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How Important Are Sectoral Shocks? Corrigendum†

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This corrigendum corrects an error in equation (12) of Atalay (2017), "How Important Are Sectoral Shocks?" published in the *American Economic Journal: Macroeconomics* 9(4): 254–80. In Section I of this corrigendum, we discuss the context in which this error appears. We derive the corrected equation (12) in Section II.

I. Background

The erroneous equation, equation (12), appears within Section ID of Atalay (2017). The goal of Section ID of Atalay (2017) is to explain—by means of a simpler version of the main quantitative model, introduced in Atalay's (2017) Sections IA and IB—that gross output co-movement depends not only on the correlation among industries' productivity shocks but also on how complementary industries' products are. In this way, Section ID motivates the Section II estimation of the model's key elasticities of substitution.

According to the model outlined in Section ID of Atalay (2017), industry gross output is log-linearly related to industry-specific productivity shocks and common productivity shocks:

$$\log Q_{tI} \approx F\left(A_{tI}, \bar{A}_t\right) \equiv \frac{1}{1-\mu}\log\left(\frac{1}{1-\mu}\right) + \log\left(\frac{1}{N}\right) + \left[\mu\varepsilon_M + (1-\mu)\varepsilon_D\right]\log A_{tI} + \frac{1}{N}\left[\left(\frac{1}{1-\mu}\right)^2 - \left(\mu\varepsilon_M + (1-\mu)\varepsilon_D\right)\right]\bar{A}_t. \tag{12}$$

In this equation, Q_{tI} represents the gross output of industry I in year t, A_{tI} the total factor productivity of industry I in year t, and N the number of industries. The parameter μ gives the importance of intermediate inputs in industries' gross output production functions. The two key elasticities of substitution, ε_D and ε_M, respectively parameterize how substitutable different industry outputs are in consumers' preferences and in industries' intermediate input bundles. Finally, within this corrigendum, we refer to $\bar{A}_t \equiv \sum_J \log A_{tJ}$ as "common" productivity shocks.

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The main takeaway that Atalay (2017) draws from equation (12) is that "a given amount of observed output co-movement could arise either from low elasticities of substitution and correlated shocks or, alternatively, high elasticities of substitution and relatively uncorrelated shocks." (p. 261) To see this, equation (12) indicates that the pass through of industry-specific TFP (the A_{tI} term appearing in the second line of the equation) to industry gross output (Q_{tI}) is increasing in ε_M and ε_D, and is 0 when the two elasticities are equal to 0.[1] When ε_M and ε_D both equal zero, gross output is perfectly correlated across industries.

The corrected version of equation (12) is

$$(12') \quad \log Q_{tI} \approx G\left(A_{tI}, \bar{A}_t\right) \equiv \log\left(\frac{1}{1-\mu}\right) + \log\left(\frac{1}{N}\right) + \left[\mu\,\varepsilon_M + (1-\mu)\,\varepsilon_D\right]\log A_{tI} + \frac{1}{N}\left[\frac{1+\mu\left(\varepsilon_Q - 1\right)}{1-\mu} - \left(\mu\varepsilon_M + (1-\mu)\,\varepsilon_D\right)\right]\bar{A}_t.$$

The main difference between the original equation (12) and the corrected equation (12′) is in the slope of the relationship between industry gross output and the common productivity term. Clearly, the pass through of common productivity to industry output is increasing in ε_Q—the elasticity of substitution in industries' production functions between capital and labor on the one hand and intermediate inputs on the other—in equation (12′) but not in equation (12). As in the original equation (12), however, the slope of the relationship between industry-specific productivity and industry output is 0 when ε_M and ε_D are 0, and is increasing in ε_M and ε_D. So, the main conclusion drawn from equation (12)—namely that complementarity across industries' products (when constructing either the consumption bundle or the intermediate input bundle) implies co-movement in output—still follows from equation (12′).

Furthermore, we note that the calculations in the remaining sections (Section IA–IC, Sections II–IV, Appendix F.1–F.5) of Atalay (2017) are separate from those in Section ID and do not suffer from the error discussed here.

II. Derivation of the Corrected Equation (12)

Step 3 of the derivation within Appendix F.6 of Atalay (2017) contains the equation

$$(\text{C.1}) \quad Q_I = C_I + \sum_{J=1}^{N} M_{I \to J} = C_I + \frac{\mu}{N}\left(P_I\right)^{-\varepsilon_M}\sum_{J=1}^{N} Q_J\left(P_J^{in}\right)^{\varepsilon_M - \varepsilon_Q} P_J^{\varepsilon_Q} A_J^{\varepsilon_Q - 1}.$$

[1] Using the F function defined in equation (12), this pass through refers to $\partial F(A_{tI}, \bar{A}_t)/\partial A_{tI} = \mu\varepsilon_M + (1-\mu)\varepsilon_D$.

(We have dropped the time subscripts for now for notational simplicity.) Within this equation, P_J and P_J^{in} represent the price of good J and the unit cost of the intermediate input bundle for good J producers. The first line of equation (C.1) gives the market-clearing condition for good J: Output is used either for consumption (C_I) or as an intermediate input ($M_{I\to J}$) in downstream industries (indexed by J). The second line plugs the first-order condition for $M_{I\to J}$ into the market-clearing condition. Appendix F.6 of Atalay (2017) had mistakenly written the final terms in the summand, $P_J^{\varepsilon_Q} A_J^{\varepsilon_Q - 1}$, as $P_J^{\varepsilon_Q - 1}$.

With this correction, we broadly follow the remaining steps of the derivation within Appendix F.6. Take the log-linear approximation of equation (C.1) around the point at which all productivity terms are equal to 1:

$$
\begin{aligned}
\text{(C.2)}\quad \log Q_I \approx{}& (1-\mu)\log\left(\frac{1}{1-\mu}\right) + (1-\mu)\log C_I - \mu\varepsilon_M \log P_I + \frac{\mu}{N}\sum_{J=1}^{N}\log Q_J \\
&+ \left(\frac{\mu}{N}\sum_{J=1}^{N}\left(\varepsilon_Q \log P_J + (\varepsilon_Q - 1)\log A_J + (\varepsilon_M - \varepsilon_Q)\log P_J^{in}\right)\right) \\
\approx{}& (1-\mu)\log\left(\frac{1}{1-\mu}\right) + (1-\mu)\log C_I + \mu\varepsilon_M \log A_I \\
&+ \frac{\mu}{N}\sum_{J=1}^{N}\log Q_J + \frac{\mu^2\varepsilon_M}{N(1-\mu)}\sum_{J=1}^{N}\log A_J \\
&+ \left(\frac{\mu}{N}\sum_{J=1}^{N}\left(\varepsilon_Q \log P_J + (\varepsilon_Q - 1)\log A_J + (\varepsilon_M - \varepsilon_Q)\log P_J^{in}\right)\right).
\end{aligned}
$$

There is a second error in the derivation of equation (12) within Appendix F.6 of Atalay (2017): The term $(1-\mu)\log(1/(1-\mu))$ in the line above is mistakenly written as $\log(1/(1-\mu))$ in the original paper. This end result of this error is that the $\left(1/(1-\mu)\right)\log(1/(1-\mu))$ term appearing in equation (12) is replaced by a $\log(1/(1-\mu))$ term in equation (12′).

We substitute approximations for $\log P_J$ and $\log P_J^{in}$ into equation (C.2):

$$
\log P_J \approx -\log A_J - \frac{\mu}{N(1-\mu)}\sum_{I=1}^{N}\log A_I \quad\text{and}\quad \log P_J^{in} \approx -\frac{1}{N(1-\mu)}\sum_{I=1}^{N}\log A_I,
$$

$$
\begin{aligned}
\log Q_I \approx{}& (1-\mu)\log\left(\frac{1}{1-\mu}\right) + (1-\mu)\log C_I + \mu\varepsilon_M \log A_I \\
&+ \frac{\mu}{N}\sum_{J=1}^{N}\log Q_J + \frac{\mu(\varepsilon_Q - 1 - \varepsilon_M)}{N}\sum_{J=1}^{N}\log A_J.
\end{aligned}
$$

We then substitute an approximation for $\log C_I$ into the previous equation:

$$\log C_I \approx \log\left(\frac{1}{N}\right) + \varepsilon_D \log A_I + \frac{1-(1-\mu)\,\varepsilon_D}{N(1-\mu)}\sum_{J=1}^{N}\log A_J,$$

$$\begin{aligned}\log Q_I \approx\ & (1-\mu)\log\left(\frac{1}{1-\mu}\right) + (1-\mu)\log\left(\frac{1}{N}\right)\\ & + \left[\mu\,\varepsilon_M + (1-\mu)\varepsilon_D\right]\log A_I + \frac{\mu}{N}\sum_{J=1}^{N}\log Q_J\\ & + \frac{1+\mu(\varepsilon_Q-1)-\left(\mu\varepsilon_M+(1-\mu)\,\varepsilon_D\right)}{N}\sum_{J=1}^{N}\log A_J.\end{aligned}$$

We write the previous equation in a more compact form:

$$\begin{aligned}&\begin{bmatrix} 1-\frac{\mu}{N} & -\frac{\mu}{N} & \cdots & -\frac{\mu}{N}\\ \vdots & & \ddots & \vdots \\ -\frac{\mu}{N} & -\frac{\mu}{N} & \cdots & 1-\frac{\mu}{N}\end{bmatrix}\begin{bmatrix}\log \mathrm{Q}_1\\ \log \mathrm{Q}_2\\ \vdots\\ \log \mathrm{Q}_N\end{bmatrix}\\ &\approx \left\{(1-\mu)\log\left(\frac{1}{1-\mu}\right) + (1-\mu)\log\left(\frac{1}{N}\right)\right.\\ &\qquad \left. + \frac{1+\mu(\varepsilon_Q-1)-\left(\mu\varepsilon_M+(1-\mu)\,\varepsilon_D\right)}{N}\sum_{J=1}^{N}\log A_J\right\}\begin{bmatrix}1\\1\\ \vdots\\1\end{bmatrix}\\ &\quad + \left[\mu\,\varepsilon_M + (1-\mu)\,\varepsilon_D\right]\begin{bmatrix}\log A_1\\ \log A_2\\ \vdots\\ \log A_N\end{bmatrix}.\end{aligned}$$

Inverting this matrix equation, with the time subscripts added back, yields:

$$\begin{aligned}\log Q_{tI} \approx\ & \log\left(\frac{1}{1-\mu}\right) + \log\left(\frac{1}{N}\right) + \left[\mu\varepsilon_M + (1-\mu)\varepsilon_D\right]\log A_{tI}\\ & + \frac{1}{N}\left[\frac{1+\mu(\varepsilon_Q-1)}{1-\mu} - \left(\mu\varepsilon_M + (1-\mu)\varepsilon_D\right)\right]\sum_{J=1}^{N}\log A_{tJ}.\end{aligned}$$

This is the corrected version of equation (12), as it appears in Section I of this corrigendum.

REFERENCE

Atalay, **Enghin.** (2017). "How Important Are Sectoral Shocks?" *American Economic Journal: Macroeconomics*, 9(4), 254–80.