Rural-urban Migration and Informality

Pei-Ju Liao
National Taiwan University

Ping Wang
Washington University in St. Louis & NBER

Yin-Chi Wang
National Taipei University

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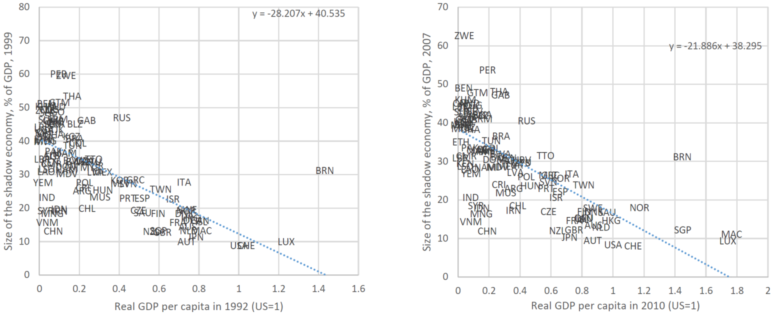
Motivation

- Large rural-urban migration in the early stage of economic development
- Sizable informality, particularly in Africa and Latin America
- High dispersion in the intensity of migration and the extent of informality
- Research questions: as informal urban sector could be a potential outlet for migrant workers,
  1. what are the interplays between rural migration and informality?
  2. what are their macroeconomic consequences?
  3. what are the implications of migration and industry policies?

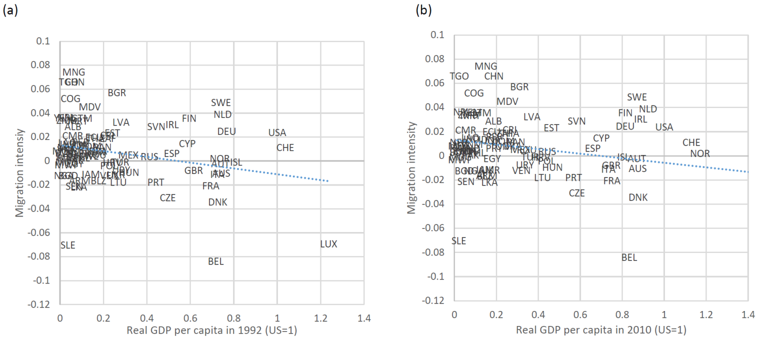
Observations: Shadow economy and real GDP

(a) (b)



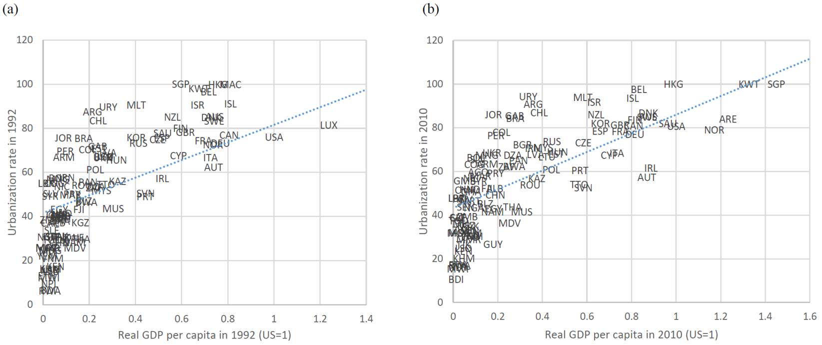
Data source: Size of shadow economy is from Medina and Schineider (2018). Real GDP per capita is from the PWT 10.0.

Observations: Migration intensity and real GDP



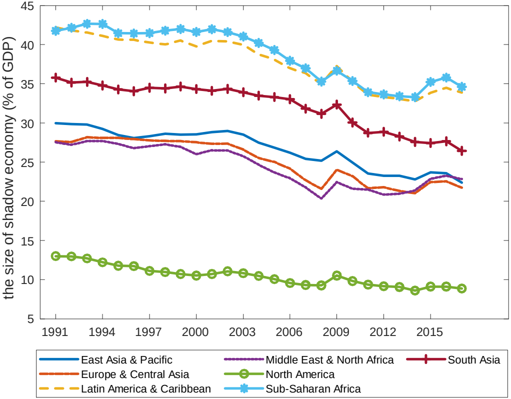
Data source: Migration intensity is computed using rural-urban employment ratio, which is taken from Global Jobs Indicator Data Base (JOIN), World Bank. See Liao, Wang, Wang and Yip (2022) for details of computation for migration intensity. Real GDP per capita data is taken from the PWT 10.0.

Observations: Urbanization and real GDP



Data source: Urbanization rates are taken from WDI. Real GDP per capita is from the PWT 10.0.

Size of shadow economy by region



Data source: The size of shadow economy is taken from Medina and Schneider (2019).

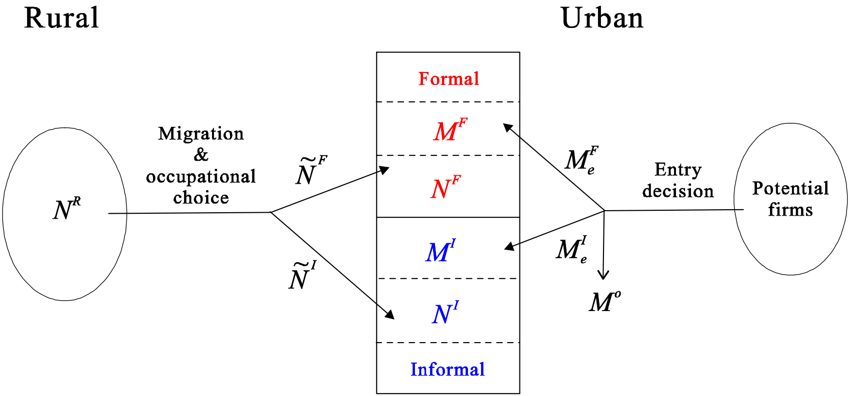
Related literature

- Rural-urban migration
  - Todaro (1969) and Harris and Todaro (1970): pioneers
  - **Lucas (2004)** studies human capital externality in urban areas
  - Liao, Wang, Wang and Yip (2022) consider tertiary education as a rural-urban migration channel
- Informal economy and development:
  - **Ulyssea (2018)** considers extensive and intensive margins of informality and finds that lower informality can be, but not necessarily, associated with higher output, TFP, or welfare.
  - Yuki (2007) emphasizes the important role of human capital accumulation in explaining the expansion of the urban formal/informal sector in the process of urbanization and development.

Environment

- Two geographical regions: Urban and rural
- Forms of production
  - Urban : { Formal sector: Melitz (2003) framework; Informal sector: DRTS technology, no fixed cost; Hand-to-mouth self-employed entrepreneurs }
  - Rural: Backyard farming
- Two groups
  - Urban firms: organizational choice of formal vs. informal; exit with a probability δ in every period
  - Workers:
    - Rural workers (our focus): migration and occupational choice decisions
    - Urban workers: passive, "inheriting" parents' occupations

Model overview



Rural households – production

- Rural working-age agents are hand-to-mouth farmers, relying on farming to make a living.
- Denote N^R as the number of rural agents cultivating rural land during a period, and Ω is total rural land. Total output in rural area is:

$$Q = z\left(N^R\right)^{\vartheta}\Omega^{1-\vartheta}, \quad z > 0,$$

  where $z > 0$ is the farming technology and $\vartheta \in (0, 1)$ is the rural labor income share.
- Normalize total rural land to one. A rural farmer's output is

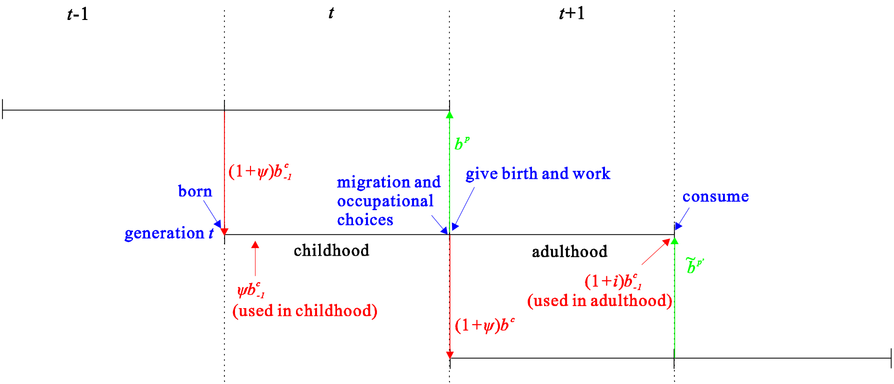
$$q = z\left(N^R\right)^{\vartheta - 1}$$

  and thereby a rural farmer's income, in value, is the total value of her output and equals $P^R q$.

Rural households

- Each agent lives for two periods: childhood and adulthood.
- Consider an agent born in period t (a generation-t agent):
  - Childhood: attached to parent, receive $(1+\psi)\,b^c_{-1}$ transfer from parents and use up ψb^c_{-1} in childhood.
  - Adulthood: own one unit of labor and work in period $t{+}1$.
- At the end of childhood, make migration and occupational choice decisions:
  - If staying in rural: produce rural goods of q and earn an income of $P^R q$.
  - If migrating to cities: $\begin{cases} \text{earn } w^F \text{ if choosing the formal sector.} \\ \text{earn } w^I \text{ if choosing the informal sector.} \end{cases}$
- After the migration and the occupational choice decisions, they become adults:
  - Upon becoming adults: give remittance of b^P to parents.
  - Given birth to a child and transfer $(1+\psi)\,b^c$ to child, with ψb^c being paid as child-rearing cost.
  - Right before the end of the adulthood, receive transfer of $\tilde{b}^{p\prime}$ from children, consume c, and exit the market.

Timeline


t-1
t
t+1
b^p
$(1+\psi)b^c_{-1}$
born
generation t
migration and occupational choices
give birth and work
consume
childhood
adulthood
ψb^c_{-1}
(used in childhood)
$(1+i)b^c_{-1}$
(used in adulthood)
$(1+\psi)b^c$
$\tilde{b}^{p'}$

Rural households - staying in rural

- Denote V^R as the value function of staying in rural area:

$$V^R \equiv \max_{c^R, b^c, b^p} c^R + \beta^c u\left(b^c\right) + \beta^p u\left(b^p\right)$$

  - c^R : consumption
  - β^c (β^p): the altruistic factor towards child (parent)
  - $u' > 0, u'' < 0$

- A generation-t rural agent's lifetime budget constraint:

$$P^R c^R + (1+\psi)\, b^c + b^p = P^R q + (1+i)\, b^c_{-1} + \tilde{b}^p$$

  - ψ : child-rearing cost markup
  - P^R : price of rural-produced goods
  - b^c : transfer to child; $(1+\psi)\, b^c_{-1}$: total transfer from parents, with b^c_{-1} being carried over from childhood to adulthood
  - b^p: transfer to parent measured in generation-t's value unit
  - $\tilde{b}^{p'}$: amount of transfer from children received by generation-t agents

Rural households - migrating to cities

- The value of being a worker in urban formal sector V^F is:

$$V^F = \max_{c^U, b^c, b^p} c^U + \beta^c u(b^c) + \beta^p u(b^p)$$

$$\text{where } c^U \equiv c^F + \lambda c^I, \quad \lambda \in (0,1)$$

$$s.t. \;\; P^F c^F + P^I c^I + (1+\psi) b^c + b^p = (1-\tau^w) w^F + (1+i) b^c_{-1} + \tilde{b}^{p'}$$

  - c^U : consumption on urban traded goods
  - c^F : consumption on formal goods
  - c^I : consumption on informal goods
  - λ : quality of informal goods relative to formal goods perceived by urban agents
  - w^F : urban formal wage
  - τ^w : labor income tax rate

- The value of being a worker in urban informal sector V^I takes the same form as that for V^F except the income $(1-\tau^w) w^F$ in the budget constraint is replaced by w^I.

Migration decision and occupational choice

- Rural workers are heterogeneous in migration disutility $(1/\mu)$ and work-effort disutility $(1/\epsilon)$ in the formal sector and take the draws of μ and ϵ from Pareto distribution $G_\mu(\mu)$ and $G_\epsilon(\epsilon)$ at birth.

- Besides incurring disutility when working, formal workers need to pay income taxes τ^w, while informal workers do not have to pay taxes. Hence, w^F must be higher than w^I so that, at least some rural migrants are willing to work for the formal sector.

- A rural worker makes migration decision, plus occupational choice if needed, before entering adulthood:

  The 1st stage decision $(\mathbb{I}^M)$ Whether to migrate to cities?
  The 2nd stage decision $(\mathbb{I}^W)$ If migrating to cities, which sector to devote to?

- We solve rural workers' problem backwardly by solving the 2nd stage problem first.

The 2nd stage problem: Occupational choice 1

- The value function V^M of a migrant worker is given by

$$V^M(\epsilon) = \max_{\mathbb{I}^W \in \{0,1\}} \mathbb{I}^W \left(V^F - \frac{\chi_\epsilon}{\epsilon} \right) + \left(1 - \mathbb{I}^W\right) V^I$$

where $\chi_\epsilon > 0$ is the relative disutility of being a formal worker and $\mathbb{I}^W$ is an indicator function such that

$$\mathbb{I}^W = \begin{cases} 1 & \text{if the rural migrant works in the formal sector,} \\ 0 & \text{if the rural migrant works in the informal sector.} \end{cases}$$

That is

$$\mathbb{I}^{W*} = \arg \max_{\mathbb{I}^W \in \{0,1\}} \mathbb{I}^W \left(V^F - \frac{\chi_\epsilon}{\epsilon} \right) + \left(1 - \mathbb{I}^W\right) V^I$$

- To focus on the nondegenerate equilibrium, we impose the following condition:

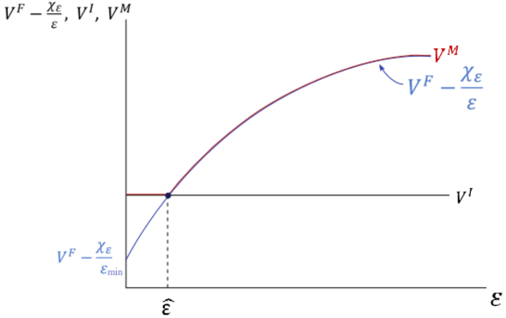
Condition F $\epsilon_{\min} < \frac{\chi_\epsilon}{V^F - V^I}$.

The 2nd stage problem: Occupational choice 2

- Under **Condition F**, since $V^F - \frac{\chi_\epsilon}{\epsilon}$ is strictly increasing in ϵ and V^I is constant in ϵ, $\exists$ a single cutoff $\hat{\epsilon}$ such that

$$\mathbb{I}^{W^*} = \begin{cases} 1 & \text{if } \epsilon \geq \hat{\epsilon}, \\ 0 & \text{if } \epsilon < \hat{\epsilon}, \end{cases}$$

with $\hat{\epsilon} = \frac{\chi_\epsilon}{V^F - V^I}$.



The 1st stage problem: Migration decision

- Denote V as the value function for a rural agent with disutility (μ, ϵ):

$$V(\mu, \epsilon) = \max_{\mathbb{I}^M \in \{0,1\}} \mathbb{I}^M \cdot \left[V^M(\epsilon) - \frac{\chi_\mu}{\mu} \right] + \left(1 - \mathbb{I}^M\right) V^R$$

where $\chi_\mu > 0$ is the relative magnitude of migration disutility and $\mathbb{I}^M$ is an indicator function such that

$$\mathbb{I}^M = \begin{cases} 1 & \text{if the rural worker decides to migrate,} \\ 0 & \text{if the rural worker decides to stay.} \end{cases}$$

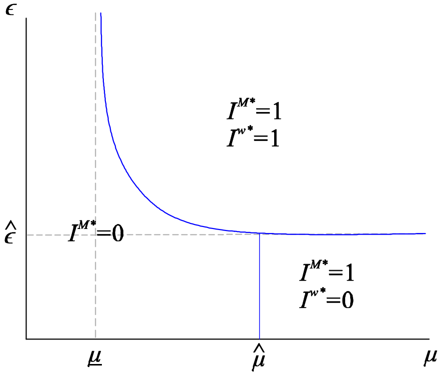
- **Condition IM** $\mu_{\min} < \frac{\chi_\mu}{V^I - V^R}$.
- Combining the two stages implies:

$$\left(\mathbb{I}^{M*}, \mathbb{I}^{W*}\right) = \begin{cases} (0, \cdot) & \text{for } \Gamma(\mu, \epsilon) < 0, \mu < \hat{\mu}, \\ (1, 0) & \text{for } \Gamma(\mu, \epsilon) < 0, \mu \geq \hat{\mu}, \\ (1, 1) & \text{for } \Gamma(\mu, \epsilon) \geq 0. \end{cases} \quad \text{(IB)}$$

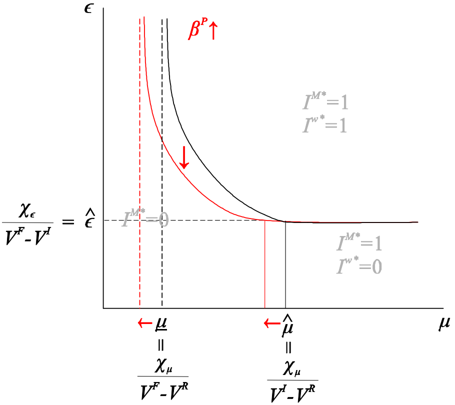
We are ready to write down the (IB) and the optimal migration and occupational choices for rural potential workers.

The IB for migration and occupational choice

- Let $\underline{\mu} = \frac{\chi_\mu}{V^F - V^R}$ be the smallest μ such that an agent with $\epsilon \to \infty$ is willing to migrate to cities.
- The figure shows the indifference boundary (IB) and the optimal decisions:



The IB with more remittance enjoyment or less migration disutility



Note: The figure shows the case where the price of goods in urban is relative expensive than goods in rural, $P^F > P^R$.

Workers' laws of motion

- The joint distribution of (μ,ϵ) is

$$\begin{aligned} \Lambda^R &= \{(\mu,\epsilon)\,|\Gamma(\mu,\epsilon)<0,\mu<\hat{\mu}\}, \\ \Lambda^I &= \{(\mu,\epsilon)\,|\Gamma(\mu,\epsilon)<0,\mu\geq\hat{\mu}\}, \\ \Lambda^F &= \{(\mu,\epsilon)\,|\Gamma(\mu,\epsilon)\geq 0\}=1-\Lambda^R-\Lambda^I. \end{aligned}$$

- Denote N_t^F, N_t^I and N_t^R the masses of workers in the formal sector, the informal sector, and rural agricultural sector at the beginning of period t.
- Migrant formal and informal workers and total migrant workers in period $t+1$ are:

$$\tilde{N}_{t+1}^F = N_t^R\Lambda^F,\ \tilde{N}_{t+1}^I = N_t^R\Lambda^I,\ \tilde{N}_{t+1} = \tilde{N}_{t+1}^F + \tilde{N}_{t+1}^I.$$

- Total workers in urban formal, urban informal and rural sectors evolve according to:

$$\begin{aligned} N_{t+1}^F &= N_t^F + N_t^R\Lambda^F, \\ N_{t+1}^I &= N_t^I + N_t^R\Lambda^I, \\ N_{t+1}^R &= N_t^R\left[1-\Lambda^F-\Lambda^I\right] \end{aligned}$$

Urban production – overview

- Total mass of potential urban firms equals M (exogenously given).
- Three types of organization: Upon paying a fixed cost of $\bar{f}_e = w^F f_e$ to enter (where f_e is in terms of labor), urban potential firms make productivity draw and choose to be:
  - Formal firm: Output level depends on individual specific productivity φ.
  - Informal firm: Output level does not depend on individual specific productivity.
  - Urban hand-to-mouth self-employed entrepreneur.
- An one-time managerial cost for establishing and managing a firm (one owner per firm):

$$d(\varphi) = \frac{\xi \cdot 1}{\varphi}$$

  with $\lim_{\varphi \to \varphi_{\min}} d(\varphi) = \frac{\xi \cdot 1}{\varphi_{\min}}$ and $\lim_{\varphi \to \infty} d(\varphi) = 0$.
- All potential urban firms exit with probability δ in every period.

Urban production – formal sector 1

- Following Melitz (2003), urban formal good Y^F is produced by:

$$Y^F = \left[\int_{\omega \in \Omega} y^F(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1$$

where $y^F(\omega)$ is the quantity of good ω produced by an urban formal firm, with $\rho \equiv \sigma / (\sigma - 1)$.

- Output/consumption and revenues for each variety ω:

$$y^F(\omega) = Y^F \left[\frac{p^F(\omega)}{P^F} \right]^{-\sigma} \text{ and } r^F(\omega) = R^F \left[\frac{p^F(\omega)}{P^F} \right]^{1-\sigma}$$

where $P^F \equiv \left[\int_{\omega \in \Omega} p^F(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$ and $R^F \equiv \int_{\omega \in \Omega} r^F(\omega) d\omega$, $Y^F \equiv R^F / P^F$.

- Following Krugman (1980) with labor as the only factor of production, labor requirement for production of an urban formal firm with productivity φ is:

$$\ell^F = \bar{\ell}^F + x + \ell_v^F = e^{-S} f + x + \frac{y^F}{\varphi},$$

where $\bar{\ell}^F = e^{-S} f$ is the fixed overhead cost and x the government regulatory cost.

Urban production – formal sector 2

- Denote the wage rate paid by urban formal firms as w^F. Monopolistic pricing implies

$$p^F(\varphi) = \frac{w^F}{\rho\varphi}$$

implying $r^F(\varphi) = R^F \left(P^F\rho\varphi\right)^{\sigma-1}\left(w^F\right)^{1-\sigma}$ and $y^F(\varphi) = Y^F\left(P^F\rho\varphi\right)^{\sigma}\left(w^F\right)^{-\sigma}$.

- So more productive urban firms produce more and earn higher revenues:

$$\frac{y^F(\varphi_1)}{y^F(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma} \text{ and } \frac{r^F(\varphi_1)}{r^F(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}.$$

- Subject to a corporate income tax rate of τ^C, an urban formal firm with productivity φ has the profit of:

$$\begin{aligned}\underset{+}{\pi^F(\varphi)} &\equiv \left(1-\tau^C\right)\left[r^F(\varphi) - w^F\ell^F(\varphi)\right] \\ &= \left(1-\tau^C\right)\left[\frac{R^F\left(P^F\rho\varphi\right)^{\sigma-1}\left(w^F\right)^{1-\sigma}}{\sigma} - w^F\left(e^{-S}f + x\right)\right].\end{aligned}$$

Urban production – formal sector 3

- Since all firms with productivity φ charge the same price $p^F(\varphi)$, P^F can be rearranged as

$$P^F = \left[\int_0^{+\infty} p^F(\varphi)^{1-\sigma} M^F \mu(\varphi)\, d\varphi\right]^{\frac{1}{1-\sigma}}$$

where

  - $M^F \equiv$ mass of operative formal firms in equilibrium.
  - $\mu(\varphi) \equiv$ (conditional) pdf of productivity levels of operative formal firms in equilibrium.

- Define $\bar{\varphi}^F \equiv \left[\int_0^{+\infty} \varphi^{\sigma-1}\mu(\varphi)\, d\varphi\right]^{\frac{1}{\sigma-1}}$ as the average productivity of urban operative formal firms. Then,

$$P^F = \frac{w^F M^{F\frac{1}{1-\sigma}}}{\rho\bar{\varphi}^F} = P^F\left(\bar{\varphi}^F\right).$$

$$R^F = M^F r^F\left(\bar{\varphi}^F\right), \;\; \Pi^F = M^F \pi^F\left(\bar{\varphi}^F\right), \;\; Y^F = M^{F\frac{\sigma}{\sigma-1}} y^F\left(\bar{\varphi}^F\right).$$

Urban production – informal sector

- The technology of urban informal firms is

$$y^I = a^I(\ell^I)^\gamma, \quad \gamma \in (0,1)$$

  where $a^I > 0$ is the technology scaling factor for urban informal sector.
- The profit of an informal firm is:

$$\pi^I = (1-\zeta)\left(P^I y^I - w^I \ell^I\right)$$

  where $\zeta = \zeta_0\left(\bar{\zeta} + \left(1-\bar{\zeta}\right)\right) \in (0,1)$ is the probability of being fined $(\zeta_0\bar{\zeta})$ and asked for bribes $\left(\zeta_0\left(1-\bar{\zeta}\right)\right)$, and $\bar{\zeta}$ is the share of firms being fined.
- Assume that informal firms pay their employees at a wage rate $w^I < VMPL = P^I\gamma a^I\left(\ell^I\right)^{\gamma-1}$:

$$w^I = \kappa P^I \gamma a^I \left(\ell^I\right)^{\gamma-1}$$

  with $\kappa \in (0,1]$ being the informal wage markdown.
- The profit of an informal firm can be rewritten as

$$\pi^I = (1-\zeta)\, P^I y^I (1-\kappa\gamma) > 0.$$

Government's technology

- The government provides public infrastructure that helps lowering formal firms' fixed costs of production in operation:

$$S = S_0 G_g,$$

where $S_0 > 0$ is the government's technology scaling factor, and G_g is government expenditure.

- Total taxes T collected by the government in period t is:

$$T = \tau^W w^F N^F + \tau^C M^F \bar{\pi}^F + \zeta_0 \bar{\zeta} M^I \pi^I,$$

where M^F and M^I are masses of formal and informal firms.

- Assume that the government runs a balanced budget in every period:

$$T = G_g.$$

Formal vs. informal cutoff profit 1

- In a stationary equilibrium, a firm either exits immediately, if it finds not worth running a business, or produces and earns the same profits in each period.
- The *expected value* of a firm with productivity φ is:

$$v(\varphi) = \max\{\sum_{t=0}^{\infty} (1-\delta)^t \pi^I, \sum_{t=0}^{\infty} (1-\delta)^t \pi^F(\varphi)\} = \max\{\frac{\pi^I}{\delta}, \frac{\pi^F(\varphi)}{\delta}\}$$

- Recall that $\pi^F(\varphi)$ is increasing in φ with $\lim_{\varphi\to\infty} \pi^F(\varphi) = \infty$, and $\pi^I > 0$. There exists a formal vs. informal cutoff productivity such that

$$\hat{\varphi} \equiv \inf\left\{\varphi \geq 0 : \pi^F(\varphi)/\delta \geq \pi^I/\delta\right\}.$$

- As π^I depends on w^I and P^I, an urban firm will choose to operate as an informal firm if

$$\frac{\pi^I(\hat{\varphi})}{\delta} - \frac{\xi}{\varphi} \geq v^o$$

where $v^o > 0$ is the outside option for an urban firm.

Formal vs. informal cutoff profit 2

- Denote $\tilde{\varphi} < \hat{\varphi}$ such that among non-formal firms with $\varphi < \hat{\varphi}$, $\frac{G(\tilde{\varphi})}{G(\hat{\varphi})}$ of them choose not to participate, and $\frac{G(\hat{\varphi})-G(\tilde{\varphi})}{G(\hat{\varphi})}$ of them choose to operate as informal firms.
- Under $\tilde{\varphi}$ and $\hat{\varphi}$, we can compute the conditional formal firms' productivity $\mu(\varphi)$ and the probability of being a formal firm, an informal firm, and a hand-to-mouth self-employed entrepreneur:

$$\begin{aligned}
\mu(\varphi) &= \begin{cases} \frac{g(\varphi)}{1-G(\hat{\varphi})} & \text{if } \varphi \geq \hat{\varphi}, \\ 0 & \text{if } \varphi < \hat{\varphi}. \end{cases} \\
P_r^{\text{formal}} &= 1 - G(\hat{\varphi}), \\
P_r^{\text{informal}} &= G(\hat{\varphi}) - G(\tilde{\varphi}). \\
P_r^{\text{o}} &= G(\tilde{\varphi}).
\end{aligned}$$

Formal vs. informal cutoff profit 3

- The average productivity and average profit are:

$$\bar{\varphi}^F(\hat{\varphi}) = \left[\frac{1}{1-G(\hat{\varphi})}\int_{\hat{\varphi}}^{+\infty}\varphi^{\sigma-1}g(\varphi)\,d\varphi\right]^{\frac{1}{\sigma-1}}$$

$$\bar{\pi}^F = \left(1-\tau^C\right)r^F\left[\bar{\varphi}^F(\hat{\varphi})\right]/\sigma - \left(1-\tau^C\right)w^F\left(e^{-S}f+x\right)$$

- At the cutoff $\hat{\varphi}$, we have $\pi^F(\hat{\varphi}) = \pi^I$, so $r^F(\hat{\varphi}) = \sigma\left[w^F\left(e^{-S}f+x\right)+\frac{\pi^I}{(1-\tau^C)}\right]$.
- **Formal-informal Cutoff Profit (FICP)** By plugging in the derived $r^F(\hat{\varphi})$ into $\bar{\pi}^F$, we can rewrite $\bar{\pi}^F$ as

$$\bar{\pi}^F = \left\{\left[\left(1\text{-}\tau^C\right)w^F\left(e^{-S}f+x\right)+\pi^I\right]\left[\left(\frac{\bar{\varphi}^F(\hat{\varphi})}{\hat{\varphi}}\right)^{\sigma-1}\text{-}1\right]+\pi^I\right\} \qquad \text{(FICP)}$$

- Under a given $\tilde{\varphi}$ and π^I, the FICP condition
  - is downward sloping in $\hat{\varphi}$, with $\lim_{\hat{\varphi}\to 0}\bar{\pi}^F = \infty$ and $\lim_{\hat{\varphi}\to\infty}\bar{\pi}^F = \pi^I$.
  - behaves similar to the ZCP in Melitz (2003).

Establishment condition 1

- **Assumption** (Nondegenerate) $v^o > w^F f_e$.
- The expected value of establishing an urban firm satisfies:

$$P_r^{\text{informal}} \cdot \left(\frac{\pi^I}{\delta} - \frac{\xi}{\tilde{\varphi}} \right) + P_r^{\text{formal}} \cdot \left(\frac{\bar{\pi}^F}{\delta} - \frac{\xi}{\tilde{\varphi}} \right) = v^o$$

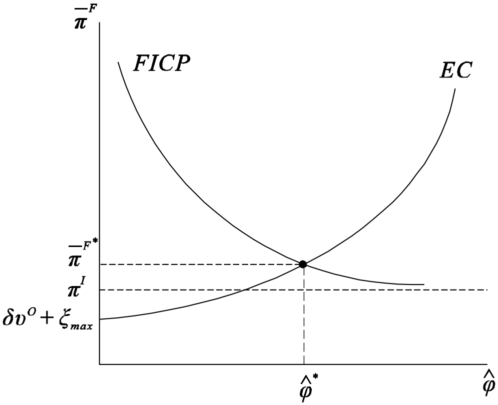
- **Establishment Condition (EC):**

$$\bar{\pi}^F = \frac{\delta}{1 - G(\hat{\varphi})} \left\{ v^o - [1 - G(\tilde{\varphi})] \left(\frac{\pi^I(\hat{\varphi})}{\delta} - \frac{\xi_{\max}}{\tilde{\varphi}/\varphi_{\min}} \right) \right\} + \pi^I(\hat{\varphi}) \qquad \text{(EC)}$$

  where $\xi_{\max} \equiv \frac{\xi}{\varphi_{\min}}$ is the maximum managerial cost for establishing and managing a firm.
- In EC,
  - $\bar{\pi}^F$ is upward sloping in $\hat{\varphi}$ under a given $\tilde{\varphi}$ and π^I;
  - $\bar{\pi}^F$ is increasing in $\tilde{\varphi}$ if the markup of informal over self-employed is less than the shaped parameter, η_{φ}.

FICP and EC

- Under a given $\tilde{\varphi}$ and π^I, the upward-sloping EC and the downward-sloping FICP intersect and determine the unique equilibrium $\left(\hat{\varphi}^*, \bar{\pi}^{F*}\right)$.



Stationary equilibrium conditions 1

Firm side

- Total potential firms as $M \equiv M^F + M^I + M^O$ is exogenously given.
- Denote M^i_- $(i = F, I)$ as the mass of firms before the death of existing firms, the entrance of new firms and the migration of rural migrant workers; M^i_e the total mass of newly entered firms and M^i_{en} as the net increase in the mass of firms:

$$\begin{aligned} M^i &= (1-\delta)\, M^i_- + M^i_e \text{ and } M^i_e = M^i_{en} + \delta M^i_- . \\ M^F(\underset{-}{\hat{\varphi}}) &= \left[1 - G(\hat{\varphi})\right] M, \\ M^I(\tilde{\varphi}, \hat{\varphi}) &= \left[G(\hat{\varphi}) - G(\tilde{\varphi})\right] M. \end{aligned}$$

- Given M^F_- and M^I_-, the net increase in formal and informal firms are:

$$M^F_{en}\left(\hat{\varphi}; M^F_-\right) = M^F(\hat{\varphi}) - M^F_- \text{ and } M^I_{en}(\underset{-}{\tilde{\varphi}}, \underset{+}{\hat{\varphi}}; \underset{-}{M^I_-}) = M^I(\tilde{\varphi}, \hat{\varphi}) - M^I_-$$

- Stationary state for firms requires that the mass of firms grows at a constant rate in order to accommodate migrant workers.

Stationary equilibrium conditions 2

Informal wage rate

- In equilibrium, $P^I = \lambda P^F(\hat{\varphi})$. For an informal firm with $\tilde{\varphi}$, its profit must satisfy $\frac{\pi^I(\hat{\varphi})}{\delta} - \frac{\xi}{\tilde{\varphi}} = v^o$. Since $\frac{\xi}{\tilde{\varphi}} = \frac{\xi_{\max}}{\tilde{\varphi}/\varphi_{\min}}$, we obtain

$$w^I(\underset{-}{\hat{\varphi}}; \underset{+}{\tilde{\varphi}}; \underset{-}{\xi_{\max}}, \underset{-}{\zeta_0}, \underset{+}{\lambda}) = \gamma \left[\frac{\delta}{(1-\gamma)(1-\zeta)} \left(v^o + \frac{\xi_{\max}}{\tilde{\varphi}/\varphi_{\min}} \right) \right]^{-\frac{1-\gamma}{\gamma}} \left(\lambda a^I P^F(\underset{-}{\hat{\varphi}}) \right)^{\frac{1}{\gamma}},$$

  where w^I is decreasing in $\hat{\varphi}$.

- From the labor demand for individual informal firms, we have:

$$\ell^I(\underset{+}{\hat{\varphi}}; \underset{-}{\tilde{\varphi}}; \underset{+}{\xi_{\max}}, \underset{+}{\zeta_0}, \underset{-}{\lambda}, \underset{+}{\kappa}) = \left(\frac{\kappa}{\gamma} \right)^{\frac{1}{1-\gamma}} \left[\frac{\delta}{(1-\gamma)(1-\zeta)} \left(v^o + \frac{\xi_{\max}}{\tilde{\varphi}/\varphi_{\min}} \right) \frac{1}{\lambda a^I P^F(\underset{-}{\hat{\varphi}})} \right]^{\frac{1}{\gamma}}$$

  with ℓ^I increasing in $\hat{\varphi}$.

Stationary equilibrium conditions 3

Informal wage rate

- Define $W^I \equiv w^I \ell^I$ as the total labor cost of an urban informal firm. We have:

$$W^I(\underset{0}{\hat{\varphi}\left(x, \tau_c, S_0\right)}; \underset{-}{\tilde{\varphi}}; \underset{+}{\xi_{\max}}, \underset{+}{\zeta_0}, \underset{0}{\lambda}, \underset{+}{\kappa}) = \left(\frac{\kappa}{\gamma^\gamma}\right)^{\frac{1}{1-\gamma}} \frac{\delta}{(1-\gamma)(1-\zeta)} \left(v^o + \frac{\xi_{\max}}{\tilde{\varphi}/\varphi_{\min}}\right).$$

That is, for given $\tilde{\varphi}$, total labor cost of an informal firm is independent of $\hat{\varphi}$ and λ, and is increasing in $\xi_{\max}$, ζ and κ.

Stationary equilibrium conditions 4

Labor market equilibrium

- Define $(i = F, I)$:
  - L^i and L^i_- (N^i and N^i_-): firm side's (household side's) total workers in sector i after and before all decisions.
  - $\tilde{N}^i$: new migrants in sector i
- (Aggregating from firm side) Define $\bar{\ell}^F_p(\hat{\varphi}) = (\sigma - 1)\frac{\bar{\pi}^F(\hat{\varphi})}{w^F} + \sigma\left(e^{-S}f + x\right)$. Total workers in sector i is :

$$
\begin{aligned}
L^F &= \left[(1-\delta) M^F_- + M^F_e\right] \bar{\ell}^F_p(\hat{\varphi}) + M^F_e f_e, \\
L^I &= (1-\delta) M^I_- \ell^I(\hat{\varphi}; \cdot) + M^I_e \left(\frac{w^F}{w^I(\hat{\varphi}; \cdot)} f_e + \ell^I(\hat{\varphi}; \cdot)\right).
\end{aligned}
$$

- (Aggregating from household side) Total workers in sector i is :

$$L^F = \tilde{N}^F + L^F_- \quad \text{and} \quad L^I = \tilde{N}^I + L^I_-,$$

where $L^F_- = M^F_- \ell^F(\bar{\varphi}^F)$ and $L^I_- = M^I_- \ell^I(\hat{\varphi}; \cdot)$.

Stationary equilibrium conditions 5

Labor market equilibrium

- Population laws of motion:

$$N^F = \tilde{N}^F + N^F_-,$$
$$N^I = \tilde{N}^I + N^I_-.$$

- Denote θ^i_j as the growth rate of j in sector i. In a stationary equilibrium,

$$\frac{\theta^F_M}{\theta^F_N} = \frac{N^F_- / M^F_-}{\bar{\ell}^F + f_e},$$

and

$$\frac{\theta^I_M}{\theta^I_N} = \frac{N^I_- / M^I_-}{\ell^I(\hat{\varphi};\cdot) + \frac{w^F}{w^I(\hat{\varphi};\cdot)} f_e}.$$

Stationary equilibrium conditions 6

Labor market equilibrium

- The labor market equilibrium conditions are:

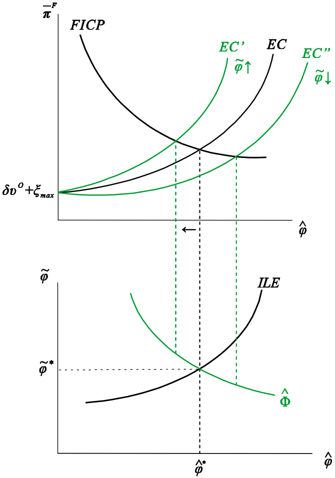
$$\tilde{N}^F = \delta M^F_- f_e + M^F_{en}\left(\bar{\ell}^F_p(\hat{\varphi}) + f_e\right), \quad \text{(FLE)}$$

$$\tilde{N}^I(\hat{\varphi}) = \frac{w^F}{w^I}\left\{\delta M^I_- f_e + \left[\Theta + \frac{[G(\hat{\varphi}) - G(\tilde{\varphi})]}{[1-G(\hat{\varphi})]}M^F_- - M^I_-\right]\left(\frac{W^I}{w^F} + f_e\right)\right\}, \quad \text{(ILE)}$$

where $w^I = w^I(\underset{-}{\hat{\varphi}}; \underset{+}{\tilde{\varphi}}; \underset{-}{\xi_{\max}}, \underset{-}{\zeta_0}, \underset{+}{\lambda})$, $W^I = W^I(\underset{-}{\hat{\varphi}}(x, \underset{0}{\tau_c}, S_0); \underset{+}{\tilde{\varphi}}; \underset{+}{\xi_{\max}}, \underset{0}{\zeta_0}, \underset{+}{\lambda}, \kappa)$, and
$\Theta = \Theta(\underset{+}{\hat{\varphi}}; \underset{-}{\xi_{\max}}, \underset{-}{x}, \underset{+}{S_0}, \underset{-}{f_e}) \equiv \frac{[G(\hat{\varphi})-G(\tilde{\varphi})]}{[1-G(\hat{\varphi})]}\frac{\tilde{N}^F - \delta M^F_- f_e}{\left(\underset{+}{\bar{\ell}^F_p(\hat{\varphi})} + f_e\right)}$.

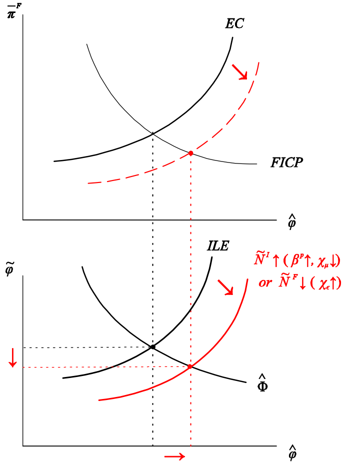
- From the **informal labor market clearing condition (ILE)** above, we can derive: $\tilde{\varphi} = \tilde{\Phi}(\underset{+}{\hat{\varphi}})$, which is positively sloped in plane-$(\hat{\varphi}, \tilde{\varphi})$.
  - The FICP, EC and ILE, together with households' optimal decisions determine the equilibrium in this economy.

The equilibrium



- Given $\{N^i_-, M^i_-\}$, $i = \{F, I\}$, EC shifts up to the left when $\tilde{\varphi}$ is higher, intersecting with FICP at a lower $\hat{\varphi}$. EC shifts down to the right when $\tilde{\varphi}$ is lower, intersecting with FICP at a higher $\hat{\varphi}$.
- We thus combine EC and FICP to derive $\tilde{\varphi} = \hat{\Phi}(\underset{(-)}{\hat{\varphi}})$, which is negatively sloped in plane-$(\hat{\varphi}, \tilde{\varphi})$.
- Also note that in equilibrium, $\pi^I(\hat{\varphi}) = \delta(v^o + \frac{\xi}{\tilde{\varphi}})$.

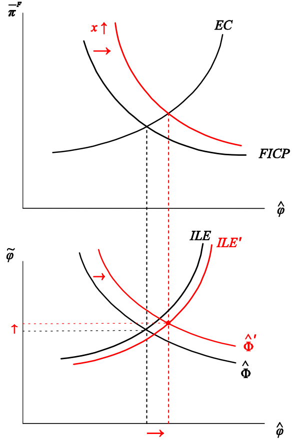
General equilibrium comparative statics 1



When households are more altruistic in remittance giving ($\beta^p \uparrow$) or migration disutility is lower ($\chi_\mu \downarrow$)

- $\tilde{N}^I$ increases, shifting ILE to the right.
- In response, EC must shift to the right.
- The result is an increase in $\hat{\varphi}$ and a decrease in $\tilde{\varphi}$ – an expansion of the informal sector.
- Positive correlation between remittance and informality, as well as migration and informality
- A higher aggregate informal output, but an ambiguous effect on aggregate formal output or aggregate output.
- Similar effect with a higher disutility from formal employment ($\chi_\epsilon \uparrow$): $\tilde{N}^F \downarrow$.

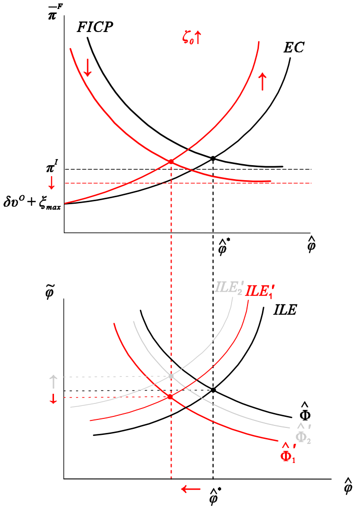
General equilibrium comparative statics 2



When the regulatory cost x increases,

- FICP rotates up to the right (keeping π^I constant), shifting $\hat{\Phi}$ up to the right.
- Less labor demand in the formal sector shifts ILE to the right.
- In equilibrium,
  - $\hat{\varphi}$ increases, meaning that running a formal firm is more "costly" and needs a higher productivity to compensate the increase in the regulatory cost.
  - $\tilde{\varphi}$ also increases, meaning that the overall threshold in running a business increases due to the increase in cost of running businesses.
- Depending on the relative increases in $\hat{\varphi}$ and $\tilde{\varphi}$, the size of the informal sector may shrink or expand.
- High dispersion in the size of informality.
- Other exercise: $S_0 \downarrow$.

General equilibrium comparative statics 3



When the probability of being fined or asked for bribes is higher ($\zeta_0 \uparrow$)

- π^I decreases, FICP shifts down, EC rotates up, and $\hat{\Phi}$ locus shifts down. Besides, ILE shifts up.
- In equilibrium,
  - $\hat{\varphi}$ decreases because the informal sector is relatively less profitable compared to the formal sector.
  - $\tilde{\varphi}$ could increase or decrease, depending on the shifts in ILE and Φ locus.
- This leads to an expansion of the formal sector but an ambiguous effect on the informal sector, though each informal firm is down-sized.
- Formal output share and formal employment share both rise.

Takeaway

- We model rural-urban migration and occupational choices and urban firm organizational choices:
  - A higher altruism on remittance or a lower migration disutility implies:
    - an expansion of the informal sector;
    - a positive correlation between remittance and informality, as well as between migration and informality;
    - a higher aggregate informal output, but an ambiguous effect on aggregate formal output or aggregate output.
  - When running a formal firm is more costly (e.g., a larger regulatory cost or a worse infrastructure provision):
    - the size of the informal sector may shrink or expand, depending on the relative changes of the two productivity cut-offs;
    - this implies a large variation in the size of informality.