# On the Elasticity of Substitution between Labor and ICT and IP Capital and Traditional Capital

Vahagn Jerbashian*

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**Abstract**

I estimate a nested CES production function for 9 European countries over 1996-2020 using EU KLEMS data, distinguishing between information and communication technologies (ICT), intellectual property (IP) capital, and traditional capital. I assume that the aggregate output is produced using labor and these capital types and allow for differences in the elasticities of substitution between labor, an aggregate of ICT and IP capital, and traditional capital. The estimated elasticity of substitution between ICT and IP capital is strictly below one implying gross complementarity. ICT and IP capital together are gross substitutes for labor while traditional capital is a gross complement. The results imply that the fast pace of technological progress and accumulation in ICT and IP capital are responsible for almost the entire fall in labor income share. The imputed labor-aggregate capital elasticity exceeds 1, rising from 1996 to 2008 and falling afterward.

**Keywords:** CES Production Function; Elasticities of Substitution; System of Equations; ICT; IP Capital; Traditional Capital

**JEL classification:** E22; E25; J23; O33

*Universitat de Barcelona, BEAT, CESifo, CREB, and GLO. Email: vahagn.jerbashian@ub.edu.

# 1 Introduction

Macroeconomic models commonly use explicit production technologies that combine labor and capital. The appropriateness of these models depends on the assumptions regarding the production technology including the elasticity of substitution between labor and capital and the direction of technological change.

I use data from the EU KLEMS database for a panel of 9 European countries and the 1996-2020 period and estimate a nested CES production function for total industrial value added together with the corresponding first order conditions. I assume that the production technology utilizes labor, an aggregate of information and communication technologies (ICT) and intellectual property (IP) capital, and the remainder of capital that I call traditional capital. IP capital includes software and patents and the assumption that ICT and IP capital enter into production jointly is motivated by, for example, that computers and software have a joint and complementary use. Moreover, the share of granted patents related to ICT in this period in sample countries was about 25% of the total, and patents related to ICT include methods of ICT applications, for example.

The estimate of the elasticity of substitution between labor and ICT and IP capital is above 1 implying that labor and ICT and IP capital are gross substitutes. The estimate of the elasticity of substitution between labor and traditional capital is below 1 implying that labor and traditional capital are gross complements. Similarly, the estimate of the elasticity of substitution between ICT and IP capital is below 1.

These results can help to explain, for example, the dynamics in the share of labor income in sample countries. I consider a counter-factual scenario where there is no ICT- and IP-related technological progress and no ICT and IP capital accumulation in a simple accounting exercise that abstracts from potential equilibrium effects. I compare the predicted labor income share averaged across countries to the average labor income share. The results from this exercise imply that most of the fall in labor share can be attributed to the fast technological progress and extensive accumulation in ICT and IP capital. In turn, labor income share would have been much higher absent technological progress and accumulation in ICT.

Finally, I derive the elasticity of substitution between labor and aggregate capital using the nested CES production function with different types of capital and compute its values using the estimated values of the parameters of this function. The imputed elasticity of substitution between labor and aggregate capital is greater than 1 implying that labor and aggregate capital are gross substitutes. It is also larger than the elasticity of substitution between labor and the aggregate of ICT and IP capital. It increases during the 1996-2008 period and declines afterward. For comparison, I estimate the elasticity of substitution between labor and aggregate capital using data from the EU KLEMS database and following a similar methodology used for the estimation of the nested CES production function with several types of capital. The estimated value of the elasticity of substitution between labor and aggregate capital appears to be below 1 and much smaller than the imputed values of this parameter. This implies that the specification of the production function can play an important role in determining the value of the estimated elasticity of substitution between labor and aggregate capital.

The values of the elasticity of substitution and the direction of technological change are important for explaining, for example, movements in factor income shares (e.g., Caballero and Hammour, 1998, Karabarbounis and Neiman, 2014). A large number of studies that focus on labor share document that it has fallen. The literature offers competing explanations for this. Karabarbounis and Neiman (2014) use cross-country data and find that labor and capital are gross substitutes. They attribute the fall in labor income share to the rapid fall in prices of capital and capital deepening. Glover and Short (2020) use similar data and challenge these estimates showing that they can be upward biased because of omitted variables. Their estimates indicate that labor and capital are gross complements.[1] The estimates of Glover and Short (2020) suggest that alternative explanations might be in order for the fall in labor income share such as, for example, the rise in product-market concentration and import competition (Autor, Dorn, Katz, Patterson and van Reenen, 2017, Grossman, Helpman, Oberfield and Sampson, 2017).

[1] Herrendorf, Herrington and Valentinyi (2015) also estimate a below one elasticity of substitution between labor and capital. Gechert, Havranek, Irsova and Kolcunova (2022) corroborate this evidence in their meta-analysis of 121 studies.

A few recent studies provide an in-depth analysis of the fall in labor income share by differentiating types of capital (Aum and Shin, 2024, Eden and Gaggl, 2018, 2019, Koh, Santaeulàlia-Llopis and Zheng, 2020). Eden and Gaggl (2018, 2019) attribute the fall in labor income share to the uptake of information and communication technologies (ICT) and the potential high substitutability of these technologies with labor because of, for example, the ease that routine tasks yield to automation (e.g., Acemoglu and Autor, 2011, Autor, Levy and Murnane, 2003, Jerbashian, 2019). ICT is a sum of ICT equipment and software in their study. They estimate the elasticity of substitution between labor and ICT using the first order conditions resulting from canonical firm's optimization problem similarly to, for example, Antràs (2004) and Karabarbounis and Neiman (2014).[2] In turn, Koh et al. (2020) perform an accounting exercise and show that the fall in labor income share can be attributed to the capitalization and the rise of compensation of intellectual property (IP) capital, R&D before 1980 and software after 1980. A very recent and independent study by Aum and Shin (2024) differentiates between ICT and software capital and considers a CES production function that, in the first step, nests labor with ICT equipment and, in the second step, nests this nest with software capital. Aum and Shin (2024) estimate first order conditions using firm-level data from South Korea. Their baseline estimation results suggest gross complementarity between ICT equipment and labor and gross substitutability between software and labor, as well as gross substitutability between software and ICT equipment. Moreover, changes in software compensation but not changes in ICT equipment compensation are largely responsible for the fall in labor income share in South Korea.

This paper differs and contributes to these studies in multiple ways. First, I follow the approach developed and implemented by Grandville (1989), Klump, McAdam and Willman (2007), and León-Ledesma, McAdam and Willman (2010) and jointly estimate a normalized CES production function and first order conditions. The normalization

[2] Krusell, Ohanian, Ríos-Rull and Violante (2000) and Ohanian, Orak and Shen (2023) estimate a nested CES production function with capital and labor inputs, and 2 levels of skills. They show that changes in factor inputs can account for most of the changes in skill premium in the US. Ohanian et al. (2023) also show that the incorporation of ICT capital in the production function can help to explain some of the movements in labor share. This study abstracts from levels of skills because of data limitations.

is motivated by the observation that the elasticity of substitution is defined as a point elasticity and its identification needs benchmark values for the level of production and factor inputs and incomes. It represents the production function in a consistent indexed number form and facilitates the identification of parameters. León-Ledesma et al. (2010) use Monte Carlo simulations to provide comprehensive evidence regarding the superiority of this estimation method for identifying elasticities of substitution together with factor-biased technological change as compared to, for example, the estimation of first-order conditions only and a translog function. The use of this estimation method then can be especially relevant for this study because it attempts to identify these parameters for ICT that have been subject to exceptionally rapid technological progress. Motivated by the joint use of computers and software, I also assume that a CES aggregate between ICT equipment and IP capital, which includes software, is a separate capital input in the production function. I nest this aggregate with labor and then nest that with traditional capital. This allows me to estimate value added production function. Moreover, it allows me to compute the implied elasticity between labor and aggregate capital. The value of this elasticity is greater than 1 though the results suggest that the fall in labor income share can be entirely attributed to progress and accumulation in ICT and IP capital. This is consistent with the juxtaposition of the results of Aum and Shin (2024), Eden and Gaggl (2018, 2019) and Karabarbounis and Neiman (2014). Finally, I draw on panel data from 9 European countries in the EU KLEMS database for my estimations. Computations based on EU KLEMS data for European Union countries (as well as the US) corroborate the findings of Koh et al. (2020), indicating that the share of capital (and labor) compensation would have remained constant if not for the increased compensation share of IP capital.

The next section describes the system of equations together with the method of estimation. Section 3 describes the data and its sources. Section 4 summarizes the results. Section 5 concludes.

## 2 The Estimation Framework and Measurement

I consider an infinitely lived firm that discounts its profits at the rate of return $r$ and produces its output $Y$ with the following technology:

$$Y_t = \left[\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK} t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1-1}}, \tag{1}$$

where

$$LIK_t = \left[\varpi_L^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}},$$

$$IK_t = \left[\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT} t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP} t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}\right]^{\frac{\varepsilon_3}{\varepsilon_3-1}},$$

and $\varpi$-s are share parameters, $\varepsilon$-s are Allen-Uzawa elasticity of susbstition parameters, $\gamma$-s are technological progress parameters, $L$ is labor, $K_{ICT}$ is ICT capital, $K_{IP}$ is intellectual property capital, and $TK$ is traditional capital.

The firm decides how much to invest in $K_{ICT}$, $K_{IP}$ and $TK$ taking the prices of investments $p_{ICT}$, $p_{IP}$, and $p_{TK}$, and interest rate $r$ as given and solves the following problem:

$$\max_{\{L_t, I_{ICT,t}, I_{IP,t}, I_{TK,t}\}_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} \left(\frac{1}{1+r_t}\right)^t \left(Y_t - w_t L_t - p_{ICT,t} I_{ICT,t} - p_{IP,t} I_{IP,t} - p_{TK,t} I_{TK,t}\right) \tag{2}$$

$s.t.$

$I_{ICT,t} = K_{ICT,t+1} - (1 - \delta_{ICT}) K_{ICT,t},$

$I_{IP,t} = K_{IP,t+1} - (1 - \delta_{IP}) K_{IP,t},$

$I_{TK,t} = TK_{t+1} - (1 - \delta_{TK}) TK_t,$

where $\delta_{ICT}, \delta_{IP}, \delta_{TK} \in (0, 1)$ are the rates of depreciation of ICT, IP, and traditional capital.

The first order conditions that follow from this problem are given by

$$\frac{r_{ICT,t}K_{ICT,t}}{Y_t} = \frac{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}} \frac{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}, \tag{3}$$

$$\frac{r_{IP,t}K_{IP,t}}{Y_t} = \frac{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}} \frac{\varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{ICT}t} K_{ICT,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left(e^{\gamma_{IP}t} K_{IP,t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}, \tag{4}$$

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \frac{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}}}, \tag{5}$$

$$\frac{r_{TK,t}TK_t}{Y_t} = \frac{\varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LIK}^{\frac{1}{\varepsilon_1}} LIK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TK}^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_{TK}t} TK_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}}, \tag{6}$$

where

$$r_{ICT,t} = \left[(1+r_t)\, p_{ICT,t-1} - (1-\delta_{ICT})\, p_{ICT,t}\right], \tag{7}$$

$$r_{IP,t} = \left[(1+r_t)\, p_{IP,t-1} - (1-\delta_{IP})\, p_{IP,t}\right], \tag{8}$$

$$r_{TK,t} = \left[(1+r_t)\, p_{TK,t-1} - (1-\delta_{TK})\, p_{TK,t}\right]. \tag{9}$$

These last three equations are essentially non-arbitrage conditions that state that the rate of return on total capital (investment) is equal to the rate of return on a unit of capital of type $i \in \{ICT, IP, TK\}$, which was purchased at the price $p_{i,t-1}$, rented out for a period and resold.

The primary focus of this study is on the estimation of Allen-Uzawa elasticity of substitution parameters, $\varepsilon$, jointly with the technological change parameters, $\gamma$. I follow the approach developed by Grandville (1989), Klump et al. (2007) and León-Ledesma et al. (2010) in the estimation methodology. Specifically, this involves the joint estimation of normalized versions of equations (1)-(6), where normalization is based on the sample averages of the variables (I use geometric averages as in Herrendorf et al., 2015).

I denote by $S_{LIK_t,Y_t}$ the share of labor, IP capital, and ICT capital compensation in value added and use $S_{IK_t,LIK_t}$ to denote the share of compensation of ICT and IP capital out of the compensation of labor and ICT and IP capital:

$$S_{LIK_t,Y_t} = \varpi_{LIK}^{\frac{1}{\varepsilon_1}} \left(\frac{LIK_t}{Y_t}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$
$$S_{IK_t,LIK_t} = \varpi_{IK}^{\frac{1}{\varepsilon_2}} \left(\frac{IK_t}{LIK_t}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}.$$

I also use $\alpha_1$, $\alpha_2$, and $\alpha_3$ to denote the geometric averages of (1) the share of labor, IP capital, and ICT capital compensation in value added, (2) the share of labor compensation out of the compensation of labor and ICT and IP capital, and (3) the share of ICT capital compensation out of the compensation of ICT and IP capital:

$$\alpha_1 = \varpi_{LIK}^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{LIK_t}{Y_t}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}}, \tag{10}$$

$$\alpha_2 = \varpi_{L}^{\frac{1}{\varepsilon_2}} \overline{\left(\frac{e^{\gamma_L t} L_t}{LIK_t}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}, \tag{11}$$

$$\alpha_3 = \varpi_{ICT}^{\frac{1}{\varepsilon_3}} \overline{\left(\frac{e^{\gamma_{ICT} t} K_{ICT,t}}{IK_t}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}}. \tag{12}$$

I use the expressions for $\alpha_1$, $\alpha_2$, and $\alpha_3$ and write the logrithm of the normalized equation for output in the following way:

$$\ln \frac{Y_t}{\overline{Y}} = \frac{\varepsilon_1}{\varepsilon_1 - 1} \ln \left[ \alpha_1 \left(\frac{LIK_t}{\overline{\overline{LIK_t}}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + (1-\alpha_1) \left(e^{\gamma_{TK} \hat{t}} \frac{TK_t}{\overline{\overline{TK_t}}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} \right], \tag{13}$$

where

$$\frac{LIK_t}{\overline{LIK_t}} = \left[\alpha_2 \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{L_t}}\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + (1-\alpha_2) \left(\frac{IK_t}{\overline{IK_t}}\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$

$$\frac{IK_t}{\overline{IK_t}} = \left[\alpha_3 \left(e^{\gamma_{ICT} \hat{t}} \frac{K_{ICT,t}}{\overline{K_{ICT,t}}}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + (1-\alpha_3) \left(e^{\gamma_{IP} \hat{t}} \frac{K_{IP,t}}{\overline{K_{IPP,t}}}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}\right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}},$$

and $\hat{t}$ is demeaned trend,

$$\hat{t} = t - \frac{1}{T}\sum t. \tag{14}$$

I further use the expressions for $S_{LIK_t,Y_t}$ and $S_{IK_t,LIK_t}$ and write the normalized first order conditions in the following way:

$$\ln r_{ICT,t} - \ln \overline{r_{ICT,t}} = \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln\left(\frac{S_{LIK_t,Y_t}}{\overline{S_{LIK_t,Y_t}}}\right) + \\ \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_1}\frac{\varepsilon_2}{\varepsilon_2 - 1}\right) \ln\left(\frac{S_{IK_t,LIK_t}}{\overline{S_{IK_t,LIK_t}}}\right) + \frac{\varepsilon_3 - 1}{\varepsilon_3}\gamma_{ICT}\hat{t} - \frac{1}{\varepsilon_3}\ln\left(\frac{K_{ICT,t}/\overline{K_{ICT,t}}}{Y_t/\overline{Y_t}}\right), \tag{15}$$

$$\ln r_{IP,t} - \ln \overline{r_{IP,t}} = \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln\left(\frac{S_{LIK_t,Y_t}}{\overline{S_{LIK_t,Y_t}}}\right) + \\ \left(1 - \frac{\varepsilon_3 - 1}{\varepsilon_3}\frac{\varepsilon_2}{\varepsilon_2 - 1}\right) \ln\left(\frac{S_{IK_t,LIK_t}}{\overline{S_{IK_t,LIK_t}}}\right) + \frac{\varepsilon_3 - 1}{\varepsilon_3}\gamma_{IP}\hat{t} - \frac{1}{\varepsilon_3}\ln\left(\frac{K_{IP,t}/\overline{K_{IP,t}}}{Y_t/\overline{Y_t}}\right), \tag{16}$$

$$\ln w_t - \ln \overline{w_t} = \left(1 - \frac{\varepsilon_2 - 1}{\varepsilon_2}\frac{\varepsilon_1}{\varepsilon_1 - 1}\right) \ln\left(\frac{S_{LIK_t,Y_t}}{\overline{S_{LIK_t,Y_t}}}\right) + \frac{\varepsilon_2 - 1}{\varepsilon_2}\gamma_L\hat{t} - \frac{1}{\varepsilon_2}\ln\left(\frac{L_t/\overline{L_t}}{Y_t/\overline{Y_t}}\right), \tag{17}$$

$$\ln r_{TK,t} - \ln \overline{r_{TK,t}} = \frac{\varepsilon_1 - 1}{\varepsilon_1}\gamma_{TK}\hat{t} - \frac{1}{\varepsilon_1}\ln\left(\frac{TK_t/\overline{TK_t}}{Y_t/\overline{Y_t}}\right). \tag{18}$$

I use equations (13)-(18) in the empirical estimations. The value of $r_t$ is needed for the estimations. The values of $r_t$ can be obtained using the zero profit condition,

$$1 + r_t = \frac{Y_t - w_t L_t}{p_{ICT,t-1}K_{ICT,t} + p_{IP,t-1}K_{IP,t} + p_{TK,t-1}TK_t} + \\ \frac{(1-\delta_{ICT})\, p_{ICT,t}K_{ICT,t} + (1-\delta_{IP})\, p_{IP,t}K_{IP,t} + (1-\delta_{TK})\, p_{TK,t}K_{TK,t}}{p_{ICT,t-1}K_{ICT,t} + p_{IP,t-1}K_{IP,t} + p_{TK,t-1}TK_t}, \tag{19}$$

assuming the data contain information on real value added, labor compensation, and prices of investments, stocks and depreciation rates of ICT, IP, and traditional capital.

# 3 Data

The data are from the 2023 version of the EU KLEMS database for 9 European countries from the Euro Area (EA) and the 1996-2020 period for most of the countries. Panel A.1 of Table 1 offers the averages and initial and final sample values of the key variables used in the estimations. A few notable observations are in order. The prices of ICT investments relative to value added prices have fallen significantly during the study period. This contrasts with the prices of investments in IP and traditional capital and likely reflects the substantial technological advancements in ICT. Moreover, the stocks of ICT capital have increased at a much higher rate than the stocks of IP and traditional capital. As reported in Table 2, the average yearly growth rate of ICT investment prices relative to value added prices in sample countries is -4.6%, whereas the corresponding growth rates for IP and traditional capital investments are -0.3% and 0.1%, respectively. The average yearly growth in real stock of ICT capital is 4.9% across sample countries, while the corresponding growth rates in real stocks of IP capital and traditional capital are 3.4% and 1.6%, respectively.

Panel A.2 of Table 1 reports the computed annual rates of depreciation across the different types of capital. ICT and IP capital depreciate at very high rates of around 20% and 24%, respectively, whereas traditional capital depreciates at the rate of 3.6%. I use these figures and prices of investments in the types of capital to compute the rate of return on total capital and the rates of return on each type of capital from non-arbitrage conditions (7)-(9) and the zero profit condition (19). Panel B.1 of Table 1 reports the results. The rate of return on traditional capital averaged across countries has fallen from 8.5% in 1996 to 8.1% in 2020. In turn, the rate of return on ICT capital was much higher in 1996 and fell much more sharply from 131% in 1996 to 24% in 2020. The computed rate of return on ICT capital is very large at the beginning of the sample period because

**Table 1:** Basic Statistics

| *Panel A.1* | | | | *Panel A.2* | |
|---|---|---|---|---|---|
| Variable | (1) Mean | (3) 1996 | (4) 2020 | Parameter | |
| Value Added, EUR, current, bn | 839 | 627 | 804 | $\delta_{ICT}$ | 0.201 |
| Total Labor Income, EUR, current, bn | 555 | 427 | 535 | $\delta_{IP}$ | 0.241 |
| Total Hours Worked, bn | 24 | 24 | 19 | $\delta_{TK}$ | 0.036 |
| ICT Capital Stock, EUR, current, bn | 27 | 25 | 24 | | |
| IP Capital Stock, EUR, current, bn | 131 | 83 | 150 | | |
| TK Stock, EUR, current, bn | 2876 | 2032 | 3025 | | |
| Total Capital Stock (K), EUR, current, bn | 3034 | 2139 | 3200 | | |
| Value Added Price Index | 0.902 | 0.739 | 1.080 | | |
| ICT Investment Price Index | 1.410 | 2.450 | 0.979 | | |
| IP Investment Price Index | 0.906 | 0.751 | 1.050 | | |
| TK Investment Price Index | 0.905 | 0.717 | 1.090 | | |
| Total Capital (K) Investment Price Index | 0.914 | 0.749 | 1.080 | | |

| *Panel B.1* | | | | *Panel B.2* | |
|---|---|---|---|---|---|
| Derived Variable | (1) Mean | (3) 1996 | (4) 2020 | Derived Parameter | |
| $r_{ICT}$ | 0.518 | 1.310 | 0.237 | $\alpha_1$ | 0.709 |
| $r_{IP}$ | 0.294 | 0.290 | 0.279 | $\alpha_2$ | 0.923 |
| $r_{TK}$ | 0.085 | 0.081 | 0.075 | $\alpha_3$ | 0.212 |
| $w$ | 25 | 24 | 27 | | |

Note: This table offers the averages and sample initial and final values of the variables used in estimations. Nominal variables are in national currency units (EUR). Price indices are normalized to 1 in 2015. Sample countries are Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Portugal, and Spain and the period is from 1996-2020 except for Germany (1996-2019) and Portugal (2001-2020). Panel B.2 of Table 1 reports the computed values of $\alpha_1$, $\alpha_2$, and $\alpha_3$ from equations (10), (11), and (12) averaged across countries. ICT capital includes information technologies (IT) and communication technologies (CT). IP capital includes software, organizational capital, and R&D capital. See Table A in the Data Description Appendix for complete descriptions and sources of variables.

of the observed sharp fall in ICT investment prices. Firms investing in ICT capital in a given year should have had high returns on it since they could have waited for a year and invested in it at much lower prices. The rate of return on ICT capital is comparable to the rate of return on IP capital in 2020 and these two are higher than the rate of return on traditional capital. However, the rates of return on these different types of capital are not that different in 2020 once the differences in depreciation rates are taken into account.

**Table 2:** Growth in Real Investment Prices and Capital Stocks

| Variable | ICT | IP | TK | K |
|---|---|---|---|---|
| Investment Price Index | -0.046 | -0.003 | 0.001 | -0.001 |
| Capital Stock | 0.049 | 0.034 | 0.016 | 0.019 |

Note: This table offers the average annual growth rates in investment price indices for ICT, IP, and traditional capital (TK) as well as the average annual growth rates in (real) stocks of these types of capital. See Table A in the Data Description Appendix for complete descriptions and sources of variables.

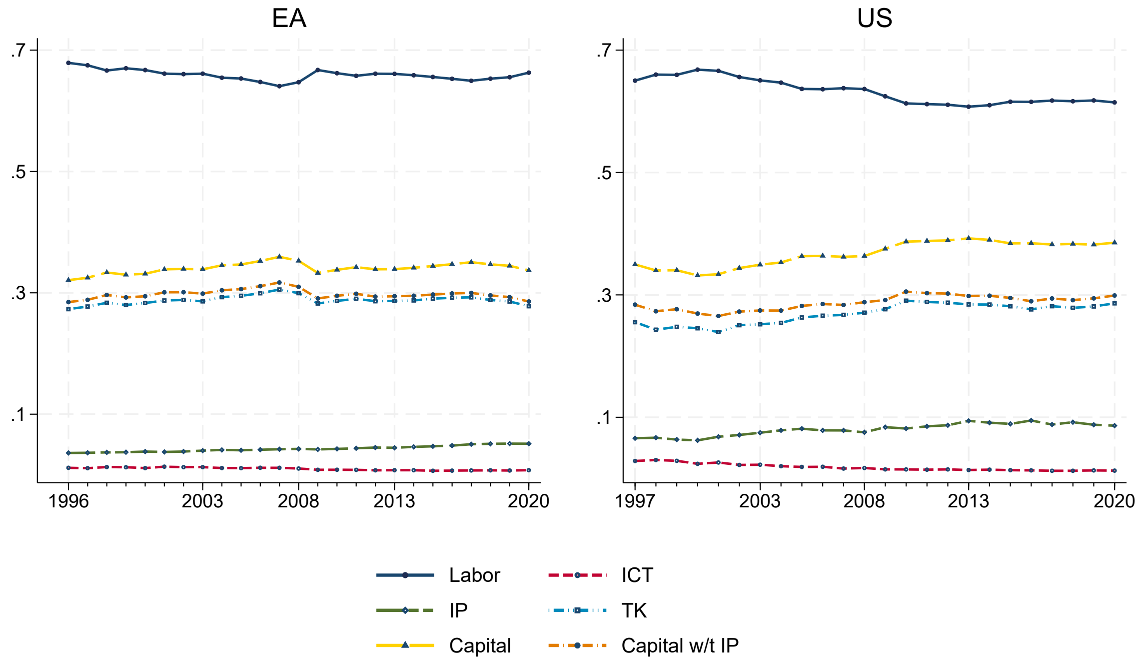
Panel B.2 of Table 1 reports the computed values of $\alpha_1$, $\alpha_2$, and $\alpha_3$ averaged across sample countries. The average share of labor, IP capital, and ICT capital compensation in value added is close to 71%. This value, together with the usual values of labor income share, suggests that labor compensation is much larger than the compensation of IP and ICT capital. The value of the average share of labor compensation out of the compensation of labor and ICT and IP capital $\alpha_2$ reflects this as it is close to 92%. In turn, the average share of ICT capital compensation out of the compensation of ICT and IP capital is close to 21% implying that the compensation of ICT capital is much smaller than the compensation of IP capital.

Finally, Figure 1 illustrates labor income share and income shares of ICT, IP, and traditional capital in sample European countries (EA). I compute the latter using the rates of return on ICT, IP, and traditional capital. Table 3 reports the values of these shares at the beginning of the sample period and the end. Figure 1 and Table 3 also offer these values for the US.[3]

Labor income share has fallen during the sample period from about 68% to 66% in

[3]Table I in the Data Appendix offers the basic statistics for each sample European country and the US.

**Figure 1:** The Shares of Compensation of Labor, ICT, IP Capital, and Traditional Capital




Note: This figure illustrates the shares of compensation of labor, ICT, IP capital, traditional capital, total capital, and capital without IP capital out of value added (Capital w/t IP) in sample countries.

**Table 3:** Sample Initial and Final Values of Income Shares

*Panel A: EA*

| Year | Labor | ICT | IP | TK |
|---|---|---|---|---|
| 1996 | 0.679 | 0.012 | 0.0361 | 0.273 |
| 2020 | 0.663 | 0.008 | 0.0514 | 0.278 |

*Panel B: US*

| Year | Labor | ICT | IP | TK |
|---|---|---|---|---|
| 1997 | 0.650 | 0.029 | 0.066 | 0.256 |
| 2020 | 0.615 | 0.013 | 0.086 | 0.286 |

Note: This table offers the average labor income share and income shares of ICT, IP, and traditional capital in sample European (EA) countries at the beginning of the sample period and the end. It also offers the values of labor income share and income shares of ICT, IP, and traditional capital in the US in 1997 and 2020 in Panel B. EA stands for European countries. These countries are part of the Euro Area. See Table A in the Data Description Appendix for complete descriptions and sources of variables.

the sample European countries. This fall can be largely accounted for by the rise in the compensation share of IP capital. Labor income share would be virtually constant absent this raise, i.e., if either IP capital compensation or the raise in it could be attributed to labor income. A similar result holds for the US where labor income share has fallen by about 3.5 percentage points during the sample period and, absent the rise in intellectual property compensation share, it would have fallen by only 1.5 percentage points. These results corroborate the results of Koh et al. (2020) that the compensation share of labor/capital is virtually flat in the US absent the rise in the compensation of IP capital. Nevertheless, the share of joint compensation of ICT and IP does not increase as much as the compensation share of IP capital. Labor income share would still fall in European countries as well as in the US absent changes in these two, i.e., if either ICT and IP capital compensation or the raise in these two could be attributed to labor income.

## 4 Results

I employ the feasible generalized non-linear least-squares estimation method in all estimations, accounting for arbitrary heteroscedasticity and serial correlation in the residuals. I use the country-year-level shares of hours of employment out of the total hours of employment across sample countries as weights in these regressions. Further, I use multiple starting/initial points in estimations for the elasticity of substitution parameters and select the results that yield the best fit based on Log Likelihood, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and R-squared for each equation.[4]

Column 1 of Table 4 presents the results from the estimation of the system of equations (13), (15), (16), (17), and (18). The estimated elasticity of substitution between labor and traditional capital, $\varepsilon_1$, is 0.75 and it is statistically significantly below 1 implying that labor and traditional capital are gross complements. The estimated elasticity of substitution between labor and ICT and IP capital, $\varepsilon_2$, is 1.19 and it is statistically significantly above 1 implying that labor and ICT and IP capital are gross substitutes.[5]

[4] In these estimations, I disregard results that produce excessively high values for the elasticities of substitution, as well as those with significantly negative values for $\gamma_{ICT}$.

[5] Eden and Gaggl (2018) have similar a finding for ICT capital that includes software using US data and

In turn, the estimate of the elasticity of substitution between ICT and IP capital, $\varepsilon_3$, is 0.96 and it is significantly below 1 implying that these two types of capital are gross complements.

The estimate of the labor augmenting technical change parameter $\gamma_L$ is small and positive, 0.003. The estimate of the ICT capital augmenting technical change parameter $\gamma_{ICT}$ is positive and $\gamma_{ICT}$ is several orders of magnitude larger than $\gamma_L$. The high value of $\gamma_{ICT}$ can possibly reflect the rapid technological progress in information and communication technologies. The estimate of IP capital augmenting technical change parameter $\gamma_{IP}$ is negative and its absolute value is smaller than $\gamma_{ICT}$. In turn, the estimate of the traditional capital augmenting technical change parameter $\gamma_{TK}$ is negative though small and statistically insignificant.

Several papers estimate a negative technical change parameter for (total) capital (e.g., Herrendorf et al., 2015, Mućk, 2017). Nevertheless, the negative values of $\gamma_{IP}$ and $\gamma_{TK}$ are not easily explained within a neoclassical framework. Such values suggest that the estimates of $\gamma_{IP}$ and $\gamma_{TK}$ might capture processes beyond technological progress. In this regard, a potential explanation can be that some parts of the accumulated IP and traditional capital may not be fully utilized in the near term, though the returns on these types of capital continue adhering to non-arbitrage conditions (8) and (9). Admittedly, equations (1) and (3)-(6) do not readily support this interpretation. Jiang and León-Ledesma (2018), drawing on data from De Loecker, Eeckhout and Unger (2020), incorporate variable markups into their estimation of value added function and first order conditions for labor and total capital. They show that this can reverse the negative sign of the estimate of (total) capital augmenting technological change. While promising, applying this strategy in the multi-country context of the current study is challenging due to data limitations. Moreover, the results of this study suggest the need to measure markups that vary across different types of capital, such as IP and traditional capital. A further potential drawback is that the data used in this study come from a single source that ensures consistency across variables and introducing markup measures from external

single equation/first order condition with no biased technological progress parameters. Antràs (2004) shows that this can introduce an upward bias in the estimates of the elasticity of substitution.

**Table 4:** Estimation Results

| *A. Estimates* | | | | |
|---|---|---|---|---|
| Parameter | (1)<br>Main | (2)<br>Eq. (20) | (3)<br>Eq. (21) | (4)<br>US |
| $\varepsilon_1$ | 0.745*** | 1.582*** | 1.212*** | 0.724*** |
| | (0.009) | (0.142) | (0.045) | (0.024) |
| $\varepsilon_2$ | 1.187*** | 0.993*** | 1.127*** | 1.712*** |
| | (0.010) | (0.004) | (0.015) | (0.133) |
| $\varepsilon_3$ | 0.961*** | 0.954*** | 0.969*** | 0.922*** |
| | (0.003) | (0.004) | (0.003) | (0.017) |
| $\gamma_L$ | 0.003*** | 0.051*** | 0.009*** | 0.018*** |
| | (0.001) | (0.003) | (0.001) | (0.001) |
| $\gamma_{ICT}$ | 0.725*** | 0.663*** | 0.880*** | 0.431*** |
| | (0.065) | (0.045) | (0.099) | (0.119) |
| $\gamma_{IP}$ | -0.111*** | -0.167*** | -0.101*** | -0.113*** |
| | (0.018) | (0.014) | (0.022) | (0.027) |
| $\gamma_{TK}$ | -0.002 | -0.103*** | -0.021*** | -0.020*** |
| | (0.002) | (0.006) | (0.002) | (0.003) |
| Obs. (per eq.) | 219 | 219 | 219 | 24 |
| *B. Measures of Fit* | | | | |
| Log Likelihood | 2262 | 2456 | 2177 | 301 |
| AIC | -4510 | -4898 | -4341 | -588 |
| BIC | -4486 | -4874 | -4317 | -580 |
| R2 | | | | |
| $ln\left(Y_t/\overline{Y}\right)$ | 0.889 | 0.960 | 0.909 | 0.986 |
| $ln\left(r_{ICT,t}/\overline{r_{ICT}}\right)$ | 0.864 | 0.872 | 0.861 | 0.987 |
| $ln\left(r_{IP,t}/\overline{r_{IP}}\right)$ | 0.212 | 0.295 | 0.020 | 0.830 |
| $ln\left(r_{TK,t}/\overline{r_{TK}}\right)$ | 0.638 | 0.617 | 0.968 | -0.104 |
| $ln\left(w_t/\overline{w}\right)$ | 0.968 | 0.873 | 0.867 | 0.935 |

Note: This table offers the results from the estimation of normalized and logarithmed production function together with the first order conditions. Panel *A* offers the estimates of the parameters and the corresponding number of observations in each equation. Sample years are given by the availability of data in the EU KLEMS database. Panel *B* offers various measures of fit including Log Likelihood, AIC, BIC, and R-squared of each equation. Column 1 offers the main results from the estimation of equations (13), (15), (16), (17), and (18). Columns 2 and 3 offer results from the estimation of other CES nests, (20) and (21) [see Technical Appendix for the first order conditions in these columns]. Column 4 offers results from the estimation of (13), (15), (16), (17), and (18) for the US. Negative R-squared in an equation means that the residual sum of squares is larger than the total sum of squares in that equation. Figure I shows the fit for the first equation in European countries and the US. All regressions use the feasible generalized non-linear least-squares estimation method and the country-year-level shares of hours of employment out of the total hours of employment across sample countries as weights. Initial/starting points for estimations are $\varepsilon_i = \{0.5, 1.5\}$ for $i = 1, 2, 3$ and $\gamma_j = 0.02$ for $j = L, ICT, IP, TK$ in columns 1-3 and the estimates from column 1 in column 4. Standard errors are in parentheses and are robust to arbitrary heteroscedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Data Description Appendix offers further details about the data.

sources could raise additional measurement concerns.

## 4.1 The Elasticity of Substitution between Labor and ICT and IP Capital: Alternative Nests

I follow the literature on automation and labor demand and write the CES nests in the production function in equation (1) so that the production function permits a difference in the elasticities of substitution between labor and traditional capital and labor and ICT and IP capital. By construction, the elasticity of substitution between labor and traditional capital and the elasticity of substitution between ICT and IP capital and traditional capital are the same in equation (1). A rationale for this is that the aggregate of ICT and IP capital, being a substitute for labor, is used in tasks that can be performed by labor (e.g., routine tasks). I explore two alternative nests in this section to assess the elasticity of substitution between labor and ICT and IP capital. To save space, I will reuse the letters $\varepsilon$ and $\gamma$ and their indices albeit this involves an abuse of notation. In particular, $\varepsilon_i$ for $i = 1, 2, 3$ will correspond to the i-th nest starting from the most outer nest, while $\gamma_j$ for $j = L, ICT, IP, TK$ will pertain to the corresponding factor input.

First, I assume that the elasticity of substitution between labor and ICT and IP capital is equal to the elasticity of substitution between traditional capital and ICT and IP capital. Moreover, the aggregate of ICT and IP capital is either a complement or a substitute for the combination of labor and traditional capital. I further assume that the production function is given by

$$Y_t = \left( \varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right)^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{20}$$

where

$$LTK_t = \left[ \varpi_L^{\frac{1}{\varepsilon_2}} \left( e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left( e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$

$$IK_t = \left[ \varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}.$$

I estimate the parameters using normalized and logarithmed $Y_t$ from equation (20) and the corresponding normalized and logarithmed first order conditions [see equations

(35)-(39) in the Technical Appendix]. Column 2 of Table 4 reports the results. The estimated elasticity of substitution between labor, as well as traditional capital, and the CES aggregate of ICT and IP capital, $\varepsilon_1$, is large and significantly above 1. Similarly to the results from column 1, the estimates of $\varepsilon_2$ and $\varepsilon_3$ imply that the pairs of traditional capital and labor and ICT and IP capital are gross complements. The estimates of the parameters $\gamma$ are also in line with the results from column 1 with a few notable differences. The estimate of $\gamma_L$ is now 0.05 and the estimate of $\gamma_{TK}$ is negative and large in absolute value, -0.1.

Another specification nests first the different types of capital and then nests these with labor. It is given by

$$Y_t = \left[ \varpi_L^{\frac{1}{\varepsilon_1}} \left( e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{21}$$

where

$$\begin{aligned} IKTK_t &= \left[ \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left( e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}}, \\ IK_t &= \left[ \varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}. \end{aligned}$$

This specification can be thought to be a generalization of a specification that combines total capital, $K$, with labor as it permits imperfect substitutability between different types of capital as well as well as technological change parameters specific to each type of capital.

Column 3 of Table 4 reports the results from the estimation of the normalized and logarithmed $Y_t$ from equation (21) and the corresponding normalized and logarithmed first order conditions [see equations (45)-(49) in the Technical Appendix]. The estimated elasticity of substitution between labor and the CES aggregate of ICT, IP and traditional capital, $\varepsilon_1$, is large and significantly above 1. The estimates of $\varepsilon_2$ imply that traditional capital and the CES aggregate of ICT and IP capital are gross substitutes and that ICT and IP capital are gross complements. Similarly to the results in column 2, the estimates of the parameters $\gamma$ are in line with the results from column 1.

The results from estimations of these two CES nests, similar to the main results, suggest that ICT and IP capital are gross substitutes for labor. Admittedly, these nests seem to be less appealing than the nest in (1) as they are not strongly in line with the literature on automation and labor demand and imply a gross substitutability between traditional capital and the aggregate of ICT and IP capital.

Taken together, these results can have important implications for the dynamics in labor income share. They can also have implications regarding the elasticity of substitution between labor and aggregate capital.

### 4.2 Labor Income Share

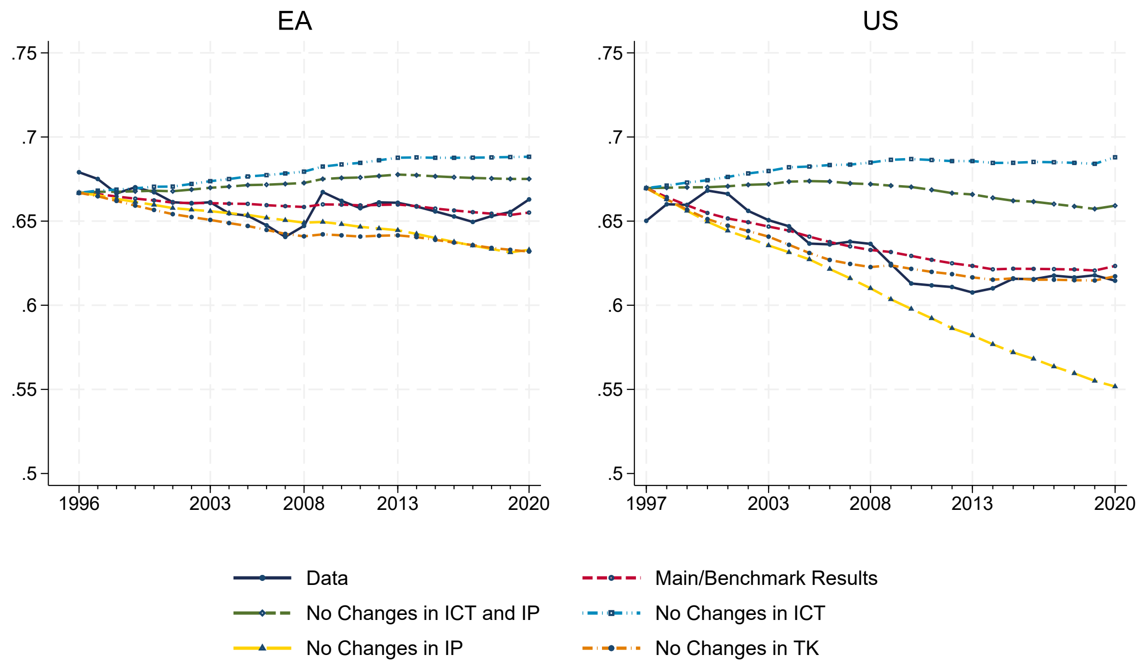
The labor income share has fallen during the sample years in European countries as well as in the US according to Table 3. This fall is visible in Figure 2 which offers the variation in labor income share in the EU KLEMS data as well as the predicted labor income share using equation (5) and parameter estimates from column 1 of Table 4 for European countries (EA). It also illustrates labor income share in the US and its predicted values using parameter estimates from column 5 of Table 4. In this column, I present the results from the estimation of equations (13), (15), (16), (17), and (18) using US data from the EU KLEMS database.[6] These results slightly under-predict the fall in labor share in European countries and over-predict it in the US but are very close to the data. Panel A of Table 3 provides the exact numbers.

Figure 2 and Panel B of Table 3 also offer simple counterfactual predictions for cases when there is no technological progress and accumulation/changes in (1) ICT and IP capital, (2) ICT, (3) IP capital, and (4) traditional capital. I fix the corresponding trend index to its sample initial value to have no technological progress and set the value of capital stock equal to its sample initial value to have no changes in it. A rough interpretation of the counterfactual exercise, for example, for ICT is that it corresponds

[6] The results for the US are sensitive to the choice of initial values in the estimation, possibly due to the very limited sample size.

to fixing the number of computers and their productivity.[7]

**Figure 2:** Labor Income Share: Data, Predicted, and Counterfactual




Note: This figure illustrates the labor income share in European countries (EA) and in the US computed using the data from the EU KLEMS database. It also illustrates the predicted labor income share using equation (5) and parameter estimates from columns 1 and 5 of Table 4 for European countries and the US, correspondingly. The counterfactual predictions are for cases when there is no technological progress and changes in (1) ICT and IP capital, (2) ICT, (3) IP capital, and (4) traditional capital. The corresponding trend index is fixed to its sample initial value to have no technological progress in the type of capital and the value of capital stock is set equal to its sample initial value to have no changes in its level. The averages across European countries are weighted by the country-year-level shares of employment hours out of the total hours of employment across sample European countries.

These counterfactual exercises suggest that rapid technological progress and the accumulation of ICT and IP capital have been key factors in the decline of the labor income share according to column 1 of Panel B in Table 3. Absent these developments, the labor income share would have slightly increased in European countries and declined by about 75% less in the US over the sample period. This is consistent with the results of, for example, Aum and Shin (2024), Eden and Gaggl (2018), Ohanian et al. (2023), and Koh et al. (2020).

Columns 2-3 offer the results from counterfactual predictions for cases when there is no technological progress and changes in ICT, IP capital, and traditional capital separately.

[7]These counterfactual exercises do not accommodate potential adjustments in the supply of the free factors of production.

**Table 5:** Labor Income Share: Data, Predicted, and Counterfactual

*A. Data and Main/Benchmark Prediction*

| European Countries (EA) | | | US | | |
|---|---|---|---|---|---|
| Year | (1) Data | (2) Main | Year | (1) Data | (2) Main |
| 1996 | 0.666 | 0.655 | 1997 | 0.650 | 0.670 |
| 2020 | 0.647 | 0.642 | 2020 | 0.615 | 0.623 |

*B. Counterfactual: No Technological Progress and Capital Accumulation*

| European Countries (EA) | | | | | US | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Year | (1) ICT and IP | (2) ICT | (3) IP | (4) TK | Year | (1) ICT and IP | (2) ICT | (3) IP | (4) TK |
| 2020 | 0.675 | 0.688 | 0.633 | 0.632 | 2020 | 0.670 | 0.688 | 0.552 | 0.617 |
| % of Predicted $\Delta_{2020-1996}$ | -0.667 | -1.750 | 2.833 | 2.917 | % of Predicted $\Delta_{2020-1997}$ | 0.000 | -0.383 | 2.511 | 1.128 |

Note: Panel A of this table offers sample initial and final values of the labor income share in European countries (EA) and in the US computed using the data from the EU KLEMS database in. It also offers the predicted labor income share using equation (5) and parameter estimates from columns 1 and 5 of Table 4 for European countries and the US, correspondingly. Panel B of this table offers results from counterfactual prediction. The counterfactual predictions are for cases when there is no technological progress and changes in (1) traditional capital, (2) ICT and IP capital, (3) ICT, and (4) IP capital. The corresponding trend index is fixed to its sample initial value to have no technological progress in the type of capital and the value of capital stock is set equal to its sample initial value to have no changes in its level. The averages across European countries are weighted by the country-year-level shares of employment hours out of the total hours of employment across sample European countries.

These results further suggest that the key factors in the decline in labor income share are rapid technological progress and changes/accumulation in ICT. Absent these advances, the labor income share would have significantly increased in European countries over the sample period. It would have also increased in the US albeit less. In turn, labor income share would have declined substantially more absent trends corresponding to IP and traditional capital in European countries.

## 4.3 The Elasticity of Substitution between Labor and Aggregate Capital

What do the estimates of the elasticities of substitution in column 1 of Table 4 imply regarding the elasticity of substitution between labor and total/aggregate capital? In an attempt to answer this question, I consider Hicks's original definition of the elasticity of substitution between labor and capital:

$$\varepsilon_{L,K} = \frac{Y_L Y_K}{Y_{L,K} Y}. \tag{22}$$

I treat the geometric averages of ICT, IP, and traditional capital as parameters and write changes in $K_t/\overline{K}$ in the following way

$$\partial \frac{K_t}{\overline{K}} = \frac{p_{K_{ICT,t}}}{p_{K_t}} \frac{\overline{K_{ICT}}}{\overline{K}} \partial \frac{K_{ICT,t}}{\overline{K_{ICT}}} + \frac{p_{K_{IP,t}}}{p_{K_t}} \frac{\overline{K_{IP}}}{\overline{K}} \partial \frac{K_{IP,t}}{\overline{K_{IP}}} + \frac{p_{TK_t}}{p_{K_t}} \frac{\overline{TK}}{\overline{K}} \partial \frac{TK_t}{\overline{TK}}. \tag{23}$$

It is straightforward to show that the elasticity of substitution between labor and aggregate capital is given by the following expression using equations (13), (22), and (23):

$$\begin{aligned} \varepsilon_{L,K} = \varepsilon_1 \Bigg[ & S_{ICTIP} S_{IKL} S_{LIKTK} + \frac{p_{K_{ICT,t}} K_{ICT,t}}{p_{K_{IP,t}} K_{IP,t}} \left(1 - S_{ICTIP}\right) S_{IKL} S_{LIKTK} + \\ & \frac{p_{K_{ICT,t}} K_{ICT,t}}{p_{TK_t} TK_t} \left(1 - S_{LIKTK}\right) \Bigg] \Big/ \Bigg\{ S_{ICTIP} S_{IKL} \left[ \left( \frac{\varepsilon_1}{\varepsilon_2} - 1 \right) + S_{LIKTK} \right] + \\ & \frac{p_{K_{ICT,t}} K_{ICT,t}}{p_{K_{IP,t}} K_{IP,t}} \left(1 - S_{ICTIP}\right) S_{IKL} \left[ \left( \frac{\varepsilon_1}{\varepsilon_2} - 1 \right) + S_{LIKTK} \right] + \\ & \frac{p_{K_{ICT,t}} K_{ICT,t}}{p_{TK_t} TK_t} \left(1 - S_{LIKTK}\right) \Bigg\}, \end{aligned} \tag{24}$$

where

$$S_{LIKTK} = \frac{\alpha_1 \left(\frac{LIK_t}{\overline{LIK_t}}\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\alpha_1 \left(\frac{LIK_t}{\overline{LIK_t}}\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1-\alpha_1)\left(e^{\gamma_{TK}\hat{t}}\frac{TK_t}{\overline{TK_t}}\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}, \tag{25}$$

$$S_{IKL} = \frac{(1-\alpha_2)\left(\frac{IK_t}{\overline{IK_t}}\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}{\alpha_2 \left(e^{\gamma_L \hat{t}}\frac{L_t}{\overline{L_t}}\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + (1-\alpha_2)\left(\frac{IK_t}{\overline{IK_t}}\right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}, \tag{26}$$

$$S_{ICTIP} = \frac{\alpha_3 \left(e^{\gamma_{ICT}\hat{t}}\frac{K_{ICT,t}}{\overline{K_{ICT,t}}}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}{\alpha_3 \left(e^{\gamma_{ICT}\hat{t}}\frac{K_{ICT,t}}{\overline{K_{ICT,t}}}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + (1-\alpha_3)\left(e^{\gamma_{IP}\hat{t}}\frac{K_{IP,t}}{\overline{K_{IPP,t}}}\right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}, \tag{27}$$

are the share of compensation of labor and $IK$ out of value added, the share of $IK$ in the compensation of $IK$ and labor, and the share of compensation of ICT capital in the compensation of $IK$, respectively.[8]

The equation for $\varepsilon_{L,K}$ (24) implies that if the elasticities of substitution between labor and capital are the same for the traditional capital and the aggregate of ICT and IP capital, $\varepsilon_1 = \varepsilon_2$, then the elasticity of substitution between aggregate capital is not different than these two, $\varepsilon_{L,K} = \varepsilon_1 = \varepsilon_2$. It also implies that $\partial \varepsilon_{L,K}/\partial \varepsilon_2 > 0$ so that when $\varepsilon_2 > \varepsilon_1$ then $\varepsilon_{L,K} > \varepsilon_1$. If, in addition to $\varepsilon_2 > \varepsilon_1$, the following inequality holds:

$$\frac{p_{K_{ICT,t}} K_{ICT,t}}{p_{TK_t} TK_t} - S_{ICTIP} S_{IKL} - \frac{p_{K_{ICT,t}} K_{ICT,t}}{p_{K_{IP,t}} K_{IP,t}} \left(1 - S_{ICTIP}\right) S_{IKL} < 0, \tag{28}$$

then the equation (24) implies that $\varepsilon_{L,K} > \varepsilon_2$.

I use equations (24)-(27) and the estimated values of parameters from column 1 of Table 4 to compute the value of $\varepsilon_{L,K}$ for each sample year taking the (weighted) average across sample European countries. Column 1 of Table 6 offers the results. The values of the elasticity of substitution between labor and aggregate capital are greater than 1. They also fall within the range of the estimated $\varepsilon_1$ in column 3 of Table 4, which can be interpreted as the elasticity of substitution between labor and aggregate capital, where aggregate capital is a CES composite of traditional, ICT, and IP capital (these values

[8] Table II and Table III in the Data Appendix offer the values of the shares $S_{ICTIP}$, $S_{IKL}$, and $S_{LIKTK}$ from equations (25)-(27) and ratios $p_{K_{ICT}} K_{ICT}/p_{K_{IP}} K_{IP}$ and $p_{K_{ICT}} K_{ICT}/p_{TK} TK$ in the sample European countries and the US.

align with the estimates reported by Karabarbounis and Neiman, 2014). Moreover, the values of $\varepsilon_{L,K}$ are greater than the estimated value of the elasticity of substitution between labor and the aggregate of ICT and IP capital $\varepsilon_2$. This is because $p_{K_{ICT,t}} K_{ICT,t}/p_{TK_t} TK_t$ attains relatively low values in the data.

The elasticity of substitution $\varepsilon_{L,K}$ also varies over time because of changes in the compensation shares $S_{LIKTK}$, $S_{IKL}$, and $S_{ICTIP}$, as well as changes in ratios $p_{K_{ICT}} K_{ICT}/p_{TK} TK$ and $p_{K_{ICT}} K_{ICT}/p_{K_{IP}} K_{IP}$. It increases during the period 1996-2008 and declines afterward.

**Table 6:** The Imputed Elasticity of Substitution between Labor and Aggregate Capital

| | European Countries (EA) | | | | | | | US | |
|---|---|---|---|---|---|---|---|---|---|
| Year | (1) Main | (2) $S_{ICTIP}$ | (3) $S_{IKL}$ | (4) $S_{LIKTK}$ | (5) $\frac{p_{K_{ICT}} K_{ICT}}{p_{TK} TK}$ | (6) $\frac{p_{K_{ICT}} K_{ICT}}{p_{K_{IP}} K_{IP}}$ | (7) Data | (8) Main | (9) Data |
| 1996 | 1.367 | 1.367 | 1.367 | 1.367 | 1.367 | 1.367 | 1.351 | | |
| 1997 | 1.366 | 1.367 | 1.363 | 1.366 | 1.365 | 1.369 | 1.351 | 2.144 | 2.281 |
| 1998 | 1.366 | 1.369 | 1.360 | 1.365 | 1.366 | 1.370 | 1.368 | 2.168 | 2.255 |
| 1999 | 1.364 | 1.368 | 1.355 | 1.363 | 1.364 | 1.372 | 1.361 | 2.172 | 2.239 |
| 2000 | 1.361 | 1.366 | 1.349 | 1.359 | 1.357 | 1.377 | 1.358 | 2.158 | 2.137 |
| 2001 | 1.362 | 1.369 | 1.348 | 1.361 | 1.360 | 1.377 | 1.374 | 2.178 | 2.173 |
| 2002 | 1.364 | 1.372 | 1.347 | 1.363 | 1.366 | 1.375 | 1.374 | 2.213 | 2.196 |
| 2003 | 1.368 | 1.378 | 1.349 | 1.369 | 1.376 | 1.370 | 1.379 | 2.265 | 2.265 |
| 2004 | 1.373 | 1.384 | 1.352 | 1.373 | 1.384 | 1.367 | 1.385 | 2.290 | 2.269 |
| 2005 | 1.376 | 1.389 | 1.353 | 1.377 | 1.389 | 1.364 | 1.389 | 2.334 | 2.327 |
| 2006 | 1.381 | 1.396 | 1.357 | 1.383 | 1.399 | 1.360 | 1.404 | 2.336 | 2.336 |
| 2007 | 1.386 | 1.403 | 1.360 | 1.387 | 1.408 | 1.355 | 1.417 | 2.355 | 2.299 |
| 2008 | 1.388 | 1.407 | 1.361 | 1.390 | 1.414 | 1.351 | 1.405 | 2.346 | 2.317 |
| 2009 | 1.387 | 1.407 | 1.357 | 1.390 | 1.414 | 1.345 | 1.366 | 2.341 | 2.32 |
| 2010 | 1.386 | 1.408 | 1.355 | 1.390 | 1.416 | 1.342 | 1.376 | 2.319 | 2.349 |
| 2011 | 1.386 | 1.410 | 1.354 | 1.389 | 1.419 | 1.338 | 1.381 | 2.308 | 2.334 |
| 2012 | 1.384 | 1.409 | 1.350 | 1.388 | 1.418 | 1.334 | 1.373 | 2.287 | 2.333 |
| 2013 | 1.383 | 1.411 | 1.348 | 1.388 | 1.422 | 1.327 | 1.375 | 2.287 | 2.327 |
| 2014 | 1.384 | 1.415 | 1.348 | 1.388 | 1.426 | 1.322 | 1.381 | 2.279 | 2.332 |
| 2015 | 1.381 | 1.414 | 1.342 | 1.383 | 1.425 | 1.320 | 1.372 | 2.275 | 2.309 |
| 2016 | 1.378 | 1.414 | 1.338 | 1.380 | 1.424 | 1.320 | 1.376 | 2.273 | 2.315 |
| 2017 | 1.377 | 1.414 | 1.335 | 1.377 | 1.423 | 1.320 | 1.383 | 2.246 | 2.289 |
| 2018 | 1.376 | 1.415 | 1.333 | 1.375 | 1.424 | 1.319 | 1.380 | 2.236 | 2.304 |
| 2019 | 1.374 | 1.415 | 1.330 | 1.373 | 1.423 | 1.318 | 1.372 | 2.217 | 2.316 |
| 2020 | 1.369 | 1.411 | 1.322 | 1.375 | 1.410 | 1.345 | 1.371 | 2.203 | 2.328 |

Note: Column 1 of this table offers the values of $\varepsilon_{L,K}$ computed using equations (24)-(27) and estimated values of parameters from column 1 of Table 4. Columns 2-6 compute the values of $\varepsilon_{L,K}$ fixing correspondingly the shares $S_{ICTIP}$, $S_{IKL}$, and $S_{LIKTK}$, and ratios $p_{K_{ICT}} K_{ICT}/p_{TK} TK$ and $p_{K_{ICT}} K_{ICT}/p_{K_{IP}} K_{IP}$ to their sample initial values. Column 7 computes the value of $\varepsilon_{L,K}$ using the data counterparts of $S_{ICTIP}$, $S_{IKL}$, and $S_{LIKTK}$. Columns 8 and 9 present the results for the US. The averages across European countries are weighted by the country-year-level shares of employment hours out of the total hours of employment across sample European countries. Figure II in the Data Appendix illustrates these values. The Data Description Appendix offers further details about the data.

I fix the values of the shares $S_{ICTIP}$, $S_{IKL}$, and $S_{LIKTK}$, and ratios $p_{K_{ICT}}K_{ICT}/p_{TK}TK$ and $p_{K_{ICT}}K_{ICT}/p_{K_{IP}}K_{IP}$ to their sample initial values in columns (2)-(6) when computing $\varepsilon_{L,K}$. The elasticity of substitution between labor and aggregate capital increases by about 0.08 points when there are no changes in $S_{ICTIP}$ and $p_{K_{ICT}}K_{ICT}/p_{TK}TK$. Most of the increase happens in the 1996-2008 period. It declines almost exactly as much when there are no changes in $S_{IKL}$ and $p_{K_{ICT}}K_{ICT}/p_{IP}K_{IP}$ and most of the decline is during the period 2008-2020. In turn, dynamics in it are almost unaffected when I fix the values of $S_{LIKTK}$ to the sample initial value of $S_{LIKTK}$.

I have used the values of all the estimated parameters from column 1 of Table 4 to compute the values of compensation shares (25)-(27). I compute data counterparts of the compensation shares using the computed rates of return on different types of capital (7), (8), and (9), recompute the elasticity of substitution between labor and aggregate capital using these shares and present the results in column 7 of Table 4. The values in column 2 are essentially the same as the values in column 1 except for higher variability and a pronounced fall during 2008-2009.

I also compute the elasticity of substitution between labor and aggregate capital for the US using equations (24)-(27) and parameter estimates from column 5 of Table 4 as well as the data counterparts of compensation shares for the US. I offer the results in columns 8 and 9 of Table 6. The values of $\varepsilon_{L,K}$ tend to be larger in the US than in the European countries similarly to the estimate of $\varepsilon_2$. Nevertheless, $\varepsilon_{L,K}$ shows similar dynamics to the one computed for the European countries. It increases till 2007 and declines afterward.

Finally, I estimate a function that combines total capital $K$ with labor and is given by

$$Y_t = \left[ \varpi_L^{\frac{1}{\varepsilon_1}} \left( e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_K^{\frac{1}{\varepsilon_1}} \left( e^{\gamma_K t} K_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{29}$$

where $K$ includes all types of capital. I consider 3 cases for $\varepsilon_1$ in (29): (1) $\varepsilon_1$ is constant (CES); (2) it linearly depends on the shares $S_{ICTIP}$, $S_{IKL}$, and ratios $p_{K_{ICT}}K_{ICT}/p_{TK}TK$

and $p_{K_{ICT}}K_{ICT}/p_{K_{IP}}K_{IP}$; and (3) it is a quadratic polynomial of time. I estimate the parameters in equation (29) for all sample countries using normalized and logarithmed $Y_t$ from equation (29) and the corresponding normalized and logarithmed first order conditions [see equations (51), (52) and (53) in the Technical Appendix].

Table 7 reports the results for European countries in columns 1-3 and the US in columns 4-6. The estimation results in columns 1 and 4 are for the case when $\varepsilon_1$ is assumed to be constant. The estimates of $\varepsilon_1$ are significantly below one in both columns and are in line with the estimates reported by Herrendorf et al. (2015), for example. Columns 2 and 5 report the estimation results for the case when the elasticity of substitution $\varepsilon_1$ is assumed to be a linear function of $S_{ICTIP}$, $S_{IKL}$, $p_{K_{ICT}}K_{ICT}/p_{TK}TK$, and $p_{K_{ICT}}K_{ICT}/p_{K_{IP}}K_{IP}$. The estimates in these columns imply that time varying elasticity of substitution between labor and aggregate capital fits the data better albeit the estimation results for the US are not very precise potentially owning to the low number of observations. Columns 3 and 6 report the estimation results for the case when $\varepsilon_1$ is assumed to be a quadratic polynomial of time. The estimates in these columns also imply that time varying elasticity of substitution between labor and aggregate capital fits the data better.[9]

Figure 3 plots the estimated values of $\varepsilon_1$ from Table 7 for European countries as well as the US. The estimated values of $\varepsilon_1$ are below 1. When the estimates for European countries imply that $\varepsilon_1$ has an inverted-U shape the estimates for the US imply the contrary.[10]

The inverted-U shape is in line with the imputed elasticity of substitution in Table 6 but $\varepsilon_1$ from Table 7 has a much lower level. The imputed elasticity of substitution $\varepsilon_{L,K}$ could attain values close to the values of $\varepsilon_1$ in Table 7 only when $\varepsilon_2$ in equation (24) is close to 0.9 (for the value of $\varepsilon_1$ from column 1 in Table 4) since the inequality (28) holds in the data. This would not be consistent however with the literature on automation as well as the values of elasticity of substitution between labor and ICT and labor and software estimated by Aum and Shin (2024), Eden and Gaggl (2018, 2019).

[9] Koh and Santaeulàlia-Llopis (2022) estimate time-varying elasticity of substitution between labor and capital at business cycle frequencies and find that it is countercyclical.

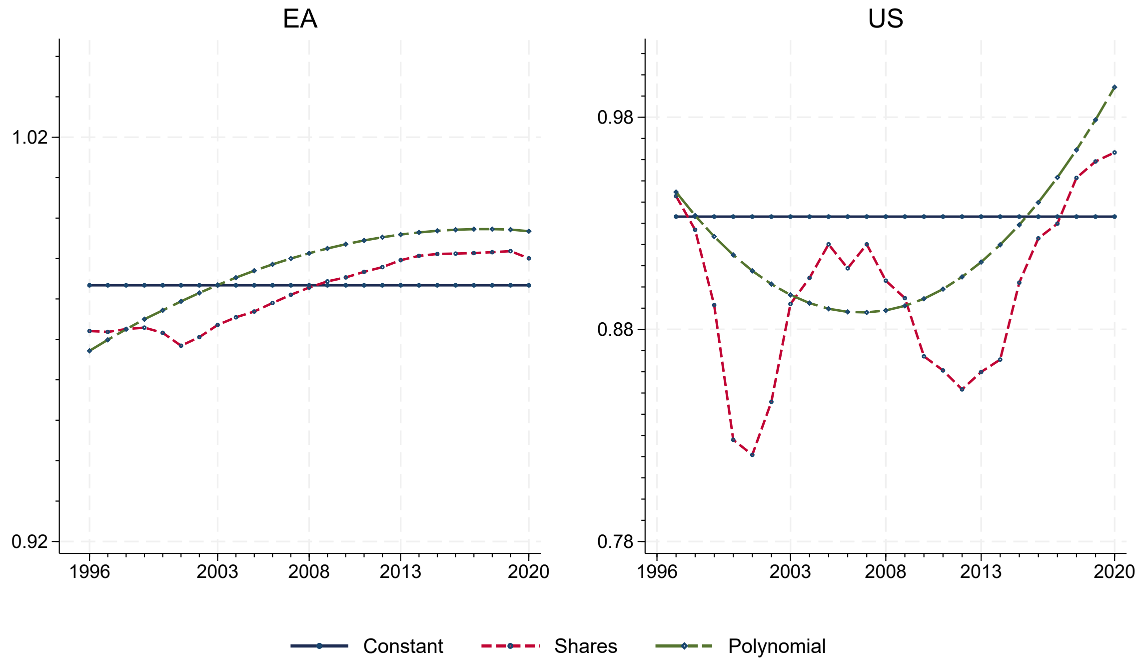
[10] This might be again because of the low number of observations.

**Table 7:** The Elasticity of Substitution and Aggregate Capital

| *A. Estimates* | European Countries (EA) | | | US | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Parameter | CES | Shares | Polynomial | CES | Shares | Polynomial |
| $\varepsilon_1$ | 0.984*** | 1.008*** | 0.988*** | 0.930*** | 1.893*** | 0.915*** |
| | (0.003) | (-0.024) | (-0.004) | (0.015) | (0.115) | (0.013) |
| $\gamma_L$ | 0.052*** | 0.052*** | 0.054*** | 0.055*** | 0.052*** | 0.064*** |
| | (-0.004) | (-0.004) | (-0.004) | (0.010) | (0.006) | (0.008) |
| $\gamma_K$ | -0.091*** | -0.092*** | -0.096*** | -0.082*** | -0.078*** | -0.100*** |
| | (-0.008) | (-0.007) | (-0.007) | (0.018) | (0.011) | (0.014) |
| $\varepsilon_{1,S_{ICTIP}}$ | | -0.017 | | | 0.031 | |
| | | (-0.069) | | | (0.156) | |
| $\varepsilon_{1,S_{IKL}}$ | | 0.045 | | | -4.652*** | |
| | | (0.310) | | | (0.716) | |
| $\varepsilon_{1,\frac{p_{K_{ICT}}K_{ICT}}{p_{K_{IP}}K_{IP}}}$ | | -0.110*** | | | 2.177** | |
| | | (0.039) | | | (0.921) | |
| $\varepsilon_{1,\frac{p_{K_{ICT}}K_{ICT}}{p_{TK}TK}}$ | | 0.306 | | | -42.851*** | |
| | | (1.119) | | | (12.261) | |
| $\varepsilon_{1,t}$ | | | 0.003** | | | -0.013*** |
| | | | (-0.001) | | | (0.003) |
| $\varepsilon_{1,t^2}$ | | | -0.0001 | | | 0.001*** |
| | | | (0.0001) | | | (0.000) |
| Obs. (per eq.) | 219 | 219 | 219 | 24 | 24 | 24 |
| *B. Measures of Fit* | | | | | | |
| Log Likelihood | 1977 | 1994 | 1990 | 256 | 277 | 268 |
| AIC | -3948 | -3974 | -3970 | -506 | -539 | -527 |
| BIC | -3938 | -3951 | -3953 | -502 | -531 | -521 |
| R2 | | | | | | |
| $ln\left(Y_t/\overline{Y}\right)$ | 0.940 | 0.941 | 0.940 | 0.995 | 0.994 | 0.994 |
| $ln\left(r_{K,t}/\overline{r_K}\right)$ | 0.860 | 0.893 | 0.867 | 0.954 | 0.982 | 0.972 |
| $ln\left(w_t/\overline{w}\right)$ | 0.672 | 0.755 | 0.691 | 0.616 | 0.884 | 0.782 |

Note: This table offers the results from the estimation of normalized and logarithmed function (29) together with the corresponding first order conditions [see equations (51), (52) and (53) in the Technical Appendix] for European countries (EA) in columns 1-3 and the US in columns 4-6. Panel *A* offers the estimates of the parameters and the corresponding number of observations in each equation. Sample years are given by the availability of data in the EU KLEMS database. Panel *B* offers various measures of fit including Log Likelihood, AIC, BIC, and R-squared of each equation. Columns 1 and 3 offer the results when $\varepsilon_1$ is assumed to be constant (CES). In columns 2 and 5, $\varepsilon_1$ is assumed to linearly depend on the shares $S_{ICTIP}$, $S_{IKL}$, and $S_{LIKTK}$, and ratios $p_{K_{ICT}}K_{ICT}/p_{TK}TK$ and $p_{K_{ICT}}K_{ICT}/p_{K_{IP}}K_{IP}$. Finally, it is assumed to be a quadratic polynomial of time in columns 3 and 6. I use a demeaned measure for the time index for estimations in these columns. All regressions use the feasible generalized non-linear least-squares estimation method and the country-year-level shares of hours of employment out of the total hours of employment across sample countries as weights. Initial/starting points for estimations are $\varepsilon_L = 0.5$ and $\gamma_j = 0.02$ for $j = L, K$. Standard errors are in parentheses and are robust to arbitrary heteroscedasticity and serial correlation. *** indicates significance at the 1% level, ** at the 5% level, and * at the 10% level. The Data Description Appendix offers further details about the data.

**Figure 3:** The Estimated Elasticity of Substitution between Labor and Aggregate Capital




Note: This figure illustrates the labor income share in European countries (EA) and in the US computed using the data from the EU KLEMS database. It also illustrates the predicted labor income share using equation (5) and parameter estimates from columns 1 and 5 of Table 4 for European countries and the US, correspondingly. The counterfactual predictions are for cases when there is no technological progress and changes in (1) ICT and IP capital, (2) ICT, (3) IP capital, and (4) traditional capital. The corresponding trend index is fixed to its sample initial value to have no technological progress in the type of capital and the value of capital stock is set equal to its sample initial value to have no changes in its level. Prediction results for European countries use parameter estimates from column 1 of Table 4. The averages across European countries are weighted by the country-year-level shares of employment hours out of the total hours of employment across sample European countries.

On the other hand, real wages have increased and returns on capital and real investment prices have fallen during the sample period. A significant part of the fall, in particular, in the investment prices is due to the fall of investment prices in ICT (see Table IV in the Data Appendix). The imputed value of the elasticity of substitution $\varepsilon_{L,K}$ and its implied dynamics in labor income share, given these observations, are in line with the juxtaposition of the results of Aum and Shin (2024), Eden and Gaggl (2018, 2019), and Karabarbounis and Neiman (2014).

# 5 Conclusion

In this study, I explore the elasticities of substitution between labor, information and communication technologies (ICT) and intellectual property (IP) capital, and traditional capital. I use data from the EU KLEMS database for a panel of 9 European countries and the estimation methodology developed and applied by Grandville (1989), Klump et al. (2007) and León-Ledesma et al. (2010). In particular, I estimate a nested CES production technology that utilizes labor, an aggregate of ICT and IP capital, and traditional capital.

The estimate of the elasticity of substitution between labor and ICT and IP capital is above 1 implying that labor and ICT and IP capital are gross substitutes. The estimate of the elasticity of substitution between labor and traditional capital is below 1 implying that labor and traditional capital are gross complements. Similarly, the estimate of the elasticity of substitution between ICT and IP capital is below 1.

These findings offer insights into the decline in labor income share, suggesting that much of the fall in labor share across European countries can be attributed to rapid technological progress and the accumulation of ICT and IP capital. Moreover, labor income share would be significantly higher absent technological progress and the accumulation of ICT capital.

These results also have implications for the elasticity of substitution between labor and aggregate capital. I derive it using the nested CES production function with different types of capital and compute its values using the function's estimated parameters. The imputed elasticity of substitution between labor and aggregate capital is greater than 1. This implies that labor and aggregate capital are gross substitutes. This value is also larger than the elasticity of substitution between labor and the aggregate of ICT and IP capital. It increases during the 1996-2008 period and declines in the following years. For comparison, I estimate the elasticity of substitution between labor and aggregate capital using data from the EU KLEMS database and following a similar methodology. The estimated value of the elasticity of substitution between labor and aggregate capital is much lower than 1 and the imputed values of this parameter. This implies that the specification of the production function can play an important role in determining the value of the estimated elasticity of substitution between labor and aggregate capital. In turn, the imputed elasticity of substitution, along with its implications for labor income share dynamics, aligns with the juxtaposition of findings by Aum and Shin (2024), Eden and Gaggl (2018, 2019), Karabarbounis and Neiman (2014). Finally, I show that all these results can also be extended to the US.

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# A Data Description Appendix

**Table A:** Definitions and Sources of Variables

| Variable Name | Definition and Source |
|---|---|
| ICT Capital Stock, NAC, current, bn | The nominal value of ICT capital stock in billions of national accounts currency (EUR in European/EA countries, USD in the US). ICT capital includes computers, communication devices, and peripheral equipment. Source: EU KLEMS database. |
| IP Capital Stock, NAC, current, bn | The nominal value of IP capital stock in billions of national accounts currency (EUR in European/EA countries, USD in the US). IP capital includes software, databases, patents, and other forms of intellectual property. Source: EU KLEMS database. |
| TK Stock, EUR, current, bn | The nominal value of traditional capital stock in billions of national accounts currency (EUR in European/EA countries, USD in the US). Traditional capital (TK) includes all types of capital but ICT and IP capital, such as machinery, transport, and construction equipment and structures. For example, it includes machinery and transport equipment. Source: EU KLEMS database. |
| ICT Investment Price Index | Price of investments in ICT capital. It is computed using a weighted average of prices of investments in information technology (IT) capital and communication technology (CT) capital. The weights are the averages of the shares of IT capital and CT capital out of the sum of IT and CT capital. The averages are taken over time within each country. Source: Authors' calculations using data from the EU KLEMS database. |
| IP Investment Price Index | Price of investments in IP capital. It is computed using a weighted average of prices of investments in R&D capital/patents and other types of intellectual property (IP) capital. The weights are the averages of the shares of different types of IP capital out of the total sum of IP capital types. The averages are taken over time within each country. Source: Authors' calculations using data from the EU KLEMS database. |

**Table A – (Continued)**

| Variable Name | Definition and Source |
|---|---|
| TK Investment Price Index | Price of investments in traditional capital, TK. It is computed using a weighted average of prices of investments in different types of traditional capital. The weights are the averages of the shares of different types of traditional capital out of the total sum of traditional capital types. The averages are taken over time within each country. Source: Authors' calculations using data from the EU KLEMS database. |
| $\delta_{ICT}$ | Annual depreciation rate of ICT capital. It is computed using a weighted average of depreciation rates of information technology (IT) capital and communication technology (CT) capital. The weights are the averages of the shares of IT capital and CT capital out of the sum of IT and CT capital. The averages are taken over time within each country. Source: Authors' calculations using data from the EU KLEMS database. |
| $\delta_{IP}$ | Annual depreciation rate of IP capital. It is computed using a weighted average of depreciation rates of R&D capital/patents and other types of intellectual property (IP) capital. The weights are the averages of the shares of different types of IP capital out of the total sum of IP capital types. The averages are taken over time within each country. Source: Authors' calculations using data from the EU KLEMS database. |
| $\delta_{TK}$ | Annual depreciation rate of traditional capital, TK. It is computed using a weighted average of depreciation rates of machinery, transport, and construction equipment and structures. The weights are the averages of the shares of different types of traditional capital (including machinery, transport, and construction equipment and structures) out of the total sum of traditional capital types. The averages are taken over time within each country. Source: Authors' calculations using data from the EU KLEMS database. |
| $r_{ICT}$ | Returns on ICT capital. Source: Authors' calculations using data from the EU KLEMS database and equation (7). |
| $r_{IP}$ | Returns on IP capital. Source: Authors' calculations using data from the EU KLEMS database and equation (8). |

**Table A – (Continued)**

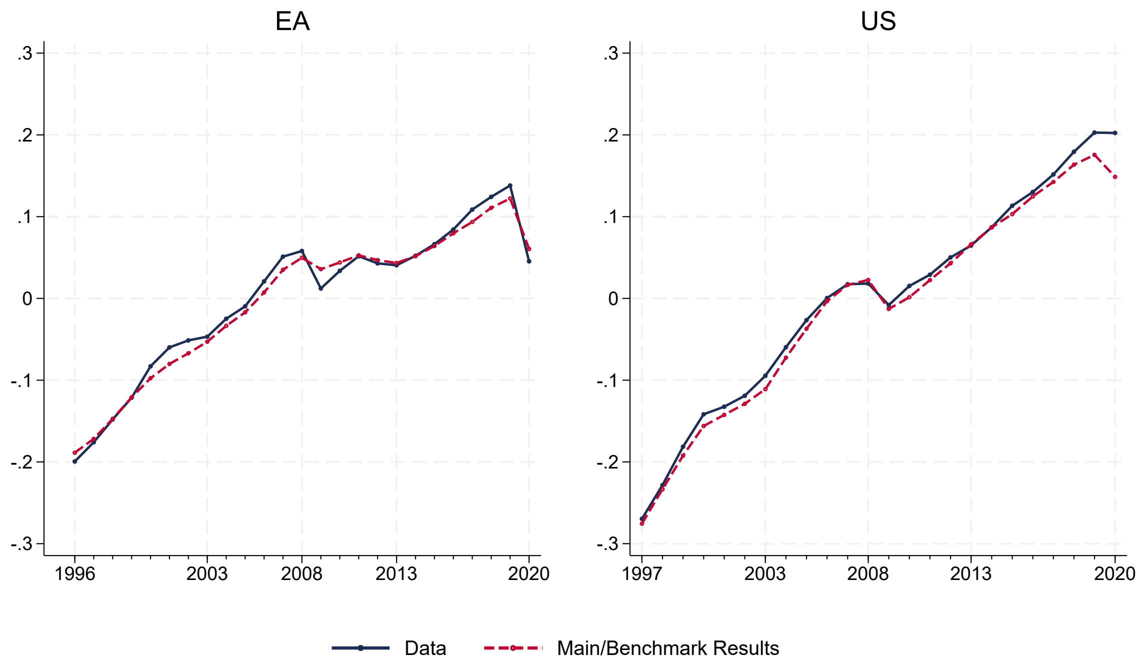
| Variable Name | Definition and Source |
|---|---|
| $r_{TK}$ | Returns on TK capital. Source: Authors' calculations using data from the EU KLEMS database and equation (9). |
| $w$ | Hourly wages. It is computed from the ratio of labor compensation and hours of work. Source: Authors' calculations using data from the EU KLEMS database. |

*Data Sources:* All data are from the 2023 release of the EU KLEMS database by the Luiss Lab of European Economics at Luiss University in Rome, Italy accessible at https://euklems-intanprod-llee.luiss.it/ (last accessed: 15.10.2024). This database includes information on investment in capital stocks across both tangible and intangible assets. It is often used to study productivity and output, employment, and labor income dynamics.

*Countries:* The main sample countries are Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Portugal, and Spain, and the period is from 1996-2020 except for Germany (1996-2019) and Portugal (2001-2020). The sample also includes data for the US (1997-2020).

# B Data Appendix

**Figure I:** Normalized and Logarithmed Real Value Added: Data and Prediction/Fit




Note: This figure illustrates the evolution of the normalized and logarithmed real value added in sample countries computed using data from the EU KLEMS database. It also illustrates the predicted values of this variable using equation (??) and parameter estimates from Table 4.

**Table I:** Country-Level Basic Statistics

| *Panel A* | Austria | | | Belgium | | | Finland | | | France | | | Germany | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) |
| Variable | Mean | 1996 | 2020 | Mean | 1996 | 2020 | Mean | 1996 | 2020 | Mean | 1996 | 2020 | Mean | 1996 | 2019 |
| Value Added, NAC, current, bn | 250 | 163 | 342 | 302 | 192 | 413 | 153 | 89 | 206 | 1661 | 1117 | 2061 | 2271 | 1742 | 3130 |
| Total Labor Income, NAC, current, bn | 165 | 115 | 233 | 189 | 125 | 248 | 99 | 61 | 128 | 1134 | 772 | 1425 | 1526 | 1204 | 2122 |
| Total Hours Worked, mn | 7 | 7 | 7 | 7 | 6 | 7 | 4 | 4 | 4 | 41 | 38 | 40 | 59 | 58 | 62 |
| ICT Capital Stock, NAC, current, bn | 23 | 18 | 28 | 15 | 10 | 21 | 4 | 2 | 6 | 31 | 21 | 42 | 88 | 104 | 74 |
| IP Capital Stock, NAC, current, bn | 46 | 20 | 88 | 44 | 19 | 84 | 30 | 12 | 39 | 304 | 186 | 460 | 402 | 262 | 639 |
| TK Stock, NAC, current, bn | 1005 | 622 | 1540 | 930 | 597 | 1333 | 516 | 310 | 781 | 5437 | 3307 | 7389 | 7674 | 5965 | 10700 |
| Total Capital Stock (K), NAC, current, bn | 1074 | 659 | 1656 | 989 | 626 | 1438 | 549 | 324 | 826 | 5771 | 3513 | 7890 | 8164 | 6331 | 11400 |
| Value Added Price Index | 0.885 | 0.749 | 1.100 | 0.898 | 0.743 | 1.090 | 0.872 | 0.708 | 1.060 | 0.914 | 0.776 | 1.060 | 0.914 | 0.840 | 1.070 |
| ICT Investment Price Index | 1.030 | 1.110 | 0.999 | 1.110 | 1.390 | 0.941 | 1.770 | 3.910 | 1.020 | 1.420 | 2.040 | 1.010 | 1.750 | 3.380 | 0.991 |
| IP Investment Price Index | 0.910 | 0.795 | 1.060 | 0.890 | 0.754 | 1.070 | 0.839 | 0.628 | 1.070 | 0.943 | 0.825 | 1.020 | 0.921 | 0.843 | 1.060 |
| TK Investment Price Index | 0.864 | 0.700 | 1.100 | 0.893 | 0.711 | 1.080 | 0.885 | 0.675 | 1.110 | 0.885 | 0.721 | 1.070 | 0.896 | 0.808 | 1.110 |
| Total Capital (K) Investment Price Index | 0.880 | 0.735 | 1.090 | 0.901 | 0.747 | 1.070 | 0.883 | 0.688 | 1.110 | 0.901 | 0.750 | 1.060 | 0.924 | 0.881 | 1.090 |

| *Panel B* | Austria | | | Belgium | | | Finland | | | France | | | Germany | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) | (14) | (15) |
| Derived Variable | Mean | 1996 | 2020 | Mean | 1996 | 2020 | Mean | 1996 | 2020 | Mean | 1996 | 2020 | Mean | 1996 | 2019 |
| $r_{ICT}$ | 0.239 | 0.345 | 0.169 | 0.407 | 0.650 | 0.275 | 0.724 | 2.100 | 0.238 | 0.440 | 0.660 | 0.203 | 0.614 | 1.180 | 0.249 |
| $r_{IP}$ | 0.289 | 0.286 | 0.255 | 0.294 | 0.285 | 0.270 | 0.265 | 0.224 | 0.274 | 0.321 | 0.318 | 0.284 | 0.276 | 0.253 | 0.282 |
| $r_{TK}$ | 0.066 | 0.058 | 0.053 | 0.101 | 0.094 | 0.100 | 0.089 | 0.075 | 0.088 | 0.077 | 0.080 | 0.067 | 0.078 | 0.071 | 0.077 |
| $w$ | 27 | 24 | 32 | 30 | 28 | 32 | 28 | 24 | 29 | 30 | 26 | 34 | 28 | 25 | 32 |

Note: This table offers the averages and sample initial and final values of the variables in each sample European country and the US. Nominal variables are in national currency units. EUR in the EU countries and USD in the US. See Table A in the **??** for complete descriptions and sources of variables.

**Table I:** Country-Level Basic Statistics — (Continued)

| *Panel A* | Italy | | | Netherlands | | | Portugal | | | Spain | | | US | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (16) | (17) | (18) | (19) | (20) | (21) | (22) | (23) | (24) | (25) | (26) | (27) | (28) | (29) | (30) |
| Variable | Mean | 1996 | 2020 | Mean | 1996 | 2020 | Mean | 1996 | 2020 | Mean | 1996 | 2020 | Mean | 1997 | 2020 |
| Value Added, NAC, current, bn | 1332 | 948 | 1502 | 523 | 311 | 710 | 150 | 119 | 175 | 831 | 452 | 1020 | 13300 | 8000 | 20200 |
| Total Labor Income, NAC, current, bn | 851 | 621 | 949 | 355 | 217 | 514 | 96 | 83 | 113 | 529 | 299 | 673 | 8315 | 5201 | 12400 |
| Total Hours Worked, mn | 43 | 41 | 39 | 12 | 11 | 13 | 9 | 10 | 9 | 31 | 24 | 30 | 252 | 240 | 261 |
| ICT Capital Stock, NAC, current, bn | 32 | 20 | 39 | 13 | 8 | 16 | 7 | 6 | 7 | 28 | 18 | 36 | 668 | 506 | 909 |
| IP Capital Stock, NAC, current, bn | 149 | 88 | 208 | 98 | 48 | 164 | 11 | 5 | 16 | 81 | 27 | 144 | 4015 | 1919 | 6607 |
| TK Stock, NAC, current, bn | 4612 | 2871 | 5806 | 1628 | 991 | 2255 | 599 | 437 | 767 | 3229 | 1591 | 4327 | 37800 | 20700 | 60700 |
| Total Capital Stock (K), NAC, current, bn | 4793 | 2978 | 6054 | 1739 | 1047 | 2435 | 617 | 449 | 791 | 3339 | 1636 | 4507 | 42500 | 23200 | 68200 |
| Value Added Price Index | 0.890 | 0.700 | 1.060 | 0.911 | 0.724 | 1.090 | 0.945 | 0.783 | 1.090 | 0.900 | 0.673 | 1.060 | 0.914 | 0.726 | 1.140 |
| ICT Investment Price Index | 1.100 | 1.240 | 1.010 | 2.020 | 5.320 | 0.900 | 1.340 | 2.080 | 0.984 | 1.150 | 1.190 | 0.973 | 2.010 | 4.590 | 0.726 |
| IP Investment Price Index | 0.899 | 0.743 | 0.998 | 0.912 | 0.755 | 1.090 | 0.970 | 0.821 | 1.070 | 0.881 | 0.663 | 1.040 | 0.949 | 0.853 | 1.070 |
| TK Investment Price Index | 0.877 | 0.679 | 1.040 | 0.917 | 0.695 | 1.110 | 0.960 | 0.804 | 1.140 | 0.978 | 0.743 | 1.080 | 0.613 | 0.713 | 1.200 |
| Total Capital (K) Investment Price Index | 0.884 | 0.697 | 1.030 | 0.928 | 0.761 | 1.100 | 0.972 | 0.844 | 1.100 | 0.965 | 0.732 | 1.070 | 0.944 | 0.818 | 1.110 |

| *Panel B* | Italy | | | Netherlands | | | Portugal | | | Spain | | | US | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (16) | (17) | (18) | (19) | (20) | (21) | (22) | (23) | (24) | (25) | (26) | (27) | (34) | (35) | (36) |
| Derived Variable | Mean | 1996 | 2020 | Mean | 1996 | 2020 | Mean | 1996 | 2020 | Mean | 1996 | 2020 | Mean | 1997 | 2020 |
| $r_{ICT}$ | 0.380 | 0.565 | 0.244 | 1.090 | 4.520 | 0.309 | 0.414 | 0.846 | 0.223 | 0.338 | 0.484 | 0.234 | 0.903 | 2.860 | 0.181 |
| $r_{IP}$ | 0.317 | 0.369 | 0.283 | 0.294 | 0.310 | 0.279 | 0.308 | 0.293 | 0.304 | 0.284 | 0.274 | 0.284 | 0.277 | 0.322 | 0.247 |
| $r_{TK}$ | 0.092 | 0.098 | 0.082 | 0.083 | 0.073 | 0.065 | 0.082 | 0.077 | 0.075 | 0.092 | 0.097 | 0.070 | 0.065 | 0.097 | 0.100 |
| $w$ | 22 | 22 | 23 | 31 | 27 | 35 | 11 | 11 | 12 | 19 | 18 | 21 | 35 | 30 | 42 |

Note: This table offers the averages and sample initial and final values of the variables in each sample European country and the US. Nominal variables are in national currency units. EUR in the EU countries and USD in the US. See Table A in the **??** for complete descriptions and sources of variables.

**Table II:** Shares and Ratios

| Year | $S_{ICTIP}$ | $S_{IKL}$ | $S_{LIKTK}$ | $\frac{p_{K_{ICT}} K_{ICT}}{p_{K_{IP}} K_{IP}}$ | $\frac{p_{K_{ICT}} K_{ICT}}{p_{TK} TK}$ |
|---|---|---|---|---|---|
| 1996 | 0.245 | 0.065 | 0.713 | 0.341 | 0.012 |
| 1997 | 0.239 | 0.065 | 0.713 | 0.345 | 0.012 |
| 1998 | 0.233 | 0.066 | 0.712 | 0.340 | 0.012 |
| 1999 | 0.227 | 0.068 | 0.712 | 0.338 | 0.012 |
| 2000 | 0.221 | 0.068 | 0.711 | 0.353 | 0.013 |
| 2001 | 0.227 | 0.068 | 0.710 | 0.381 | 0.013 |
| 2002 | 0.222 | 0.069 | 0.710 | 0.362 | 0.012 |
| 2003 | 0.216 | 0.069 | 0.711 | 0.333 | 0.011 |
| 2004 | 0.211 | 0.070 | 0.711 | 0.316 | 0.011 |
| 2005 | 0.205 | 0.071 | 0.711 | 0.302 | 0.010 |
| 2006 | 0.200 | 0.071 | 0.711 | 0.282 | 0.010 |
| 2007 | 0.195 | 0.072 | 0.711 | 0.263 | 0.009 |
| 2008 | 0.189 | 0.073 | 0.711 | 0.247 | 0.009 |
| 2009 | 0.184 | 0.074 | 0.713 | 0.234 | 0.008 |
| 2010 | 0.179 | 0.075 | 0.714 | 0.226 | 0.008 |
| 2011 | 0.174 | 0.076 | 0.714 | 0.214 | 0.008 |
| 2012 | 0.168 | 0.077 | 0.715 | 0.204 | 0.008 |
| 2013 | 0.164 | 0.077 | 0.716 | 0.188 | 0.008 |
| 2014 | 0.160 | 0.078 | 0.715 | 0.179 | 0.007 |
| 2015 | 0.156 | 0.078 | 0.714 | 0.176 | 0.008 |
| 2016 | 0.152 | 0.078 | 0.713 | 0.176 | 0.008 |
| 2017 | 0.148 | 0.079 | 0.712 | 0.176 | 0.008 |
| 2018 | 0.144 | 0.079 | 0.711 | 0.174 | 0.008 |
| 2019 | 0.140 | 0.079 | 0.710 | 0.173 | 0.008 |
| 2020 | 0.134 | 0.077 | 0.710 | 0.189 | 0.008 |

Note: This table offers the values of the shares $S_{ICTIP}$, $S_{IKL}$, and $S_{LIKTK}$ from equations (25)-(27) and ratios $p_{K_{ICT}} K_{ICT}/p_{K_{IP}} K_{IP}$ and $p_{K_{ICT}} K_{ICT}/p_{TK} TK$. The averages across European countries are weighted by the country-year-level shares of employment hours out of the total hours of employment across sample European countries.

**Table III:** Shares and Ratios in the US

| Year | $S_{ICTIP}$ | $S_{IKL}$ | $S_{LIKTK}$ | $\frac{p_{K_{ICT}}K_{ICT}}{p_{K_{IP}}K_{IP}}$ | $\frac{p_{K_{ICT}}K_{ICT}}{p_{TK}TK}$ |
|---|---|---|---|---|---|
| 1997 | 0.284 | 0.106 | 0.749 | 0.264 | 0.024 |
| 1998 | 0.272 | 0.110 | 0.747 | 0.259 | 0.024 |
| 1999 | 0.259 | 0.114 | 0.745 | 0.267 | 0.025 |
| 2000 | 0.248 | 0.118 | 0.743 | 0.276 | 0.026 |
| 2001 | 0.238 | 0.122 | 0.742 | 0.266 | 0.026 |
| 2002 | 0.230 | 0.125 | 0.743 | 0.242 | 0.023 |
| 2003 | 0.222 | 0.128 | 0.742 | 0.209 | 0.020 |
| 2004 | 0.213 | 0.131 | 0.742 | 0.202 | 0.019 |
| 2005 | 0.206 | 0.134 | 0.740 | 0.173 | 0.017 |
| 2006 | 0.197 | 0.136 | 0.738 | 0.171 | 0.017 |
| 2007 | 0.190 | 0.137 | 0.736 | 0.157 | 0.016 |
| 2008 | 0.182 | 0.139 | 0.735 | 0.153 | 0.016 |
| 2009 | 0.174 | 0.141 | 0.735 | 0.152 | 0.016 |
| 2010 | 0.167 | 0.142 | 0.733 | 0.152 | 0.017 |
| 2011 | 0.160 | 0.141 | 0.730 | 0.147 | 0.016 |
| 2012 | 0.153 | 0.140 | 0.727 | 0.147 | 0.017 |
| 2013 | 0.147 | 0.140 | 0.725 | 0.140 | 0.016 |
| 2014 | 0.142 | 0.139 | 0.722 | 0.132 | 0.016 |
| 2015 | 0.135 | 0.135 | 0.719 | 0.132 | 0.015 |
| 2016 | 0.130 | 0.134 | 0.718 | 0.131 | 0.015 |
| 2017 | 0.124 | 0.131 | 0.716 | 0.136 | 0.015 |
| 2018 | 0.119 | 0.129 | 0.713 | 0.137 | 0.015 |
| 2019 | 0.114 | 0.127 | 0.711 | 0.135 | 0.015 |
| 2020 | 0.109 | 0.128 | 0.715 | 0.138 | 0.015 |

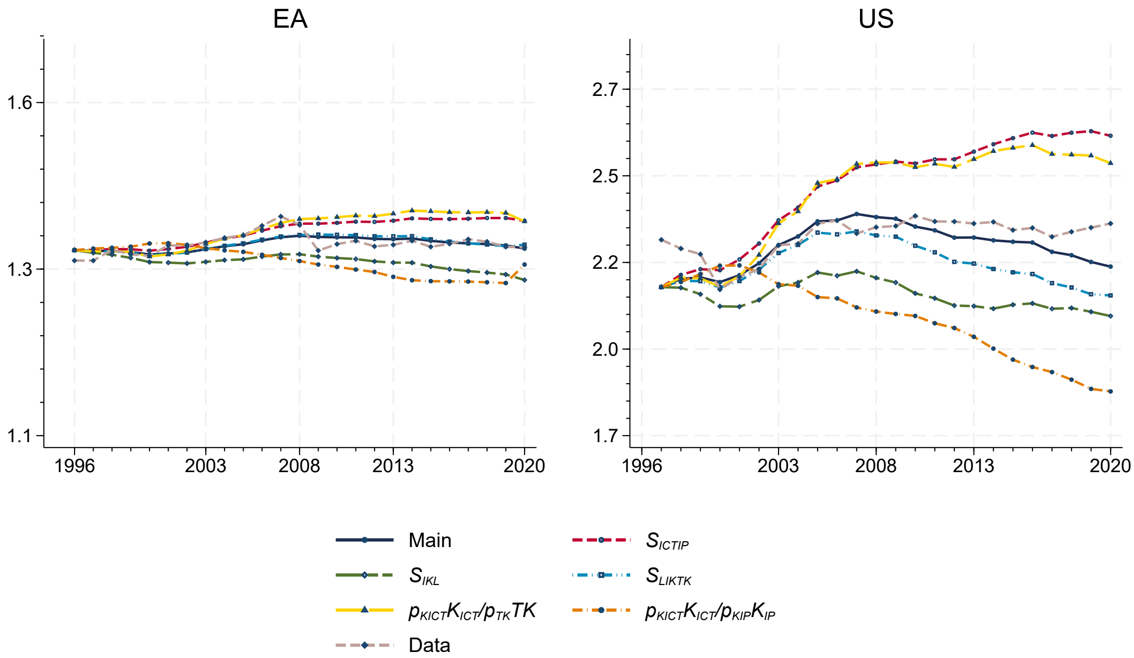
Note: This table offers the values of the shares $S_{ICTIP}$, $S_{IKL}$, and $S_{LIKTK}$ from equations (25)-(27) and ratios $p_{K_{ICT}}K_{ICT}/p_{K_{IP}}K_{IP}$ and $p_{K_{ICT}}K_{ICT}/p_{TK}TK$ in the US.

**Table IV:** Growth Rate of Real Investment Prices in Aggregate Capital and Real Wages

| | European Countries (EA) | | | | US | | |
|---|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | | (4) | (5) | (6) |
| | $p_K$ | $p_K$ w/t $\Delta$ in $p_{ICT}$ | $w$ | | $p_K$ | $p_K$ w/t $\Delta$ in $p_{ICT}$ | $w$ |
| $g_{1996-2020}$ | -0.025 | -0.011 | 0.132 | $g_{1997-2020}$ | -0.147 | -0.035 | 0.332 |

Note: This table offers the growth rates of real prices of investments over the sample period in total/aggregate capital $p_K$ in columns 1, 2, 4, and 5 and real wages $w$ in columns 3 and 6. I compute $p_K$ as $p_{K,t} = p_{ICT,t} \times K_{ICT,t}/K_t + p_{IP,t} \times K_{IP,t}/K_t + p_{TK,t} \times TK_t/K_t$ and set $p_{ICT,t}$ to its sample initial value for all countries and years in columns 2 and 5. The Data Description Appendix offers further details about the data.

**Figure II:** The Imputed Elasticity of Substitution and Counterfactuals




Note: This figure illustrates values of $\varepsilon_{L,K}$ computed using equations (24)-(27) and estimated values of parameters from column 1 of Table 4. Counterfactuals illustrate the values of $\varepsilon_{L,K}$ when the values of the shares $S_{ICTIP}$, $S_{IKL}$, and $S_{LIKTK}$, and ratios $p_{K_{ICT}}K_{ICT}/p_{TK}TK$ and $p_{K_{ICT}}K_{ICT}/p_{K_{IP}}K_{IP}$ are fixed at their sample initial values. This figure also illustrates the values of $\varepsilon_{L,K}$ computed using the data counterparts of $S_{ICTIP}$, $S_{IKL}$, and $S_{LIKTK}$ instead of using equations (25)-(27).

# C Technical Appendix

In this section, I offer the estimated equations for alternative specifications of the production function (20), (21), and (29).

## C.A Separate Nests for Labor and Traditional Capital and for ICT and IP

The firm solves problem (2) where production function is now given by

$$Y_t = \left( \varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right)^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{30}$$

where

$$LTK_t = \left[ \varpi_L^{\frac{1}{\varepsilon_2}} \left( e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left( e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$

$$IK_t = \left[ \varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}.$$

The first order conditions that follow from this problem are given by

$$\frac{r_{ICT,t} K_{ICT,t}}{Y_t} = \frac{\varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} \times \frac{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}, \tag{31}$$

$$\frac{r_{IP,t} K_{IP,t}}{Y_t} = \frac{\varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} \times \frac{\varpi_{IP}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}, \tag{32}$$

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK} t} TK_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}, \tag{33}$$

and

$$\frac{r_{TK,t} TK_t}{Y_t} = \frac{\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_{LTK}^{\frac{1}{\varepsilon_1}} LTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{IK}^{\frac{1}{\varepsilon_1}} IK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \frac{\varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK} t} TK_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{L}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK} t} TK_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}. \tag{34}$$

I denote by $S_{LTK_t,Y_t}$ and $S_{IK_t,Y_t}$ the shares of compensation of labor and traditional capital and ICT and IP capital in value added:

$$S_{LTK_t,Y_t} = \varpi_{LTK}^{\frac{1}{\varepsilon_1}} \left(\frac{LTK_t}{Y_t}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$
$$S_{IK_t,Y_t} = \varpi_{IK}^{\frac{1}{\varepsilon_2}} \left(\frac{IK_t}{Y_t}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}.$$

I also use $\alpha_{1,R3}$, $\alpha_{2,R3}$, and $\alpha_{3,R3}$ to denote the following expressions:

$$\alpha_{1,R3} = \varpi_{LTK}^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{LTK_t}{Y_t}\right)}^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$
$$\alpha_{2,R3} = \varpi_{L}^{\frac{1}{\varepsilon_2}} \overline{\left(\frac{e^{\gamma_L t} L_t}{LTK_t}\right)}^{\frac{\varepsilon_2-1}{\varepsilon_2}},$$
$$\alpha_{3,R3} = \varpi_{ICT}^{\frac{1}{\varepsilon_3}} \overline{\left(\frac{e^{\gamma_{ICT} t} K_{ICT,t}}{IK_t}\right)}^{\frac{\varepsilon_3-1}{\varepsilon_3}}.$$

I use $\alpha_{1,R3}$, $\alpha_{2,R3}$, and $\alpha_{3,R3}$ and write the logrithm of the normalized equation for output in the following way:

$$\ln \frac{Y_t}{\overline{Y_t}} = \frac{\varepsilon_1}{\varepsilon_1 - 1} \ln \left[\alpha_{1,R3} \left(\frac{LTK_t}{\overline{LTK_t}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + (1 - \alpha_{1,R3}) \left(\frac{IK_t}{\overline{IK_t}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right], \tag{35}$$

where

$$\frac{LTK_t}{\overline{LTK_t}} = \left[\alpha_{2,R3}\left(e^{\gamma_L \hat{t}}\frac{L_t}{\overline{L_t}}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + (1-\alpha_{2,R3})\left(\frac{TK_t}{\overline{TK_t}}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}},$$

$$\frac{IK_t}{\overline{IK_t}} = \left[\alpha_{3,R3}\left(e^{\gamma_{ICT}\hat{t}}\frac{K_{ICT,t}}{\overline{K_{ICT,t}}}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + (1-\alpha_{3,R3})\left(e^{\gamma_{IP}\hat{t}}\frac{K_{IP,t}}{\overline{K_{IPP,t}}}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}\right]^{\frac{\varepsilon_3}{\varepsilon_3-1}},$$

and $\hat{t}$ is demeaned trend.

I write the normalized equations corresponding to the first order conditions in the following way:

$$\ln r_{ICT,t} - \ln \overline{r_{ICT,t}} = \left(1 - \frac{\varepsilon_3-1}{\varepsilon_3}\frac{\varepsilon_1}{\varepsilon_1-1}\right)\ln\left(\frac{S_{IK_t,Y_t}}{\overline{S_{IK_t,Y_t}}}\right) + \frac{\varepsilon_3-1}{\varepsilon_3}\gamma_{ICT}\hat{t} - \frac{1}{\varepsilon_3}\ln\left(\frac{K_{ICT,t}/\overline{K_{ICT,t}}}{Y_t/\overline{Y_t}}\right), \tag{36}$$

$$\ln r_{IP,t} - \ln \overline{r_{IP,t}} = \left(1 - \frac{\varepsilon_3-1}{\varepsilon_3}\frac{\varepsilon_1}{\varepsilon_1-1}\right)\ln\left(\frac{S_{IK_t,Y_t}}{\overline{S_{IK_t,Y_t}}}\right) + \frac{\varepsilon_3-1}{\varepsilon_3}\gamma_{IP}\hat{t} - \frac{1}{\varepsilon_3}\ln\left(\frac{K_{IP,t}/\overline{K_{IP,t}}}{Y_t/\overline{Y_t}}\right), \tag{37}$$

$$\ln w_t - \ln \overline{w_t} = \left(1 - \frac{\varepsilon_2-1}{\varepsilon_2}\frac{\varepsilon_1}{\varepsilon_1-1}\right)\ln\left(\frac{S_{LTK_t,Y_t}}{\overline{S_{LTK_t,Y_t}}}\right) + \frac{\varepsilon_2-1}{\varepsilon_2}\gamma_L\hat{t} - \frac{1}{\varepsilon_2}\ln\left(\frac{L_t/\overline{L_t}}{Y_t/\overline{Y_t}}\right), \tag{38}$$

and

$$\ln r_{TK,t} - \ln \overline{r_{TK,t}} = \left(1 - \frac{\varepsilon_2-1}{\varepsilon_2}\frac{\varepsilon_1}{\varepsilon_1-1}\right)\ln\left(\frac{S_{LTK_t,Y_t}}{\overline{S_{LTK_t,Y_t}}}\right) + \frac{\varepsilon_2-1}{\varepsilon_2}\gamma_{TK}\hat{t} - \frac{1}{\varepsilon_2}\ln\left(\frac{TK_t/\overline{TK_t}}{Y_t/\overline{Y_t}}\right). \tag{39}$$

## C.B One Nest for Capital Types

The firm solves problem (2) where production function is now given by

$$Y_t = \left[ \varpi_L^{\frac{1}{\varepsilon_1}} \left( e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} \right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}, \tag{40}$$

where

$$IKTK_t = \left[ \varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left( e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} \right]^{\frac{\varepsilon_2}{\varepsilon_2 - 1}},$$

and

$$IK_t = \left[ \varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} \right]^{\frac{\varepsilon_3}{\varepsilon_3 - 1}}.$$

The first order conditions that follow from this problem are given by

$$\begin{aligned} \frac{r_{ICT,t} K_{ICT,t}}{Y_t} = & \frac{\varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left( e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} \times \\ & \frac{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left( e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}} \times \\ & \frac{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}, \end{aligned} \tag{41}$$

$$\begin{aligned} \frac{r_{IP,t} K_{IP,t}}{Y_t} = & \frac{\varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left( e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} \times \\ & \frac{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}}{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2 - 1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left( e^{\gamma_{TK} t} TK_t \right)^{\frac{\varepsilon_2 - 1}{\varepsilon_2}}} \times \\ & \frac{\varpi_{IP}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}{\varpi_{ICT}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{ICT} t} K_{ICT,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}} + \varpi_{IP}^{\frac{1}{\varepsilon_3}} \left( e^{\gamma_{IP} t} K_{IP,t} \right)^{\frac{\varepsilon_3 - 1}{\varepsilon_3}}}, \end{aligned} \tag{42}$$

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_L^{\frac{1}{\varepsilon_1}} \left( e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left( e^{\gamma_L t} L_t \right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}, \tag{43}$$

and

$$\frac{r_{TK,t}TK_t}{Y_t} = \frac{\varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} IKTK_t^{\frac{\varepsilon_1-1}{\varepsilon_1}}} \times \tag{44}$$
$$\frac{\varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK} t} TK_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}{\varpi_{IK}^{\frac{1}{\varepsilon_2}} IK_t^{\frac{\varepsilon_2-1}{\varepsilon_2}} + \varpi_{TK}^{\frac{1}{\varepsilon_2}} \left(e^{\gamma_{TK} t} TK_t\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}}.$$

I denote by $S_{IKTK_t,Y_t}$ and $S_{IK_t,IKTK_t}$ the following expressions:

$$S_{IKTK_t,Y_t} = \varpi_{TKIK}^{\frac{1}{\varepsilon_1}} \left(\frac{IKTK_t}{Y_t}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$
$$S_{IK_t,IKTK_t} = \varpi_{IK}^{\frac{1}{\varepsilon_2}} \left(\frac{IK_t}{IKTK_t}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}.$$

I also use $\alpha_{1,R4}$, $\alpha_{2,R4}$, and $\alpha_{3,R4}$ to denote the following expressions:

$$\alpha_{1,R4} = \varpi_L^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{e^{\gamma_L t} L_t}{Y_t}\right)}^{\frac{\varepsilon_1-1}{\varepsilon_1}},$$
$$\alpha_{2,R4} = \varpi_{IK}^{\frac{1}{\varepsilon_2}} \overline{\left(\frac{IK_t}{IKTK_t}\right)}^{\frac{\varepsilon_2-1}{\varepsilon_2}},$$
$$\alpha_{3,R4} = \varpi_{ICT}^{\frac{1}{\varepsilon_3}} \overline{\left(\frac{e^{\gamma_{ICT} t} K_{ICT,t}}{IK_t}\right)}^{\frac{\varepsilon_3-1}{\varepsilon_3}}.$$

I use $\alpha_{1,R4}$, $\alpha_{2,R4}$, and $\alpha_{3,R4}$ and write the logarithm of the normalized equation for output in the following way:

$$\ln \frac{Y_t}{\overline{\overline{Y_t}}} = \frac{\varepsilon_1}{\varepsilon_1 - 1} \ln \left[\alpha_{1,R4} \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{\overline{L_t}}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}} + (1 - \alpha_{1,R4}) \left(\frac{IKTK_t}{\overline{\overline{IKTK_t}}}\right)^{\frac{\varepsilon_1-1}{\varepsilon_1}}\right], \tag{45}$$

where

$$\frac{IKTK_t}{\overline{\overline{IKTK_t}}} = \left[\alpha_{2,R4} \left(\frac{IK_t}{\overline{\overline{IK_t}}}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}} + (1 - \alpha_{2,R4}) \left(e^{\gamma_{TK} \hat{t}} \frac{TK_t}{\overline{\overline{TK_t}}}\right)^{\frac{\varepsilon_2-1}{\varepsilon_2}}\right]^{\frac{\varepsilon_2}{\varepsilon_2-1}},$$
$$\frac{IK_t}{\overline{\overline{IK_t}}} = \left[\alpha_{3,R4} \left(e^{\gamma_{ICT} \hat{t}} \frac{K_{ICT,t}}{\overline{\overline{K_{ICT,t}}}}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}} + (1 - \alpha_{3,R4}) \left(e^{\gamma_{IP} \hat{t}} \frac{K_{IP,t}}{\overline{\overline{K_{IP,t}}}}\right)^{\frac{\varepsilon_3-1}{\varepsilon_3}}\right]^{\frac{\varepsilon_3}{\varepsilon_3-1}}.$$

I write the normalized equations corresponding to the first order conditions in the following way:

$$\ln r_{ICT,t} - \ln \overline{r_{ICT,t}} = \left(1 - \frac{\varepsilon_1}{\varepsilon_1 - 1}\frac{\varepsilon_3 - 1}{\varepsilon_3}\right) \ln\left(\frac{S_{IKTK_t,Y_t}}{\overline{\overline{S_{IKTK_t,Y_t}}}}\right) + \left(1 - \frac{\varepsilon_2}{\varepsilon_2 - 1}\frac{\varepsilon_3 - 1}{\varepsilon_3}\right) \ln\left(\frac{S_{IK_t,IKTK_t}}{\overline{\overline{S_{IK_t,IKTK_t}}}}\right) + \frac{\varepsilon_3 - 1}{\varepsilon_3}\gamma_{ICT}\hat{t} - \frac{1}{\varepsilon_3}\ln\left(\frac{K_{ICT,t}/\overline{K_{ICT,t}}}{Y_t/\overline{Y_t}}\right), \tag{46}$$

$$\ln r_{IP,t} - \ln \overline{r_{IP,t}} = \left(1 - \frac{\varepsilon_1}{\varepsilon_1 - 1}\frac{\varepsilon_3 - 1}{\varepsilon_3}\right) \ln\left(\frac{S_{IKTK_t,Y_t}}{\overline{\overline{S_{IKTK_t,Y_t}}}}\right) + \left(1 - \frac{\varepsilon_2}{\varepsilon_2 - 1}\frac{\varepsilon_3 - 1}{\varepsilon_3}\right) \ln\left(\frac{S_{IK_t,IKTK_t}}{\overline{\overline{S_{IK_t,IKTK_t}}}}\right) + \frac{\varepsilon_3 - 1}{\varepsilon_3}\gamma_{IP}\hat{t} - \frac{1}{\varepsilon_3}\ln\left(\frac{K_{IP,t}/\overline{K_{IP,t}}}{Y_t/\overline{Y_t}}\right), \tag{47}$$

$$\ln w_t - \ln \overline{w_t} = \frac{\varepsilon_1 - 1}{\varepsilon_1}\gamma_L\hat{t} - \frac{1}{\varepsilon_1}\ln\left(\frac{L_t/\overline{L_t}}{Y_t/\overline{Y_t}}\right), \tag{48}$$

and

$$\ln r_{TK,t} - \ln \overline{r_{TK,t}} = \left(1 - \frac{\varepsilon_1}{\varepsilon_1 - 1}\frac{\varepsilon_3 - 1}{\varepsilon_3}\right) \ln\left(\frac{S_{IKTK_t,Y_t}}{\overline{\overline{S_{IKTK_t,Y_t}}}}\right) + \frac{\varepsilon_2 - 1}{\varepsilon_2}\gamma_{TK}\hat{t} - \frac{1}{\varepsilon_2}\ln\left(\frac{TK_t/\overline{TK_t}}{Y_t/\overline{Y_t}}\right). \tag{49}$$

## C.C One Type of Capital

I consider an infinitely lived firm that has the following production technology

$$Y_t = \left[\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_K^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_K t} K_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right]^{\frac{\varepsilon_1}{\varepsilon_1 - 1}}. \tag{50}$$

where $K$ is total capital stock.

The firm discounts its profits at the rate of return $r$ and solves the following problem:

$$\max_{\{L_t,I_t\}_{t=0}^{+\infty}} \sum_{t=0}^{+\infty} \left(\frac{1}{1+r_t}\right)^t (Y_t - w_t L_t - p_t I_t)$$

$$s.t.$$

$$I_t = K_{t+1} - (1-\delta) K_t.$$

The first order conditions that follow from this problem are given by

$$\frac{w_t L_t}{Y_t} = \frac{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_K^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_K t} K_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}$$

$$\frac{r_{K,t} K_t}{Y_t} = \frac{\varpi_K^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_K t} K_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}{\varpi_L^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_L t} L_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + \varpi_K^{\frac{1}{\varepsilon_1}} \left(e^{\gamma_K t} K_t\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}}$$

I use $\alpha_{1,R1}$ to denote

$$\alpha_{1,R1} = \varpi_L^{\frac{1}{\varepsilon_1}} \overline{\left(\frac{A_{L,t} L_t}{Y_t}\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}} = \overline{\frac{w_t L_t}{Y_t}},$$

and the estimations use the following system of normalized equations:

$$\ln \frac{Y_t}{\overline{Y}} = \frac{\varepsilon_1}{\varepsilon_1 - 1} \ln \left[\alpha_{1,R1} \left(e^{\gamma_L \hat{t}} \frac{L_t}{\overline{\overline{L_t}}}\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}} + (1 - \alpha_{1,R1}) \left(e^{\gamma_K \hat{t}} \frac{K_t}{\overline{\overline{K_t}}}\right)^{\frac{\varepsilon_1 - 1}{\varepsilon_1}}\right], \tag{51}$$

$$\ln w_t - \ln \overline{w_t} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \gamma_L \hat{t} - \frac{1}{\varepsilon_1} \ln \left(\frac{L_t / \overline{L_t}}{Y_t / \overline{Y_t}}\right), \tag{52}$$

$$\ln r_{K,t} - \ln \overline{r_{K,t}} = \frac{\varepsilon_1 - 1}{\varepsilon_1} \gamma_K \hat{t} - \frac{1}{\varepsilon_1} \ln \left(\frac{K_t / \overline{K_t}}{Y_t / \overline{Y_t}}\right). \tag{53}$$

The values of $r$ are determined using $r_t = (Y_t - w_t L_t) / K_t$.