Online Appendix to

Macroeconomic Implications of COVID-19: Can Negative Supply Shocks Cause Demand Shortages?

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A Consumption functions

In this section we derive the individual consumption functions used for the plots in Figures 4, and derive condition (15).

The marginal utility of good B is

$$U_{c_B}\left(c_{At}, c_{Bt}\right) = (1-\phi)^{\frac{1}{\epsilon}}\, c_t^{\frac{1}{\epsilon}-\frac{1}{\sigma}} c_{Bt}^{-\frac{1}{\epsilon}},$$

where

$$c_t = \left(\phi^{\frac{1}{\epsilon}} c_{At}^{\frac{\epsilon-1}{\epsilon}} + (1-\phi)^{\frac{1}{\epsilon}}\, c_{Bt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}.$$

The Euler equation of unconstrained consumers, with $c_{A0} = 0$ and $\beta\left(1+r_0\right) = 1$, then takes the following form

$$c_0^{\frac{1}{\epsilon}-\frac{1}{\sigma}} c_{B0}^{-\frac{1}{\epsilon}} = c_1^{\frac{1}{\epsilon}-\frac{1}{\sigma}} c_{B1}^{-\frac{1}{\epsilon}}.$$

Substituting $c_0 = (1-\phi)^{\frac{1}{\epsilon-1}}\, c_{B0}$ and $c_{1B} = (1-\phi)\, c_1$ and rearranging, gives

$$c_1 = (1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}\, c_{B0}.$$

Taking into account that $c_t = c_1$ for $t = 1, 2, \ldots$, the intertemporal budget constraint is

$$c_{B0} + \frac{\beta}{1-\beta} c_1 = y_0 + \frac{\beta}{1-\beta}.$$

Solving, we obtain the consumption function

$$c_{B0} = \frac{(1-\beta)\, y_0 + \beta}{1-\beta+\beta\,(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}. \tag{30}$$

For constrained consumers we can follow similar steps, allowing for the Euler equation to hold as an inequality, and obtain

$$c_{B0} = \min\left\{y_0, \frac{(1-\beta)\, y_0 + \beta}{1-\beta+\beta\,(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}\right\}. \tag{31}$$

The consumption functions without the shock can be derived in similar manner and are

$$c_{B0} = (1-\phi)\left((1-\beta)\, y_0 + \beta\right) \tag{32}$$

for unconstrained consumers, and

$$c_{Bt} = (1-\phi)\min\left\{y_0, (1-\beta)\, y_t + \beta\right\} \tag{33}$$

for constrained consumers. Notice that in the last expression the factor $(1-\phi)$ appears before the min operator, because before the shock the consumers allocate a fraction ϕ of their spending to good A, whether or not the constraint is binding. These are the consumption functions plotted in Figure 4.

Suppose now that the income of the consumers in sector B remains at 1. The total change in consumption following the shock is

$$\frac{(1-\beta)\,(1-\phi)+\beta\,(1-\mu\phi)}{1-\beta+\beta\,(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}} - (1-\phi)\,. \tag{34}$$

This expression is negative iff condition (13) holds.

The expression above can be decomposed in three terms:

1. The shift in the consumption function at income $y_0 = 1$:

$$\frac{1}{1-\beta+\beta\,(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}} - (1-\phi)\,;$$

2. The change in consumption of the unconstrained consumers hit by the shock, due to the income loss:

$$-(1-\mu)\,\phi\left(\frac{1-\beta}{1-\beta+\beta\,(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}\right);$$

3. The change in consumption of the constrained consumers hit by the shock, due to the income loss:

$$-\mu\phi\left(\frac{1}{1-\beta+\beta\,(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}\right).$$

The marginal propensities to consume are

$$\frac{1-\beta}{1-\beta+\beta\,(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}$$

for the first group and

$$\frac{1}{1-\beta+\beta\,(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}$$

for the second groups, so the average MPC of A workers is

$$\overline{MPC}^A \equiv (1-\mu)\frac{1-\beta}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}+\mu\frac{1}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}.$$

The reduction in consumption of A good is equal to ϕ for all agents so

$$\left[\frac{\Delta c_B}{\Delta c_A}\right]^{shutdown}=\frac{\frac{1}{1-\beta+\beta(1-\phi)^{-\frac{\sigma-1}{\epsilon-1}}}-(1-\phi)}{\phi}.$$

We conclude that the expression in (34) is negative iff

$$\phi\left[\frac{\Delta c_B}{\Delta c_A}\right]^{shutdown}-\phi\overline{MPC}^A<0$$

which gives (15) in the main text.

B Partial shutdown

B.1 Proof of Proposition 5

Let us rewrite the equilibrium conditions derived in the text, using the notation $p = P_{A0}/W^*$ and $P = P_0/W^*$ and dropping time subscripts:

$$Y_A=\phi(1-\delta)=\phi p^{-\epsilon}\left(\mu\phi P^{\epsilon-1}(1-\delta)+(1-\mu\phi)P^{\epsilon-\sigma}\right),\tag{35}$$

$$Y_B=(1-\phi)\left(\mu\phi P^{\epsilon-1}(1-\delta)+(1-\mu\phi)P^{\epsilon-\sigma}\right).\tag{36}$$

Taking ratios side by side and using $Y_B^*=1-\phi$ yields

$$n_B=\frac{Y_B}{Y_B^*}=p^{\epsilon}(1-\delta).\tag{37}$$

From the CPI (7) we get

$$P=\left(\phi p^{1-\epsilon}+1-\phi\right)^{\frac{1}{1-\epsilon}}.$$

The equilibrium value of p can then be found substituting P in (35) and solving:

$$1-\delta=p^{-\epsilon}\left(\mu\phi\left(\phi p^{1-\epsilon}+1-\phi\right)^{-1}(1-\delta)+(1-\mu\phi)\left(\phi p^{1-\epsilon}+1-\phi\right)^{\frac{\epsilon-\sigma}{1-\epsilon}}\right).\tag{38}$$

It can be shown that this equation has a unique solution p, strictly increasing in δ. Substituting in (37) gives n_B and Y_B.

To complete the equilibrium characterization, we need to check that sector A workers with no credit access are indeed constrained, that is, that their Euler equation holds as an inequality, which requires

$$\frac{1-\delta}{P} < P^{-\sigma}.$$

Aggregating (35) (multiplied by p) and (36) side by side and using the definition of the CPI yields the following

$$\phi n_A + (1-\phi)\, n_B + (p-1)\, Y_A = pY_A + Y_B = \mu\phi\,(1-\delta) + (1-\mu\phi)P^{1-\sigma}.$$

Using $n_A = 1-\delta$ and $n_B = p^{\epsilon}\,(1-\delta)$ we then get

$$P^{1-\sigma} = \frac{\phi\,(1-\mu)\,(1-\delta) + (1-\phi)\,p^{\epsilon}\,(1-\delta) + (p-1)\,Y_A}{1-\mu\phi} > 1-\delta,$$

where the second inequality follows from $p > 1$.

To derive the frontier of the KSS region, we impose $n_B = 1$ in (37) to obtain

$$p = (1-\delta)^{-\frac{1}{\epsilon}}.$$

Substituting in (38) yields

$$1 = \mu\phi\frac{1-\delta}{\phi\,(1-\delta)^{1-\frac{1}{\epsilon}} + 1-\phi} + (1-\mu\phi)\left(\phi\,(1-\delta)^{1-\frac{1}{\epsilon}} + 1-\phi\right)^{\frac{\epsilon-\sigma}{1-\epsilon}}.$$

Solving this equation for σ gives the level $\hat{\sigma}$ that yields exactly $n_B = 1$, given all other parameters. The expression for $\hat{\sigma}$ is equal to the right-hand side of (17).

To complete the argument, we need to show that when $\sigma > \hat{\sigma}$ the pair (n_B, p) that solves (37) and (38) satisfies $n_B < 1$. To do so we keep all parameters fixed and do comparative statics with respect to σ. Inspecting (38) shows that increasing σ reduces p. It follows that n_B from (37) is decreasing in σ, completing the argument.

To derive the limit case for $\delta \to 0$ notice that a linear approximation of (38) at $\delta = 0$ gives

$$-d\delta = -\epsilon dp - \mu\phi\delta + \mu\phi\,(\epsilon-1)\,dP + (1-\mu\phi)\,(\epsilon-\sigma)\,dP,$$

substituting $dP = \phi dp$ and rearranging gives

$$dp = \frac{1-\mu\phi}{(1-\phi)\,\epsilon + \phi\,(\mu\phi + (1-\mu\phi)\,\sigma)}d\delta.$$

Approximating (37) and substituting dp gives

$$dn_B = \epsilon dp - d\delta = \left(\epsilon\frac{1-\mu\phi}{(1-\phi)\,\epsilon + \phi\,(\mu\phi + (1-\mu\phi)\,\sigma)} - 1\right) d\delta.$$

Therefore we get $dn_B < 0$ iff the expression in parenthesis is negative, which gives

$$\sigma > \frac{\epsilon(1-\mu) - \mu\phi}{1-\mu\phi}.$$

The same expression can be obtained by applying L'Hopital's rule to (17).

B.2 Derivation for the limit case $\delta \to 1$

Notice that as $\delta \to 1$ we have $p \to \infty$. If $\epsilon < 1$ we also have $P \to \infty$. Inspecting the expression (36) shows that the term with $P^{\epsilon-1}$ goes to zero. The term with $P^{\epsilon-\sigma}$ goes to zero if $\epsilon < \sigma$, in which case we have a KSS that leads to a complete shutdown of both sectors A and B. The term $P^{\epsilon-\sigma}$ goes to ∞ if $\epsilon > \sigma$, in which case we have full employment in sector B. Using this limit argument, in the case $\epsilon < 1$, the frontier of the KSS region is $\sigma = \epsilon$, as plotted in Figure 3.

C Preference shocks and health

C.1 Preference shocks

We want to characterize an equilibrium in which both sector A and sector B are demand constrained so $P_{A0} = P_{B0} = P^*$. The Euler equations of the unconstrained consumers are then

$$\phi^{\frac{1}{\epsilon}} \theta^{\frac{1}{\epsilon}} \mathbf{c}_0^{\frac{1}{\epsilon}-\frac{1}{\sigma}} \mathbf{c}_{A0}^{-\frac{1}{\epsilon}} = 1,$$
$$(1-\phi)^{\frac{1}{\epsilon}} \mathbf{c}_0^{\frac{1}{\epsilon}-\frac{1}{\sigma}} \mathbf{c}_{B0}^{-\frac{1}{\epsilon}} = 1,$$

which can be solved to give

$$\mathbf{c}_{A0} = \phi\theta (\phi\theta + 1 - \phi)^{\frac{\epsilon-\sigma}{1-\epsilon}},$$
$$\mathbf{c}_{B0} = (1-\phi)(\phi\theta + 1 - \phi)^{\frac{\epsilon-\sigma}{1-\epsilon}}.$$

For constrained agents with income n_{A0} we get

$$c_{A0} = \phi\theta (\phi\theta + 1 - \phi)^{-1} n_{A0},$$
$$c_{B0} = (1-\phi)(\phi\theta + 1 - \phi)^{-1} n_{A0}.$$

Aggregating, we obtain

$$\frac{Y_{A0}}{Y_A^*} = \theta \left[\mu\phi \left(\phi\theta + 1 - \phi\right)^{-1} \frac{Y_{A0}}{Y_A^*} + (1 - \mu\phi) \left(\phi\theta + 1 - \phi\right)^{\frac{\epsilon-\sigma}{1-\epsilon}} \right],$$
$$\frac{Y_{B0}}{Y_B^*} = \mu\phi \left(\phi\theta + 1 - \phi\right)^{-1} \frac{Y_{A0}}{Y_A^*} + (1 - \mu\phi) \left(\phi\theta + 1 - \phi\right)^{\frac{\epsilon-\sigma}{1-\epsilon}}.$$

It immediately follows that

$$\frac{Y_{A0}}{Y_A^*} = \theta \frac{Y_{B0}}{Y_B^*}, \tag{39}$$

and, in particular, $\frac{Y_{A0}}{Y_A^*} < \frac{Y_{B0}}{Y_B^*}$. We can substitute (39) into the second equation to arrive at

$$\frac{Y_{B0}}{Y_B^*} = \frac{(1 - \phi(1-\theta))^{\frac{1-\sigma}{1-\epsilon}}}{1 - \phi \frac{1-\mu}{1-\mu\phi} (1 - \theta)}.$$

Notice that $1 - \phi \frac{1-\mu}{1-\mu\phi} (1-\theta) > 0$ because $\theta > 0$, so the expression above is always positive. We have an equilibrium in which both sectors are demand constrained iff the expression on the right-hand side is less than 1. This proves the following result.

Proposition 8. *Consider the incomplete markets economy, with rigid wages and the nominal rate set at $i_0 = i^*$. A temporary preference shock $\theta_0 = \theta < 1$ causes a contraction in activity in both sectors A and B, with a larger contraction in sector A, if*

$$\sigma > \epsilon - (1-\epsilon) \frac{\ln\left(1 - \mu\phi \frac{\theta}{\phi\theta + 1 - \phi}\right) - \ln(1 - \mu\phi)}{\ln(\phi\theta + 1 - \phi)}. \tag{40}$$

Notice the similarity with the condition for a supply shock causing a partial shutdown in Proposition 5. In particular, if we define $p = \theta^{-1/(\epsilon-1)}$ as the effective price of good A in terms of future consumption we can define the effective CPI (in terms of future consumption) as

$$P_0 = W^* \left(\phi\theta + 1 - \phi\right)^{\frac{1}{1-\epsilon}}.$$

Output in sector B can then be written as

$$\frac{Y_{B0}}{Y_B^*} = \left(\frac{W^*}{P_0}\right)^{-\epsilon} \left(\mu\phi \frac{W^*}{P_0} \frac{Y_{A0}}{Y_A^*} + (1 - \mu\phi) \left(\frac{P_0}{P^*}\right)^{-\sigma} \right),$$

which mirrors the expression (16) for the partial shutdown model and captures the three forces at work: intratemporal substitution, intertemporal substitution, income losses of constrained consumers. The only difference, when solving for condition (40), is that the ratio of output gaps in the two sectors $\frac{Y_{A0}/Y_A^*}{Y_{B0}/Y_B^*}$ is θ instead of $p^{-\epsilon}$.

C.2 Health model

We characterize the model with health in the utility function. Assume $\phi - \eta > 0$ so there is positive consumption in sector A and define

$$\theta \equiv \frac{\phi - \eta}{\phi}.$$

To set the stage for the analysis in Section IIC,we introduce the transfer $\rho\left(1 - n_{j0}\right)$ as in Section II financed by government debt

$$D = \rho\left[\phi\left(1 - n_{A0}\right) + \left(1 - \phi\right)\left(1 - n_{B0}\right)\right],$$

and we introduce a tax τ on consumption of good A, which is rebated lump sum. From the Euler equations, the average consumption of unconstrained consumers is now

$$\mathbf{c}_{A0} = \frac{\theta\phi}{1+\tau}\left(1 + \frac{\mu\phi}{1-\mu\phi}rD\right),$$

$$\mathbf{c}_{B0} = \left(1 - \phi\right)\left(1 + \frac{\mu\phi}{1-\mu\phi}rD\right),$$

where $r = 1/\beta - 1$. For constrained consumers we get

$$c_{A0} = \frac{\theta\phi}{1+\tau}\frac{n_{A0} + \rho\left(1 - n_{A0}\right)}{\frac{\theta\phi}{1+\tau} + 1 - \phi},$$
$$c_{B0} = \left(1 - \phi\right)\frac{n_{B0} + \rho\left(1 - n_{B0}\right)}{\frac{\theta\phi}{1+\tau} + 1 - \phi},$$

as long as the borrowing constraint is binding, which happens iff the following condition holds

$$n_{A0} + \rho\left(1 - n_{A0}\right) < \left(\frac{\theta\phi}{1+\tau} + 1 - \phi\right)\left(1 - rD\right).$$

If the borrowing constraint above is binding, the goods market equilibrium conditions are

$$Y_{A0} = \frac{\theta\phi}{1+\tau}\left[\mu\phi\frac{n_{A0} + \rho\left(1 - n_{A0}\right)}{\frac{\theta\phi}{1+\tau} + 1 - \phi} + 1 - \mu\phi + \mu\phi rD\right],$$
$$Y_{B0} = \left(1 - \phi\right)\left[\mu\phi\frac{n_{A0} + \rho\left(1 - n_{A0}\right)}{\frac{\theta\phi}{1+\tau} + 1 - \phi} + 1 - \mu\phi + \mu\phi rD\right].$$

Combining the conditions above shows that the equilibrium features binding borrowing constraints and unemployment in sector B, $n_{B0} < 1$, if

$$\rho < \hat{\rho} \equiv \frac{1}{1 + r\phi\left(\frac{\theta\phi}{1+\tau} + 1 - \phi\right)} \frac{1 - n^*_{A0} + \frac{\theta\phi}{1+\tau} - \phi}{1 - n^*_{A0}} < 1,$$

where $n^*_{A0} = \frac{\theta}{1+\tau}$. If $\rho \geq \hat{\rho}$ the borrowing constraint is not binding for any consumer, the goods market equilibrium conditions are

$$Y_{A0} = \frac{\theta\phi}{1+\tau}, \quad Y_{B0} = 1 - \phi,$$

and there is full employment in sector B. The fact that the conditions for a non-binding constraint and for full employment in B coincide is due to the fact that the log case satisfies condition $\sigma = \epsilon$, so the natural rate is equal to $1/\beta - 1$ under complete markets.

D Fiscal Policy

We characterize an equilibrium with fiscal policy. Consider first an equilibrium in which the borrowing constraint of sector A workers with no credit access is binding, which requires

$$\rho < (1-\phi)^{\frac{\sigma-1}{\epsilon-1}} \left(1 - \frac{\zeta}{\mu} r^* D\right).$$

The average consumption of unconstrained consumers in periods $t = 1, 2, \ldots$ is

$$\mathbf{c}_1 = 1 + \frac{r^* D}{1-\mu\phi} - \frac{\phi(1-\mu)\frac{1-\zeta}{1-\mu} r^* D + (1-\phi)(1-\mu)\frac{1-\zeta}{1-\mu} r^* D + (1-\phi)\mu\frac{\zeta}{\mu} r^* D}{1-\mu\phi}$$

given that the stock of debt D is entirely held by the group of $1-\mu\phi$ unconstrained consumers. Rearranging gives

$$\mathbf{c}_1 = 1 + \frac{\zeta\phi}{1-\mu\phi} r^* D.$$

Using the Euler equation their consumption of good B at date 0 is

$$\mathbf{c}_{B0} = (1-\phi)^{\frac{\sigma-1}{\epsilon-1}} \left(1 + \frac{\zeta\phi}{1-\mu\phi} r^* D\right),$$

and total demand in sector B is

$$Y_{B0} = G + \mu\phi\rho + (1-\mu\phi)(1-\phi)^{\frac{\sigma-1}{\epsilon-1}} \left(1 + \frac{\zeta\phi}{1-\mu\phi} r^* D\right), \tag{41}$$

where

$$D = G + \phi\rho + (1-\phi)\,\rho\,(1-n_{B0})\,.$$

D.1 Proof of Proposition (6)

Set $\zeta = 0$. At $G = \rho = 0$ we get

$$dY_{B0} = dG + \mu\phi d\rho,$$

and the effect of $d\rho$ on total transfers is

$$dT = [\phi + (1-\phi)\,(1-n_{B0})]\,d\rho.$$

The expressions for the multipliers follow from these two equations.

D.2 Proof of Proposition 7

Substituting the expression for D in (41) and rearranging we get

$$n_{B0} = \frac{\mu\phi\rho}{1-\phi} + (1-\phi)^{\frac{\sigma-\epsilon}{\epsilon-1}}\left[1 - \mu\phi + \zeta\phi r^{*}\left(\phi\rho + (1-\phi)\,\rho\,(1-n_{B0})\right)\right].$$

It is possible to show that as ρ varies in $[0,1]$ the value of n_{B0} that solves this equation is increasing in ρ and so is the expression

$$\rho - (1-\phi)^{\frac{\sigma-1}{\epsilon-1}}\left(1 - \frac{\zeta}{\mu}r^{*}D\right), \tag{42}$$

and both are continuous. Let the cutoff $\hat{\rho}$ be the smallest ρ for which either $n_{B0} = 1$ or (42) is zero. Notice that if (42) is negative, then all consumers are unconstrained and demand for good B is given by

$$Y_{B0} = (1-\phi)^{\frac{\sigma-1}{\epsilon-1}}\,.$$

Therefore, by the definition of $\hat{\rho}$, when $\rho > \hat{\rho}$ the equilibrium value of n_{B0} is constant and either equal to 1 or equal to $(1-\phi)^{\frac{\sigma-\epsilon}{\epsilon-1}} < 1$.

Setting $\zeta = \mu$ and $\rho = 1$ implies that all agents receive income after transfers equal to 1 in period 0 and pay tax $r^{*}D$ in all future periods. Therefore, it achieves perfect insurance and replicates the complete market allocation. If $\sigma < \epsilon$ the complete market allocation also achieves full employment in sector B, so it is first best optimal. If $\sigma > \epsilon$ the complete market allocation with real rate equal to $1/\beta$ is not first best optimal as there is unemployment in sector B. However, the planner cannot increase output above $(1-\phi)^{\frac{\sigma-1}{\epsilon-1}}$ with the fiscal instruments allowed (ρ and ζ) and the complete market allocation maximizes the utilitarian planner objective conditional on $Y_{B0} \leq (1-\phi)^{\frac{\sigma-1}{\epsilon-1}}$. So in both cases setting $\zeta = \mu$ and $\rho = 1$ is optimal.