i,t+h}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{FPE_h}. \tag{A9}$$

The coefficients CF_h and PG_h give a quantitative measure of how much dispersion in price-earnings ratios is accounted for by dispersion in expected earnings growth and how much is accounted for by dispersion in expected price growth. We can now estimate this equation using the exact expectations of price growth without an approximation.

Table AI shows that the results of this exact decomposition are very similar to the main decomposition results in Table I. We find that 10.3% of dispersion in price-earnings ratios is accounted for by differences in one-year future earnings growth and 13.2% is accounted for by differences in one-year price growth. Just as in the main decomposition, differences in earnings growth are overestimated, with expected earnings growth accounting for nearly a third (33.1%) of all dispersion in price-earnings ratios. Differences in price growth are underestimated, with expected price growth accounting for only 3.3% of all dispersion in price-earnings ratios. A similar pattern can be observed at the four-year horizon. Overall, all the coefficients closely align with those reported in Table I.

We can also estimate an exact version of the unexpected anomaly return decomposition (20). Just as in the main identity, we normalize all anomalies $\tilde{a}_{i,t}$ so that they have variance 1 and positively comove with future price growth. From equation (A7), we have the identity

$$\begin{aligned} \underbrace{Cov\left(\Delta\tilde{p}_{i,t+1} - E_t^*\left[\Delta\tilde{p}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,p}} &= \underbrace{Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,x}} \\ &+ \underbrace{Cov\left(\tilde{px}_{i,t+1} - E_t^*\left[\tilde{px}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,px}}. \end{aligned} \tag{A10}$$

Here, the values for $\sigma_{a,x}$ and $\sigma_{a,px}$ indicate how much the predictable price growth forecast

Table AI

Decomposition of dispersion in price-earnings ratios using price growth

This table decomposes the variance of price-earnings ratios using equation (AI). The FIRE column report the elements CF_h, PG_h and FPE_h of the decomposition using future earnings growth, future price growth and future price-earning ratios. The Expected column report the elements of the decomposition using expected earnings growth, expected price growth and expected price-earning ratios. The Forecast Errors column reports the contribution of the forecast errors of each element. For instance, $CF_1 = Cov\left(\Delta\tilde{x}_{i,t+1}, \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ is shown in the FIRE column. This component can be split into its expected component $Cov\left(E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ and its error component $Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$. The results for the one-year decomposition are shown on the left and the results for the four year decomposition are shown on the right. The sample period is 1999 to 2020. Driscoll-Kraay standard errors are calculated and clustered at the year level. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.

		One-year horizon ($h = 1$)			One-to-four year horizon ($h = 4$)		
		FIRE	Expected	Forecast errors	FIRE	Expected	Forecast errors
1999-2020	CF_h	0.103***	0.331***	−0.228***	0.100***	0.439***	−0.340***
		[0.028]	[0.022]	[0.022]	[0.039]	[0.018]	[0.034]
1999-2020	PG_h	0.132**	0.033**	0.100*	0.292***	0.135***	0.157**
		[0.054]	[0.013]	[0.056]	[0.062]	[0.036]	[0.063]
1999-2020	FPE_h	0.765***	0.636***	0.129**	0.608***	0.426***	0.182***
		[0.052]	[0.020]	[0.062]	[0.048]	[0.023]	[0.054]

Table AII

Unexpected Anomaly Price Growth

This table measures and decomposes unexpected anomaly price growth. For each anomaly $\tilde{a}_{i,t}$, we sort stocks into five equal-value portfolios based on the anomaly variable. The table shows the coefficients of regressing each of the dependent variables on a specific anomaly variable. The three dependent variables are the unexpected price growth $\Delta\tilde{p}_{i,t+1} - E_t^*\left[\Delta\tilde{p}_{i,t+1}\right]$, the earnings growth forecast errors $\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]$, and the price-earnings ratio forecast errors $\left(\tilde{px}_{i,t+1} - E_t^*\left[\tilde{px}_{i,t+1}\right]\right)$. For the anomaly variables, Value is measured using the price-earnings ratio. Profitability is measured using gross profitability. Investment is measured using net stock issuance. The Representative Anomaly is the average ranking of each stock across 22 different anomalies. Each anomaly variable is scaled to have a standard devition of 1 and signed to positively comove with future price growth. The sample period is 1999 to 2020. Driscoll-Kraay standard errors are clustered at the portfolio and year level. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.

	Dependent variable		
Anomaly $\tilde{a}_{i,t}$	$\Delta\tilde{p}_{i,t+1} - E_t^*\left[\Delta\tilde{p}_{i,t+1}\right]$	$\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]$	$\left(\tilde{px}_{i,t+1} - E_t^*\left[\tilde{px}_{i,t+1}\right]\right)$
Value	0.0301^{*} [0.0169]	0.0687^{***} [0.0066]	-0.0389^{**} [0.0187]
Profitability	0.0107 [0.0080]	0.0091 [0.0113]	0.0016 [0.0111]
Investment	0.0242^{***} [0.0090]	0.0287^{**} [0.0127]	−0.0045 [0.0071]
Representative anomaly	0.0293^{***} [0.0098]	0.0655^{***} [0.0165]	-0.0362^{***} [0.0094]

errors are explained by predictable errors in next-year earnings growth expectations and expectations of the future price-earnings ratio. Table AII shows the results for the four anomalies studied in Section II.D. For each anomaly, we estimate a positive value of $\sigma_{a,p}$, meaning that investors do not fully anticipate the high growth on high $\tilde{a}_{i,t}$ stocks. For three of the four anomalies, the predictable errors in one-year earnings growth expectations are more than large enough to account for the unexpected one-year price growth (i.e., $\sigma_{a,x}$ is greater than $\sigma_{a,p}$). For the profitability anomaly, the predictable errors in earnings growth account for more than 85% of the unexpected price growth.

D. FIRE model simulations

For each model, we simulate the cross-section of firms. We set the number of firms based on the original calculations in each paper. Specifically, we use 50, 5,000, and 200 firms for Berk et al. (1999), Zhang (2005), and Lettau and Wachter (2007), respectively. We set every sample to a length of 20 years and we run 1,000 simulations for each model. All parameter values are taken from the original papers.

For Berk et al. (1999) and Zhang (2005), we sort firms into five portfolios based on their price-book ratios. For Berk et al. (1999), we treat profits as our measure of earnings and for Zhang (2005), we treat profits after the cost of new capital and adjustment costs as our measure of earnings.[32] For Lettau and Wachter (2007), the only firm variables are price and dividends, so we treat dividends as our measure of earnings and sort firms into five portfolios based on their price-dividend ratios. We then estimate the finite-horizon and infinite horizon decomposition in equation (18) for each model.

D.1. Berk, Green, and Naik 1999

Each firm has some existing projects which generate cash flows. Each period, the firm draws a new potential project, which it can pay a fixed cost to undertake. The value of the firm comes from its existing projects as well as the option to undertake future projects ("growth options"). As the term "growth options" implies, future earnings growth plays a key role in this model. The ratio of the firm's price to its current earnings reflects how much of the firm's value comes from existing projects versus growth options. Firms with high price-earnings ratios derive most of their value from their expected future projects rather than existing projects, and future earnings growth accounts for most dispersion in price-earnings ratios ($CF_4 = 0.84$).

The model features a time-varying risk-free rate which also generates differences in risk

[32]We find nearly identical results if we use profits as our measure of earnings for Zhang (2005).

premia.[33] Compared to existing projects, the value of growth options is less sensitive to changes in the risk-free rate, as the firm can endogenously change its decision to exercise the option (i.e., it only undertakes the potential project if the risk-free rate is low). Because of this, the agent requires a higher risk premium for firms with low price-earnings ratios. Quantitatively, the difference in risk premia is only a small part of the dispersion in price-earnings ratios ($DR_4 = 0.03$, $DR_\infty = 0.04$).

D.2. Zhang 2005

In this model, firm earnings are

$$X_{i,t} = e^{x_t + z_{i,t} + p_t} k_{i,t}^{\alpha} - f - i_{i,t} - h\left(i_{i,t}, k_{i,t}\right)$$

where x_t is aggregate productivity, $z_{i,t}$ is idiosyncratic productivity, p_t is the aggregate price level, $k_{i,t}$ is firm-level capital, f is a fixed cost, $i_{i,t}$ is investment in capital, and $h\left(i_{i,t}, k_{i,t}\right)$ is an adjustment cost. Differences across firms are due to differences in their sequence of idiosyncratic productivity $\{z_{i,\tau}\}_{\tau=0}^{t}$. Because idiosyncratic productivity is AR(1), future earnings growth is partly predictable and dispersion in price-earnings ratios largely predicts differences in future earnings growth ($CF_4 = 0.69$).

The model also features differences in discount rates. Because of adjustment costs, it is costly for firms to lower their capital to the new optimal level after a negative shock to aggregate productivity x_t. Therefore, the agent requires a higher risk premium for firms with high capital relative to total firm value, as they are more sensitive to negative aggregate shocks. Quantitatively, these differences in risk premia are small relative to the dispersion in price-earnings ratios ($DR_4 = -0.03$).[34]

In order to calculate CF_h and DR_h, we have to address the issue that model earnings are

[33] The risk-free rate is closely tied to the agent's stochastic discount factor.

[34] In the model, high price-earnings ratio firms have *low* price-capital ratios. A 1% increase in $e^{z_{i,t}}$ does not change the current capital ($k_{i,t}$), increases the current earnings by 1%, and increases the current price by less than 1% since the increase in productivity is persistent but not permanent. Thus, an increase in $z_{i,t}$ raises the price-capital ratio and lowers the price-earnings ratio. This is why discount rate news is slightly negative, as the model predicts that high price-capital ratio firms will have lower future returns, which means that high price-earnings ratio firms will have *higher* future returns.

often negative, even at the portfolio level, which is not compatible with the Campbell-Shiller decomposition.[35] To use the decomposition, we want to think about an investor that makes a one-time payment to buy a claim to the company, never pays anything more in the future, and receives some cash flows in the future. Thus, we will think of an investor that holds some share $\theta_{i,t}$ of the company. When the company has positive cash flows, the investor does not change her share in the company and receives these cash flows. When the company has negative cash flows, we assume the investor sells a part of her stake in the company to cover this. Specifically, this investor receives cash flows $\hat{X}_{i,t} \equiv \theta_{i,t} \max\{X_{i,t}, 0\}$, where $\theta_{i,t} = \theta_{i,t-1}\left(1 + \min\{X_{i,t}, 0\}/P_{i,t}\right)$. Intuitively, rather than receiving a negative cash flow, this investor dilutes her claim to the future (on average positive) cash flows. This investor receives the same return as someone who owned the entire firm and received the negative cash flows, $\frac{\theta_{i,t}P_{i,t}+\hat{X}_{i,t}}{\theta_{i,t-1}P_{i,t-1}} \equiv \frac{P_{i,t}+X_{i,t}}{P_{i,t-1}}$.

D.3. Lettau and Wachter 2007

In this model, each firm receives some share $s_{i,t}$ of the aggregate earnings. The value of $s_{i,t}$ goes through a fixed cycle, increasing from $\underline{s}$ to a peak value of $\bar{s}$ and then decreasing back to $\underline{s}$. The cross-section of firms is populated with firms at different points in this share cycle. Because all firms receive a share of the same aggregate earnings, the cross-sectionally demeaned log earnings growth $\Delta\tilde{x}_{i,t}$ is simply the log share growth $\log(s_{i,t}) - \log(s_{i,t-1})$.

In the model, the stochastic discount factor is exposed to shocks that are partly reversed over time, which means that the agent requires a lower risk premium for longer horizon cash flows. Because of this, firms with high price-earnings ratios (i.e., firms with a low current share $s_{i,t}$) earn slightly lower returns for the first few years ($DR_4 = 0.06$). However, the quantitatively larger component is that high price-earnings ratio firms experience higher earnings growth as their share increases ($CF_4 = 0.24$). Over time, the firms with low current $s_{i,t}$ eventually become the firms with high $s_{i,t+h}$ and require a higher risk premium, as their

[35] After a large aggregate shock, nearly all firms will substantially change their capital which requires paying large adjustment costs.

cash flows are now front-loaded. Thus, discount rate news is small and ambiguous in terms of sign at long horizons, $DR_\infty = -0.03$ (0.09).

E. Model parameter estimation

This section derives the cash flow parameters ϕ, σ_u, and σ_v from the standard deviation and autocorrelation of aggregate earnings growth $\sigma\left(\Delta x_t^{agg}\right)$ and $AC\left(\Delta x_t^{agg}\right)$ and the average across portfolios of the standard deviation over time of earnings growth $\sigma\left(\Delta \tilde{x}_{i,t+1}\right)$.

According to equation (2), we can express aggregate earnings growth as:

$$\Delta x_t^{agg} = \phi \Delta x_{t-1}^{agg} - u_{t-1} + u_t. \quad \text{(A11)}$$

Taking covariance of equation (A11) with current earnings growth on both sides results in:

$$\begin{aligned} Cov(\Delta x_t^{agg}, \Delta x_{t-1}^{agg}) &= \phi Var(\Delta x_{t-1}^{agg}) - \sigma_u^2 \\ AC(\Delta x_t^{agg}) &= \phi - \frac{\sigma_u^2}{Var(\Delta x_t^{agg})}. \end{aligned} \quad \text{(A12)}$$

Taking the variance of equation (A11) on both sides gives:

$$\begin{aligned} Var(\Delta x_t^{agg}) &= \phi^2 Var(\Delta x_t^{agg}) + 2\sigma_u^2 - 2\phi\sigma_u^2 \\ Var(\Delta x_t^{agg}) &= \frac{2\sigma_u^2}{1+\phi}. \end{aligned} \quad \text{(A13)}$$

From equations (A12) and (A13), we have:

$$\begin{aligned} \phi &= 1 + 2AC\left(\Delta x_t^{agg}\right) \\ \sigma_u &= \left(\frac{1+\phi}{2}\right)^{1/2} \sigma\left(\Delta x_t^{agg}\right). \end{aligned}$$

Finally, to estimate the individual variance, we use equation (3) to obtain the value for σ_v in terms of idiosyncratic earnings growth:

$$\sigma_v = \frac{\sigma\left(\Delta x_{i,t}\right)}{\sqrt{2}}.$$

From the empirical values of $\sigma\left(\Delta x_t^{agg}\right) = 0.057$, $AC\left(\Delta x_t^{agg}\right) = -0.086$ and a portfolio-average of $\sigma\left(\Delta x_{i,t}\right) = 0.149$ we infer $\phi = 0.828$, $\sigma_u = 0.337$ and $\sigma_v = 0.104$.