Online appendix: Platform governance

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This Online Appendix contains omitted details from the main paper and the details of extensions in Section 6.

A Discrete governance

In this section, we consider the case in which the design choice a is not a continuous variable. In what follows, we replicate the results in Sections 3 - 4. Let $\Theta \subseteq \mathbb{R}^n$ be a finite subset of n-dimensional real vector space. Each design choice is denoted as a vector $a \in \Theta$ and corresponds to a given level of gross transaction value $V(a)$ and markup $M(a)$.

Given that the choice set is finite, we can equivalently reformulate the design problem as directly choosing a pair of markup level and transaction value $(M(a), V(a))$, or $(m, v(m))$, as in the analysis in the main text. In cases in which a given $M(a)$ corresponds to multiple possible levels of $V(a)$, we can select the highest $V(a)$ among them without loss of generality.

Given the reformulation, we note that the proofs of Proposition 1, 3, 5, and 6 in the main text do not rely on a being a continuous variable, except when establishing the strict inequalities (in which we have explicitly used the first-order conditions). Therefore, these results carry over immediately with weak inequalities. As for Propositions 2 and 4, the following propositions deliver similar insights:

Proposition A.1 *(Exogenous proportional fee) Suppose the platform charges an exogenous proportional fee r. There exist thresholds $\bar{c}^l$ and $\bar{c}^h$, where $0 < \bar{c}^l \leq \bar{c}^h$, such that:*

- *If $c < \bar{c}^l$, then $m^p \geq m^w$.*
- *If $c > \bar{c}^h$, then $m^p \leq m^w$.*

Proof. From

$$\begin{aligned}
\Pi(m) &= r\left(\frac{c}{1-r} + m\right) Q\left(v(m) - m - \frac{c}{1-r}\right) \\
W(m) &= \left(\frac{c}{1-r} + m - c\right) Q\left(v(m) - m - \frac{c}{1-r}\right) + \int_{-\infty}^{v(m)-m-\frac{c}{1-r}} Q(t)\, dt,
\end{aligned}$$

we make the following two observations: (i) $v(m) - m < v(m^p) - m^p$ for all $m > m^p$; and (ii) $v(m) - m < v(m^w) - m^w$ for all $m > m^w$. Otherwise, m^p and m^w cannot be maximizers. Consider the following function:

$$\psi(x) \equiv -cQ(x) + \int_{-\infty}^{x} Q(t)\, dt, \tag{A.1}$$

the derivative of which is $\frac{\partial \psi}{\partial x} = \left(\frac{Q(x)}{Q'(x)} - c\right) Q'(x)$. Rewrite the welfare function as

$$W(m) = \frac{1}{r}\Pi(m) + \psi\left(v(m) - m - \frac{c}{1-r}\right).$$

Let $m^h \equiv \arg\max_m \{v(m) - m\}$ and $m^l \equiv \arg\min_m \{v(m) - m\}$, both of which are well defined by the compactness of the domain and the continuity of $v(m) - m$. Let $\bar{c}^i$, $i \in \{l, h\}$ be the solution to

$$\frac{Q(v(m^i) - m^i - \frac{\bar{c}^i}{1-r})}{Q'(v(m^i) - m^i - \frac{\bar{c}^i}{1-r})} = \bar{c}^i.$$

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The existence and uniqueness of $\bar{c}^i$ follows from the intermediate value theorem and log-concavity of Q.

Suppose $c < \bar{c}^l$. This implies $\frac{\partial \psi}{\partial x} > 0$ for all $x \geq v(m^l) - m^l - \frac{c}{1-r}$, i.e., $\psi\left(v(m) - m - \frac{c}{1-r}\right)$ is increasing for all possible m. Given that we know $v(m) - m < v(m^p) - m^p$ for all $m > m^p$, it follows that

$$W(m) < \frac{1}{r}\Pi(m) + \psi\left(v(m^p) - m^p - \frac{c}{1-r}\right) \leq W(m^p)$$

for all $m > m^p$, implying $m^w \leq m^p$.

Suppose $c > \bar{c}^h$. This implies $\frac{\partial \psi}{\partial x} < 0$ for all $x \leq v(m^h) - m^h - \frac{c}{1-r}$, i.e. $\psi\left(v(m) - m - \frac{c}{1-r}\right)$ is decreasing for all possible m. Given that we know $v(m) - m < v(m^w) - m^w$ for all $m > m^w$, it follows that

$$\begin{aligned}
\Pi(m) &= rW(m) - r\psi\left(v(m) - m - \frac{c}{1-r}\right) \\
&< rW(m^w) - r\psi\left(v(m^w) - m^w - \frac{c}{1-r}\right) = \Pi(m^w),
\end{aligned}$$

implying $m^p \leq m^w$. ■

Proposition A.2 *(Endogenous proportional fee) Suppose the social planner can control the platform's governance design, but cannot control the proportional fee set by the platform. There exist thresholds $\tilde{c}^l$ and $\tilde{c}^h$, where $0 < \tilde{c}^l \leq \tilde{c}^h$, such that:*

- *If $c < \tilde{c}^l$, then $m^p \geq m^{sb}$.*
- *If $c > \tilde{c}^h$, then $m^p \leq m^{sb}$.*

Proof. We first make the following two observations regarding m^{sb}:

Claim 1: For all $m > m^{sb}$, we have $\tilde{r}(m) < \tilde{r}\left(m^{sb}\right)$. By contradiction, suppose there is some $m' > m^{sb}$ such that $\tilde{r}(m') \geq \tilde{r}\left(m^{sb}\right)$. Using these two inequalities, (12) implies $v(m') - p_{\tilde{r}}(m') \geq v\left(m^{sb}\right) - p_{\tilde{r}}(m^{sb})$. From the welfare function, this implies

$$\begin{aligned}
\tilde{W}(m') &\geq \left(\frac{c}{1-\tilde{r}(m')} + m' - c\right) Q\left(v\left(m^{sb}\right) - p_{\tilde{r}}(m^{sb})\right) + \int_{-\infty}^{v\left(m^{sb}\right)-p_{\tilde{r}}(m^{sb})} Q(t)\, dt \\
&> \tilde{W}(m^{sb}),
\end{aligned}$$

a contradiction, which proves the claim.

Claim 2: For all $m > m^{sb}$, we have $v(m) - p_{\tilde{r}}(m) < v\left(m^{sb}\right) - p_{\tilde{r}}(m^{sb})$. By contradiction, suppose there is some $m' > m^{sb}$ such that $v(m') - p_{\tilde{r}}(m') \geq v\left(m^{sb}\right) - p_{\tilde{r}}(m^{sb})$. Then, (15) implies $p_{\tilde{r}}(m') > p_{\tilde{r}}(m^{sb})$. From the welfare function, this implies $W(m') > W(m^{sb})$, a contradiction, which proves the claim.

To prove the proposition suppose $c > \tilde{c}^h$, where

$$\tilde{c}^h \equiv \frac{Q(v(m^h) - m^h)}{Q'(v(m^h) - m^h)} \tag{A.2}$$

and $m^h \equiv \arg\max_m \{v(m) - m\}$. In this case, $\psi(x)$, as defined in (A.1), is decreasing for all $x \leq$

$v(m^h) - m^h$. For all $m > m^{sb}$, we have

$$\begin{aligned}
\tilde{\Pi}(m) &= \tilde{r}(m)\tilde{W}(m) - \tilde{r}(m)\psi(v(m) - p_{\tilde{r}}(m)) \\
&< \tilde{r}\left(m^{sb}\right)\tilde{W}(m) - \tilde{r}\left(m^{sb}\right)\psi(v(m) - p_{\tilde{r}}(m)) \\
&< \tilde{r}\left(m^{sb}\right)\tilde{W}(m) - \tilde{r}\left(m^{sb}\right)\psi\left(v\left(m^{sb}\right) - p_{\tilde{r}}(m^{sb})\right) \\
&< \tilde{r}\left(m^{sb}\right)\tilde{W}(m) - \tilde{r}\left(m^{sb}\right)\psi\left(v(m^h) - m^h\right) \\
&= \tilde{\Pi}(m^w).
\end{aligned}$$

where the first inequality is due to Claim 1, the second inequality is due to Claim 2, and the last inequality is due to $v\left(m^{sb}\right) - p_{\tilde{r}}(m^{sb}) = v\left(m^{sb}\right) - m^{sb} - \frac{c}{1-r} \leq v(m^h) - m^h$ (by Claim 2 and the definition of m^h). We conclude that $m^p \leq m^{sb}$.

To establish the existence of the lower threshold $\tilde{c}^l$, it suffices to consider the case of $c \to 0$. When $c \to 0$, we first note that $\psi(x)$, as defined in (A.1), is increasing for all x. Moreover, $c \to 0$ implies (12) $\tilde{r}(m) \to 1$ and $\tilde{\Pi}(m) \to mQ(v(m) - m)$, which implies that $v(m) - m < v(m^p) - m^p$ for all $m > m^p$ (otherwise m^p is not a maximizer). So,

$$\begin{aligned}
\tilde{W}(m) &\to \tilde{\Pi}(m) + \psi\left(v(m) - m - \frac{c}{1-r}\right) \\
&\leq \tilde{\Pi}(m) + \psi\left(v(m^p) - m^p - \frac{c}{1-r}\right) \leq W(m^p)
\end{aligned}$$

for all $m > m^p$, implying $m^{sb} \leq m^p$. ■

B Derivations of Examples 1-3

This section provides the derivations of the examples in Section 2. In what follows, we do not specify the exact fee instrument used by the platform. Instead, we focus on deriving how the platform's governance design influences the buyer-seller interactions in each of the examples.

B.1 Example 1: Variety choice by the platform

Example 1 can be summarized with the following timing: (i) The platform announces the number of sellers $N - a$ that it will admit; (ii) Sellers and buyers decide whether to enter the platform; (iii) Sellers set their prices; (iii) $N - a$ sellers are admitted, and the buyer observes the prices and match values of these sellers and purchases accordingly. We focus on symmetric pure strategy Nash equilibrium in which all sellers set the same price p (for any $N - a$ chosen by the platform).

Let Q be the number of participating buyers, which is exogenous from individual sellers' point of view. To derive seller pricing, consider a deviating seller i who sets price $p_i \neq p$. For each seller, the effective demand is

$$\begin{aligned}
&\Pr\left(x_i - p_i \geq \max_{j \neq i}\{x_j - p\}\right) \\
= &\int_{-\infty}^{\infty} (1 - F(x - p + p_i))\, dF(x)^{N-a-1}.
\end{aligned}$$

The demand for seller i's product is

$$Q_i(p_i) = Q \times \int_{-\infty}^{\infty} (1 - F(x - p + p_i))\, dF(x)^{N-a-1}.$$

From the profit function $(p_i - c) Q_i (p_i)$, we can derive the symmetric equilibrium price as

$$p = c + M(a) \equiv c + \frac{1}{(N-a)\int_{-\infty}^{\infty} f(x)\, dF^{N-a-1}(x)}.$$

B.2 Example 2: Information design by the platform

Example 2 can be summarized with the following timing: (i) The platform announces the information structure parameterized by $a \in [0,1]$; (ii) Sellers and buyers decide whether to enter the platform; (iii) Sellers set their prices; (iii) Each buyer observes the realized signal and prices and purchases accordingly.

Let $\bar{s}$ be the signal received by the buyer who is indifferent between seller 1 and 2, i.e., $p_1 + t\mathbf{E}(x_1|\bar{s}) = p_2 + t\mathbf{E}(x_2|\bar{s})$, or $\bar{s} = \frac{1}{2} + \frac{p_2 - p_1}{ta}$. Given s is drawn from uniform distribution over $[0,1]$, seller 1's demand is $\Pr(s < \bar{s}) = \frac{1}{2} + \frac{p_2 - p_1}{ta}$. Therefore, the price competition is the same as Example 1, except that the transportation cost parameter is replaced by ta. It follows that $p = c + ta$. To ensure that the market is fully covered, we assume $V_0 - t\mathbf{E}(x_1|s = 1/2) - p > 0$ for all a, or $V_0 > c + \frac{3t}{2}$.

From a buyer's ex-ante perspective, with probability a the signal is informative and the buyer gets the preferred product, with expected mismatch cost $t/4$; With probability $1-a$ the signal is uninformative so that the buyer effectively gets a randomly chosen product, with expected mismatch cost $t/2$; Hence, the ex-ante expected mismatch cost is $\frac{t(2-a)}{4}$, as stated in the main text.

B.3 Example 3: Quality control by the platform

Example 3 can be summarized with the following timing: (i) The platform announces a; (ii) Buyers and sellers decide whether to enter the platform; (iii) Sellers with $q_i \geq 1 - a$ set their prices, and buyers who have has entered the platform carry out sequential search. We focus on symmetric perfect Bayesian equilibria (PBE) in which all sellers set the same price p. As is standard in the search literature, buyers keep the same (passive) beliefs about the distribution of future prices on and off the equilibrium path.

The following derivation follows from Eliaz and Spiegler (2011). We first derive buyers' search strategy for each given a set by the platform. Define the reservation value $V(a)$ as the solution to

$$\mathbf{E}(q_i | q_i \geq 1 - a) \int_V^{\bar{\epsilon}} (\epsilon - V)\, dF(\epsilon) = s. \tag{B.1}$$

The left-hand side of (B.1) represents the incremental expected benefit from one more search, while the right-hand side represents the incremental search cost. There is, at most, one solution to (B.1), since the left-hand side is strictly decreasing in v.

It is well known in the consumer search literature (Wolinsky, 1986; Anderson and Renault, 1999) that buyers' optimal search rule in this environment is stationary and described by the standard cutoff rule. When searching, each buyer employs the following strategy: (i) she stops and buys form seller i if the product is not defective and $\epsilon_i - p_i \geq V(a) - p$; and (ii) she continues to search the next seller otherwise. Following the standard result, the buyer's expected surplus from initiating a search is $V(a) - p$. Then, a buyer with intrinsic participation cost d enters the platform if and only if $d < p - V(a)$. Provided that search cost is not too large, there is a symmetric price equilibrium in which a strictly positive measure of buyers join the platform.

Compared with a standard search model, notice from (3) that a search pool with a higher expected quality $\mathbf{E}(q_i | q_i \geq 1 - a)$ is analogous to a lower effective search cost for buyers. This reflects that each buyer searches less and consequently incurs a lower total expected search cost of $s/\mathbf{E}(q_i | q_i \geq 1 - a)$ before reaching a non-defective match. Given that $\mathbf{E}(q_i | q_i > 1 - a)$ decreases with a, a more relaxed quality standard set by the platform is analogous to increasing the effective search cost of buyers. Thus, it follows that $V(a)$ is a decreasing function of a.

From the buyer search rule above, the derivation of demand is straightforward. The mass of buyers initiating search is $Q(V(a)-p)$, which is exogenous from each firm's point of view. Conditional on these buyers, the demand of a deviating firm i with type q_i follows the standard search model and is given by

$$q_i\left(1-F\left(V\left(a\right)-p+p_i\right)\right)\sum_{z=0}^{\infty}F\left(V\left(a\right)\right)^z=\left(\frac{1-F\left(V\left(a\right)-p+p_i\right)}{1-F\left(V\left(a\right)\right)}\right)q_i.$$

The log-concavity assumption on $1-F$ ensures that the usual first-order condition determines a unique optimal price. The symmetric equilibrium price is given by $p=c+\frac{1-F(V(a))}{f(V(a))}$.

C Endogeneity of fee instrument

We prove Proposition 7 stated in the main text. Recall that the platform's optimal fee instrument is a two-part tariff with a subsidy on buyer participation. It optimally sets its governance design at $m^{p+}=m^*$ to maximize $v\left(m\right)$ and then adjusts the transaction-based fee components and the buyer-side participation subsidies accordingly to achieve the maximal monopoly profit, which we denote as $\Pi^*\equiv\left(p^*\left(m^*\right)-c\right)Q\left(v\left(m^*\right)-p^*\left(m^*\right)\right)$.

Meanwhile, the social planner optimally restricts the platform to charge sellers a pure lump-sum participation fee. Notice that allowing the platform to impose participation fees on both sellers and buyers is ineffective, because the platform would set a positive buyer participation fee in an attempt to replicate Π^*, which results in deadweight losses. The welfare function under the restriction of seller participation fees is

$$\tilde{W}\left(m\right)=mQ\left(v\left(m\right)-m-c\right)+\int_{-\infty}^{v(m)-m-c}Q\left(t\right)dt.$$

For all $m>m^*$, we have

$$\begin{aligned}\tilde{W}\left(m\right)&\leq mQ\left(v\left(m^*\right)-m-c\right)+\int_{-\infty}^{v(m^*)-m-c}Q\left(t\right)dt\\&< m^*Q\left(v\left(m^*\right)-m^*-c\right)+\int_{-\infty}^{v(m^*)-m^*-c}Q\left(t\right)dt\\&=\tilde{W}\left(m^*\right),\end{aligned}$$

which implies $m^{sb+}\leq m^*=m^{p+}$.

D Costly governance

The analysis in this section corresponds to Section 6.2 of the main text. We focus on extending the results in Section 4 to the case in which the platform's fixed cost is an increasing and convex function $K=K(v(m))\geq 0$. Denote $k\geq 0$ as the derivative of K with respect to its argument, where k is increasing by the convexity assumption.

Proposition D.1 *(Endogenous per-transaction fees) If value and markup are always positively correlated, then Proposition 3 continues to hold.*

Proof. If we rewrite the profit function in terms of price, we have

$$\tilde{\Pi}\left(m\right)=\left(p_{\tilde{\tau}}(m)-c-m\right)Q\left(v\left(m\right)-p_{\tilde{\tau}}(m)\right)-K(v(m)),$$

where $p_{\tilde{\tau}}(m)$ is implicitly defined by $p_{\tilde{\tau}}(m) = c + m + \frac{Q(v(m)-m-c-\tilde{\tau})}{Q'(v(m)-m-c-\tilde{\tau})}$. Recall that the log-concavity of Q implies that $v(m) - p_{\tilde{\tau}}(m)$ is increasing in m if and only if $v(m) - m$ is increasing in m.

We first claim that $v(m) - p_{\tilde{\tau}}(m) \leq v(m^p) - p_{\tilde{\tau}}(m^p)$ for all $m < m^p$. By contradiction, suppose there is some $m' < m^p$ such that $v(m') - p_{\tilde{\tau}}(m') > v(m^p) - p_{\tilde{\tau}}(m^p)$; then the definition of $p_{\tilde{\tau}}$ implies $p_{\tilde{\tau}}(m') - m' > p_{\tilde{\tau}}(m^p) - m^p$. Given that $v(m)$ is increasing, $K(v(m')) \leq K(v(m^p))$, and so

$$\begin{aligned}\tilde{\Pi}(m') &> (p_{\tilde{\tau}}(m^p) - c - m^p)Q(v(m^p) - p_{\tilde{\tau}}(m^p)) - K(v(m')) \\ &\geq \tilde{\Pi}(m^p),\end{aligned}$$

which contradicts the definition of m^p being a maximizer. Hence, the claim is proven.

From the welfare function,

$$\tilde{W}(m) = \tilde{\Pi}(m) + mQ(v(m) - p_{\tilde{\tau}}(m)) + \int_{-\infty}^{v(m)-p_{\tilde{\tau}}(m)} Q(t)\,dt.$$

the proven claim implies $\tilde{W}(m) < \tilde{W}(m^p)$ for all $m < m^p$, implying $m^{sb} \geq m^p$. The final step in ruling out the equality is the same as the corresponding step in the proof of Proposition 3, and hence omitted here. ■

Proposition D.2 *(Endogenous proportional fee) Suppose the social planner can control the platform's governance design, but cannot control the proportional fee set by the platform. Suppose welfare function (D.1) is unimodal, and denote*

$$\tilde{\Psi}(m) \equiv \frac{\left(\frac{p_{\tilde{r}}}{\tilde{r}} + \frac{K}{\tilde{r}^2}\right)\frac{d\tilde{r}}{dm}}{\frac{dp_{\tilde{r}}}{dm} - \frac{dv}{dm}}.$$

- *Suppose $c \leq \frac{Q(v(m^p)-p_{\tilde{r}}(m^p))}{Q'(v(m^p)-p_{\tilde{r}}(m^p))}(1 + \tilde{\Psi}(m^p))$. If value and markup are always negatively correlated, then $m^p \geq m^{sb}$.*
- *Suppose $c \geq \frac{Q(v(m^p)-p_{\tilde{r}}(m^p))}{Q'(v(m^p)-p_{\tilde{r}}(m^p))}(1 + \tilde{\Psi}(m^p))$. If value and markup are always positively correlated, then $m^p \leq m^{sb}$.*

Proof. Recall

$$\tilde{\Pi}(m) = \tilde{r}(m)\,p_{\tilde{r}}(m)Q(v(m) - p_{\tilde{r}}(m)) - K(v(m)),$$

where $p_{\tilde{r}}(m)$ is defined in (15). The welfare function is

$$\begin{aligned}\tilde{W}(m) &= (p_{\tilde{r}}(m) - c)Q(v(m) - p_{\tilde{r}}(m)) - K(v(m)) + \int_{-\infty}^{v(m)-p_{\tilde{r}}(m)} Q(t)dt \qquad \text{(D.1)} \\ &= \frac{\tilde{\Pi}(m) + K(v(m))}{\tilde{r}(m)} - K(v(m)) - cQ(v(m) - p_{\tilde{r}}(m)) + \int_{-\infty}^{v(m)-p_{\tilde{r}}(m)} Q(t)dt\end{aligned}$$

and

$$\frac{d\tilde{W}}{dm} = \frac{1}{\tilde{r}}\frac{d\tilde{\Pi}}{dm} + \left(\frac{1-\tilde{r}}{\tilde{r}}\right)k\frac{dv}{dm} + \left(\frac{Q}{Q'}(1 + \tilde{\Psi}(m^p)) - c\right)\left(\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm}\right)Q'.$$

Suppose $c \leq \frac{Q(v(m^p)-p_r(m^p))}{Q'(v(m^p)-p_r(m^p))}(1 + \tilde{\Psi}(m^p))$ and $\frac{dv}{dm} \leq 0$ for all m. Lemma 4 implies $\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm} \leq 0$. Hence, $\frac{d\tilde{W}}{dm}|_{m=m^p} \leq 0$ whenever $\frac{d\tilde{\Pi}}{dm}|_{m=m^p} = 0$ and $\frac{d\tilde{W}}{dm}|_{m=m^p} < 0$ whenever $\frac{d\tilde{\Pi}}{dm}|_{m=m^p} < 0$. The unimodality of $\tilde{W}(m)$ implies $m^{sb} \leq m^p$ as required.

Suppose $c \geq \frac{Q(v(m^p)-p_r(m^p))}{Q'(v(m^p)-p_r(m^p))}(1 + \tilde{\Psi}(m^p))$ and $\frac{dv}{dm} \geq 0$ for all m. If $(\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm})|_{m=m^p} \leq 0$, then we have $\frac{d\tilde{W}}{dm}|_{m=m^p} \geq 0$ whenever $\frac{d\tilde{\Pi}}{dm}|_{m=m^p} = 0$, and $\frac{d\tilde{W}}{dm}|_{m=m^p} > 0$ whenever $\frac{d\tilde{\Pi}}{dm}|_{m=m^p} > 0$. These imply

$m^{sb} \geq m^p$. Suppose instead that $(\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm})|_{m=m^p} > 0$; then we get from (D.1):

$$\frac{d\tilde{W}}{dm}|_{m=m^p} = (p_{\tilde{r}}(m) - c)Q' \left(\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm}\right)|_{m=m^p} + (Q-k)\frac{dv}{dm}|_{m=m^p} > 0,$$

and the unimodality of $\tilde{W}(m)$ implies $m^{sb} \geq m^p$. ■

Proposition D.3 *(lump-sum fees) If value and markup are always negatively correlated, then Proposition 5 continues to hold.*

Proof. The platform's profit is $\tilde{\Pi}(m) = mQ(v(m) - m - c) - K(v(m))$, while the welfare function is

$$\tilde{W}(m) = \tilde{\Pi}(m) + \int_{-\infty}^{v(m)-m-c} Q(t)dt.$$

We first claim that $v(m) - m \leq v(m^p) - m^p$ for all $m > m^p$. By contradiction, suppose there is some $m' > m^p$ such that $v(m') - m' > v(m^p) - m^p$. Given that $v(m)$ is decreasing, $K(v(m')) \leq K(v(m^p))$, and so $\tilde{\Pi}(m') > m^p Q(v(m^p) - m^p) - K(v(m')) \geq \tilde{\Pi}(m^p)$, which contradicts the definition of m^p being a maximizer thus proving the claim.

From the welfare function, the proven claim implies that $\tilde{W}(m) < \tilde{W}(m^p)$ for all $m > m^p$, and so we have $m^{sb} \leq m^p$, as required. The final step in ruling out the equality is the same as the corresponding step in the proof of Proposition 5, and hence omitted here. ■

Proposition D.4 *(Two-part tariff) If value and markup are always negatively correlated, then Proposition 6 continues to hold.*

Proof. Recall from the proof of Proposition 6 that we denote the profit-maximizing induced price as $\tilde{p}(m) = \max\{m + c, p^*(m)\}$ (the existence of the fixed cost does not affect the optimal pricing). Denote

$$\begin{aligned} m_k^* &\equiv \arg\max_m \tilde{\Pi}(m) \\ &= \arg\max_m \{(p^*(m) - c)Q(v(m) - p^*(m)) - K(v(m))\}. \end{aligned}$$

Similar to the proof of Proposition 6, we denote $\hat{m}$ as the solution to $\hat{m} + c = p^*(\hat{m})$. By construction, $\tilde{p}(m) = p^*(m)$ for all $m > \hat{m}$ given that $v(m)$ is decreasing for all m.

Case 1. If $c \leq p^*(m_k^*) - m_k^*$, then $p^*(m_k^*)$ is implementable. We know by the envelope theorem that $\tilde{\Pi}(m)$ is maximized at $m^p = m_k^*$. For all $m > m^p$, we know $\tilde{p}(m) = p^*(m)$ because $m^p = m_k^* > \hat{m}$. We first claim that $v(m) - p^*(m) \leq v(m^p) - p^*(m^p)$ for all $m > m^p$. By contradiction, suppose there is some $m' > m^p$ such that $v(m') - p^*(m') > v(m^p) - p^*(m^p)$; then the definition of p^* implies $p^*(m') > p^*(m^p)$. Therefore,

$$\begin{aligned} \tilde{\Pi}(m^p) &< (p^*(m') - c)Q(v(m') - p^*(m')) - K(v(m^p)) \\ &\leq \tilde{\Pi}(m'), \end{aligned}$$

where the second inequality is due to $K(v(m')) \leq K(v(m^p))$ given that $v(m)$ is decreasing. This contradicts the definition of m^p being a maximizer. Hence, the claim is proven. Then, from the welfare function, the proven claim implies $\tilde{W}(m) < \tilde{W}(m^p)$ for all $m > m^p$, and so we have $m^{sb} \leq m^p$.

Case 2. Suppose $c > p^*(m_k^*) - m_k^*$, then from the preceeding case we know that $p^*(m_k^*)$ is no longer implementable. Similar to the proof of Proposition 6, we know $m^p < \hat{m}$ and $\tilde{p}(m^p) = c + m^p$. We first claim that $v(m) - \tilde{p}(m) \leq v(m^p) - m^p - c$ for all $m > m^p$. By contradiction, suppose there is some

$m' > m^p$ such that $v(m') - \tilde{p}(m') > v(m^p) - m^p - c$. The definition of $\tilde{p}$ implies $\tilde{p}(m') \geq m' + c > m^p + c$. So,

$$\tilde{\Pi}(m') > (m^p - c)Q(v(m^p) - m^p) - K(v(m')) \geq \tilde{\Pi}(m^p),$$

which contradicts the definition of m^p being a maximizer. Hence, the claim is proven. Then, from the welfare function, the proven claim implies that $\tilde{W}(m) < \tilde{W}(m^p)$ for all $m > m^p$, and so we have $m^{sb} \leq m^p$. The final step in ruling out the equality is the same as the corresponding step in the proof of Proposition 6. ■

E Consumer surplus benchmark

For all given fee instruments, we know that $m^{bs} = \arg\max\{v(m) - p(m)\}$. Following the previous analysis, when the platform uses per-transaction fees (regardless of whether it is exogenous or endogenous), $m^p = \arg\max\{v(m) - p(m)\}$, so $m^p = m^{bs}$. Likewise, if the platform uses lump-sum fees, two-part tariffs, or exogenous proportional fees, from the expressions of the platform's profit function it is obvious that $m^p \geq m^{bs}$ because a higher m increases the platform's margin.

The only non-obvious case is when the platform uses endogenous proportional fees. Let $m^{bs} = \arg\max\{v(m) - p_{\tilde{r}}(m)\}$. If $m^p = \bar{m}$, then $m^p \geq m^{bs}$ trivially holds. If $m^p < \bar{m}$, then either it is an interior solution or $m^p = \underline{m}$. In both cases, from (8) we have $\frac{dv}{dm}|_{m=m^p} \leq 1 - \frac{Q}{p_{\tilde{r}}Q'}|_{m=m^p}$. Recall from Lemma 4 that $\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm} < 0$ whenever $\frac{dv}{dm} \leq 1 - \frac{Q}{p_{\tilde{r}}Q'}$. Hence, we have $\left(\frac{dv}{dm} - \frac{dp_{\tilde{r}}}{dm}\right)|_{m=m^p} < 0$, implying $m^p \geq m^{bs}$.