

Sources of Return Predictability

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Outline

- ① Excess (residual) volatility present on N- but absent on A-days
 - variance-in-mean relationship present on A-days
 - reversal present on N-days
- ② Justifies separating predictors into “variance-in-mean” and “reversal” ones
- ③ Why the dichotomy? One (our) answer: disagreement

Excess volatility puzzle (Shiller, 1981) [2]

Each of the relationships can be re-written in terms of *ex-post* rational (**detrended**) price series P_t^* (p_t^*) as the present value of actual subsequent (detrended) dividends:

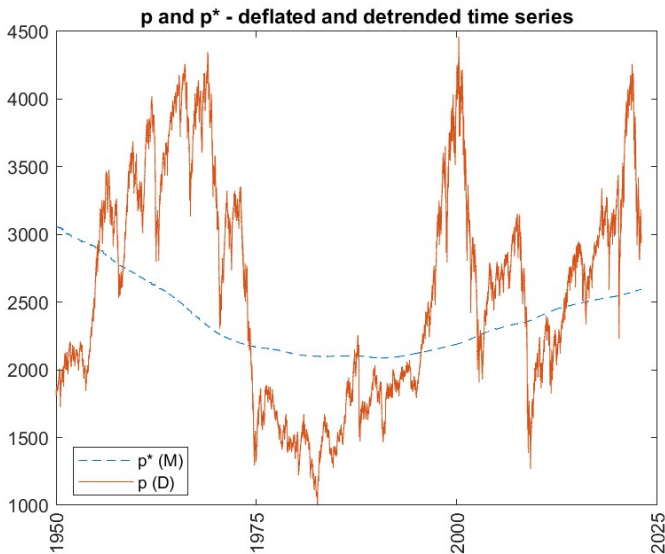
$$P_t^* = \sum_{k=1}^{\infty} \gamma^k D_{t+k} \quad \left(p_t^* = \sum_{k=1}^{\infty} \bar{\gamma}^k d_{t+k} \right) \quad (3)$$

Subject to the choice of terminal value of the *ex-post* rational price, p_T^* , the entire time series can be determined recursively by

$$P_t^* = \gamma (P_{t+1}^* + D_{t+1}^*) \quad (p_t^* = \bar{\gamma} (p_{t+1}^* + d_{t+1}^*))$$

and working backwards from the base year.

Excess volatility puzzle (Shiller, 1981) [3]



Q: Are markets excessively volatile on all days?

- Log change in deflated (detrended) fundamental value (FV):

$$Y_{t+1} = \log(P_{t+1}^*) - \log(P_t^*)$$

$$Y_{t+1} = \log(p_{t+1}^*) - \log(p_t^*)$$

- Does it justify A-day (N-day) price movements?

$$Y_{t+1} = \beta_0 + \beta_1 rA_t + \beta_2 rN_t$$

- Particularly, is $\beta_1 > \beta_2$?
 - $Y_{t+1} = \delta_0 + \delta_1(rA_t + rN_t) + \delta_2(rA_t - rN_t)$
 - $\beta_1 > \beta_2$ if and only if $\delta_2 > 0$

Q: Are markets excessively volatile on all days? A: NO! [1]

Forecasting changes in fundamental value using real ex-post rational price P_{t+1}^* as its proxy (FV: 1953 – 2010)

$Y_{t+1} = \beta_0 + \beta_1 r_t^A + \beta_2 r_t^N$					$Y_{t+1} = \delta_0 + \delta_1(r_t^A + r_t^N) + \delta_2(r_t^A - r_t^N)$				
β_0	β_1	β_2	Adj.R ²	N	δ_0	δ_1	δ_2	Adj.R ²	N
Panel 1A: Monthly data					Panel 2A: Monthly data				
0.003	0.004	-0.003	0.08%	695	0.003	0.001	0.003	0.08%	695
[31.49]	[0.85]	[-1.37]			[31.49]	[0.26]	[1.30]		
Panel 1B: Quarterly data					Panel 2B: Quarterly data				
0.008	0.014	-0.001	0.61%	231	0.008	0.006	0.008	0.61%	231
[34.61]	[1.81]	[-0.53]			[34.61]	[1.56]	[1.82]		
Panel 1C: Annual data					Panel 2C: Annual data				
0.030	0.046	-0.012	3.72%	57	0.030	0.017	0.029	3.72%	57
[16.02]	[1.70]	[-1.16]			[16.02]	[1.19]	[1.99]		

- N-day price movements are *"too big to be justified by future dividends"*... **[excess vol.]**
- ... but A-day price movements are not! **[NO excess vol.]**

Q: Are markets excessively volatile on all days? A: NO! [2]

Forecasting changes in fundamental value using *detrended* real ex-post rational price p_{t+1}^* as its proxy (FV: 1953 – 2010)

$Y_{t+1} = \beta_0 + \beta_1 r_t^A + \beta_2 r_t^N$					$Y_{t+1} = \delta_0 + \delta_1(r_t^A + r_t^N) + \delta_2(r_t^A - r_t^N)$				
β_0	β_1	β_2	Adj.R ²	N	δ_0	δ_1	δ_2	Adj.R ²	N
Panel 1A: Monthly data					Panel 2A: Monthly data				
0.000	0.003	-0.002	0.07%	695	0.000	0.001	0.003	0.07%	695
[-4.34]	[0.83]	[-1.36]			[-4.34]	[0.24]	[1.28]		
Panel 1B: Quarterly data					Panel 2B: Quarterly data				
-0.001	0.010	-0.001	0.56%	231	-0.001	0.005	0.006	0.56%	231
[-5.47]	[1.80]	[-0.46]			[-5.47]	[1.57]	[1.78]		
Panel 1C: Annual data					Panel 2C: Annual data				
-0.004	0.035	-0.009	3.69%	57	-0.004	0.013	0.022	3.69%	57
[-2.81]	[1.67]	[-1.20]			[-2.81]	[1.14]	[1.97]		

- N-day price movements are *"too big to be justified by future dividends"*... **[excess vol.]**
- ... but A-day price movements are not! **[NO excess vol.]**

Q: Is excess volatility concentrated in N-days?

Realizing that

$$r_{t+1} = r_{t+1}^A + r_{t+1}^N$$

and building on the Campbell-Shiller variance decomposition:

$$r_{t+1} = \frac{1 - \rho^{T+1}}{1 - \rho} k + \sum_{j=0}^T \rho^j \Delta d_{t+1+j} - \sum_{j=1}^T \rho^j r_{t+1+j} + (\rho^{T+1} p d_{t+T+1} - p d_t)$$

$$\overline{pd} = E[pd_t] \quad \rho = (1 + \exp(-\overline{pd}))^{-1} \quad k = -\ln \rho - (1 - \rho) \ln(1/\rho - 1),$$

we derive the contribution of A-day and N-day returns to the total stock market volatility.

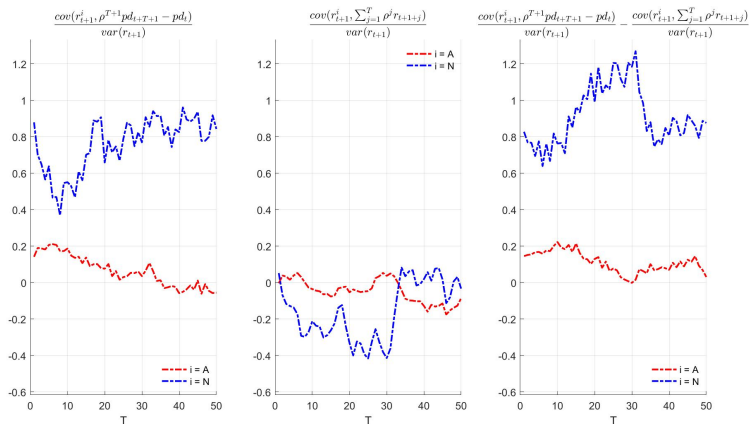
Excess volatility relative to the Div Discount Model

For a high value of T , the share of excess (with respect to DDM) volatility of returns under each regime is given by

$$S_{EV}^A = \frac{\text{Cov}[r_{t+1}^A, \rho^{T+1}pd_{t+T+1} - pd_t]}{\text{Var}[r_{t+1}]} - \frac{\text{Cov}[r_{t+1}^A, \sum_{j=1}^T \rho^j r_{t+1+j}]}{\text{Var}[r_{t+1}]}$$

$$S_{EV}^N = \frac{\text{Cov}[r_{t+1}^N, \rho^{T+1}pd_{t+T+1} - pd_t]}{\text{Var}[r_{t+1}]} - \frac{\text{Cov}[r_{t+1}^N, \sum_{j=1}^T \rho^j r_{t+1+j}]}{\text{Var}[r_{t+1}]}$$

Q: Is excess volatility concentrated in N-days? A: YES!



- The excess volatility (relative to the DDM) is almost entirely due to N-day returns.

Residual volatility relative to the Conditional CAPM

- If you believe Conditional CAPM, returns responding to changes in risk-free rates and measures of stock market variance is not *irrational*
- CCAPM benchmark can vary across regimes

$$r_{t+1}^d = r_{f,t+1}^d + \gamma^d \text{Var}_t[r_{t+1}] + v_{t+1}^d \quad d = A, N$$

- It's the residual volatility (v_{t+1}^d) that requires explanation.
- *Is it (also) concentrated in N-day returns?*

Residual volatility relative to the Conditional CAPM

At a high value of T , the share of *residual* (with respect to CCAPM) volatility of returns under each regime is given by

$$S_{RV}^A = \frac{\text{Cov}[r_{t+1}^A, \rho^{T+1}pd_{t+T+1} - pd_t]}{\text{Var}[r_{t+1}]} - \frac{\text{Cov}[r_{t+1}^A, \sum_{j=1}^T \rho^j v_{t+1+j}^A]}{\text{Var}[r_{t+1}]}$$

$$S_{RV}^N = \frac{\text{Cov}[r_{t+1}^N, \rho^{T+1}pd_{t+T+1} - pd_t]}{\text{Var}[r_{t+1}]} - \frac{\text{Cov}[r_{t+1}^N, \sum_{j=1}^T \rho^j v_{t+1+j}^N]}{\text{Var}[r_{t+1}]}.$$

A- and N-day returns – why the difference?

Savor and Wilson (2013) results suggest:

- A-days: variance in mean
- N-days: reversal

This motivates separating predictors into “variance-in-mean” and “reversal” ones.

Variables I

OurName	Descr	Authors
tbl	t-bill	Campbell
logPD	dividend price ratio	Campbell, Shiller
logSP500d12e12	dividend payout	Campbell, Shiller
logSP500e12p	earnings price ratio	Campbell, Shiller
dfr	default return spread	Fama, French
dfy	default yield spread	Fama, French
ltr	long govt return	Fama, French
lty	long govt yield	Fama, French
tms	term spread	Fama, French
ik	invstmt/capital	Cochrane
bm	b/m	Kothari, Shanken
cayGW	cnsmt, wlth, incm	Lettau, Ludvigson
svar	σ^2	Guo
GWZ_A.house	housing/consumption	Piazzesi, Schneider, Tuzel
ntis	net equity issuance	Boudoukh, Michaely, Richardson, Roberts
GWZ_M_wtexas	oil price changes	Driesprong, Jacobsen, Maat

Continued on next slide

Variables II

OurName	Descr	Authors
GWZ_M_wtexas	oil price changes	Driesprong, Jacobsen, Maat
GWZ_M_ogap	prdctn-output gap	Cooper, Priestley
GWZ_M_avgcor	acvg corr stock returns	Pollett, Wilson
GWZ_M_dtoat	to Dow all-time high	Li, Yu
GWZ_M_dtoy	to Dow 52-week high	Li, Yu
GWZ_M_ygap	stock-bond yield gap	Maio
GWZ_M_fbm	b/m x-sect factor	Kelly, Pruitt
GWZ_Q_govik	public sector investmt	Belo, Yu
GWZ_A_gip	yearend econ growth	Møller, Rangvid
GWZ_A_gpce	yearend econ growth	Møller, Rangvid
GWZ_M_tail	x-sect tail risk	Kelly, Jiang
GWZ_M_tchi	14 technical indicators	Neely, Rapach, Tu, Zhou
GWZ_M_rdsp	stock return dispersion	Maio
GWZ_S_skew	skewness	Colacito, Ghysels, Meng, Siwasarit
GWZ_M_lzrt	9 illiq measures	Chen, Eaton, Paye
GWZ_M_skvw	avg stock skewness	Jondeau, Zhang, Zhu
GWZ_Q_pce	consumption/trend	Atanasov, Møller, Priestley

Univariate Regressions I

Univariate regressions of quarterly returns on predictor variables.

$$r_{t+1}^i = \alpha + \beta x_t, \quad i = (A\&N, A, N),$$

Variable	Quarterly returns			A-day quarterly returns			N-day quarterly returns		
	β	$t(\beta)$	R^2	β	$t(\beta)$	R^2	β	$t(\beta)$	R^2
Panel A: Announcement day return predictors									
dfy	1.36	[1.18]	0.1%	0.79	[1.90]	0.9%	0.57	[0.55]	-0.3%
svar	0.90	[1.94]	1.0%	0.58	[3.54]	4.0%	0.32	[0.76]	-0.2%
GWZ_M_wtexas	0.02	[0.33]	-0.3%	0.04	[1.77]	0.8%	-0.02	[-0.35]	-0.3%
GWZ_M_dtoy	-0.14	[-2.05]	1.2%	-0.06	[-2.42]	1.7%	-0.08	[-1.32]	0.3%

Univariate Regressions II

Univariate regressions of quarterly returns on predictor variables.

$$r_{t+1}^i = \alpha + \beta x_t, \quad i = (A\&N, A, N),$$

Variable	Quarterly returns			A-day quarterly returns			N-day quarterly returns		
	β	$t(\beta)$	R^2	β	$t(\beta)$	R^2	β	$t(\beta)$	R^2
Panel B: Non-announcement day return predictors									
logPD	-0.02	[-1.42]	0.4%	0.00	[0.45]	-0.3%	-0.02	[-1.78]	0.8%
lty	-0.30	[-1.74]	0.7%	0.01	[0.19]	-0.4%	-0.31	[-2.03]	1.1%
tbl	-0.38	[-2.39]	1.7%	-0.01	[-0.22]	-0.3%	-0.36	[-2.59]	2.0%
tms	0.63	[1.78]	0.8%	0.11	[0.87]	-0.1%	0.52	[1.64]	0.6%
ltr	0.22	[2.40]	1.7%	0.01	[0.30]	-0.3%	0.21	[2.57]	2.0%
ik	-5.31	[-3.34]	3.6%	0.17	[0.29]	-0.3%	-5.48	[-3.89]	4.9%
GWZ_M_logap	-0.31	[-4.25]	5.9%	0.00	[-0.17]	-0.4%	-0.30	[-4.73]	7.2%
GWZ_Q_pce	-0.47	[-3.49]	4.0%	-0.06	[-1.11]	0.1%	-0.41	[-3.45]	3.9%
GWZ_S_skew	-0.01	[-1.39]	0.3%	0.00	[0.70]	-0.2%	-0.01	[-1.84]	0.9%
GWZ_A_gpce	-3.62	[-3.57]	4.2%	-0.28	[-0.76]	-0.2%	-3.34	[-3.70]	4.5%
GWZ_A_gip	-0.91	[-3.49]	4.0%	-0.03	[-0.32]	-0.3%	-0.88	[-3.80]	4.7%

Univariate Regressions III

Univariate regressions of quarterly returns on predictor variables.

$$r_{t+1}^i = \alpha + \beta x_t, \quad i = (A\&N, A, N),$$

Variable	Quarterly returns			A-day quarterly returns			N-day quarterly returns		
	β	$t(\beta)$	R^2	β	$t(\beta)$	R^2	β	$t(\beta)$	R^2
Panel C: Announcement and non-announcement day return predictors									
GWZ_M_dtoat	-0.15	[-2.87]	2.6%	-0.03	[-1.77]	0.8%	-0.11	[-2.49]	1.9%
GWZ_M_avgcor	0.18	[4.17]	5.6%	0.05	[3.00]	2.8%	0.13	[3.41]	3.7%

Univariate Regressions IV

Univariate regressions of quarterly returns on predictor variables.

$$r_{t+1}^i = \alpha + \beta x_t, \quad i = (A\&N, A, N),$$

Variable	Quarterly returns			A-day quarterly returns			N-day quarterly returns		
	β	$t(\beta)$	R^2	β	$t(\beta)$	R^2	β	$t(\beta)$	R^2
Panel D: Variables not predicting either announcement or non-announcement returns									
cayGW	-0.03	[-0.23]	-0.35%	0.04	[0.92]	-0.06%	-0.08	[-0.63]	-0.22%
ntis	-0.25	[-0.99]	-0.01%	-0.04	[-0.39]	-0.31%	-0.22	[-0.95]	-0.04%
dfr	0.08	[0.40]	-0.31%	-0.04	[-0.54]	-0.26%	0.12	[0.67]	-0.20%
bm	0.01	[0.26]	-0.34%	0.00	[-0.59]	-0.24%	0.01	[0.53]	-0.26%
logSP500d12e12	0.02	[1.25]	0.20%	0.01	[1.62]	0.59%	0.01	[0.74]	-0.16%
logSP500e12p	0.00	[0.26]	-0.34%	-0.01	[-1.34]	0.29%	0.01	[0.83]	-0.11%
GWZ_M.lzrt	-0.01	[-0.51]	-0.27%	-0.01	[-0.91]	-0.06%	0.00	[-0.20]	-0.35%
GWZ_M.skvw	-0.06	[-0.61]	-0.23%	-0.01	[-0.14]	-0.36%	-0.06	[-0.62]	-0.22%
GWZ_M.tail	0.13	[1.41]	0.36%	0.00	[0.12]	-0.36%	0.13	[1.53]	0.49%
GWZ_M.fbm	0.02	[0.38]	-0.31%	0.02	[1.06]	0.04%	0.00	[-0.01]	-0.37%
GWZ_M.ygap	0.01	[0.44]	-0.30%	-0.01	[-1.37]	0.32%	0.01	[1.05]	0.04%
GWZ_M.rdsp	-0.06	[-0.15]	-0.36%	0.18	[1.21]	0.17%	-0.24	[-0.66]	-0.20%
GWZ_M.tchi	0.00	[0.46]	-0.29%	0.00	[-0.67]	-0.20%	0.00	[0.79]	-0.14%
GWZ_Q.govik	0.24	[0.33]	-0.33%	-0.16	[-0.59]	-0.24%	0.40	[0.61]	-0.23%
GWZ_A.house	0.60	[1.12]	0.10%	-0.11	[-0.56]	-0.25%	0.71	[1.49]	0.45%

Period covered: 1953 – 2021.

Possible explanation? Disagreement. I

Building on [Andrei, Cujean, and Wilson \(2023\)](#), we can show:

$$\text{Var}_{i,t}[\tilde{R}_{M,t+1}^e] = \text{Var}_t[\tilde{R}_{M,t+1}^e] - \text{Var}_t[\bar{E}[\tilde{R}_{M,t+1}^e]] - \text{Var}_t[E_i[\tilde{R}_{M,t+1}^e] - \bar{E}[\tilde{R}_{M,t+1}^e]],$$

where:

$\text{Var}_{i,t}[\tilde{R}_{M,t+1}^e]$: variance of the market given agent's i information set $F_{i,t}$

$\text{Var}_t[\tilde{R}_{M,t+1}^e]$: unconditional date- t variance

$\text{Var}_t[\bar{E}[\tilde{R}_{M,t+1}^e]]$: conditional variance of consensus returns

$\text{Var}_t[E_i[\tilde{R}_{M,t+1}^e] - \bar{E}[\tilde{R}_{M,t+1}^e]]$: cross-sectional variance of expected market returns

Possible explanation? Disagreement. II

Suppose a conditional CAPM holds with respect to each agent's information set $F_{i,t}$:

$$E_i[\tilde{R}_{M,t+1}^e] = \gamma \text{Var}_{i,t}[\tilde{R}_{M,t+1}^e].$$

Then with respect to consensus beliefs:

$$\bar{E}[\tilde{R}^e] = \frac{1}{N} \sum_{i=1}^N E_i[\tilde{R}^e] = \frac{\gamma}{N} \sum_{i=1}^N \text{Var}_{i,t}[\tilde{R}_{M,t+1}^e] = \gamma \text{Var}_{i,t}[\tilde{R}_{M,t+1}^e]$$

Possible explanation? Disagreement. III

Plugging in from the previous slide we get:

$$\bar{E}[\tilde{R}^e] = \gamma \text{Var}_t[\tilde{R}_{M,t+1}^e] - \gamma \text{Var}_t[\bar{E}[\tilde{R}_{M,t+1}^e]] - \gamma \text{Var}_t[E_i[\tilde{R}_{M,t+1}^e] - \bar{E}[\tilde{R}_{M,t+1}^e]].$$

- Standard tests for a variance-in-mean relation for market returns include only the first term on the RHS but not the second two.
- In small time intervals (monthly or quarterly data):
 - second term is unlikely to be large
 - third term is likely to be large when there is dispersion of beliefs, and also somewhat variable over time

Crucially for our results: when agents observe dispersed signals $F_{i,t}$ the last term is likely to be much larger than when they observe a common signal.

Dispersion of beliefs

How to measure $Var_t[E_i[\tilde{R}_{M,t+1}^e] - \bar{E}[\tilde{R}_{M,t+1}^e]]?$

- This is the variance of time-t deviations of brokers' expected returns on the market from the consensus expectation of the return on the market
- We compute it using IBES expected price data; forecast length = 12 months ahead; lookback period = 6 months;
- We focus on brokers covering at least 100 stocks but results are robust to other cut-offs

Results I

Regress returns on market variance and disagreement:

$$r_{t+1}^{type} = \alpha + \beta_1 Var_t[\tilde{R}_{M,t+1}^e] + \beta_2 Var_t[E_i[\tilde{R}_{M,t+1}^e] - \bar{E}[\tilde{R}_{M,t+1}^e]]$$

Panel A: Returns accrued on all days

	Annual			Semiannual			Quarterly		
α	-0.01	0.16	0.18	0.01	0.07	0.08	0.01	0.03	0.03
	-0.11	1.80	2.49	0.36	2.21	2.45	0.92	1.43	1.41
β_1	17.43		58.64	1.24		11.30	-2.57		-1.31
	0.92		3.10	0.21		1.77	-0.96		-0.45
β_2		-121.24	-271.91		-58.80	-89.32		-22.30	-19.05
		-1.62	-3.49		-2.19	-2.85		-1.38	-1.07
N	19	19	19	38	38	38	76	76	76
$R^2(\%)$	-0.91	8.31	39.08	-2.65	9.27	14.32	-0.09	1.18	0.10

Results II

Regress returns on market variance and disagreement:

$$r_{t+1}^{type} = \alpha + \beta_1 Var_t[\tilde{R}_{M,t+1}^e] + \beta_2 Var_t[E_i[\tilde{R}_{M,t+1}^e] - \bar{E}[\tilde{R}_{M,t+1}^e]]$$

Panel B: Returns accrued on A-days

	Annual			Semiannual			Quarterly		
α	-0.01	0.00	0.00	-0.01	0.00	0.01	-0.01	0.00	0.00
	-0.62	-0.08	0.08	-0.78	0.18	0.47	-1.05	-0.40	-0.33
β_1	11.88		15.70	5.62		7.72	3.17		3.35
	1.64		1.67	2.22		2.56	2.76		2.65
β_2		15.14	-25.21		2.22	-18.63		5.55	-2.73
		0.48	-0.65		0.17	-1.26		0.75	-0.35
N	19	19	19	38	38	38	76	76	76
$R^2(\%)$	8.63	-4.49	5.42	9.56	-2.70	11.02	8.09	-0.58	6.98

Panel C: Returns accrued on N-days

	Annual			Semiannual			Quarterly		
α	0.01	0.16	0.18	0.02	0.07	0.07	0.02	0.03	0.03
	0.13	2.15	2.67	0.75	2.28	2.31	1.61	1.85	1.82
β_1	5.55		42.94	-4.38		3.58	-5.74		-4.67
	0.32		2.46	-0.80		0.58	-2.53		-1.89
β_2		-136.37	-246.70		-61.02	-70.69		-27.85	-16.32
		-2.13	-3.44		-2.44	-2.33		-1.97	-1.07
N	19	19	19	38	38	38	76	76	76
$R^2(\%)$	-5.26	16.49	35.66	-0.98	11.77	10.11	6.72	3.70	6.92

Summary

- ① Develop an approach to determine whether predictors represent proxies for fundamental risk (i.e. linked to new information about economic conditions) or excess volatility
- ② Show that (with few exceptions) predictors forecast returns on *either* announcement *or* non-announcement days
 - Commonly used predictors are not fundamentals-based ...
 - ... but are rather forecasting excess volatility of the market.

References

Daniel Andrei, Julien Cujean, and Mungo Wilson. The Lost Capital Asset Pricing Model. *The Review of Economic Studies*, page rdad013, 02 2023. ISSN 0034-6527. doi: 10.1093/restud/rdad013. URL <https://doi.org/10.1093/restud/rdad013>.

Pavel Savor and Mungo Wilson. How much do investors care about macroeconomic risk? Evidence from scheduled economic announcements. *Journal of Financial and Quantitative Analysis*, 48(2):343–375, 2013.