# Online Appendix

# A Theory of Chosen Preferences

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We have condensed the details of the proofs to conserve space. Readers can also consult the uncondensed proofs in our working paper, Bernheim et al., 2019.

## Proof of Proposition 1

Throughout the following, we define the optimal action correspondence $z^*(\alpha)$ as follows: $z^*(\alpha) = 1$ for $\alpha > \bar{\alpha}$, $z^*(\alpha) = 2$ for $\alpha < \bar{\alpha}$, and $z^*(\alpha) \in \{1, 2\}$ for $\alpha = \bar{\alpha}$. Furthermore, let $z$ be any selection from $z^*$.

We begin with a lemma.

**Lemma 1.** *Consider the following problem. For fixed $\alpha$, solve*

$$\max_{\alpha' \in [0,1]} (1-\lambda)U(\alpha', z(\alpha')) + \lambda U(\alpha, z(\alpha'))$$

*For $\alpha < \alpha^*$, the solution is $\alpha' = 0$. For $\alpha > \alpha^*$, the solution is $\alpha' = 1$. For $\alpha = \alpha^*$, the solution is $\alpha' \in \{0, 1\}$.*

*Proof:* First we claim that the optimum is either $\alpha' = 0$ or $\alpha' = 1$. It is easily checked that if $z(\alpha') = 1$ and $\alpha' < 1$ (resp. $z(\alpha') = 2$ and $\alpha' > 0$), then one can strictly increase the objective function by switching $\alpha'$ to 1 (resp. to 0). The switch increases the first term and leaves the second unchanged.

One can then easily show that the expression $((1-\lambda)\, U(0,2) + \lambda U(\alpha, 2)) - ((1-\lambda)\, U(1,1) + \lambda U(\alpha, 1))$ is strictly positive iff $\alpha < \alpha^*$, strictly negative iff $\alpha > \alpha^*$, and equal to 0 iff $\alpha = \alpha^*$. □

Now we prove the proposition.

Step 1: Verify that we can construct an MPE with the indicated properties.

Assuming the proposed strategies $(\phi, z)$ govern future actions, choosing $\alpha_{t+1} = 0$ produces the sequence of worldview-action pairs $\sigma^0 = ((\alpha_t, z(\alpha_t)), (0, 2), (0, 2), ...)$, while choosing $\alpha_{t+1} = 1$ produces the sequence $\sigma^1 = ((\alpha_t, z(\alpha_t)), (1, 1), (1, 1), ...)$. Any other choice yields a sequence of the form $\sigma^2 = ((\alpha_t, z(\alpha_t)), (\alpha_{t+1}, z(\alpha_{t+1})), ...)$. It follows from Lemma 1 that the best available outcome is $\sigma^0$ if $\alpha_t < \alpha^*$, $\sigma^1$ if $\alpha_t > \alpha^*$, and both $\sigma^0$ and $\sigma^1$ if $\alpha_t = \alpha^*$.

Step 2: In all stationary MPE, $\phi(\alpha) = 0$ for $\alpha < \alpha^*$, $\phi(\alpha) \in \{0, 1\}$ for $\alpha = \alpha^*$, and $\phi(\alpha) = 1$ for $\alpha > \alpha^*$. We prove this step through a series of claims.

Claim (i): $\phi(0) = 0$. Imagine that $\phi(0) \neq 0$. Suppose $\alpha_t = 0$ and consider a defection to $\alpha_{t+1} = 0$. Using the fact that $U(0, 2) > (1-\lambda)U(\alpha', z(\alpha')) + \lambda U(0, z(\alpha'))$ for all $\alpha' \neq 0$ (an implication of Lemma 1), one can easily show such a defection is attractive because it delays the outcome trajectory from choosing $\phi(0)$ by one period while maximizing the payoff from the period $t+1$ outcome according to the period $t$ assessment.

Claim (ii): $\phi(\alpha) = 0$ for $\alpha < \alpha^*$, and $\phi(\alpha) \in \{0, 1\}$ for $\alpha = \alpha^*$. Suppose $\alpha_t \leq \alpha^*$. From claim (i), choosing $\alpha_{t+1} = 0$ produces the sequence of worldview-action pairs $\sigma^0 = ((\alpha_t, z(\alpha_t)), (0, 2), (0, 2), ...)$, while any other choice yields a distinct sequence of the form $((\alpha_t, z(\alpha_t)), (\alpha_{t+1}, z(\alpha_{t+1})), ...)$. It follows immediately from Lemma 1 that $\alpha_{t+1} = 0$ yields a strictly better outcome from the period-$t$ perspective than all $\alpha_{t+1} \in (0,1)$, as well as a strictly better outcome than $\alpha_{t+1} = 1$ as long as $\alpha_t < \alpha^*$.

Claim (iii): If $\alpha^* < 1$, then $\phi(1) = 1$. Imagine that $\phi(1) \neq 1$. Suppose $\alpha_t = 1$ and consider a defection to $\alpha_{t+1} = 1$. Using the fact that $U(1,1) > (1-\lambda)U(\alpha', z(\alpha')) + \lambda U(1, z(\alpha'))$ for all $\alpha' \neq 1$ (an implication of Lemma 1), one can easily show such a defection is attractive because it delays the outcome trajectory from choosing $\phi(1)$ by one period while maximizing the payoff from the period $t+1$ outcome according to the period $t$ assessment.

Claim (iv): $\phi(\alpha) = 1$ for $\alpha > \alpha^*$. Suppose $\alpha_t > \alpha^*$. From claim (iii), choosing $\alpha_{t+1} = 1$ produces the sequence of worldview-action pairs $\sigma^1 = ((\alpha_t, z(\alpha_t)), (1,1), (1,1), ...)$, while any other choice yields a distinct sequence of the form $((\alpha_t, z(\alpha_t)), (\alpha_{t+1}, z(\alpha_{t+1})), ...)$. It follows immediately from Lemma 1 that $\alpha_{t+1} = 1$ yields a strictly better outcome from the period-$t$ perspective than all $\alpha_{t+1} < 1$. □

## Proof of Proposition 2

Define $u_M(x) = \max_{j \in J} u_j(x)$. That is, $u_M(x)$ is the maximum utility achievable for action $x$ under any worldview. Let worldview $i$ satisfy $u_i(z^*(\alpha(i))) > u_j(z^*(\alpha(i)))$ for all $j \neq i$. Since $z^*(\alpha(i))$ is unique, there exists $\lambda_i < 1$ such that, for all $\lambda > \lambda_i$, $u_i(z^*(\alpha(i))) > (1-\lambda)u_M(\hat{x}) + \lambda u_i(\hat{x})$ for all actions $\hat{x} \neq z^*(\alpha(i))$ (and equality for $\hat{x} = z^*(\alpha(i))$).

Imagine there is a stationary MPE in which $\phi(\alpha(i)) \neq \alpha(i)$ when $\lambda > \lambda_i$. Suppose $\alpha_0 = \alpha(i)$. The MPE must then yield a sequence of worldview-action pairs of the form $\sigma_i = \big((\alpha(i), z^*(\alpha(i))), (\alpha_1, z(\alpha_1)), \dots\big)$, where $\alpha_1 \neq \alpha(i)$ and $z$ is some selection from the correspondence $z^*$. A one-period defection from $\alpha_1$ to $\alpha(i)$ changes period-0 utility by:

$$
\begin{aligned}
\Delta &= \delta u_i(z^*(\alpha(i))) - (1-\delta)\sum_{k=1}^{\infty} \delta^k \big[(1-\lambda)U(\alpha_k, z(\alpha_k)) + \lambda u_i(z(\alpha_k))\big] \\
&\geq \delta u_i(z^*(\alpha(i))) - (1-\delta)\sum_{k=1}^{\infty} \delta^k \big[(1-\lambda)u_M(z(\alpha_k)) + \lambda u_i(z(\alpha_k))\big] \\
&\geq \delta u_i(z^*(\alpha(i))) - (1-\delta)\sum_{k=1}^{\infty} \delta^k u_i(z^*(\alpha(i))) = 0
\end{aligned}
$$

where the first inequality follows by definition of $u_M$, and the second inequality follows from $\lambda > \lambda_i$. We claim that one of these two inequalities must be strict. If $z(\alpha_1) = z^*(\alpha(i))$, then $U(\alpha_1, z(\alpha_1)) < U(\alpha(i), z(\alpha_1)) = u_M(z(\alpha_1))$, which means the first inequality is strict. For $z(\alpha_1) \neq z^*(\alpha(i))$, $\lambda > \lambda_i$ implies $u_i(z^*(\alpha(i))) > (1-\lambda)u_M(z(\alpha_1)) + \lambda u_i(z(\alpha_1))$, which means the second inequality is strict. Therefore, in any stationary MPE, worldview $i$ must map back to itself. □

## Proof of Proposition 3

It is useful to define the following sequence: $\left\{\alpha^{(\tau)}\right\}_{\tau=0}^{\infty}$ where $\alpha^{(0)}=0$, $\alpha^{(1)}=\bar{\alpha}$, $\alpha^{(2)}=\alpha^{(1)}+\frac{1-\lambda}{\lambda}\left(\frac{u_2(2)-U(\alpha^{(1)},1)}{(u_2(2)-u_1(2))-(u_2(1)-u_1(1))}\right)$ and recursively (for $\tau>2$), $\alpha^{(\tau)}=\alpha^{(\tau-1)}+\frac{\Phi}{\delta^{\tau-2}}\left[\alpha^{(\tau-1)}-\alpha^{(\tau-2)}\right]$ where $\Phi=\frac{1-\lambda}{\lambda}\frac{[u_2(1)-u_1(1)]}{[u_2(2)-u_1(2)]-[u_2(1)-u_1(1)]}>0$. It is straightforward to show that, $\forall\,(\delta,\lambda)\in(0,1)^2$, $\left\{\alpha^{(\tau)}\right\}_{\tau=0}^{\infty}$ is a strictly increasing sequence, and that there exists $\bar{\tau}>1$ such that $\alpha^{(\bar{\tau})}<1$ and $\alpha^{(\bar{\tau}+1)}>1$. We prove Proposition 3 through a series of lemmas.

**Lemma 2.** $\forall\,(\delta,\lambda)\in(0,1)^2$ *, the following Markov policy function is an MPE:*

$$(\phi(\alpha),z(\alpha))=\begin{cases}(\alpha^{(0)},2) & if\ \alpha\in\left[\alpha^{(0)},\alpha^{(1)}\right)\\ (\alpha^{(0)},1) & if\ \alpha\in\left[\alpha^{(1)},\alpha^{(2)}\right)\\ (\alpha^{(1)},1) & if\ \alpha\in\left[\alpha^{(2)},\alpha^{(3)}\right)\\ \vdots & \vdots\\ (\alpha^{(\bar{\tau}-2)},1) & if\ \alpha\in\left[\alpha^{(\bar{\tau}-1)},\alpha^{(\bar{\tau})}\right)\\ (\alpha^{(\bar{\tau}-1)},1) & if\ \alpha\in\left[\alpha^{(\bar{\tau})},1\right]\end{cases}$$

*Proof:*

Step 1: By construction, $z(\alpha)$ is optimal for each $\alpha$. (Trivial.)

Step 2: Assuming future behavior is governed by $\phi$, then for every worldview $\alpha$, the individual strictly prefers $\alpha^{(\tau)}$ to any $\alpha\in(\alpha^{(\tau)},\alpha^{(\tau+1)})\equiv I^{(\tau)}$ for all $\tau\in\{0,...,\bar{\tau}-1\}$, and $\alpha^{(\bar{\tau})}$ to any $\alpha\in(\alpha^{(\bar{\tau})},1]\equiv I^{\bar{\tau}}$.

Consider any $\tau$. By construction, the continuation sequence of mixed worldviews and actions is identical for all $\alpha\in\{\alpha^{(\tau)}\}\cup I^{(\tau)}$. Because worldview 2 happiness-dominates worldview 1, the current payoff is monotonically decreasing in $\alpha$ within this interval. The claim follows directly.

Step 3: Assuming future behavior is governed by $\phi$, then with mixed worldview $\alpha^{(\tau)}$, $\tau\in\{2,...,\bar{\tau}\}$, the individual is indifferent between choosing $\alpha^{(\tau-1)}$ and $\alpha^{(\tau-2)}$.

Consider an agent with initial worldview $\alpha$. Equating the continuation payoffs after choosing $\alpha^{(\tau-1)}$ and $\alpha^{(\tau-2)}$ and solving for $\alpha$ yields

$$\alpha=\alpha^{(1)}+\frac{1-\lambda}{\lambda}\left[\left[\frac{U(0,2)-U(\alpha^{(1)},1)}{[u_2(2)-u_1(2)]-[u_2(1)-u_1(1)]}\right]+\sum_{k=1}^{\tau-2}\left(\frac{1}{\delta^{\tau-k-1}}\right)\left(\frac{U\left(\alpha^{(\tau-k-1)},1\right)-U\left(\alpha^{(\tau-k)},1\right)}{[u_2(2)-u_1(2)]-[u_2(1)-u_1(1)]}\right)\right]$$

It is immediate that $\alpha^{(2)}$ satisfies this equation for $\tau=2$, and it easily verified that if $\alpha^{(\tau)}$ satisfies it for $\tau\geq 2$, then $\alpha^{(\tau+1)}$ satisfies it for $\tau+1$. The desired conclusion follows directly.

Step 4: Assuming future behavior is governed by $\phi$, if the individual weakly prefers $\alpha^{(r)}$ to $\alpha^{(s)}$ for $r>s$ with worldview $\alpha$, then the individual strictly prefers $\alpha^{(r)}$ to $\alpha^{(s)}$ with worldview $\alpha'>\alpha$. Likewise, if the individual weakly prefers $\alpha^{(r)}$ to $\alpha^{(s)}$ for $r<s$ with worldview $\alpha$, then the individual strictly prefers $\alpha^{(r)}$ to $\alpha^{(s)}$ with worldview $\alpha'<\alpha$.

Assume $r > s$. We can decompose the difference between the continuation payoff following from the selection of $\alpha^{(r)}$, and the continuation payoff following from the selection of $\alpha^{(s)}$, into two terms, as follows: $K(r,s) + \lambda \sum_{t=s}^{\tau-1} \delta^t \left(\alpha\left((u_1(1) - u_1(2)) + (u_2(2) - u_2(1))\right) + (u_2(1) - u_2(2))\right)$. The first term depends on $r$ and $s$ but not on $\alpha$, and the second is strictly increasing in $\alpha$. The desired conclusion follows immediately. An analogous argument applies in the case of $r < s$.

Step 5: $\phi$ is a MPE.

From step 2, we know that the individual will always choose $\alpha^{(\tau)}$ for some $\tau$. Combining steps 3 and 4, we see that the individual strictly prefers $\alpha^{(\tau+1)}$ to $\alpha^{(\tau)}$ for $\alpha > \alpha^{(\tau+2)}$, and strictly prefers $\alpha^{(\tau)}$ to $\alpha^{(\tau+1)}$ for $\alpha < \alpha^{(\tau+2)}$. It follows that the unique optimum is $\alpha^{(\tau)}$ for all $\alpha \in \left(\alpha^{(\tau+1)}, \alpha^{(\tau+2)}\right)$, and that the optima are $\left\{\alpha^{(\tau)}, \alpha^{(\tau+1)}\right\}$ for $\alpha = \alpha^{(\tau+2)}$. □

**Lemma 3.** *All stationary MPE policy functions coincide with the one described in Lemma 2 on a set of full measure.*

*Proof:* We will use $(\psi, y)$ to denote the generic stationary MPE. Our objective is to show that $(\psi, y)$ coincides with $(\phi, z)$ on a set of full measure.

Step 1: $y(\alpha) = 2$ for $\alpha < \alpha^{(1)}$, and $y(\alpha) = 1$ for $\alpha > \alpha^{(1)}$. (Trivial. Notice the implication: $y$ must coincide with $z$ everywhere except possibly at $\alpha^{(1)}$.)

Step 2: $\psi(\alpha) \leq \alpha$ for all $\alpha$.

The argument will make use of the following notation: $V_{\psi,y}(\alpha', \alpha)$ denotes the discounted continuation payoff (ignoring the current period) resulting from choosing $\alpha'$ under worldview $\alpha$ when future choices are governed by the MPE $(\psi, y)$ (defining $\psi^1(\alpha) = \psi(\alpha)$ and, recursively, $\psi^t(\alpha) = \psi\left(\psi^{t-1}(\alpha)\right)$ for $t > 1$):

$$V_{\psi,y}(\alpha', \alpha) = \lambda U(\alpha, y(\alpha')) + (1-\lambda)U(\alpha', y(\alpha') + \sum_{t=1}^{\infty} \delta^t [\lambda U(\alpha, y(\psi^t(\alpha'))) + (1-\lambda)U(\psi^t(\alpha'), y(\psi^t(\alpha'))]$$

Assume contrary to the claim that there exists $\alpha'$ with $\psi(\alpha') > \alpha'$. Then choosing $\psi(\alpha')$ leaves the individual at least as well off as deviating to $\alpha'$ (which then induces the same continuation path): $(1-\delta)V_{\psi,y}(\psi(\alpha'), \alpha') \geq U(\alpha', y(\alpha'))$. We can then write

$$V_{\psi,y}(\psi(\alpha'), \alpha') - \delta V_{\psi,y}(\psi^2(\alpha'), \alpha') = (1-\lambda)U(\psi(\alpha'), y(\psi(\alpha'))) + \lambda U(\alpha', y(\psi(\alpha'))) < U(\alpha', y(\alpha')) \leq (1-\delta)V_{\psi,y}(\psi(\alpha'), \alpha')$$

where the first inequality follows from the assumption that $\psi(\alpha') > \alpha$. Rearranging the preceding expression, we obtain $V_{\psi,y}(\psi(\alpha'), \alpha') < V_{\psi,y}(\psi^2(\alpha'), \alpha')$. But then the individual can increase discounted payoffs by deviating from

$\psi(\alpha')$ to $\psi^2(\alpha')$, a contradiction.

Step 3: $\psi(\alpha) = 0$ for $\alpha \in [0, \alpha^{(1)}]$.

Step 2 implies that $\psi(0) = 0$. Therefore, choosing $\psi(\alpha) = 0$ generates the continuation path $((0,2),(0,2),(0,2),...)$. It is easy to check that, from the perspective of $\alpha \in (0, \alpha^{(1)}]$, this trajectory is strictly superior to any other.

Step 4: $\psi(\alpha) = 0$ for $\alpha \in (\alpha^{(1)}, \alpha^{(2)})$.

It is easily shown that the agent would rather chooose 0 than any $\alpha' < \alpha^{(1)}$. Furthermore, from step 2 of this lemma, if $\psi(\alpha) \geq \alpha^{(1)}$ for some $\alpha \in (\alpha^{(1)}, \alpha^{(2)})$, then there exists some $T \geq 1$ (possibly $+\infty$) such that $\psi^t(\alpha) = 0$ for $t > T$, and $\psi^t(\alpha) \geq \alpha^{(1)}$ for $t \leq T$. This trajectory generates a constant payoff per period of no more than $(1-\lambda)U\left(\alpha^{(1)}, 1\right) + \lambda U(\alpha, 1)$ for the first $T$ periods, followed by a constant payoff of $(1-\lambda)U(0,2) + \lambda U(\alpha, 2)$ in all subsequent periods. From steps 3 and 4 of the proof of Lemma 2, we have $(1-\lambda)U\left(\alpha^{(1)}, 1\right) + \lambda U(\alpha, 1) < (1-\lambda)U(0,2) + \lambda U(\alpha, 2)$ for $\alpha \in (\alpha^{(1)}, \alpha^{(2)})$. But this inequality shows that the trajectory that follows from choosing $\alpha' = 0$ generates a strictly higher discounted payoff than the trajectory that follows from choosing any $\alpha' \geq \alpha^{(1)}$.

Step 5: Assume $(\psi, y)$ coincides with $(\phi, z)$ on $[0, \alpha^{(\tau)})/\{\alpha^{(\tau-1)}\}$ for $\tau \geq 2$, and $\psi\left(\alpha^{(\tau-1)}\right) \in \{\alpha^{(\tau-3)}, \alpha^{(\tau-2)}\}$. Then $(\psi, y)$ coincides with $(\phi, z)$ on $[0, \alpha^{(\tau+1)})/\{\alpha^{(\tau)}\}$ and $\psi\left(\alpha^{(\tau)}\right) \in \{\alpha^{(\tau-2)}, \alpha^{(\tau-1)}\}$.

Suppose for the moment that $(\psi, y)$ also coincides with $(\phi, z)$ at $\alpha^{(\tau-1)}$ (and therefore on $[0, \alpha^{(\tau)})$). Consider $\alpha \in [\alpha^{(\tau)}, \alpha^{(\tau+1)})$. Choosing any $\alpha' < \alpha^{(\tau)}$ yields the same continuation path as with $\phi$. Consequently we know that the best choice within this set is $\alpha^{(\tau-1)}$ for $\alpha \in (\alpha^{(\tau)}, \alpha^{(\tau+1)})$, and is an element of $\{\alpha^{(\tau-2)}, \alpha^{(\tau-1)}\}$ for $\alpha^{(\tau)}$; furthermore, this restricted best choice yields a continuation payoff of $V_{\phi,z}(\alpha^{(\tau-1)}, \alpha)$. Assume toward a contradition that $\psi(\alpha) \geq \alpha^{(\tau)}$ for some $\alpha \in (\alpha^{(\tau)}, \alpha^{(\tau+1)})$. Furthermore, from step 2 (which guarantees that $\psi^t(\alpha)$ remains in $[\alpha^{(\tau)}, \alpha^{(\tau+1)})$ as long as it does not fall below $\alpha^{(\tau)}$), there exists some $T \geq 1$ (possibly $+\infty$) such that $\psi^{T+1}(\alpha) = \alpha^{(\tau-1)}$, and $\psi^t(\alpha) \geq \alpha^{(\tau)}$ for $t \leq T$. This trajectory generates a payoff of no more than $(1-\lambda)U\left(\alpha^{(\tau)}, 1\right) + \lambda U(\alpha, 1)$ per period for the first $T$ periods, followed by a continuation payoff of $V_{\phi,z}(\alpha^{(\tau-1)}, \alpha)$. Therefore, $V_{\psi,y}(\psi(\alpha), \alpha) \leq \frac{1-\delta^{T+1}}{1-\delta}\left[(1-\lambda)U\left(\alpha^{(\tau)}, 1\right) + \lambda U(\alpha, 1)\right] + \delta^{T+1} V_{\phi,z}(\alpha^{(\tau-1)}, \alpha)$. Combining this inequality with the fact that $V_{\phi,z}(\alpha^{(\tau-1)}, \alpha) > V_{\phi,z}(\alpha^{(\tau)}, \alpha) = \left[(1-\lambda)U\left(\alpha^{(\tau)}, 1\right) + \lambda U(\alpha, 1)\right] + \delta V_{\phi,z}(\alpha^{(\tau-1)}, \alpha)$ (see Lemma 2, steps 3 and 4), which implies $(1-\delta)V_{\phi,z}(\alpha^{(\tau-1)}, \alpha) > \left[(1-\lambda)U\left(\alpha^{(\tau)}, 1\right) + \lambda U(\alpha, 1)\right]$, we obtain $V_{\psi,y}(\psi(\alpha), \alpha) < V_{\phi,z}(\alpha^{(\tau-1)}, \alpha) = V_{\psi,y}(\alpha^{(\tau-1)}, \alpha)$. It follows that the individual would deviate from $\psi(\alpha)$ to $\alpha^{(\tau-1)}$, a contradiction.

Now suppose that $(\psi, y)$ does not coincide with $(\phi, z)$ at $\alpha^{(\tau-1)}$. This supposition implies either that $y(\alpha^{(1)}) = 2$ (rather than 1) in the case of $\tau = 2$, or $\psi(\alpha^{(\tau-1)}) = \alpha^{(\tau-3)}$ (rather than $\alpha^{(\tau-2)}$) in the case of $\tau > 2$. Both alternatives make the choice of $\alpha^{(\tau-1)}$ strictly less attractive from the perspective of any $\alpha > \alpha^{(\tau-1)}$ (Lemma 2, steps 3 and 4). As a result, for any $\alpha \in (\alpha^{(\tau)}, \alpha^{(\tau+1)})$, the continuation payoff is increasing as $\alpha' \downarrow \alpha^{(\tau-1)}$, but falls discontinuously at $\alpha^{(\tau-1)}$. It follows that an optimal choice does not exist, which contradicts the hypothesis that $(\psi, y)$ is an equilibrium. □

Applying induction to step 5, we see that $(\psi, y)$ coincides with $(\phi, z)$ everywhere except possibly for $\alpha^{(\overline{\tau})}$. The additional properties described in the proposition can be verified by inspection. □

## Proof of Proposition 4

Because a naif acts as if she can select an execute any desired trajectory $\sigma_t$ from period $t$ forward (subject to the constraint that $x_k \in z^*(\alpha_k)$ for each $k \geq t$), and because her utility is time-separable, her choice for period $\alpha_{t+1}$ satisfies

$$\max_{\alpha_{t+1}, x_{t+1} \in z^*(\alpha_{t+1})} (1-\lambda) U(\alpha_{t+1}, x_{t+1}) + \lambda U(\alpha_t, x_{t+1}) \tag{1}$$

She incorrectly anticipates sticking with this choice forever after choosing it for period $t+1$.

Step 1: The solution to (1) is a pure worldview. Assume the solution is not a pure worldview and that it involves action $x$. By assumption, $(\alpha^I(x), x)$ is feasible and $u_{\alpha^I(x)}(x) > u_i(x)$ for $i \neq \alpha^I(x)$, which means it yields a strictly higher value of the objective function.

Step 2: The choices of a naive decision maker cannot cycle among pure worldviews. Suppose the consumer switches from $(\alpha^I(x_t), x_t)$ in some period $t$ to $(\alpha^I(x_{t+1}), x_{t+1})$ in period $t+1$, where $x_{t+1} \neq x_t$. Then it must be the case that $(1-\lambda)U(\alpha^I(x_{t+1}), x_{t+1}) + \lambda U(\alpha^I(x_t), x_{t+1}) \geq (1-\lambda)U(\alpha^I(x_t), x_t) + \lambda U(\alpha^I(x_t), x_t)$. Rearranging this inequality, we obtain $(1-\lambda)\left[U(\alpha^I(x_{t+1}), x_{t+1}) - U(\alpha^I(x_t), x_t)\right] \geq \lambda \left[U(\alpha^I(x_t), x_t) - U(\alpha^I(x_t), x_{t+1})\right]$. Using the fact that $U(\alpha^I(x_t), x_t) > U(\alpha^I(x_t), x_{t+1})$, we see that $U(\alpha^I(x_{t+1}), x_{t+1}) > U(\alpha^I(x_t), x_t)$. Ranking the actions according to the value of $U(\alpha^I(x), x)$, we see that it is only possible to move upward in this ranking. Consequently, there can be no cycles. With a finite number of actions, the consumer must stop changing worldviews after a finite number of periods. □

## Proof of Proposition 5

(i) Let $\alpha$ denote the weight on worldview 1, and let $\phi$ denote the Markov policy function. It is easy to show that $\phi(0) = 0$ using an argument similar to the one in Step 2, Claim 1 of the proof of Proposition 1. It follows that, beginning with any mixed worldview $\alpha$, choosing pure worldview 2 yields a continuation payoff of $\sum_{t=1}^{\infty} \delta^t \left[(1-\lambda)u_2(1) + \lambda U(\alpha, 1)\right]$. Any other choice reduces the first term and leaves the second unchanged. Therefore, the consumer places zero weight on worldview 1 after the first period in all stationary MPE.

(ii) In this setting, mixed worldviews belong to the set $S = \left\{(\alpha^1, \alpha^2) \mid 0 \leq \alpha^1 + \alpha^2 \leq 1, \alpha^1 \geq 0, \alpha^2 \geq 0\right\}$ where $\alpha^3 = 1 - \alpha^1 - \alpha^2$, and the Markov policy function $\phi$ maps $S$ into $S$.

It is easy to show that $\phi(0,0) = (0,0)$, once again by an argument similar to that of Step 2, Claim 1 of the proof of Proposition 1. We claim that $\phi(0, \alpha^2) = (0,0)$ for all $\alpha^2 \in (0,1]$. Given that $\phi(0,0) = (0,0)$, if the consumer chooses $(0,0)$, her flow utility (according to worldview $(0,\alpha^2)$) will be $\lambda\left[\alpha^2 u_2(2)+(1-\alpha^2)u_3(2)\right]+(1-\lambda)u_3(2)$ in all subsequent periods, which is maximal contingent on choosing action 2. Her flow utility contingent on choosing action 1 is bounded above by $u_2(1)$. Note that

$$\lambda\left[\alpha^2 u_2(2)+(1-\alpha^2)u_3(2)\right]+(1-\lambda)u_3(2)>\lambda u_2(2)+(1-\lambda)u_3(2)>u_2(1)$$

Thus, choosing $(0,0)$ generates a strictly higher continuation payoff than any other choice.

Let $P$ denote the total discounted payoff achieved in an MPE by a consumer who starts out with pure worldview 1 ($\alpha^1=1$), evaluated from that perspective. If this consumer chooses pure worldview 1 for $t=1$, her discounted payoff will be $u_1(1)+\delta P$. Incentive compatibility requires $P\geq u_1(1)+\delta P$, which implies $P\geq\frac{u_1(1)}{1-\delta}$. Let $T$ denote the first period in which the consumer places zero weight on pure worldview 1. From the preceding arguments, we know she will choose pure worldview 3 and action 2 in every subsequent period, so her flow utility from period $T+1$ forward will be no higher than $\lambda u_1(2)+(1-\lambda)u_3(2)$. From period 1 to period $T$, her flow utility is bounded by $u_3(2)$ (the highest overall flow utility). Thus,

$$\left(\frac{1-\delta^{T+1}}{1-\delta}\right)u_3(2)+\left(\frac{\delta^{T+1}}{1-\delta}\right)\left[\lambda u_1(2)+(1-\lambda)u_3(2)\right]\geq P\geq\frac{u_1(1)}{1-\delta}.$$

It follows that

$$\delta^{T+1}\leq\frac{u_3(2)-u_1(1)}{\lambda\left[u_3(2)-u_1(2)\right]}.$$

Both numerator and denominator are strictly positive. Thus, defining $K_{T^*}=u_3(2)-\frac{u_3(2)-u_1(1)}{\lambda\delta^{T^*+1}}$, we see that if $u_1(2)<K_{T^*}$, worldview 1 cannot receive zero weight in the first $T^*$ periods. As $T^*\to\infty$, we have $K_{T^*}\to-\infty$. □

## Proof of Proposition 6

Consider any wordview 2 satisfying the following constraints: $u_2(k)\in(u_1(1),u_3(2))$ for $k=1,2$, and $u_2(2)>u_2(1)$. Define $S$ as in the proof of Proposition 5, and let $U(\alpha^1,\alpha^2,x)$ denote the flow utility obtained from action $x$ under worldview $\left(\alpha^1,\alpha^2\right)\in S$.

Next define the sequence of values $\left\{\alpha^{(\tau)}\right\}_{\tau=0}^{\infty}$ as follows:

$$\alpha^{(1)}=\frac{u_2(2)-u_2(1)}{\left[u_2\left(2\right)-u_2\left(1\right)\right]+\left[u_1\left(1\right)-u_1\left(2\right)\right]}=\bar{\alpha}$$

$$\alpha^{(2)}=\alpha^{(1)}+\left(\frac{1-\lambda}{\lambda}\right)\left(\frac{U(0,0,2)-U(\alpha^{(1)},1-\alpha^{(1)},1)}{\left[u_2\left(2\right)-u_2\left(1\right)\right]+\left[u_1\left(1\right)-u_1\left(2\right)\right]}\right)$$

and recursively for $\tau > 2$

$$\alpha^{(\tau)} = \alpha^{(\tau-1)} + \frac{\phi}{\delta^{\tau-2}} \left[\alpha^{(\tau-1)} - \alpha^{(\tau-2)}\right]$$

where $\phi = \frac{1-\lambda}{\lambda} \frac{[u_2(1)-u_1(1)]}{[u_2(2)-u_2(1)]+[u_1(1)-u_1(2)]} > 0$. This sequence resembles the one used in the proof of Proposition 3, and here one can also show that $\forall(\delta, \lambda) \in (0,1)^2$, $\exists\ \bar{\tau} \geq 1$ s.t. $\alpha^{(\bar{\tau})} \leq 1$ and $\alpha^{(\bar{\tau}+1)} > 1$.

Consider the following Markov policy function. We partition the set of possible mixed worldviews into three sets. Set 1 ($S_1$) consists of those for which $\alpha_3 = 0$ and action 1 is optimal ($\alpha^1 \geq \alpha^{(1)}$). Set 2 ($S_2$) consists of those for which $\alpha_3 > 0$ and action 1 is optimal. Set 3 ($S_3$) consists of those for which action 2 is optimal. We can incorporate the boundary between $S_2$ and $S_3$ in either set. For all worldviews in $S_1$, let

$$\phi(\alpha^1, 1-\alpha^1) = \begin{cases} (0,0) & if\ \alpha^1 \in [\alpha^{(1)}, \alpha^{(2)}) \\ (\alpha^{(1)}, 1-\alpha^{(1)}) & if\ \alpha^1 \in [\alpha^{(2)}, \alpha^{(3)}) \\ (\alpha^{(2)}, 1-\alpha^{(2)}) & if\ \alpha^1 \in [\alpha^{(3)}, \alpha^{(4)}) \\ \vdots & \vdots \\ (\alpha^{(\bar{\tau}-2)}, 1-\alpha^{(\bar{\tau}-2)}) & if\ \alpha^1 \in [\alpha^{(\bar{\tau}-1)}, \alpha^{(\bar{\tau})}) \\ (\alpha^{(\bar{\tau}-1)}, 1-\alpha^{(\bar{\tau}-1)}) & if\ \alpha^1 \in [\alpha^{(\bar{\tau})}, 1] \end{cases}$$

For all worldviews in $S_2$, let $\phi(\alpha^1, \alpha^2)$ be the best choice from the set

$$\left\{(0,0), (\alpha^{(1)}, 1-\alpha^{(1)}), (\alpha^{(2)}, 1-\alpha^{(2)}), \ldots, (\alpha^{(\rho(1-\alpha^2)-1)}, 1-\alpha^{(\rho(1-\alpha^2)-1)})\right\},$$

under the assumption that $\phi$ will govern subsequent choices (producing stepwise convergence to $(0,0)$),[1]where $\rho(\alpha)$ is the integer $\rho$ satisfying $\alpha \in [\alpha^{(\rho)}, \alpha^{(\rho+1)})$. For all worldviews in $S_3$, let $\phi(\alpha^1, \alpha^2) = (0,0)$.

We now prove that $\phi$ (along with optimal action choices) is an MPE.

*Step 1*: Assuming $\phi$ governs future choices, the best current choice as of period $t$ (for period $t+1$) belongs to the set $T = \left\{(0,0), (\alpha^{(1)}, 1-\alpha^{(1)}), (\alpha^{(2)}, 1-\alpha^{(2)}), \ldots, (\alpha^{(\bar{\tau}-1)}, 1-\alpha^{(\bar{\tau}-1)})\right\}$.

Points in $S \setminus T$ fall into three categories, which we consider in turn.

(i) Consider any $\left(\alpha^1, 1-\alpha^1\right) \in S_1 \setminus T$. By construction, $\phi\left(\alpha^1, 1-\alpha^1\right) = \left(\alpha^{(\rho(\alpha^1)-1)}, 1-\alpha^{(\rho(\alpha^1)-1)}\right)$ (or $(0,0)$ in the case where $\rho(\alpha^1)-1=0$). We also have $\phi\left(\alpha^{(\rho(\alpha^1))}, 1-\alpha^{(\rho(\alpha^1))}\right) = \left(\alpha^{(\rho(\alpha^1)-1)}, 1-\alpha^{(\rho(\alpha^1)-1)}\right)$ (or $(0,0)$ in the case where $\rho(\alpha^1)-1=0$). Therefore, continuation paths from period $t+2$ forward are the same whether the agent chooses $\left(\alpha^1, 1-\alpha^1\right)$ or $\left(\alpha^{(\rho(\alpha^1))}, 1-\alpha^{(\rho(\alpha^1))}\right)$. Both lead to action 1 in period $t+1$. However,

[1]If there is more than one best choice, we make an arbitrary selection.

because $u_2(1) > u_1(1)$ and $\alpha_1 > \alpha^{(\rho(\alpha^1))}$, $\left(\alpha^{(\rho(\alpha^1))}, 1-\alpha^{(\rho(\alpha^1))}\right)$ generates strictly higher continuation utility.

(ii) Consider any $(\alpha^1, \alpha^2) \in S_2$ for which $\phi(\alpha^1, \alpha^2) \neq (0,0)$ . By construction, $\phi(\alpha^1, \alpha^2) = (\alpha^{(k)}, 1-\alpha^{(k)})$ for some $k \leq \rho(1-\alpha^2) - 1$. Since $1-\alpha^2 \in [\alpha^{(\rho(1-\alpha^2))}, \alpha^{(\rho(1-\alpha^2)+1)})$, we have $1-\alpha^2 \geq \alpha^{(\rho(1-\alpha^2))} \geq \alpha^{(k+1)}$, or $\alpha^2 \leq 1-\alpha^{(k+1)}$, and we also have $\phi\left(\alpha^{(k+1)}, 1-\alpha^{(k+1)}\right) = \left(\alpha^{(k)}, 1-\alpha^{(k)}\right)$. Therefore, choosing either $(\alpha^1, \alpha^2)$ or $\left(\alpha^{(k+1)}, 1-\alpha^{(k+1)}\right)$ for period $t+1$ yields the same continuation paths from period $t+2$ forward and, with respect to period $t+1$, both lead to action 1. However, because $u_2(1) > u_1(1) > u_3(1)$ and $\alpha^2 \leq 1-\alpha^{(k+1)}$, $\left(\alpha^{(k+1)}, 1-\alpha^{(k+1)}\right)$ generates strictly higher continuation utility.

(iii) Consider any $(\alpha^1, \alpha^2) \notin S_1$ for which $\phi(\alpha^1, \alpha^2) = (0,0)$. Since $\phi(0,0) = (0,0)$, the continuation path from period $t+2$ forward involves worldview $(0,0)$ in every period, along with action 2.

Supposing $(\alpha^1, \alpha^2) \in S_3$, $(0,0)$ produces the same outcome as $(\alpha^1, \alpha^2)$ from period $t+2$ forward, and both lead to action 2 in period $t+1$. However, because $u_3(2)$ is the highest possible flow utility, $(0,0)$ generates strictly higher overall continuation utility.

Supposing $(\alpha^1, \alpha^2) \in S_2$, $(\alpha^{(1)}, 1-\alpha^{(1)})$ produces the same outcome as $(\alpha^1, \alpha^2)$ from period $t+2$ forward, and both lead to action 1 in period $t+1$. It is straightforward to verify that $(\alpha^{(1)}, 1-\alpha^{(1)})$ solves $\max_{(\alpha^1,\alpha^2)\in S} U\left(\alpha^1, \alpha^2, 1\right)$ subject to $1 \in z^*(\alpha^1, \alpha^2)$, which means that $(\alpha^{(1)}, 1-\alpha^{(1)})$ yields higher flow utility from action 1 in period $t+1$, and hence higher overall continuation utility.

*Step 2*: Assuming that $\phi$ governs future choices, $\phi$ prescribes the optimal choice in period $t$.

For $\left(\alpha^1, 1-\alpha^1\right) \in S_1$: The conclusion follows from arguments similar to those used to prove Proposition 3.

For $(\alpha^1, \alpha^2) \in S_2$: We claim that, assuming $\phi$ governs future choices, the best choice within $T$ from the perspective of worldview $(\alpha^1, \alpha^2) \in S_2$ is either $(0,0)$ or $(\alpha^{(k)}, 1-\alpha^{(k)})$ with $k \leq \rho(1-\alpha^2) - 1$.

To prove this claim, consider worldview $(1-\alpha^2, \alpha^2)$. By construction, $\phi(1-\alpha^2, \alpha^2) = \left(\alpha^{(\rho(1-\alpha^2)-1)}, 1-\alpha^{(\rho(1-\alpha^2)-1)}\right)$. We can write the difference between the continuation payoff when choosing $(\alpha^{(\rho(\alpha^1)-1)}, 1-\alpha^{(\rho(\alpha^1)-1)})$, and when choosing $(\alpha^{(m)}, 1-\alpha^{(m)})$ for any $m > \rho(1-\alpha^2) - 1$, as

$$\Delta_1 = W(\rho(1-\alpha^2)-1) - W(m) + \sum_{t=\rho(1-\alpha^2)}^{m} \lambda\delta^t \left(U(1-\alpha^2, \alpha^2, 2) - U(1-\alpha^2, \alpha^2, 1)\right) \geq 0,$$

where $W(k)$ is proportional to the continuation payoff associated with the trajectory starting from $(\alpha^{(k)}, 1-\alpha^{(k)})$ assuming perfect mindset flexibility. From the perspective of worldview $(\alpha^1, \alpha^2)$, the corresponding difference is

$$\Delta_2 = W(\rho(1-\alpha^2)-1) - W(m) + \sum_{t=\rho(1-\alpha^2)}^{m} \lambda\delta^t \left(U(\alpha^1, \alpha^2, 2) - U(\alpha^1, \alpha^2, 1)\right).$$

Notice that

$$\Delta_2 - \Delta_1 = \sum_{t=\rho(1-\alpha^2)}^{m} \lambda\delta^t \left[ \left( U(1-\alpha^2, \alpha^2, 1) - U(\alpha^1, \alpha^2, 1) \right) + \left( U(\alpha^1, \alpha^2, 2) - U(1-\alpha^2, \alpha^2, 2) \right) \right].$$

In light of the fact that $u_1(1) > u_3(1)$, we have $U(1-\alpha^2, \alpha^2, 1) - U(\alpha^1, \alpha^2, 1) > 0$. Moreover, in light of the fact that $u_1(2) < u_3(2)$, we have $U(\alpha^1, \alpha^2, 2) - U(1-\alpha^2, \alpha^2, 2) > 0$. Therefore, $\Delta_2 - \Delta_1 > 0$, and the conclusion follows.

For $(\alpha^1, \alpha^2) \in S_3$: Placing all weight on worldview 3 and picking action 2 in all subsequent periods yields the highest feasible payoff from the perspective of $(\alpha^1, \alpha^2)$, and $\phi$ achieves this bound. □

## Proof of Proposition 7

A stationary Markov-perfect equilibrium involves a function $\phi : [0,1] \times \Lambda \to [0,1] \times \Lambda$ mapping from today's worldview and flexibility parameter to tomorrow's: $\phi_1(\alpha_t, \lambda_t) = \alpha_{t+1}$ and $\phi_2(\alpha_t, \lambda_t) = \lambda_{t+1}$.

Because the sequence $\left\{\alpha^{(\tau)}\right\}_{\tau=0}^{\infty}$ defined in Proposition 3 depends on $\lambda$, we write it here as $\left\{\alpha_\lambda^{(\tau)}\right\}_{\tau=0}^{\infty}$. (The values $\alpha^{(0)}$ and $\alpha^{(1)}$ are the same regardless of $\lambda$, and therefore do not need to be indexed.) We define $\bar{\tau}_\lambda$ similarly.

For any $\alpha \in [0,1]$, we define $\tau^*(\alpha)$ as the value of $\tau$ satisfying $\alpha \in [\alpha_\lambda^{(\tau)}, \alpha_\lambda^{(\tau+1)})$.

**Lemma 4.** *Consider the continuation trajectories $A = (\alpha_1, \alpha_2, ...)$ and $A' = (\alpha_1^{'}, \alpha_2^{'}, ...)$, such that $\alpha_k \leq \alpha_k^{'}$ for all $k > 0$, with strict inequality for some $k$, along with an optimal action mapping, $z$. Suppose a consumer with the current perspective $(\alpha, \lambda)$, $\alpha > \alpha^{(1)}$, weakly prefers $A$ to $A'$. Then a consumer with the current perspective $(\alpha, \lambda')$ with $\lambda' < \lambda$ strictly prefers $A$ to $A'$.*

Proof: Let $V(\alpha, \lambda, A)$ denote the continuation payoff for trajectory $A$ from the perspective of $(\alpha, \lambda)$. Let $M = \left\{ t > 0 \mid z(\alpha_t) = 2 \text{ and } z(\alpha_t^{'}) = 1 \right\}$. Note there is no $t$ for which $z(\alpha_t) = 1$ and $z(\alpha_t^{'}) = 2$. Then

$$V(\alpha, \lambda, A) - V(\alpha, \lambda, A') = (1-\lambda) \sum_{t=1}^{\infty} \delta^t \left[ U(\alpha_t, z(\alpha_t)) - U(\alpha_t^{'}, z(\alpha_t^{'})) \right] + \lambda \sum_{t \in M} \delta^t \left[ U(\alpha, 2) - U(\alpha, 1) \right] \qquad (2)$$

One can easily show that $U(a, z(a))$ is strictly decreasing in $a$. It follows that $U(\alpha_t, z(\alpha_t)) - U(\alpha_t^{'}, z(\alpha_t^{'})) \geq 0$, with strict inequality for some $t$. Moreover, with $\alpha > \alpha^{(1)}$, we have $U(\alpha, 2) - U(\alpha, 1) < 0$. Thus, $V(\alpha, \lambda, A) - V(\alpha, \lambda, A')$ is decreasing in $\lambda$. The claim follows. □

**Lemma 5.** *Any Markov policy mapping satisfying the following restrictions is an MPE:*

*(i) $z(\alpha) = 1$ for $\alpha \geq \alpha^{(1)}$ and 2 otherwise.*[2]

[2]Technically, an MPE allows for action functions of the form $z(\alpha, \lambda)$. However, incentive compatibility ties down $z$ as a function of $\alpha$ everywhere but $\bar{\alpha}$, at which point the consumer is indifferent irrespective of $\lambda$. Thus, we are free to look for MPE within the class of policy functions for which actions depend only on $\alpha$.

*(ii) If $\tau^*(\alpha) < 2$, then $\phi_1(\alpha, \lambda) = \alpha^{(0)}$.*

*(iii) If $\tau^*(\alpha) \geq 2$ and $\lambda = \bar{\lambda}$, then $\phi_1(\alpha, \lambda) = \alpha_{\bar{\lambda}}^{(\tau^*(\alpha)-1)}$.*

*(iv) If $\tau^*(\alpha) \geq 2$, $\lambda < \bar{\lambda}$, and $\alpha = \alpha_{\bar{\lambda}}^{(\tau^*(\alpha))}$, then $\phi_1(\alpha, \lambda)$ is the best choice from the set $\{\alpha^{(0)}, \alpha^{(1)}, \alpha_{\bar{\lambda}}^{(2)}, \ldots, \alpha_{\bar{\lambda}}^{(\tau^*(\alpha)-2)}\}$ from the perspective of worldview $(\alpha, \lambda)$, assuming that in the future (i)-(iii) will govern the consumer's choices of actions and worldviews, and that she will be maximally mindset inflexible ($\lambda = \bar{\lambda}$).*

*(v) If $\tau^*(\alpha) \geq 2$, $\lambda < \bar{\lambda}$, and $\alpha > \alpha_{\bar{\lambda}}^{(\tau^*(\alpha))}$, then $\phi_1(\alpha, \lambda)$ is the best choice from the set $\{\alpha^{(0)}, \alpha^{(1)}, \alpha_{\bar{\lambda}}^{(2)}, \ldots, \alpha_{\bar{\lambda}}^{(\tau^*(\alpha)-1)}\}$ from the perspective of worldview $(\alpha, \lambda)$, assuming that in the future (i)-(iii) will govern the consumer's choices of actions and worldviews, and that she will be maximally mindset inflexible ($\lambda = \bar{\lambda}$).*

*(vi) If $\phi_1(\alpha, \lambda) > \alpha^{(1)}$, then $\phi_2(\alpha, \lambda) = \bar{\lambda}$.*

*Proof:* By construction, $z(\alpha)$ is optimal for each $(\alpha, \lambda)$.

Let $\mathcal{A}$ denote the set of trajectories of the form $(\alpha, \alpha_{\bar{\lambda}}^{(k)}, \alpha_{\bar{\lambda}}^{(k-1)}, ..., \alpha^{(1)}, \alpha^{(0)}, \alpha^{(0)}, ...)$ where $k \leq \tau^*(\alpha)-1$. By construction, $\mathcal{A}$ contains all one-period deviation trajectories that are feasible under $\phi$. Let $\mathcal{A}_\text{s} \subset \mathcal{A}$ denote the set of trajectories of the form $(\alpha_{\bar{\lambda}}^{(k)}, \alpha_{\bar{\lambda}}^{(k-1)}, ..., , ..., \alpha^{(1)}, \alpha^{(0)}, \alpha^{(0)}, ...)$ for $k < \bar{\tau}_{\bar{\lambda}}$. All of these continuation trajectories are feasible (without deviations) under $\phi$.

We claim that every optimal feasible continuation trajectory lies within $\mathcal{A}_\text{s}$. Consider any sequence $A = (\alpha, \alpha_{\bar{\lambda}}^{(k)}, \alpha_{\bar{\lambda}}^{(k-1)}, ..., \alpha^{(1)}, \alpha^{(0)}, \alpha^{(0)}, ...) \in \mathcal{A} \setminus \mathcal{A}_S$ (with $k \leq \tau^*(\alpha) - 1$), as well as the alternative sequence $A^{k+1} = (\alpha_{\bar{\lambda}}^{(k+1)}, \alpha_{\bar{\lambda}}^{(k)}, \alpha_{\bar{\lambda}}^{(k-1)}, ..., \alpha^{(1)}, \alpha^{(0)}, \alpha^{(0)}, ...) \in \mathcal{A}_S$. With $k \leq \tau^*(\alpha)-1$, we must have $\alpha_{\bar{\lambda}}^{(k+1)} < \alpha$, where the strictness of the inequality follows from the fact that $A$ does not lie in $\mathcal{A}_S$. It follows that $A^{k+1}$ yields a higher continuation payoff than $A$.

Next we claim that $\phi$ prescribes an optimal choice for $(\alpha, \lambda)$, given that it governs subsequent choices:

Suppose the consumer starts at $(\alpha, \bar{\lambda})$. The proof of Proposition 3 shows, in effect, that the optimal continuation path in $\mathcal{A}_S$ is $A^{\tau^*(\alpha)-1}$, which is generated by repeatedly applying $\phi$. The claim follows.

Next suppose the consumer starts at $(\alpha, \lambda)$ such that $\lambda < \bar{\lambda}$, $\tau^*(\alpha) \geq 2$, and $\alpha > \alpha_{\bar{\lambda}}^{(\tau^*(\alpha))}$. We know that $V(\alpha, \bar{\lambda}, A^{\tau^*(\alpha)-1}) > V(\alpha, \bar{\lambda}, A^k)$ for all $k > \tau^*(\alpha) - 1$. The claim follows by applying Lemma 4.

A similar argument applies in the case where a consumer starts at $(\alpha, \lambda)$ such that $\lambda < \bar{\lambda}$, $\tau^*(\alpha) \geq 2$ and $\alpha = \alpha_{\bar{\lambda}}^{(\tau^*(\alpha))}$, and in the case a consumer starts at $(\alpha, \lambda)$ such that $\lambda < \bar{\lambda}$ and $\tau^*(\alpha) < 2$. □

**Lemma 6.** *Every stationary MPE policy mapping coincides with one belonging to the class described in Lemma 5 on a set of full measure.*

*Proof.* Let $(\psi, y)$ denote a generic stationary MPE. We will show that these functions coincide with some $(\phi, z)$ satisfying the restrictions described in Lemma 5 on a set of full measure.

Step 1: (i) $y(\alpha) = 2$ for $\alpha < \alpha^{(1)}$ and $y(\alpha) = 1$ for $\alpha > \alpha^{(1)}$, (ii) $\psi_1(\alpha, \lambda) \leq \alpha$ for all $\alpha$, and (iii) $\psi_1(\alpha, \lambda) = 0$ for all $\alpha \in [0, \alpha^{(1)}]$. The arguments are essentially the same as in Steps 1-3 of Lemma 3.

Step 2: $\psi_1(\alpha, \lambda) = 0$ for $\alpha \in (\alpha^{(1)}, \alpha_{\bar{\lambda}}^{(2)})$.

Since $\alpha_{\hat{\lambda}}^{(2)}$ is decreasing in $\hat{\lambda}$, $\alpha < \alpha_{\bar{\lambda}}^{(2)}$ implies $\alpha < \alpha_{\lambda}^{(2)}$; the conclusion follows using the same argument as in Step 4 of Lemma 3.

Step 3: Suppose that for some integer $\tau \geq 2$, $(\psi, y)$ satisfies the characterization given in (ii)-(vi) of Lemma 5 for all pairs $(\alpha, \lambda) \in [0, \alpha_{\bar{\lambda}}^{(\tau)}) \times \Lambda \setminus \left(\alpha^{(\tau-1)}, \bar{\lambda}\right)$, and that $\psi_1\left(\alpha^{(\tau-1)}, \bar{\lambda}\right) \in \{\alpha_{\bar{\lambda}}^{(\tau-3)}, \alpha_{\bar{\lambda}}^{(\tau-2)}\}$.[3] Then $(\psi, y)$ satisfies the characterization given in (ii)-(vi) of Lemma 5 for all pairs $(\alpha, \lambda) \in [0, \alpha_{\bar{\lambda}}^{(\tau+1)}) \times \Lambda \setminus \left(\alpha^{(\tau)}, \bar{\lambda}\right)$, and $\psi_1\left(\alpha^{(\tau)}, \bar{\lambda}\right) \in \{\alpha_{\bar{\lambda}}^{(\tau-2)}, \alpha_{\bar{\lambda}}^{(\tau-1)}\}$; furthermore, $z(\bar{\alpha}, \lambda) = 1$ for at least one value of $\lambda \in \Lambda$.

For the moment, suppose that $(\psi, y)$ also satisfies the characterization given in (ii)-(vi) of Lemma 5 at $(\alpha^{(\tau-1)}, \bar{\lambda})$, and hence on $[0, \alpha_{\bar{\lambda}}^{(\tau)}) \times \Lambda$. Also suppose that $z(\bar{\alpha}, \lambda) = 1$ for at least one value of $\lambda \in \Lambda$. In that case, the choice of any $(\alpha', \lambda')$ with $\alpha' < \alpha^{(\tau)}$ yields an element of $\mathcal{A}$ as the continuation path. It follows from the arguments in the proof of Lemma 5 that to show $(\psi, y)$ has the desired properties for $\alpha \in [\alpha_{\bar{\lambda}}^{(\tau)}, \alpha_{\bar{\lambda}}^{(\tau+1)})$, we only need to show that we cannot have $\psi_1(\alpha, \lambda) \geq \alpha_{\bar{\lambda}}^{(\tau)}$. We separately consider two cases: (i) $\lambda = \bar{\lambda}$, and (ii) $\lambda < \bar{\lambda}$. In either case, there must exist some $T \geq 1$ (possibly $+\infty$) such that $\psi_1^{T+1}(\alpha, \bar{\lambda}) < \alpha^{(\tau)}$, and $\psi_1^t(\alpha, \bar{\lambda}) \geq \alpha^{(\tau)}$ for $t \leq T$.

For case (i), $\psi_1(\alpha, \lambda) < \alpha_{\bar{\lambda}}^{(\tau)}$ follows from arguments similar to those in Step 5 of Lemma 3.

Now consider case (ii), where $\alpha \in [\alpha_{\bar{\lambda}}^{(\tau)}, \alpha_{\bar{\lambda}}^{(\tau+1)})$ and $\lambda < \bar{\lambda}$. Suppose toward a contradiction that $\psi_1(\alpha, \lambda) \geq \alpha_{\bar{\lambda}}^{(\tau)}$. We claim that $T < \tau - 1$ and $\psi_1^{T+1}(\alpha, \lambda) < \alpha_{\bar{\lambda}}^{(\tau-T-1)}$. Because we have assumed that the characterization from Lemma 5 applies for $\alpha < \alpha^{(\tau)}$, it follows that $\psi_1^{T+1}(\alpha, \lambda) = \alpha_{\bar{\lambda}}^{(m)}$ for some $m < \tau$, and consequently that the continuation trajectory from period $T+1$ forward is $A^m$. Were it not the case that $T < \tau - 1$ and $\psi_1^{T+1}(\alpha, \lambda) < \alpha_{\bar{\lambda}}^{(\tau-T-1)}$, a preference for the trajectory $A^{(\tau-1)}$ over the trajectory induced by $\psi$ from the perspective of $\left(\alpha, \bar{\lambda}\right)$ (which we established in case (i)) would (by Lemma 4) imply a strict preference from the perspective of $(\alpha, \lambda)$, a contradiction that establishes the claim. In light of the fact that $\alpha_{\bar{\lambda}}^{(m)} = \psi_1^{T+1}(\alpha, \lambda) < \alpha_{\bar{\lambda}}^{(\tau-T-1)}$, we must have $m < \tau - T - 1$. Because $m + T + 1 < \tau$, we know that choosing $(\alpha^{(m+T+1)}, \bar{\lambda})$ induces the continuation trajectory $A^{m+T+1}$. Note that this trajectory coincides with $\left(\psi_1(\alpha, \lambda), \psi_1^2(\alpha, \lambda), ...\right)$ from period $T+1$ forward, but $A^{m+T+1}$ yields a strictly higher continuation payoff in the first $T$ periods, a contradiction.

Finally, to prove that $z(\bar{\alpha}, \lambda) = 1$ for at least one value of $\lambda \in \Lambda$, and that $\psi\left(\alpha^{(\tau-1)}, \bar{\lambda}\right) = \alpha_{\bar{\lambda}}^{(\tau-2)}$, one can use a continuity argument similar to the one in Step 5 of Lemma 3. Unless both of these conditions hold, an optimal choice does not exist for some worldviews, which contradicts the hypothesis that $(\psi, y)$ is an equilibrium. □

The Proposition's first two claims follow directly. Lemma 5 establishes existence of a stationary MPE. Lemma

[3] In the case where $\tau = 2$, $\alpha_{\bar{\lambda}}^{(-1)}$ is undefined, so the condition implies $\psi_1\left(\alpha^{(1)}, \bar{\lambda}\right) = \alpha_{\bar{\lambda}}^{(0)}$, which we have already established.

6 and condition (vi) of Lemma 5 guarantee that, in all stationary MPE, a consumer who chooses $\alpha_{t+1} > \alpha^{(1)} = \bar{\alpha}$ in period $t$ also selects maximal inflexibility ($\lambda_{t+1} = \bar{\lambda}$).

Turning to the Proposition's final statement, one can easily show using continuity that for any $\alpha > \bar{\alpha}$, there exists a $\lambda_\alpha < 1$ for which $\alpha^{(3)}_{\lambda_\alpha} = \alpha$. Moreover, it is straightforward to establish that a consumer with worldview $(\alpha, \lambda)$, where $\lambda > \lambda_\alpha$, strictly prefers $A^2$ to both $A^1$ and $A^0$ (first by showing that a consumer with worldview $\left(\alpha^{(3)}_{\lambda_\alpha}, \lambda_\alpha\right)$ strictly prefers $A^2$ to both $A^1$ and $A^0$, and then by applying Lemma 4). Thus, fixing any $\alpha > \bar{\alpha}$, if we take $\underline{\lambda} = \lambda_\alpha$ and assume that $\bar{\lambda} > \underline{\lambda}$, then for all $\lambda \in (\underline{\lambda}, \overline{\lambda}]$, an individual with worldview $(\alpha, \lambda)$ chooses $\alpha' > \bar{\alpha}$. □

## Proof of Proposition 8

To prove this proposition, we transform this model into one we have already analyzed. Let $v_1(1) = \theta u_1(2) + u_1(1)$, $v_2(1) = \theta u_2(2) + u_2(1)$, $v_1(2) = (1+\theta)u_1(2)$, and $v_2(2) = (1+\theta)u_2(2)$.

Notice that $v_1(1) - v_1(2) = u_1(1) - u_1(2) > 0$ and $v_2(2) - v_2(1) = u_2(2) - u_2(1) > 0$. It follows that action $k$ maximizes $v_k$. In addition, $v_2(2) - v_1(1) = \theta\left[u_2(2) - u_1(2)\right] + \left[u_2(2) - u_1(1)\right] > 0$, because we have assumed that $u_2(2) > u_1(1)$. Next notice that $v_1(1) - v_2(1) = \theta\left[u_1(2) - u_2(2)\right] + \left[u_1(1) - u_2(1)\right]$. Therefore, worldview 2 happiness-dominates worldview 1 in the modified model iff

$$\theta > \frac{u_1(1) - u_2(1)}{u_2(2) - u_1(2)} \equiv \theta_1 > 0.$$

It follows that, if $\theta < \theta_1$, the characterization in Proposition 1 applies to the modified model, while if $\theta > \theta_1$, the characterization in Proposition 3 applies.

We are interested in the existence of cases in which $\theta > \theta_1$ and $\alpha^{(2)} < 1$, because in those cases the transition to worldview 2 will be gradual. It is straightforward to check that $\alpha^{(2)} < \alpha^*$ when worldview 2 happiness-dominates worldview 1. Furthermore, for the modified model, the condition $\alpha^* < 1$ becomes $\lambda(1+\theta)u_1(2) + (1-\lambda)(1+\theta)u_2(2) < \theta u_1(2) + u_1(1)$ or

$$\theta < \frac{u_1(1) - \lambda u_1(2) - (1-\lambda)\,u_2(2)}{(1-\lambda)\left(u_2(2) - u_1(2)\right)} \equiv \theta_2$$

Notice that $\theta < \theta_2$ guarantees $\alpha^{(2)} < 1$. Notice also that $\theta_2$ varies continuously with $\lambda$ for $\lambda < 1$. To satisfy both $\theta > \theta_1$ and $\theta < \theta_2$, we must have $\theta_2 > \theta_1$. From inspection of the last formula (given $u_1(1) > u_1(2)$), we have $\lim_{\lambda \to 1} \theta_2 = +\infty$. Therefore, $\theta_2 > \theta_1$ holds for $\lambda$ sufficiently large. In light of the foregoing, the statements in the proposition and footnote 26 follow directly from Propositions 1 and 3. □

## Proof of Proposition 9

Part (a): We will prove the proposition for the case of $i = 1$. (The arguments when $i = 2$ are completely analogous.) By Proposition 1, if $\alpha_t < \alpha^*$ for $t = K$, then $\alpha_{t+k} = 0$ for all $k \geq 1$. A simple backward induction argument then establishes that the same statement holds for all $t < K$.

We claim that for any $\alpha_0$, there exists a finite integer $C$ such that if $K > C$, the consumer chooses $\alpha_1 = 0$. From Proposition 1, we know that any sequence of choices for the first $K$ periods will yield one of two continuation paths from period $K+1$ forward: either $((0,2),(0,2),...)$ or $((1,1),(1,1),...)$. Among the choice sequences that yield $((0,2),(0,2),...)$ from period $K+1$ forward, the best one (from the perspective of worldview $\alpha_0$ in period 0) is plainly $((0,2),(0,2),...)$ from period 1 forward, which is achieved by choosing $\alpha_1 = 0$. Let $V = \frac{\delta}{1-\delta}\left[\lambda U(\alpha_0, 2) + (1-\lambda)U(0,2)\right]$ denote the resulting payoff from the perspective of worldview $\alpha_0$ in period 0. According to our opening observation, all sequences of choices that yield $((1,1),(1,1),...)$ from period $K+1$ onwards (to the extent they exist) must have the property that $\alpha_t \geq \alpha^*$ for $t \leq K$. The resulting payoff from the perspective of worldview $\alpha_0$ in period 0 is bounded above by

$$W = \left(\frac{\delta - \delta^{K+1}}{1-\delta}\right)\left[\lambda U(\alpha_0, 2) + (1-\lambda)U(\alpha^*, 2)\right] + \left(\frac{\delta^{K+1}}{1-\delta}\right)\left[\lambda U(\alpha_0, 1) + (1-\lambda)U(1,1)\right]$$

Observe that

$$V - W = \left(\frac{\delta - \delta^{K+1}}{1-\delta}\right)(1-\lambda)\left[U(0,2) - U(\alpha^*, 2)\right]$$

$$+ \left(\frac{\delta^{K+1}}{1-\delta}\right)\left(\left[\lambda U(\alpha_0, 2) + (1-\lambda)U(0,2)\right] - \left[\lambda U(\alpha_0, 1) + (1-\lambda)U(1,1)\right]\right)$$

For $K$ sufficiently large, $V - W > 0$, which completes the proof.

Part (b): Because the subsidy or tax induces the consumer to choose action $j$ in the first $K$ periods regardless of her worldviews, the problem is isomorphic to the one in which action $i$ is banned for the first $K$ periods. □

## Proof of Proposition 10

A stationary Markov policy function $\phi$ is now a mapping $\phi(\alpha_t, M_t)$ from the period-$t$ worldview and the period-$t$ restriction ($M_t = 1$ indicates the restriction is in force, while $M_t = 0$ indicates it is not) to the period-$t+1$ worldview.

Define the sequence $\left\{\alpha^{(\kappa)}\right\}_{\kappa=0}^{\infty}$ as follows: $\alpha^{(0)} = 1$, $\alpha^{(1)} = \alpha^*$, $\alpha^{(2)} = \alpha^{(1)} - \frac{(1-\lambda)(1-\delta)\left[u_1(1) - U(\alpha^{(1)},1)\right]}{\lambda p \delta([u_2(2)-u_1(2)]-[u_2(1)-u_1(1)])}$, and recursively (for $\kappa > 2$), $\alpha^{(\kappa)} = \alpha^{(\kappa-1)} + \frac{\Psi}{\delta^{\kappa-1}(1-p)^{\kappa-2}}\left(\alpha^{(\kappa-1)} - \alpha^{(\kappa-2)}\right)$ where $\Psi = \frac{(1-\lambda)(1-\delta)}{\lambda p}\frac{[u_1(1)-u_2(1)]}{[u_2(2)-u_1(2)]-[u_2(1)-u_1(1)]} > 0$. This sequence is analogous to the one described in Proposition 3. Using induction, one can easily show that $\forall\ (\delta, \lambda, p) \in (0,1)^3$, $\left\{\alpha^{(\kappa)}\right\}_{\kappa=0}^{\infty}$ is a strictly decreasing sequence (note that $\alpha^{(2)} < \alpha^{(1)}$ because we are in Case 1, so $u_1(1) > U(\alpha^{(1)}, 1)$). One can also show there exists $\bar{\kappa} \geq 1$ such that $\alpha^{(\bar{\kappa})} \geq 0$ and $\alpha^{(\bar{\kappa}+1)} < 0$.

To prove the proposition, we show that, for all $(\delta, \lambda, p) \in (0,1)^3$, the following Markov policy functions constitute an MPE:

$$\phi(\alpha, 1) = \begin{cases} \alpha^{(\bar{\kappa}-1)} & if \ \alpha \in [0, \alpha^{(\bar{\kappa})}] \\ \alpha^{(\bar{\kappa}-2)} & if \ \alpha \in (\alpha^{(\bar{\kappa})}, \alpha^{(\bar{\kappa}-1)}] \\ \vdots & \vdots \\ \alpha^{(1)} & if \ \alpha \in (\alpha^{(3)}, \alpha^{(2)}] \\ \alpha^{(0)} & if \ \alpha \in (\alpha^{(2)}, 1] \end{cases}$$

and

$$\phi(\alpha, 0) = \begin{cases} 0 & \alpha \leq \alpha^* \\ 1 & \alpha > \alpha^* \end{cases}$$

Finally, the consumer takes action 1 when the restriction is in force ($z(\alpha, 1) = 1$). When it is not in force, $z(\alpha, 0) = 2$ if $\alpha < \bar{\alpha}$; otherwise $z(\alpha, 0) = 1$.

Once $M_t = 0$, the characterization given in Proposition 1 applies. Therefore, we can focus on the case of $M_t = 1$. Define $V(\alpha^{(\kappa)}, \alpha)$ as the discounted continuation payoff under worldview $\alpha$ resulting from choosing $\alpha^{(\kappa)}$ and following the MPE thereafter (assuming $\kappa \geq 1$):

$$V(\alpha^{(\kappa)}, \alpha) = \sum_{n=0}^{\kappa-1} \delta^n (1-p)^n \left[ \lambda U(\alpha, 1) + (1-\lambda) U(\alpha^{(\kappa-n)}, 1) \right]$$
$$+ (1-p)^\kappa \delta^\kappa \Big( \frac{\lambda U(\alpha, 1) + (1-\lambda) u_1(1)}{1-\delta} \Big) + \sum_{n=0}^{\kappa-1} p(1-p)^n \delta^{n+1} \Big( \frac{\lambda U(\alpha, 2) + (1-\lambda) u_2(2)}{1-\delta} \Big)$$

We now proceed in a series of steps that are analogous to those in Lemma 2:

*Step 1*: Assuming future behavior is governed by $\phi$, then for every worldview $\alpha$, the individual strictly prefers $\alpha^{(\kappa)}$ to any $\alpha \in (\alpha^{(\kappa+1)}, \alpha^{(\kappa)}) \equiv I^{(\kappa)}$ for all $\kappa \in \{0, ..., \bar{\kappa} - 1\}$, and $\alpha^{(\bar{\kappa})}$ to any $\alpha \in [0, \alpha^{(\bar{\kappa})}) \equiv I^{\overline{\kappa}}$.

We use the same argument as in Step 2 of Lemma 2; all continuation sequences are the same for any choice of $\alpha \in I^{(\kappa)} \cup \alpha^{(\kappa)}$, regardless of future state realizations $M_t$. Given that the individual must choose action 1 tomorrow, choosing $\alpha^{(\kappa)}$ will yield the highest payoff.

*Step 2*: An individual with worldview $\alpha^{(\kappa)}$, where $\kappa \geq 2$, is indifferent between choosing $\alpha^{(\kappa-1)}$ and $\alpha^{(\kappa-2)}$ for next period.

Consider an individual with worldview $\alpha$. We equate continuation payoffs after choosing $\alpha^{(\kappa-1)}$ and $\alpha^{(\kappa-2)}$, and solve for $\alpha$. After some manipulation, we obtain:

$$\alpha = \frac{u_2(2) - u_2(1)}{[u_2(2) - u_1(2)] - [u_2(1) - u_1(1)]} + (\frac{1-\lambda}{\lambda})(\frac{u_2(2) - u_1(1)}{[u_2(2) - u_1(2)] - [u_2(1) - u_1(1)]})$$
$$+ \frac{(1-\delta)(1-\lambda)}{p\lambda(1-p)^{\kappa-2}} \sum_{n=0}^{\kappa-2} \frac{(1-p)^n}{\delta^{\kappa-1-n}} \frac{U(\alpha^{(\kappa-1-n)}, 1) - U(\alpha^{(\kappa-2-n)}, 1)}{[u_2(2) - u_1(2)] - [u_2(1) - u_1(1)]}$$

It is immediate that $\alpha^{(2)}$ satisfies this equation for $\kappa = 2$, and it is easily verified that if $\alpha^{(\kappa)}$ satisfies it for $\kappa \geq 2$, then $\alpha^{(\kappa+1)}$ satisfies it for $\kappa + 1$. The claim follows.

*Step 3*: Given that $\phi$ governs behavior for all future periods, if an individual with worldview $\alpha$ is indifferent between $\alpha^{(r)}$ and $\alpha^{(r-1)}$, then an individual with worldview $\alpha' < \alpha$ strictly prefers $\alpha^{(r)}$ to $\alpha^{(r-1)}$, while an individual with worldview $\alpha' > \alpha$ strictly prefers $\alpha^{(r-1)}$ to $\alpha^{(r)}$.

It is easy to verify that one can express $V(\alpha^{(r)}, \alpha) - V(\alpha^{(r-1)}, \alpha)$, the difference in continuation payoffs from selecting $\alpha^{(r)}$ instead of $\alpha^{(r-1)}$, as $K(p, r) + p(1-p)^{r-1}\delta^{r-2}\lambda\,[U(\alpha, 2) - U(\alpha, 1)]$, where $K(p, r)$ is a term that does not depend on $\alpha$. The desired conclusion follows from the fact that this difference is strictly decreasing in $\alpha$.

*Step 4*: $\phi$ is an MPE. We know from step 1 that an individual will always chooses $\alpha^{(\kappa)}$ for some value $\kappa$. From step 2 we know that an individual with worldview $\alpha^{(\kappa+2)}$ is indifferent between $\alpha^{(\kappa)}$ and $\alpha^{(\kappa+1)}$, while an individual with worldview $\alpha^{(\kappa+3)}$ is indifferent betwen $\alpha^{(\kappa+1)}$ and $\alpha^{(\kappa+2)}$. From step 3 it follows that the unique optimum is $\alpha^{(\kappa+1)}$ for all $\alpha \in (\alpha^{(\kappa+3)}, \alpha^{(\kappa+2)})$, and the optima are $\{\alpha^{(\kappa+1)}, \alpha^{(\kappa)}\}$ for $\alpha = \alpha^{(\kappa+2)}$. It also follows that the unique optimum is $\alpha^{(0)}$ for $\alpha > \alpha^{(2)}$. □

## Proof of Proposition 11

We begin by solving for $U(x, \bar{\theta}(x))$ analytically. It is straightforward to show that the first- and second-order conditions for an interior solution to $\max_\theta U(x, \theta)$ can only be satisfied if either (a) $\alpha < 1$, $k > 0$, and $\eta > 1$, or (b) $\alpha > 1$, $k < 0$, and $\eta < 1$. Using the first-order condition to solve for $\bar{\theta}(x)$, we then derive the following expression for $\overline{U}(x) \equiv U(x, \bar{\theta}(x))$:

$$\bar{U}(x) = \left(\frac{1}{(1-\alpha)k}\right)^{\frac{1}{\eta-1}} \left(\frac{\eta - 1}{(1-\alpha)\,\eta}\right) x^{\frac{(1-\alpha)\eta}{\eta-1}}$$

This is a CRRA utility function with relative risk aversion parameter $1 - \frac{(1-\alpha)\eta}{\eta-1} = \frac{\alpha\eta-1}{\eta-1}$. In case (a), this parameter converges to $\alpha$ as $\eta \to \infty$, and it converges to $-\infty$ as $\eta \downarrow 1$. In case (b), this parameter converges to $\alpha$ as $\eta \to -\infty$, and it converges to $-\infty$ as $\eta \uparrow 1$. In either case, it is always strictly less than $\alpha$.

**Lemma 7.** *For any $y > 0$, there exists $\bar{c} > 0$ with the following property: for any $c < \bar{c}$, we can select finite numbers, $\sigma_S$ and $\sigma_L$ with $0 < \sigma_S \leq \sigma_L$ such that the consumer does not change her worldview if $|x - y| < \sigma_S$ and* *does change her worldview if* $|x-y| > \sigma_L$. *For* $\alpha < 1$, *we can take* $\bar{c} = +\infty$.

Proof: Note that

$$U(x,\bar{\theta}(x)) - U(x,\bar{\theta}(y)) = \left(\frac{1}{(1-\alpha)k}\right)^{\frac{1}{\eta-1}} \left(\frac{x^{1-\alpha}}{1-\alpha}\right)\left[\left(\frac{\eta-1}{\eta}\right)x^{\frac{1-\alpha}{\eta-1}} - y^{\frac{1-\alpha}{\eta-1}}\right] + \frac{k}{\eta}\left[\left(\frac{y^{1-\alpha}}{(1-\alpha)k}\right)^{\frac{1}{\eta-1}}\right]^{\eta} \tag{3}$$

which is continuous and equals zero when $x = y$. Accordingly, there always exists $\sigma_S > 0$ such that the consumer does not change her worldview if $|x-y| < \sigma_S$.

Differentiating $U(x,\bar{\theta}(x)) - U(x,\bar{\theta}(y))$, we obtain:

$$\frac{d}{dx}\left[U(x,\bar{\theta}(x)) - U(x,\theta_1)\right] = \left(\frac{1}{(1-\alpha)k}\right)^{\frac{1}{\eta-1}} x^{-\alpha}\left[x^{\frac{1-\alpha}{\eta-1}} - y^{\frac{1-\alpha}{\eta-1}}\right]$$

Recalling that $\frac{1-\alpha}{\eta-1} > 0$, we see that the bracketed term must be strictly greater than zero when $x > y$, and strictly less than zero when $x < y$. Consequently, as $x$ moves away from $y$ in either direction, if there comes a point at which $U(x,\bar{\theta}(x)) - U(x,\bar{\theta}(y)) > c$, then this inequality continues to hold as $x$ moves further from $y$. Select any $x > y$, and let $\bar{c} = U(x,\bar{\theta}(x)) - U(x,\bar{\theta}(y))$. By the Intermediate Value Theorem, for any $c < \bar{c}$, there exists $x' \in (y,x)$ such that $U(x',\bar{\theta}(x')) - U(x',\bar{\theta}(y)) = c$. Let $x'' < y$ satisfy $U(x'',\bar{\theta}(x'')) - U(x'',\bar{\theta}(y)) = c$ when a solution exists, and let $x'' = 0$ otherwise. It then follows that $U(x,\bar{\theta}(x)) - U(x,\bar{\theta}(y)) > c$ (and hence the consumer changes her worldview) for $|x-y| > \sigma_L \equiv \max\{x'-y, y-x''\}$.

When $\alpha < 1$, $k > 0$, and $\eta > 1$, it is easy to check that $U(x,\bar{\theta}(x)) - U(x,\bar{\theta}(y))$ increases in $x$ without bound. It follows that one can take $\bar{c} = +\infty$ in this case. □

It follows from the preceding lemma that decisions involving only small stakes ($|x-y| < \sigma_S$ for every possible outcome $x$) are governed by the objective function $U(x,\bar{\theta}(y))$, and that decisions involving only large stages ($|x-y| > \sigma_L$ for every possible outcome $x$) are governed by the objective function $W$. Note that

$$r_W(x) = -x\frac{\lambda U_{xx}(x,\theta_1) + (1-\lambda)\bar{U}_{xx}(x)}{\lambda U_x(x,\theta_1) + (1-\lambda)\bar{U}_x(x)}$$

It follows that $\lim_{\lambda\to 1} r_W(x) = r_U(x) = \alpha$ and $\lim_{\lambda\to 0} r_W(x) = r_{\bar{U}}(x) = \frac{\alpha\eta-1}{\eta-1}$. One can also show that

$$\frac{dr_W(x)}{d\lambda} = -x\left[\frac{U_{xx}(x,\theta_1)\bar{U}_x(x) - \bar{U}_{xx}(x)U_x(x,\theta_1)}{\left[\lambda U_x(x,\theta_1) + (1-\lambda)\bar{U}(x)\right]^2}\right]$$

Using the fact that $r_{\bar{U}}(x) < r_U$, we see that $\bar{U}_{xx}(x)U_x(x,\theta_1) > U_{xx}(x,\theta_1)\bar{U}_x(x)$, from which it follows that $\frac{dr_W(x)}{d\lambda} > 0$, as claimed. □