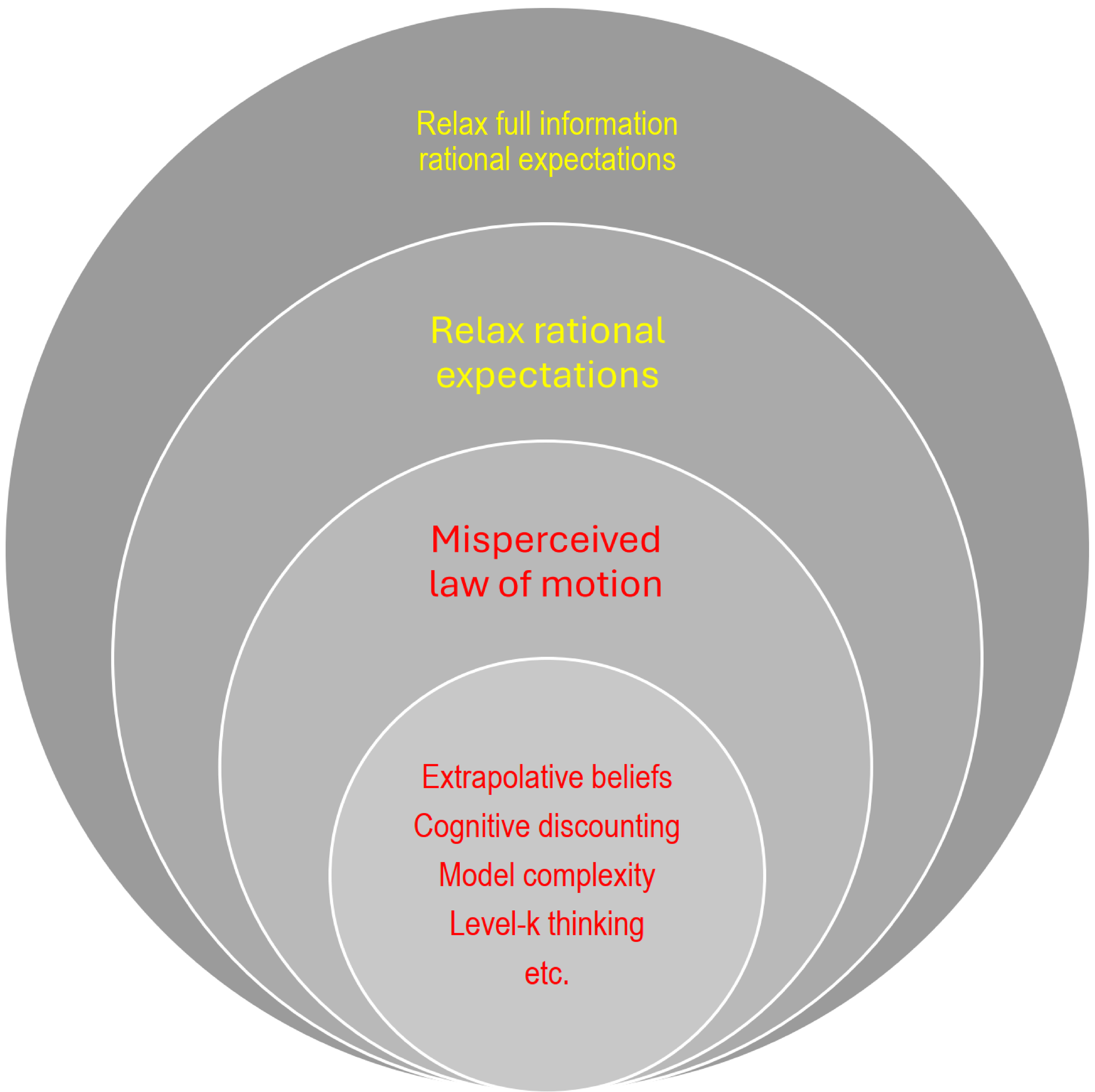
Misperceived Law of Motion in Macroeconomic Expectations

Zhesheng Qiu, the Hong Kong University of Science and Technology
Email: zheshqiu@ust.hk Website: https://www.zheshengqiu.com/

Refining Expectations Formation



General Framework

- **Model:** use the historical noisy signals $\mathbf{w}_i^t$ on $\mathbf{y}^t$ to forecast vector $\mathbf{y}_{t+h}$ via linear Gaussian signal extraction under perceived covariance $\widetilde{cov}_i$ and variance $\widetilde{var}_i$

- **Best linear unbiased forecaster:**
$$\mathbf{F}_{i,t}\mathbf{y}_{t+h} = \widetilde{cov}_i(\mathbf{y}_{t+h}, \mathbf{w}_i^t) \cdot \widetilde{var}_i(\mathbf{w}_i^t)^{-1} \cdot \mathbf{w}_i^t$$

- **Relax full information but keep rational expectations:**
$$\widetilde{cov}_i(\mathbf{y}_{t+h}, \mathbf{w}_i^t) = cov(\mathbf{y}_{t+h}, \mathbf{w}_i^t), \qquad \widetilde{var}_i(\mathbf{w}_i^t) = var(\mathbf{w}_i^t) > var(\mathbf{y}^t)$$
  ① Sticky-information á la Mankiw and Reis (2002)
  ② Noisy-information á la Woodford (2003)
  ③ Rational inattention á la Sims (2003)
  ④ Sentiment á la Angeletos and La'O (2013)

- **Relax rational expectations but keep law of motion correctly perceived:**
$$\widetilde{cov}_i(\mathbf{y}_{t+h}, \mathbf{w}_i^t) = cov(\mathbf{y}_{t+h}, \mathbf{w}_i^t), \qquad \widetilde{var}_i(\mathbf{w}_i^t) \neq var(\mathbf{w}_i^t)$$
  ⑤ Overconfidence á la Daniel et al. (1998)
  ⑥ Diagnostic expectations á la Bordalo et al. (2020)
  ⑦ Imprecise memory á la da Silveira et al. (2024)
  ⑧ Ambiguity aversion about the mean á la Huo et al. (2024)

- **Relax the correctly perceived law of motion** (note that $\mathbf{w}_i^t = \mathbf{y}^t + \textbf{noise}$):
$$\widetilde{cov}_i(\mathbf{y}_{t+h}, \mathbf{w}_i^t) \neq cov(\mathbf{y}_{t+h}, \mathbf{w}_i^t)$$
  ⑨ Over-extrapolation á la Angeletos et al. (2020)
  ⑩ Cognitive discounting á la Gabaix (2020)
  ⑪ Reduced model complexity á la Molavi et al. (2024)
  ⑫ Level-k thinking á la García-Schmidt and Woodford (2019)

Self-Adjoint Method

- **Method:** detect misperceived law of motion via rejecting the null hypothesis
$$cov(\mathbf{F}_t\mathbf{y}_{t+h}, \mathbf{y}_t) = cov(\mathbf{y}_{t+h}, \mathbf{F}_t\mathbf{y}_t)$$
  where $\mathbf{F}_t$ denotes the consensus version of $\mathbf{F}_{i,t}$

- **Byproducts:**
  - Characterizing the **actual law of motion** via autocovariance $cov(\mathbf{y}_{t+h}, \mathbf{y}_t)$
  - **Test of information rigidity** via $cov(\mathbf{y}_{t+h}, \mathbf{y}_t) = cov(\mathbf{y}_{t+h}, \mathbf{F}_t\mathbf{y}_t)$
  - **Test of** Kohlhas and Walther (2021) via $cov(\mathbf{y}_{t+h}, \mathbf{y}_t) = cov(\mathbf{F}_t\mathbf{y}_{t+h}, \mathbf{y}_t)$

- **Special case:** AR(1) law of motion with misperceived persistence $\tilde{\rho}$
$$\left(\frac{\tilde{\rho}}{\rho}\right)^h = \frac{cov(\mathbf{F}_t\mathbf{y}_{t+h}, \mathbf{y}_t)}{cov(\mathbf{y}_{t+h}, \mathbf{F}_t\mathbf{y}_t)}$$

- **General properties of this method:**
  - No need of data on all elements of $\mathbf{F}_t\mathbf{y}_{t+h}$
  - No need of individual level expectations $\mathbf{F}_{i,t}\mathbf{y}_{t+h}$
  - No need of prior knowledge on the actual law of motion
  - No need of a constant or homogeneous perceived signal precision

Application to the SPF

- **Focus** on the 3-month treasury bill rate and the expectations of it in the Survey of Professional Forecasters.

- Figure 1 provides the **raw data** showing that forecasts errors are persistent over multiple horizons. Forecasters tend to overestimate the outcome in the declining phase but underestimate it in the rising phase.

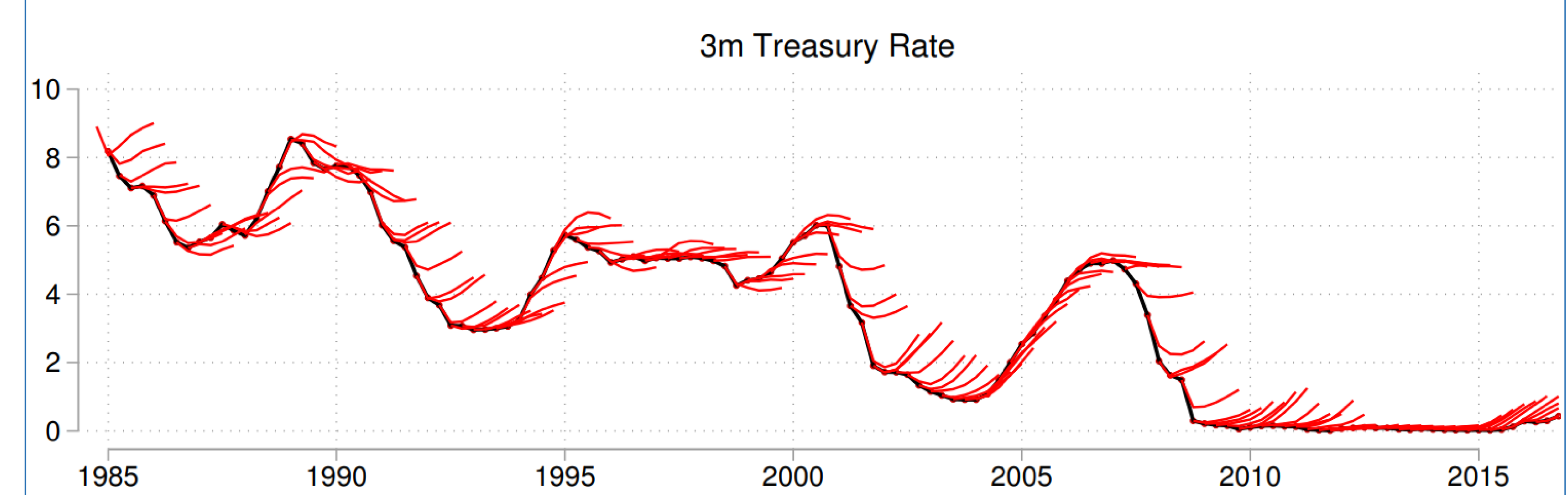


Figure 1. The realized outcomes of 3-month treasury rate (%, long black) and the expectations of it over multiple horizons (%, short red) from the Survey of Professional Forecasters. The differences between the outcomes and the forecasts are forecast errors.

- Table 1 provides three types of **moments in the data** for the quarterly changes of the 3-month treasury bill rate.
  - M_1 is declining over horizon h, exhibiting an autocorrelated law of motion.
  - M_2 is persistently lower than 1, indicating non-trivial information rigidity.
  - M_3 is increasingly lower than 1, implying misperceived law of motion.

- Table 1 provides the same set of **moments in a VAR(2) model** that also includes the quarterly changes of unemployment rate as control variable. The forecasters observe the outcomes with noise and forecast the two variables with potentially misperceived parameters on the law of motion.

- The VAR is estimated via **simulated method of moments** targeting $\{M_1, M_2, M_3\}$.

- The **comparison between model and data** in **Table 1** indicates that
  - M_1: the model captures the actual law of motion quite well.
  - M_2: the model captures the information rigidity reasonably well.
  - M_3: the model misses the perceived law of motion in medium horizon.

- **Takeaway:** the professional forecasters overcomplicate the dynamics of 3-month treasury bill rates.

Table 1. Comparison between model vs. data

	$M_1 = cov(y_{t+h}, y_t)$		$M_2 = \frac{cov(y_{t+h}, F_t y_t)}{cov(y_{t+h}, y_t)}$		$M_3 = \frac{cov(F_t y_{t+h}, y_t)}{cov(y_{t+h}, F_t y_t)}$	
h	Data	Model	Data	Model	Data	Model
0	0.166	0.165	0.811	0.756	1.000	1.000
1	0.105	0.109	0.723	0.743	0.586	0.674
2	0.065	0.065	0.759	0.742	0.226	0.122
3	0.050	0.040	0.660	0.747	-0.235	-0.291
4	0.021	0.027	0.472	0.751	-1.560	-0.334

Implications for Modeling

- Strong evidence in favor of under-extrapolation as M_3 is lower than 1 in the data for the quarterly changes of 3-month treasury bill rate for all $h \in \{1,2,3,4\}$.

- Some evidence in favor of cognitive discounting due to the same pattern for the quarterly changes of unemployment rate as well.

- No evidence of reduced model complexity among professional forecasters as M_3 is matched much worse in the model than M_1, when the complexity levels of the perceived and actual law of motion are identical.

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