When Green Investors Are Green Consumers

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Abstract

We introduce investors with preferences for green assets to a general equilibrium setting in which they also prefer consuming green goods. Their preference for green goods induces *consumption premia* on expected returns, which counterbalance the *green premium* stemming from their preferences for green assets. Because they provide a hedge when green goods become expensive, brown assets command lower consumption premia, while green investors allocate a larger share of their portfolios towards them. Empirically, the green-minus-brown consumption premia differential reached 30-40 basis points annually, and contributes to explaining the limited impact of green investing on the cost of capital of polluting firms.

Keywords: Sustainable Finance, Asset Pricing, Portfolio Choice.

JEL codes: G11, G12.

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1 Introduction

The same proportion of U.S. individual investors surveyed by the Morgan Stanley Institute for Sustainable Investing (2019), namely, 33% of them, declare both that they "screen their investments according to their interests and values" and "purchase from a brand particularly because of the company's environmental or social impact." This survey suggests that the ethical motives underpinning green investors' capital allocation decisions (Riedl and Smeets, 2017; Hartzmark and Sussman, 2019; Krüger et al., 2020) are also—at least partly—reflected in their consumption practices.

Recent research has characterized a *green premium* on expected asset returns, which is induced by pro-environmental investment preferences (Pastor et al., 2021; Pedersen et al., 2021; Zerbib, 2022). This premium is higher on brown assets than green assets because green investors require a higher expected return to hold the assets they dislike in equilibrium. The existence of a green premium is of major importance, especially for investors willing to have an impact on corporate practices, because it can incentivize companies to mitigate their environmental footprints so as to decrease their cost of capital. However, the literature is silent on the effects of preferences for green consumption. How do pro-environmental preferences for consumption translate into investment decisions and expected returns? How do they interact with pro-environmental preferences for investment? Answering these questions is key to understanding whether the impact of green investors—once their preference for green consumption is taken into account—on the cost of capital of companies can be an effective channel for reducing their environmental footprint.

In this paper, we address these questions by building a general equilibrium model that features a green and a neutral investor, as well as a green and a brown equity asset that produce a green and a brown good, respectively. The green investor has preferences towards both investing in the green asset and consuming the green good, while the neutral investor has no preferences for tilting his investment portfolio or consumption basket.[1] We show that the green investor's preference for consuming the green good gives rise to *consumption premia* on expected returns.[2] Because the brown asset has a higher payoff when the green good becomes expensive, it offers a good hedge for the green investor, and thus, commands lower consumption premia than the green asset in equilibrium, as detailed below. These consumption premia counterbalance the green premium that stems from the green investor's preference for the green asset. This effect arises as the green investor allocates a larger share of her wealth to the brown asset compared to the case with no preferences for green consumption. Empirically as well as in the model, the impact can offset almost entirely the green premium.

There are two consumption premia. The first and more substantial one is related to the relative supply of the green good. It is associated with the willingness of the green investor to hedge against a decline in the relative supply of the green good or, equivalently, an increase in its relative price. This risk may materialize as a result of the election of a new government (e.g., the withdrawal from the Paris climate agreement, the repeal of the Clean Power Act, and the suspension of federal subsidies to the renewable energy sector following the election of Donald Trump in the U.S. in 2017), a contraction of international trade (e.g., the 300% increase in the price of silicon—an essential component of solar panels, mostly produced by China—between August 2021 and October 2021 due to the Covid-19 crisis), the outbreak of an armed

[1]Our framework also accommodates a more general setup in which investors with preferences for green goods are not necessarily the ones with preferences for green assets.

[2]An equivalent term, closer to the terminology in asset pricing, is *hedging premia*.

conflict (e.g., the increase in the share of coal in electricity production in Germany following the restrictions imposed on Russian gas imports since March 2022), or global energy shortages and the fear of an economic slump (e.g., the increase in coal production in China by 10% in the first two months of 2022 compared with the same period in 2021).

The mechanism of the first premium is as follows. When the green good becomes scarcer, the relative payoff of the brown asset increases. This effect occurs because the decrease in the relative supply of the green good is only partially compensated by the increase in its price, provided that the elasticity of substitution between the goods is not too low (specifically, greater than one), as suggested by empirical evidence (Papageorgiou et al., 2017). Consequently, as a hedge against the adverse event that the green good becomes scarce or its price increases, the average investor in the economy[3] overweights the brown asset in her portfolio because it comoves positively with the price of the green good that she favors. This is also the reason why the consumption premium associated with shocks to the relative supply of the green good is larger on the green asset than on the brown asset in equilibrium. This premium is driven by a larger beta on that risk for the green asset compared to the brown asset, and a positive price of risk. In short, from the perspective of this consumption premium, the green asset is riskier.

The second premium is related to the wealth share of the green investor. When her wealth increases, the resulting buying pressure increases the relative price of the green good, which penalizes the average consumer in the economy who has a preference for that good. However, in that situation, the green asset provides a good hedge because its returns increase with the price of the green good. Consequently,

[3]The average consumption preferences of the agents in the economy are tilted towards the green good because the neutral investor has equal preferences for green and brown goods.

this second consumption premium is lower for the green asset than that for the brown asset. As a result, on average, this premium works in the opposite direction to the first consumption premium in the baseline calibration. Although this effect is more pronounced when the supply of the green good is low, it is generally very small, of the order of a few basis points.[4] Therefore, the effect of consumption preferences on asset returns is largely dominated by the premium associated with the relative supply of the green good.

Methodologically, we build on general equilibrium models with multiple heterogeneous agents, multiple equity assets, multiple consumption goods, and general preferences, such as recently developed in Sauzet (2022a). We augment this setting by embedding preferences for specific assets, in the spirit of Pastor et al. (2021), Pedersen et al. (2021), and Zerbib (2022). This setup allows us to (*i*) derive explicit expressions for risk premia, portfolios, and other variables; (*ii*) study these variables not only on average but also in their dynamic evolution with the state of the economy, which is a key aspect in our analysis; and (*iii*) highlight the significant impact of various parameters such as the elasticity of substitution across goods, preference for green consumption, and preference for green investing. Each of those advances is made possible by the unique combination, proposed in Sauzet (2022a), of general preferences, the use of continuous time, and a global solution method.

We provide empirical evidence supporting the existence of the consumption premia on U.S. stock returns. Empirically as well as in the model, the difference in consumption premia between the greenest and brownest companies reaches up to 40

[4] Formally, the sign of this relative premium depends on the comovements between the wealth share of the green investor and the relative supply of the green good. These comovements are, in turn, determined by the bias in the portfolio of the green investor towards the green asset, and therefore, by the relative strength of her preference for green investing versus green consumption. Taken together, this leads to the lower wealth share premium on the green asset in the baseline calibration. In all cases, this wealth share premium remains small regardless of its sign.

basis points (bps) per year, which counterbalances and offsets a substantial part of the green premium that has been estimated in the recent literature (Pastor et al., 2022; Zerbib, 2022). We also validate that the comovements of green good prices with brown asset returns are higher than those with green asset returns.

To test the predictions of the model, we estimate the beta-representation of the equilibrium equation for risk premia stemming from the model using U.S. common stocks from 2006 to 2019 at a monthly frequency. Since the factor associated with the relative supply of the green good is equivalent to a factor capturing the relative *price* of the green good—the two being negatively related[5]—we construct this factor based on the prices of goods from Producer Price Indexes at the industry level from the U.S. Bureau of Labor Statistics, and companies' carbon intensities provided by S&P–Trucost. By identifying green funds invested in U.S. equities from Bloomberg data, we define the wealth share factor as the ratio of their U.S. stocks under management, obtained from FactSet, to the total market capitalization of their investment universe. Finally, we use the environmental rating from MSCI to build the green factor (Pastor et al., 2022) because the carbon intensity is a partial and limited metric to construct a green asset allocation (as explained in Section 4). The green factor is the portfolio taking a long position on the greenest companies and shorting the brownest ones, which reflects green investors' preferences for green assets. We also control for the five Fama and French (2015) factors and the momentum factor (Carhart, 1997). Through four main results, we confirm the predictions of our asset pricing model regarding the risk of an increase in the relative price of green goods: (*i*) the price of this risk is highly significant and negative[6] across all specifications, and (*ii*) the betas are lower

[5] Using prices as opposed to produced quantities has the benefit of being a much easier, cleaner, and higher frequency measure to come by.

[6] The price of this risk is negative, as expected, because the relative *prices* of the green goods are negatively related to the relative *supply* of the green goods.

for brown than for green stocks across all months, emphasizing the better hedge that brown assets provide against the increase in green good prices. As a result, (*iii*) the consumption premium related to the price of green goods has gradually increased for green assets with respect to brown assets to reach a spread of 30 to 40 bps per year, thereby counterbalancing the green premium. Thus, green assets are riskier than brown assets from the perspective of consumption premia, as implied by the model. In addition, *(iv)* regardless of the environmental rating of the assets considered, the consumption premium between the assets with the 33$^{\text{rd}}$ percentile beta (high $\beta_{\mathcal{E}}$) and those with the 66$^{\text{th}}$ percentile beta (low $\beta_{\mathcal{E}}$) is substantial: assets that can hedge shocks to the relative price of green goods carry lower returns of up to 1.5% annually.

The results in this study have implications not only for asset pricing, but also in terms of the real impact of sustainable investing. Through their preferences for green goods, green investors reduce their upward pressure on the cost of capital of polluting firms. Therefore, the consumption premia help explain the low impact of green investing on mitigating the environmental footprints of companies through the cost of capital channel, as suggested by the literature (Berk and van Binsbergen, 2021; De Angelis et al., 2022). Instead, investors' preferences for green consumption make the case for a stronger focus on shareholder engagement to impact companies' practices for two reasons: by allocating a larger share of their wealth to brown assets, green investors (*i*) reduce their impact on the cost of capital of brown firms, and simultaneously, (*ii*) increase their ability to actively engage with them. Several tools are available to policymakers to counteract the effect of the consumption premia, including a price cap on green goods or the implementation of a tax on dividends from brown firms, which we briefly discuss in Section 5.

Related literature. This paper contributes to several strands of the literature in asset pricing and sustainable finance. First, to the best of our knowledge, this is the first paper that studies the effects that investors' preferences towards sustainable consumption have on asset prices and investors' asset allocation. The construction of a general equilibrium model allows us to uncover these effects. From a theoretical viewpoint, Pastor et al. (2021), Pedersen et al. (2021), and Zerbib (2022) characterize the green premium driven by investors' preferences for green assets in equilibrium on financial markets. This premium is higher on brown assets than on green assets, and corresponds to the compensation required by sustainable investors for holding the assets they like least. Empirical evidence supports the existence of a green premium that is higher on the stock returns of the carbon-intensive companies (Bolton and Kacperczyk, 2021, 2022), polluting companies (Hsu et al., 2022), companies most exposed to climate change risk (Bansal et al., 2016; Barnett, 2022), and least held by green funds (Zerbib, 2022) than on the stock returns of green companies. A similar effect is documented on the cost of equity (ElGhoul et al., 2011; Chava, 2014), expected returns approximated from option-implied information (Sautner et al., 2022), bond yields (Chava, 2014; Baker et al., 2018; Zerbib, 2019; Painter, 2020; Goldsmith-Pinkham et al., 2021; Huynh and Xia, 2021; Seltzer et al., 2022), venture capital funds (Barber et al., 2021), and real estate prices (Bernstein et al., 2019; Baldauf et al., 2020; Giglio et al., 2021). However, opposite effects can emerge in a dynamic setting: the cost of capital of green firms may increase because investors' preferences for green assets reduce asset price informativeness (Goldstein et al., 2021) or as a result of shocks on preferences for green assets (Avramov et al., 2021). In addition, performing empirical analyses on more recent time frames, Ardia et al. (2021) and Pastor et al. (2022) find higher green premia on the greenest stock returns driven by recent capital inflows, reflecting changes in investors' preferences in a transitory phase. Distinct from the work on the green premium, Albuquerque et al. (2019) show that green assets have lower systemic risk than brown assets and that this effect is stronger for firms with high product differentiation. Another body of the literature on sustainable asset pricing studies the impact of climate risks on asset prices (e.g., Hong et al., 2019; Alok et al., 2020; De Angelis et al., 2022). Notably, a couple of recent papers analyze climate-related financial risks in general equilibrium. Barnett (2022) shows that the price of climate risk is significantly negative, particularly driven by the risk of transition to a low-carbon economy. Hambel et al. (2022) highlight that investors' willingness to diversify their assets complements the attempt to mitigate economic damages from climate change in the short run, while in the longer run, a trade-off between diversification and climate action emerges. Engle et al. (2020) and Alekseev et al. (2021) propose portfolio construction methods that allow to efficiently hedge these climate risks. In this study, we depart from the asset pricing literature on the impact and hedging of environmental risks, and we focus on the preferences of green investors for green *consumption* by constructing a two-investor, two-tree, two-good general equilibrium model with heterogeneous preferences for investment and consumption. We provide the first theoretical and empirical evidence for the existence of significant consumption premia that counterbalance the effect of the green premium on asset returns.

Second, this paper contributes more broadly to the literature on theoretical general equilibrium asset pricing with multiple heterogeneous agents, multiple equity assets, multiple consumption goods, and general preferences, such as recently developed in Sauzet (2022a,b). This framework, in turn, combines models with multiple agents—they have a long and distinguished history since the seminal contributions of Dumas (1989, 1992), Wang (1996), Basak and Cuoco (1998), Dumas et al. (2000), Dumas and

Uppal (2001), Chan and Kogan (2002), and more recently Brunnermeier and Pedersen (2009), Weinbaum (2009), Bhamra and Uppal (2009, 2014), Brunnermeier and Sannikov (2014), Gârleanu and Pedersen (2011), Chabakauri (2013), Gârleanu and Panageas (2015), Drechsler et al. (2018), Borovička (2020)—with settings with multiple equity securities but one investor such as Cochrane et al. (2008), Martin (2013), and two consumption goods (Fang, 2019). In other words, the framework generalizes the contributions of Zapatero (1995), Pavlova and Rigobon (2007, 2008, 2010), Stathopoulos (2017), to non-log preferences, and a general aggregation of goods. The unique combination, proposed in Sauzet (2022a,b), of general preferences, the use of continuous time, and a global solution method, is key in allowing us to derive most of our results. We also add the preferences for specific assets: in our case, the green investor prefers the green asset, but the formulation is general and could be used in other contexts.[7]

This paper also contributes to the literature on environmental and ecological economics. Specifically, using two goods to capture green and brown consumption is in the spirit of Guesnerie (2004), Hoel and Sterner (2007), Sterner and Persson (2008), Gollier (2010), Traeger (2011), Barro and Misra (2016), and Gollier (2019), in which the two goods are taken to represent aggregate economic capital (physical capital, labor, scientific knowledge, *etc.*) on the one hand, and various ecosystem services generated by natural capital on the other. While most of these contributions are based on a representative agent or social planner, we bring this intuition to a general

[7]On the theoretical front, our study is also related to contributions introducing recursive preferences in continuous-time, for example, Duffie and Epstein (1992), and contributions focusing on the existence and uniqueness of equilibria in the presence of multiple agents, and possibly multiple goods and incomplete markets, for example, Polemarchakis (1988), Geanakoplos and Polemarchakis (1986), Geanakoplos and Mas-Colell (1989), Geanakoplos (1990), Duffie et al. (1994), Berrada et al. (2007), Anderson and Raimondo (2008), Hugonnier et al. (2012a), Ehling and Heyerdahl-Larsen (2015, 2017).

equilibrium economy with several investors. The investors are heterogenous in their (general) preferences for consumption and investment, and we solve for the decentralized equilibrium, which allows a meaningful discussion of portfolios, in addition to risk premia, and other variables. Broadly speaking, our study is also related to contributions in environmental macroeconomics such as Pindyck and Wang (2013), Golosov et al. (2014), Cai and Lontzek (2019), van den Bremer and van der Ploeg (2021) on the theoretical side, and Papageorgiou et al. (2017) on the empirical side.

Fourth, and importantly, this paper contributes to the literature on impact investing. Building on the seminal paper by Heinkel et al. (2001), De Angelis et al. (2022) find that the increase in the cost of capital driven by green investing has a limited impact on the practices of the most polluting companies. This conclusion is consistent with Berk and van Binsbergen (2021) who show that the effect of impact investing on the cost of capital is too small to meaningfully affect real investment decisions. Through two different approaches, Oehmke and Opp (2019) and Green and Roth (2020) emphasize the importance of investor coordination to finance the companies that need it most and increase their impact on the economy as a whole. In addition, Landier and Lovo (2020) highlight the effects of search frictions in capital markets, which increase the impact of investors on corporate practices. From an impact perspective, Broccardo et al. (2020) suggest that in most cases, shareholder engagement is more effective than the effect of sustainable investors' asset allocation on companies' cost of capital. This paper reinforces that suggestion for a different reason: green investors' preferences for green goods weaken the cost of capital channel via the consumption premia and increase the allocation of green investors towards the brownest companies, which are the preferred targets for shareholder engagement campaigns.

Outline. The paper is organized as follows. Section 2 describes the set-up of the economy and introduces the two state variables that drive economic mechanisms—the wealth share of the green investor and the relative supply of the green good. Section 3 revisits the impact of green investors on asset prices when they also have preferences for green goods. Section 4 provides empirical evidence supporting our findings. Section 5 discusses the results in light of impact investing challenges and Section 6 concludes. Proofs and additional material are provided in Appendix.

2 The Economy

This section presents the theoretical setup. We introduce a pure-exchange economy comprising a green and a neutral investor ($i \in \{G, N\}$), and a green and a brown tree ($j \in \{g, b\}$). The trees produce differentiated green and brown goods, respectively, and are traded as equity assets à la Lucas (1978). The green investor has preferences not only for investing in the green asset (Pastor et al., 2021; Pedersen et al., 2021; Zerbib, 2022), but also for consuming the green good (Sauzet, 2022a). We show that the equilibrium can be characterized as a function of two state variables: the relative wealth of the green investor, x_t, and the relative supply of the green good, y_t. This setup is summarized in Figure B.1 of the Appendix. Appendix A gathers additional results that are omitted in the main text.

Time is continuous and the horizon is infinite, $t \in [0, \infty)$. Uncertainty is represented by a probability space $(\Omega, \mathcal{F}, \mathbb{F}, P)$ supporting a two-dimensional Brownian motion $\vec{Z} \equiv (Z_g, Z_b)^T \in \mathbb{R}^2$. The filtration $\mathbb{F} = (\mathcal{F}_t)_{t\in[0,\infty)}$ is the usual augmentation of the filtration generated by the Brownian motions, and $\mathcal{F} \equiv \mathcal{F}_\infty$.

2.1 Endowments, prices, assets

The two trees produce differentiated, green and brown, goods. Their outputs follow geometric Brownian motions[8]

$$\frac{dY_{j,t}}{Y_{j,t}} = \mu_{Y_j} dt + \sigma_{Y_j}^{\top} d\vec{Z}_t, \quad j \in \{g, b\}.$$

The prices of the green and brown goods are $p_{g,t}$ and $p_{b,t}$, respectively. We also define the terms of trade $q_t \equiv p_{g,t}/p_{b,t}$, which is the relative price of the green good, and the real exchange rate $\mathcal{E}_t \equiv P_t^G/P_t^N$, which is the relative price of the consumption basket of the green investor. All prices are defined with respect to a numéraire taken to be a CES-basket with weight $a = 1/2$ on both goods.[9]

The green and brown trees are traded as equity assets, with returns given by

$$dR_{j,t} = \frac{dQ_{j,t}}{Q_{j,t}} + \frac{p_{j,t}Y_{j,t}}{Q_{j,t}}dt = \frac{d\left(p_{j,t}Y_{j,t}/F_{j,t}\right)}{p_{j,t}Y_{j,t}/F_{j,t}} + F_{j,t}dt \equiv \mu_{j,t}dt + \sigma_{j,t}^{\top} d\vec{Z}_t, \quad j \in \{g, b\}, \tag{1}$$

where $Q_{j,t}$ are the equity prices, and $F_{j,t} \equiv p_{j,t}Y_{j,t}/Q_{j,t}$ are the dividend yields, for both assets. We obtain drifts $\mu_{j,t}$, which measure the conditional expected returns, and diffusion terms $\sigma_{j,t}$, which measure the loadings on the shocks and therefore the conditional volatilities, from Itô's Lemma and present them in Appendix A.

The supply of each equity asset is normalized to unity. There also exists a bond

[8]Extending the framework to a production economy is beyond the scope of this paper, but is a promising avenue that we are exploring in ongoing research. It could also introduce the possibility that the greenness of firms be determined endogenously, and could allow us to study some aspects of greenwashing on the part of firms. Likewise, studying the impact of uncertainty shocks could prove interesting.

[9]Specifically, we normalize $\left[(1/2)p_{g,t}^{1-\theta} + (1/2)p_{b,t}^{1-\theta}\right]^{1/(1-\theta)}$ to unity.

in net zero supply, which is locally riskless in units of numéraire. Its price is B_t, and the corresponding instantaneous interest rate is r_t, so that $dB_t/B_t = r_t dt$.

2.2 Preferences

Investors have recursive preferences à la Duffie and Epstein (1992) that are defined over consumption, but also over the weights on each asset in their portfolios, $\boldsymbol{w}^i$. Specifically, for the green and neutral investors, $i \in \{G, N\}$,

$$V_t^i = \max_{\{C_{g,u}^i, C_{b,u}^i, w_{g,u}^i, w_{b,u}^i\}_{u=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} f^i \left(C_u^i, V_u^i, \boldsymbol{w_u^i} \right) du \right], \tag{2}$$

$$f^i(C, V, \boldsymbol{w}) \equiv \left(\frac{1-\gamma}{1-1/\psi} \right) V \left[\left(\frac{C}{[(1-\gamma)V]^{1/(1-\gamma)}} \right)^{1-1/\psi} - \rho + \Phi^i(\boldsymbol{w}) \right],$$

where γ is the coefficient of relative risk aversion, ψ the elasticity of intertemporal substitution (EIS), and ρ is the discount rate.

Recursive preferences are relevant for two reasons. First, contrary to the case with *log* utility, investors are not myopic and hedging terms arise, which are important drivers of risk premia and portfolios. Second, the coefficient of relative risk aversion is not equal to the reciprocal of the EIS, $\psi \neq 1/\gamma$. This matters quantitatively to obtain risk premia that are closer to their empirical counterparts as well as for the quantitative impact of a potential tax on brown assets as discussed in Section 5. In what follows, parameters γ, ψ, ρ are taken to be identical for both investors. However, the resolution allows for any value so that exploring additional asymmetries stemming from these parameters could be an interesting avenue for future work.

The green investor expresses her pro-environmental motives, in part, by displaying a preference towards the green asset. In this general equilibrium context, we introduce it as functions of the portfolio weights for both investors, $\Phi^i(\boldsymbol{w})$, where $\boldsymbol{w}_t^i \equiv \left(w_{g,t}^i, w_{b,t}^i\right)$, and $w_{g,t}^i$ ($w_{b,t}^i$) is the share of wealth on the green (brown) asset in the portfolio of investor $i \in \{G, N\}$. Specifically, we take[10]

$$\Phi^i(\boldsymbol{w}^i) \equiv (1 - 1/\psi)\left(w_g^i \phi_g^i + w_b^i \phi_b^i\right). \tag{3}$$

Parameter $\phi_g^G \equiv \phi > 0$ captures the additional value that the green investor derives from holding the green asset, in the spirit of Pastor et al. (2021) and Zerbib (2022). Without loss of generality, we assume that that the neutral investor has no preference for the green asset ($\phi_g^N = 0$), and that neither investors have a preference for the brown asset ($\phi_b^G = \phi_b^N = 0$). In Section 3, we show that the preference of the green investor for the green asset gives rise to a green premium, which reduces the expected return on the green asset.[11]

In terms of consumption, the basket of each investor is composed of the green and brown goods, which are combined according to an aggregator with constant elasticity of substitution θ, and bias in consumption α^i,

$$C_t^i = \left[{\alpha^i}^{\frac{1}{\theta}} {C_{g,t}^i}^{\frac{\theta-1}{\theta}} + (1 - \alpha^i)^{\frac{1}{\theta}} {C_{b,t}^i}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}. \tag{4}$$

[10]The $(1 - 1/\psi)$ factor serves purely as a normalization so that parameter ϕ drives the green premium like in the literature (cf. Section 3.2).

[11]Because the focus of this study is the effect of preferences for green consumption, and for the sake of avoiding further complexity in the model, we consider the preferences of the green investor for green investments (ϕ) as being constant over time. Similarly, the preference for green consumption (α) is constant. An economy in which ϕ and α are stochastic is beyond the scope of this paper, but is a promising avenue to study more closely the effect of demand shocks. We are exploring it in ongoing research. Microfounding those preferences further, for example, by relying on directional or source-dependent risk aversion as used in Hugonnier et al. (2012b), could also be an interesting avenue for future work.

While the neutral investor has no particular preference towards either of the goods ($\alpha^N = 1/2$), the green investor expresses her pro-environmental preferences by also tilting her consumption towards the green good ($\alpha^G > 1/2$). This preference for green consumption is the key novel element in this paper. In the theoretical characterization of Section 3, we show that it underpins large consumption premia on expected returns, which can offset the effect of the green premium stemming from green asset preferences.

It is also important to allow for a general elasticity of substitution across goods, θ, because its value determines the relative magnitude of movement in the relative price of the goods for a given shock to the relative supply. In turn, this relative magnitude governs the movements in the relative dividends of the two assets, and ultimately the tilt in portfolios and consumption premia.[12]

From the share of wealth that investors allocate to the green and brown equity assets, $w^i_{g,t}, w^i_{b,t}$, they earn expected returns $\mu_{g,t}, \mu_{b,t}$. They allocate the remainder of their wealth $(1 - w^i_{g,t} - w^i_{b,t})$ to the riskless bond. They use the proceeds of their investments to purchase their desired baskets of consumption $c^i_t \equiv C^i_t / W^i_t$, at price P^i_t. In other words, investors $i \in \{G, N\}$ choose their consumption and portfolios to maximize (2) subject to the following budget constraint

$$\frac{dW^i_t}{W^i_t} = \left(r_t + w^i_{g,t}\left(\mu_{g,t} - r_t\right) + w^i_{b,t}\left(\mu_{b,t} - r_t\right) - P^i_t c^i_t\right) dt + \left(w^i_{g,t}\sigma_{g,t} + w^i_{b,t}\sigma_{b,t}\right)^\top d\vec{Z}_t. \tag{5}$$

[12]For instance, the common Cobb-Douglas case ($\theta = 1$) leads the relative price of the goods to move exactly enough to compensate relative supply so that relative dividends are unaffected. The relative payoffs of the two assets can then be perfectly correlated, and the portfolio choice between them indeterminate, at least without additional preferences for specific assets. Empirically, $\theta > 1$, which will drive the direction of the hedging terms as discussed in Section 3.

To complete the definition of their optimization problems, investors are subject to a standard transversality condition, and W_0^i is given. Note also that $W_t^i \geqslant 0$.

This framework also allows for additional ingredients, such as taxes on the dividends of each asset. This extension is discussed in Section 5.

2.3 Equilibrium and state variables

The definition of the equilibrium is standard: (1) investors solve their optimization problems by taking the aggregate stochastic processes as given, and (2) goods and equity markets clear. The detailed definition of the equilibrium is given in Appendix A.4. The bond market clears by Walras' law, which gives rise to the following useful relationship: $W_t^G + W_t^N = Q_{g,t} + Q_{b,t}$. In words, the total wealth has to be held in the form of an aggregate of the two equity assets.

Stationary recursive Markovian equilibrium. Most importantly, the equilibrium can be recast as a stationary recursive Markovian equilibrium in which all the variables of interest are expressed as a function of a pair of state variables $X_t \equiv (x_t, y_t)'$, whose dynamics are also solely a function of X_t. The variable x_t is the wealth share of the green investor and y_t is the relative supply of the green good.[13] Both are defined below.

Section 3 focuses on the characterization of the solution as a system of coupled algebraic and second-order partial differential equations. For now, let us discuss the intuition behind both state variables. Note that the ratio of the green equity price to total wealth, $w_{g,t}^M$, is an additional variable, which is not a state variable *per se* but is

[13]Formally, this is shown using a guess and verify approach like, for example, in Gârleanu and Panageas (2015). The variables of interest are: $\{c_{g,t}^G, c_{b,t}^G, c_{g,t}^N, c_{b,t}^N, w_{g,t}^G, w_{b,t}^G, w_{g,t}^N, w_{b,t}^G, \mu_{R_g,t}, \mu_{R_b,t}, r_t, F_{g,t}, F_{b,t}, p_{g,t}, p_{b,t}, P_t^G, P_t^N, q_t, \mathcal{E}_t\}$.

useful throughout. It captures the weight of the green asset in the market portfolio. Thus, it can be shown that[14]

$$w_{g,t}^M \equiv \frac{Q_{g,t}}{Q_{g,t} + Q_{b,t}} = \left(1 + \left(\frac{F_{g,t}}{F_{b,t}}\right) q_t^{-1} \left(\frac{1 - y_t}{y_t}\right)\right)^{-1}. \tag{6}$$

Wealth share. The wealth share of the green investor is defined as

$$x_t \equiv \frac{W_t^G}{W_t^G + W_t^N}. \tag{7}$$

In this setting, the wealth share is neither constant nor solely a monotonic function of the current relative supply of the green good, y_t. Therefore, it is required as an additional state variable, even when risk sharing is perfect (i.e., even when there are no taxes on the dividends). This occurs because the preferences are recursive, and as a result of the fundamental heterogeneity stemming from the green investor's bias towards consuming and investing green.

Relative supply. The relative supply of the green good captures the effect of the current fundamentals and is defined as[15]

$$y_t \equiv \frac{Y_{g,t}}{Y_{g,t} + Y_{b,t}}. \tag{8}$$

The relative supply is a key driver of the marginal values of wealth of both investors

[14]Because the bond is in zero net supply $b_t^M = 0$, the weight of the brown asset in the market portfolio is $w_{b,t}^M = 1 - w_{g,t}^M$ in equilibrium.

[15]Note that the ratio involves *quantities* of the two different goods. This poses no particular theoretical issue and is used because it simplifies the characterization of the equilibrium. This definition is a monotonic transformation of $Y_{b,t}/Y_{g,t}$: $y_t \equiv (1 + Y_{b,t}/Y_{g,t})^{-1}$, which ensures that the state variable evolves in the bounded interval $[0, 1]$. $Y_{b,t}/Y_{g,t}$ has the clear interpretation of the output of brown good produced per unit of green good. An economic intuition is that one compares the economy to the symmetric point in which relative prices are $q_t = \mathcal{E}_t = 1$.

due to their desire to consume both goods, which stems from their CES consumption baskets. This is particularly true for the green investor who has a strong preference to consume more green goods, as discussed in Section 3. For the same reason, we show in Section 3 that the relative supply is also the main driver of the relative price of the green good, q_t, and consequently, of the relative price of the consumption basket of the green investor, $\mathcal{E}_t$. This strong monotonic relationship between y_t and $\mathcal{E}_t$ justifies using $\mathcal{E}_t$—a much easier, cleaner, and higher frequency measure—as a proxy to test our mechanisms empirically in Section 4.[16]

As discussed in the introduction, a decline in the relative supply of green goods or, equivalently, an increase in the relative price of green goods, may result from a variety of political and economic risk factors such as energy shortages, contraction of international trade, the election of a new government, or the outbreak of an armed conflict.

Note that because $W_t^i \geqslant 0$ and $Y_{j,t} \geqslant 0$, x_t and y_t both evolve in the bounded interval $[0, 1]$. This has the advantage that solving for unknown functions on a bounded domain is numerically more stable. Conceptually, as x_t gets closer to either of the boundaries, the economy converges (continuously) to a natural one-investor environment. As y_t gets closer to either of the boundaries, the economy converges to a one-good one-equity asset economy, but this has consequences in terms of the marginal values of wealth as the investors still want to consume both goods.

Throughout, we focus on the solution to the decentralized, that is, Radner equilib-

[16] Similarly, the international finance literature has emphasized mechanisms related to the relative supply, for example, Coeurdacier (2009) and Coeurdacier and Rey (2013). Those have been tested empirically mostly using relative prices or exchange rates, and have become known as "real exchange rate hedging" mechanisms. The model could be equivalently recast using q_t as a state variable. We focus on y_t because it makes the intuition sharper, the equations simpler, and because it is exogenous as opposed to q_t that also depends on x_t.

rium instead of relying on the social planner's problem. The existence and uniqueness of the equilibrium should be guaranteed, for instance, following Duffie and Epstein (1992), who use partial differential equation techniques to prove them in an infinite-horizon Markov diffusion setting with stochastic differential utility, or Chabakauri (2013) and Bhamra and Uppal (2014), who do so constructively for economies with heterogeneous agents and incomplete and complete markets, respectively. Both are also shown in situations with potentially dynamically complete markets[17] using a planner solution in Anderson and Raimondo (2008), and under complete markets with a full set of Arrow-Debreu securities in Hugonnier et al. (2012a). As has been known since the seminal example of Hart (1975), however, the introduction of multiple goods could complicate the matter, for instance, because markets can become dynamically incomplete even if the number of assets should technically be sufficient to span risks. Those multiple-good contexts are discussed, for example, in Berrada et al. (2007) and Ehling and Heyerdahl-Larsen (2015), again for the most part through the lens of the Pareto efficient allocation obtained from a social planner. Overall, equilibrium existence and uniqueness in the context of this paper with multiple goods, a bias in consumption and investment, potential imperfect risk sharing (when there exists a tax on dividends), and a decentralized Radner solution, could therefore be analyzed further from a theoretical perspective. This represents an interesting avenue for further research.

2.4 Computation of the equilibrium

Section 3 characterizes all the variables of interest as a function of the state variables, $X_t = (x_t, y_t)'$, and a set of unknown functions $\mathcal{G} \equiv \{J_t^G,\ J_t^N,\ F_{g,t},\ F_{b,t},\ q_t,\ w_{g,t}^G,$

[17] A securities market is potentially dynamically complete if the number of securities with non-colinear payoffs is equal to one plus the number of risk factors (Brownian motions) to be spanned.

$w^G_{b,t}\}$.[18] Due to the stationary recursive Markovian structure of the equilibrium, these unknown functions are themselves solely functions of X_t, and are determined by a set of coupled algebraic and second-order partial differential equations.

The resolution is based on projection methods and orthogonal collocation. Specifically, each unknown function $g : [0,1]^2 \to \mathcal{D}^g \subseteq \mathbb{R}$ in $\mathcal{G}$ is approximated using Chebyshev polynomials and the equilibrium is solved on a grid based on the zeros of the Chebyshev polynomials. Details are provided in Sauzet (2022a).

The main appeal of this approach is that this is a global solution method, which allows us to trace out the evolution of our variables of interest as a function of the state of the economy. Combined with continuous-time, it makes it possible to cleanly express and solve for the exact subcomponents of the main variables—risk premia, portfolios, goods prices—, as well as our mechanisms of interest, in particular, hedging components induced by the consumption preferences. Our methodology will prove crucial, for example, when discussing the dynamic aspects of those mechanisms, and how they can be state-dependent.[19]

[18] J^G_t, J^N_t are introduced in Section 3 and capture (an increasing monotonic transformation of) the marginal values of wealth of each investor. In addition, as a point of notation, for any function g, g_t simply denotes $g(X_t)$, not the time-derivative of g (which is zero because the model is stationary due to the infinite horizon).

[19] Projection methods are also well-suited to contexts with multiple state variables. Some settings with additional state variables could become computationally too costly, such as those that might arise when generalizing the framework. In these cases, extensions of those methods to higher-dimensional settings could prove necessary. One such method consists in naturally extending the concept of projection approaches, but to replace the Chebyshev polynomials in the approximation with neural networks, which are designed specifically to handle high-dimensional (and non-linear) contexts. These "projection methods via neural networks" for continuous-time models are proposed in Sauzet (2022c).

3 Characterization of the Equilibrium

We now characterize the equilibrium theoretically. In Section 3.1, we start by discussing the marginal values of wealth of both investors, consumption, and goods prices, all of which are important underpinnings for the other variables of the economy. Section 3.2 discusses asset prices, and we show that a preference for green consumption gives rise to consumption premia, which counterbalance the green premium on green assets stemming from the preference for green investing. Section 3.3 focuses on the portfolios and describes how a preference for green consumption leads investors to allocate a larger share of their wealth to brown assets than when they have solely preferences for green investing. Appendix A discusses additional theoretical results such as the evolution of the state variables (Appendix A.5).

Assumption 1 (Baseline calibration)**.** *Unless otherwise specified, the results in this section are obtained based on the following calibration,* $i \in \{G, N\}, j \in \{g, b\}$*:*

- *Preference for green consumption:* $\alpha^G = \alpha = 0.85$, $\alpha^N = 1/2$,
- *Preference for green investing:* $\phi_g^G = \phi = 1\%$.
- *Elasticity of substitution across goods:* $\theta^i = \theta = 2$,
- *Numéraire basket:* $a = 1/2$,
- *Risk aversion:* $\gamma^i = \gamma \in \{15, 25, 50\}$,
- *Elasticity of intertemporal substitution:* $\psi^i = \psi = 1.25$,
- *Discount rate:* $\rho^i = \rho = 1\%$,
- *Output:* $\mu_{Y_j} = \mu_Y = 2\%, \sigma_{Y_1} = (4.1\%, 0)^\top, \sigma_{Y_2} = (0, 4.1\%)^\top$ *(no fundamental correlation).*

What matters for the preference for green consumption of the green investor is that $\alpha^G > 1/2$. Similarly, what matters for her preference for green investing is that $\phi > 0$.

Their exact values have a quantitative impact that is discussed below. In practice, we pick $\alpha^G = 0.85$, and $\phi_g^G = \phi = 1\%$ to broadly match the green premium that has been estimated in the recent literature (Pastor et al., 2022; Zerbib, 2022) and the consumption premia obtained empirically in Section 4. The elasticity of substitution across goods, θ, is also of particular interest for the direction of portfolio biases and risk premia in equilibrium (see Sections 3.2 and 3.3). We follow the estimations in the environmental economics literature and set $\theta = 2 > 1$. In an influential contribution, Papageorgiou et al. (2017) provide evidence that this parameter significantly exceeds unity, a condition that is favorable for promoting green growth.[20]

The values of other parameters mostly have a quantitative impact as long as (i) risk aversion γ is above 1, so that there are hedging terms, and (ii) risk aversion is not equal to the reciprocal of the EIS, $\gamma \neq 1/\psi$, so that the preferences are recursive. We pick a relatively large risk aversion of $\gamma \in \{15, 25, 50\}$, to obtain average risk premia that are in line with the data. Indeed, as is well-known, consumption-based asset pricing models tend to generate somewhat modest risk premia. However the effect is purely quantitative and it predominantly impacts the "market" component of risk premia, which is not our focus. Our novel consumption premia arise, regardless of the exact value of γ, and remain quantitatively large.[21] Similarly, we pick $\psi = 1.25$ to keep a relatively low average riskfree rate r_t. Consistent with the literature (e.g., Bansal

[20]Some particular subsets of goods could be closer substitutes, for example, coal energy versus green energy from the perspective of the end consumer. We stay conservative, however, and pick a moderate $\theta = 2$, because some goods and services in the basket are also likely to be very different (e.g., health services versus car purchases). The elasticity could also differ in the short and long-run, an aspect that could be interesting to explore empirically. Finally, such a calibration with $\theta > 1$ is also consistent with the elasticity of substitution across goods in other settings. For instance, this is the case in an international context, as discussed in Imbs and Méjean (2015), among others.

[21]An alternative could be to introduce additional elements such as consumption habits. We refrain from doing so in this paper because the main benefit of such additions in this context would be to increase average risk aversion, as we do, while they could obscure the main mechanisms that we uncover. However, this represents an interesting avenue for future work.

and Yaron, 2004), $\psi > 1$, and investors prefer the early resolution of uncertainty ($\gamma > 1/\psi$). In what follows, parameters γ, ψ, ρ are also taken to be identical for both investors. However, the resolution allows for any value; hence, exploring additional asymmetries stemming from them could be an interesting avenue for future work.

3.1 Marginal values of wealth, consumption, goods prices

The marginal values of wealth of the investors underly many decisions in the economy. To characterize them, note that due to the homotheticity of preferences, the value functions of the investors $i \in \{G, N\}$ can be expressed as

$$V^i(W_t^i, x_t, y_t) = \left(\frac{W_t^{i1-\gamma}}{1-\gamma} \right) J^i(x_t, y_t)^{\frac{1-\gamma}{1-\psi}}. \tag{9}$$

As W_t^G, W_t^N mostly have an impact in levels, the marginal values are primarily driven by functions J_t^G, J_t^N. Thus, in the remainder of the text, we refer to them as (monotonic transformations of) the marginal values of wealth. These quantities underpin the dynamics of the stochastic discount factors of both investors in the economy, which in turn determine the portfolios, asset prices, and other economic decisions.[22]

The evolutions of $J_t^i, i \in \{G, N\}$, are governed by two Hamilton-Jacobi-Bellman equations, summarized in Proposition A.4 of the Appendix. Figure 1 shows the baseline calibration result for both investors, represented as a function of the relative

[22]The stochastic discount factors for investors $i \in \{G, N\}$ are given by

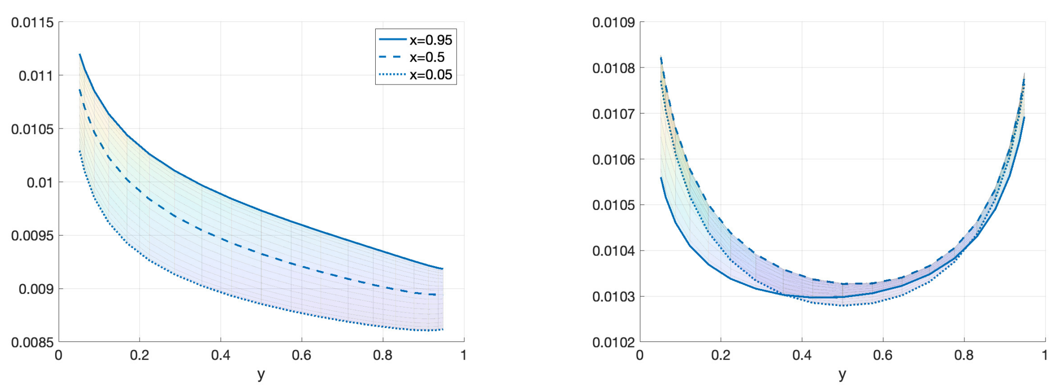
$$\xi_t^i \equiv \xi_0^i \exp\left\{ \int_0^t \left(\Theta_1 P_u^{i1-\psi} J_u^i + \Theta_2 \right) du \right\} W_t^{i-\gamma} J_t^{i\frac{1-\gamma}{1-\psi}},$$

with $\Theta_1 \equiv -(\gamma - 1/\psi)/(1 - 1/\psi)$ and $\Theta_2 \equiv \rho(\gamma - 1)/(1 - 1/\psi)$.

supply of the green good (y_t; shown on the horizontal axis) and the wealth share of the green investor (x_t; shown as different curves).

Figure 1: Marginal values of wealth

(a) Green investor (J_t^G) (b) Neutral investor (J_t^N)



Notes: Based on the calibration of Assumption 1. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

The intuition is as follows, and will be at the core of the consumption premia and portfolio biases.

As the green good becomes relatively scarce, that is, as y_t decreases, both investors have to switch some of their consumption to the brown good. The green investor is particularly affected negatively: she prefers consuming more of the green good ($\alpha^G > 1/2$), but cannot do so due to its low relative supply, or equivalently its high relative price. Therefore, her marginal value of consumption, which is the same as her marginal value of wealth J_t^G following a standard envelope argument, strongly increases. The neutral investor does not have a specific preference for the green good ($\alpha^N = 1/2$), but still likes consuming both due to his CES consumption basket. Therefore, he is also negatively impacted and his marginal value of wealth J_t^N increases as any of the goods become relatively scarce ($y_t \to 0$ or $y_t \to 1$) because he would prefer a more balanced basket, that is, a more comparable relative supply or relative price of both goods. However, this effect is substantially muted for the neutral investor.

Similarly, as her share of wealth x_t increases, the preference of the green investor for green consumption puts an upward pressure on the price of her preferred green good. This induces her to reluctantly tilt her consumption slightly towards the brown good, and her marginal value of wealth J_t^G increases. On the other hand, because he has no particular bias in consumption, the marginal value of wealth for the neutral investor J_t^N is scarcely affected by x_t. Therefore, in practice, the changes in the economy-wide marginal value of wealth $\widetilde{J}_t \equiv x_t J_t^G + (1 - x_t) J_t^N$ are dominated by those of J_t^G.

From the Hamilton-Jacobi-Bellman equations in A.4, a first set of first-order conditions yield expressions for consumptions, summarized in Proposition A.5, which emphasize once again the underlying role of J_t^i: $c_t^i \equiv C_t^i / W_t^i = P_t^{i-\psi} J_t^i$. The details are shown in Appendix A.7. On combining with the market-clearing conditions, one obtains Equation (10) for the relative price of the green good q_t, as shown in Proposition 1.

Proposition 1. *The relative price of the green good, $q_t = q(X_t) \equiv p_{g,t}/p_{b,t}$, solves the following non-linear equation*

$$q_t = S_t^{1/\theta} \left(\frac{1 - y_t}{y_t} \right)^{1/\theta}, \tag{10}$$

where

$$S_t = \frac{\alpha^G J_t^G x_t P_t^{G\theta-\psi} + \alpha^N P_t^{N\theta-\psi} J_t^N (1 - x_t)}{(1 - \alpha^G) P_t^{G\theta-\psi} J_t^G x_t + (1 - \alpha^N) P_t^{N\theta-\psi} J_t^N (1 - x_t)}.$$

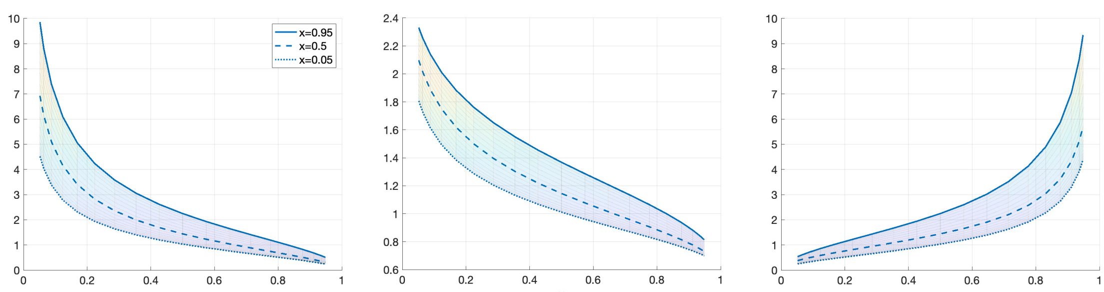
Prices $p_{g,t}, p_{b,t}, P_t^G, P_t^N, \mathcal{E}_t$ follow from the definition of the numéraire and Propo-

sition A.5, and are shown in Proposition A.6.

Figure 2 shows the resulting relative price in the baseline calibration of Assumption 1. As expected, the relative price of the green good, q_t, strongly decreases as the green good becomes more abundant, that is, as y_t increases [Panel (a)]. The pattern is similar for the relative price of the consumption basket of the green investor, $\mathcal{E}_t \equiv P_t^G/P_t^N$ [Panel (b)], whose evolutions are driven by q_t as shown in Proposition A.6. Given this strong monotonic relationship, y_t, q_t, $\mathcal{E}_t$ can be used interchangeably. Hence, we base the empirical tests of our model in Section 4 on $\mathcal{E}_t$—a much easier, cleaner, and higher frequency measure.[23] q_t and $\mathcal{E}_t$ also increase as the share of wealth of the green investor x_t increases, due to the upward pressure on the price of the green good stemming from the green investor's preference for consuming green.

Figure 2: Relative prices and dividends

(a) Green good ($q_t \equiv p_{g,t}/p_{b,t}$) (b) Green consumption basket ($\mathcal{E}_t \equiv P_t^G/P_t^N$) (c) Relative dividends ($p_{g,t}Y_{g,t}/(p_{b,t}Y_{b,t})$)



Notes: Based on the calibration of Assumption 1. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

[23]Similarly, the international finance literature has emphasized mechanisms related to the relative supply, for example, Coeurdacier (2009) and Coeurdacier and Rey (2013). Those have been tested empirically mostly using relative prices or exchange rates, and have become known as "real exchange rate hedging" mechanisms. The model could be equivalently recast using q_t as a state variable. We focus on y_t because it makes the intuition sharper, the equations simpler, and because it is exogenous as opposed to q_t that also depends on x_t.

Beyond the relative prices, which drive relative consumption decisions, the relative dividends of the green asset are also of particular interest. They are shown in Panel (c) of Figure 2 and are obtained as

$$\frac{p_{g,t}Y_{g,t}}{p_{b,t}Y_{b,t}} = q_t \left(\frac{y_t}{1-y_t}\right) = S_t^{\frac{1}{\theta}} \left(\frac{1-y_t}{y_t}\right)^{\frac{1-\theta}{\theta}}. \tag{11}$$

Let us consider a situation in which the green good becomes scarce (y_t decreases). In that case, the relative quantity of output of the green tree, $Y_{g_t}/Y_{b,t} = y_t/(1-y_t)$, decreases. As discussed above, the relative price of the green good, $p_{g,t}/p_{b,t}$, therefore, increases. However, because the green and brown goods remain substitutable enough ($\theta > 1$), the effect on the relative price remains muted and the relative dividends of the green tree see an overall decrease. In other words, the relative dividends and relative supply move in the same direction—an observation that will prove important for the direction of portfolio biases and risk premia. Indeed, as we discuss in Section 3.2, relative dividends are the main drivers of the relative returns on the two assets, while changes in dividend yields (i.e., equity prices relative to fundamentals) play a limited role.

The case in which green and brown goods are very poor substitutes (broadly $\theta < 1$)[24] would have the counterintuitive implication that the payoff of an asset would be *low* when the quantity of goods that it produces is *high*. Most importantly, it is also inconsistent with empirical estimates in the environmental economics literature that put θ strongly above unity, a condition that is also favorable for promoting green growth (see, for instance, Papageorgiou et al., 2017).

Finally, the relative dividends of the green asset also increase as the wealth share

[24]Coeurdacier (2009) shows in a CRRA context based on zero-order approximations that the exact value at which the switch occurs is a non-linear function of all parameters, although it is close to 1.

of the green investor increases, consistent with her preference for green consumption, which puts an upward pressure on the relative price of the green good. However this effect is much more muted in the baseline calibration.

3.2 Asset prices

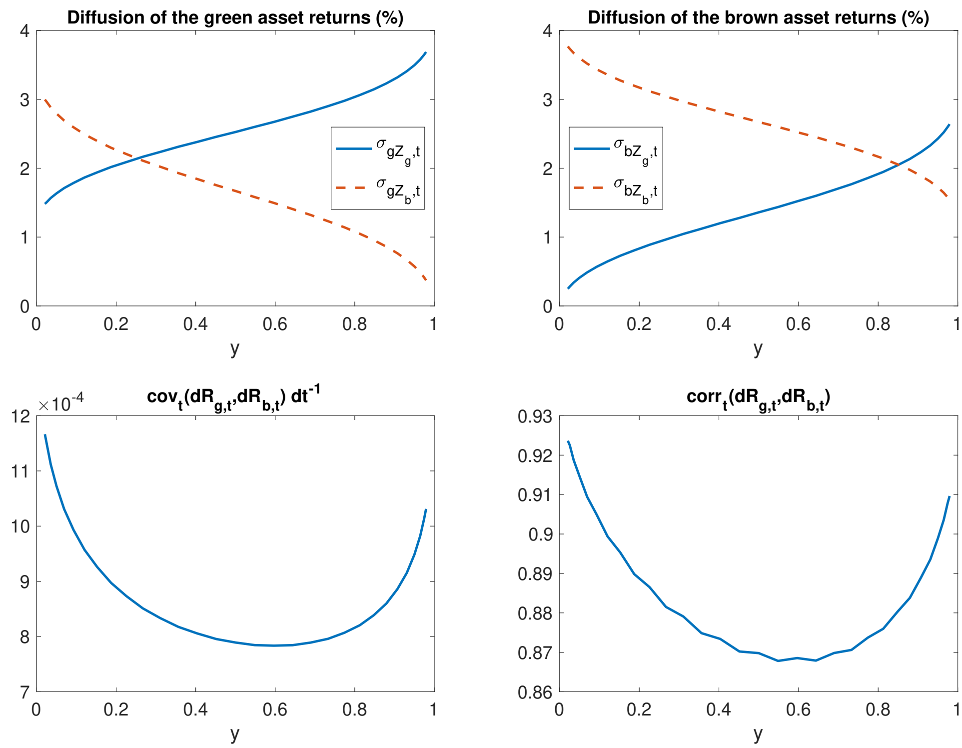
Second moments. Let us start with second moments, which underpin part of the intuition on risk premia and portfolios.

Recall that the diffusion terms for both asset returns, $j \in \{g, b\}$, are $\sigma_{j,t} \equiv \sigma_{p_j,t} + \sigma_{Y_j} - \sigma_{F_j,t}$.[25] They capture how returns load on the different shocks in the economy. In practice, although the dividend yields, $F_{j,t}$, are time-varying in our setting with recursive preferences and heterogeneity, the effects of their changes via $\sigma_{F_j,t}$ remain comparatively muted. Therefore, the patterns in returns diffusions are mostly driven by $\sigma_{p_j,t}, \sigma_{Y_j}$, that is, by movements in the relative dividends of both assets. In turn, the relative dividends of the green asset, $p_{g,t}Y_{g,t}/(p_{b,t}Y_{b,t})$, evolve in the same direction as relative supply, $Y_{g,t}/Y_{b,t}$. As described above in Section 3.1, the intuition is that the two goods are substitutable enough ($\theta > 1$). As a result, the effect on the relative price of the goods, $p_{g,t}/p_{b,t}$, is moderate enough to not overturn the impact of the relative supply, $Y_{g,t}/Y_{b,t}$. This implies that for most of the state space, the returns on the green asset tend to load more on the output shocks to the green tree ($\sigma_{gZ_g,t} > \sigma_{gZ_b,t}$) because those shocks increase the relative dividends of the green tree. Similarly, the returns on the brown asset load more on the output shocks to the brown tree ($\sigma_{bZ_b,t} > \sigma_{bZ_g,t}$) because those shocks increase the relative dividends of the brown tree.[26]

[25] This expression comes from an application of Itô's Lemma to the definition of returns in Equation (1), $dR_{j,t} = dQ_{j,t}/Q_{j,t} + F_{j,t}dt$, with $Q_{j,t} = p_{j,t}Y_{j,t}/F_{j,t}$. Cf. Section 2 and Appendix A.

[26] Note that if θ were to be below unity, movements in the relative prices of the goods would be so

Figure 3: Second moments of returns



Notes: Based on the calibration of Assumption 1. x_t is the wealth share of the green investor. y_t is the relative supply of the green good. The figure shows a cut in which $x_t = 1/3$, consistent with empirical estimates.

In short: the green asset tends to earn higher returns when the green good is relatively more abundant, that is, when positive shocks to the supply of the green tree occur, which increase its relative supply y_t. Figure 3, which shows the diffusion terms for the returns on both assets in the baseline calibration, confirms that this is indeed the case: on average, the loading of the green asset returns on the green output shock ($\sigma_{gZg,t}$, blue curve) is larger than their loading on the brown output

extreme that relative dividends would move invertedly with relative supply, so that the returns on the *green* asset would ultimately load more on the output shock to the *brown* tree. This implication is both counterintuitive and inconsistent with empirical estimates in the environmental economics literature, which puts θ strongly above unity, a condition that is also favorable for promoting green growth (see, for instance, Papageorgiou et al., 2017) and on which we focus.

shock ($\sigma_{gZb,t}$, orange curve), and vice versa for the brown asset returns.

Beyond the main patterns discussed above, the returns on both assets also load on both shocks, albeit in a more limited way. This leads to a strong correlation between the assets: on average, $\text{corr}_t(dR_{g,t}, dR_{b,t}) \approx 0.9$, even though the outputs of the trees themselves have no fundamental correlation ($\sigma_{Y_g Z_b} = \sigma_{Y_b Z_g} = 0$). In other words, the large correlation between asset returns emerges endogenously. This phenomenon is driven by movements in the relative prices of the goods and in the allocation of wealth, as well as by the patterns of the marginal values of wealth of both investors J_t^i (and hence patterns of their stochastic discount factors). Economically, this emphasizes the financial contagion spreading through the asset markets in this economy: a shock to the output of a given tree has a sizable impact on the returns of the other tree, and can therefore impact both investors beyond its impact on the goods markets. The bottom panels of Figure 3, together with Figure B.6 in the Appendix, also emphasize that second moments are inherently time-varying, and depend on the current state of the economy.

Risk premia. Proposition 2 presents the expected excess returns on the green and brown assets. Proposition A.1 in the Appendix generalizes these expressions to the case in which risk aversion and EIS differ across investors, and in which both investors have preferences towards both assets, $\phi_j^i \neq 0$ for $i \in \{G, N\}$, $j \in \{g, b\}$. In that case, the economy-wide risk aversion also becomes state-dependent, γ_t.

Proposition 2. *The expected risk premia on the green and brown equity assets are*

$$\begin{aligned} \mu_{g,t} - r_t &= \gamma \sigma_{g,t}^\top \sigma_{\widetilde{W},t} - x_t \phi - \gamma \sigma_{g,t}^\top \sigma_{\tilde{J},t}, \\ \mu_{b,t} - r_t &= \gamma \sigma_{b,t}^\top \sigma_{\widetilde{W},t} \qquad\quad - \gamma \sigma_{b,t}^\top \sigma_{\tilde{J},t}, \end{aligned} \tag{12}$$

where

$$\sigma_{\widetilde{W},t} \equiv w^M_{g,t}\sigma_{g,t} + (1 - w^M_{g,t})\sigma_{b,t},$$
$$\sigma_{\widetilde{J},t} \equiv \left(\frac{1}{\gamma}\right)\left(\frac{1-\gamma}{1-\psi}\right)\left(x_t\sigma_{J^G,t} + (1-x_t)\sigma_{J^N,t}\right),$$

and $\widetilde{W}_t$ *is the total wealth,* $\widetilde{J}_t$ *is the economy-wide marginal value of wealth, and* $\sigma_{J^G,t}, \sigma_{J^N,t}$ *are the geometric diffusion terms of* J^G_t, J^N_t *obtained as in Remark A.1.*

The expressions for risk premia are composed of three terms.

The first term is a *total wealth component*, which is driven by the covariance of each risky asset return with the total wealth in the economy $\widetilde{W}_t$. It can be thought of as a "market" component. Intuitively, an asset that comoves a lot with the total wealth provides little diversification benefits, is therefore risky, and commands a high risk premium in equilibrium. This is the usual financial diversification component that exists even when investors are myopic, and makes them want to hold some of both assets to maximize the Sharpe ratio of their portfolios.

The second term is the green premium characterized by Pastor et al. (2021) and Zerbib (2022), among others. As the green investor has a preference for investing in the green asset ($\phi > 0$), she accepts a lower expected return to hold it. Thus, the expected returns on that asset decrease. Additionally, this effect scales with the wealth share of the green investor, x_t. Because we set the preferences of both investors for the brown asset to zero ($\phi^G_b = \phi^N_b = 0$), the brown asset does not display any such premium. This is without loss of generality and the green premium term should be understood in a relative sense between the green and brown assets.

The third term is a hedging component that constitutes our novel consumption premia, and deserves more emphasis.

From a broad perspective, this term is driven by the comovement of asset returns with the economy-wide wealth-weighted marginal value of wealth, $\widetilde{J}_t \equiv x_t J_t^G + (1 - x_t) J_t^N$. Intuitively, an asset whose returns are large when $\widetilde{J}_t$ is large is a good hedge because it pays when it is most valuable for the average investor, that is, for the economy as a whole. Such an asset is therefore less risky, and commands a lower risk premium in equilibrium.[27]

Importantly, note that such hedging components—and, hence, our novel consumption premia—would be completely absent with log, mean-variance, or CARA preferences because investors would be myopic under those specifications.

Our Markovian setting allows us to provide a better intuition of the effects of the hedging terms by breaking them down as follows:[28]

$$\begin{aligned} -\gamma \sigma_{j,t}^{\top} \sigma_{\widetilde{J},t} = &- \sigma_{j,t}^{\top} \sigma_{x,t} x_t \left(\frac{1-\gamma}{1-\psi} \right) \left\{ x_t \frac{J_{x,t}^G}{J_t^G} + (1 - x_t) \frac{J_{x,t}^N}{J_t^N} \right\} \\ &- \sigma_{j,t}^{\top} \sigma_{y,t} y_t \left(\frac{1-\gamma}{1-\psi} \right) \left\{ x_t \frac{J_{y,t}^G}{J_t^G} + (1 - x_t) \frac{J_{y,t}^N}{J_t^N} \right\}. \end{aligned} \tag{13}$$

In words, the novel consumption premia comprise a wealth-hedging premium (hedging of movements in the wealth share of the green investor, x_t) and a relative-supply-hedging premium (hedging of movements in the relative supply of the green good, y_t, or, equivalently, of its relative price).

Establishing the sign of these consumption premia for green versus. brown assets requires eliciting the patterns of quantities of risk, and of the prices of those risks.

[27]In the terminology of the asset pricing literature, those consumption premia embed the desire of investors to hedge against changes in their investment opportunities, captured here by the state variables $X_t = (x_t, y_t)'$.

[28]Again, the framework allows for potentially different γ^i, ψ^i, ρ^i for both investors. In that case, the economy-wide risk aversion is state-dependent, $\gamma_t \equiv \left(x_t/\gamma^G + (1 - x_t)/\gamma^N \right)^{-1}$, and the weighting in the economy-wide marginal value of wealth $\widetilde{J}_t$ also reflects differences in those parameters.

The quantities of risk are driven by the (instantaneous) covariances of the asset returns with the state variables, x_t, y_t, which fully characterize the state of the economy: $\text{cov}_t\left(dR_{j,t}, dx_t\right) dt^{-1} = \sigma_{j,t}^{\top}\sigma_{x,t}x_t$ and $\text{cov}_t\left(dR_{j,t}, dy_t\right) dt^{-1} = \sigma_{j,t}^{\top}\sigma_{y,t}y_t$. On average, we expect the latter covariance, $\text{cov}_t\left(dR_{j,t}, dy_t\right) dt^{-1}$, to be positive for the green asset, and negative for the brown asset. That is, we expect the returns on the green (brown) asset to increase (decrease) with the relative supply of the green good, y_t. This is because, as explained above, the returns on the green asset tend to load more on shocks to the green output $dZ_{g,t}$, which also increase $Y_{g,t}$ and, hence, increase the relative supply of the green good, $y_t \equiv Y_{g,t}/(Y_{g,t} + Y_{b,t})$. Conversely, the returns on the brown asset tend to load more on $dZ_{b,t}$, which also increase $Y_{b,t}$ and, hence, decrease y_t. The sign of $\text{cov}_t\left(dR_{j,t}, dx_t\right) dt^{-1}$ depends on the covariance between x_t and y_t, which is endogenous and depends on investors' portfolios, which in turn, depend on ϕ. It is discussed below.

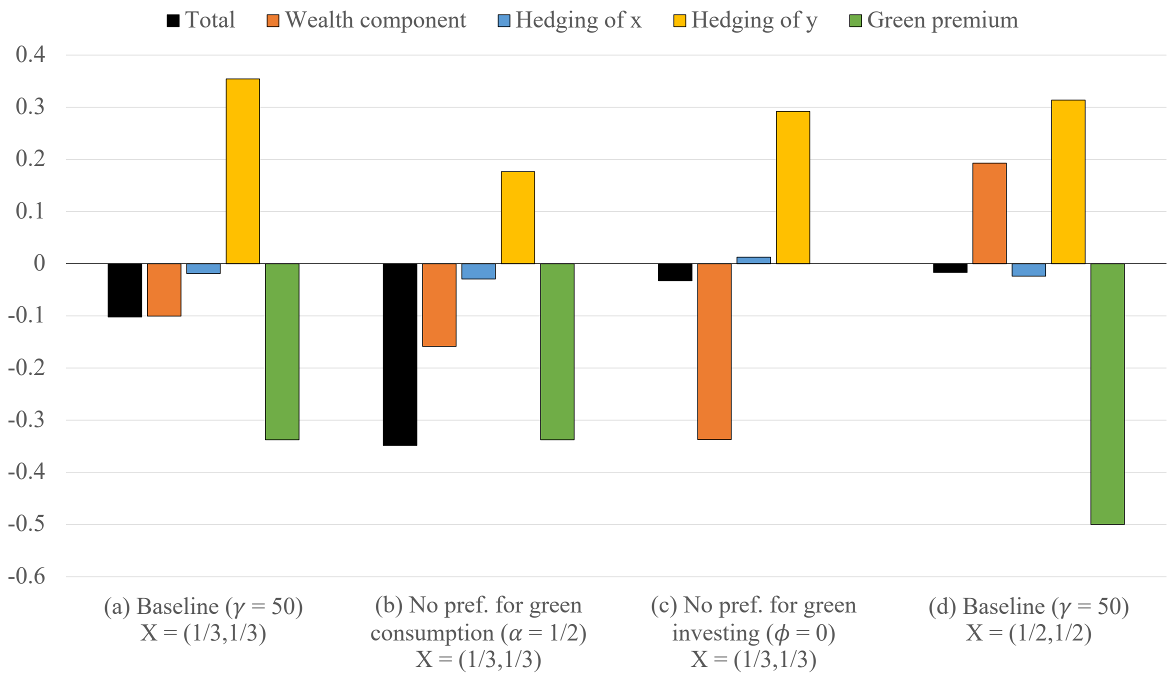
The remaining pieces are the prices of these risks, which summarize how investors *value* an asset with certain quantities of risk. These prices of risk are driven by preference parameters γ, ψ, and crucially, by how the economy-wide wealth-weighted marginal value of wealth evolves with the state variables. This is captured by $J_{x,t}^i$, $J_{y,t}^i$, which represent the derivatives of the marginal values of wealth of both investors $i \in \{G, N\}$ with respect to each state variable. As the economy is composed of an investor with a preference for green consumption and an investor who is neutral, the "average investor" has, on average, a tilt towards preferring the green good. In other words, because J_t^G strongly decreases with the relative supply of the green good ($J_{y,t}^G \ll 0$), the economy-wide wealth-weighted marginal value of wealth $\tilde{J}_t$ is, on average, a decreasing function of y_t. Therefore, situations in which y_t is low—hence, the relative price of the green good is high—are adverse states of the world, and the price of y_t-risk is positive (recall the minus sign in Equation (13)). Consequently, an asset that comoves with the relative supply of the green good y_t is risky, because it is a poor hedge against these adverse states and hence, commands a higher risk premium in equilibrium. Conversely, the price of x_t-risk is expected to be negative on average. Indeed, as shown in Section 3.1, the evolution of $\widetilde{J}_t$ with x_t is again dominated by the marginal value of wealth of the green investor J_t^G, which tends to increase with her wealth share x_t ($J_{x,t}^G > 0$) due to the upward pressure she puts on the price of her preferred green good.

Thus, taken together, we expect the green asset (whose returns comove positively with y_t) to be riskier in terms of relative supply risk than the brown asset (whose returns comove negatively with y_t). Therefore, the green asset is expected to command a higher relative-supply-hedging premium on average. The sign of the hedging of the wealth share risk is more ambiguous and is discussed below. It turns out as negative but small in our benchmark.

Panel (a) of Figure 4 shows the *difference* in the expected returns between the green and brown assets, $(\mu_{g,t} - r_t) - (\mu_{b,t} - r_t)$, as well as its components, in the baseline calibration of Assumption 1. To get a sense of the average premia differentials, they are shown at the point at which the green investor holds one third of the wealth ($x_t = 1/3$), and the relative supply of the green good is one third ($y_t = 1/3$), which is broadly consistent with empirical estimates in Morgan Stanley Institute for Sustainable Investing (2019). The wealth component, $\gamma\sigma_{j,t}^{\top}\sigma_{\widetilde{W},t}$, which can be understood as a market component, is important to derive risk premia that are quantitatively in line with the data: on average, $\mu_{j,t} - r_t \approx 4.2\%$, which is only slightly lower than their empirical counterparts.[29]

[29]Getting such values for the average risk premia is the main reason for which we pick a high calibration of risk aversion γ. Indeed, as is well-known, consumption-based asset pricing models tend to generate somewhat modest risk premia. Cf. footnote 20.

Figure 4: Average difference in risk premia between green and brown asset $(\mu_{g,t} - r_t) - (\mu_{b,t} - r_t)$ (%)



Notes: Based on the calibration of Assumption 1, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good. The figure shows the difference between risk premia on the green and brown asset, and their components, at $X_t \equiv (x_t, y_t)' = (1/3, 1/3)$ for Panels (a), (b), (c), and at $X_t = (1/2, 1/2)$ for Panel (d).

In practice, this market component largely depends on how dominant a given asset is in the total wealth, that is, on the weights of the assets in the market portfolio $(w^M_{g,t}, w^M_{b,t})$. The wealth component therefore drives the overall shape of the risk premia on both assets with the state of the economy, especially with respect to the relative supply.[30] However, as this term is more common, it is not our focus in this paper.

[30]The market weights $(w^M_{g,t}, w^M_{b,t})$ are inherently related to y_t, the relative supply of both goods, as shown in Panels (a) and (b) of Figure B.10 in the Appendix. They are equal for both assets broadly around the point at which their relative supply is equal, $y_t = 1/2$, although the preference for green investing leads the green asset to be slightly overvalued so that its weight in the market portfolio $(w^M_{g,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t}))$ is, on average, slightly larger. In other words, the wealth component is,

The green premium and the novel consumption premia are of greater interest.

When the green investor holds about one third of total wealth, as in the data, the green premium is $-x_t\phi \approx -0.333\%$. Recall that this green premium should be interpretated in a relative manner, so that, on average, the expected excess returns on the green asset are 33.3 basis points smaller than those on the brown asset, when we focus purely on the effect of the preference of investors for green investing. This is also visible from Panel (c), which sets $\phi = 0$, and is consistent with recent empirical estimates (Pastor et al., 2022; Zerbib, 2022).

Most importantly—and this is our main result—the consumption premia broadly compensate the green premium: when we focus purely on the effect of the preference of investors for green consumption, on average [i.e., at $X_t = (1/3, 1/3)$], the expected excess returns on the green asset are 36.6 basis points larger than those on the brown asset. In other words, the only reason why $\mu_{g,t} - r_t$ is larger at $X_t = (1/3, 1/3)$ is the mostly mechanical total wealth component. This is visible in our baseline calibration [Panel (a)], and it can be compared to the case wherein the green investor has no preference for green consumption ($\alpha = 0.5$), so that the green premium dominates,[31] as well as the case wherein she has no preference for green investing ($\phi = 0$), so that the consumption premia dominate. The effect can become larger than the green premium for a larger risk aversion γ, an EIS closer to $\psi = 1$, a larger bias towards consumption α, and in some parts of the state space, as discussed below. Consistent with our intuition above, this is driven by a positive relative-supply-hedging premium (35.5 basis points), which constitutes the bulk of the consumption premia.

on average, slightly larger for the green asset, even though the difference is dominated by variations with the state of the economy.

[31] Note that there still exist consumption premia even without preference for green consumption ($\alpha = 1/2$). This is because investors still want to consume both goods, and therefore remain sensitive to movements in relative prices (i.e. they still want to hedge against changes in their investment opportunities). The effect is quantitatively more muted, but continues to show the appeal of bringing green investors to this general equilibrium context in which they also consume, even in that case.

The hedging of relative wealth risk is negative for both assets, and in particular, slightly more negative for the green asset. This effect stems from the larger covariance of green asset returns with the wealth share x_t, that is, from a larger quantity of "x_t"-risk (Figure B.4), combined with the negative price for that risk described above (Figure B.5).[32] In practice, this conclusion depends on the magnitude of the bias in portfolio holdings, and therefore on the calibration. For instance, this premium is positive if the preferences for green consumption (α) are strong enough, while the preferences for green investing (ϕ) are moderate enough, like in Panel (c) of Figure 4. However, in most cases, and regardless of its sign, the magnitude of this effect remains quantitatively small (e.g., -1.9 basis points in the baseline), and therefore, it is neither our main focus here nor in the empirical part (Section 4).[33]

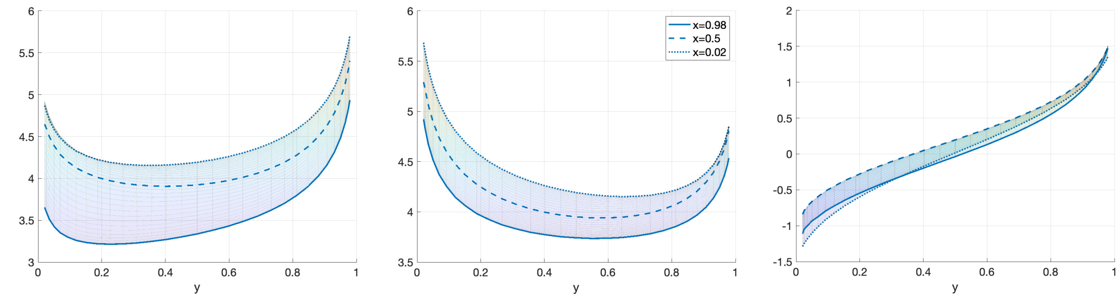
Dynamics of risk premia. Interestingly, the patterns of the risk premia and their subcomponents also vary strongly with the state of the economy—an aspect that our global solution allows to explore.

[32] In more details, the intuition for this negative relative premium is as follows. In the baseline, the wealth share loads more on shocks to the output of the green tree ($dZ_{g,t}$): Figures A.3 and A.4 in the Appendix show that $\sigma_{xZ_g,t}x_t > \sigma_{xZ_b,t}x_t$ in magnitude, and $\sigma_{xZ_g,t}x_t > 0$ for any X_t, while the loading of x_t on shocks to the brown output $\sigma_{xZ_b,t}x_t$ flips its sign, for example, as y_t increases. As shown in Proposition A.3 of the Appendix, the loadings of the wealth share are themselves endogenous and follow these patterns provided that the portfolio of the green investor is biased enough towards the green asset ($w^G_{g,t} - w^M_{g,t} > w^G_{b,t} - w^M_{b,t}$). In short, in the baseline, a positive shock to the output of the green tree tends to increase the wealth share x_t. (Relatedly, because the relative supply y_t also tends to increase with positive shocks to the green output, the wealth share x_t and relative supply y_t are positively correlated in the baseline.) Because such a shock also tends to increase the returns on the green asset more, as explained previously, this leads the covariance of the green asset returns with x_t to be larger than that of the brown asset returns. In other words, the quantity of x_t "risk" is larger for the green asset (Figure B.4). Combined with the negative price of this risk described above (Figure B.5), this leads to the negative relative wealth-share-hedging premium for the green asset in the baseline calibration. In most cases however, the magnitude of this effect remains small.

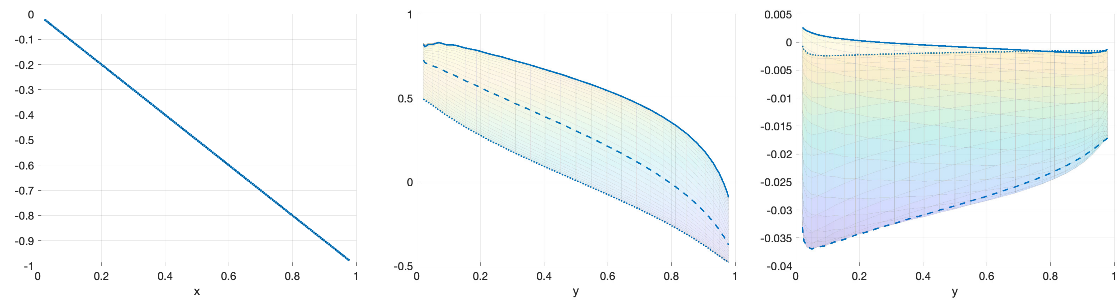
[33] The introduction of a tax on dividends as discussed in Section 5 can reinforce the impact of x_t-hedging, because it can lead to imperfect risk sharing across investors.

Figure 5: Returns

(a) Total risk premium on green asset ($\mu_{g,t} - r_t$) (b) Total risk premium on brown asset ($\mu_{b,t} - r_t$) (c) Relative "market" component ($g - b$)



(d) Relative green premium ($-x_t\phi, g - b$) (e) Relative y_t-hedging premium ($g - b$) (f) Relative x_t-hedging premium ($g - b$)



Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Panel (a) of Figure 5 shows that the expected risk premium on the green asset $\mu_{g,t} - r_t$ increases as the relative supply of the underlying green tree, y_t, increases. This pattern is driven by the wealth component shown in Panel (c): as y_t increases, the green good starts to dominate the economy, and consequently, the green asset also starts to dominate the total wealth ($w^M_{g,t} \equiv Q_{g,t}/[Q_{g,t} + Q_{b,t}]$, the weight of the green asset in the market portfolio, increases towards 1). In such situations, the risk of the green asset is difficult to diversify away. As a result, the green asset is risky and commands a higher risk premium.[34] Conversely, the expected returns on the brown

[34]In other words, because the green asset dominates total wealth as y_t becomes large, the covari-

asset increase as y_t decreases, as shown in Panel (b).[35]

The expected excess returns on both assets also decrease with the share of wealth held by the green investor, x_t [Panels (a) and (b)]. For the green asset, this is largely driven by the increasing impact of the preference for green investing (ϕ) as the green investor becomes larger in the economy, that is, by an increasing green premium. This is seen in Panel (d), which plots the green premium on the green asset relative to the brown asset as a function of x_t. However, for the brown asset, this pattern is driven by the state-dependence in the hedging of relative supply risk, which becomes more strongly negative for the brown asset as the green investor (who is more worried about this risk) increasingly holds more wealth. The riskfree interest rate also increases with x_t, which is consistent with the pattern of borrowing and saving discussed in Section 3.3.

Panels (e) and (f) confirm that the consumption premia are themselves very time-varying. The hedging of the relative supply risk for the green asset relative to the brown asset is positive and large for most of the state-space, as shown in Panel (e). Again, it increases as the green investor, who is particularly worried about this risk, becomes larger in the economy, that is, as x_t increases. This positive relative premium on the green asset also strongly increases as the relative supply of the green good, y_t, decreases; for example, it reaches close to 1% for large x_t and small y_t. This is

ance of this asset with total wealth is large because it is broadly equal to the covariance of the asset with itself. This leads the wealth component of the risk premia, which is driven by the covariance with total wealth, to be large for the green asset.

[35]Figure B.3 in the Appendix also shows that the states of the world in which one of the goods becomes scarce (low or high y_t) are associated with a lower riskfree interest rate r_t. This is consistent with higher precautionary saving motives. Note that in some calibrations, for example, with $\gamma = 50$, r_t is negative. This is in line with the real interest rates being empirically negative in the recent period even for longer maturities (e.g., Figure B.2 in the Appendix shows that this is the case for the 10-year market yield on inflation-indexed U.S. Treasury Securities since 2019). This has no particular impact on the equilibrium. For instance, Figure B.3 in the Appendix shows that $r_t > 0$ for $\gamma = 15$, and risk premia and portfolios in that case are similar to those with a larger γ except in terms of magnitude.

consistent with the green investor being *especially* worried about the relative supply risk when her preferred good becomes very scarce, suggesting that hedging terms can grow and continue to compensate the green premium even as the latter grows when the green investor becomes dominant. Finally, as discussed above and shown in Panel (f), the relative premium on the green asset that stems from wealth share hedging is negative on average in the baseline, albeit more muted (and dependent on the calibration). Its magnitude is largest around $x_t = 1/2$, the point at which the identity of the investor dominating the economy flips.[36]

3.3 Portfolios

We conclude this characterization by a discussion of the optimal portfolios of both investors. Proposition 3 shows that these are Merton (1973)-type portfolios that are made up of two components.[37]

The first term is similar for both investors and corresponds to the myopic portfolio that would be chosen by a one-period mean-variance investor. This is the usual financial diversification component driven by the risk premia on both assets, normalized by volatilities, and is partly related to the market portfolio ($w^M_{g,t}$, $w^M_{b,t}$).

However, in this context, the first term also embeds the preference of the green investor for green assets (ϕ). Equation (14) shows that it is isomorphic to the expected returns on the green asset being perceived as (relatively) larger by the green investor. As expected, this term makes her tilt her portfolio allocation towards the green as-

[36]The patterns of the x_t- and y_t-premia are in turn driven by changes in both the quantities and prices of risk with the state of the economy $X_t = (x_t, y_t)'$, as shown in Figures B.4 and B.5 in the Appendix.

[37]Again, Proposition A.2 in the Appendix generalizes the expressions to the case wherein risk aversion and EIS differ across investors, and where both investors have preferences for both assets, $\phi^i_j \neq 0$ for $i \in \{G, N\}$, $j \in \{g, b\}$. In this case, the economy-wide risk aversion also becomes state-dependent, γ_t.

set in equilibrium. In other words, it is the manifestation of the green premium for portfolios.

Proposition 3. *The optimal portfolios of the green and neutral investors $j \in \{G, N\}$ are given by*

$$\begin{pmatrix} w^G_{g,t} \\ w^G_{b,t} \end{pmatrix} = \frac{1}{\gamma} \left(\Sigma_t^\top \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t + \phi \\ \mu_{b,t} - r_t \end{pmatrix} + \left(\frac{1-\gamma}{1-\psi}\right) \Sigma_t^\top \left(\frac{J^G_{x,t}}{J^G_t} x_t \sigma_{x,t} + \frac{J^G_{y,t}}{J^G_t} y_t \sigma_{y,t} \right) \right\},$$
$$b^G_t = 1 - w^G_{g,t} - w^G_{b,t}, \tag{14}$$

$$\begin{pmatrix} w^N_{g,t} \\ w^N_{b,t} \end{pmatrix} = \frac{1}{\gamma} \left(\Sigma_t^\top \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t \\ \mu_{b,t} - r_t \end{pmatrix} + \left(\frac{1-\gamma}{1-\psi}\right) \Sigma_t^\top \left(\frac{J^N_{x,t}}{J^N_t} x_t \sigma_{x,t} + \frac{J^N_{y,t}}{J^N_t} y_t \sigma_{y,t} \right) \right\},$$
$$b^N_t = 1 - w^N_{g,t} - w^N_{b,t}, \tag{15}$$

where $w^i_{g,t}, w^i_{b,t}$, and b^i_t are the portfolio weights (as a share of wealth) allocated to the green equity asset, the brown equity asset, and the riskless bond, and $\Sigma_t \equiv \begin{bmatrix} \sigma_{g,t} & \sigma_{b,t} \end{bmatrix}$.

The second component corresponds to the hedging terms, absent with log or myopic preferences (such as CARA). They are the counterpart of the consumption risk premia for portfolios, and capture the way investors tilt their allocation to insure against changes in the state of the economy, summarized by $X_t = (x_t, y_t)'$. Investors do so by overweighting assets whose payoffs are large when they find it most valuable, that is, when their individual marginal values of wealth are high, so that hedging terms are governed by the covariance between the risky returns and the individual marginal values of wealth, J^G_t, J^N_t.

Overall, the common term drives the broad pattern of the portfolios of both investors throughout the state space. It is corrected for the preference of the green investor for green investing (ϕ), while the hedging term captures how investors differentially deviate from this broad pattern. Therefore, the hedging terms are a prime quantity of interest in our economy with heterogeneous investors.

Figure 6 shows the corresponding portfolio weights of the risky assets for each investor as a percentage of their wealth ($w^i_{g,t}$, $w^i_{b,t}$ for $i \in \{G, N\}$), as well as their components, for various calibrations.[38] As with risk premia, to get a sense of average portfolios, all the variables are shown at the point where the green investor holds one third of the wealth ($x_t = 1/3$), and the relative supply of the green good is one third ($y_t = 1/3$). These are broadly consistent with empirical estimates in Morgan Stanley Institute for Sustainable Investing (2019) (except for Panel (d) where $X_t = (1/2, 1/2)$).

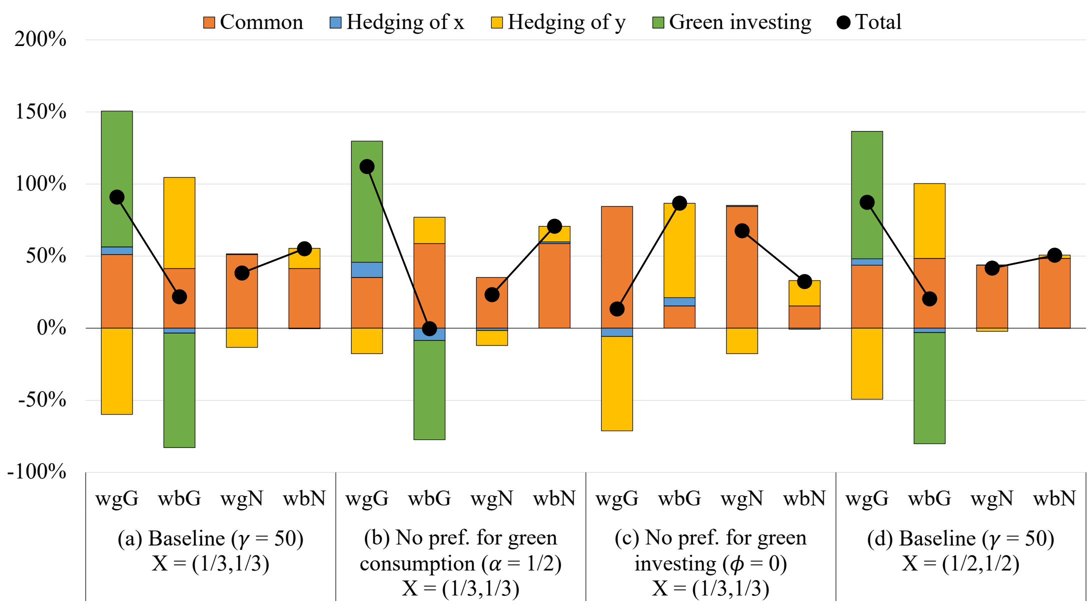
Panel (a) shows that in the baseline calibration, the green investor significantly tilts her allocation towards the green asset: on average [i.e., at $X_t = (1/3, 1/3)$], she invests $w^G_{g,t} \approx 91\%$ of her wealth in it, as opposed to $w^G_{b,t} \approx 22\%$ in the brown asset. This is significantly more biased towards the green asset than the market portfolio, $w^M_{g,t} \approx 56\%$, $w^M_{b,t} \approx 44\%$ (Figure B.7 in the Appendix). The neutral investor, being less sensitive to changes in relative supply and wealth share, is willing to take the other side of this trade: on average, he invests more of his wealth in the brown asset, $w^N_{b,t} = 55\%$, than in the green asset, $w^N_{g,t} = 38\%$.

As expected, the overweighting of the green asset in the portfolio of the green investor is driven by her preference towards green investing ($\phi > 0$), shown in green in Figure 6. In the baseline, this component taken separately would lead her to overweight the green asset by an *additional* 94% of her wealth. This is substantially beyond the 51% dictated by the common component (shown in orange) that is iden-

[38]We plot the portfolio weight that each investor allocates to the riskless bond to borrow or save (b^i_t) in Figure B.8 of the Appendix. Figure B.7 also shows the weights in the *market* portfolio for comparison: $w^M_{g,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t})$, $w^M_{b,t}$. Recall that the bond is in zero net supply so that for the market overall, $b^M_t = 0$, and $w^M_{b,t} = 1 - w^M_{g,t}$.

tical for both investors. Conversely, it would lead her to underweight the brown asset by 79% of her wealth, compared to the 41% dictated by the common component.

Figure 6: Portfolios at $X_t = (1/3, 1/3)$, and $X_t = (1/2, 1/2)$



Notes: Based on the calibration of Assumption 1, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good. The figure shows portfolios and their components at $X_t \equiv (x_t, y_t)' = (1/3, 1/3)$ for Panels (a), (b), (c), and at $X_t = (1/2, 1/2)$ for Panel (d). $w^i_{g,t}, w^i_{b,t}$ are the weights (as % of wealth) on the green and brown asset in the portfolio of investor $i \in \{G, N\}$.

Most importantly, our novel result in terms of portfolios shows that the impact of green investing is again strongly counterbalanced once it is brought to our general equilibrium context. Indeed, the hedging term associated with the relative supply (shown in yellow in Figure 6), which mostly stems from the preferences of the green investor towards green consumption ($\alpha > 1/2$), leads her to *under*weight the green asset by about 60% of her wealth. This arises because the returns on the green asset are comparatively smaller when the relative supply of the green good, y_t, is low (i.e., when the relative price of green good is high, cf. Section 3.2), that is, when the green investor values it most because her marginal value of wealth J_t^G is high in these states of the world (Section 3.1).

Overall, this leads the green investor to cut the overweighting stemming from green investing by about two-third. This is also visible in Panel (c), which shows that when she has no preference for green investing ($\phi = 0$), the green investor invests much more of her wealth in the brown asset ($w_{g,t}^G \approx 13\%$, $w_{b,t}^G \approx 87\%$). Therefore, compared to the market portfolio, the green investor picks a portfolio that is biased towards the *brown* asset in equilibrium. Conversely, the counterbalancing impact of the hedging terms is also visible in Panel (b): without any preference for green consumption ($\alpha = 1/2$), the green investor invests an even larger share of her wealth in the green asset ($w_{g,t}^G \approx 112\%$, $w_{b,t}^G \approx 0\%$).

A few further comments on the portfolios are in order.

First, as with risk premia, the impact of the hedging of the wealth share depends on the calibration, but remains muted in most cases. In the baseline, it leads the green investor to slightly increase the weight of the green asset in her portfolio as shown in Figure 6 (blue component).

Second, as the green investor is more sensitive to the risks associated with the consumption preferences, especially the one related to the relative supply, she is more eager to strongly tilt her portfolio according to her preferences. In practice, she is willing to borrow in the riskless bond to *leverage* her risky portfolio weights slightly: at $X_t = (1/3, 1/3)$, she borrows $|b_t^G| = |1 - w_{g,t}^G - w_{b,t}^G| = 13\%$ of her wealth (Figure B.8 in the Appendix). The remaining investor, being neutral, is willing to accommodate the green investor by lending $b_t^N = 6\%$ of his wealth. These patterns of borrowing and lending are also reflected in the riskfree rate: r_t increases as x_t increases (or as the green investor, who is a borrower, gets a larger share of total wealth). Introducing portfolio constraints, such as borrowing or shorting limits, could be an interesting avenue for further research towards enriching these phenomena.

Third, because of the preference for green investing and green consumption of the "average investor" in the baseline, the equity price of the green asset $Q_{g,t}$ is slightly overvalued compared to an economy with $\phi = 0$ and $\alpha = 1/2$. In other words, the weight of the green asset in the market portfolio (which is its equity price divided by the total wealth $w^M_{g,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t})$) is slightly larger than the weight of the brown asset in the market portfolio, $w^M_{b,t} \equiv Q_{b,t}/(Q_{g,t} + Q_{b,t})$.[39] This is visible in Figure B.7 of the Appendix, which plots the market portfolios in various cases, especially in Panel (d), which focuses on the symmetric point $X_t = (1/2, 1/2)$. At this point, the weight in the market portfolio is $w^M_{g,t} = w^M_{b,t} = 50\%$ for $\alpha = 0.5$ (or more generally symmetric αs) and $\phi = 0$, as opposed to $w^M_{g,t} = 65\%, w^M_{b,t} = 35\%$ in the baseline calibration. Similarly, in the absence of any preference for green consumption and green investing, the market weights at $X_t = (1/3, 1/3)$ would be strongly tilted towards the *brown* asset, unlike Panels (a), (b), (c) of Figure B.7, wherein either $\phi > 0$, $\alpha^G > 1 - \alpha^N = 1/2$, or both.

Finally, the portfolio weights, as well as how their biases with respect to the market portfolio ($w^M_{g,t} \equiv Q_{g,t}/[Q_{g,t} + Q_{b,t}]$, $w^M_{b,t} = 1 - w^M_{g,t}$), are also strongly state-dependent. This is shown in Figures B.9 and B.10 of the Appendix, which plot both as a function of the state of the economy $X_t = (x_t, y_t)'$ in the baseline calibration. For instance, both investors increase the share of their wealth invested in the brown asset as the relative supply of the green good, y_t, decreases (consistent with the market

[39] In equilibrium, the latter is $w^M_{b,t} = 1 - w^M_{g,t}$ because the bond is in zero net supply.

portfolio).[40]

Overall, and even though these time variations in portfolios are not the focus of our empirical analysis in Section 4, they could provide interesting avenues for further tests and research.

4 Empirical Evidence

In this section, we provide empirical evidence for the effect of the consumption premia on asset returns. Strongly supporting our theoretical results, we find that, in recent years, the annual relative-supply consumption premium on a basket of green assets has steadily increased to 30 to 40 bps higher than that on a basket of brown assets, with a highly significant price of risk. More generally, assets that can hedge shocks to the relative-price of green goods, regardless of their environmental ratings, carry lower returns of up to 1.5% annually.

4.1 Data and factor construction

We test the existence of the consumption premia empirically by estimating the beta-representation implied by the equilibrium equations for expected returns in Proposition 2. Given the strong negative monotonic relationship between the relative supply of the green good, y_t, and the relative price of the green consumption basket, $\mathcal{E}_t$ (discussed in Section 3.1 and shown in Panel (b) of Figure 2), we focus on estimating the

[40] They do so because of the heightened relative-supply hedging motives, and despite the fact that the common component should make them want to *decrease* their portfolio weight in that asset. The green investor also increases her weight on the green asset as y_t decreases because the impact of her preference for green assets is heightened by a strongly increasing correlation across assets. However, as the weight of the green asset in the market portfolio *decreases* at the same time, the green investor has to rely on an increasing amount of borrowing in the riskfree bond $|b_t^G|$ to tilt her risky portfolio as she desires in what she perceives as bad times (i.e., when y_t decreases).

following beta-representation for all assets j:[41]

$$\mu_{j,t} - r_t = \alpha_{j,t} + \lambda_{M,t}\beta_{j,M,t} + \lambda_{x,t}\beta_{j,x,t} + \lambda_{\mathcal{E},t}\beta_{j,\mathcal{E},t} + \lambda_{GMB,t}\beta_{j,GMB,t} + \varepsilon_{j,t}. \tag{16}$$

GMB_t is the green-minus-brown factor used in the literature (see, e.g., Pastor et al., 2022) to capture the green premium related to investors' preferences for green assets. It is discussed below. The quantities of risk are defined as

$$\begin{aligned} \beta_{j,x,t} &\equiv \frac{\text{cov}_t\left(dR_{j,t}, dx_t\right)}{\text{var}_t(dx_t)}\ ,\ \beta_{j,\mathcal{E},t} \equiv \frac{\text{cov}_t\left(dR_{j,t}, d\mathcal{E}_t\right)}{\text{var}_t(d\mathcal{E}_t)}, \\ \beta_{j,\widetilde{W},t} &\equiv \frac{\text{cov}_t\left(dR_{j,t}, dR_{M,t}\right)}{\text{var}_t(dR_{M,t})}\ ,\ \beta_{j,GMB,t} \equiv \frac{\text{cov}_t\left(dR_{j,t}, GMB_t\right)}{\text{var}_t(GMB_t)}, \end{aligned} \tag{17}$$

where $dR_{j,t}$ and $dR_{M,t}$ are the excess returns on asset j and the market, respectively. We refer to $d\mathcal{E}_t$ and dx_t as the relative-price factor and wealth factor, respectively. The theoretical expressions for the prices of risk $\lambda_{M,t}, \lambda_{x,t}, \lambda_{\mathcal{E},t}$, and $\lambda_{GP,t}$ can also be derived theoretically from Equation (12) in Proposition 2.

Given the magnitude of the consumption premium associated with the relative supply of the green good suggested by our theoretical results, and the strong negative relationship between y_t and $\mathcal{E}_t$, we expect the price of risk associated with the relative price of green goods, $\lambda_{\mathcal{E},t}$, to be significantly negative. Indeed, the average investor in the economy values the assets for which returns are positively correlated with the prices of green goods, because those assets offer a hedge against those adverse states of the world. On the other hand, the small magnitude and the change in sign of the consumption premium associated with the wealth share of the green investor suggested by the model do not lead us to have a strong prior on the estimate of the

[41] Using prices as opposed to produced quantities has the benefit of being a much easier, cleaner, and higher frequency measure to come by.

price of risk $\lambda_{x,t}$, although we expect it to be small.

We start our analysis from all the common stocks (share type codes 10 and 11) listed on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX), and National Association of Securities Dealers Automated Quotations exchange (NASDAQ; exchange codes 1, 2, and 3) in the CRSP database. We then map them to the 6-digit North American Industry Classification System (NAICS).

We construct the relative-price factor using the carbon intensity of companies provided by S&P–Trucost, as sustainable consumers are primarily concerned with the climate footprint of their consumption (Schanes et al., 2016), in line with the goals of the Paris Agreement. The carbon intensity of a company is defined as the annual amount of greenhouse gases emitted by the company across its value chain, normalized by its annual revenues.[42] The firms producing the greenest (brownest) goods are, therefore, those with the lowest (highest) carbon intensity. Given our specific focus on supply shocks, we use the Producer Price Indexes constructed by the U.S. Bureau of Labor Statistics as proxies for the prices of goods. As those indexes are available by 6-digit NAICS industry only, we compute the carbon intensity of each industry as the market-value weighted carbon intensity of all firms in that industry.[43] After normalizing all prices to 100 in December 2006, we construct $\mathcal{E}_t$ as the ratio of the average production price of the 33% greenest industries to the average production

[42]We use Trucost's default emission scope, which includes direct and first-tier indirect emissions, that is, for a given firm, the emissions related to the its activity (scope 1), induced by the generation of its purchased energy (scope 2), and those of its suppliers (upstream scope 3).

[43]When a price index is not available for a given N-digit NAICS industry, we use the price index for the N-1-digit industry that includes it.

price of all industries in the economy for each month:

$$\mathcal{E}_t \equiv \frac{\frac{1}{|\Omega_{33\%,t}|} \sum_{i \in \Omega_{33\%,t}} P_t^i}{\frac{1}{|\Omega_t|} \sum_{i \in \Omega_t} P_t^i},$$

where Ω_t and $\Omega_{33\%,t}$ stand for the set of all industries and the set of the 33% greenest industries in t, respectively. $|\Omega_t|$ and $|\Omega_{33\%,t}|$ are the cardinalities of Ω_t and $\Omega_{33\%,t}$, respectively. From the model's perspective, using this ratio corresponds to defining the goods produced by the 33% greenest firms as the green good. In turn, the factor $d\mathcal{E}_t$ is defined as the change in $\mathcal{E}_t$ between two consecutive months. We also refer to $d\mathcal{E}_t$ as the $Price\_Hedge_t$ factor.

Following Zerbib (2022), we construct the green investor wealth share factor by first identifying 453 funds whose asset management mandates include environmental guidelines (flagged as "environmentally friendly," "climate change," and "clean energy"), of which the investment asset classes are defined as "equity," "mixed allocation," and "alternative," and with the geographical investment scope including the United States, using data from Bloomberg as of December 2019. We obtain their assets under management on a quarterly basis using FactSet, and for each quarter, we compute the ratio of the market value of the U.S. stocks in the 453 green funds to the market value of their investment universe. We then interpolate this ratio for each month using a polynomial of degree 2, and construct x_t by smoothing this interpolation.[44] The variable x_t approximates the wealth dynamics of U.S. investors with pro-environmental preferences, and the factor dx_t is defined as the change in x_t between two consecutive months. We also refer to dx_t as the $Wealth\_Hedge_t$ factor.

[44] We filter out from the series the seasonal component, which we calculate in the following standard way. First, we extract the trend of the series using a convolution filter. Then, we remove the trend from the series, and we calculate the seasonal component as the average of the detrended series for each period.

We construct the green factor (also referred to as green-minus-brown factor, GMB_t, in the literature) using the environmental rating provided by MSCI. We do not use carbon intensity data for this purpose because building an asset allocation by minimizing its carbon intensity leads to skewing the portfolio towards a few low-emitting sectors (e.g., banking, insurance, business services, entertainment, healthcare, telecommunications), which poses a dual problem for green investors: (*i*) some key sectors for the ecological transition, with higher emissions, are left out of the allocation (e.g., utilities, electrical equipment, construction materials), and (*ii*) the portfolio loses much in sectoral diversification. This is why other environmental metrics are often used by green investors in combination with the carbon intensity, such as environmental ratings, the green share (Mirova, 2021) or the portfolio alignment to a temperature trajectory (Raynaud et al., 2020). In addition, MSCI is the world's largest provider of ESG ratings (Eccles and Stroehle, 2020) and covers more firms than the other ESG raters (Berg et al., 2022). In practice, we closely follow Pastor et al. (2022) and construct the green factor (GMB_t) as a green minus brown value-weighted portfolio that is long on the tercile of the greenest firms and short on the tercile of the brownest firms, excluding firms without ratings. Given the fact that green investing rose only recently (Zerbib, 2022) and the availability of environmental ratings, we begin our analysis in March 2006.

Finally, we proxy for the market component by using excess returns on the market, that is, the standard market factor (market return minus riskfree rate) from Fama and French (2015). In the estimations, as is usual, we also control for the small-minus-big (SMB), high-minus-low (HML), conservative-minus-aggressive (CMA), robust-minus-weak (RMW) factors (Fama and French, 2015), and the momentum (MOM) factor (Carhart, 1997). We obtain all those factors from Kenneth French's website.[45]

[45] The URL is: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_

All in all, we work with a scope of 3388 stocks and estimate the specification of Equation (16) using a two-pass (Fama and MacBeth, 1973) regression from March 2006 to December 2019. In the second pass, we run cross-sectional regressions of the time-series average of each asset returns on a constant and the betas. The returns are winsorized at the 1% level. Table 1 provides the descriptive statistics of the variables used.

Table 1: Summary statistics (%, monthly)

Variable	$\mu_{j,t} - r_t$	$Price\_Hedge_t$	$Wealth\_Hedge_t$	GMB_t	$Mkt - RF_t$	HML_t	SMB_t	RMW_t	CMA_t	MOM_t
Mean	0.185	-0.010	0.001	0.118	0.736	-0.141	0.102	0.266	0.019	0.045
Standard deviation	12.052	0.097	0.001	2.241	4.124	2.581	2.393	1.606	1.412	4.570
Min	-36.112	-0.377	-0.002	-5.648	-17.230	-11.110	-4.920	-3.880	-3.230	-34.300
25th percentile	-6.216	-0.048	0.000	-1.103	-1.270	-1.670	-1.710	-0.660	-1.010	-1.550
Median	-0.036	0.000	0.001	0.220	1.060	-0.310	0.180	0.340	-0.010	0.300
75th percentile	6.082	0.019	0.001	1.662	3.130	1.120	1.610	1.250	0.900	2.560
Max	42.961	0.330	0.003	9.476	11.350	8.210	7.040	4.940	3.700	12.750
Count	359,969	359,969	359,969	359,969	359,969	359,969	359,969	359,969	359,969	359,969

Notes: $Price\_Hedge_t$ and $Wealth\_Hedge_t$ refer to $d\mathcal{E}_t$ and dx_t in the main text, respectively. Cf. Section 4.1.

4.2 Estimation

Consistent with the characterization of the model, the results of the estimation strongly support the existence of the relative-price consumption premium in the cross-section of stock returns. They are summarized in Table 2.

First, the price of risk, $\widehat{\lambda}_{\mathcal{E}}$, associated with the relative price of green goods is negative and highly significant in all estimated specifications (Panel A, specifications [1] to [8]): it ranges from -0.7 bps to -2.6 bps per month with t-stats ranging from -2.7 to -3.7.

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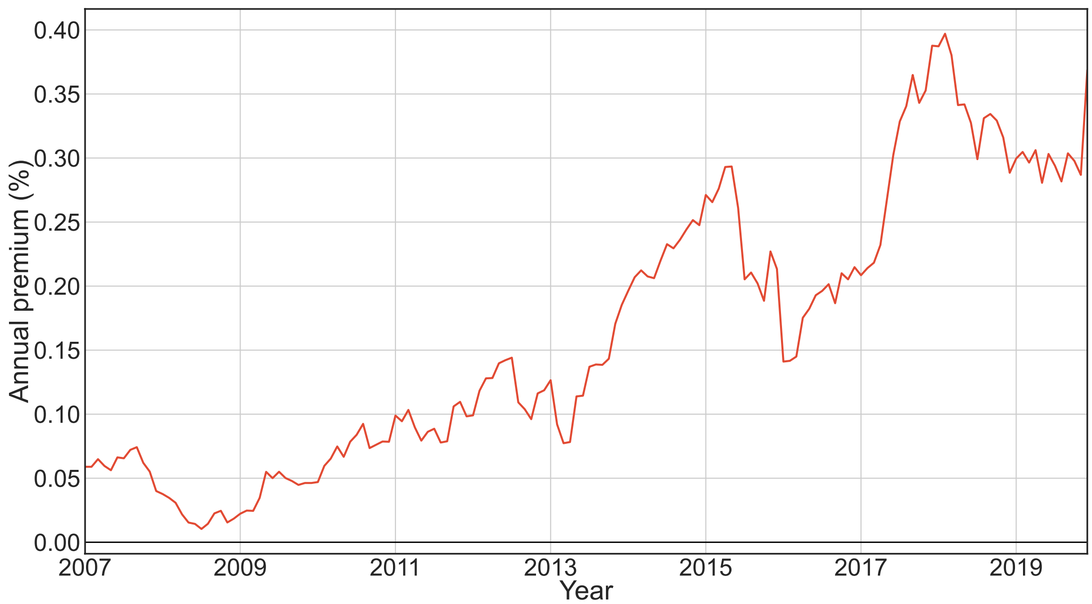
Table 2: Empirical estimation of consumption premia

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Prices of risk ($\widehat{\lambda}$, monthly, %)								
$Price\_Hedge_t$	-0.022	-0.021	-0.026	-0.009	-0.009	-0.007	-0.008	-0.007
	(-3.355)	(-3.002)	(-3.682)	(-3.733)	(-3.126)	(-2.659)	(-2.75)	(-2.829)
$Wealth\_Hedge_t$		0.000	0.000	0.000	0.000	0.000	0.000	0.000
		(1.616)	(1.008)	(1.371)	(2.763)	(2.573)	(1.68)	(0.576)
GMB_t			0.135	0.102	0.194	0.218	0.087	0.152
			(2.353)	(1.951)	(2.727)	(3.5)	(1.438)	(2.578)
Panel B: Premia difference (annual, %)								
$Price\_Hedge_t$	-1.245	-1.217	-1.483	-0.953	-0.929	-0.802	-0.801	-0.798
$Wealth\_Hedge_t$		0.325	0.202	0.269	0.328	0.335	0.203	0.073
GMB_t			0.938	0.705	1.365	1.505	0.604	1.094
Controls				CAPM	FF3	FF3MOM	FF5	FF5MOM

Notes: The $Price\_Hedge_t$ and $Wealth\_Hedge_t$ factors refer to $d\mathcal{E}_t$ and dx_t in the main text, respectively. Variables are defined in Section 4.1. Newey-West t-stats are in parenthesis. Full sample: Mar. 2006-Dec. 2019. Returns are winsorized at the 1% level. The premium difference for each variable is computed as the annual premium on assets with a loading in the 33$^{\text{rd}}$ highest percentile (high $\hat{\beta}$), minus that on assets with a loading in the 33% lowest percentile (low $\hat{\beta}$).

Second, as they provide a hedge against an increase in the prices of green goods, the assets that covary most with the relative-price factor (representing the change in relative prices of green goods), $d\mathcal{E}_t$, have a lower risk premium than the assets that covary least or negatively with it. Panel B gives the difference in the risk premium between the assets with loading in the 33$^{\text{rd}}$ percentile (high $\widehat{\beta}_{\mathcal{E}}$) and those with loading in the 66$^{\text{th}}$ percentile (low $\widehat{\beta}_{\mathcal{E}}$). The difference in risk premium is substantial: it ranges from -80 bps to -1.48% per year depending on the specification considered, and suggests that investors value assets whose payoffs are large when green goods are expensive.

Figure 7: Relative-price premium differential between green and brown assets



Notes: This figure depicts the difference in relative-price premium between the assets with the highest environmental rating (top 33%) and those with the lowest environmental rating (bottom 33%), $(\widehat{\beta}_{g,\mathcal{E},t} - \widehat{\beta}_{b,\mathcal{E},t})\widehat{\lambda}_{\mathcal{E}}$.

Third, focusing on the MSCI environmental score of the assets, the 33% greenest assets had a lower average beta than the 33% brownest assets across all months between March 2006 and December 2019. The beta gap widened, and between 2017 and 2019, the average beta of green assets reached 3 units lower than that of brown assets, that is, $\widehat{\beta}_{g,\mathcal{E},t} - \widehat{\beta}_{b,\mathcal{E},t} \approx -3$. As a result, the risk premium between the 33% greenest assets and the 33% brownest assets, $(\widehat{\beta}_{g,\mathcal{E},t} - \widehat{\beta}_{b,\mathcal{E},t})\widehat{\lambda}_{\mathcal{E}}$, has been positive throughout the sample, and gradually increased to reach 30 bps to 40 bps between

2017 and 2020, as shown on Figure 7.[46] This effect strongly supports the predictions of the model: brown assets provide a financial hedge against the risk of rising green consumption good prices, and conversely, green assets are riskier from the perspective of consumption premia. As such, the steady growth of this premium and its current level could be consistent with a recent and growing awareness of this effect by green investors.

Regarding the wealth consumption premium, we do not find convincing evidence of a significant price of risk across all specifications (t-stats ranging from 1 to 2.8), which is consistent with the mixed and more muted results in the model.

Finally, the price of risk associated with the GMB factor is significant for almost all specifications, but contrary to what the theory predicts in equilibrium, the premium for the 33% greenest firms is higher than that for the 33% brownest firms by 70 bps to 1.50% per year. As documented by Pastor et al. (2021) and Bolton and Kacperczyk (2022), this effect is due to the unexpected increase in investors' preferences for green assets over the last few years, which has pushed up the realized returns and does not permit the capture of green premium on expected returns. Several papers have developed methods to control for this effect (e.g., Ardia et al., 2021; Pastor et al., 2022; Sautner et al., 2022; Zerbib, 2022). For example, Pastor et al. (2022) find that the green premium on U.S. equities—corresponding to the difference between the premium on the expected returns of green stocks and the premium on the expected returns of brown stocks—ranges from −0.50% to −2% between 2013 and 2020. Zerbib (2022) estimates the premium at an average of −1.50% between 2013 and 2019. As a result, the 30 bps to 40 bps relative-price premium that we estimate between 2017

[46] We do not plot the differences in beta itself, $\widehat{\beta}_{g,\mathcal{E},t} - \widehat{\beta}_{b,\mathcal{E},t}$, because it is simply the mirror image of the risk premium in Figure 7 given that the price of risk, $\widehat{\lambda}_{\mathcal{E}}$, is constant. The gap widened from $\widehat{\beta}_{g,\mathcal{E},t} - \widehat{\beta}_{b,\mathcal{E},t} \approx -0.5$ in 2007 to $\widehat{\beta}_{g,\mathcal{E},t} - \widehat{\beta}_{b,\mathcal{E},t} \approx -3$ in 2019.

and 2019 accounts for 15% to 80% of the green premium estimated by Pastor et al. (2022), thereby significantly counterbalancing its effect.

Overall, the results of our estimations strongly support the model predictions. Through the relative-price factor, the consumption premia can offset a substantial part of the green premium. As such, the consumption premia, related to pro-environmental preferences for green goods, help explain the limited effect of green investing on the cost of capital of brown firms as discussed in Section 5.

5 Implications for Impact Investing

Impact investing covers several investment strategies that aim at encouraging companies to change their practices. By inducing a green premium that increases the cost of capital of polluting companies, investors' preferences for green assets are supposed to incentivize companies to mitigate their environmental footprints. Yet, empirical evidence suggests that the real impact is low. De Angelis et al. (2022) find that by internalizing the climate externalities of the companies in which they invest, green investors drive companies to reduce their carbon footprint at a substantially low rate, in the range of 1% to 3% per year. In addition, Oehmke and Opp (2019), Landier and Lovo (2020), and Green and Roth (2020) emphasize that green investors do not maximize their global impact by internalizing only the environmental footprints of the companies in which they invest.[47]

Our findings have dual implications from the perspective of impact investing. First, by showing that the green premium is counterbalanced by green investors' pref-

[47] The impact is larger when they internalize the environmental footprints of all firms in the economy, irrespective of whether they invest in them (Oehmke and Opp, 2019; Green and Roth, 2020), and by prioritizing firms where the inefficiencies induced by the externalities are particularly acute and the capital search frictions are strong (Landier and Lovo, 2020).

erences for green consumption, we contribute to explaining why the impact of green investors on the cost of capital and practices of polluting firms is limited. Second, the overweighting of polluting companies in green investors' portfolios is an opportunity to leverage their shareholder position so as to increase their engagement with these companies (e.g., private or public communications, votes in general assemblies) and push them to become greener. This conclusion reinforces the findings of Broccardo et al. (2020) who suggest that shareholder engagement is often more effective than green investment without accounting for consumption preferences.

Be it to accelerate the ecological transition in general, or specifically to mitigate the effect of the consumption premia on firms' cost of capital, policymakers have different options, such as capping green good prices or introducing a dividend tax. For example, we can show (see Sauzet, 2022a, for details) that when investors pay a tax τ on dividends from brown firms, the expected returns are written as follows:

$$\begin{aligned}\mu_{g,t} - r_t &= \gamma\sigma_{j,t}^{\top}\sigma_{\widetilde{W},t} - \gamma\sigma_{g,t}^{\top}\sigma_{\tilde{J},t} - x_t\phi^G, \\ \mu_{b,t} - r_t &= \gamma\sigma_{j,t}^{\top}\sigma_{\widetilde{W},t} - \gamma\sigma_{b,t}^{\top}\sigma_{\tilde{J},t} + \tau F_{b,t}.\end{aligned} \tag{18}$$

Tax on the dividends counterbalances the consumption premia on the assets of brown firms through $\tau F_{b,t}$ and hence, increases their cost of capital. From a quantitative viewpoint, dividend taxation has a substantial impact on expected returns if the dividend yields are sufficiently high, or equivalently, if asset prices at a given dividend level are sufficiently low, that is, when firms' cost of capital is high.[48] Therefore, introducing a dividend tax is all the more effective because the brownest companies are subject to transition risks (environmental regulations, carbon prices increases,

[48]Technically, this occurs for instance when the elasticity of intertemporal substitution is not too large.

changes in consumer preferences, technological and reputational risks, etc.), which increase their cost of capital relative to green companies.

6 Conclusion

In this paper, we show how investors' preferences for green consumption substantially moderate the effect of the green premium associated with their preferences for green assets on expected asset returns. Indeed, green assets are riskier from the perspective of consumption premia, while brown assets provide a financial hedge against states in which green goods are expensive. In addition to being relevant for asset pricing and capital allocation, the main effect documented in this paper has the following implications for investors willing to contribute to the ecological transition: the increase in the cost of capital of brown firms is dampened as soon as green goods are subject to shocks that may increase their relative prices. Thus, the allocation of a larger share of green investors' capital to brown firms could provide a welcome opportunity to reinforce their engagement with the most polluting firms.

The construction of general equilibrium models in sustainable finance, as we propose in this paper, opens up multiple avenues for future research. For instance, it could allow to study the effects of stochastic preferences for green investments and demand for green consumption on firms' cost of capital and investors' wealth allocation. It would also be valuable to analyze alternative forms of investments and account for shareholder engagement with a view to maximizing investors' impacts in a general equilibrium model. Another promising avenue is to include environment-related financial risks (van den Bremer and van der Ploeg, 2021; Hambel et al., 2022; Barnett, 2022) and production into a general equilibrium model that features green consumption and investment preferences.[49] Finally, constructing portfolios that are hedged against several types of risks, notably climate risks but also the risk of a rise in green good prices, by building on Engle et al. (2020) and Alekseev et al. (2021), constitute an interesting direction for future work.

[49] We are exploring stochastic demand and production economies in ongoing work.

References

Albuquerque, Rui, Yrjö Koskinen, and Chendi Zhang, 2019, Corporate Social Responsibility and Firm Risk: Theory and Empirical Evidence, *Management Science* 65, 4451–4469.

Alekseev, Georgij, Stefano Giglio, Quinn Maingi, Julia Selgrad, and Johannes Stroebel, 2021, A quantity-based approach to constructing climate risk hedge portfolios, *Working Paper* .

Alok, Shashwat, Nitin Kumar, and Russell Wermers, 2020, Do fund managers misestimate climatic disaster risk, *Review of Financial Studies* 33, 1146–1183.

Anderson, Robert M., and Roberto C. Raimondo, 2008, Equilibrium in continuous-time financial markets: Endogenously dynamically complete markets, *Econometrica* 76, 841–907.

Ardia, David, Keven Bluteau, Kris Boudt, and Koen Inghelbrecht, 2021, Climate change concerns and the performance of green versus brown stocks, *Working Paper,* SSRN.

Avramov, Doron, Abraham Lioui, Yang Liu, and Andrea Tarelli, 2021, Dynamic ESG Equilibrium, *Working Paper,* SSRN.

Baker, M., D. Bergstresser, G. Serafeim, and J. Wurgler, 2018, Financing the Response to Climate Change: The Pricing and Ownership of U.S. Green Bonds, *Working Paper,* NBER.

Baldauf, Markus, Lorenzo Garlappi, Constantine Yannelis, and José Scheinkman, 2020, Does climate change affect real estate prices? only if you believe in it, *Review of Financial Studies* 33, 1256–1295.

Bansal, Ravi, Dana Kiku, and Juan Ochoa, 2016, Price of long-run temperature shifts in capital markets, *Working Paper,* NBER.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *The Journal of Finance* 59, 1481–1509.

Barber, Brad M., Adair Morse, and Ayako Yasuda, 2021, Impact investing, *Journal of Financial Economics* 139, 162–185.

Barnett, Michael, 2022, Climate Change and Uncertainty: An Asset Pricing Perspective, *Working Paper,* Stanford Libraries.

Barro, Robert J., and Sanjay Misra, 2016, Gold Returns, *The Economic Journal* 126, 1293–1317.

Basak, Suleyman, and Domenico Cuoco, 1998, An Equilibrium Model with Restricted Stock Market Participation, *The Review of Financial Studies* 11, 309–341.

Berg, Florian, Julian F Kölbel, and Roberto Rigobon, 2022, Aggregate Confusion: The Divergence of ESG Ratings, *Review of Finance* rfac033.

Berk, Jonathan B., and Jules H. van Binsbergen, 2021, The Impact of Impact Investing, *Working Paper,* SSRN.

Bernstein, Asaf, Matthew T. Gustafson, and Ryan Lewis, 2019, Disaster on the horizon: The price effect of sea level rise, *Journal of Financial Economics* 134, 253–272.

Berrada, Tony, Julien Hugonnier, and Marcel Rindisbacher, 2007, Heterogeneous preferences and equilibrium trading volume, *Journal of Financial Economics* 83, 719 – 750.

Bhamra, Harjoat S., and Raman Uppal, 2009, The Effect of Introducing a Non-Redundant Derivative on the Volatility of Stock-Market Returns When Agents Differ in Risk Aversion, *The Review of Financial Studies* 22, 2303–2330.

Bhamra, Harjoat S., and Raman Uppal, 2014, Asset Prices with Heterogeneity in Preferences and Beliefs, *The Review of Financial Studies* 27, 519–580.

Bolton, Patrick, and Marcin Kacperczyk, 2021, Do investors care about carbon risk?, *Journal of Financial Economics* 142, 517–549.

Bolton, Patrick, and Marcin Kacperczyk, 2022, Global pricing of carbon-transition risk, *Working Paper,* SSRN.

Borovička, Jaroslav, 2020, Survival and long-run dynamics with heterogeneous beliefs under recursive preferences, *Journal of Political Economy* 128, 206–251.

Broccardo, Eleonora, Oliver Hart, and Luigi Zingales, 2020, Exit vs. Voice, *Working Paper,* NBER.

Brunnermeier, Markus, and Yuliy Sannikov, 2014, A macroeconomic model with a financial sector, *American Economic Review* 104, 379–421.

Brunnermeier, Markus K., and Lasse Heje Pedersen, 2009, Market liquidity and funding liquidity, *The Review of Financial Studies* 22, 2201–2238.

Cai, Yongyang, and Thomas S. Lontzek, 2019, The Social Cost of Carbon with Economic and Climate Risks, *Journal of Political Economy* 127, 2684–2734.

Carhart, M. M., 1997, On Persistence in Mutual Fund Performance, *Journal of Finance* 52, 57–82.

Chabakauri, Georgy, 2013, Dynamic Equilibrium with Two Stocks, Heterogeneous Investors, and Portfolio Constraints, *The Review of Financial Studies* 26, 3104–3141.

Chan, Yeung Lewis, and Leonid Kogan, 2002, Catching up with the joneses: Heterogeneous preferences and the dynamics of asset prices, *Journal of Political Economy* 110, 1255–1285.

Chava, S., 2014, Environmental externalities and cost of capital, *Management Science* 60, 2223–2247.

Cochrane, John H., Francis A. Longstaff, and Pedro Santa-Clara, 2008, Two Trees, *The Review of Financial Studies* 21, 347–385.

Coeurdacier, Nicolas, 2009, Do trade costs in goods market lead to home bias in equities?, *Journal of International Economics* 77, 86 – 100.

Coeurdacier, Nicolas, and Hélène Rey, 2013, Home bias in open economy financial macroeconomics, *Journal of Economic Literature* 51, 63–115.

De Angelis, Tiziano, Peter Tankov, and Olivier David Zerbib, 2022, Climate Impact Investing, *Management Science,* Forthcoming.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, 2018, A model of monetary policy and risk premia, *The Journal of Finance* 73, 317–373.

Duffie, D., J. Geanakoplos, A. Mas-Colell, and A. McLennan, 1994, Stationary markov equilibria, *Econometrica* 62, 745–781.

Duffie, Darrell, and Larry Epstein, 1992, Stochastic differential utility, *Econometrica* 60, 353–94.

Dumas, Bernard, 1989, Two-Person Dynamic Equilibrium in the Capital Market, *The Review of Financial Studies* 2, 157–188.

Dumas, Bernard, 1992, Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World, *The Review of Financial Studies* 5, 153–180.

Dumas, Bernard, and Raman Uppal, 2001, Global Diversification, Growth, and Welfare with Imperfectly Integrated Markets for Goods, *The Review of Financial Studies* 14, 277–305.

Dumas, Bernard, Raman Uppal, and Tan Wang, 2000, Efficient intertemporal allocations with recursive utility, *Journal of Economic Theory* 93, 240–259.

Eccles, Robert G., and Judith C. Stroehle, 2020, Exploring social origins in the construction of esg measures, *Working Paper,* SSRN.

Ehling, Paul, and Christian Heyerdahl-Larsen, 2015, Complete and incomplete financial markets in multi-good economies, *Journal of Economic Theory* 160, 438 – 462.

Ehling, Paul, and Christian Heyerdahl-Larsen, 2017, Correlations, *Management Science* 63, 1919–1937.

ElGhoul, S., O. Guedhami, C. C.Y. Kowk, and D. R. Mishra, 2011, Does corporate social responsibility affect the cost of capital?, *Journal of Banking and Finance* 35, 2388–2406.

Engle, Robert F, Stefano Giglio, Bryan Kelly, Heebum Lee, and Johannes Stroebel, 2020, Hedging Climate Change News, *Review of Financial Studies* 33, 1184–1216.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fama, Eugene F., and James D. MacBeth, 1973, Risk, return and equilibrium: Empirical tests, *Journal of Political Economy* 81, 607–636.

Fang, Xiang, 2019, Intermediary leverage and currency risk premium, *Working Paper,* SSRN.

Gârleanu, Nicolae, and Stavros Panageas, 2015, Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing, *Journal of Political Economy* 123, 670 – 685.

Gârleanu, Nicolae, and Lasse Heje Pedersen, 2011, Margin-based asset pricing and deviations from the law of one price, *The Review of Financial Studies* 24, 1980–2022.

Geanakoplos, John, 1990, An introduction to general equilibrium with incomplete asset markets, *Journal of Mathematical Economics* 19, 1 – 38.

Geanakoplos, John, and Andreu Mas-Colell, 1989, Real indeterminacy with financial assets, *Journal of Economic Theory* 47, 22 – 38.

Geanakoplos, John D, and Heraklis M Polemarchakis, 1986, Existence, regularity, and constrained suboptimality of competitive allocations, *Essays in Honor of Kenneth J. Arrow: Volume 3, Uncertainty, Information, and Communication* 3, 65.

Giglio, Stefano, Matteo Maggiori, Krishna Rao, Johannes Stroebel, Andreas Weber, and Stijn Van Nieuwerburgh, 2021, Climate Change and Long-Run Discount Rates: Evidence from Real Estate [Abrupt climate change], *Review of Financial Studies* 34, 3527–3571.

Goldsmith-Pinkham, Paul, Matthew T. Gustafson, Ryan C. Lewis, and Michael Schwert, 2021, Sea Level Rise Exposure and Municipal Bond Yields, *Working Paper,* SSRN.

Goldstein, Itay, Alexandr Kopytov, Lin Shen, and Haotian Xiang, 2021, On esg investing: Heterogeneous preferences, information, and asset prices, *Working Paper,* SSRN.

Gollier, Christian, 2010, Ecological discounting, *Journal of Economic Theory* 145, 812–829, Judgment Aggregation.

Gollier, Christian, 2019, Valuation of natural capital under uncertain substitutability, *Journal of Environmental Economics and Management* 94, 54–66.

Golosov, Mikhail, John Hassler, Per Krusell, and Aleh Tsyvinski, 2014, Optimal Taxes on Fossil Fuel in General Equilibrium, *Econometrica* 82, 41–88.

Green, Daniel, and Benjamin Roth, 2020, The allocation of socially responsible capital, *Working Paper,* SSRN.

Guesnerie, Roger, 2004, Calcul économique et développement durable, *Revue économique* 55, 363–382.

Hambel, Christoph, Holger Kraft, and Frederick van der Ploeg, 2022, Asset Diversification versus Climate Action, *Working Paper,* SSRN.

Hart, Oliver D, 1975, On the optimality of equilibrium when the market structure is incomplete, *Journal of Economic Theory* 11, 418 – 443.

Hartzmark, Samuel, and Abigail Sussman, 2019, Do Investors Value Sustainability? A Natural Experiment Examining Ranking and Fund Flows, *Journal of Finance* 74, 2789–2837.

Heinkel, R., A. Kraus, and J. Zechner, 2001, The effect of green investment on corporate behaviour, *Journal of Financial and Quantitative Analysis* 36, 377–389.

Hoel, Michael, and Thomas Sterner, 2007, Discounting and relative prices, *Climatic Change* 84, 265–280.

Hong, Harrison, Frank Weikai Li, and Jiangmin Xu, 2019, Climate risks and market efficiency, *Journal of Econometrics* 208, 265–281.

Hsu, Po-Hsuan, Kai Li, and Chi-Yang Tsou, 2022, The Pollution Premium, *Journal of Finance,* Forthcoming.

Hugonnier, J., S. Malamud, and E. Trubowitz, 2012a, Endogenous completeness of diffusion driven equilibrium markets, *Econometrica* 80, 1249–1270.

Hugonnier, Julien, Florian Pelgrin, and Pascal St-Amour, 2012b, Health and (Other) Asset Holdings, *The Review of Economic Studies* 80, 663–710.

Huynh, Thanh D., and Ying Xia, 2021, Climate Change News Risk and Corporate Bond Returns, *Journal of Financial and Quantitative Analysis* 56, 1985–2009.

Imbs, Jean, and Isabelle Méjean, 2015, Elasticity optimism, *American Economic Journal: Macroeconomics* 7, 43–83.

Krüger, Philipp, Zacharias Sautner, and Laura Starks, 2020, The Importance of Climate Risk for Institutional Investors, *Review of Financial Studies,* 33, 1067–1111.

Landier, Augustin, and Stefano Lovo, 2020, ESG Investing: How to Optimize Impact?, *Working Paper,* SSRN.

Lucas, Robert, 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–45.

Martin, Ian, 2013, The lucas orchard, *Econometrica* 81, 55–111.

Merton, Robert, 1973, An intertemporal capital asset pricing model, *Econometrica* 41, 867–87.

Mirova, 2021, Our approach to ESG assessment, Technical report, Mirova.

Morgan Stanley Institute for Sustainable Investing, 2019, Sustainable Signals: Individual Investor Interest Driven by Impact, Conviction and Choice, Technical report, Morgan Stanley.

Oehmke, Martin, and Marcus Opp, 2019, A Theory of Socially Responsible Investment, *Working Paper,* SSRN.

Painter, Marcus, 2020, An inconvenient cost: The effects of climate change on municipal bonds, *Journal of Financial Economics* 135, 468–482.

Papageorgiou, Chris, Marianne Saam, and Patrick Schulte, 2017, Substitution between Clean and Dirty Energy Inputs: A Macroeconomic Perspective, *The Review of Economics and Statistics* 99, 281–290.

Pastor, Lubos, Robert Stalbaugh, and Lucian Taylor, 2022, Dissecting Green Returns, *Journal of Financial Economics,* Forthcoming.

Pastor, Lubos, Robert F. Stambaugh, and Lucian A. Taylor, 2021, Sustainable Investing in Equilibrium, *Journal of Financial Economics,* Forthcoming.

Pavlova, Anna, and Roberto Rigobon, 2007, Asset Prices and Exchange Rates, *The Review of Financial Studies* 20, 1139–1180.

Pavlova, Anna, and Roberto Rigobon, 2008, The Role of Portfolio Constraints in the International Propagation of Shocks, *The Review of Economic Studies* 75, 1215–1256.

Pavlova, Anna, and Roberto Rigobon, 2010, An asset-pricing view of external adjustment, *Journal of International Economics* 80, 144 – 156, Special Issue: JIE Special Issue on International Macro-Finance.

Pedersen, Lasse Heje, Shaun Fitzgibbons, and Lukasz Pomorski, 2021, Responsible Investing: The ESG-Efficient Frontier, *Journal of Financial Economics,* Forthcoming.

Pindyck, Robert S., and Neng Wang, 2013, The economic and policy consequences of catastrophes, *American Economic Journal: Economic Policy* 5, 306–39.

Polemarchakis, Heraklis M, 1988, Portfolio choice, exchange rates, and indeterminacy, *Journal of Economic Theory* 46, 414–421.

Raynaud, Julie, Peter Tankov, and Stéphane Voisin, 2020, Portfolio Alignment to a 2°C Trajectory: Science or Art?, *Working Paper,* SSRN.

Riedl, Arno, and Paul Smeets, 2017, Why Do Investors Hold Socially Responsible Mutual Funds?, *Journal of Finance* 72, 2505–2550.

Sautner, Zacharias, Laurence van Lent, Grigory Vilkov, and Ruishen Zhang, 2022, Pricing Climate Change Exposure, *Management Science,* Forthcoming.

Sauzet, Maxime, 2022a, Two investors, two trees, two goods, Working paper.

Sauzet, Maxime, 2022b, Asset prices, global portfolios, and the international financial system, Working paper.

Sauzet, Maxime, 2022c, Projection methods via neural networks for continuous-time models, Working paper.

Schanes, Karin, Stefan Giljum, and Edgar Hertwich, 2016, Low carbon lifestyles: A framework to structure consumption strategies and options to reduce carbon footprints, *Journal of Cleaner Production* 139, 1033–1043.

Seltzer, Lee, Laura T. Starks, and Qifei Zhu, 2022, Climate Regulatory Risks and Corporate Bonds, *Working Paper,* SSRN.

Stathopoulos, Andreas, 2017, Asset Prices and Risk Sharing in Open Economies, *The Review of Financial Studies* 30, 363–415.

Sterner, Thomas, and U. Martin Persson, 2008, An even sterner review: Introducing relative prices into the discounting debate, *Review of Environmental Economics and Policy* 2, 61–76.

Traeger, Christian P., 2011, Sustainability, limited substitutability, and non-constant social discount rates, *Journal of Environmental Economics and Management* 62, 215–228.

van den Bremer, Ton S., and Frederick van der Ploeg, 2021, The Risk-Adjusted Carbon Price, *American Economic Review* 111, 2782–2810.

Wang, Jiang, 1996, The term structure of interest rates in a pure exchange economy with heterogeneous investors, *Journal of Financial Economics* 41, 75 – 110.

Weinbaum, David, 2009, Investor heterogeneity, asset pricing and volatility dynamics, *Journal of Economic Dynamics and Control* 33, 1379 – 1397.

Zapatero, Fernando, 1995, Equilibrium asset prices and exchange rates, *Journal of Economic Dynamics and Control* 19, 787–811.

Zerbib, Olivier David, 2019, The effect of pro-environmental preferences on bond prices: Evidence from green bonds, *Journal of Banking and Finance* 98, 39–60.

Zerbib, Olivier David, 2022, A Sustainable Capital Asset Pricing Model (S-CAPM): Evidence from green investing and sin stock exclusion, *Review of Finance,* Forthcoming.

Appendix

A Additional theoretical results

A.1 Drift and diffusion terms for any variable

Remark A.1. *By Itô's Lemma, the geometric drift and diffusion term for any function $g_t = g(X_t)$ are given by:*

$$\frac{dg_t}{g_t} = \frac{dg(X_t)}{g(X_t)} \equiv \mu_{g,t}dt + \sigma_{g,t}^{\top} d\vec{Z}_t \tag{A.1}$$

where:

$$\mu_{g,t} = \frac{g_{x,t}}{g_t} x_t \mu_{x,t} + \frac{g_{y,t}}{g_t} y_t \mu_{y,t} + \frac{1}{2}\frac{g_{xx,t}}{g_t} x_t^2 \sigma_{x,t}^{\top}\sigma_{x,t} + \frac{1}{2}\frac{g_{yy,t}}{g_t} y_t^2 \sigma_{y,t}^{\top}\sigma_{y,t} + \frac{g_{xy,t}}{g_t} x_t y_t \sigma_{x,t}^{\top}\sigma_{y,t} \tag{A.2}$$

$$\sigma_{g,t} = \frac{g_{x,t}}{g_t} x_t \sigma_{x,t} + \frac{g_{y,t}}{g_t} y_t \sigma_{y,t} \tag{A.3}$$

This result is used repeatedly throughout the paper.

As a point of notation, recall that for any function g, g_t simply denotes $g(X_t)$, not the time-derivative of g (which is zero because the model is stationary due to infinite horizon). $g_{x,t}, g_{y,t}, g_{xx,t}, g_{yy,t}, g_{xy,t}$ denote the partial derivatives of $g(X_t)$.

A.2 Returns, and risk premia

The (geometric) drifts and diffusion terms for asset returns are obtained from Itô's Lemma and are as follows, for $j \in \{g, b\}$

$$
\begin{aligned}
dR_{j,t} &= \mu_{j,t} dt + \sigma_{j,t}^{\top} d\vec{Z}_t \qquad \text{(A.4)} \\
&\equiv \left(F_{j,t} + \mu_{p_j,t} + \mu_{Y_j} + \sigma_{p_j,t}^{\top} \sigma_{Y_j} - \mu_{F_j,t} + \sigma_{F_j,t}^{\top} \sigma_{F_j,t} - \left(\sigma_{p_j,t} + \sigma_{Y_j} \right)^{\top} \sigma_{F_j,t} \right) dt \\
&+ \left(\sigma_{p_j,t} + \sigma_{Y_j} - \sigma_{F_j,t} \right)^{\top} d\vec{Z}_t
\end{aligned}
$$

where $\mu_{p_j,t}$, $\mu_{F_j,t}$, $\sigma_{p_j,t}$, $\sigma_{F_j,t}$ are obtained using Remark A.1 above.

Proposition A.1 generalizes Proposition 2 to the case in which investors have different risk aversions, $\gamma^G \neq \gamma^N$, different elasticity of intertemporal substitutions, $\psi^G \neq \psi^N$, and in which both investors have preferences towards both assets, $\phi_j^i \neq 0$ for $i \in \{G, N\}$, $j \in \{g, b\}$. In that case, the economy-wide risk aversion also becomes state-dependent, γ_t. This poses no particular problem for the resolution, as our method allows for any value of the parameters. Exploring additional asymmetries stemming from those could be an interesting avenue for future work.

Proposition A.1. *The expected risk premia on the green and brown equity assets are*

$$
\begin{aligned}
\mu_{g,t} - r_t &= \gamma_t \sigma_{g,t}^{\top} \sigma_{\widetilde{W},t} - \gamma_t \sigma_{g,t}^{\top} \sigma_{\widetilde{J},t} - \gamma_t \left(x_t \frac{\phi_g^G}{\gamma^G} + (1 - x_t) \frac{\phi_g^N}{\gamma^N} \right) \qquad \text{(A.5)} \\
\mu_{b,t} - r_t &= \gamma_t \sigma_{b,t}^{\top} \sigma_{\widetilde{W},t} - \gamma_t \sigma_{b,t}^{\top} \sigma_{\widetilde{J},t} - \gamma_t \left(x_t \frac{\phi_b^G}{\gamma^G} + (1 - x_t) \frac{\phi_b^N}{\gamma^N} \right)
\end{aligned}
$$

where $\widetilde{W}_t$ is the total wealth, $\widetilde{J}_t$ is the economy-wide marginal value of wealth, γ_t is the wealth-weighted risk aversion, $\sigma_{J^G,t}, \sigma_{J^N,t}$ are the geometric diffusion terms of J_t^G, J_t^N

obtained as in Remark A.1 above, and

$$
\begin{aligned}
\sigma_{\widetilde{W},t} &\equiv w^M_{g,t}\sigma_{g,t} + (1 - w^M_{g,t})\sigma_{b,t} \\
\sigma_{\tilde{J},t} &\equiv x_t \left(\frac{1}{\gamma^G}\right)\left(\frac{1-\gamma^G}{1-\psi^G}\right)\sigma_{J^G,t} + (1-x_t)\left(\frac{1}{\gamma^N}\right)\left(\frac{1-\gamma^N}{1-\psi^N}\right)\sigma_{J^N,t} \\
\gamma_t &\equiv \left(\frac{x_t}{\gamma^G} + \frac{1-x_t}{\gamma^N}\right)^{-1}
\end{aligned}
$$

A.3 Portfolios

Proposition A.2 generalizes Proposition 3 to the case in which investors have different risk aversions, $\gamma^G \neq \gamma^N$, different elasticity of intertemporal substitutions, $\psi^G \neq \psi^N$, and in which both investors have preferences towards both assets, $\phi_j^i \neq 0$ for $i \in \{G, N\}$, $j \in \{g, b\}$. In that case, the economy-wide risk aversion also becomes state-dependent, γ_t. This poses no particular problem for the resolution, as our method allows for any value of the parameters. Exploring additional asymmetries stemming from those could be an interesting avenue for future work.

Proposition A.2. *The optimal portfolios of the green and neutral investors $j \in \{G, N\}$ are given by*

$$\begin{pmatrix} w_{g,t}^G \\ w_{b,t}^G \end{pmatrix} = \frac{1}{\gamma^G} \left(\Sigma_t^\top \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t + \phi_g^G \\ \mu_{b,t} - r_t + \phi_b^G \end{pmatrix} + \left(\frac{1-\gamma^G}{1-\psi^G}\right) \Sigma_t^\top \left(\frac{J_{x,t}^G}{J_t^G} x_t \sigma_{x,t} + \frac{J_{y,t}^G}{J_t^G} y_t \sigma_{y,t} \right) \right\}$$
$$b_t^G = 1 - w_{g,t}^G - w_{b,t}^G \tag{A.6}$$

$$\begin{pmatrix} w_{g,t}^N \\ w_{b,t}^N \end{pmatrix} = \frac{1}{\gamma^N} \left(\Sigma_t^\top \Sigma_t\right)^{-1} \left\{ \begin{pmatrix} \mu_{g,t} - r_t + \phi_g^N \\ \mu_{b,t} - r_t + \phi_b^N \end{pmatrix} + \left(\frac{1-\gamma^N}{1-\psi^N}\right) \Sigma_t^\top \left(\frac{J_{x,t}^N}{J_t^N} x_t \sigma_{x,t} + \frac{J_{y,t}^N}{J_t^N} y_t \sigma_{y,t} \right) \right\}$$
$$b_t^N = 1 - w_{g,t}^N - w_{b,t}^N \tag{A.7}$$

where $w_{g,t}^i, w_{b,t}^i, b_t^i$ are the portfolio weights (as a share of wealth) allocated to the green equity asset, the brown equity asset, and the riskless bond, and $\Sigma_t \equiv \begin{bmatrix} \sigma_{g,t} & \sigma_{b,t} \end{bmatrix}$.

A.4 Equilibrium

The definition of the equilibrium is standard.

Definition 1. *A competitive equilibrium is a set of aggregate stochastic processes adapted to the filtration generated by $\vec{Z}$: the price of the equity asset $(Q_{g,t}, Q_{b,t})$, and the interest rate (r_t), together with a set of individual stochastic processes for each investor: consumption of each good $(C^G_{g,t}, C^G_{b,t}, C^N_{g,t}, C^N_{b,t})$, wealth (W^G_t, W^N_t), and portfolio shares $(w^G_{g,t}, w^N_{b,t}, w^G_{g,t}, w^N_{b,t})$, such that, given the output of the two endowment trees $(Y_{g,t}, Y_{b,t})$:*

1. *Given the aggregate stochastic processes, individual choices solve the investor optimization problem given in Section 2.*
2. *Markets clear.*
   (a) *Good markets:*

$$C^G_{g,t} + C^N_{g,t} = Y_{g,t} \tag{A.8}$$
$$C^G_{b,t} + C^N_{b,t} = Y_{b,t}$$

   (b) *Equity markets:*

$$w^G_{g,t} W^G_t + w^N_{g,t} W^N_t = Q_{g,t} \tag{A.9}$$
$$w^G_{b,t} W^G_t + w^N_{b,t} W^N_t = Q_{b,t}$$

Most importantly, as shown in Section 2.3 of the main text, the equilibrium can be recast as a stationary recursive Markovian equilibrium in which all variables of interest are expressed as a function of a pair of state variables $X_t \equiv (x_t, y_t)'$, whose dynamics are also solely a function of X_t. x_t is the wealth share of the green investor, and y_t is the relative supply of the green good.

A.5 Evolutions of the state variables

Due to the Markovian nature of the equilibrium, the laws of motion of the state variables underlie the dynamics of the economy. They are summarized in Proposition A.3.

Proposition A.3. *The laws of motion for the wealth share of the green investor x_t, and the relative supply of the green good y_t are*

$$\frac{dx_t}{x_t} \equiv \mu_{x,t}dt + \sigma_{x,t}^{\top} d\vec{Z}_t \tag{A.10}$$
$$\frac{dy_t}{y_t} \equiv \mu_{y,t}dt + \sigma_{y,t}^{\top} d\vec{Z}_t$$

where

$$\begin{aligned}
\mu_{x,t} &= \left(w_{g,t}^G - w_{g,t}^M\right)(\mu_{g,t} - r_t) + \left(w_{b,t}^G - w_{b,t}^M\right)(\mu_{b,t} - r_t) \\
&\quad + \left(F_{g,t} w_{g,t}^M + w_{b,t}^M F_{b,t}\right) - P_t^G c_t^G \\
&\quad - \left((w_{g,t}^G - w_{g,t}^M)\sigma_{g,t} + (w_{b,t}^G - w_{b,t}^M)\sigma_{b,t}\right)^{\top} \left(w_{g,t}^M \sigma_{g,t} + w_{b,t}^M \sigma_{b,t}\right) \\
\sigma_{x,t} &= \left(\left(w_{g,t}^G - w_{g,t}^M\right)\sigma_{g,t} + \left(w_{b,t}^M - w_{b,t}^M\right)\sigma_{b,t}\right) \\
\mu_{y,t} &= (1 - y_t)\left(\mu_{Y_g} - \mu_{Y_b}\right) - (1 - y_t)\left(\sigma_{Y_g} - \sigma_{Y_b}\right)^{\top}\left(y_t \sigma_{Y_g} + (1 - y_t)\sigma_{Y_b}\right) \\
\sigma_{y,t} &= (1 - y_t)\left(\sigma_{Y_g} - \sigma_{Y_b}\right)
\end{aligned}$$

and $w_{g,t}^M \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t}), w_{b,t}^M \equiv Q_{b,t}/(Q_{g,t} + Q_{b,t})$ are the weights of the green and brown equity assets in the market portfolio, with $w_{g,t}^M$ defined in Equation (6) and $w_{b,t}^M = 1 - w_{g,t}^M$ in equilibrium because the bond is zero net supply.

Figure A.1 show the drift and diffusion terms for y_t, the relative supply of the green good. They do not depend on the wealth share of the green investor x_t or on parameters beyond $\mu_{Y_g}, \mu_{Y_b}, \sigma_{Y_g}, \sigma_{Y_b}$, because y_t is purely determined by the outputs of the green and brown trees.

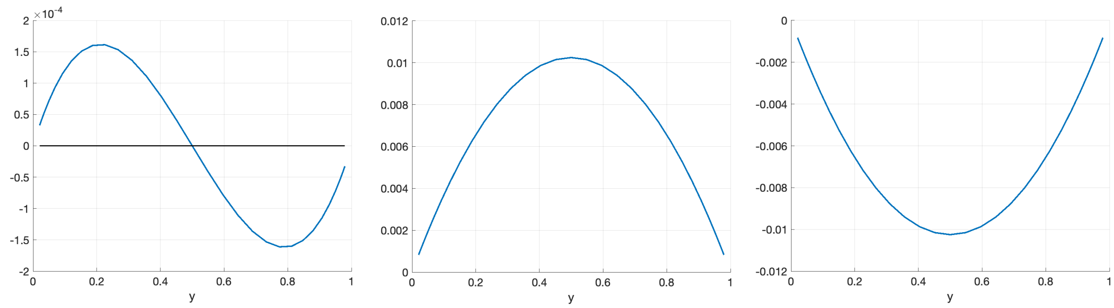
Figures A.2, A.3, A.4 show the drift and diffusion terms for x_t, the wealth share of the green investor, for various calibrations. As mentioned in the main text, the diffusion terms for x_t, and therefore the covariance between state variables, are inherently dependent on the portfolio bias of the green investor, which in turn depends strongly on her preference for green consumption (α) and green investing (ϕ).

Figure A.1: Drift and diffusion terms for the relative supply of the green good y_t

(a) Drift of y_t: $\mu_{y,t} y_t$

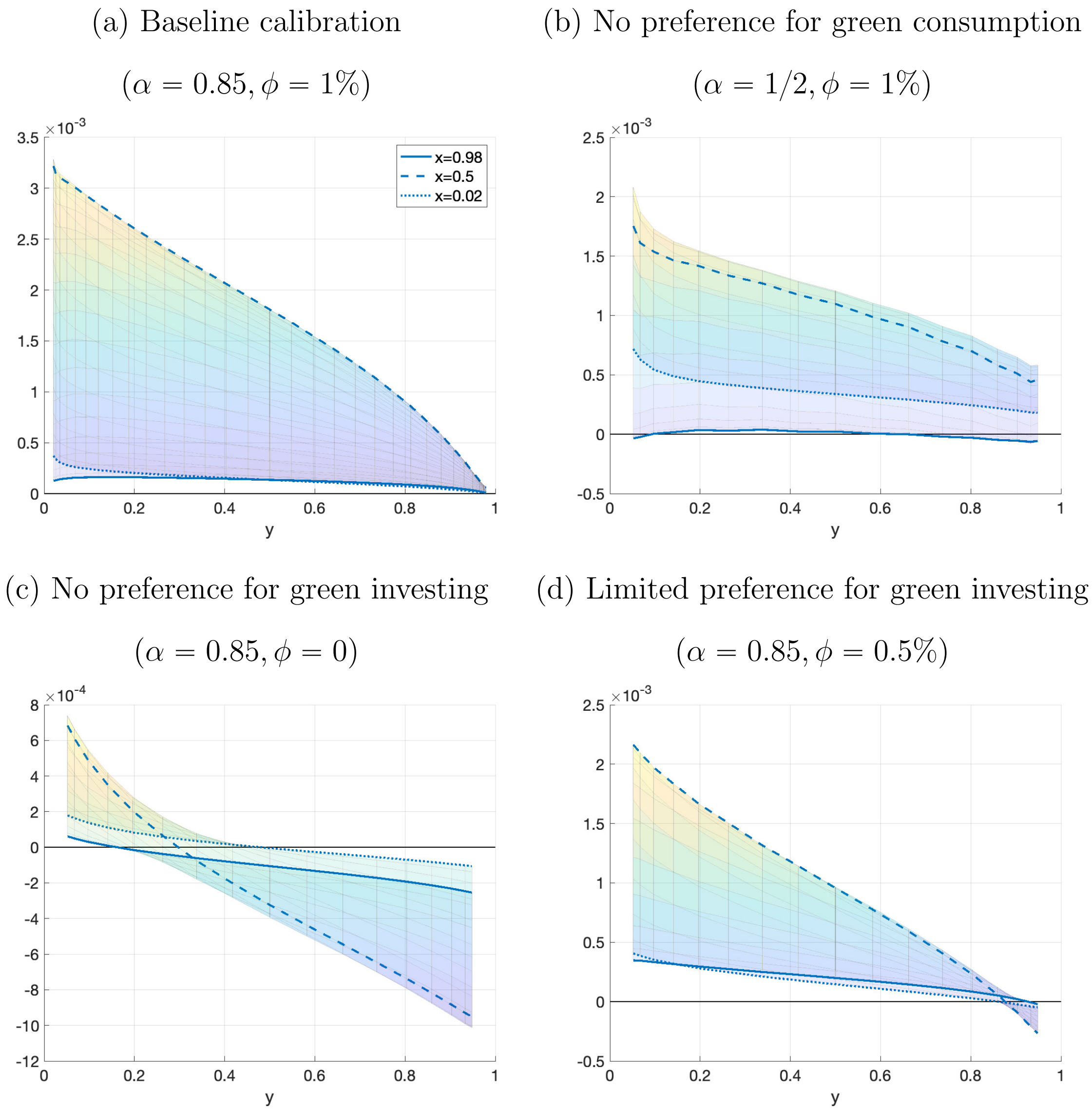
(b) Loading of y_t on shock to green output: $\sigma_{yZ_g,t} y_t$

(c) Loading of y_t on shock to brown output: $\sigma_{yZ_b,t} y_t$



Notes: Based on the calibration of Assumption 1. y_t is the relative supply of the green good, which is exogenous so that its drift and diffusion terms do not depend on the wealth share of the green investor x_t.

Figure A.2: Drift for the wealth share of the green investor x_t: $\mu_{x,t}x_t$



(a) Baseline calibration ($\alpha = 0.85, \phi = 1\%$)

(b) No preference for green consumption ($\alpha = 1/2, \phi = 1\%$)

(c) No preference for green investing ($\alpha = 0.85, \phi = 0$)

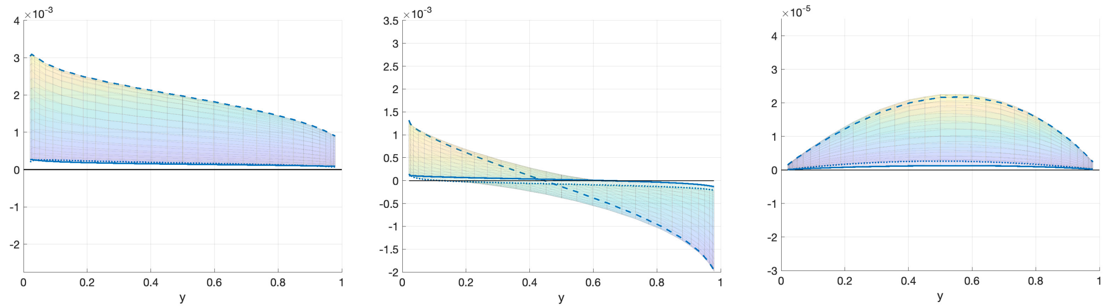
(d) Limited preference for green investing ($\alpha = 0.85, \phi = 0.5\%$)

Notes: Based on the calibration of Assumption 1, with $\gamma = 50$, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure A.3: Diffusion terms for the wealth share of the green investor x_t

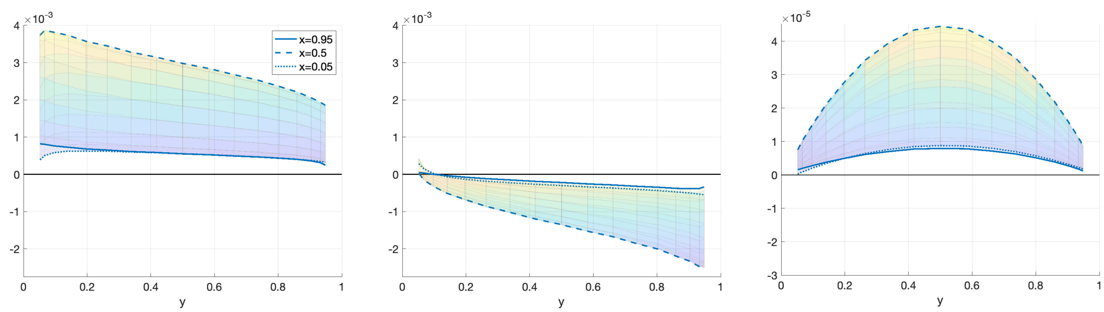
(a) Baseline calibration ($\alpha = 0.85, \phi = 1\%$)

Loading of x_t on shock to green output: $\sigma_{xZ_g,t}y_t$	Loading of x_t on shock to green output: $\sigma_{xZ_g,t}y_t$	Cov. of state variables: $\text{cov}_t(dx_t, dy_t)dt^{-1}$



(b) No preference for green consumption ($\alpha = 1/2, \phi = 1\%$)

Loading of x_t on shock to green output: $\sigma_{xZ_g,t}y_t$	Loading of x_t on shock to green output: $\sigma_{xZ_g,t}y_t$	Cov. of state variables: $\text{cov}_t(dx_t, dy_t)dt^{-1}$

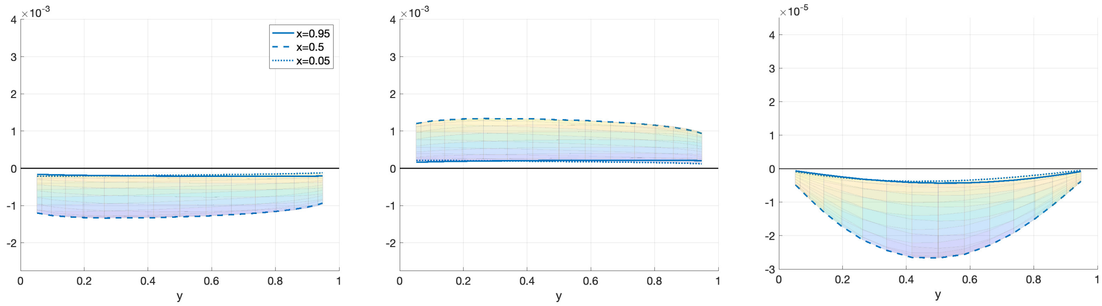


Notes: Based on the calibration of Assumption 1, with $\gamma = 50$, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure A.4: Diffusion terms for the wealth share of the green investor x_t

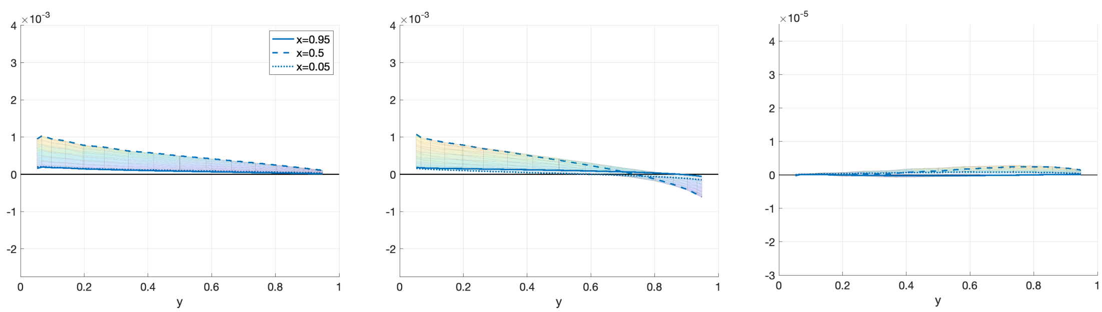
(c) No preference for green investing ($\alpha = 0.85, \phi = 0$)

Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$	Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$	Cov. of state variables: $\text{cov}_t(dx_t, dy_t)dt^{-1}$



(d) Limited preference for green investing ($\alpha = 0.85, \phi = 0.5\%$)

Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$	Loading of x_t on shock to green output: $\sigma_{xZ_g,t} y_t$	Cov. of state variables: $\text{cov}_t(dx_t, dy_t)dt^{-1}$



Notes: Based on the calibration of Assumption 1, with $\gamma = 50$, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

A.6 Marginal values of wealth and Hamilton-Jacobi-Bellman equations

Proposition A.4. *J_t^G, J_t^N satisfy the Hamilton-Jacobi-Bellman equations for $i \in \{G, N\}$*

$$0 = \left(\frac{1}{\psi - 1}\right) P_t^{i1-\psi} J_t^i - \left(\frac{1}{1 - 1/\psi}\right) \rho + r_t + \frac{\gamma}{2} \left(w_{g,t}^i \sigma_{g,t} + w_{b,t}^i \sigma_{b,t}\right)^\top \left(w_{g,t}^i \sigma_{g,t} + w_{b,t}^i \sigma_{b,t}\right) + \left(\frac{1}{1 - \psi}\right) \mu_{J^i,t} + \frac{1}{2} \left(\frac{1}{1 - \psi}\right) \left(\frac{\psi - \gamma}{1 - \psi}\right) \sigma_{J^i,t}^\top \sigma_{J^i,t} \tag{A.11}$$

where $\mu_{J^i,t}, \sigma_{J^i,t}$ are the geometric drift and diffusion terms of J_t^i obtained as in Remark A.1:

$$\frac{dJ_t^i}{J_t^i} \equiv \mu_{J^i,t} dt + \sigma_{J^i,t}^\top d\vec{Z}_t \tag{A.12}$$

A.7 Consumptions, goods prices

Proposition A.5. *The consumption of each investor $i \in \{G, N\}$ is given by*

$$c_t^i \equiv \frac{C_t^i}{W_t^i} = P_t^{i-\psi} J_t^i \tag{A.13}$$

$$c_{g,t}^i = \alpha^i \left(\frac{p_{g,t}}{P_t^i}\right)^{-\theta} c_t^i \tag{A.14}$$

$$c_{b,t}^i = (1 - \alpha^i) \left(\frac{p_{b,t}}{P_t^i}\right)^{-\theta} c_t^i \tag{A.15}$$

$$P_t^i = \left[\alpha^i p_{g,t}^{1-\theta} + (1 - \alpha^i) p_{b,t}^{1-\theta}\right]^{1/(1-\theta)} \tag{A.16}$$

Proposition A.6. *The relative price of the green good, $q_t = q(X_t) \equiv p_{g,t}/p_{b,t}$, solves the following non-linear equation*

$$q_t = S_t^{1/\theta} \left(\frac{1 - y_t}{y_t} \right)^{1/\theta} \tag{A.17}$$

where

$$S_t = \frac{\alpha^G J_t^G x_t P_t^{G\theta-\psi} + \alpha^N P_t^{N\theta-\psi} J_t^N (1 - x_t)}{(1 - \alpha^G) P_t^{G\theta-\psi} J_t^G x_t + (1 - \alpha^N) P_t^{N\theta-\psi} J_t^N (1 - x_t)}$$

Using the defintion of the numéraire, with $a = 1/2$, prices follow

$$p_{g,t} = \left(a + (1 - a) q_t^{\theta-1}\right)^{1/(\theta-1)} \tag{A.18}$$

$$p_{b,t} = p_{g,t} q_t^{-1} = \left(a q_t^{1-\theta} + (1 - a)\right)^{1/(\theta-1)} \tag{A.19}$$

$$P_t^i = \left[\alpha^i p_{g,t}^{1-\theta} + (1 - \alpha^i) p_{b,t}^{1-\theta}\right]^{1/(1-\theta)} \tag{A.20}$$

$$\mathcal{E}_t = P_t^G / P_t^N \tag{A.21}$$

B Additional figures

B.1 Economic set-up

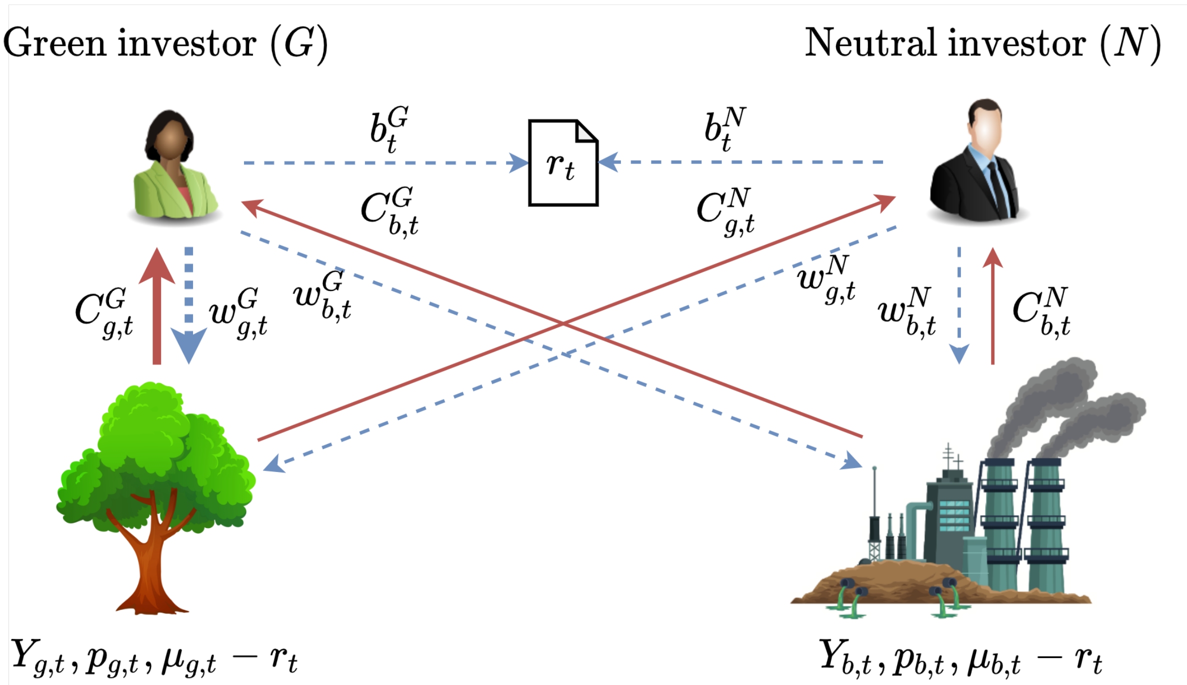
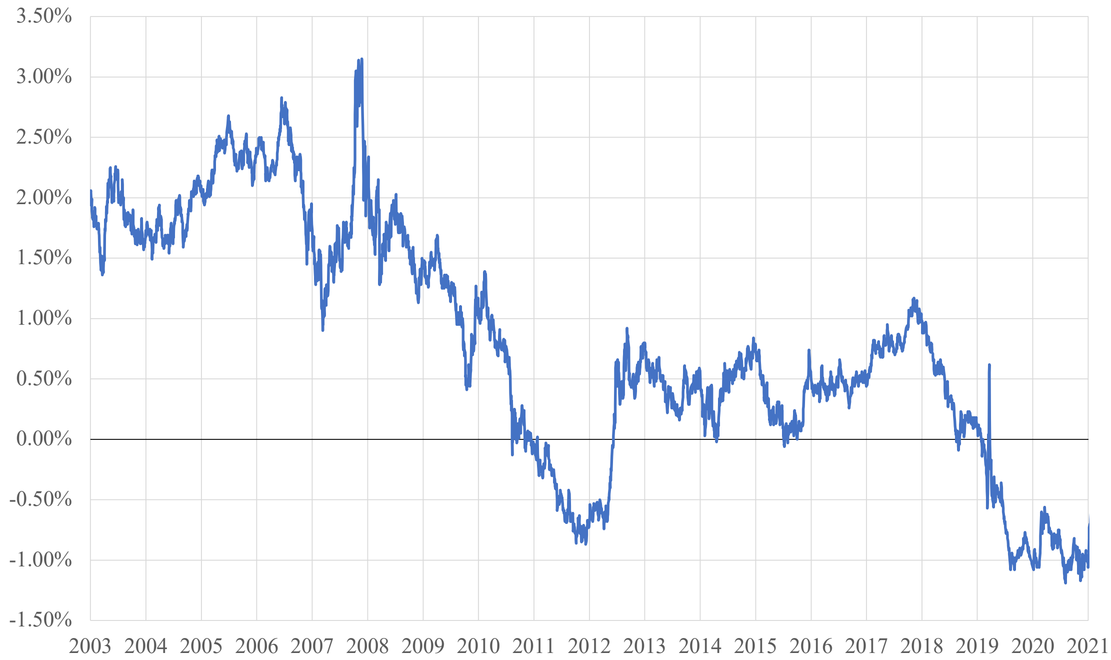


Figure B.1: The Economy

Source: Vecteezy.com. Back to main text: Section 2.

B.2 Riskfree interest rate

Figure B.2: Market Yield on U.S. Treasury Securities at 10-Year Constant Maturity, Inflation-Indexed



Source: Federal Reserve Economic Data (FRED), Federal Reserve Bank of St. Louis.

Figure B.3: Riskfree interest rate (r_t) for various calibrations

(a) Baseline ($\gamma = 50, \alpha = 0.85, \phi = 1\%$)

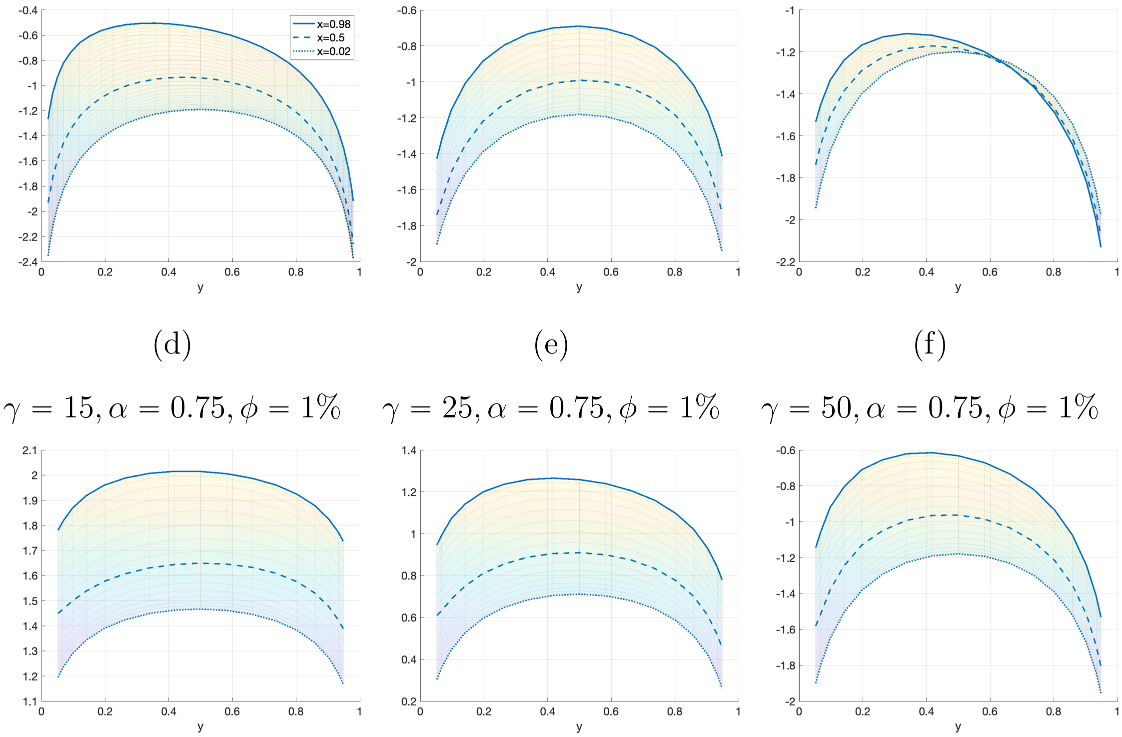
(b) No pref. green cons. ($\gamma = 50, \alpha = 1/2, \phi = 1\%$)

(c) No pref. green inv. ($\gamma = 50, \alpha = 0.85, \phi = 0\%$)

(d) $\gamma = 15, \alpha = 0.75, \phi = 1\%$

(e) $\gamma = 25, \alpha = 0.75, \phi = 1\%$

(f) $\gamma = 50, \alpha = 0.75, \phi = 1\%$



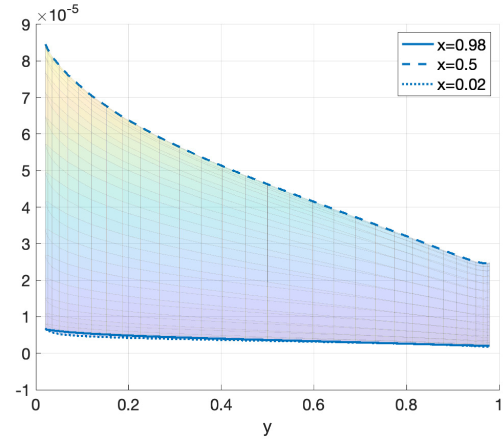
Notes: Based on the calibration of Assumption 1, except for the specified parameters. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

B.3 Quantities and prices of risk

Figure B.4: Quantities of risk

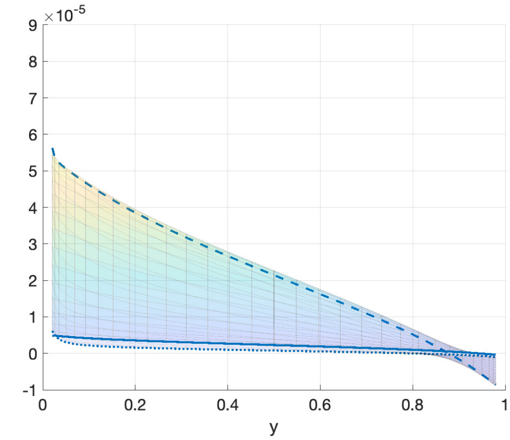
(a) Green asset returns on x_t risk

$$\text{cov}_t(dR_{g,t}, dx_t)dt^{-1} = \sigma_{g,t}^\top \sigma_{x,t} x_t$$



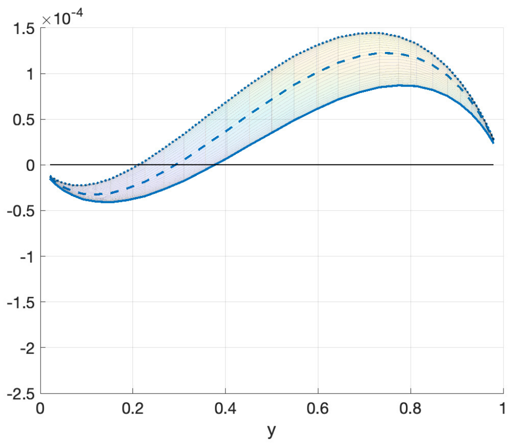
(b) Brown asset returns on x_t risk

$$\text{cov}_t(dR_{b,t}, dx_t)dt^{-1} = \sigma_{b,t}^\top \sigma_{x,t} x_t$$



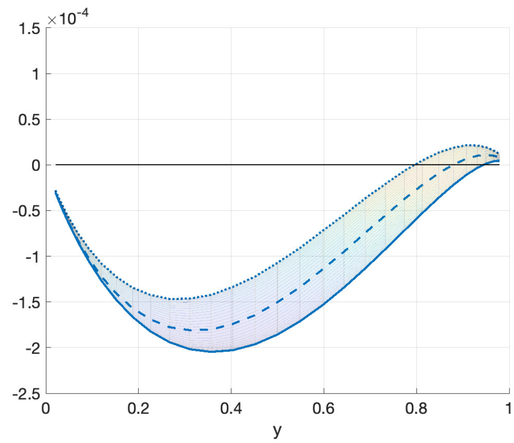
(c) Green asset returns on y_t risk

$$\text{cov}_t(dR_{g,t}, dy_t)dt^{-1} = \sigma_{g,t}^\top \sigma_{y,t} y_t$$



(d) Brown asset returns on y_t risk

$$\text{cov}_t(dR_{b,t}, dy_t)dt^{-1} = \sigma_{b,t}^\top \sigma_{y,t} y_t$$

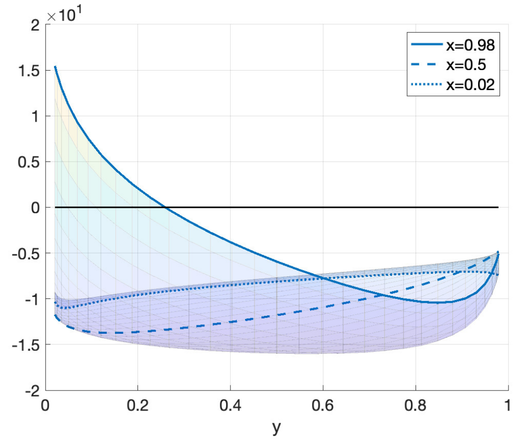


Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure B.5: Quantities of risk

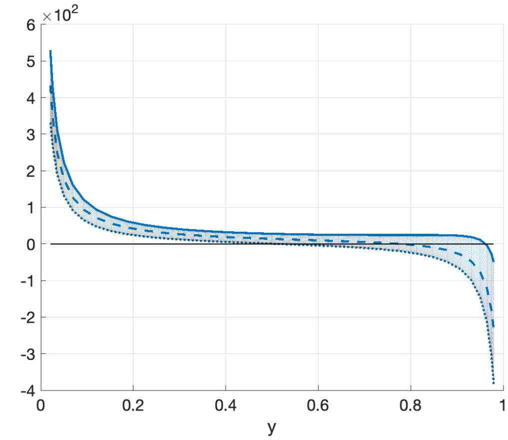
(a) Price of x_t risk

$$-\left(\frac{1-\gamma}{1-\psi}\right)\left\{x_t\frac{J^G_{x,t}}{J^G_t}+(1-x_t)\frac{J^N_{x,t}}{J^N_t}\right\}$$



(b) Price of y_t risk

$$-\left(\frac{1-\gamma}{1-\psi}\right)\left\{x_t\frac{J^G_{y,t}}{J^G_t}+(1-x_t)\frac{J^N_{y,t}}{J^N_t}\right\}$$



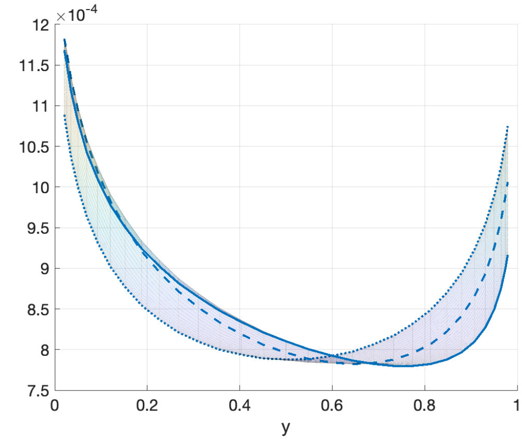
Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

B.4 Second moment of returns

Figure B.6: (Instantaneous) Second moment of returns

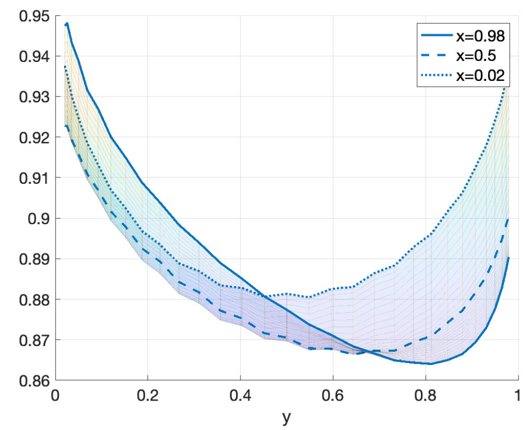
(a) Covariance of returns

$$\text{cov}_t(dR_{g,t}, dR_{b,t})dt^{-1} = \sigma_{g,t}^\top \sigma_{b,t}$$



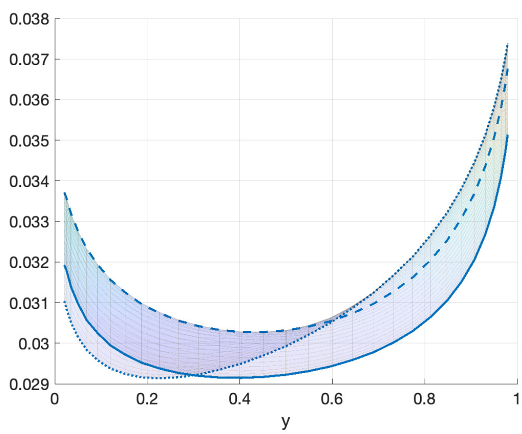
(b) Correlation of returns

$$\text{corr}_t(dR_{b,t}, dR_{b,t})dt^{-1}$$



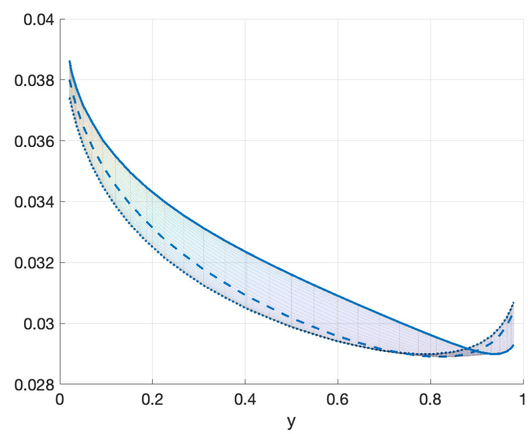
(c) Volatility of green asset returns

$$\text{vol}_t(dR_{g,t}) = \left(\sigma_{g,t}^\top \sigma_{g,t}\right)^{1/2}$$



(d) Volatility of brown asset returns

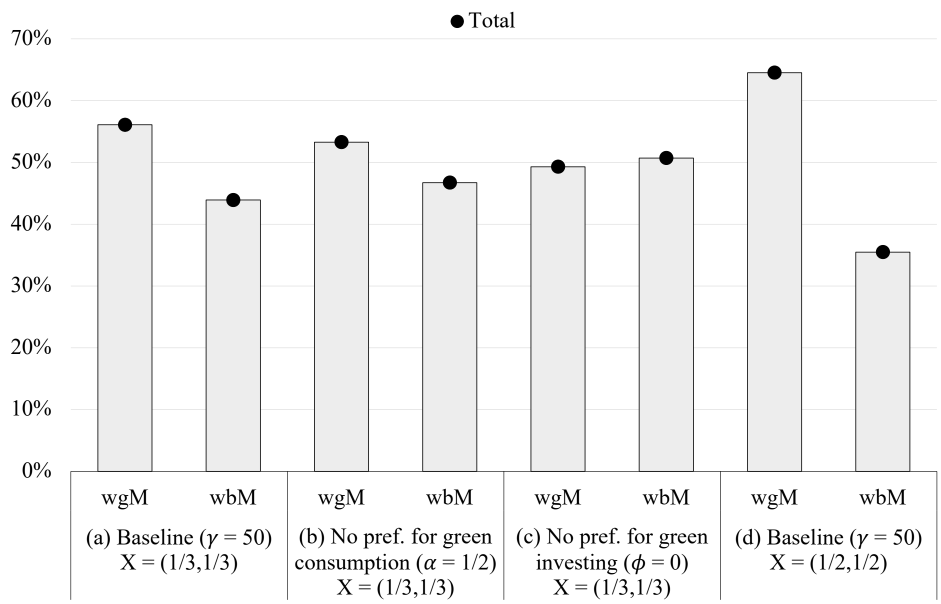
$$\text{vol}_t(dR_{b,t}) = \left(\sigma_{b,t}^\top \sigma_{b,t}\right)^{1/2}$$



Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

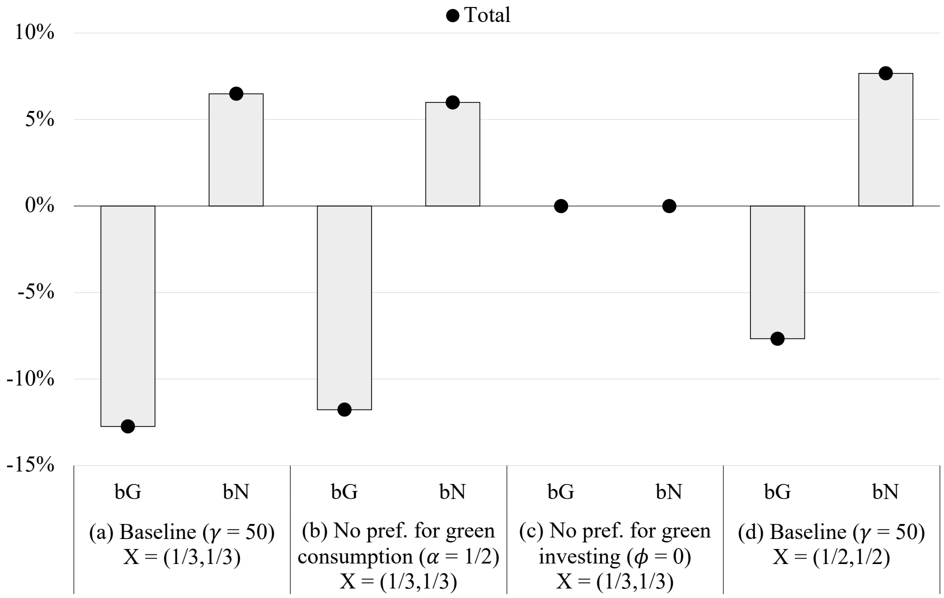
B.5 Portfolios

Figure B.7: Market portfolio at $X_t = (1/3, 1/3)$, and $X_t = (1/2, 1/2)$



Notes: Based on the calibration of Assumption 1, except for the specified parameters. The figure shows the market portfolio at $X_t \equiv (x_t, y_t)' = (1/3, 1/3)$ for Panels (a), (b), (c), and at $X_t = (1/2, 1/2)$ for Panel (d). $w^M_{g,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t}), w^M_{b,t} \equiv Q_{g,t}/(Q_{g,t} + Q_{b,t})$ are the weights (as % of wealth) on the green and brown asset in the market portfolio. In equilibrium, $w^M_{b,t} = 1 - w^M_{g,t}$ because the bond is in zero net supply.

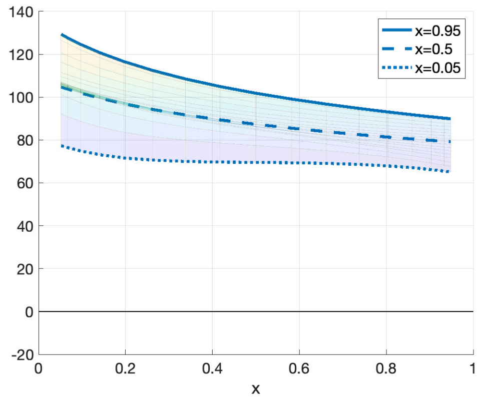
Figure B.8: Borrowing and saving in the riskless bond at $X_t = (1/3, 1/3)$, and $X_t = (1/2, 1/2)$



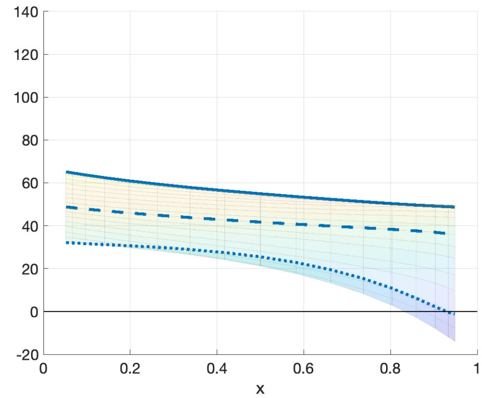
Notes: Based on the calibration of Assumption 1, except for the specified parameters. The figure shows $b_t^i = 1 - w_{g,t}^i - w_{b,t}^i$, the weight (as % of wealth) allocated to the riskfree bond by each investor, $i \in \{G, N\}$. $b_t^i > 0$ corresponds to saving in the bond, $b_t^i < 0$ corresponds to borrowing.

Figure B.9: Portfolios of both investors, $i \in \{G, N\}$ (% of wealth)

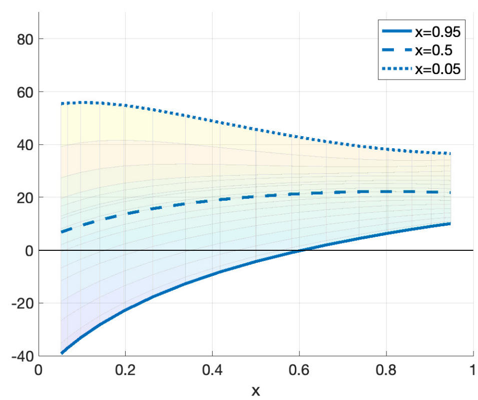
(a) Green asset in green portfolio ($w^G_{g,t}$)



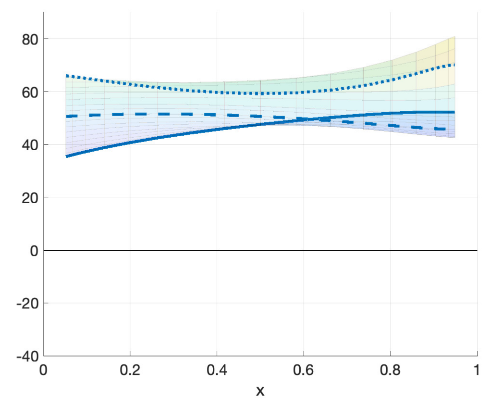
(b) Green asset in neutral portfolio ($w^N_{g,t}$)



(c) Brown asset in green portfolio ($w^G_{b,t}$)

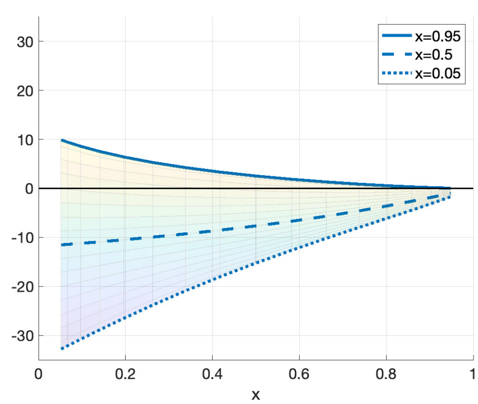


(d) Brown asset in neutral portfolio ($w^N_{b,t}$)



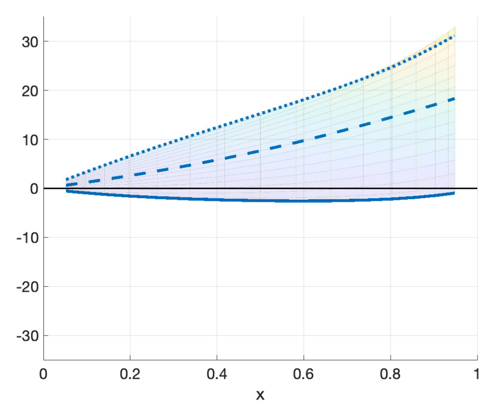
(e) Riskfree bond in green portfolio

$(b^G_t = 1 - w^G_{g,t} - w^G_{b,t})$



(f) Riskfree bond in neutral portfolio

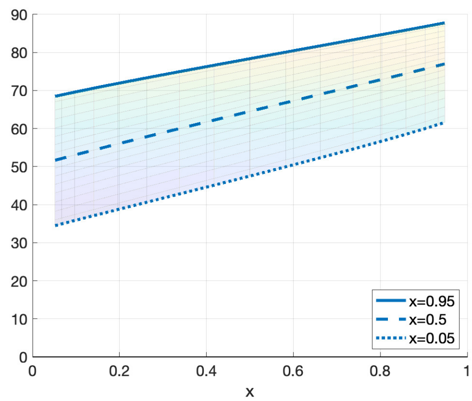
$(b^N_t = 1 - w^N_{g,t} - w^N_{b,t})$



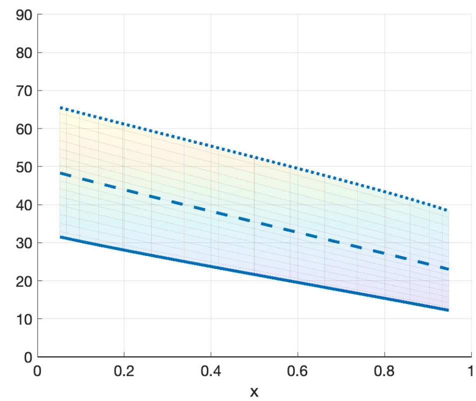
Notes: Based on the calibration of Assumption 1, with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.

Figure B.10: Portfolios of both investors, $i \in \{G, N\}$, vs. market portfolio (%)

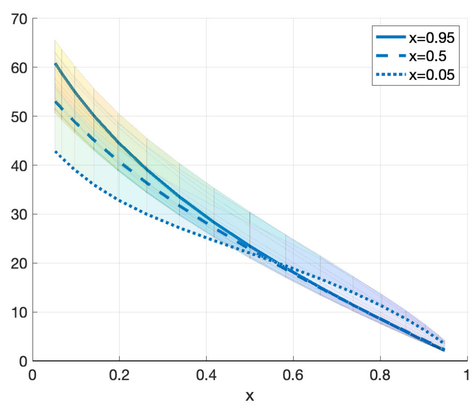
(a) Green asset in market portfolio ($w_{g,t}^{M}$)



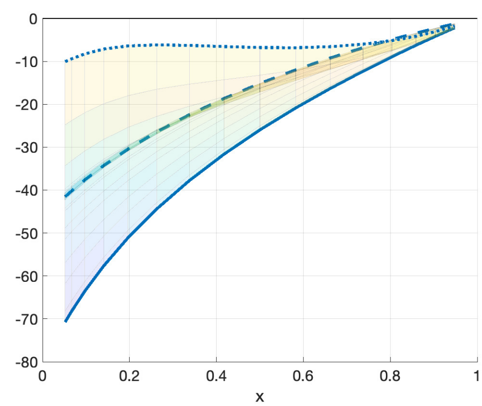
(b) Brown asset in market portfolio ($w_{g,t}^{M}$)



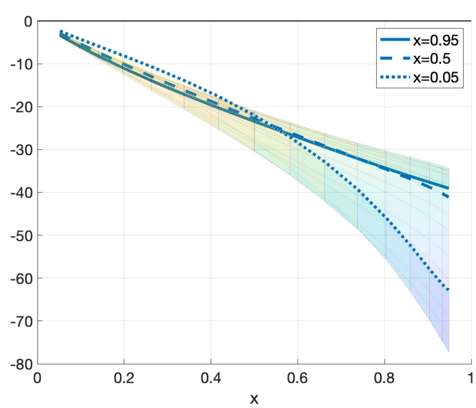
(c) Bias on green asset in green vs. market portfolio ($w_{g,t}^{G} - w_{g,t}^{M}$)



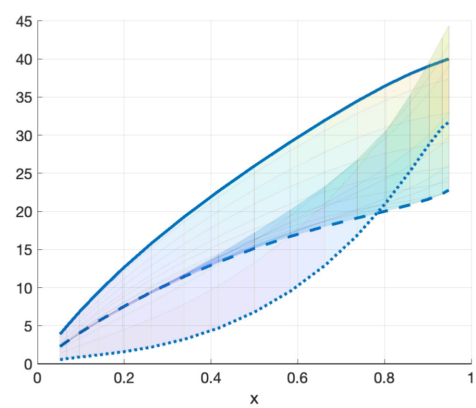
(d) Bias on brown asset in green vs. market portfolio ($w_{b,t}^{G} - w_{b,t}^{M}$)



(e) Bias on green asset in neutral vs. market portfolio ($w_{g,t}^{N} - w_{g,t}^{M}$)



(f) Bias on brown asset in neutral vs. market portfolio ($w_{b,t}^{N} - w_{b,t}^{M}$)



Notes: Based on the calibration of Assumption 1 with $\gamma = 50$. x_t is the wealth share of the green investor. y_t is the relative supply of the green good.