Online Appendix:

Skilled Labor Productivity and Cross-country Income Differences

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August 5, 2021

No Capital-skill Complementarity

The following derivations apply to all models without capital-skill complementarity. Skill bias can be exogenous or chosen from a technology frontier.

A1. Notation

It is useful to define commonly used notation at the outset.

1) $\mathcal{R}(x_j) = x_{j,r}/x_{j,p}$ denotes the rich-to-poor country ratio of $x_{j,c}$.
2) $\mathcal{S}(x_c) = x_{s,c}/x_{u,c}$ denotes the skilled-to-unskilled ratio of $x_{j,c}$.
3) $\mathcal{RS}(x) = \mathcal{R}(\mathcal{S}(x)) = \mathcal{S}(\mathcal{R}(x)) = \frac{x_{s,r}/x_{u,r}}{x_{s,p}/x_{u,p}}$ denotes the relative abundance of skilled versus unskilled x in the rich compared with the poor country.
4) The income share of an input is denoted by $IS_{a,c} = \text{income}_{a,c}/Y_c$.
5) The income ratio of two inputs is denoted by $IR_{a/b,c} = \text{income}_{a,c}/\text{income}_{b,c}$. In particular, $IR_{L_s/L_u} = \mathcal{S}(W)$.

A number of useful properties of the rich-to-poor and skilled–to-unskilled ratios are worth noting. For any constant ϕ, we have:

1) $\mathcal{R}\left(x^{\phi}\right) = \mathcal{R}(x)^{\phi}$ and $\mathcal{S}\left(x^{\phi}\right) = \mathcal{S}(x)^{\phi}$.

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2) The order of rich-to-poor and skilled-to-unskilled ratios is interchangeable:

$$\mathcal{R}\left(\mathcal{S}\left(x^{\phi}\right)\right)=\mathcal{S}\left(\mathcal{R}\left(x^{\phi}\right)\right)=\left[\frac{x_{s,r}/x_{u,r}}{x_{s,p}/x_{u,p}}\right]^{\phi}. \tag{A1}$$

A2. CES Results

It is useful to state a number of known properties of cost minimization with CES production. These results will be used repeatedly in the derivations below.

Consider the generic cost minimization problem

$$\min_{x_j}\sum_{j=1}^{J}p_jx_j+\lambda\left[\bar{y}-\left[\sum_{j=1}^{J}(\gamma_jx_j)^{\rho}\right]^{1/\rho}\right]. \tag{A2}$$

The cost-minimizing input ratios are given by

$$\left[\frac{x_i}{x_j}\right]^{1-\rho}=\left[\frac{\gamma_i}{\gamma_j}\right]^{\rho}\frac{p_j}{p_i}. \tag{A3}$$

The ratio of factor incomes is then given by

$$\frac{p_ix_i}{p_jx_j}=\left[\frac{\gamma_ix_i}{\gamma_jx_j}\right]^{\rho} \tag{A4}$$

$$=\left[\frac{\gamma_i}{\gamma_j}\right]^{\frac{1}{1-\rho}}\left[\frac{p_j}{p_i}\right]^{\frac{\rho}{1-\rho}}. \tag{A5}$$

The income share of each input is given by

$$\frac{p_jx_j}{\bar{y}}=\left[\frac{\gamma_jx_j}{\bar{y}}\right]^{\rho}. \tag{A6}$$

The minimized cost per unit of output is given by

$$p_y = \left[\sum_{j=1}^{J} (\gamma_j p_j)^{\frac{\rho}{1-\rho}} \right]^{\frac{1-\rho}{\rho}} . \tag{A7}$$

A3. Firm First-order Conditions

The firm's first-order conditions for labor inputs are given by

$$p_{j,c} = (1-\alpha)\, z_c^{1-\alpha} K_c^{\alpha} L_c^{1-\rho-\alpha} \theta_{j,c}^{\rho} L_{j,c}^{\rho-1}. \tag{A8}$$

If skill bias is endogenous, the first-order condition for $\theta_{j,c}$ is given by

$$\frac{\partial Y_c}{\partial \theta_{j,c}} = \lambda_c \omega \kappa_j^{\omega} \theta_{j,c}^{\omega-1} \tag{A9}$$

where λ_c is the Lagrange multiplier on the technology frontier constraint and

$$\frac{\partial Y_c}{\partial \theta_{j,c}} = (1-\alpha)\, K_c^{\alpha} z_c^{1-\alpha} L_c^{1-\rho-\alpha} L_{j,c}^{\rho} \theta_{j,c}^{\rho-1}. \tag{A10}$$

From (A4), the wage bill ratio is given by

$$W_{s,c}/W_{u,c} = \mathcal{S}\,(p_c L_c) = \mathcal{S}\,(\theta_c L_c)^{\rho} \tag{A11}$$

Since $\rho > 0$, an increase in the relative supply of type j labor increases its income share.

Endogenous Skill Bias

The derivations in this section apply for the model with endogenous skill bias.

The first-order conditions equation (A9) imply the optimal skill bias ratio

$$\mathcal{S}\left(\theta_c\right)^{\omega-\rho} = \mathcal{S}\left(\kappa^{-\omega} L_c^{\rho}\right). \tag{B1}$$

PROPOSITION 5: *Optimal skill bias levels are given by*

$$\theta_{u,c}^{\omega} = \frac{B_c}{\kappa_u^{\omega}\Lambda_c} \tag{B2}$$

with

$$\Lambda_c = \sum_j \left(\frac{\kappa_u}{\kappa_j}\frac{L_{j,c}}{L_{u,c}}\right)^{\Psi}. \tag{B3}$$

This holds whether or not B_c is chosen by firms.

PROOF:

Starting from the technology frontier, we have

$$B_c^{\omega} = \sum_j \left(\kappa_j \theta_{j,c}\right)^{\omega} \tag{B4}$$

$$= \left(\kappa_u \theta_{u,c}\right)^{\omega} \sum_j \left(\frac{\kappa_j}{\kappa_u}\frac{\theta_{j,c}}{\theta_{u,c}}\right)^{\omega}. \tag{B5}$$

Substituting in the condition for optimal relative skill bias (B1) yields

$$B_c^{\omega} = \left(\kappa_u \theta_{u,c}\right)^{\omega} \sum_j \left(\frac{\kappa_j}{\kappa_u}\right)^{\omega} \left[\left(\frac{L_{j,c}}{L_{u,c}}\right)^{\rho}\left(\frac{\kappa_u}{\kappa_j}\right)^{\omega}\right]^{\frac{\omega}{\omega-\rho}}. \tag{B6}$$

Note that $\omega - \frac{\omega^2}{\omega-\rho} = \frac{-\rho\omega}{\omega-\rho} = -\Psi$. This implies

$$B_c^{\omega} = (\kappa_u \theta_{u,c})^{\omega} \sum_j \left(\frac{\kappa_u}{\kappa_j} \frac{L_{j,c}}{L_{u,c}}\right)^{\Psi} = (\kappa_u \theta_{u,c})^{\omega} \Lambda_c. \tag{B7}$$

PROPOSITION 6: *When skill bias is endogenous, the skill premium is given by*

$$\mathcal{S}(p_c) = (\mathcal{S}(L_c))^{\Psi-1} \mathcal{S}(\kappa)^{-\Psi}. \tag{B8}$$

PROOF:

From equation (A3) we have

$$\mathcal{S}(p_c) = \mathcal{S}(L_c)^{\rho-1} \mathcal{S}(\theta_c)^{\rho}. \tag{B9}$$

Applying the optimal skill bias ratio (B1) implies

$$\mathcal{S}(\theta_c^{\rho}) = \mathcal{S}\left(\kappa^{-\Psi} L_c^{\frac{\rho^2}{\omega-\rho}}\right). \tag{B10}$$

Combining both expressions yields

$$\mathcal{S}(p_c) = \mathcal{S}\left(\kappa^{-\Psi}\right) \mathcal{S}(L_c)^{\rho+\frac{\rho^2}{\rho-\omega}-1} \tag{B11}$$

Using $\rho + \frac{\rho^2}{\omega-\rho} = \frac{\rho\omega}{\omega-\rho} = \Psi$ gives equation (B8).

B2. Reduced Form Labor Aggregator

PROOF:

(Proposition 1)

The following hold regardless of how B_c is determined (endogenous or fixed). The definition of the labor aggregator (2) implies

$$L_c = \theta_{u,c} L_{u,c} \left(\sum_j \left[\frac{\theta_{j,c}}{\theta_{u,c}} \frac{L_{j,c}}{L_{u,c}} \right]^{\rho} \right)^{1/\rho} \tag{B12}$$

Substituting in the condition for the optimal choice of relative skill bias (B10) yields

$$L_c = \theta_{u,c} L_{u,c} \left(\sum_j \left[\frac{L_{j,c}}{L_{u,c}} \right]^{\frac{\rho^2}{\omega-\rho}} \left[\frac{\kappa_j}{\kappa_u} \right]^{-\Psi} \left[\frac{L_{j,c}}{L_{u,c}} \right]^{\rho} \right)^{1/\rho} \tag{B13}$$

The exponent on labor inputs is given by

$$\frac{\rho^2}{\omega - \rho} + \rho = \frac{\omega\rho}{\omega - \rho} = \Psi \tag{B14}$$

Then the summation term becomes Λ_c, defined in (B3), and we have

$$L_c = \theta_{u,c} L_{u,c} \Lambda_c^{1/\rho} \tag{B15}$$

Then using (B2), we have

$$L_c = B_c \kappa_u^{-1} \Lambda_c^{-1/\omega} L_{u,c} \Lambda_c^{1/\rho} \tag{B16}$$

Note that

$$1/\rho - 1/\omega = \frac{\omega - \rho}{\omega\rho} = 1/\Psi \tag{B17}$$

so that

$$L_c = B_c\left(1/\kappa_u\right)L_{u,c}\Lambda_c^{1/\Psi} \tag{B18}$$
$$= B_c\left(1/\kappa_u\right)L_{u,c}\left[\sum_j\left(\kappa_j^{-1}L_{j,c}\right)^{\Psi}\right]^{1/\Psi}\kappa_u/L_{u,c}$$

Cancelling terms yields equation (9).

B3. Closed Form Solution

PROOF:

(Proposition 2)

Using the labor aggregator (9) with $\kappa_j = 1$, we have

$$L_c = B_c L_{u,c}\left(1 + \mathcal{S}\left(L_c\right)^{\Psi}\right)^{1/\Psi}. \tag{B19}$$

Taking the rich-to-poor country ratio yields

$$\mathcal{R}\left(L\right) = \mathcal{R}\left(L_u\right)\mathcal{R}\left(1 + \mathcal{S}\left(L\right)^{\Psi}\right)^{1/\Psi} \tag{B20}$$

Since equation (A11) also applies to the reduced form labor aggregator, we have

$$W_{s,c}/W_{u,c} = \mathcal{S}\left(L_c\right)^{\Psi} \tag{B21}$$

Using this to replace $\mathcal{S}\left(L\right)^{\Psi}$ in (B20) with W_s/W_u yields

$$\mathcal{R}\left(L\right) = \mathcal{R}\left(L_u\right)\mathcal{R}\left(W/W_u\right)^{1/\Psi} \tag{B22}$$

If $\mathcal{S}\left(L_r\right) > \mathcal{S}\left(L_p\right)$, then $\mathcal{R}\left(1 + W_s/W_u\right) > 1$ and $\mathcal{R}\left(L\right) > \mathcal{R}\left(L_u\right)$. Since $W_{j,c} =$

$p_{j,c}L_{j,c}$, the ratio of unskilled labor inputs is given by

$$\mathcal{R}(L_u) = \frac{\mathcal{R}(W_u)}{\mathcal{R}(p_u)} \tag{B23}$$

$$= \frac{\mathcal{R}(W_u)}{\mathcal{R}(p_u)} \frac{\mathcal{R}(Y)\,\mathcal{R}(1-\alpha)}{\mathcal{R}((1-\alpha)\,Y)} \tag{B24}$$

$$= \frac{\mathcal{R}(Y)}{\mathcal{R}(p_u)} \frac{\mathcal{R}(1-\alpha)}{\mathcal{R}(W/W_u)} \tag{B25}$$

Substituting this into (B22) and rearranging yields

$$\mathcal{R}(L) = \frac{\mathcal{R}(Y)}{\pi_u} \mathcal{R}(W/W_u)^{1/\Psi - 1} \mathcal{R}(1-\alpha)\,. \tag{B26}$$

Taking logarithms gives

$$\Delta \ln(L) = \Delta \ln(Y) - \ln \pi_u + \left(\frac{1}{\Psi} - 1\right) \Delta \ln(W/W_u) + \Delta \ln(1-\alpha)\,. \tag{B27}$$

Dividing by $\Delta \ln(Y)$ and assuming that labor shares do not differ across countries yields equation (11).

The solution for Ψ follows from (B21) which implies $\mathcal{RS}(W) = \mathcal{RS}(L)^{\Psi}$. Taking logarithms and rearranging yields equation (12).

Exogenous Skill Bias

C1. Closed Form Solution

PROOF:

(Proposition 3)

Define $contrib_L^{poor}$ as the increase in L due to replacing $L_{j,p}$ with $L_{j,r}$, holding $\theta_{j,p}$

fixed:

$$contrib_L^{poor} = \frac{\left[\sum_j \left(\theta_{j,p} L_{j,r}\right)^\rho\right]^{1/\rho}}{\left[\sum_j \left(\theta_{j,p} L_{j,p}\right)^\rho\right]^{1/\rho}} \tag{C1}$$

$$= \frac{\theta_{u,p} L_{u,p} \left[\mathcal{R}\left(L_u\right)^\rho + \left(\frac{\theta_{s,p}}{\theta_{u,p}} \frac{L_{s,r}}{L_{u,p}}\right)^\rho\right]^{1/\rho}}{\theta_{u,p} L_{u,p} \left[1 + W_{s,p}/W_{u,p}\right]^{1/\rho}} \tag{C2}$$

$$= \frac{\left[\mathcal{R}\left(L_u\right)^\rho + \left(\frac{\theta_{s,p}}{\theta_{u,p}} \frac{L_{s,p}}{L_{u,p}} \mathcal{R}\left(L_s\right)\right)^\rho\right]^{1/\rho}}{\left[1 + W_{s,p}/W_{u,p}\right]^{1/\rho}} \tag{C3}$$

$$= \frac{\left[\mathcal{R}\left(L_u\right)^\rho + W_{s,p}/W_{u,p}\left(\mathcal{R}\left(L_s\right)\right)^\rho\right]^{1/\rho}}{\left[1 + W_{s,p}/W_{u,p}\right]^{1/\rho}} \tag{C4}$$

This uses (A11) to replace $\mathcal{S}(\theta L)^\rho$ with W_s/W_u. Pulling out $\mathcal{R}(L_u)$ yields

$$contrib_L^{poor} = \mathcal{R}\left(L_u\right)\left[\frac{1 + W_{s,p}/W_{u,p}\mathcal{R}\mathcal{S}\left(L\right)^\rho}{1 + W_{s,p}/W_{u,p}}\right]^{1/\rho}. \tag{C5}$$

Replacing $\mathcal{R}(L_u)$ using (B25) gives

$$contrib_L^{poor} = \frac{\mathcal{R}\left(Y\right)\mathcal{R}\left(1-\alpha\right)}{\pi_u \mathcal{R}\left(W/W_u\right)}\left[\frac{1 + W_{s,p}/W_{u,p}\mathcal{R}\mathcal{S}\left(L\right)^\rho}{1 + W_{s,p}/W_{u,p}}\right]^{1/\rho}. \tag{C6}$$

Since $share_L^{poor} \equiv \ln\left(contrib_L^{poor}\right)/\Delta\ln\left(Y\right)$, taking logarithms, setting $\mathcal{R}(1-\alpha) = 1$, and dividing by $\Delta\ln(Y)$ yields (19). To see that $contrib_L^{poor} \in \left(\mathcal{R}\left(L_u\right), \mathcal{R}\left(L_s\right)\right)$, note that

$$contrib_L^{poor} = \mathcal{R}\left(L_s\right)\frac{\left[\mathcal{R}\mathcal{S}\left(L\right)^{-\rho} + \mathcal{S}\left(W_p\right)\right]^{1/\rho}}{\left[1 + W_{s,p}/W_{u,p}\right]^{1/\rho}} \tag{C7}$$

If $\mathcal{R}(L_s) > \mathcal{R}(L_u)$, then $\mathcal{R}(L_u) < contrib_L^{poor} < \mathcal{R}(L_s)$; otherwise $\mathcal{R}(L_s) < contrib_L^{poor} < \mathcal{R}(L_u)$.

Using rich country skill bias, we have

$$contrib_L^{rich} = \frac{\left[\sum_j \left(\theta_{j,r} L_{j,r}\right)^{\rho}\right]^{1/\rho}}{\left[\sum_j \left(\theta_{j,r} L_{j,p}\right)^{\rho}\right]^{1/\rho}} \tag{C8}$$

$$= \frac{\theta_{u,r} L_{u,r} \left[1 + \left(\frac{\theta_{s,r}}{\theta_{u,r}} \frac{L_{s,r}}{L_{u,r}}\right)^{\rho}\right]^{1/\rho}}{\theta_{u,r} L_{u,r} \left[\mathcal{R}\left(L_u\right)^{-\rho} + \left(\frac{\theta_{s,r}}{\theta_{u,r}} \frac{L_{s,r}}{L_{u,r}} \frac{L_{s,p}}{L_{s,r}}\right)^{\rho}\right]^{1/\rho}} \tag{C9}$$

$$= \frac{\left[1 + W_{s,r}/W_{u,r}\right]^{1/\rho}}{\left[\mathcal{R}\left(L_u\right)^{-\rho} + W_{s,r}/W_{u,r} \mathcal{R}\left(L_s\right)^{-\rho}\right]^{1/\rho}}$$

Assuming that labor shares do not differ across countries, pulling out $\mathcal{R}\left(L_u\right)$ and replacing it using (B25) yields (20).

Investment in the Frontier

We consider a model where firms can expend resources to shift the technology frontier outwards, as in Acemoglu (2007). The representative firm solves

$$\max_{K_c, L_{j,c}, \theta_{j,c}, B_c} Y_c - q_c K_c - \sum_j p_{j,c} L_{j,c} - C(B_c) \tag{D1}$$

subject to (1), (2), and (3), taking factor prices as given. We assume the cost function $C(B_c) = b_c B_c^{\omega}$ as in Acemoglu (2007)'s example 1. The firm takes $b_c > 0$ as given. We assume $\omega > 1$ to ensure that optimal skill weights are finite. We normalize all $\kappa_j = 1$. We also assume that labor shares do not differ across countries.

Compared with the fixed frontier case studied in Section I, the only change is the endogeneity of B_c. Conditional on its value all quantities and prices are the same as in the endogenous technology model.

If we treat b_c as a parameter, the model has increasing returns to scale. We show that, in this case, $share_L$ is magnified by the factor $\frac{\omega}{\omega-1}$ compared with the endogenous technology model. If b_c scales appropriately with Y_c so that the model has constant returns to scale, we show that the development accounting results of the endogenous technology model remain unchanged.

D1. Reduced Form Labor Aggregator

PROPOSITION 7: *The labor aggregator is given by*

$$\hat{L}_c = \left((1-\alpha)\, z_c^{1-\alpha} K_c^{\alpha} \omega^{-1} b_c^{-1}\right)^{\frac{1}{\omega+\alpha-1}} \tilde{L}_c^{\frac{\omega}{\omega+\alpha-1}} \tag{D2}$$

where $\tilde{L}_c$ is the reduced form labor aggregator with a fixed frontier, given by (9).

PROOF:

The firm's problem may be written as

$$\max_{K_c, L_{j,c}, \theta_{j,c}} Y_c - q_c K_c - \sum_j p_{j,c} L_{j,c} - b_c \sum_j (\kappa_j \theta_{j,c})^{\omega} \tag{D3}$$

The firm's first-order condition for $\theta_{j,c}$ is again given by (A9), except that now $\lambda_c = b_c$ so that

$$\theta_{j,c}^{\omega-\rho} = X_{j,c} L_{j,c}^{\rho} L_c^{1-\alpha-\rho} \tag{D4}$$

where

$$X_{j,c} = \frac{(1-\alpha)\, z_c^{1-\alpha} K_c^{\alpha}}{b_c \omega \kappa_j^{\omega}} \tag{D5}$$

Together with $1 + \rho / (\omega - \rho) = \omega / (\omega - \rho)$ this implies

$$\theta_{u,c} L_{u,c} = X_{u,c}^{\frac{1}{\omega-\rho}} L_{u,c}^{\frac{\omega}{\omega-\rho}} L_c^{\frac{1-\alpha-\rho}{\omega-\rho}} \tag{D6}$$

From equation (B15) we have $\theta_{u,c} L_{u,c} = L_c \Lambda_c^{-1/\rho}$. From equation (B18) we obtain $\tilde{L}_c = \frac{L_c}{\kappa_u} \Lambda_c^{1/\Psi}$ and therefore

(D7) $$\theta_{u,c} L_{u,c} = L_c \left(L_u \kappa_u^{-1} / \tilde{L}_c \right)^{\Psi/\rho}$$

Setting both expressions for $\theta_{u,c} L_{u,c}$ equal and noting that $\Psi/\rho = \omega / \left(\omega - \rho\right)$, we have

(D8) $$L_c^{1 - \frac{1-\alpha-\rho}{\omega-\rho}} = \left(\kappa_u \tilde{L}_c \right)^{\frac{\omega}{\omega-\rho}} X_{u,c}^{\frac{1}{\omega-\rho}}$$

Since

(D9) $$1 - \frac{1-\alpha-\rho}{\omega-\rho} = \frac{\omega+\alpha-1}{\omega-\rho}$$

we have

(D10) $$L_c = \left(\kappa_u^{\omega} X_{u,c} \right)^{\frac{1}{\omega+\alpha-1}} \tilde{L}_c^{\frac{\omega}{\omega+\alpha-1}}$$

D2. Reduced Form Production Function

PROPOSITION 8: *The reduced form production function is given by*

(D11) $$Y_c = \left(K_c^{\alpha} \left(\hat{A}_c z_c \tilde{L}_c \right)^{1-\alpha} \right)^{\frac{\omega}{\omega+\alpha-1}}$$

where $\tilde{L}_c$ is given by equation (9) and $\hat{A}_c = \left(\frac{1-\alpha}{\omega b_c} \right)^{1/\omega}$ is a constant.

PROOF:

Substituting the reduced form labor aggregator (D10) into the production function, we have

$$Y_c = K_c^{\alpha}\left(z_c L_c\right)^{1-\alpha} \tag{D12}$$

$$= K_c^{\alpha} z_c^{1-\alpha}\left(\kappa_u^{\omega} X_{u,c}\right)^{\frac{1-\alpha}{\omega+\alpha-1}}\left(\tilde{L}_c\right)^{\frac{\omega(1-\alpha)}{\omega+\alpha-1}} \tag{D13}$$

$$= \tilde{A}_c\left(z_c^{1-\alpha} K_c^{\alpha}\right)^{1+\frac{1-\alpha}{\omega+\alpha-1}}\left(\tilde{L}_c\right)^{\frac{\omega(1-\alpha)}{\omega+\alpha-1}} \tag{D14}$$

where

$$\tilde{A}_c = \left(\frac{1-\alpha}{\omega b_c}\right)^{\frac{1-\alpha}{\omega+\alpha-1}} \tag{D15}$$

collects all constant terms. Then

$$Y_c = \left(K_c^{\alpha}\left(\hat{A}_c z_c \tilde{L}_c\right)^{1-\alpha}\right)^{\frac{\omega}{\omega+\alpha-1}} \tag{D16}$$

This is true because the exponent on $z_c^{1-\alpha} K_c^{\alpha}$ is

$$1 + \frac{1-\alpha}{\omega+\alpha-1} = \frac{\omega}{\omega+\alpha-1} \tag{D17}$$

If b_c is fixed, the model has increasing returns to scale due to scale effects. Increasing any factor input or increasing TFP raises the benefits from investing in B_c, but not the cost. The optimal level of B_c increases, amplifying the effect on output. The imperfect substitutes term is governed by the exponent $\frac{\omega}{\omega+\alpha-1}$.

The scale effect is eliminated if the cost of investing in B_c scales appropriately with output. Specifically, if $b_c \propto Y_c$, the production function reverts to the one for the fixed frontier, except that the TFP level z_c is multiplied by a constant. In that case, investment in the frontier has no impact on development accounting.

PROPOSITION 9: *The reduced form production function (D11) satisfies*

$$Y_c = \left[(K_c/Y_c)^{\alpha/(1-\alpha)} \hat{A}_c z_c \tilde{L}_c \right]^{\frac{\omega}{\omega-1}} \tag{D18}$$

PROOF:

Write equation (D11) as

$$Y_c = \left[(K_c/Y_c)^{\alpha} \left(\hat{A}_c z_c \tilde{L}_c \right)^{1-\alpha} \right]^{\frac{\omega}{\omega+\alpha-1}} Y_c^{\frac{\alpha\omega}{\omega+\alpha-1}} \tag{D19}$$

and note that the exponent on Y_c becomes

$$1 - \frac{\alpha\omega}{\omega+\alpha-1} = \frac{\omega+\alpha-1-\alpha\omega}{\omega+\alpha-1} \tag{D20}$$

$$= (\alpha-1) \frac{1-\omega}{\omega+\alpha-1} \tag{D21}$$

Then

$$Y_c = \left[(K_c/Y_c)^{\alpha} \left(\hat{A}_c z_c \tilde{L}_c \right)^{1-\alpha} \right]^{\phi} \tag{D22}$$

with $\phi = \frac{\omega}{\omega+\alpha-1} \times \frac{\omega+\alpha-1}{(1-\alpha)(\omega-1)}$. Simplify exponents to arrive at equation (D18).

Now the only difference relative to the case where B_c is fixed is the exponent $\omega/(\omega-1)$. To perform development accounting, it is necessary to know the values of ω and ρ, not just the reduced form elasticity governed by Ψ. Identifying both values requires an additional data moment. Relative to the model with a fixed frontier, the contribution of labor inputs to output gaps is amplified by a constant factor, $\omega/(1-\omega)$.

PROPOSITION 10: *The share of cross-country output gaps accounted for by* *labor inputs is given by*

$$share_L = \frac{\omega}{\omega - 1} \frac{\ln \mathcal{R}\left(\tilde{L}\right)}{\Delta \ln (Y)} \tag{D23}$$

where $\tilde{L}_c$ *takes on the same value as in the model without investment in the frontier.*

PROOF:

Let $A_c = (K_c/Y_c)^{\alpha/(1-\alpha)} \hat{A}_c z_c$ collect all country specific terms other than labor inputs. Then $Y_c = \left[A_c \tilde{L}_c\right]^{\frac{\omega}{\omega-1}}$ and

$$\frac{\omega - 1}{\omega} \Delta \ln (Y) = \Delta \ln (A) + \Delta \ln \left(\tilde{L}\right) \tag{D24}$$

This implies (D23). Since the calibrated values of $h_{j,c}$ and Ψ do not depend on whether or not B_c is endogenous, the labor aggregator is the same as in the model with fixed B_c.

Capital-skill Complementarity

E1. Equipment and Structures Data

Calibrating the model with capital-skill complementarity requires additional data moments related to equipment and structures that are constructed as follows. All data are constructed for year 2011, which is the latest and most comprehensive benchmark year for the International Comparison Program. From the Penn World Tables, we obtain:

1) output per worker Y as `cgdpo/emp`.

2) capital per worker K as `ck/emp`.

3) the price levels of capital `pl_k` and consumption `pl_c`.

4) the value of the equipment stock at local prices as `Kc_Mach + Kc_TraEq`.

5) the value of the structures stock at local prices as `Kc_Struc + Kc_Other` (from the capital detail file).

From ICP we obtain the PPP prices (series S03) of equipment (classification C20 Machinery and equipment) and structures (classification C21 Construction).

We define the stock of equipment as $E_c = (\text{Kc\_Mach}+\text{KcTraEq})\,/\text{emp}/PPP_{C20}$ and the stock of structures as $S_c = (\text{Kc\_Struc}+\text{Kc\_Other})\,/\text{emp}/PPP_{C21}$.

Before computing the calibration targets, we drop countries with missing output or employment data or with population (`pop`) < 1m. We also drop 6 countries with capital or consumption prices above 10 times the sample median. Finally, we drop 7 countries for which the discrepancy between k and $E+S$ is above 20 percent.

E2. Preliminaries

This section contains results that are used in subsequent derivations. They hold for endogenous and exogenous skill bias.

Firm first-order conditions. — The firm's first-order conditions are:

$$S : \alpha Y_c / S_c = q_{s,c} \tag{E1}$$

$$E : \frac{\partial Y_c}{\partial L_c}\frac{\partial L_c}{\partial Z_c} Z_c^{1-\phi}\mu_e^{\phi}E_c^{\phi-1} = q_{e,c} \tag{E2}$$

(E3) $$L_s : \frac{\partial Y_c}{\partial L_c}\frac{\partial L_c}{\partial Z_c} Z_c^{1-\phi} \mu_s^{\phi} L_{s,c}^{\phi-1} = p_{s,c}$$

(E4) $$L_u : \frac{\partial Y_c}{\partial L_c} L_c^{1-\rho} \theta_{u,c}^{\rho} L_{u,c}^{\rho-1} = p_{u,c}$$

where

(E5) $$\frac{\partial Y_c}{\partial L_c} = (1-\alpha)\, Y_c / L_c$$

(E6) $$\frac{\partial L_c}{\partial Z_c} = L_c^{1-\rho} \theta_{s,c}^{\rho} Z_c^{\rho-1}$$

If there is a technology frontier, we also have

(E7) $$\theta_{u,c} : \frac{\partial Y_c}{\partial L_c} L_c^{1-\rho} \theta_{u,c}^{\rho-1} L_{u,c}^{\rho} = \lambda_c \omega \kappa_u^{\omega} \theta_{u,c}^{\omega-1}$$

(E8) $$\theta_{s,c} : \frac{\partial Y_c}{\partial L_c} L_c^{1-\rho} \theta_{s,c}^{\rho-1} Z_c^{\rho} = \lambda_c \omega \kappa_s^{\omega} \theta_{s,c}^{\omega-1}$$

which implies that the optimal skill bias ratio is a constant elasticity function of relative inputs:

$$\mathcal{S}(\theta)^{\omega-\rho} = \mathcal{S}(\kappa)^{-\omega}(Z/L_u)^{\rho}. \tag{E9}$$

This is analogous to equation (B1) in the endogenous technology model.

Income ratios and shares. — Applying the generic CES expression (A4) yields the income ratios of skilled labor to equipment

$$IR_{L_s/e} = \left(\frac{\mu_s L_s}{\mu_e E}\right)^{\phi} \tag{E10}$$

and of Z versus L_u

$$IR_{Z/L_u} = \left(\frac{\theta_s Z}{\theta_u L_u}\right)^{\rho} \tag{E11}$$

The income ratio of skilled versus unskilled labor is then given by

$$W_s/W_u = IR_{L_s/e} IR_{Z/L_u} = \left(\frac{\mu_s L_s}{Z}\right)^{\phi}\left(\frac{\theta_s Z}{\theta_u L_u}\right)^{\rho} \tag{E12}$$

The income share of equipment is given by $IS_e = IS_L IR_{Z/L} IR_{E/Z}$. Again applying the generic CES expressions yields

$$IS_E = (1-\alpha)\left[\frac{\theta_s Z}{L}\right]^{\rho}\left[\frac{\mu_e E}{Z}\right]^{\phi} \tag{E13}$$

E3. Endogenous Skill Bias

Reduced form labor aggregator. — PROOF:

(Proposition 4)

We may think of the firm as solving its problem in two steps. First, the firm chooses $L_{s,c}/E_c$ to minimize the cost of Z. This is a standard CES cost minimization problem with the solution (according to equation (A3))

$$\left[\frac{L_s}{E}\right]^{1-\phi} = \frac{q_E}{p_s}\left[\frac{\mu_s}{\mu_e}\right]^{\phi} \tag{E14}$$

and the unit cost

$$p_Z = \left[(\mu_e q_E)^{\frac{\phi}{1-\phi}} + (\mu_s p_s)^{\frac{\phi}{1-\phi}}\right]^{\frac{1-\phi}{\phi}} \tag{E15}$$

In the second step, the firm solves

$$\max_{L_{u,c},Z_c,\theta_{j,c},S_c} S^{\alpha}\left[z_c L_c\right]^{1-\alpha} - q_S S - p_u L_u - p_Z Z \tag{E16}$$

subject to the labor aggregator (22) and the frontier constraint (3). This problem has the same structure as the one solved by the firm in the endogenous technology model, except that the firm chooses structures instead of capital and Z instead of L_2. It follows directly that the labor aggregator takes on the same form as in the endogenous technology model.

Joint Contribution of Labor Inputs and Equipment. — We derive a closed form solution for the joint contribution of labor inputs and equipment to cross-country output gaps, $share_{L+E}$.

PROPOSITION 11: *The joint contribution of labor inputs and equipment to cross-country output gaps is given by*

$$share_{L+E} = \underbrace{1 - \frac{\ln\left(\frac{p_{u,r}}{p_{u,p}}\right)}{\Delta\ln(Y)}}_{\textit{perfect substitutes}} + \underbrace{\frac{\frac{1}{\Psi}\Delta\ln\left(1 + IR_{Z/L_u}\right) - \Delta\ln\left(1 + W_s/W_u\right)}{\Delta\ln(Y)}}_{\textit{imperfect substitutes}} \tag{E17}$$

where the reduced form curvature is given by

$$\Psi = \frac{\Delta \ln \left(IR_{Z/L_u} \right)}{\Delta \ln \left(Z/L_u \right)} \tag{E18}$$

and the curvature of the Z aggregator is given by

$$\phi = \frac{\Delta \ln \left(IR_{L_s/e} \right)}{\Delta \ln \left(L_s/E \right)} \tag{E19}$$

In terms of observable data moments, the reduced form curvature may be written as

$$\Psi = \frac{\ln \mathcal{RS}\left(W \right) + \Delta \ln \left(1 + IR_{e/L_s} \right)}{\ln \mathcal{RS}\left(L \right) + \frac{1}{\phi} \Delta \ln \left(1 + IR_{e/L_s} \right)} \tag{E20}$$

Throughout, $IR_{a/b,}$ denotes the ratio of incomes received by inputs a and b.

PROOF:

The labor aggregator equation (25) may be written as

$$L_c = L_{u,c} \left[1 + \left(Z_c/L_{u,c} \right)^{\Psi} \right]^{1/\Psi}, \tag{E21}$$

where B_c is normalized to 1. Applying the generic CES expressions for income shares and income ratios to the reduced form labor aggregator yields

$$\left(\frac{Z_c}{L_{u,c}} \frac{\kappa_u}{\kappa_s} \right)^{\Psi} = W_{s,c}/W_{u,c} \left(1 + IR_{e/L_s,c} \right) \tag{E22}$$

$$= IR_{Z/L_u,c} \tag{E23}$$

where κ_j may be normalized to one. Using equation (E23) we have

$$L_c = L_{u,c} \left[1 + W_{s,c}/W_{u,c} \left(1 + IR_{e/L_s,c} \right) \right]^{1/\Psi} \tag{E24}$$

Taking logarithms and replacing $\mathcal{R}(L_u)$ using equation (B25) yields (E17). If labor shares differ across countries, the perfect substitutes term is increased by $\frac{\Delta \ln(1-\alpha)}{\Delta \ln(Y)}$, consistent with the other models.

The solution for Ψ is obtained by taking the rich-to-poor country ratio of (E23) in logarithms which yields

$$\Psi = \frac{\Delta \ln \left(W_s/W_u \left(1 + IR_{e/L_s}\right)\right)}{\Delta \ln (Z/L_u)} \tag{E25}$$

$$= \frac{\Delta \ln \left(IR_{Z/L_u}\right)}{\Delta \ln (Z/L_u)} \tag{E26}$$

where $\mathcal{R}(Z)$ follows from

$$\mathcal{R}(Z) = \mathcal{R}(Z/(\mu_e E))\, \mathcal{R}(E) \tag{E27}$$

$$= \mathcal{R}\left(\left[1 + IR_{s/e}\right]^{1/\phi}\right) \mathcal{R}(E) \tag{E28}$$

$$= \mathcal{R}\left(\left[1 + IR_{e/L_s}\right]^{1/\phi}\right) \mathcal{R}(L_2) \tag{E29}$$

The solution for Ψ can be expressed in a form that is closer to the endogenous technology model. From equation (E23), we have

$$\frac{Z}{L_u} = \frac{Z}{L_s} \mathcal{S}(L) = \mathcal{S}(L) \left(1 + IR_{e/L_s}\right)^{1/\phi} \tag{E30}$$

Therefore

$$\Psi = \frac{\Delta \ln (W_s/W_u) + \Delta \ln \left(1 + IR_{e/L_s}\right)}{\ln \mathcal{RS}(L) + \frac{1}{\phi} \Delta \ln \left(1 + IR_{e/L_s}\right)} \tag{E31}$$

The data moments used in the calibration imply that skilled labor and equipment are complements ($\phi < 0$).[22] This is consistent with U.S. time series evidence (see

[22] The numerator in (E19) is positive because $\mathcal{R}(IS_e) = 1$ and $\mathcal{R}(IS_{L_s}) > 1$. The denominator is negative because equipment stocks vary across countries more than labor inputs. Hence $\phi < 0$.

Krusell et al. 2000).

Since we assume that the income share of equipment is the same in rich and poor countries, IR_{e/L_s} is lower in rich compared with poor countries. Together with $\phi < 0$, it follows that the long-run elasticity of substitution between skilled and unskilled labor is lower than in the endogenous technology model. This increases the imperfect substitutes term.

The expression for $share_{L+E}$ is similar in structure to the endogenous technology model's (11). The perfect substitutes term is the same, again reflecting the contribution of human capital in a single skill model. The imperfect substitutes term now depends on the ratio of incomes received by Z (by skilled labor and equipment jointly) to unskilled labor. When equipment is "unimportant," so that $IR_{e/L_s} \approx 0$, the values of Ψ and $\mathcal{R}(L)$ approach those of the endogenous technology model.

E4. Exogenous Skill Bias

Our final model treats variation in skill bias $\theta_{j,c}$ across countries as exogenous. Except for dropping the technology frontier, the model is identical to the one described in Section IV.A.

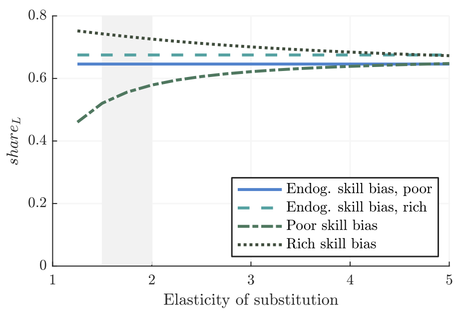
Development Accounting. — We define the contribution of labor inputs to cross-country output variation as the change in steady state output that results from increasing $L_{j,p}$ to $L_{j,r}$, holding capital rental prices and skill bias $\theta_{j,c}$ constant. It follows that $share_L$ depends on the fixed levels of q_E (but not on q_S) and now also on those of $\theta_{j,c}$. We consider two cases:

1) $share_L^{poor}$ fixes skill bias and q_E at poor country levels. This corresponds to increasing labor inputs in the poor country.

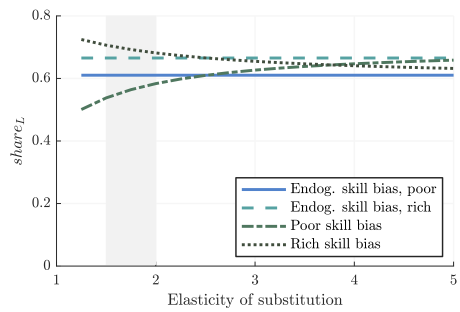
2) $share_L^{rich}$ fixes skill bias and q_E at rich country levels. This corresponds to reducing labor inputs in the rich country.

Relative to the model with the technology frontier, one additional parameter needs to be calibrated because counterfactual output depends on the values of ρ and ω, not only on the reduced form curvature Ψ. The development accounting results therefore require fixed values of ρ.

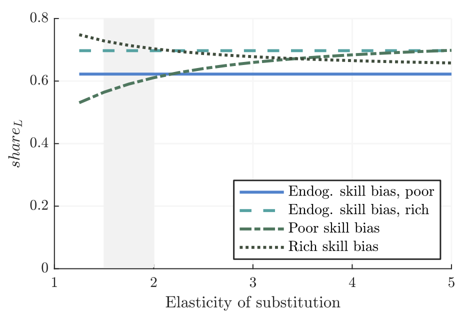
Quantitative Results. — figure Figure E1 provides a compact visual summary of the results. When poor country $\theta_{j,c}$ and q_E are used, the results are very similar to the endogenous technology model. $share_L^{poor}$ is smaller than $share_L$ when $\rho < \Psi$. It increases with the elasticity of substitution and the skill cutoff. Values below 0.5 are associated with very large cross-country differences in relative skill bias (at least factor 10^5).



(a) SHS Skill Cutoff



(b) HSG Skill Cutoff



(c) SC Skill Cutoff

Figure E1. : $share_L$: Capital-skill Complementarity

With rich country $\theta_{j,c}$ and q_E, $share_L^{rich}$ is higher than $share_L$ when $\rho < \Psi$. Its value decreases with the elasticity of substitution and the skill cutoff. For conventional values of the elasticity, we find $share_L^{rich}$ between 0.64 and 0.74 (compared with 0.59 to 0.74 in the endogenous technology model).