Effective Demand Failures and the Limits of Monetary Stabilization Policy

Michael Woodford*

October 25, 2021

Abstract

The challenge for stabilization policy presented by the COVID-19 pandemic stems above all from disruption of the circular flow of payments, resulting in a failure of what Keynes (1936) calls "effective demand." As a consequence, economic activity in many sectors can be inefficiently low, and interest-rate policy cannot eliminate the distortions — not because of a limit on the extent to which interest rates can be reduced, but because interest-rate reductions fail to stimulate demand of the right sorts. Fiscal transfers are instead well-suited to addressing the fundamental problem, and can under certain circumstances achieve a first-best allocation of resources.

The COVID-19 pandemic has presented substantial challenges both to policymakers and to macroeconomists, and these go beyond the simple fact that the disturbance to economic life has been unprecedented in both its severity and its suddenness. The nature of the disturbance has also been different from those typically considered in discussions of business cycles and stabilization policy, and this has raised important questions about how to think about an appropriate policy response.

Among the more notable features of the economic crisis resulting from the pandemic has been the degree to which its effects have been concentrated in particular sectors of the economy, with some activities having to shut down completely for the sake of public health, while others continue almost as normal. A consequence of this asymmetry is a significant disruption of the "circular flow" of payments between sectors of the economy. In a stationary

*Department of Economics, Columbia University. Email: michael.woodford@columbia.edu. An earlier version of this paper was presented at the 2020 NBER Summer Institute under the title "Pandemic Shocks, Effective Demand, and Stabilization Policy." I would like to thank Aloísio Araújo, Gauti Eggertsson, Guido Lorenzoni, Argia Sbordone, Ludwig Straub, Harald Uhlig, and Iván Werning for helpful comments, and Yeji Sung for research assistance.

equilibrium of the kind to which an economy tends in the absence of shocks, each economic unit's payment outflows are balanced by its inflows, over any interval of time; this makes it possible for the necessary outflows to be financed at all times, without requiring the household or firm to maintain any large liquid asset balances.

Economic disturbances, regardless of whether these are "supply shocks" or "demand shocks," do not change this picture, as long as they affect all sectors of the economy in the same way: whether activity of all types is temporarily higher or lower, as long as the co-movement of the different sectors is sufficiently close, it continues to be the case that inflows and outflows should balance, so that financing constraints do not bind, even when many individual units maintain low liquid asset balances. Under such circumstances, the market mechanism should do a good job of ensuring an efficient allocation of resources. It is only necessary for policy to ensure that intertemporal relative prices (i.e., real interest rates) incentivize economic units to allocate expenditure over time in a way that is in line with variations in the efficient level of aggregate activity; in an economy where the prices of goods and services are fixed in advance in monetary units, this requires the central bank to manage the short-term nominal interest rate in an appropriate way. But it is often supposed that a reasonably efficient allocation of resources can be assured as long as interest-rate policy is adjusted in response to aggregate disturbances in a suitable way.

A disturbance like the COVID-19 creates difficulties of a different kind. The efficient level of some activities is now different, once public health concerns are taken into account. But in addition, the cessation of payments for the activities that are no longer safe interrupts the flow of payments that would ordinarily be used to finance other activities, even though these latter activities are still socially desirable (if one compares the utility that consumers can get from them to the disutility required to supply them). As a result, many activities may take place at a lower than efficient level, owing to insufficiency of what Keynes (1936) calls "effective demand" — the ability of people to signal in the marketplace the usefulness of goods to them, through their ability to pay for them. While it may well be efficient for restaurants or theaters to suspend the supply of their services for a period (because their usual customers cannot safely consume these services while the disease is rampant),the loss of their normal source of revenue may leave them unable to pay their rent; the loss of rental income may then require the real-estate management companies to dismiss their maintenance staff and fail to pay their property taxes; the furloughed maintenance staff may be unable to buy food or pay their own rent, the municipal government that does not receive a normal level of tax revenue may have to lay off city employees, and so on.[1] The later steps in this chain of effects are all suspensions of economic transactions that are in no way required by the inability to continue supplying in-restaurant meals and theater performances.

An effective demand failure of this kind can result in a reduction in economic activity that is much greater than would occur in an efficient allocation of resources, even taking into account the public health constraint. Yet the problem is not simply that aggregate demand is too low, at existing (predetermined) prices, relative to the economy's aggregate productive capacity; in such a case one would expect the problem to be cured by a monetary policy that sufficiently reduces the real rate of interest. But as Leijonhufvud (1973) stresses,

[1] See, for example, Goodman and Magder (2020) and Gopal (2020) on the problems created by effective demand failures of this kind in New York City during the current crisis.

in a situation of sufficiently generalized effective demand failure, arising because financing constraints have temporarily become binding for a large number of economic units, the usual mechanisms of price adjustment in a market economy do not suffice to achieve an efficient allocation. The market-determined real rate of interest in a flexible-price economy will not achieve this; and neither, in the more realistic case of an economy with nominal rigidities, will a central bank that adjusts its policy rate to bring about the real rate of interest that would be associated with a flexible-price equilibrium, be able to do so.

Here we present a simple (and highly stylized) model to illustrate the nature of the problem presented by a disturbance like the COVID-19 pandemic. In our model, the fact that economic activity is much lower than in an optimal allocation of resources, in the absence of any policy response, does not necessarily imply that interest rates need to be reduced. While the model is one in which (owing to nominal rigidities) a reduction of the central bank's policy rate increases economic activity, the particular ways in which it increases activity need not correspond at all closely with the particular activities that it would most enhance welfare to increase. Instead, fiscal transfers directly respond to the fundamental problem preventing the effective functioning of the market mechanism, and can bring about a much more efficient equilibrium allocation of resources, even when they are not carefully targeted. And when fiscal transfers of a sufficient size are made in response to the pandemic shock, there is no longer any need for interest-rate cuts, which instead will lead to excessive current demand.

We are not the first to note that a crucial feature of the COVID-19 pandemic has been the degree to which its effects are sectorally concentrated; in particular, this is emphasized by both Guerrieri *et al.* (2020) and Baqaee and Farhi (2020). Indeed, the framework used here to consider alternative possible responses to a pandemic owes much of its structure to the pioneering work of Guerrieri *et al.* The emphases here are somewhat different, however, than in either of those earlier studies. We abstract altogether from either preference-based or technological complementarities between sectors, of the kind emphasized in the papers just cited, in order to focus more clearly on the consequences of the network structure of payments even in the absence of those other reasons for spillovers between activity in different sectors of the economy to exist. Because a key issue examined here is the effects of different possible network structures of payments, we consider a model in which there can be more than two sectors (and hence more than one sector still active in the case of a pandemic), unlike the baseline model of Guerrieri *et al.* And unlike either of these papers, we do not assume that all consumers choose to consume the same basket of goods; as we show below, non-uniformity in the way expenditure is allocated across goods by economic units that also have different sources of income can play an important role in amplifying the magnitude of the effective demand shortfall resulting from a pandemic.[2]

[2]Other papers that stress the importance of network structure for the propagation of economic disturbances include Acemoglu *et al.,* (2012), Bigio and La'O (2020), Elliott *et al.* (2021), Ghassibe (2021), La'O and Tahbaz-Salehi (2021), Ozdagli and Weber (2017), Pastén *et al.* (2020), and Rubbo (2020). These papers analyze the effects of production linkages between sectors (the input-output structure), which we abstract from here in order to emphasize the importance of the network structure of payments even in the absence of production linkages. Somewhat more closely related to our concerns here are the papers of Acemoglu *et al.* (2015) and Elliott *et al.* (2014), emphasizing the consequences of networks of financial obligations for the fragility of the financial system.

The macroeconomics of a shock like COVID-19 is the subject of a rapidly expanding literature, already too large to easily summarize. Many interesting contributions focus on different issues than those of concern here. For example, Bigio *et al.* (2020) do not consider what can be achieved with conventional interest-rate policy, instead comparing the effects of lump-sum transfers with those of central-bank credit policies, in a model which emphasizes the endogeneity of borrowing limits (not analyzed here). Araújo and Costa (2021) similarly discuss how a modification of bankruptcy law in response to a pandemic shock can substitute for fiscal transfers. Caballero and Simsek (2020) consider the possible amplification of the effects of the shock through the effects of income reductions on endogenous financial constraints, from which we abstract here. Céspedes *et al.* (2020) primarily emphasize the longer-run costs of firms having to shed workers during the crisis; here we abstract from such effects, and show that transfers can be beneficial even when they are not taken into account. Auerbach *et al.* (2020) similarly emphasize the increased effects of transfer policies when there is endogenous exit of firms, and focus on channels through which transfers matter even in the absence of financing restrictions. None of these papers give much attention to the effects of conventional interest-rate policy in the case of a pandemic shock.

The paper proceeds as follows. Section I explains the structure of the model, and derives the first-best allocation of resources, both for the case of shocks that affect all sectors identically, and for asymmetric disturbances such as a pandemic shock. This section also shows that if there are only aggregate shocks, interest-rate policy suffices to achieve the first-best allocation as a decentralized equilibrium outcome, while lump-sum transfers are not only unnecessary, but also ineffective as a tool of aggregate demand management. Section II analyzes the effects of an asymmetric disturbance such as a pandemic shock in the absence of any monetary or fiscal policy response, showing how a collapse of effective demand can occurs. Section III shows how either fiscal transfers or government credit policy can mitigate the effects of such a disturbance by reducing the degree of effective demand shortfall, even when interest-rate policy does not respond to the shock at all; and shows that at least under some circumstances, the first-best allocation can be achieved without any change in interest-rate policy. Finally, section IV considers what can be achieved by adjusting the central bank's interest-rate target in response to the asymmetric disturbance, if the fiscal policy response is insufficient. It explains why this is more modest than might be expected, and offers examples in which ex-ante welfare is not improved by any reduction in interest rates at all, though the level of economic activity remains inefficiently low. Section V concludes.

I An N-sector Model

Let us consider an N-sector "yeoman farmer" model, in which the economy is made up of producer-consumers that each supply goods or services for sale (subject to a disutility of supplying them), and also purchase and consume the goods or services supplied by other such units. Each such unit belongs to one of N sectors (where $N \geq 2$) and specializes in the supply of the good produced by that sector, but consumes the goods produced by multiple sectors.[3] We assume that there is a continuum of unit length of infinitesimal units in each

[3]These "sectors" need not be interpreted as separate industries (e.g., travel and hospitality), though the direct impact of the COVID-19 shock was indeed extremely different across industries. They might equally

of the sectors. We further order the sectors on a circle, and use modulo-N arithmetic when adding or subtracting numbers from sectoral indices (thus "sector $N+1$" is the same as sector 1, "sector -2" is the same as sector $N-2$, and so on).

I.A Preferences and the network structure of payments

A producer-consumer in sector j seeks to maximize the ex-ante expected value of a discounted sum of utilities

$$\sum_{t=0}^{\infty} \beta^t U^j(t) \tag{1}$$

where $0 < \beta < 1$ is a common discount factor for all sectors, and the utility flow each period is given by

$$U^j(t) \;=\; \sum_{k \in K_j(t)} \theta_k^j(t) u(c_k^j(t)/\theta_k^j(t);\, \xi_t) \;-\; v(y_j(t);\, \xi_t), \tag{2}$$

where $c_k^j(t)$ is the quantity consumed in period t of the goods produced by sector k, and $y_j(t)$ is the unit's production of its own sector's good. The non-negative coefficients $\{\theta_k^j(t)\}$ allow a given sector to have asymmetric demands for the goods produced by the other sectors; $K_j(t)$ is the subset of subset of sectors k for which $\theta_k^j(t) > 0$ (so that j wishes to consume goods produced in sector k in period t). The vector ξ_t represents aggregate disturbances that may shift either the utility from consumption or the disutility of supplying goods (or both);[4] note that these shocks are assumed to affect all goods and all consumers in the same way, as in standard one-sector New Keynesian models.

For any possible vector of aggregate shocks ξ, the utility functions are assumed to satisfy the following standard conditions: $u(0) = 0$, and $u'(c) > 0$, $u''(c) < 0$ for all $c > 0$; $\lim_{c\to 0} u'(c) = \infty$, and $\lim_{c\to\infty} u'(c) = 0$; and finally, $v(0) = 0$, and $v'(y) > 0$, $v''(y) \geq 0$ for all $y > 0$. The Inada conditions imply that the socially optimal supply of each good will be positive but finite (unless $\theta_k^j(t) = 0$ for all j in some period). Moreover, they imply that $\lim_{c\to\infty} u(c)/c = 0$, so that $\theta u(c/\theta)$ has a well-defined limiting value (equal to 0 for any $c > 0$) as $\theta \to 0$. Hence our assumption in (2) that we simply omit terms for $k \notin K_j(t)$ results in a utility function that varies continuously with the coefficients $\{\theta_k^j(t)\}$.

Furthermore, the additively separable form (2) implies that closing down one sector (preventing either production or consumption of that good) has no effect on either the utility from consumption or disutility of supplying any of the other goods. Thus we abstract entirely from complementarities between sectors owing either to preferences or production technologies, of the kind stressed by Guerrieri *et al.* (2020), in order to focus more clearly on the linkages between sectors resulting from the circular flow of payments.

The coefficients $\{\theta_k^j(t)\}$ are important for our analysis, as they determine the network structure of the flow of payments in the economy; random variation in these coefficients is also the only kind of asymmetric disturbance that we consider. In this paper, we further

well be understood as regions, or other ways in which economic units may be differentiated, that matter both for (i) the way they are impacted by some important disturbances, and (ii) the way in which other economic units allocate their spending.

[4] These may include aggregate productivity shocks, represented here as a shift in the disutility of effort required to produce a given quantity of output.

specialize to the case in which $\theta_k^j(t) = \phi_k(t) \cdot \alpha_{k-j}$ for all j, k, and t. The only kind of random disturbance that we consider is a shock to the multiplicative factor $\phi_k(t)$ that affects the taste for sector k's products by everyone in the economy. (This allows us to consider a disturbance like the COVID-19 pandemic.) The constant coefficients $\{\alpha_h\}$ instead determine the degree to which the consumption preferences of units in different sectors j are different. We assume that $\alpha_h \geq 0$ for each h, and also that $\alpha_0, \alpha_1 > 0$ (given an appropriate ordering of the sectors), to ensure that the network structure of payments is indecomposable.[5] We also assume that in any state of the world in any period t, $\phi_k(t) \geq 0$ for all k, and there is at most one sector k for which $\phi_k(t) = 0$.[6]

We further normalize the $\{\alpha_h\}$ so that $\sum_{h=0}^{N-1} \alpha_h = 1$. Then if $\phi_k(t) = 1$ for each sector k (which we will call the "normal case"), and in addition all goods have the same price in period t, the optimal intra-temporal allocation of expenditure by any sector j will be given by

$$c_k^j(t) \;=\; \alpha_{k-j} \cdot c^j(t) \tag{3}$$

for each good k, where $c^j(t) \equiv \sum_{k=1}^{N} c_k^j(t)$ is total real expenditure by the sector in period t. Thus in this case the coefficients $\{\alpha_h\}$ correspond to expenditure shares. In the more general case where $\phi_k(t) \neq 1$ for one or more sectors (but still assuming that all goods have the same price), the optimal allocation of expenditure will be of the form

$$c_k^j(t) \;=\; A_{kj}(t) \cdot c^j(t) \tag{4}$$

where

$$A_{kj}(t) \;\equiv\; \frac{\phi_k(t)\alpha_{k-j}}{\omega_j(\phi(t))}, \qquad \omega_j(\phi(t)) \;\equiv\; \sum_k \phi_k(t)\alpha_{k-j}. \tag{5}$$

(Our assumptions above guarantee that for any sector j and any period t, there is at least one sector $k \in K_j(t)$, so that $\omega_j(\phi(t)) > 0$, and the coefficients $A_{kj}(t)$ are well-defined.)

The coefficients $\{\alpha_h\}$ are assumed to be the same for all sectors j; this means that in the "normal case", the model has a rotational symmetry: it is invariant under any relabeling of the sectors in which each sector j is relabeled $j + r \pmod N$, for some integer r. Figure 1 illustrates two of the possible network structures allowed by our notation, for the case $N = 5$. Note that in either case, the numbers on the arrows leaving any sector sum to 1; these indicate the share of that sector's spending allocated to each of the sectors to which arrows lead, in the "normal case" and when all goods prices are the same. Because of the rotational symmetry, the numbers on the arrows leading to any sector also sum to 1. If in addition to prices being the same, each sector spends the same amount, then all sectors' revenues will be the same, and each sector's inflows and outflows will be balanced. This illustrates the balanced "circular flow" of payments in an equilibrium in which only aggregate shocks occur (discussed further below).

[5] We exclude, for example, cases in which N is even and even-numbered sectors purchase only from other even-numbered sectors, while odd-numbered sectors purchase only from odd-numbered sectors.

[6] Admitting the case in which $\phi_k(0) = 0$ in some single sector k allows us to consider the effects of a pandemic shock of the kind discussed by Guerrieri *et al.* (2020). We could also allow $\phi_k(0)$ to be zero in more than one sector, if we impose other restrictions that imply that the coefficients $\{\omega_j\}$ defined in (5) are nonetheless all positive.

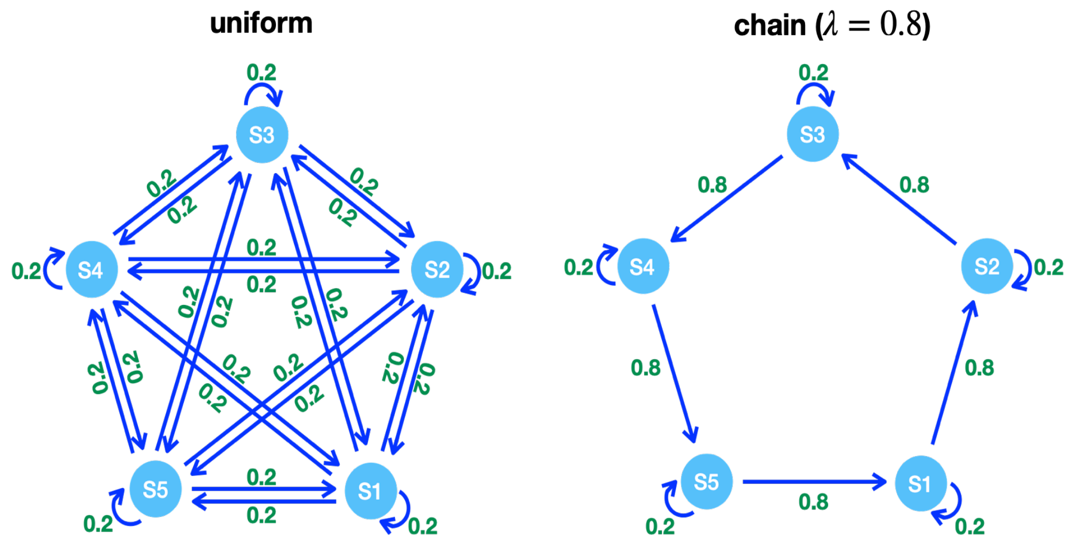

Figure 1: Two possible network structures when $N = 5$. The number on the arrow from sector j to sector k indicates the value of the coefficient α_{k-j}.

The left panel shows the case of a uniform network, in which $\alpha_h = 1/N$ for all h. In this case, each sector has the same preferences over consumption bundles as any other sector, and these preferences treat all goods symmetrically; if the prices of all goods are the same, each individual unit will purchase the same quantity from each sector. The right panel instead shows the case of a "chain" network, in which $\alpha_0 = 1 - \lambda$ and $\alpha_1 = \lambda$, for some $0 < \lambda < 1$, while all other α_h are zero. In this case, each sector purchases only from its own sector and the sector immediately following it on the circle. In the numerical example shown in the figure, $\lambda = 0.8$, so that in both examples the fraction of own-sector purchases (in the case that prices of all goods are equal) is the same (i.e., 20 percent). But in the left panel, out-of-sector purchases are uniformly distributed over all of the other sectors, while in the right panel they are concentrated on one other sector. We show below that the network structure has important consequences for both the effects of a pandemic shock and the effects of fiscal transfers in response to the shock.

We further assume that the entire sequences $\xi \equiv \{\xi_t\}$ and $\phi \equiv \{\phi_k(t)\}$ for all periods $t \geq 0$ are revealed at time $t = 0$; for simplicity, we suppose there is no further uncertainty about these disturbances to reveal after that. (This greatly simplifies our discussion of the short-run real effects of monetary policy.) We also assume for simplicity that asymmetric disturbances occur only in period $t = 0$; that is, we assume that $\phi_k(t) = 1$ for all k in all periods $t \geq 1$. (Thus we analyze only the effects of unanticipated asymmetric disturbances.)

Finally, we assume that the ex ante distribution of possible realizations of $\phi(0)$ exhibits a rotational symmetry. Let R be the matrix that maps an arbitrary N-vector v to a vector $v' = Rv$, where $v'_k = v_{k-1}$ for each k. Then we assume that any non-negative vector $\phi(0)$ has exactly the same ex-ante probability as the rotation $R\phi(0)$. (In the case of a pandemic shock that requires one sector to shut down while the others are unaffected, this assumption means that ex ante it is equally likely that any of the N sectors could be the one impacted by the pandemic.) This means that despite our allowing for the possibility of asymmetric disturbances (such as a pandemic shock) in period zero, our model is rotationally symmetric ex ante. This is convenient both because it simplifies the solution for equilibrium outcomes, and because it provides us with an unambiguous ex ante welfare ranking of the outcomes associated with different stabilization policies, despite the differing situations of producer-consumers in the different sectors ex post.

I.B The first-best optimal allocation of resources

As a benchmark for discussion of what stabilization policy can achieve, it is useful to define the first-best allocation that would be chosen by a social planner, given only the constraints of preferences and technology. For any possible realization of the disturbances (ξ, ϕ), let the set of N possible rotations

$$(\xi, \phi), \quad (\xi, R\phi), \quad \ldots, \quad (\xi, R^{N-1}\phi) \tag{6}$$

constitute the "rotation family" to which the particular realization (ξ, ϕ) belongs.[7] We can separately consider optimal policy for each possible rotation family.

We further consider only rotationally-invariant allocations of resources, that is, ones under which the consumption allocation $\{c_k^j(t; \xi, \phi)\}$ associated with given disturbance sequences (ξ, ϕ) satisfies

$$c_k^j(t; \xi, R\phi) \;=\; c_{k-1}^{j-1}(t; \xi, \phi)$$

for all j, k, t and any possible disturbance sequences (ξ, ϕ). Thus we do not allow policies that favor a particular sector, *except* to the extent that this results from the asymmetric impact of the exogenous disturbance ϕ on this sector; it must be the case that if the asymmetric disturbance were a rotation of the one that has actually occurred, the equilibrium allocation under the policy would have been correspondingly rotated. In the case of purely aggregate disturbances (the "normal case"), policies must treat units in all sectors identically. If we let $U^j(t; \xi, \phi)$ be the flow utility (2) in the case of disturbances (ξ, ϕ) under such a policy, it follows that

$$U^j(t; \xi, R\phi) \;=\; U^{j-1}(t; \xi, \phi)$$

for each sector j.

Then considering only the possible outcomes associated with a particular rotation family (6), the terms in the ex ante expected value of (1) associated with these outcomes are proportional to

$$\frac{1}{N}\sum_{h=0}^{N-1}\sum_{t=0}^{\infty}\beta^t U^j(t; \xi, R^h\phi) \;=\; \frac{1}{N}\sum_{h=0}^{N-1}\sum_{t=0}^{\infty}\beta^t U^{j-h}(t; \xi, \phi).$$

This sum is the same for each sector j, in addition to being the same for each of the sequences (ξ, ϕ) in a given rotation family. This immediately yields the following conclusion.[8]

[7] Here for any sequence of N-vectors ϕ, $\phi' = R\phi$ means the alternative sequence such that $\phi'(t) = R\phi(t)$ for each $t \geq 0$. Note that in the "normal case" in which $\phi_k(0) = 1$ for all k, so that there is no asymmetric disturbance, the rotation family of the realization (ξ, ϕ) consists only of (ξ, ϕ) itself.

[8] Proofs of all numbered lemmas and propositions are given in the online appendix.

Lemma 1. *Units in all sectors j agree about the ex ante ranking of alternative feasible rotationally-invariant allocations of resources. With regard to policy for a particular rotation family of possible disturbances, they unanimously prefer one allocation to another if in the case of any of the sequences (ξ, ϕ) of exogenous disturbances (once uncertainty is resolved at date $t = 0$) in this family, the first allocation achieves a higher value of*

$$\sum_{t=0}^{\infty} \beta^t \left[\sum_{j=1}^{N} U^j(t) \right], \tag{7}$$

where $U^j(t)$ is defined in (2).

Given this, there is an obvious welfare objective to use in comparing alternative policies. We define the "first-best" optimal allocation in the case of disturbance sequences (ξ, ϕ) as the one that maximizes (7) under the constraints that $\sum_j c_k^j(t) = y_k(t)$ for each sector k at each date t.

The welfare objective (7) can be written as a sum of separate terms for each good k at each date t. We thus obtain a separate problem for each k, t for which $\phi_k(t) > 0$, in which we must choose $y_k(t)$ and the $\{c_k^j(t)\}$ for $j = 1, \ldots, N$ to maximize

$$\sum_{h \in H} \phi_k(t) \alpha_h u(c_k^{k-h}(t)/(\alpha_h \phi_k(t));\, \xi_t) \;-\; v(y_k(t);\, \xi_t),$$

where H is the set of indices h for which $\alpha_h > 0$, subject to the constraints that $\sum_j c_k^j(t) = y_k(t)$. (If k is a sector such that $\phi_k(t) = 0$, then the problem is trivial, and all quantities must equal zero.) The solution to this static problem is easily characterized.

Lemma 2. *In the case of any disturbance sequences (ξ, ϕ), the unique first-best allocation of resources involves sectoral output levels $y_k(t) = y^*(\phi_k(t); \xi_t)$, where for any aggregate disturbance vector ξ and any $\phi > 0$, $y^*(\phi; \xi)$ is the unique solution to the equation*

$$u'(y^*/\phi;\, \xi) \;=\; v'(y^*;\, \xi). \tag{8}$$

This supply of each good k is then allocated to consumers according to

$$c_k^j(t) \;=\; \alpha_{k-j} \cdot y_k(t) \tag{9}$$

for each sector j.

Thus the efficient level of output differs across sectors only as a result of asymmetry in the vector $\phi(0)$. In the "normal case" in which $\phi_k(0) = 1$ for all k, and there are only aggregate shocks, the first-best allocation requires that $y_k(t) = y_t^*$ for each sector, where y_t^* is defined by the same condition,

$$u'(y_t^*; \xi_t) \;=\; v'(y_t^*; \xi_t), \tag{10}$$

as determines the "natural rate of output" in the familiar one-sector model. In addition, in this case the optimal allocation shares are given by the coefficients $\{\alpha_h\}$, as in the examples in Figure 1.

In the case of an asymmetric disturbance to the vector $\phi(0)$, the optimal output in period zero differs across sectors. But it is only optimal for output to differ across sectors in the period in which the vector $\phi(t)$ is asymmetric (i.e., only in period $t = 0$), and even in period $t = 0$ it is only optimal for $y_k(0)$ to differ from y_0^* to the extent that $\phi_k(0) \neq 1$ in that sector. Thus if we model a pandemic shock as a situation in which $\phi_p(0) < 1$ in some sector p only, while we continue to have $\phi_k(0) = 1$ for all $k \neq p$, under the first-best resource allocation, output in sector p should decrease (to a greater extent the greater the reduction in $\phi_p(0)$), while the efficient level of output in all sectors $k \neq p$ remains unchanged.[9] In this respect, a pandemic shock can be considered a "negative supply shock," as in the discussion by Guerrieri *et al.* (2020).

I.C The decentralized economy

Because all uncertainty is resolved at time $t = 0$, the allocation of resources from period zero onward (conditional on the shocks revealed at that time) can be modeled as a perfect foresight equilibrium of a deterministic model. Each period, there are spot markets for the goods produced by all of the sectors for which $\phi_k(t) > 0$, with $p_k(t)$ the money price of good k in period t. There is also trading in a one-period nominal bond, that pays a nominal interest rate $i(t)$ between periods t and $t + 1$.[10] The price $p_k(t)$ of each sectoral good is assumed to be predetermined one period in advance, at a level that is expected at that earlier time to clear the market for good k in period t; this temporary stickiness of prices allows monetary policy to affect real activity in period zero.

Let $a^j(t)$ be the nominal asset position of units in sector j at the beginning of period t (after any taxes or transfers), and $b^j(t)$ the nominal asset position at the end of the period (after period t payments for goods are settled). Then in any period $t \geq 0$, a unit in sector j chooses expenditures $\{c_k^j(t)\}$ (for $k \in K_j(t)$) and end-of-period assets $b^j(t)$ subject to the flow budget constraint

$$\sum_{k \in K_j(t)} p_k(t) c_k^j(t) \; + \; b^j(t) \;\; = \;\; a^j(t) \; + \; p_j(t) y_j(t) \tag{11}$$

and the borrowing constraint

$$b^j(t) \;\; \geq \;\; \underline{b}^j(t), \tag{12}$$

where $y_j(t)$ is the quantity sold by the unit of its product, and $\underline{b}^j(t) \leq 0$ is a (possibly sector-specific) borrowing limit.

The borrowing constraint (12) is crucial to our analysis, and in particular for the possibility of a collapse of effective demand, as analyzed in section II. We treat $\underline{b}^j(t)$ as a quantity determined by government policy (credit policy). We suppose that units are unable to credibly promise to repay, except to the extent that the government allows them to issue debt up to a certain limit, the repayment of which is guaranteed by the government. (We assume also that the government is able to force borrowers to repay these guaranteed debts,

[9] See Figure 2 below for examples.

[10] Because there is only one possible future path for the economy conditional on the state in period zero, allowance for more than one financial asset in any period $t \geq 0$ would be redundant.

rather than bearing any losses itself.) Thus we shall refer to the case in which $\underline{b}^j(t) = 0$ for all j as the case of "no credit policy."[11]

In period zero, $a^j(0) \geq 0$ is given as an initial condition for each sector; this quantity reflects not only wealth brought into the period (before shocks are realized), but also any transfers from the government in response to the shocks realized at time $t = 0$, and the payoffs from any private insurance contracts conditional on those shocks. In any subsequent period, $a^j(t+1)$ is given by

$$a^j(t+1) \;=\; (1+i(t))b^j(t) \;-\; \tau(t+1)/N, \tag{13}$$

where $\tau(t+1)$ is total lump-sum nominal tax collections at the beginning of period $t+1$, assumed to equally divided among all sectors. (We consider the possibility of sector-specific taxes or transfers only in period zero, in response to an asymmetric shock.) A unit in sector j takes as given the value of $a^j(0)$, and the sequences $\{\xi_t, p_k(t), y_j(t), i(t), \underline{b}^j(t), \tau(t+1)\}$ for all $t \geq 0$, and chooses sequences $\{c^j_k(t), b^j(t)\}$ consistent with constraints (11)–(13) for all $t \geq 0$ so as to maximize (1).

In equilibrium, the sales by units in each sector are given by

$$y_k(t) \;=\; \sum_{j=1}^{N} c^j_k(t). \tag{14}$$

The assumption that prices are set in advance at a level expected to clear markets means that for each j, the sequence $\{y_j(t)\}$ for $t \geq 1$ must be the sequence that a unit in sector j would choose, if it were also to choose that sequence at time $t = 0$, taking as given the values of $a^j(0)$ and $y_j(0)$ and the sequences $\{\xi_t, p_k(t), i(t), \underline{b}^j(t), \tau(t+1)\}$ for all $t \geq 0$. The value of $y_j(0)$, however, need not be the one that units in sector j would choose, given the shocks realized at $t = 0$, because the price $p_k(0)$ is determined prior to the realization of those shocks.[12] Because of the ex ante symmetry of our model, the predetermined prices $p_k(0)$ will all be set equal to some common price $\bar{p} > 0$. The exact determinants of $\bar{p}$ are not relevant to our results below; it is only important that the price is the same for all sectors, and that it cannot be changed by any policy response to shocks realized at time $t = 0$.

Conditions (11), (13) and (14) imply that the total supply of liquid assets $a(t)$ must evolve according to a law of motion

$$a(t+1) \;=\; (1+i(t))a(t) \;-\; \tau(t+1) \tag{15}$$

for all $t \geq 0$, which can be regarded as a flow budget constraint of the government. We shall consider only cases in which $a(t) > 0$ for all $t \geq 0$, so that there exist public-sector liabilities, the interest rate on which is controlled by the central bank.[13]

[11] It is not essential to our conclusions that the borrowing limit be zero in the absence of government credit policy; what is important for our discussion below is that borrowing limits are unaffected by the changes in monetary or fiscal policy that we consider, other than an explicit change in credit policy.

[12] We suppose that when the pre-determined price $p_k(0)$ is set, the suppliers in sector k agree that they will each supply an equal share of whatever quantity of good k turns out to be demanded at that price.

[13] See Woodford (2003, chap. 2) for discussion of the conduct of monetary policy by setting the nominal interest yield on an outside nominal asset of this kind.

We can now define an (ex-post) *perfect foresight equilibrium* given the exogenous disturbance sequences (ξ, ϕ) that have been realized in period zero, and the specifications of interest-rate policy, credit policy, and fiscal policy in response to such disturbances. This is a specification of the path of interest rates $\{i(t)\}$ for $t \geq 0$, prices $\{p_k(t)\}$ for $t \geq 1$, the allocation of resources $\{c_k^j(t), y_k(t)\}$ in each period $t \geq 0$, end-of-period balances $\{b^j(t)\}$ for periods $t \geq 0$, and associated beginning-of-period balances $\{a^j(t)\}$ in periods $t \geq 1$, such that (i) for each sector j, the plan specifying $\{c_k^j(t), b^j(t)\}$ for each $t \geq 0$ and $\{a^j(t), y_j(t)\}$ for each $t \geq 1$ maximizes (1), given $a^j(0)$ (determined by fiscal policy), $y_j(0)$ (determined by spending decisions in aggregate), the path of tax obligations $\{\tau(t)\}$ for $t \geq 1$ (determined by fiscal policy), the path of borrowing limits $\{\underline{b}^j(t)\}$ for $t \geq 0$ (determined by credit policy), and the endogenous paths of interest rates and prices; (ii) for each sector k, the quantity $y_k(t)$ produced satisfies (14) in each period $t \geq 0$; and (iii) total liquid asset holdings satisfy $\sum_j a^j(t) = a(t)$ in each period $t \geq 1$, where $\{a(t)\}$ is the path for the public debt determined by fiscal policy.

I.D Optimal policy if only aggregate shocks occur

Let us consider first the "normal case" in which $\phi_k(0) = 1$ for all k, but different sequences $\{\xi_t\}$ for the aggregate disturbances may be revealed in period $t = 0$. Can the first-best allocation of resources (characterized above) be supported as an equilibrium, using only the policy instruments listed in the previous section? It is easily seen that this is possible, regardless of the predetermined price level $\bar{p}$ and the particular aggregate shock sequence $\{\xi_t\}$.

It suffices to consider policy regimes of the following kind. First, the central bank sets the interest rate in accordance with a Taylor rule of the form

$$\log(1 + i(t)) \;=\; \log(1 + r_t^*) \;+\; \pi^*(t+1) \;+\; \psi \cdot [\log(P(t)/P(t-1)) - \pi^*(t)], \tag{16}$$

for each $t \geq 0$, assuming that the right-hand side of the right-hand side of this formula is non-negative, and sets $i(t) = 0$ otherwise.[14] Here r_t^* is the "natural rate of interest,"

$$1 + r_t^* \;\equiv\; \frac{1}{\beta} \frac{u'(y_t^*;\, \xi_t)}{u'(y_{t+1}^*;\, \xi_{t+1})}, \tag{17}$$

a uniquely defined function of the exogenous disturbances; the price index $P(t)$ is equal to $(1/N) \sum_k p_k(t)$; and $\{\pi^*(t)\}$ is a sequence of target inflation rates for $t \geq 0$. Second, the fiscal authority chooses period-zero transfers and a uniform tax obligation $\tau(t+1)$ each period after that so as to make (15) consistent with a target path $\{a(t)\}$ for the nominal public debt, where $a(t) > 0$ each period, so that riskless debt exists in positive supply. Finally, there is no credit policy: $\underline{b}^j(t) = 0$ each period for each sector j. A regime of this kind supports the first-best optimal allocation as an equilibrium outcome under the following conditions.

[14] Here we assume that it is only feasible for the central bank to enforce a non-negative riskless nominal interest rate, though we do not model the institutional reasons for this to be the case (e.g., the possibility of storing non-interest-earning currency). But neither the assumption that the effective lower bound is exactly zero, nor even the assumption that there is a lower bound, matters for any of our conclusions below.

Proposition 1. *Suppose that* $\phi_k(0) = 1$ *for all* k*, so that only aggregate disturbances exist. And suppose (i) that interest-rate policy is determined by a Taylor rule (16), where the target inflation rate* $\pi^*(0)$ *is chosen to equal the predetermined inflation rate* $\log(\bar{p}/P(-1))$*, and subsequent targets are chosen so that*

$$\log(1 + r_t^*) \; + \; \pi^*(t + 1) \; \geq \; 0 \tag{18}$$

for all $t \geq 0$*; (ii) that there is no credit policy; (iii) that there are no taxes or transfers in period zero (so that* $a^j(0) = a(0)/N$ *for each* j*); and (iv) that the target path* $\{a(t)\}$ *for the nominal public debt satisfies* $a(t) > 0$ *for all* $t \geq 0$*, and*

$$\lim_{t\to\infty} \beta^t u'(y_t^*; \, \xi_t) \frac{a(t)}{P^*(t)} \; = \; 0, \tag{19}$$

where $\{P^*(t)\}$ *is the path for the price level implied by the sequence of inflation targets (i.e., the requirement that* $\log(P(t + 1)/P(t)) = \pi^*(t + 1)$ *for all* $t \geq 0$*) and the initial condition* $P^*(0) = \bar{p}$*. Then there exists an equilibrium in which the price of all goods is the same each period (*$p_k(t) = P(t)$ *for all* k*); the central bank's inflation target is achieved each period (*$P(t) = P^*(t)$ *for all* $t \geq 0$*); and the equilibrium allocation of resources is the first-best optimal allocation:* $y_k(t) = y_t^*, c_k^j(t) = \alpha_{k-j} y_t^*$*, for all* j, k *in each period* $t \geq 0$*.*

Note that under this optimal policy regime, neither the borrowing limits $\{\underline{b}^j(t)\}$, nor the initial sector-specific asset positions $\{a^j(0)\}$, nor the subsequent target path $\{a(t + 1)\}$ for the public debt needs to respond to the particular sequence of aggregate disturbances $\{\xi_t\}$ that has been realized. Only the interest-rate reaction function (16) responds to the disturbances, through their effect on the implied path of the natural rate of interest. Thus all of the work of stabilization of the economy in response to exogenous disturbances is done by interest-rate policy, while fiscal policy is purely "passive."

Moreover, the model is one in which lump-sum transfers would not be useful for stabilization purposes. The equilibrium paths of both prices and quantities in Proposition 1 are the same, regardless of the exact path of the public debt $\{a(t)\}$. Thus making the path of lump-sum transfers and taxes state-contingent would have no effect, as long as we continue to assume that under any realization of the exogenous disturbances, (i) each sector j receives the same transfers and is subject to the same tax liabilities, and (ii) the transversality condition (19) is satisfied. (Sector-specific transfers that redistribute income between sectors would change the equilibrium allocation of resources, but not in a way that increases the welfare objective (7).)

Credit policy would similarly have no effect. We have written Proposition 1 to emphasize that the first-best can be achieved without credit policy; but we would obtain the same result in the case of any other paths $\{\underline{b}^j(t)\}$ for the borrowing limits, as long as $\underline{b}^j(t) \leq 0$ each period, and the borrowing limits satisfy a transversality condition similar to (19), to exclude the possibility of "Ponzi schemes." The reason is that as long as there are only aggregate disturbances, we have an equilibrium each period in which $p_k(t) = P(t)$ for each k, $c^j(t)$ is the same level of total spending for each sector, and each unit's expenditure allocation satisfies (3). It follows that each period we must have

$$\sum_k p_k(t) c_k^j(t) \; = \; \sum_k p_j(t) c_j^k(t) \; = \; p_j(t) y_j(t)$$

for each sector j, so that there is a balanced circular flow of payments (as in the examples in Figure 1). This in turn implies that asset balances remain uniformly distributed across sectors (given that we start from an initial uniform distribution), and that $b^j(t) = a^j(t) = a(t)/N > 0$ each period. Hence the borrowing limit never binds for units in any sector, and loosening the borrowing limits will not change any units' desired behavior, assuming that Ponzi schemes continue to be infeasible.

Thus in the case of only aggregate shocks (and fiscal policies that affect all sectors uniformly), fiscal transfers and credit policy are irrelevant as tools of stabilization policy, while monetary policy alone suffices to allow the first-best allocation of resources to be supported as an equilibrium outcome. As we shall see, our conclusions about the relative usefulness of these different types of policy are quite different in the case of a severely asymmetric disturbance, such as a pandemic.

In the case that there are only aggregate shocks, there is also a direct relationship between the degree to which output is above or below its efficient level and the degree to which monetary policy sets the interest rate at a level that is too low or too high, given the nature of the aggregate shock. This is illustrated by the following generalization of Proposition 1.

Proposition 2. *Suppose that $\phi_k(0) = 1$ for all k, so that only aggregate disturbances exist. Suppose also that fiscal policy and credit policy are specified as in Proposition 1, and that interest-rate policy is the same as is specified in Proposition 1 for all periods $t \geq 1$, but that $i(0)$ need not satisfy (16). Then there is again an equilibrium in which $p_k(t) = P^*(t)$ for all k and all t, and in which the allocation of resources in each period $t \geq 1$ is the one specified in Proposition 1. In period $t = 0$, the allocation of resources continues to be rotationally symmetric:*

$$y_k(0) \;=\; y(0), \qquad c_k^j(0) \;=\; \alpha_{k-j}y(0)$$

for all j, k. However, the common level of sectoral output $y(0)$ will in general not equal y_0^* (the efficient level, given the disturbance ξ_0). Instead, the equilibrium involves $y(0) < y_0^*$ if and only if $i(0)$ is higher than the value specified by (16), while $y(0) > y_0^*$ if $i(0)$ is lower than the right-hand side of (16).

This result implies that (under the conditions assumed in the proposition) an observation that output is inefficiently low in period $t = 0$ (the period for which prices have been predetermined, before realization of the disturbance sequence ξ) implies that the interest rate is too high in that period, relative to the disturbance sequence that has occurred (which determines the value of r_0^*). This conclusion is independent of the exact specification of fiscal policy and credit policy, within a wide range of possible specifications of the latter policies. It suggests that a measure of the aggregate "output gap" might be enough to indicate that the central bank's interest-rate target should be reduced in response to a particular type of real disturbance, regardless of the nature of that disturbance. (The conclusion in Proposition 2 holds whether ξ_0 represents a shock to the degree of impatience to consume, a shock to the disutility of working, or a shock to labor productivity, among other possibilities.) But as we shall see, the same is not true in the case of an asymmetric disturbance.

II Asymmetric Disturbances and Effective Demand Failure

In the case of an asymmetric disturbance, the policy specified in Proposition 1 no longer suffices to ensure the existence of an equilibrium in which the allocation of resources is the first-best optimal allocation characterized in Lemma 2. One might suppose that this is because we have assumed that prices are fixed a period in advance, and the uniform prices ($p_k(0) = \bar{p}$ for all k) that are chosen ex ante will in general imply the wrong relative prices ex post, in the case of an asymmetric disturbance. This is indeed generally an obstacle to attainment of the first-best allocation. But as shown below, there are cases in which the first-best allocation can be achieved using only the policy instruments specified in section I.C above, despite the occurrence of an asymmetric disturbance. Yet in these cases, not only is the optimal policy not the one specified in Proposition 1, it is not a policy under which interest-rate policy alone responds to the disturbance. Moreover, it is not necessarily even the case that a disturbance that results in an inefficiently low level of output implies that matters would be improved by reducing the interest rate.

II.A Equilibrium with an asymmetric disturbance

We begin by discussing how equilibrium is different in the case of a disturbance that causes $\phi_k(0)$ to differ across sectors, such as a pandemic (modeled as an exogenous reduction of $\phi_k(0)$ in some, but not all sectors, owing to health concerns that temporarily preclude consumption of certain goods and services). Since aggregate disturbances create no problems that cannot be dealt with using monetary policy alone, as shown in Proposition 1, from here on we abstract from them: we assume that $\xi_t = \bar{\xi}$ for all $t \geq 0$, and let $\bar{y} = y^*(1; \bar{\xi})$ be the natural rate of output associated with these constant values for the disturbance parameters.[15]

Our primary interest is in the effects of alternative possible policy responses in period $t = 0$ to asymmetric shock to the vector $\phi(0)$. Hence we shall simplify our discussion by assuming that policy from period $t = 1$ onward is of a particular sort.

Assumption 1. *In all periods $t \geq 1$, monetary and fiscal policy are assumed to satisfy conditions (i) and (iv) of Proposition 1 in all periods. The borrowing limits satisfy $\underline{b}^j(t) \leq 0$ for all j and all $t \leq 1$, together with the requirement that*

$$\lim_{t \to \infty} \beta^t \frac{\underline{b}_t^j}{P^*(t)} \;=\; 0 \tag{20}$$

for each sector j. In addition, let

$$\tilde{a}^j(1) \;\equiv\; (1 + i(0)) \frac{b^j(0)}{P^*(1)} \tag{21}$$

denote the real pre-tax wealth carried into period 1 by each of the units in sector j, if the period-1 inflation rate conforms to the central bank's target. Then the paths $\{a(t), \underline{b}^j(t)\}$

[15] It would be straightforward to generalize our results below to the case on non-zero aggregate shocks, at the cost of more complex formulas, but without changing our conclusions about the effects of shocks to $\phi(0)$.

satisfy

$$\frac{(1/N)a(t) - \underline{b}^j(t)}{P^*(t)} \;\geq\; \beta \left[(1/N) \sum_{\ell=1}^{N} \tilde{a}^\ell(1) - \tilde{a}^j(1) \right] \tag{22}$$

for each j and all $t \geq 1$.

Condition (20) implies that even if credit policy relaxes borrowing constraints for some or all sectors, the level of allowable borrowing does not grow so fast as to make a Ponzi scheme feasible: debt must eventually be repaid. On the other hand, condition (22) implies that borrowing limits will also not be contracted too quickly. While the condition is stated in Assumption 1 purely as a constraint on variables in periods $t \geq 1$, a sufficient condition for (22) to hold is that

$$\left[\frac{(1/N)a(t) - \underline{b}^j(t)}{P^*(t)} \right] \;\geq\; \beta(1 + i(0)) \frac{P^*(0)}{P^*(1)} \left[\frac{(1/N)a(0) - \underline{b}^j(0)}{P^*(0)} \right]$$

for all $t \geq 1$. That is, the condition holds if we assume that in the case of any increase of aggregate liquidity $a(0)$ through fiscal transfers, or relaxation of the borrowing limit $\underline{b}^j(0)$ of any sector in period 0 will be permanent (in real terms). This implies that the government does not create liquidity difficulties for any sector in periods $t \geq 1$ by insisting that either public or private borrowing must be rapidly repaid.

Because we assume that economic fundamentals are the same in all periods $t \geq 1$, despite the occurrence of an asymmetric disturbance in period zero, it is natural to expect a perfect foresight equilibrium in which quantities and relative prices are constant for all $t \geq 1$. And because the equilibrium in periods $t \geq 1$ must be the same as if all goods prices were fully flexible, the resource allocation and relative prices in each of the periods $t \geq 1$ should correspond to those of a static (one-period) competitive equilibrium. Let $\boldsymbol{f}$ be a vector of net transfers to the units in the different sectors (where the element f^j is the net transfer to sector j), measured in units of a composite good consisting of one unit of each of the N goods, and such that $\sum_j f^j = 0$. For any such vector, consider a one-period competitive equilibrium model in which units in each sector j choose quantities $\{c^j_k\}$ and y_j to maximize (2), subject to the budget constraint

$$\sum_{h \in H} q_{j+h} c^j_{j+h} \;=\; q_j y_j \;+\; f^j, \tag{23}$$

where $q_k \equiv p_k/P$ is the relative price of good k. Competitive equilibrium requires that the relative prices be such that markets clear, i.e., such that (14) holds for each k.

Standard methods[16] allow one to show that the collection of endowment vectors $\boldsymbol{f}$ and corresponding equilibrium relative prices $\boldsymbol{q}$ form a smooth manifold of dimension $N - 1$. Because of the rotational symmetry of the static model, one point on this manifold must be $(\mathbf{0}, \bar{\boldsymbol{q}})$, where $\bar{q}_k = 1/N$ for all k, the equilibrium in which $c^j_k = \alpha_{k-j}\bar{y}$, $y_k = \bar{y}$ for all j, k.[17] It follows that for some neighborhood U of the vector $\mathbf{0}$, we can define a continuously

[16] See, for example, Balasko (2009).

[17] Note that this point corresponds to the stationary equilibrium in each period $t \geq 1$ in the intertemporal equilibria characterized in Propositions 1 and 2.

differentiable function $\boldsymbol{q}^*(\boldsymbol{f})$ such that $\boldsymbol{q}^*(\mathbf{0}) = \bar{\boldsymbol{q}}$, and such that for any $\boldsymbol{f} \in U$, the pair $(\boldsymbol{f}, \boldsymbol{q}^*(\boldsymbol{f}))$ also belongs to the equilibrium manifold — that is, such that $\boldsymbol{q}^*(\boldsymbol{f})$ is an equilibrium vector of relative prices in the case of an endowment vector $\boldsymbol{f}$. In this way, for any endowment vector close enough to $\mathbf{0}$, we select a competitive equilibrium price vector that is correspondingly close to the uniform vector of relative prices $\bar{\boldsymbol{q}}$. Corresponding to any such equilibrium price vector there is also a uniquely defined competitive equilibrium allocation, given by the solution to the problem of maximizing (2) subject to the static budget constraint (23), for each sector j.

We can then use this solution for competitive equilibrium in the static model to define a perfect foresight equilibrium for periods $t \geq 1$, as a function of the net asset position of units in each of the sectors at the beginning of period $t = 1$. Let $\tilde{\boldsymbol{a}}(1)$ be the vector specifying the net asset position of each of the sectors at the beginning of period 1. Then as long as the vector $\tilde{\boldsymbol{a}}(1)$ indicates a wealth distribution at the beginning of period $t = 1$ that is not too non-uniform, we can compute a perfect foresight equilibrium for periods $t \geq 1$ that depends only on the vector $\tilde{\boldsymbol{a}}(1)$.

Lemma 3. *Let pre-tax net asset positions at the beginning of period $t = 1$ be specified by a vector $\tilde{\boldsymbol{a}}(1)$, and suppose that the implied vector of sectoral net incomes*

$$f^j \;=\; (1-\beta)[\tilde{a}^j(1) - (1/N)\sum_{\ell=1}^{N} \tilde{a}^\ell(1)] \tag{24}$$

satisfies $\boldsymbol{f} \in U$, where U is the neighborhood on which the static equilibrium selection is defined. Suppose furthermore that policy satisfies Assumption 1 in all periods $t \geq 1$.

Then there exists a flexible-price perfect-foresight equilibrium for periods $t \geq 1$, consistent with the assumed pre-tax net asset positions, of the following kind. Relative prices each period are given by $\boldsymbol{q}^*(\boldsymbol{f})$, and the allocation of resources each period is the stationary allocation that, for each sector j, solves the problem of maximizing (2) subject to the constraint (23), given these relative prices and the transfer f^j specified in (24). The price index each period is given by the central bank's inflation target, $P(t) = P^*(t)$, and the real interest rate each period is given by

$$(1+i(t))\frac{P(t)}{P(t+1)} \;=\; 1+r^* \;=\; \frac{1}{\beta} \;>\; 1. \tag{25}$$

In this equilibrium, the borrowing constraint (12) never binds for units in any sector, in any period $t \geq 1$.

Finally, we can use this solution for the perfect foresight equilibrium for all periods $t \geq 1$, conditional on the assets carried into period 1, to characterize equilibrium behavior in period $t = 0$ as well, as a function of the asymmetric disturbance. Suppose that the real pre-tax assets carried into period 1 by units in general are described by a vector $\tilde{\boldsymbol{a}}(1)$, but that an individual unit in sector j contemplates carrying some other level of real pre-tax assets $\tilde{a}$ into period 1. We can define the optimization plan for this individual unit from $t = 1$ onward, if its initial assets are $\tilde{a}$ and it faces the relative prices and real interest rates implied by the aggregate decisions $\tilde{\boldsymbol{a}}(1)$ (as characterized in Lemma 3).

Let $V^j(\tilde{a}; \tilde{\boldsymbol{a}}(1))$ be the discounted utility flow for this individual unit in periods $t \geq 1$ (discounted back to period 1). The unit's optimization problem in period zero (after the realization of the disturbances and any policy response) can then be written as a choice of an expenditure plan $\{c_k^j(0)\}$ for all $k \in K_j(0)$ and end-of-period asset balance $b^j(0)$ consistent with (11)–(12), so as to maximize the objective

$$U^j(0) \; + \; \beta V^j(\tilde{a};\, \tilde{\boldsymbol{a}}(1)), \tag{26}$$

where $U^j(0)$ is given by (2) and $\tilde{a} = (1 + i(0))b^j(0)/P^*(1)$.[18] This defines an optimization problem for units in any sector j that depends on the period-zero policy variables $a^j(0)$ (reflecting possible transfers in response to the disturbance), $\underline{b}^j(0)$, and $i(0)$.

Given policy choices $\{a^j(0), \underline{b}^j(0), i(0)\}$, an equilibrium is then a plan $\{c_k^j(0), b^j(0)\}$ for units in each sector j, and a vector $\tilde{\boldsymbol{a}}(1)$, such that (i) for each j, the plan $\{c_k^j(0), b^j(0)\}$ solves the optimization problem for an individual unit (just stated), given the policy variables and the vector $\tilde{\boldsymbol{a}}(1)$ of aggregate decisions; and (ii) $\tilde{\boldsymbol{a}}(1)$ is the vector of sectoral net asset positions at the beginning of period 1 implied by these plans, using (21).

Moreover, the optimal plan for each sector j is easily characterized. Since $p_k(0) = \bar{p}$ for each of the goods with $\phi_k(0) > 0$, an optimal consumption plan must be of the form (4), for some level of total expenditure $c^j(0)$. The optimal level of total expenditure must satisfy the Euler condition

$$u'\left(\frac{c^j(0)}{\omega^j(\phi(0))}\right) \;\; \geq \;\; \beta(1 + i(0))\frac{\bar{p}}{P^*(1)}\, \Lambda^j(\tilde{a}^j(1);\, \tilde{\boldsymbol{a}}(1)), \tag{27}$$

where

$$\Lambda^j(\tilde{a};\, \tilde{\boldsymbol{a}}(1)) \;\equiv\; \frac{\partial V^j(\tilde{a};\, \tilde{\boldsymbol{a}}(1))}{\partial \tilde{a}}$$

measures the marginal utility of additional real wealth at the beginning of period 1, for units in sector j. An optimal plan must satisfy both inequalities (12) and (27), and at least one of these must hold with equality for each sector j.

The following characterization of the function $\Lambda^j(\tilde{a}^j(1);\, \tilde{\boldsymbol{a}}(1))$ is useful for constructing examples of possible equilibria.

Lemma 4. *Let the utility functions $u(c; \bar{\xi})$, $v(y; \bar{\xi})$ be given, as well as the coefficients $\{\alpha_{k-j}\}$. Then we can define a vector function $\boldsymbol{\Lambda}^*(\boldsymbol{f})$ for all $\boldsymbol{f} \in U$, such that for each sector j, and any vector $\tilde{\boldsymbol{a}}(1)$ consistent with the hypothesis of Lemma 3,*

$$\Lambda^j(\tilde{a}^j(1);\, \tilde{\boldsymbol{a}}(1)) \;=\; \Lambda^{*j}(\boldsymbol{f}),$$

where $\boldsymbol{f}$ is the vector of transfers defined by (24). The function $\boldsymbol{\Lambda}^*(\boldsymbol{f})$ is independent of the value of β, which matters only through the way in which it enters (24). Furthermore,

$$\lim_{\boldsymbol{f} \to \boldsymbol{0}} \boldsymbol{\Lambda}^*(\boldsymbol{f}) \;=\; \boldsymbol{\Lambda}^*(\boldsymbol{0}) \;=\; u'(\bar{y}; \bar{\xi}) \cdot \boldsymbol{e},$$

where $\boldsymbol{e}$ is a vector of ones.

This means that the Euler condition (27) takes an especially simple form when the vector $\boldsymbol{f}$ defined by (24) is small in all of its elements. The next two sections discuss two special cases in which this approximation can be used.

[18] In this expression, $b^j(0)$ means the end-of-period asset balance chosen by the individual unit, which may differ from the aggregate choice of units in sector j, reflected in the quantity $\tilde{a}^j(1)$.

II.B Effective demand failure when liquidity is scarce

We now consider the equilibrium allocation of resources in the case of an asymmetric disturbance such as a pandemic, under the assumption that policy continues to be of the kind assumed in Proposition 1. While monetary policy responds to shocks under such a regime, it is assumed to respond only to *aggregate* shocks (determinants of the "natural rate of output" y_t^*). Thus (for now) we assume that there is no response of macroeconomic policy to the asymmetric disturbance.

The resulting impairment of effective demand is most dramatic when liquid asset balances are low. In this section, we simplify the analysis by considering the limiting case in which $a(0) \to 0$. We also continue to assume that $\underline{b}_j(0) = 0$ for all sectors: there is no credit policy. Note that even in this limiting case, no inefficiency would result as long as only aggregate shocks occur (Proposition 1); thus we can imagine an economy choosing to operate with a very low level of liquid assets, if the ex-ante probability assigned to the occurrence of a pandemic shock has been quite small.

As explained in section I.A, the fact that $p_k(0) = \bar{p}$ for all sectors implies that each sector's spending will be allocated in accordance with (4), where however the factors $\{\omega_j\}$ will differ from 1 in the case of an asymmetric disturbance.[19] It follows that the total demand for the product of any sector k will be given by

$$y_k(0) \;=\; \sum_{j=1}^{N} c_k^j(0) \;=\; \sum_{j=1}^{N} A_{kj} \cdot c^j(0).$$

In vector notation, we can write

$$\boldsymbol{y}(0) \;=\; \boldsymbol{A}\boldsymbol{c}(0), \tag{28}$$

where $\boldsymbol{y}(0)$ is the N-vector indicating the output of each of the N sectors, $\boldsymbol{c}(0)$ is the N-vector indicating the total real spending of each of the sectors, and $\boldsymbol{A}$ is the $N \times N$ matrix with element A_{kj} in row k and column j.

Clearing of the asset market in period zero requires that in equilibrium, $\sum_{j=1}^{N} b_j(0) = a(0)$. It follows that if $a(0) \to 0$, the only way in which constraint (12) can be satisfied for all j, with $\underline{b}_j(0) = 0$ for all sectors, is if $b^j(0) \to 0$ for each sector. Thus in equilibrium, each sector must spend exactly its income, so that $c^j(0) = y^j(0)$ for each j. It then follows from (28) that $\boldsymbol{c}(0) = \boldsymbol{A}\boldsymbol{c}(0)$. Thus $\mathbf{c}(0)$ must be a right eigenvector of $\boldsymbol{A}$, with an associated eigenvalue of 1.

Such an eigenvector must exist. Using the properties of stochastic matrices discussed in Gantmacher (1959, sec. XIII.6),[20] we can further establish that 1 is the maximal eigenvalue of $\boldsymbol{A}$ (all of its $N-1$ other eigenvalues have modulus less than 1), and that the right eigenvector $\boldsymbol{\pi}$ associated with this maximal eigenvalue is non-negative in all elements.[21] If we normalize the eigenvector so that $\boldsymbol{e}'\boldsymbol{\pi} = 1$, then $\boldsymbol{\pi}$ corresponds to the stationary probability distribution of an N-state Markov chain for which $\boldsymbol{A}$ defines the transition probabilities.

[19] From here onward, we simply write ω_j and A_{kj} for the coefficients defined in (5). We omit the argument $\phi(0)$, since we only consider the alternative possible equilibria associated with different policies in the case of a particular asymmetric disturbance $\phi(0)$.

[20] See the proof of Proposition 3 in the appendix.

[21] Of course, the left eigenvector associated with the maximal eigenvalue is $\boldsymbol{e}'$, the vector of 1s.

Since this is the unique right eigenvector with an associated eigenvalue of 1, equilibrium requires that $\boldsymbol{c}(0) = \Omega\boldsymbol{\pi}$ for some scalar coefficient $\Omega \geq 0$. In order to determine the value of Ω, we recall that intertemporal optimization requires that the Euler condition (27) must hold for each sector j, and must hold with equality for any sector with $b^j(0) > 0$. In the limiting case considered in this section, $\boldsymbol{f} \to \boldsymbol{0}$, so that Lemma 4 implies that the Euler condition reduces to

$$u'(c^j(0)/\omega_j; \bar{\xi}) \;\geq\; u'(\bar{y}; \bar{\xi}). \tag{29}$$

Because of the strict concavity of $u(c)$, condition (29) can equivalently be written as

$$c^j(0) \;\;\leq\;\; c^{*j} \;\equiv\; \omega_j \bar{y}. \tag{30}$$

The value of Ω must be small enough for each element of $\boldsymbol{c}(0)$ to be consistent with this upper bound; but at the same time it must be large enough for the inequality to hold with equality for at least one sector. Thus we must have

$$\frac{1}{\Omega} \;=\; \max_j \frac{\pi_j}{\omega_j} \cdot \frac{1}{\bar{y}} \;>\; 0. \tag{31}$$

Note that here the quantities $\{\pi_j, \omega_j\}$ depend on the disturbance vector $\phi(0)$; thus we obtain a solution $\Omega = \Omega(\phi(0))$. We can summarize our results as follows.

Proposition 3. *Suppose that, despite the occurrence of an asymmetric disturbance $\phi(0)$ in period $t = 0$, all policies remain as specified in Proposition 1. And suppose further that, for a given specification of the disturbance vector, we let $a(0) \to 0$. Then in this limit, the equilibrium level of spending in each sector in period $t = 0$ is given by $\boldsymbol{c}(0) = \Omega\boldsymbol{\pi}$, where $\boldsymbol{\pi}$ is the maximal right eigenvector of the matrix $\boldsymbol{A}$ and Ω is given by (31). The equilibrium level of production by each sector is given by $\boldsymbol{y}(0) = \boldsymbol{c}(0)$, and the allocation of each sector's spending across the different goods is given by (4).*

In all periods $t \geq 1$, the equilibrium allocation of resources continues to be the one specified in Proposition 1. And all equilibrium prices (including the interest rate $i(0)$) remain those specified in Proposition 1.

In this solution, there will necessarily be at least one sector (the sector or sectors j for which the maximum value is achieved in the problem on the right-hand side of (31)) for which the borrowing constraint does not bind, and as a consequence period zero expenditure $c^j(0)$ is at the level c^{*j}. But at the same time, in the case of an asymmetric disturbance, there will generally be some sectors that are borrowing-constrained, and hence consume less than this amount. This provides a further reason for the consumption and production of some goods to be inefficiently low — not just lower than in the "normal case", but lower than would be the case in the first-best optimal allocation, taking into account the disturbance — if the shock reduces $\phi_k(0)$ in some sectors.

Corollary 1. *Suppose that $0 \leq \phi_k(0) \leq 1$ for all sectors, and that policy (and the initial level of liquid assets) are as assumed in Proposition 3. Then the quantity $c_k^j(0)$ of goods of type k consumed in sector j is necessarily no greater than the quantity associated with the first-best optimal resource allocation given the disturbance (characterized in Lemma 2). Furthermore, it will be strictly less than the first-best level (meaning also that the production $y_k(0)$ of good k will be inefficiently low), if either (i) sector j is borrowing-constrained, and $\alpha_{k-j} > 0$, or (ii) $0 < \phi_k(0) < 1$, and the function $v(y; \bar{\xi})$ is strictly convex.*

The corollary follows directly from (30), which together with (4) implies that in the absence of a binding borrowing constraint for sector j,

$$c_k^j(0) \;=\; \alpha_{k-j}\phi_k(0)\bar{y}. \tag{32}$$

In the case that $v(y;\bar{\xi})$ is strictly convex, (8) implies that $\phi\bar{y} < y^*(\phi)$ for all $0 < \phi < 1$, so that this will be an inefficiently low level of consumption of good k, if k is a good for which $0 < \phi_k(0) < 1$. (Non-borrowing-constrained consumers fail to take into account that the social cost of consuming more of good k is proportional to $v'(y_k(0);\bar{\xi})$, which is lower than the value $v'(\bar{y};\bar{\xi})$ in the "normal case" if v is strictly convex.) Moreover, even if v is linear in y (or ϕ is exactly 0 or 1), $\phi\bar{y}$ is never larger than $y^*(\phi)$. Hence the non-borrowing-constrained level of consumption is less than or equal to the first-best level for each good k. At the same time, (30) implies that a borrowing constraint can only lower spending relative to the non-borrowing-constrained level. Hence there can be no good for which consumption is too high, under the assumptions of the corollary, and it will be inefficiently low if *either* the borrowing constraint binds for sector j or $\phi_k(0)\bar{y} < y^*(\phi_k(0))$ for sector k.

One source of inefficiency in the general case is the fact that prices in period zero are pre-determined at a level that was expected to clear markets in the "normal case," but that no longer correctly reflect supply costs given the disturbance. This is not, however, the only reason why equilibrium is generally inefficient in the case of an asymmetric disturbance. We can set aside the issue of distortions owing to the fact that prices are pre-determined by considering the case of a pandemic shock in which it becomes impossible to safely consume the product of one sector, while the value of consuming other goods is unchanged. We shall (without loss of generality) suppose that sector 1 is the one impacted by the pandemic, and consider a disturbance in which $\phi_1(0) = 0$, while $\phi_k(0) = 1$ for all $k \neq 1$. In this special case, the first-best level of production continues to be $\bar{y}$ in all sectors $k \neq 1$, while it falls to zero in sector 1; thus (32) coincides with the first-best level of consumption of each good.

Nonetheless, consumption will be inefficiently low in the case of any sector j for which the borrowing constraint binds; and there will necessarily be at least one such sector (sector 1, which necessarily has no income). And in this case the inefficiency cannot be attributed to the fact that prices are not flexible. The pre-determined prices still imply relative prices for all of the goods that anyone purchases (goods $k \neq 1$) that correspond to the marginal rates of substitution in the first-best allocation (and as discussed below, it is possible in this case to achieve the first-best outcome despite the fact that prices are pre-determined, without having to use taxes or subsidies that can effectively change the prices of goods). Nonetheless, equilibrium will be inefficient, under the policies assumed in Proposition 3.

The severity of the inefficiency depends not only on the the disturbance vector $\phi(0)$, but also (crucially) on the network structure of payments. The two cases shown in Figure 1 provide contrasting examples. Figure 2 shows the equilibrium consumption vector $\boldsymbol{c}(0)$ for each of these numerical examples, in the case of the pandemic shock just discussed. In each panel of the figure, the five columns represent total expenditure by units in each of the five sectors. The height of the dashed black border indicates the "normal" level of expenditure (equal to $\bar{y}$ for each sector) — the equilibrium level of expenditure if no pandemic shock occurs, which is also the optimal allocation in that case. The height of the solid red outline for each sector indicates the level c^{*j} that would be optimal given the occurrence of the

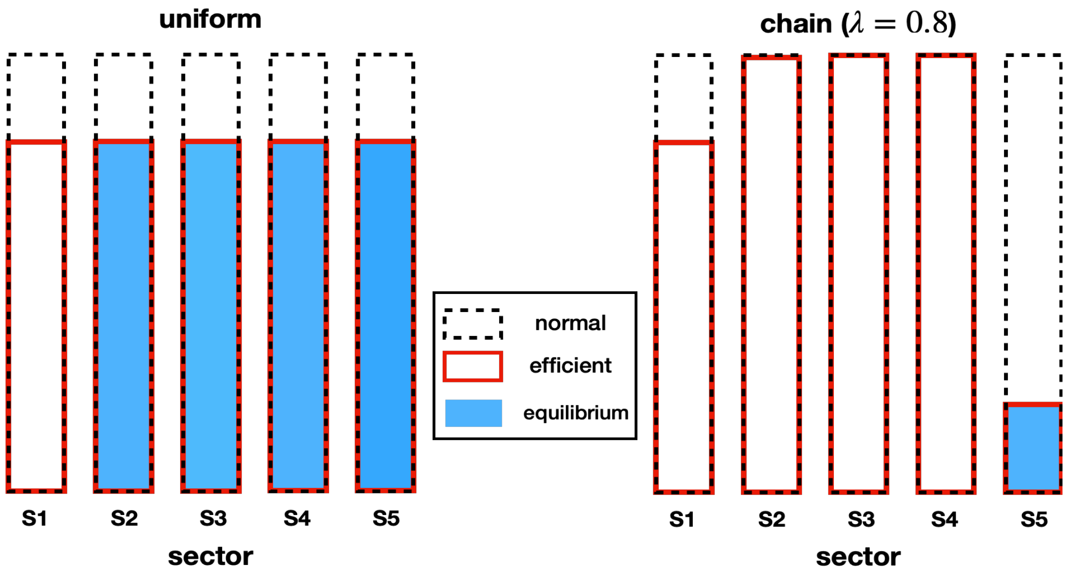

Figure 2: Equilibrium and first-best optimal sectoral expenditure levels in the case of a pandemic shock that requires sector 1 to be shut down, in the case of the two network structures shown in Figure 1.

pandemic shock. (This is necessarily no higher than the normal level and smaller for at least some sectors; thus it is optimal for expenditure and production to decline if a pandemic shock occurs.) The height of the filled blue bar instead indicates the equilibrium level of expenditure $c^j(0)$. This is necessarily no higher than the optimal level for any sector, in accordance with Corollary 1.[22]

In the case of a uniform network structure, expenditure collapses completely in sector 1 (which no longer receives any income), but it is reduced in sectors $j \neq 1$ only to the extent that it is efficient for these sectors to reduce their spending (given that they no longer can or should buy sector-1 goods). The collapse of effective demand is much more severe (and the inefficiency much greater) in the case of a "chain" network. Once again, sector 1 cannot spend at all, because it receives no income. But in the case of the "chain" network, this means that sector 2 receives no income other than its own within-sector spending, and the only consistent result is one with zero spending by sector 2 as well. Continuing iteratively in this way, one can show that every sector but sector N must have zero expenditure.

These two cases illustrate two extremes with regard to the degree of collapse of aggregate expenditure and output in the limiting case in which $a(0) \to 0$. Clearly the network structure makes an important difference. But even in the most benign case (the uniform network of payments), aggregate spending and output are inefficiently low, because spending by sector 1 is inefficiently low.[23]

[22] See the appendix, section B.5, for the algebraic calculations used in the figure.

[23] It is also clear that the empirically relevant network structure is not completely uniform, so that sector 1 will almost certainly not be the only borrowing-constrained sector in the limits as $a(0) \to 0$. See, for example, Danieli and Olmstead-Rumsey (2020), who document for the US economy that the sectoral composition of the reduction of demand in the COVID-19 crisis was non-uniform in ways that go beyond the simple fact that people in contact-intensive occupations were no longer able to work.

II.C Effective demand failure when discounting is minimal

Another case in which we can use Lemma 4 to simplify the form of the Euler condition, even when there is a severely asymmetric disturbance in period $t = 0$, is when β is very close to 1. This might be understood to represent a case in which our discrete "periods" are short in terms of calendar time, so that the asymmetric disturbance with which we are concerned is quite transitory in its effects (though possibly severe while it lasts, as in the case of the disruption of normal patterns of economic activity by COVID-19). Here we fix the single-period utility functions u and v, the coefficients $\{\alpha_h\}$ describing the within-period network structure, and the vector $\phi(0)$ specifying the nature of the asymmetric disturbance, but consider the limiting equilibrium as $\beta \to 1$.[24]

Regardless of the values $\{b^j(0)\}$ chosen by units in the various sectors, in the limit as $\beta \to 1$ we must have $\boldsymbol{f} \to \mathbf{0}$. Hence by Lemma 4, the equilibrium allocation and relative prices in periods $t \geq 1$ will involve $y_j = \bar{y}, q_j^* = 1$ for all sectors, and the Euler condition will again reduce to the simpler form (29). Because in this case we need not have $b^j(0)$ near zero for all sectors in order for this simplification to be possible, we can now consider equilibria in which the quantity of liquid assets in period zero need not be negligible, and in which borrowing may be possible as well. This allows us to consider the effects of policies that include transfers in period zero in response to the asymmetric disturbance, or credit policy that relaxes borrowing limits.

Let units in each sector j have initial liquid assets $a^j(0) \geq 0$ in period zero (including any fiscal transfers, which may make the asset balances of different sectors unequal), and a borrowing limit $\underline{b}^j(0) \leq 0$ (possibly modified by credit policy). In this case, the allocation of each unit's spending across different goods continues to be given by (4), so that (28) still holds. Because the Euler condition again reduces to (29), the upper bound on sectoral expenditure for each sector (i.e., the level of spending if the borrowing limit does not bind) continues to be given by (30). However, the level of total spending by any sector j allowed by its borrowing limit is now given by

$$c^j(0) \;\leq\; y_j(0) + \frac{a^j(0) - \underline{b}^j(0)}{\bar{p}} \;=\; \sum_k A_{jk} c^k(0) + \frac{a^j(0) - \underline{b}^j(0)}{\bar{p}}. \tag{33}$$

where the second term in each of these expressions can now differ from zero, either because $a^j(0)$ is a non-negligible amount or because borrowing is possible.

At least one of the inequalities (30) and (33) must hold with equality for each sector; this implies a consumption function

$$c^j(0) \;=\; \min\left\{ \frac{a^j(0) - \underline{b}^j(0)}{\bar{p}} + \sum_k A_{jk} c^k(0),\; c^{*j} \right\} \tag{34}$$

for each sector j. The collection of these conditions, one for each j, can be written in vector

[24] It doesn't matter whether we assume that the numerical value of the inflation target changes as "periods are made shorter." The numerical path of $\{\pi^*(t)\}$ has no consequences for the equilibrium allocation of resources, as long as we continue to assume that $i(0)$ is determined by a policy rule (16) that involves that involves the value of $P^*(1)$ implied by the target.

form as

$$\boldsymbol{c}(0) \;=\; \mathbf{min}\left\{\frac{1}{\bar{p}}\boldsymbol{\delta} \,+\, \boldsymbol{A}\boldsymbol{c}(0),\; \boldsymbol{c}^*\right\}, \tag{35}$$

where **min** is the operator that maps two N-vectors to an N-vector, each element of which is the minimum of the corresponding elements of its arguments,[25] $\boldsymbol{\delta}$ is the N-vector with elements $\delta^j \equiv a^j(0) - \underline{b}^j(0)$, and $\boldsymbol{c}^*$ is the vector of optimal expenditure levels $\{c^{*j}\}$. For any vector $\boldsymbol{\delta}$ measuring the tightness of liquidity constraints, equilibrium requires that $\boldsymbol{c}(0)$ be a fixed point of (35). This is a multidimensional generalization of the "Keynesian cross" diagram commonly used to explain the derivation of the fiscal multipliers in Keynes (1936, chap. 3).[26]

For any vector $\boldsymbol{\delta} >> \mathbf{0}$, the right-hand side of (35) defines a positive concave mapping of the kind for which the results summarized in Cavalcante *et al.* (2016) allow one to establish that there is a unique fixed point.[27] Thus equation (35) has a unique solution $\boldsymbol{c}(0) = \bar{\boldsymbol{c}}(\boldsymbol{\delta})$. In fact, the piecewise linearity of the mapping allows us to give a closed-form solution to the fixed-point problem, which can also be extended to all vectors $\boldsymbol{\delta} \geq \mathbf{0}$.

Let C be any conjecture about the subset of sectors that are borrowing-constrained (in the sense that $c^j(0) < c^{*j}$) in the solution to (35); this includes the possibility that no sectors are borrowing-constrained ($C = \emptyset$). We can restrict the set of conjectures that we need to consider by observing that for vectors $\boldsymbol{\delta}$ near enough to $\mathbf{0}$, the set of unconstrained sectors U_0 is the set of j for which the maximum value is achieved in the problem on the right-hand side of (31). Hence at such points the set of constrained sectors will be C_0, the complement of U_0. We can further show that increasing any element of $\boldsymbol{\delta}$ can only reduce the set of sectors that are borrowing-constrained; hence for any $\boldsymbol{a}(0) \geq \mathbf{0}$, the set of constrained sectors C is necessarily an element of $\mathcal{C}$, the set of all subsets of C_0 (including the empty set).

Now for any conjecture about the subset C, we can replace each of the equations in (35) by either an equation that states that (33) holds with equality (if $j \in C$), or an equation that states that (30) holds with equality (if $j \neq C$). We then obtain a system of linear equations to solve for $\boldsymbol{c}(0)$, that can be written in the form

$$\boldsymbol{\Phi}\boldsymbol{c}(0) \;=\; \boldsymbol{\Psi}_1\boldsymbol{\delta} \,+\, \boldsymbol{\Psi}_2\boldsymbol{c}^*, \tag{36}$$

where the coefficients of the matrices depend on the choice of C. We can further show the following.

Proposition 4. *For any $C \in \mathcal{C}$, the matrix $\boldsymbol{\Phi}$ is invertible, so that the system of equations (36) has a unique linear solution,*

$$\boldsymbol{c}^{loc}(\boldsymbol{\delta};\, C) \;=\; \boldsymbol{M}\boldsymbol{\delta} \,+\, \boldsymbol{N}\boldsymbol{c}^*, \tag{37}$$

where all elements of the matrices $\boldsymbol{M}$ and $\boldsymbol{N}$ are non-negative. Moreover, the fixed-point problem (35) has a unique solution, which is just the lower envelope of the finite collection of these candidate local solutions:

$$\bar{\boldsymbol{c}}(\boldsymbol{\delta}) \;=\; \mathbf{min}_{C\in\mathcal{C}}\boldsymbol{c}^{loc}(\boldsymbol{\delta}; C). \tag{38}$$

[25]That is, it is the meet of the two vectors, if $\mathbb{R}^n$ is treated as a lattice with the partial order $\leq$.

[26]Auclert *et al.* (2018) similarly propose a multidimensional generalization of the Keynesian cross, but for the allocation of spending across time, rather than across sectors at a given point in time.

[27]See the proof of Proposition 4 in the appendix for details.

And associated with this unique solution for $\boldsymbol{c}(0)$ *is a unique solution for the vector of sectoral output levels* $\boldsymbol{y}(0)$, *given by (28).*

We can now consider again the consequences of an asymmetric disturbance in period zero, under the assumption that policy remains as specified in Proposition 1, but now allowing for a non-trivial initial level of liquid assets $a(0) > 0$, again divided equally across the N sectors. One conclusion from the previous section that remains unchanged is Corollary 1.

Corollary 2. *Suppose again that* $0 \leq \phi_k(0) \leq 1$ *for all sectors, and that all policies remain as specified in Proposition 1, though the initial asset holdings* $\{a^j(0)\}$ *need not be negligible in size. And for a fixed specification of the intra-period utility functions, the aggregate disturbance vector* $\bar{\xi}$, *and the asymmetric disturbance vector* $\phi(0)$, *consider equilibrium in the limit as* $\beta \to 1$. *In this limit, the quantity* $c^j_k(0)$ *of goods of type* k *consumed in sector* j *is necessarily no greater than the quantity associated with the first-best optimal resource allocation given the disturbance (characterized in Lemma 2). Furthermore, it will be strictly less than the first-best level (meaning also that the production* $y_k(0)$ *of good* k *will be inefficiently low), if either (i) sector* j *is borrowing-constrained, and* $\alpha_{k-j} > 0$, *or (ii)* $0 < \phi_k(0) < 1$, *and the function* $v(y; \bar{\xi})$ *is strictly convex.*

Thus we conclude again that an asymmetric disturbance will generally result in inefficiently low production and consumption of at least some goods, and once again the inefficiency will be greater, the larger the number of sectors that are borrowing-constrained. The existence of a higher level of initial assets $a(0)$ makes no difference (in the $\beta \to 1$ limit) to the consumption of sectors that are not borrowing-constrained; the only difference is that, for given borrowing limits, a higher level of initial assets makes it less likely that sectors are borrowing-constrained. If the level of initial assets is high enough, no sector will be borrowing-constrained, and in this case, the equilibrium allocation of resources will be the first-best optimal one. How large the required level of initial assets is depends on the degree to which $\phi_k(0)$ is reduced in the impacted sectors. (We have shown in Proposition 1 that negligible liquid assets suffice in the case that $\phi_k(0) = 1$ for all sectors.)

But when $a(0)$ is positive but not extremely large, there can still be inefficiently low production and consumption in period zero in the event of a sufficiently severe asymmetric disturbance. Here we consider again the example of a pandemic shock that requires suspension of all consumption of the product of sector 1, in the case of the two network structures illustrated in Figures 1 and 2. Figure 3 shows how the equilibrium solution for total expenditure $c^j(0)$ for each of the sectors varies with the level of total initial liquid assets $a(0)$, under the assumption that initial liquid assets are equally distributed across sectors ($a^j(0) = a(0)/N$ for each j). The two panels present the results for the two different network structures in shown in the corresponding panels of Figure 1. In each panel, the upper envelope plots $c^{agg}(0) \equiv \sum_j c^j(0)$ as a function of $a(0)$, and the differently shaded regions decompose aggregate expenditure into the contributions from expenditure by each of the sectors.

We observe that there is a finite level of initial liquid assets (the level denoted $\hat{a}_4$ in both panels), such that $a(0) \geq \hat{a}_4$ is a sufficient condition for no sector to be borrowing-constrained. In the case of the kind of asymmetric disturbance considered here, this in turn

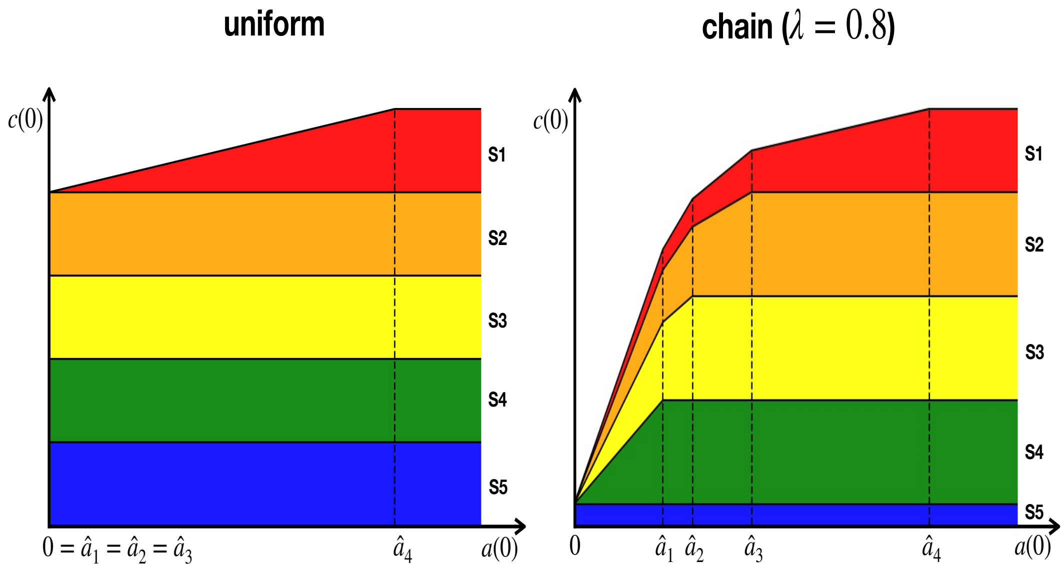

Figure 3: Equilibrium expenditure as a function of total liquid assets $a(0)$ after any transfers in response to the shock, in the case of the two network structures shown in Figure 1. Here liquid assets are assumed to be equally divided among the 5 sectors.

implies that the equilibrium allocation of resources will coincide with the first-best optimal allocation.[28] This critical level of initial assets is characterized in the following result.

Corollary 3. *Under the assumptions of Corollary 2, the unique equilibrium discussed in Proposition 4 involves no binding borrowing constraint in any sector if and only if*

$$\delta^j \;\geq\; [\sum_k \alpha_{k-j}\phi_k(0) - \phi_j(0)] \cdot \bar{p}\bar{y} \tag{39}$$

for every sector j. In this case, the equilibrium allocation of resources in period $t = 0$ is given by

$$c^j_k(0) \;=\; \alpha_{k-j}\phi_k(0)\bar{y}, \qquad y_k(0) \;=\; \phi_k(0)\bar{y} \tag{40}$$

for all j, k. In the special case that $\underline{b}^j(0) = 0$ for all j (no credit policy) and $a^j(0) = a(0)/N$ for all j (no sectorally-targeted transfers), condition (39) holds if and only if the level of initial liquid assets satisfies

$$a(0) \;\geq\; N\bar{p}\bar{y} \cdot \max_j\{\sum_k \alpha_{k-j}\phi_k(0) - \phi_j(0)\}. \tag{41}$$

Corollary 3 shows that the degree of asymmetry of the disturbance in period zero is critical for generating an effective demand failure — that is, a situation in which the inefficiency of the allocation of resources is increased by the consequences of binding borrowing constraints,

[28]It is actually not exactly exactly equal to the first-best allocation, but approaches the first-best allocation in the limit as $\beta \to 1$. Fully achieving the first-best allocation when $\beta < 1$ would require fiscal transfers that redistribute income between the sectors; see Proposition 5. But when $\beta \to 1$, the effect of this redistribution on spending in any individual period becomes negligible.

which reduce the spending both of the constrained sectors themselves and and also of any borrowing-constrained sectors that would otherwise sell more to those sectors. Proposition 1 had already indicated that this is not a problem in the case of disturbances that affect all sectors identically. But we now see that borrowing constraints will also not bind in the case of a sufficiently mild asymmetry in the effects of the disturbance (assuming a non-negligible value for $a(0)$, or at least some possibility of borrowing), since the bounds in (39) and (41) difference between the value of $\phi_j(0)$ for an individual sector and the average value of $\phi_k(0)$ for the sectors from which it purchases (weighted by their importance as suppliers for sector j); this required level of liquidity becomes larger the greater the degree of asymmetry in the elements of the vector $\phi(0)$.

Figure 3 illustrates how the level of initial liquid assets determines which sectors are borrowing-constrained in the event of a pandemic shock. For values of $a(0)$ near zero, the pattern is the one already illustrated in Figure 2, determined by the maximal eigenvector of the matrix $\boldsymbol{A}$. Since a larger quantity of initial liquid assets can only result in fewer borrowing constraints binding, in the case of the uniform network structure (where we have already seen that only sector 1 is constrained, even when $a(0) \to 0$), only sector 1 has a binding borrowing constraint for any $0 < a(0) < \hat{a}_4$, as shown in the left panel of the figure. In the case of the chain network, instead, for any $0 < a(0) < \hat{a}_1$, every sector but sector 5 is borrowing-constrained; for any $a_1 \leq a(0) < \hat{a}_2$, all sectors but 4 and 5 are borrowing-constrained; and so on, with progressively more sectors ceasing to be borrowing-constrained as the initial level of liquid assets is increased.

While the inefficiency of the allocation of resources is not as severe as those shown in Figure 2 except in the limiting case of liquid assets near zero, we see that even when liquid assets exist, effective demand failure can result in significant distortions in the event of a sufficiently severe asymmetric disturbance, as in the case of a pandemic. The outcomes shown in Figure 3, however, are still for the case of no policy response to the disturbance. We turn now to the question of how stabilization policies can mitigate the effects of such a shock.

III Fiscal Transfers and Effective Demand

We first consider what can be achieved using lump-sum taxes and transfers that are adjusted in response to the occurrence of an asymmetric disturbance. In the case that there are only aggregate disturbances, we have seen (Proposition 1) not only that an optimal allocation of resources can be achieved without any adjustment of fiscal policy in response to shocks, but that (assuming that certain bounds on the path of the public debt are respected) fiscal transfers have no effect on equilibrium prices or quantities. But neither result continues to be true in the case of an asymmetric disturbance that disrupts the circular flow of payments, to a sufficient extent to cause borrowing constraints to bind for some sectors. We now reconsider the role of fiscal transfers as a tool of stabilization policy in this context.

We begin by assuming, as in Proposition 4, that there is no monetary policy response to the asymmetric disturbance, meaning that $i(0) = \bar{\imath}$, and that the central-bank reaction function (16) for periods $t \geq 1$ continues to be the one appropriate to an environment in which no such shocks occur. The possible role of interest-rate policy (with or without fiscal transfers or credit policy as well) is deferred until the following section.

III.A Fiscal policy as "retrospective insurance"

We first consider whether it is possible to achieve the first-best optimal allocation resources, even in the case of an asymmetric disturbance, through an appropriate policy response. In general, this is not possible, if the instruments of policy are restricted to the ones specified in section I.C.

Lemma 5. *Suppose that $v(y;\xi)$ is a strictly convex function of y. Then the first-best optimal allocation of resources (characterized in Lemma 2) is not achievable as an equilibrium outcome, under any specification of interest-rate policy, credit policy, or fiscal transfers, except if there is a quantity $\bar{\phi} > 0$ such that for every sector k, either $\phi_k(0) = 0$ or $\phi_k(0) = \bar{\phi}$.*[29]

The reason is that prices are pre-determined at the values $p_k(0) = \bar{p}$. This requires a consumption allocation in period zero consistent with (4), regardless of the specification of policy; but this is inconsistent with the first-best allocation in Lemma 2, except in two special cases: either $v'' = 0$ over the range of values of y in which the sectoral output levels $\{y_k(0)\}$ all fall (so that Lemma 5 does not apply), or there is only one value of $\phi_k(0)$ for all of the sectors with $\phi_k(0) > 0$.

We can however achieve a first-best outcome in either of these special cases listed in the lemma. In the case that $v(y;\xi_0)$ is strictly convex, so that Lemma 5 applies, we shall treat the disturbance as a composition of two types of disturbance: an aggregate disturbance that determines a common value of $\bar{\phi}$ for all sectors, and an asymmetric disturbance (a "pandemic shock") that sets $\phi_k(0)$ equal to zero (rather than the common level $\bar{\phi}$) for one or more sectors k (but not for all of them). When we assume that interest-rate policy in period zero is the one that would be optimal in the absence of an asymmetric disturbance, we allow it to respond to any change that may have occurred in the value of $\bar{\phi}$. The effects of a change in $\bar{\phi}$ for all sectors can furthermore be incorporated into the effects of the aggregate state vector ξ_0. Thus in treating optimal policy for this case, we can without loss of generality assume (as we have above) that in the case of a pandemic shock, the "normal" value $\phi_k(0) = 1$ prevails in all sectors except the sector or sectors that are shut down during the pandemic.

Here we also continue, for simplicity, to write our results only for the case in which there are no aggregate disturbances ($\xi_t = \bar{\xi}$ for all $t \geq 0$).[30] Optimal policy in the case of an asymmetric disturbance can then be characterized fairly simply.

Proposition 5. *Suppose that there are no aggregate disturbances ($\xi_t = \bar{\xi}$ for all $t \geq 0$), but that there is an asymmetric disturbance $\phi(0)$ in period zero. Suppose also that either (i) $v(y;\xi_0)$ is a linear function of y, $v = \nu(\xi_0) \cdot y$; or (ii) for every sector k, either $\phi_k(0) = 0$*

[29] We may allow cases in which $\phi_k(0) = 0$ for more than one sector, if we assume weights $\{\alpha_h\}$ such that $\omega_j > 0$ for every sector j despite this. In this paper, we ensure that $\omega_j > 0$ by assuming that $\phi_k(0)$ can be zero in at most one sector.

[30] Allowing for an aggregate disturbance in period zero, such as a change in the level of $\bar{\phi}$, would require a change in the specification of optimal monetary policy in period zero, of the kind described in Proposition 1, but would not change the optimal response to an asymmetric disturbance.

or $\phi_k(0) = 1$. Finally, suppose that policy in periods $t \geq 1$ is consistent with Assumption 1. Then there exists an equilibrium in which the allocation of resources is the first-best optimal allocation defined in Lemma 2, if and only if policy in period $t = 0$ is of the following kind: (a) $i(0)$ is determined in the way specified in Proposition 1); and (b) sector-specific lump-sum transfers and taxes are used to ensure that the initial (post-transfer) assets of each sector j are equal to

$$a^j(0) \;=\; \frac{a(0)}{N} \,+\, [\sum_k \alpha_{k-j}\phi_k(0) \,-\, \phi_j(0)] \cdot \bar{p}\bar{y} \tag{42}$$

for some $a(0) > 0$. (Credit policy is irrelevant: the result holds for any borrowing limits satisfying $\underline{b}^j(0) \leq 0$ for all j.) In the equilibrium associated with this policy, end-of-period balances are the same for all sectors: $b^j(0) = a(0)/N$ for all j. As a consequence, borrowing constraints do not bind in any sector, and the allocation of resources in period zero is given by (40). Equilibrium prices and quantities in all periods $t \geq 1$ are as specified in Proposition 1.

Among the cases to which this result applies is the kind of pandemic shock considered in Figures 2 and 3.

Note that achievement of the first-best optimum requires no response of monetary policy to the disturbance (in the cases where it can be achieved at all); the optimal interest-rate policy in period zero is unchanged in the absence of an aggregate shock. Nor does it require any use of credit policy; given monetary and fiscal policies of the kind described, credit policy is irrelevant, as borrowing constraints are in any event not binding for any sector. But it *is* essential that sector-specific fiscal transfers respond appropriately to the realized vector $\phi(0)$, as indicated in (42).

It is also noteworthy that condition (42) leaves the total quantity of liquid assets $a(0)$ (and hence the size of any government deficit) indeterminate; under an optimal policy, the path of the public debt $\{a(t)\}$ need not respond to the asymmetric disturbance. What the condition *does* require is redistribution between sectors, depending on the way in which they are impacted by the realization of the asymmetric disturbance.

There is furthermore a simple interpretation for equation (42). In the equilibrium that would occur in the absence of any asymmetric disturbance (i.e., if $\phi(k) = 1$ for all k), consumption would be given by $c^j_k(0) = \alpha_{k-j}\bar{y}$ for all j, k. Then if units in each sector would begin period zero with assets $a(0)/N$ in the absence of a disturbance, (42) specifies that as a result of the disturbance, each unit receives a transfer in the amount by which spending on its product is reduced after the disturbance,

$$\bar{p}\bar{y} - \sum_\ell \phi_j\alpha_{j-\ell}\bar{p}\bar{y} \;=\; (1 - \phi_j)\bar{p}\bar{y},$$

and pays a lump-sum tax equal to the amount by which it reduces its spending after the disturbance,

$$\bar{p}\bar{y} - \sum_k \phi_k\alpha_{k-j}\bar{p}\bar{y} \;=\; \left(1 - \sum_k \alpha_{k-j}\phi_k\right)\bar{p}\bar{y}.$$

These taxes and transfers balance (so that no change in the size of the public debt is required). They exactly compensate for the disruption to the circular flow of payments that the new pattern of spending would otherwise involve, so that all units end period zero with equal asset balances, and a balanced circular flow of payments continues to be possible in all subsequent periods, as in the equilibrium in Proposition 1, without any further need for sector-specific taxes or transfers.[31]

The optimal fiscal policy in this case amounts to "retrospective insurance" of the kind called for by Milne (2020);[32] fiscal policy implements the state-contingent transfers that would have been privately agreed upon in (counter-factual) ideal ex-ante contracting.[33] Note that the case for retrospective insurance made here is independent of the central argument of Milne (2020) or Saez and Zucman (2020), that it is important to prevent business failures, because of the costs involved in re-establishing such businesses once they have failed. While we do not deny that this should also be an important concern, it is not the only reason why a retrospective insurance policy would be efficient (at least if we abstract from the cost of administration of such a policy). Even abstracting from any cost of restarting economic activities in period 1 that did not take place in period zero, as in the present model, the retrospective insurance transfers increase ex-ante welfare.

Note also that while we have referred to implementation of the policy by levying state-contingent lump-sum taxes on the sectors that spend less in the event of a pandemic (because many goods that they normally enjoy are temporarily unavailable) while their incomes are not greatly impacted, the first-best outcome can also be achieved without levying sectorally-targeted taxes on anyone (if one supposes that such taxes are politically unacceptable, while sectorally-targeted transfers are acceptable). Because the value of $a(0)$ in (42) is indeterminate (except for the lower bound), it is possible to choose $a(0)$ higher than the quantity of liquid assets that people held prior to the realization of the disturbance, meaning that the government makes net transfers to the private sector (running a deficit in period zero) in response to the disturbance. By choosing a high enough value of $a(0)$, it is possible to ensure that all sectors receive non-negative transfers in period zero. The larger public debt will require correspondingly larger tax collections in later periods; and even though we assume that tax obligations must be the same for all sectors in periods $t \geq 1$, some sectors will end up receiving transfers in period zero greater than the present value of their increased future taxes, while other sectors receive less than that amount. Thus the policy still amounts to a redistribution between sectors, according to how their budgets have been impacted by the asymmetric disturbance, though there need not be sectorally-targeted taxes in any period.

[31] Of course, the fact that such taxes and transfers are not needed after $t = 0$ depends on our assumption that there are no asymmetric disturbances after $t = 0$. In a more general model, similar sectorally-targeted taxes and transfers would be needed each period to compensate for asymmetric disturbances, when they occur.

[32] See also Saez and Zucman (2020) for a similar proposal.

[33] See Woodford (2020) for an explicit analysis of the equilibrium with ex-ante contingent claims contracts, in a generalization of the model presented here. State-contingent transfers as specified in (42) would be voluntarily chosen ex ante by all units, owing to the ex-ante rotational symmetry of their situations.

III.B The matrix of fiscal transfer multipliers

Let us now consider more generally what can be achieved by fiscal transfers, that are not necessarily of the precise form specified in (42). When $\beta < 1$, we will in general have $\boldsymbol{f} \neq \mathbf{0}$, in which case analytical solutions are difficult. We can again obtain simpler conclusions by considering the limit as $\beta \to 1$, as in section II.C.

Rather than there being a single "transfer multiplier," in our model there is an $N \times N$ matrix $\boldsymbol{M}$ of multipliers, the elements of which are

$$M_{jk} \;\equiv\; \bar{p} \cdot \frac{\partial c^j(0)}{\partial \delta^k},$$

understood to be right derivatives.[34] The matrix of multipliers is the same as the matrix $\boldsymbol{M}$ in (37),[35] which we now write more explicitly. For any vector $\boldsymbol{\delta}$, there is a set C of sectors that are borrowing-constrained, in the sense that $c^j(0) < c^{*j}$ in the solution (38), and a complementary set U (containing at least one sector) that is unconstrained. (Note that C may be the empty set.) If we order the sectors so that all of the sectors in C (if any) come first, then the matrices $\boldsymbol{A}$ and $\boldsymbol{M}$ can be partitioned as

$$\boldsymbol{A} \;=\; \begin{bmatrix} \boldsymbol{A}_{CC} & \boldsymbol{A}_{CU} \\ \boldsymbol{A}_{UC} & \boldsymbol{A}_{UU} \end{bmatrix}, \qquad \boldsymbol{M} \;=\; \begin{bmatrix} \boldsymbol{M}_{CC} & \boldsymbol{M}_{CU} \\ \boldsymbol{M}_{UC} & \boldsymbol{M}_{UU} \end{bmatrix}, \tag{43}$$

where submatrix $\boldsymbol{A}_{CC}$ measures the share of spending by each of the constrained sectors on the products of other constrained sectors, and so on.[36] We can then use this notation to write an explicit expression for the local linear solution (37).

Lemma 6. *For any specification $C \in \mathcal{C}$ of the set of borrowing-constrained sectors, let the matrix $\boldsymbol{A}$ be partitioned as in (43). Then $\boldsymbol{I} - \boldsymbol{A}_{CC}$ is an invertible matrix, and $[\boldsymbol{I} - \boldsymbol{A}_{CC}]^{-1} >> \mathbf{0}$. It follows that the local linear solution to the system (35) can be written in the form (37), with matrices of coefficients*

$$\boldsymbol{M} \;=\; \begin{bmatrix} (\boldsymbol{I} - \boldsymbol{A}_{CC})^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \qquad \boldsymbol{N} \;=\; \begin{bmatrix} \mathbf{0} & (\boldsymbol{I} - \boldsymbol{A}_{CC})^{-1}\boldsymbol{A}_{CU} \\ \mathbf{0} & \boldsymbol{I} \end{bmatrix},$$

and all elements of the matrices $\boldsymbol{M}$ and $\boldsymbol{N}$ are non-negative. Moreover, we can equivalently write the non-zero block of $\boldsymbol{M}$ in the form

$$\boldsymbol{M}_{CC} \;=\; \sum_{r=0}^{\infty} (\boldsymbol{A}_{CC})^r. \tag{44}$$

[34]These right derivatives everywhere well-defined, even though the right derivatives can differ from the corresponding left derivatives at values of $\boldsymbol{\delta}$ where the borrowing constraint just ceases to bind for some sector.

[35]Under our definition of C, the set C must remain the same in the case of small increases in any of the elements of $\boldsymbol{\delta}$. Hence the solution (37) continues to apply, and the matrix $\boldsymbol{M}$ in this solution is the matrix of right derivatives.

[36]Note that the ordering of sectors required for this notation, as well as the partitioning of the rows and columns, depends on the set C. Thus $\boldsymbol{A}_{CC}$ has a different meaning for different vectors $\boldsymbol{\delta}$, in the notation used here, even though the matrix $\boldsymbol{A}$ can be given a representation that is independent of $\boldsymbol{\delta}$.

Our model implies that the multiplier effects of fiscal transfers can be quite different, depending both on the sectors receiving the transfer and which sectors' expenditure we are concerned with. One reason why it matters which sectors receive the transfers is that the marginal propensity to consume out of additional income is different for units in different sectors, owing to differences in the degree to which sectors are borrowing-constrained, as stressed by Oh and Reis (2012). But it also generally matters how transfers are allocated among the different borrowing-constrained sectors, as the size of "higher-round effects" are generally different for different sectors.

Lemma 6 implies that the multipliers are all zero, except those in the block $\boldsymbol{M}_{CC}$, indicating the effects of transfers to borrowing-constrained sectors on the spending by other borrowing-constrained sectors. The solution (44) for this sub-matrix expresses the total "multiplier effect" of fiscal transfers as the sum of a "first-round" effect (a unit effect on spending by the sector receiving the transfer), a "second-round" effect (additional spending resulting from the increases in income due to the "first-round" effects), and so on. Note, however, that in our model even the "first-round" effects exist only in the case of transfers to borrowing-constrained units; "second-round" effects exist only to the extent that "first-round" spending increases involve purchases from borrowing-constrained units, and so on.[37] This implies that in the event of a pandemic shock that suspends consumption of one sector's products, multiplier effects beyond the "first-round" effect can exist only if $N > 2$;[38] hence Guerrieri *et al.* (2020) find no such effects in their baseline model.

It is possible, in principle, for fiscal transfer multipliers to be sizeable. For example, in the case of the numerical example shown in the right panel of Figure 3, the aggregate expenditure multiplier for a uniformly distributed lump-sum transfer, when pre-transfer asset balances are small enough,[39] is equal to 2.45, while the aggregate expenditure multiplier for transfers targeted to units in sector 1 only would equal 4.75. But the multipliers depend not only on how the transfers are targeted, but on the network structure of payments. In the case of the uniform network structure shown in the left panel of Figure 1, the aggregate expenditure multiplier for a uniformly distributed lump-sum transfer is only 0.20, even when initial asset balances are extremely small.

Figure 3 illustrates a general principle: transfer multipliers are largest when existing liquid asset balances are low (and borrowing constraints are tight). As the size of the transfers is increased, the multiplier effect of further transfers will generally decline, as additional sectors cease to be borrowing-constrained. The following observation is a direct consequence of Proposition 4.

Corollary 4. *Suppose that all policies remain as specified in Proposition 1, except that*

[37]The conclusion that the multiplier is zero in the case of transfers to sectors that are not borrowing-constrained depends on the simplification of considering the limiting case in which $\beta \to 1$. When $\beta < 1$, there is instead a small effect on the current spending of unconstrained units of receiving greater-than-average transfers, so that their intertemporal budget increases despite the anticipation of higher future taxes. Nonetheless, the effects on current spending by such units remain small, under realistic assumptions about the discount factor, because of their desire to smooth expenditure over a long horizon.

[38]If $N = 2$, the set of constrained sectors C must consist only of sector 1, as there must be at least one unconstrained sector. And $A_{11} = 0$, since no one can purchase sector-1 goods during the pandemic. Hence $A_{CC} = 0$, and all terms corresponding to $r \geq 1$ in (44) vanish.

[39]The calculations are explained in the appendix, section C.4.

(possibly sector-specific) lump-sum transfers occur at the beginning of period zero, in response to the asymmetric disturbance. Let post-transfer initial asset holdings be

$$a^j(0) \;=\; \alpha^j \;+\; \gamma^j \cdot s \tag{45}$$

for each sector j, where the quantities $\alpha^j \geq 0$, with $\sum_j \alpha^j > 0$, denote each sector's pre-transfer asset holdings; the coefficients $\gamma^j \geq 0$, with $\sum_j \gamma^j = 1$, indicate the share of transfers going to units in each sector j; and $s \geq 0$ indicates the total size of the transfer. For this one-parameter family of possible transfer policies, let

$$m_j(s) \;\equiv\; \bar{p} \cdot \frac{\partial c^j(0)}{\partial s}$$

(where again we mean a right derivative) measure the multiplier effect of additional transfers on spending by sector j, for any possible scale of transfers s. Then for each sector j, $m_j(s)$ is a piecewise constant, non-increasing function of s, that falls to zero for all $s \geq \bar{s}$, where $\bar{s}$ is finite.

Given a matrix $\boldsymbol{M}$ of multiplier effects on sectoral expenditure levels, the matrix $\boldsymbol{M}^Y \equiv \boldsymbol{AM}$ then indicates the multiplier effects of transfers to each of the sectors on economic activity (output) of each of the sectors. Since the elements of $\boldsymbol{M}$ are all non-negative, the output multipliers $\boldsymbol{M}^Y$ are all non-negative as well. And it follows from Corollary 4 that the output multipliers must decrease as the size of transfers is increased, falling to zero in the case of large enough transfers.

III.C Transfer policy and welfare

We have shown that the multiplier effect of lump-sum transfers on economic activity in each of the different sectors is necessarily non-negative. But a more reasonable goal of policy, of course, should be not to increase economic activity as an end in itself, but rather to increase welfare. Here we consider the effect of lump-sum transfers on the ex-ante welfare measure derived in Lemma 1.

We first note that the ex-ante welfare measure can equivalently be chosen to be

$$\sum_{j=1}^{n} U^j(0) \;+\; \frac{\beta}{1-\beta} \sum_{j=1}^{N} [\bar{U}^j - U^*], \tag{46}$$

where $\bar{U}^j$ is the stationary level of the utility flow (2) associated with the stationary allocation for all $t \geq 1$ characterized in Lemma 3, and U^* is the value of that stationary utility flow (the same for all j) in the first-best allocation of resources for periods $t \geq 1$. For a given specification of preferences, this differs from (7) only by a constant, which does not affect the welfare ranking of alternative possible allocations of resources; but it has the advantage over (7) of being a measure with a well-behaved limit as $\beta \to 1$.

Lemma 7. *Suppose that policy in all periods $t \geq 1$ is consistent with Assumption 1, and that we specify policy in period zero by values $a^j(0) \geq 0$ such that $\sum_j a^j(0) > 0$, values $b^j(0) \leq 0$,* *and a parameter* $\psi \equiv \beta(1 + i(0))\bar{p}/P^*(1) > 0$ *which measures the discrepancy between the real rate of interest and the "natural rate" defined in (17). Let us consider different values for the discount factor* β*, while holding fixed all other aspects of preferences, and the policy parameters just listed; and for all values of* β *close enough to 1, let the equilibrium for periods* $t \geq 1$ *be the stationary equilibrium characterized in Lemma 3. Then as* β *approaches 1, we have*

$$\lim_{\beta \to 1} \frac{\beta}{1-\beta} \sum_{j=1}^{N} [\bar{U}^j - U^*] \;=\; 0 \tag{47}$$

for any policy. Hence one policy implies a higher value for (46) in the limit as $\beta \to 1$ *if and only if it implies a higher value for*

$$W_0 \;\equiv\; \sum_{j=1}^{N} U^j(0). \tag{48}$$

It follows that in evaluating welfare under alternative policies in the limiting case in which $\beta \to 1$, we need consider only the effects on the policy on W_0, which involves only the allocation of resources in period zero.

We can then use our conclusions about the multiplier effects of lump-sum transfers to calculate effects of such transfers on the welfare measure W_0. For each sector j, let $w_j \equiv \bar{p} \cdot \partial W_0 / \partial a^j(0)$ be the effect on welfare per unit transfer to sector j, for fixed values of the other policy parameters (in the limit as $\beta \to 1$); and let $\boldsymbol{w}$ be the vector with these components (i.e., the welfare gradient). Then differentiation of (48) implies that

$$\boldsymbol{w}' \;=\; [\boldsymbol{g}' + \boldsymbol{h}'\boldsymbol{A}]\boldsymbol{M}, \tag{49}$$

where $\boldsymbol{g}$ and $\boldsymbol{h}$ are the vectors with elements

$$g_j \;\equiv\; u'(c^j(0)/\omega_j;\, \bar{\xi}) \;-\; u'(\bar{y};\bar{\xi}), \qquad h_j \;\equiv\; v'(\bar{y};\bar{\xi}) \;-\; v'(y_j(0);\, \bar{\xi}).$$

We further observe that $g_j > 0$ whenever $c^j(0) < c^{*j} \equiv \omega_j \bar{y}$, and that $h_j \geq 0$ whenever $y_j(0) < \bar{y}$. This allows us to sign the elements of the welfare gradient under fairly general conditions.

Proposition 6. *Suppose that* $0 \leq \phi_k(0) \leq 1$ *for all sectors, and consider the effects of small additional lump-sum transfers, starting from a situation in which policy is specified as in Lemma 7, with* $\psi = 1$ *(i.e., monetary policy as in Proposition 1). Then* $\boldsymbol{w} \geq \boldsymbol{0}$*, so that a lump-sum transfer cannot reduce welfare (in the* $\beta \to 1$ *limit), no matter how large the transfer may be, and no matter how it is distributed across sectors. Moreover,* $w_j > 0$ *(so that a transfer to sector* j *increases welfare) if and only if sector* j *is borrowing-constrained in the absence of the additional transfer (i.e., if* $j \in C$*), while* $w_j = 0$ *if the sector is unconstrained.*

Thus not only is it possible for lump-sum transfers to increase welfare; in the $\beta \to 1$ limit, they necessarily increase welfare, as long as at least some of the transfers go to sectors that would be borrowing-constrained in the absence of the transfers. Thus careful targeting of transfers is not necessary in order for them to increase welfare. Moreover, in the case that it is possible to achieve the first-best optimal allocation with some combination of policies (i.e., when this is not precluded by Lemma 5), in the $\beta \to 1$ limit this no longer requires the carefully targeted transfers specified in Proposition 5. Instead, the first-best outcome can be achieved by large enough lump-sum transfers, almost independently of how they are distributed.

Proposition 7. *Suppose again that the assumptions of Proposition 5 are satisfied. Then in the limit as $\beta \to 1$, the equilibrium allocation of resources approaches the first-best optimal allocation defined in Lemma 2, if policy in period $t = 0$ is of the following kind: (a) $i(0)$ is determined in the way specified in Proposition 1; and (b) lump-sum transfers ensure that (39) holds for every sector j. In this limiting equilibrium, borrowing constraints do not bind in any sector, and the allocation of resources in period zero is given by (40). Equilibrium prices and quantities in all periods $t \geq 1$ are as specified in Proposition 1.*

Note that condition (39) is satisfied by almost any distribution of lump-sum transfers, as long as they are large enough. In the case of a transfer policy of the form (45), the condition is satisfied for all large enough values of s, as long as $\gamma^j > 0$ for each of the sectors with $\phi_j(0) < \sum_k \alpha_{k-j}\phi_k(0)$.[40] Thus transfers need not be sectorally-targeted at all; in the case of uniform transfers (so that $a^j(0) = a(0)/N$ for all j), the condition is satisfied, and the first-best optimal allocation will be achieved, as long as $a(0)$ is large enough.[41]

When $\beta < 1$, a more specific pattern of transfers is required in order to achieve the first-best outcome, as indicated in Proposition 5, because the transfers must adjust sectoral budgets so that (i) borrowing constraints no longer bind in period zero for any sector, and (ii) all sectors end period zero with identical end-of-period balances. But in the limit as $\beta \to 1$, condition (ii) ceases to matter, as differences in the assets carried into period one by different sectors have only a negligible effect on per-period spending in each of the periods $t \geq 1$; the marginal utility of income remains essentially the same for all sectors, as indicated by Lemma 4. In this case, it is important that transfers be large enough to prevent borrowing constraints from binding in any sector, but no significant distortions result from "unnecessary" transfers to some sectors.

Note also that Proposition 7 implies that if $v(y; \bar{\xi})$ is a linear function of y, it is possible to obtain the first-best allocation of resources despite the occurrence of an asymmetric disturbance, even when policy is of the kind specified in Proposition 1, so that policy does not respond to the asymmetric disturbance at all. This result obtains if the initial level of liquid assets $a(0)$ satisfies (41) even in the absence of any fiscal transfers in response to the shock. For a given level $a(0) > 0$, the condition is satisfied as long as the sectoral non-uniformity of the disturbance $\phi(0)$ is not too great. Our result here recalls Leijonhufvud's (1973) concept of a "corridor" within which market mechanisms can be relied upon to be self-stabilizing, so that Keynesian policies are needed to restore proper functioning only in the case of disturbances large enough to move the economy outside the "corridor." Our account of the nature of effective demand failures is in many ways similar to Leijonhufvud's, but it implies that "the corridor" should be defined not as a state in which shocks to the

[40] In the case of a pandemic shock that shuts down one sector without affecting the others, this condition requires only that some of the transfers go to the sector directly impacted by the shock.

[41] In the case of no credit policy ($\underline{b}^j(0) = 0$ for all j), the required level of $a(0)$ is given by (41).

economy are sufficiently small, but rather as one in which they are sufficiently symmetric in their effects on the income and spending of different parts of the economy.

III.D Credit policy as stabilization policy

Credit policy can also be a useful response to disruption of the circular flow of payments by an asymmetric disturbance. Indeed, in our model, relaxation of sectoral borrowing limits (by allowing government-guaranteed borrowing up to a certain amount) has effects that are similar to the effects of transfers to units in that sector.[42]

In particular, in our model a uniform relaxation of the period-zero borrowing limit in each sector by a common amount Δ (reducing $\underline{b}^j(0)$ by Δ for each j) has an identical effect as a uniform lump-sum transfer of Δ (increasing $a^j(0)$ by Δ for each j). In either case, the amount that each unit can spend in period zero without violating their borrowing constraint is increased by Δ; and the present value of their future obligations (future lump-sum taxes in the case of the transfer policy, debt repayments in the case of credit policy) is increased by exactly Δ as well. Since we have shown that sufficiently large uniform transfers can achieve the first-best outcome in the $\beta \to 1$ limit (Proposition 7), it follows that a sufficiently large uniform relaxation of borrowing limits can achieve the first-best outcome in this limiting case as well.

In the case that the degree of relaxation of borrowing limits differs across sectors, the effects of credit policy are no longer identical to those of similarly distributed transfers, because the additional borrowing allowed owing to credit policy must be repaid by the units that increase their borrowing, while transfers increase the future tax obligations of all units uniformly even when the transfers are not uniformly distributed. However, in the limit as $\beta \to 1$, this difference in the distribution of repayment obligations in periods $t \geq 1$ has only a negligible effect on the allocation of resources. Hence in this limit, the effects of a policy that reduces $\underline{b}^j(0)$ by an amount Δ^j (that may differ across sectors) are identical to those of a lump-sum transfer of Δ^j to units in sector j at the beginning of period zero. This can be seen from the fact that equations (35) and the solution (38) involve only the vector $\boldsymbol{\delta}$, which is affected in the same way by sectorally-targeted transfers and sector-specific credit policy. Thus the matrix of multipliers characterized in Lemma 6 applies equally to the effects of sector-specific credit policy on expenditure in the different sectors.

There are nonetheless respects in which credit policy and transfer policy should not be regarded as perfect substitutes for one another. First, when $\beta < 1$, the differing effects on repayment obligations in periods $t \geq 1$ do matter. Hence in Proposition 5, we are able to specify sectorally-targeted transfers that are necessary in order to achieve the first-best outcome, *regardless* of what is assumed about credit policy; it would not be possible to achieve the first-best outcome (except in the limit as $\beta \to 1$) using credit policy alone. This is an advantage of fiscal transfers over credit policy as a tool of stabilization policy. At the same time, there is an advantage of credit policy as well. Our model assumes that government debt can be repaid through revenues raised by lump-sum taxes, but in practice,

[42] Bigio *et al.* (2020) also compare credit policy with fiscal transfers, in a more developed model of how borrowing constraints are determined. Araújo and Costa (2021) similarly discuss how a modification of bankruptcy law in response to a pandemic shock, or endogenous renegotiation of private loan contracts, can substitute for fiscal transfers in improving stabilization.

distorting taxes may be the only source of revenue. If so, credit policy has the advantage of not creating the distortions that would result from an increase in future taxation of labor income (for example) in order to finance the transfers. This is an advantage of using credit policy, at least to some extent, if the associated administrative costs are not too great.

IV The Role of Interest-Rate Policy

In the previous section, we have considered the effects of fiscal transfers and credit policy, under the assumption that interest-rate policy does not respond at all to the asymmetric disturbance. We now consider what can be achieved by a response of interest-rate policy to such a disturbance. Propositions 5 and 7 have shown that under certain conditions, fiscal transfers can achieve the first-best allocation of resources without any need for a change in monetary policy. But what if fiscal policy does not respond, or does not respond ideally, owing to political or administrative constraints? To what extent can monetary policy be used instead?

In the equilibria shown in Figure 3 for cases in which the initial level of liquid assets satisfies $a(0) < \hat{a}_4$, equilibrium output is below the efficient level (given the disturbance to fundamentals) in some or all of the sectors not directly affected by the pandemic shock. It is *not* inefficiently low in the impacted sector, sector 1, since it is optimal for production to cease temporarily in that sector; but there are *no* sectors in which activity is inefficiently *high*, in the absence of a policy response.

Given Proposition 2, it might seem natural to suppose that the central bank's interest-rate target should be cut in response to such a real disturbance. And our model is one in which equilibrium activity in period zero can be increased or decreased by interest-rate policy; thus to the extent that one thinks about stabilization policy in terms of an aggregate output gap, it should be possible to eliminate the gap entirely by a sufficiently large cut in interest rates, assuming that the effective interest-rate lower bound does not preclude this. Thus it is often supposed that if counter-cyclical fiscal policy is also needed, this is only because the lower bound may not allow interest rates to be reduced to the degree needed in the case of a severe disturbance. And in fact our model is one in which the zero lower bound never constrains how much it should be possible to reduce the real interest rate (which is what matters for aggregate demand), if the central bank is willing to commit itself to more inflationary policy in the future.

Nonetheless, monetary policy remains a decidedly second-best policy instrument for dealing with the inefficiencies created by an effective demand failure resulting from a pandemic shock. The problem is not that interest-rate policy cannot increase economic activity in these circumstances — the elasticity of aggregate output with respect to changes in the real interest rate is determined by the intertemporal elasticity of substitution, in the same way as in the case of the response to aggregate disturbances considered in Proposition 2. It is rather that the composition of the added expenditure that can be stimulated by interest-rate cuts will necessarily be inefficient, and (depending on the network structure of payments) may be severely so.

IV.A Interest-rate policy when liquidity is negligible

Let us first consider how the analysis in section II.B is changed if we suppose that the central bank cuts $i(0)$ in response to the asymmetric disturbance. It continues to be the case that if $a(0)$ is small and $\underline{b}^j(0) = 0$ for all j, the equilibrium from period $t = 1$ onward must be one with $\boldsymbol{f}$ near $\mathbf{0}$. However, (29) now takes the more general form

$$u'\left(c^j(0)/\omega_j;\, \bar{\xi}\right) \;\geq\; \psi u'(\bar{y};\, \bar{\xi}), \tag{50}$$

where $\psi > 0$ is the coefficient defined in Lemma 7, measuring the degree to which $i(0)$ differs from the "normal" policy assumed in Proposition 1. (A value $\psi < 1$ means that the interest-rate target is reduced in response to the asymmetric disturbance.) This in turn implies that (30) takes the more general form

$$c^j(0) \;\leq\; \hat{c}^j(\psi) \;\equiv\; \omega_j \hat{y}(\psi), \tag{51}$$

generalizing (30), where $\hat{y}(\psi)$ is the quantity implicitly defined by

$$u'(\hat{y}(\psi);\, \bar{\xi}) \;=\; \psi u'(\bar{y};\, \bar{\xi}). \tag{52}$$

Because $u(c; \bar{\xi})$ is strictly concave, $\hat{y}(\psi)$ is a monotonically decreasing function.

We then obtain the following generalization of Proposition 3.

Proposition 8. *Suppose that, despite the occurrence of an asymmetric disturbance $\phi(0)$ in period $t = 0$, all policies remain as specified in Proposition 1, except that $i(0)$ responds to the shock. And suppose further that, for a given specification of the disturbance vector and the policy response ψ, we let $a(0) \to 0$. Then in this limit, the equilibrium level of spending in each sector in period $t = 0$ is given by $\boldsymbol{c}(0) = \Omega \boldsymbol{\pi}$, where $\boldsymbol{\pi}$ is the maximal right eigenvector of the matrix $\boldsymbol{A}$ and Ω is given by*

$$\frac{1}{\Omega} \;=\; \max_j \frac{\pi_j}{\omega_j} \cdot \frac{1}{\hat{y}(\psi)} \;>\; 0, \tag{53}$$

where $\hat{y}(\psi)$ is defined in (52). The equilibrium level of production by each sector is given by $\boldsymbol{y}(0) = \boldsymbol{c}(0)$, and the allocation of each sector's spending across the different goods is given by (4).

In all periods $t \geq 1$, the equilibrium allocation of resources continues to be the one specified in Proposition 1. And all equilibrium prices (except the interest rate $i(0)$) remain those specified in Proposition 1.

Note that (53) reduces to (31) under our previous assumption that $\psi = 1$.

It follows that our model implies that $c^{agg}(0)$, and correspondingly aggregate output $y^{agg}(0)$, increases in proportion to $\hat{y}(\psi)$ if the interest-rate target is changed. This is the same interest-rate elasticity of aggregate output as exists in the case that only aggregate disturbances exist: in that case, $c^j(0) = y_j(0) = \hat{y}(\psi)$ for every sector, so that also in that case $y^{agg}(0)$ grows in proportion to $\hat{y}(\psi)$. Thus the fact that borrowing constraints may bind for many sectors need not imply any lower interest-elasticity of output in the case of an effective demand failure. And the Inada conditions assumed for the function $u(c; \bar{\xi})$ imply that $\hat{y}$ can be driven arbitrarily close to zero by raising the real interest rate enough, and made arbitrarily large by lowering the real interest rate enough;[43] thus the model implies that a very great degree of control over aggregate output (in the short run) is possible using monetary policy, even during the crisis created by a pandemic shock.

Nonetheless, monetary policy is not well-suited to correct the distortions created by a pandemic shock. While lowering interest rates should increase spending and hence output, the sectoral composition of the spending and output that are stimulated need not correspond to the kinds are most needed in order to increase welfare. In fact, an interest-rate cut need not increase welfare at all, as shown by the following (admittedly special) example.

Corollary 5. *Let the assumptions of Proposition 8 be satisfied. In addition, suppose that $v(y;\bar{\xi}) = \nu \cdot y$ for some $\nu > 0$ (that may depend on $\bar{\xi}$), and that the disturbance $\phi(0)$ is such that the sectors that are borrowing-constrained in period zero are unable to consume at all. Then W_0 is maximized when $\psi = 1$; that is, if there is no response of monetary policy to the disturbance.*

The cases shown in Figure 2 provide two examples in which the assumption in the corollary about the equilibrium pattern of consumption (which depends on the eigenvector $\boldsymbol{\pi}$) is satisfied. In such a case, there is no spending by borrowing-constrained sectors, and no production by those sectors either. A reduction of $i(0)$ increases spending only in the unconstrained sectors, and only increases production in those same sectors. Hence only types of consumption $c_k^j(0)$ increase that are already at the efficient level when $\psi = 1$; further increases in this kind of spending increases ex ante utility by an amount less than the increase in the disutility of supplying the goods, and ex-ante welfare is necessarily decreased. Thus while the allocations of resources depicted in Figure 2 are far from the first-best optimal allocation (especially in the case shown in the right panel), welfare cannot be improved in either of these cases by cutting interest rates.

IV.B Interest-rate policy when discounting is minimal

Let us next consider how the analysis in section II.C is modified if interest-rate policy responds to the asymmetric disturbance. In the limit as $\beta \to 1$, we again must have $\boldsymbol{f} \to \boldsymbol{0}$, regardless of the policy chosen in period zero. The Euler condition again reduces to (50), which again implies (51). The derivation of the consumption function (34) then proceeds as before, except that we must replace the constant c^{*j} by the upper bound in (51), that depends on ψ. The system of equations that determine the sectoral expenditure levels then takes the form

$$\boldsymbol{c}(0) \;=\; \mathbf{min}\left\{ \frac{1}{\bar{p}}\boldsymbol{\delta} \;+\; \boldsymbol{A}\boldsymbol{c}(0),\; \boldsymbol{\omega}\cdot\hat{y}(\psi) \right\}, \tag{54}$$

generalizing (35), where $\boldsymbol{\omega}$ is the vector with jth element equal to ω_j.

For a given specification of policy $(\boldsymbol{\delta}, \psi)$, we can show that the "multivariate Keynesian cross" system (54) has a unique fixed point. This allows the following characterization of equilibrium under an arbitrary monetary policy.

[43] Because of the zero lower bound on the nominal interest rate, of course, very low real interest rates can be achieved only by creating an expectation of high inflation.

Proposition 9. *Suppose that policy in all periods is consistent with Assumption 1, and that policy in period zero is specified as in Lemma 7. Then in the limit as $\beta \to 1$, there is a unique equilibrium in which both prices and the allocation of resources in all periods $t \geq 1$ are the ones specified in Proposition 1. In period zero, the vector of sectoral expenditure levels is*

$$\boldsymbol{c}(0) \;=\; \boldsymbol{c}(\boldsymbol{\delta};\psi) \;=\; \frac{\hat{y}(\psi)}{\bar{y}} \cdot \bar{\boldsymbol{c}}\left(\frac{\bar{y}}{\hat{y}(\psi)}\boldsymbol{\delta}\right), \tag{55}$$

where $\bar{\boldsymbol{c}}(\boldsymbol{\delta})$ is the solution to (35) given in (38). The pattern of spending on individual goods is then given by (4), and the sectoral levels of production are given by (14).

Given this solution, it is straightforward to compute the effects of an interest-rate reduction on the allocation of resources in period zero, given a fixed specification of both lump-sum transfers and credit policy. Aggregate output is necessarily increased, but not all types of consumption are increased equally, and the additional increases in spending become more narrowly concentrated as the degree to which the real interest rate is cut is made deeper.

Corollary 6. *Consider how the period zero allocation in Proposition 9 varies with ψ, for fixed policy parameters $\boldsymbol{\delta} \geq \mathbf{0}$. As ψ is decreased, $y^{agg}(0)$ increases; moreover, $y^{agg}(0)$ is a piecewise linear, concave function of $\hat{y}(\psi)$. Similarly, each of the components of this aggregate, $c^j_k(0)$ for each j, k, and $y_k(0)$ for each k, is a non-decreasing, piecewise linear, concave function of $\hat{y}(\psi)$. As ψ is reduced (and $\hat{y}(\psi)$ increases), the set C of borrowing-constrained sectors is non-decreasing. And there exists a $\underline{\psi} > 0$ such that for any $\psi < \underline{\psi}$, the set of borrowing-constrained sectors is equal to C_0, the set of constrained sectors in the case of negligible liquidity, identified in Proposition 3 (and the maximal element of $\mathcal{C}$). For any ψ in this range, the equilibrium vector of sectoral expenditure levels will be*

$$\boldsymbol{c}(0) \;=\; \begin{bmatrix} \boldsymbol{M}_{CC} \\ \mathbf{0} \end{bmatrix} \frac{\boldsymbol{\delta}_C}{\bar{p}} \;+\; \frac{\boldsymbol{\pi}}{\max_\ell(\pi_\ell/\omega_\ell)}\hat{y}(\psi), \tag{56}$$

where $\boldsymbol{\pi}$ is again the eigenvector referred to in Proposition 3, and we partition both $\boldsymbol{M}$ and $\boldsymbol{\delta}$ according to the sectors belonging to C_0 and U_0 respectively.

Thus for all low enough real interest rates, the solution for $\boldsymbol{c}(0)$ (and hence the solutions for consumption and production of all goods) is the sum of two terms: a term that is proportional to $\hat{y}(\psi)$, identical to the solution in Proposition 8 for the case of negligible liquidity, plus a term proportional to the elements of $\boldsymbol{\delta}_C$, that is independent of interest-rate policy. In this low-interest-rate case, the only marginal effects of further interest-rate cuts will be increases in expenditure (and similarly production) by the different sectors in proportion to the elements of the eigenvector $\boldsymbol{\pi}$, just as in Proposition 8.

The conclusion that progressively larger increases in $\hat{y}(\psi)$ have diminishing marginal effects might seem parallel to our conclusion in Corollary 4 that the multiplier effects of fiscal transfers diminish as the scale of the transfers is increased. However, the reason for the diminishing effects on economic activity of larger interventions is quite different in the two cases. In the case of fiscal transfers, larger transfers progressively *reduce* the number of sectors that continue to be borrowing-constrained; additional transfers then have less effect, because there is less of a problem of effective demand failure for them to solve. In the case of interest-rate cuts, instead, larger reductions in the real interest rate progressively *increase* the number of sectors that are borrowing-constrained, so that interest-rate policy has fewer and fewer channels through which it can increase spending. The difference is important: in the case of large transfers, while the effect on aggregate spending diminishes, the remaining effects are concentrated on the few remaining sectors that are still financially constrained; but in the case of large interest-rate cuts, the remaining effects are concentrated on the few sectors that are *not* financially constrained.

This matters for the welfare effects of policy. In the case of fiscal transfers, we have shown that (at least under the conditions assumed in Proposition 6) additional fiscal transfers can only increase ex-ante welfare, no matter how large they are, and now matter how they are targeted; and under nearly any possible distribution of the transfers, sufficiently large transfers will achieve the first-best allocation (Proposition 7). Instead, even in cases where a moderate reduction of the real interest rate will increase welfare, further interest-rate cuts will instead begin to *reduce* welfare. And the point at which interest-rate cuts become counterproductive will generally be one in which economic activity remains inefficiently low in some sectors; the continued existence of under-employed resources need not imply that further interest-rate reductions are desirable.

Indeed, it need not be desirable to cut interest rates at all, in response to an asymmetric disturbance, and despite a fiscal response that is inadequate to fully counteract the shock.

Corollary 7. *Let the assumptions of Proposition 9 be satisfied, and consider alternative specifications of monetary policy ψ for a given vector $\boldsymbol{\delta}$. In addition, suppose that $v(y;\bar{\xi}) = \nu \cdot y$ for some $\nu > 0$ (that may depend on $\bar{\xi}$), and that the disturbance $\phi(0)$ is such that the sectors that are borrowing-constrained in period zero when $\psi = 1$ (for the given specification of $\boldsymbol{\delta}$) imply a partition of the $\boldsymbol{A}$ matrix in which $\boldsymbol{A}_{CU} = \mathbf{0}$. Then W_0 is maximized when $\psi = 1$; that is, if there is no response of monetary policy to the disturbance.*

Note that the hypothesis about $\boldsymbol{A}_{CU}$ is satisfied by the examples shown in Figure 3, for any value of $a(0) > 0$. Thus these are examples in which a reduction of $i(0)$ would reduce welfare, even when $a(0) < \hat{a}_4$, so that fiscal transfers are insufficient to achieve the optimal allocation of resources.

V Conclusion

Our results imply that common views about the relative importance of interest-rate policy and fiscal transfers as tools of macroeconomic stabilization require revision in the case of an economic crisis resulting from a severely asymmetric disturbance, such as the COVID-19 pandemic. It is often thought that the existence of "Keynesian unemployment" provides a prima facie case for the desirability of interest-rate cuts to increase aggregate demand. Instead, we have seen that it is possible for a disturbance to result in inefficiently low activity in at least some parts of the economy (and inefficient over-consumption nowhere), in the absence of a policy response, and yet for the situation to be one in which the level of real interest rates required to support an efficient allocation of resources (assuming a suitable distribution of income) has not fallen as a result of the disturbance.

The contraction of economic activity relative to the efficient pattern may result not from real interest rates being too high, but rather from an effective demand failure, owing to a disruption of the circular flow of payments between different parts of the economy. In such a situation, the policy response that can achieve the ex-ante optimal outcome is one that arranges for appropriately targeted lump-sum transfers in response to the disturbance, without any reduction of interest rates necessarily being needed at all (Proposition 5).

Of course, our model does not incorporate all of the channels through which monetary policy might be beneficial in response to a pandemic shock.[44] But some of the ways in which lower interest rates could be beneficial involve effects that could also be achieved through other means, and that might better be achieved without also changing intertemporal relative prices. For example, in a more elaborate model, cutting the real interest rate should raise a variety of asset prices, and increased asset values might relax the financing constraints of the households or firms that own these assets. Borrowing constraints are at the heart of the problem of effective demand failure emphasized in this paper, and so one might argue that increased asset valuations should be helpful. Yet there are also other ways in which policy can increase the availability of credit to borrowing-constrained parts of the economy; and an advantage of direct credit policy (in addition to the fact that one does not distort intertemporal relative prices for parts of the economy that are not borrowing-constrained) is that it can be better targeted to the parts of the economy that are most impacted by the disruption of the usual circular flow of payments.

We should also stress that our analysis here concerns the role of interest-rate policy as a tool for regulating aggregate demand. Nothing in this paper should be taken to challenge the role of central banks as a lender of last resort, in the case of threats to financial stability. Threats to financial stability are likely to arise in the case of severe recessions, regardless of the nature of the disturbance that causes economic activity to contract. The model here abstracts from such issues, but is not intended to minimize them. We are concerned here only with the use of monetary policy to address an aggregate demand shortfall, in circumstances where financial institutions remain largely sound.[45]

Nor does our model imply that interest-rate reductions are never appropriate in response to a pandemic shock. Under certain relatively special circumstances (e.g., the cases described in Propositions 5 and Corollaries 5 and 7), we have seen that it is not ex-ante desirable for interest rates to be reduced at all; but more generally, a modest reduction in interest rates can increase ex-ante welfare, in the absence of a sufficiently aggressive (or sufficiently well-targeted) fiscal policy response. The more important point is that whereas a real interest-rate reduction of the right size should be able to completely solve the problems created by purely aggregate disturbances (Proposition 1), what one can hope to achieve with interest-rate policy alone is more limited in the case of a shock with significantly asymmetric effects.

[44]For example, Guerrieri *et al.* (2020) discuss a number of channels through which an interest-rate reduction might be helpful as a response to a pandemic, that are not present in the deliberately simplified model presented here.

[45]For example, we do not mean to question the appropriateness of the Federal Reserve's market interventions in March-April 2020, to address the disappearance of liquidity in the U.S. Treasury market at the onset of the COVID-19 pandemic (Fleming, 2020). But Treasury market liquidity had returned to 2019 levels within a few weeks; the Fed's aggressive monetary accommodation since then has instead been motivated by a concern to maintain aggregate demand.

In particular, it will be a mistake to assume that if a modest interest-rate reduction does not eliminate all under-employment of productive resources, this means that even deeper real interest-rate reductions (perhaps through unconventional policies) are called for. Our examples show that the ex-ante optimal degree of interest-rate reduction can be reached long before inefficient under-production and under-consumption have been eliminated in all sectors. Instead, there is an important role for fiscal transfers as a response to significantly asymmetric disturbances, for which interest-rate policy does not provide a close substitute.

And we should be clear that in our analysis there remains an important role for interest-rate policy as a tool of economic stabilization. Of course, all economic disturbances are at least somewhat asymmetric in their effects on different parts of the economy; "purely aggregate" disturbances of the kind assumed in Proposition 1 do not actually occur.[46] However, actual disturbances can be decomposed into an aggregate component (that by itself would create no imbalance in the circular flow of payments) and an asymmetric component; and interest-rate policy should be the tool of choice to respond to the aggregate component, as in the situation described in Proposition 1. If this is the more important part of how a given disturbance affects the economy, interest-rate policy may correspondingly be the main policy response that is needed.

Indeed, in the case of sufficiently small asymmetric disturbances, there may be no need for any fiscal transfers in response to the shock in order for the ex-ante optimal allocation of resources to be achieved, at least in the $\beta \to 1$ limit, as noted in our discussion of Proposition 7. We could add aggregate disturbances to the situation assumed in the discussion following Proposition 7, and conclude that the first-best outcome can be achieved with an appropriate monetary policy response, but with no response of either fiscal policy or credit policy. Thus there is a range of possible circumstances in which the conventional view of stabilization policy, according to which interest-rate policy alone should be used, will not be off-base. But there are other situations, vividly illustrated by the COVID-19 crisis, in which this view is quite inadequate. A more complete theory of stabilization policy, that also allows for the possibility of effective demand failures, is badly needed.

One consequence should be a greater willingness to use fiscal transfers as a tool of stabilization policy, at least under some circumstances. But this is not the only way in which policy can be adjusted in order to minimize the distortions created by periodic failures of effective demand. Our model implies that the size of the asymmetric disturbances required to create a situation in which fiscal transfers and/or credit policy will be needed in order to prevent borrowing constraints from distorting spending patterns depends on the quantity of liquid assets held by the various economic units, that can serve to buffer transitory variations in the flow of payments. And while the equilibrium distribution of those liquid assets depends on the choices made by the various individual economic units in the economy, in our model the aggregate supply of them depends on the size of the public debt.

Thus maintaining a larger real public debt has the virtue of reducing the extent to which financing constraints distort the equilibrium allocation of resources, as argued in Woodford (1990). Of course, a higher real public debt also has costs, when the revenues required to

[46] It should also be noted that when we refer to "asymmetric disturbances," the differential effects need not only be differences across sectors or industries; they might be asymmetric effects on different regions, or on different occupational categories within an industry.

service it can only be raised using distorting taxes; thus a balance must be struck between these costs and the benefits resulting from a higher average level of liquid balances. But it may be an important mistake to consider the optimal level of public debt without taking into account the advantages for macroeconomic stability of making it possible for people to maintain a higher level of liquid asset balances.

References

[1] Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, "The Network Origins of Aggregate Fluctuations," *Econometrica* 80: 1977-2016 (2012).

[2] Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, "Systemic Risk and Stability in Financial Networks," *American Economic Review* 105: 564-608 (2015).

[3] Araújo, Aloísio P., and Vitor C. Costa, "Bankruptcy Law as an Alternative to Fiscal Policy in a Woodford Model with Pandemic Shock," working paper, IMPA, Rio de Janeiro, April 2021.

[4] Auclert, Adrien, Matthew Rognlie, and Ludwig Straub, "The Intertemporal Keynesian Cross," NBER Working Paper no. 25020, September 2018.

[5] Auerbach, Alan J., Yuriy Gorodnichenko, and Daniel Murphy, "Fiscal Policy and COVID19 Restrictions in a Demand-Determined Economy," NBER Working Paper no. 27366, June 2020.

[6] Balasko, Yves, *The Equilibrium Manifold: Postmodern Developments in the Theory of General Economic Equilibrium,* MIT Press, 2009.

[7] Baqaee, David Rezza and Emmanuel Farhi, "Supply and Demand in Disaggregated Keynesian Economies with an Application to the Covid-19 Crisis," NBER Working Paper no. 27152, revised June 2020.

[8] Bigio, Saki, and Jennifer La'O, "Distortions in Production Networks," *Quarterly Journal of Economics* 135: 2187-2253.

[9] Bigio, Saki, Mengbo Zhang, and Eduardo Zilberman, "Transfers vs. Credit Policy: Macroeconomic Policy Trade-offs During Covid-19," NBER Working Paper no. 27118, May 2020.

[10] Caballero, Ricardo J., and Alp Simsek, "A Model of Asset Price Spirals and Aggregate Demand Amplification of a 'Covid-19' Shock," NBER working paper no. 27044, revised May 2020.

[11] Cavalcante, Renato L.G., Yuxiang Shen, and Sławomir Stańczak, "Elementary Properties of Positive Concave Mappings with Applications to Network Planning and Optimization," *IEEE Transactions on Signal Processing* 64: 1774-1783 (2016).

[12] Céspedes, Luis Felipe, Roberto Chang, and Andrés Velasco, "The Macroeconomics of a Pandemic: A Minimalist Model," NBER Working Paper no. 27228, May 2020.

[13] Coibion, Olivier, Yuriy Gorodnichenko, and Michael Weber, "How Did U.S. Consumers Spend Their Stimulus Payments?" NBER Working Paper no. 27693, August 2020.

[14] Danieli, Ana, and Jane Olmstead-Rumsey, "Sector-Specific Shocks and the Expenditure Elasticity Channel During the COVID-19 Crisis," working paper, Northwestern University, May 2020.

[15] Elliott, Matthew, Benjamin Golub, and Matthew O. Jackson, "Financial Networks and Contagion," *American Economic Review* 104: 3115-3153 (2014)

[16] Elliott, Matthew, Benjamin Golub, and Matthew V. Leduc, "Supply Network Formation and Fragility," *arXiv:* 2001.03853v4, posted February 19, 2021.

[17] Fleming, Michael, "Treasury Market Liquidity and the Federal Reserve During the COVID-19 Pandemic," Federal Reserve Bank of New York *Liberty Street Economics,* May 29, 2020.

[18] Gantmacher, F.R., *The Theory of Matrices,* volume II, Providence: AMS Chelsea Publishing, 1959.

[19] Ghassibe, Mishel, "Monetary Policy and Production Networks: An Empirical Investigation," *Journal of Monetary Economics* 119: 21-39 (2021).

[20] Goodman, Laurie, and Dan Magder, "Avoiding a COVID-19 Disaster for Renters and the Housing Market," Urban Institute policy brief, April 2020.

[21] Gopal, Prashant, "NYC Rental Market Pushed to Breaking Point by Tenant Debts," *Bloomberg Businessweek,* posted July 8, 2020.

[22] Guerrieri, Veronica, Guido Lorenzoni, Ludwig Straub, and Iván Werning, "Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?" NBER Working Paper no. 26918, April 2020.

[23] Keynes, John Maynard, *The General Theory of Employment, Interest and Money,* London: MacMillan, 1936.

[24] La'O, Jennifer, and Alireza Tahbaz-Salehi, "Optimal Monetary Policy in Production Networks," working paper, Columbia University, July 2021.

[25] Leijonhufvud, Axel, "Effective Demand Failures," *Swedish Journal of Economics* 75: 27-48 (1973).

[26] Milne, Alistair, "A Critical Covid 19 Economic Policy Tool: Retrospective Insurance," working paper, Loughborough University, UK, posted on *SSRN,* March 2020.

[27] Oh, Hyunseung, and Ricardo Reis, "Targeted Transfers and the Fiscal Response to the Great Recession," *Journal of Monetary Economics* 59: S50-S64 (2012).

[28] Ozdagli, Ali, and Michael Weber, "Monetary Policy through Production Networks: Evidence from the Stock Market," NBER Working Paper no. 23424, May 2017.

[29] Pastén, Ernesto, Raphael Schoenle, and Michael Weber, "The Propagation of Monetary Policy Shocks in a Heterogeneous Production Economy," *Journal of Monetary Economics* 116: 1-22 (2020).

[30] Rubbo, Elisa, "Networks, Phillips Curves, and Monetary Policy," working paper, Harvard University, June 2020.

[31] Saez, Emmanuel, and Gabriel Zucman, "Keeping Business Alive: The Government Will Pay," posted online at http://gabriel-zucman.eu/files/coronavirus2.pdf, March 2020.

[32] Woodford, Michael, "Public Debt as Private Liquidity," *American Economic Review* 80(2): 382-388 (1990).

[33] Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press, 2003.

[34] Woodford, Michael, "Effective Demand Failures and the Limits of Monetary Stabilization Policy," NBER Working Paper no. 27768, September 2020.