'Market Power and Innovation in the Intangible Economy' by Maarten De Ridder, Online Appendix

Appendix A. Proofs and Derivations

A.1. Derivation of the Choke Price

The choke price p_i^c of firm i is the minimum price at which a cost-minimizing producer breaks even:

$$p_i^c = \inf\left\{p > 0 : \max_{s\in(0,1]} \left(p - ws\right) Y p^{-1} - w\phi_i \left(s^{-\theta} - 1\right) \geq 0\right\},$$

where the maximized function measures the producing firm's profits, net of fixed-cost intangibles, at price p. Note that none of the terms on the right-hand side are specific to the good that a firm produces. This means that the firm's choke price, p_i^c, is homogeneous across the goods that a firm produces. Setting profits to zero yields an expression for the choke price conditional on the fraction of marginal costs that a cost-minimizing firm would incur if selling at its choke price, s_i^c:

$$p_i^c = ws_i^c(Y - w\phi_i([s_i^c]^{-\theta} - 1))^{-1} Y.$$

The expression is not closed form because optimal intangibles depend on prices. Under cost minimization, first-order condition (6) at the choke price can be written as :

$$s_i^c = \min\left[\left(p_i^c Y^{-1}\theta\phi_i\right)^{\frac{1}{\theta+1}}, 1\right]. \tag{A.1}$$

For a given wage rate w and aggregate output Y, the choke price is thus entirely determined by a firm's intangible cost parameter ϕ_i, so that $p_i^c = p^c(\phi_i)$. In closed form, the choke price is given by:

$$p^c(\phi_i) = \begin{cases} w\left(\theta^{1/(1+\theta)} + \theta^{-\theta/(1+\theta)}\right)^{(1+\theta)/\theta} \left(\left[\left(\frac{Y}{w\phi_i}\right)^{\theta/(1+\theta)}\right]\left[\frac{Y}{w\phi_i}+1\right]^{-1}\right)^{(1+\theta)/\theta} & \text{if } \phi_i < Y/(\theta w) \\ w & \text{if } \phi_i \geq Y/(\theta w), \end{cases}$$

where $Y/(\theta w)$ is the lowest value of the intangible cost parameter such that $s_i^c = 1$.

Two properties of the choke price are worth pointing out. First, the choke price is homogeneous of degree one in *(w,Y)*. This is clear from the choke-price equation above, and means that the relative choke prices across firms are constant along a balanced growth path where w and Y are growing at the same constant rate. Second, for $s_i^c < 1$, the choke price strictly increases in a firm's intangible cost parameter ϕ_i. To see this, note that the choke price strictly increases in the term

$$\left(\left[\left(\frac{Y}{w\phi_i}\right)^{\theta/(1+\theta)}\right]\left[\frac{Y}{w\phi_i}+1\right]^{-1}\right)^{(1+\theta)/\theta},$$

and that this term, in turn, increases in ϕ_i for $0 < \phi_i < Y/(\theta w)$. Hence $\partial p^c(\phi_i)/\partial\phi_i > 0$ for $\phi_i < Y/(\theta w)$.

A.2. Proof of Proposition 1

The value function is given by the following Bellman equation:

$$rV_t(\phi_i, J_i) - \dot{V}_t(\phi_i, J_i) = \max_{x_i}\left\{\begin{array}{c} \sum_{j\in J_i}\left[\begin{array}{c}\pi_t(\phi_i,\lambda_{ij}) + \\ \tau(\phi_i)\left[V_t(\phi_i, J_i\setminus\{\lambda_{ij}\}) - V_t(\phi_i,J_i)\right]\end{array}\right] \\ +x_i\mathscr{P}(\phi_i)\mathbb{E}_{\phi_i}\left[V_t(\phi_i, J_i\cup_+\lambda_{ih}) - V_t(\phi_i,J_i)\right] \\ -w_t\eta^x(x_i)^{\psi^x} n_i^{-\sigma} - F(\phi_i, n_i)\end{array}\right\}$$

Guess that the solution takes the following form:

$$V_t(\phi_i, J_i) = \sum_{j \in J_i} v_t(\phi_i, \lambda_{ij})$$

where $v_t(\cdot)$ (and hence V_t) grows at a constant rate g in the balanced growth equilibrium. Then $v_t(\phi_i, \lambda_{ij})$ can be written as:

$$\left[r - g + \tau(\phi_i)\right] v_t(\phi_i, \lambda_{ij}) = \pi_t(\phi_i, \lambda_{ij}) + \Gamma$$

where Γ is the option value of innovation adjusted for the fixed term $F(\phi_i, n_i)$:

$$\Gamma = \max_{x_i} \left[\frac{x_i}{n_i} \mathscr{P}(\phi_i) \mathbb{E}_{\phi_i} \left[v_t(\phi_i, \lambda_{ih}) \right] - w_t \eta^x (x_i)^{\psi^x} n_i^{\sigma - 1} \right] - \frac{F(\phi_i, n_i)}{n_i} \tag{A.2}$$

which is a function Γ. In order for the value function to scale with size along the guess (a simplification that is removed in Section V, Γ must not change with the number of goods that the firm produces. I achieve that by choosing $F(\phi_i, n_i)$ so that $\Gamma = 0$. To find the $F(\phi_i, n_i)$ that achieves this, use that the first-order condition satisfies:

$$\mathscr{P}(\phi_i) \mathbb{E}_{\phi_i} \left[v_t(\phi_i, \lambda_{ih}) \right] = \psi^x w_t \eta^x (x_i)^{\psi^x - 1} n_i^{\sigma}$$

so that if $\Gamma = 0$, the fixed term satisfies:

$$F(\phi_i, n_i) = (\psi^x - 1) w_t \eta^x \left[x_{n_i}(\phi_i) \right]^{\psi^x} n_i^{\sigma}$$

With this constraint, optimal research and development expenditures satisfy the equation in Proposition 1:

$$x_{n_i}(\phi_i) = \left(\mathscr{P}(\phi_i) \frac{\mathbb{E}_{\phi_i} \left[\frac{\pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)} \right]}{\eta^x \psi^x w_t} \right)^{\frac{1}{\psi^x - 1}} n_i^{\frac{\sigma}{\psi^x - 1}}$$

It follows that

$$V_t(\phi_i, J_i) = \frac{\sum_{j \in J_i} \pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)}$$

where operating profits satisfy:

$$\pi_t(\phi_i, \lambda_{ij}) = \left[1 - \frac{\left(\lambda_{ij} \frac{w_t}{Y_t} \phi_i \right)^{\frac{1}{\theta + 1}}}{\lambda_{ij}} \right] Y_t - w_t \phi_i \left(\left[\lambda_{ij} \frac{w_t}{Y_t} \phi_i \right]^{\frac{-\theta}{\theta + 1}} - 1 \right)$$

which increases at rate g along the balanced growth path, confirming the initial guess.

A.3. Derivation of Aggregate Quantities and Proof of Proposition 2

The equilibrium wage is derived as follows. Start with the definition of aggregate output when each sector is in a betrand equilibrium:

$$\ln Y = \int_0^1 \int \mathbf{1}_{j \in J_i} \ln \left(q_{ij} y_{ij} \right) di \, dj$$

Inserting the firm's production function $y_{ij} = l_{ij} / (s_{ij})$ and demand function $y_{ij} = Y / p_{ij}$ yields:

$$\ln Y = \ln Y + \int_0^1 \int \mathbf{1}_{j \in J_i} \ln \left(q_{ij} (w[s_{ij}])^{-1} \mu_{ij}^{-1} \right) di \, dj$$

Isolating wage on the left hand side gives:

$$\ln w = \int_0^1 \int \mathbf{1}_{j\in J_i} \ln \left[\frac{q_{ij}}{s_{ij}}\right] di\, dj + \int_0^1 \int \mathbf{1}_{j\in J_i} \ln \left[\frac{s_{ij}}{\lambda_{ij}}\right] di\, dj$$

The derivation of GDP is as follows. Labor market equilibrium requires:

$$L^p = \int_0^1 \int \mathbf{1}_{j\in J_i} l_{ij} di\, dj$$

Inserting the firm's production function $y_{ij} = l_{ij}/s_{ij}$ and demand function $y_{ij} = Y/p_{ij}$ yields:

$$L^p = \int_0^1 \int \mathbf{1}_{j\in J_i} Y p_{ij}^{-1} s_{ij} di\, dj$$

Isolate Y on the left hand side, insert the first-order condition for pricing, and insert the equilibrium wage to obtain:

$$Y = L^p \exp\left(\int_0^1 \int \mathbf{1}_{j\in J_i} \ln \left[\frac{q_{ij}}{s_{ij}}\right] di\, dj\right) \frac{\exp \int_0^1 \int \mathbf{1}_{j\in J_i} \ln \mu_{ij}^{-1} di\, dj}{\int_0^1 \int \mathbf{1}_{j\in J_i} \mu_{ij}^{-1} di\, dj} \tag{A.3}$$

Define total factor productivity Q_t as the terms to the right of L^p in expression (A.3). A balanced growth path equilibrium is characterized by constant type-shares $K(\phi_i)$. Given that markups equation λ_{ij}/s_{ij} where s_{ij} is given by equation 7, the law of large numbers assures that the third term in (A.3) is constant. Hence $g \equiv \partial \ln Q/\partial t$ is given by:

$$g = \int_0^1 \int \mathbf{1}_{j\in J_i} \frac{\partial \ln q_{ij}}{\partial t}\, di\, dj = \sum_{\phi_i \in \Phi} K(\phi_i)\tau(\phi_i)\mathbb{E}_{-\phi_i}(\lambda_{hj} - 1)$$

which uses that $K(\phi_i)\tau(\phi_i)$ is the fraction of goods that changes producer each instance and where initially produced by ϕ_i-type firms.

A.4. Proposition on Shape of Alternative Value Function

Proposition A.1. *The value function of a firm with intangible cost parameter ϕ_i that produces a portfolio of goods J_i with cardinality n_i grows at rate g along the balanced growth path and is given by*

$$V_t(\phi_i, J_i) = \sum_{j\in J_i} \Upsilon_t^1(\phi_i, \lambda_{ij}) + \Upsilon_{t,n_i}^2(\phi_i),$$

where Υ_1 is the present value of the profit flow from producing good j. Matching coefficients gives

$$\Upsilon_t^1(\phi_i, \lambda_{ij}) = \pi_t(\phi_i, \lambda_{ij})(r - g + \tau(\phi_i))^{-1},$$

while Υ_{2n_i} is the option value of research and development which evolves along this sequence:

$$\begin{aligned} \Upsilon_{t,n_i+1}^2(\phi_i) \quad = \quad & \left[\left((r-g)\Upsilon_{t,n_i}^2(\phi_i) + n_i\tau(\phi_i)\left[\Upsilon_{t,n_i}^2(\phi_i) - \Upsilon_{t,n_i-1}^2(\phi_i)\right]\psi^x - 1\right)(\psi^x - 1)^{-1}\right]^{\frac{\psi^x - 1}{\psi^x}} \\ & \left[\mathscr{P}(\phi_i)\right]^{-1}\psi^x\left(\eta w_t\right)^{\psi^{x-1}} n_i^{-\frac{\sigma}{\psi^x}} + \Upsilon_{t,n_i}^2(\phi_i) - \Upsilon_t^1(\phi_i, \lambda_{ij}), \end{aligned}$$

so that the first-order conditions for optimal research and development and entry read

$$x_{n_i}(\phi_i) = \left(\mathscr{P}(\phi_i)\frac{\mathbb{E}_{\phi_i}\left[\Upsilon_t^1(\phi_i, \lambda_{ij}) + \Upsilon_{t,n_i+1}^2(\phi_i) - \Upsilon_{t,n_i}^2(\phi_i)\right]}{\eta^x \psi^x w_t}\right)^{\frac{1}{\psi^x - 1}} n_i^{\frac{\sigma}{\psi^x - 1}},$$

$$e = \left(\sum_{\phi_h \in \Phi} G(\phi_h) H\left(\frac{p^c(\phi_h)}{p^c(\phi_{-i})}\right)\frac{\mathbb{E}_{\phi_h}\left[\Upsilon_t^1(\phi_h, \lambda_{hj}) + \Upsilon_{t,1}^2(\phi_h)\right]}{\eta^e \psi^e w_t}\right)^{\frac{1}{\psi^e - 1}}. \tag{A.4}$$

Proof:

The value function is given by the following Bellman equation:

$$r V_t(\phi_i, J_i) - \dot{V}_t(\phi_i, J_i) = \max_{x_i} \left\{ \begin{array}{c} \sum_{j \in J_i} \left[\begin{array}{c} \pi_t(\phi_i, \lambda_{ij}) + \\ \tau(\phi_i) \left[V_t(\phi_i, J_i \setminus \{\lambda_{ij}\}) - V_t(\phi_i, J_i) \right] \end{array} \right] \\ + x_i \mathscr{P}(\phi_i) \\ \mathbb{E}_{\phi_i} \left[V_t(\phi_i, J_i \cup_+ \lambda_{ih}) - V_t(\phi_i, J_i) \right] - w_t \eta^x (x_i)^{\psi^x} n_i^{-\sigma}) \end{array} \right\}$$

Guess that the solution takes the following form:

$$V_t(\phi_i, J_i) = \sum_{j \in J_i} \Upsilon^1_t(\phi_i, \lambda_{ij}) + \Upsilon^2_{t,n_i}(\phi_i)$$

where firm i produces a portfolio of goods J_i with cardinality n_i, and where $\Upsilon^1_t()$ and $\Upsilon^2_{t,n_i}()$ (and hence V_t) grow at a constant rate g in the balanced growth equilibrium. Grouping terms yields:

$$(r - g + \tau(\phi_i)) \Upsilon^1_t(\phi_i, \lambda_{ij}) = \pi_t(\phi_i, \lambda_{ij}) \Rightarrow \Upsilon^1_t(\phi_i, \lambda_{ij}) = \frac{\pi_t(\phi_i, \lambda_{ij})}{r - g + \tau(\phi_i)}$$

The proof of proposition 1 showed that profits grow at rate g, confirming the guess. Furthermore:

$$\left(r - g\right) \Upsilon^2_{t,n_i}(\phi_i) = \max_{x_i} \left\{ \begin{array}{c} n_i \tau(\phi_i) \left[\Upsilon^2_{t,n_i-1}(\phi_i) - \Upsilon^2_{t,n_i}(\phi_i) \right] + x_i \mathscr{P}(\phi_i) \\ \mathbb{E}_{\phi_i} \left[\Upsilon^2_{t,n_i+1}(\phi_i) - \Upsilon^2_{t,n_i}(\phi_i) + \Upsilon^1_t(\phi_i, \lambda_{ij}) \right] - w_t \eta^x (x_i)^{\psi^x} n_i^{-\sigma}) \end{array} \right\}$$

The first-order condition of the maximization reads:

$$\mathscr{P}(\phi_i) \mathbb{E}_{\phi_i} \left[\Upsilon^2_{t,n_i+1}(\phi_i) - \Upsilon^2_{t,n_i}(\phi_i) + \Upsilon^1_t(\phi_i, \lambda_{ij}) \right] = w_t \psi^x \eta^x (x_i)^{\psi^x - 1} n_i^{-\sigma}$$

Inserting the first-order condition and isolating $\Upsilon^2_{t,n_i+1}(\phi_i)$ and $\Upsilon^1_t(\phi_i, \lambda_{ij})$ on the left hand side gives the sequence for Υ^2_{t,n_i+1}:

$$\begin{aligned} \Upsilon^2_{t,n_i+1}(\phi_i) + \Upsilon^1_t(\phi_i, \lambda_{ij}) \quad = \quad & \left[\frac{(r-g) \Upsilon^2_{t,n_i}(\phi_i) + n_i \tau(\phi_i) \left[\Upsilon^2_{t,n_i}(\phi_i) - \Upsilon^2_{t,n_i-1}(\phi_i) \right]}{\psi^x - 1} \right]^{\frac{\psi^x - 1}{\psi^x}} \\ & \mathscr{P}(\phi_i)^{-1} \psi^x \left(\eta w_t \right)^{\psi^{x-1}} n_i^{-\frac{\sigma}{\psi^x}} + \Upsilon^2_{t,n_i}(\phi_i). \end{aligned}$$

A.5. Proof of Proposition 3

Part (a) I first show that, holding all else constant, a decline in ϕ_i raises a firm's cost-minimizing ratio of fixed over variable costs. To see this, consider the s^*_{ijt} that minimizes costs

$$tc_{ij} = y_{ij} s_{ij} \boldsymbol{c}(w_1, w_2, .., w_k) + f(s_{ij}, \phi_i).$$

where time subscripts are omitted for ease of exposition. Given Cobb-Douglas demand, s^*_{ij} is implicitly given through the first-order condition

$$y_{ij} \boldsymbol{c}(w_{1t}, w_{2t}, .., w_{kt}) = -\frac{\partial f(s^*_{ij}, \phi_i)}{\partial s_{ij}}. \tag{A.5}$$

for an interior solution where $s^*_{ij} < 1$. The ratio of fixed costs over total costs whenever $f(s^*_{ij}, \phi_i) > 0$ is therefore given by:

$$\frac{f(s^*_{ij}, \phi_i)}{tc_{ij}} = \frac{f(s^*_{ij}, \phi_i)}{f(s^*_{ij}, \phi_i) - s^*_{ij} \frac{\partial f(s^*_{ij}, \phi_i)}{\partial s^*_{ij}}}.$$

Dividing both the numerator and denominator by cost-minimizing fixed costs, this simplifies to:

$$\frac{f(s_{ij}^*,\phi_i)}{tc_{ij}} = \frac{1}{1-\varepsilon_{f(s_{ij}^*,\phi_i),s_{ij}^*}}. \tag{A.6}$$

where $\varepsilon_{f(s_{ij},\phi_i),s_{ij}} < 0$ is the elasticity of fixed costs with respect to s_{ij}. It follows that a reduction in intangible cost parameter ϕ_i leads to an increase to the cost-minimizing firm's fixed costs ratio if the elasticity declines in ϕ_i. This condition is satisfied in the model:

$$\begin{aligned}\frac{\partial\varepsilon_{f(s_{ij}^*,\phi_i),s_{ij}^*}}{\partial\phi_i} &= -\theta\left(\frac{\partial {s_{ij}^*}^{-\theta}/({s_{ij}^*}^{-\theta}-1)}{\partial\phi_i}\right), \\ &= -\theta\left(\frac{\theta}{\theta+1}\frac{1}{\phi_i}\frac{{s_{ij}^*}^{-\theta}}{({s_{ij}^*}^{-\theta}-1)^2}\right) < 0.\end{aligned} \tag{A.7}$$

Part (b) I next show that firms with lower intangible efficiencies ϕ_i innovate at higher rates and, on average, charger higher markups. A firm's rate of innovation is given by first-order condition (17), hence

$$\frac{\partial \ln x_{n_i}(\phi_i)}{\partial\phi_i} = \frac{1}{\psi^x-1}\left(\frac{\partial}{\partial\phi_i}\ln\mathscr{P}(\phi_i) + \frac{\partial}{\partial\phi_i}\ln\mathbb{E}_{\phi_i}\left[\frac{\pi(\phi_i,\lambda_{ij})}{r-g+\tau(\phi_i)}\right]\right).$$

Regardless of whether firms have equal or unequal intangible efficiencies, a lower ϕ_i raises innovation through profitability. Under the Cobb Douglas aggregator, profits are given by $Y - tc_{ij}$. Hence, if a lower ϕ_i reduces total costs, profits fall in ϕ_i. It is straightforward to show that this is the case. With slight abuse of notation, I denote tc_{ij}^* as a firm's total costs under cost minimization,

$$tc_{ij}^* = f(s_{ij}^*,\phi_i) - s_{ij}^*\frac{\partial f(s_{ij}^*,\phi_i)}{\partial s_{ij}^*}.$$

These costs rise in a firm's intangible cost parameter, as is clear from the following total derivative:

$$\frac{dtc_{ij}^*}{d\phi_i} = \frac{\partial f(s_{ij}^*,\phi_i)}{\partial\phi_i} + \frac{\partial f(s_{ij}^*,\phi_i)}{\partial s_{ij}^*}\frac{\partial s_{ij}^*}{\partial\phi_i} - \frac{\partial s_{ij}^*}{\partial\phi_i}\frac{\partial f(s_{ij}^*,\phi_i)}{\partial s_{ij}^*} > 0,$$

which uses that the derivative term in total costs is a constant through first-order condition (A.5). Hence low-ϕ_i firms, ceteris paribus, are more profitable. This incentivises them to spend more on R&D regardless of intangible cost parameters of other firms. If firms have heterogeneous intangible efficiencies ($|\Phi| > 1$), $x_{n_{it}}$ is higher for firms with a lower ϕ_i through two additional channels. First, the probability of success in innovation is given by

$$\mathscr{P}(\phi_i) = \textstyle\sum_{\phi_{-i}\in\Phi} K(\phi_{-i})H\left(\frac{\phi_i}{\phi_{-i}}\right),$$

we have that $\partial\mathscr{P}(\phi_i)/\partial\phi_i > 0$ as long as $|\Phi| > 1$ and $f(s_{ij}^*,\phi_i) > 0$. Second, the rate of creative destruction $\tau(\phi_i)$ in (14) obeys $\partial\tau(\phi_i)/\partial\phi_i < 0$ as long as $|\Phi| > 1$ and $f(s_{ij}^*,\phi_i) > 0$. Hence $\partial\ln x_{n_i}/\partial\phi_i > 0$. The positive relationship between markups and fixed costs follows from the first-order condition:

$$\ln\mu_{ij} = \lambda_{ij} - \ln mc_{ij}$$

where $\tilde{\lambda}_{ij}$ is the quality gap between the current and previous producer of *j*. Given that $mc_{ij} = s_{ij}\boldsymbol{c}(w_1, w_2, .., w_k)$, we have $\partial\ln\mu_{ij}/\partial\ln s_{ij} < 0$. Combined with (A.11), this confirms the proposition.

A.6. Derivation of estimation equation (30)

To derive the estimation equation, I start from the first-order equation for R&D in the model from Section I. Combining (17) with (10) and defining R&D spending $xrd_{it} = w_t rd_{it}$, I write

$$\ln\left(\frac{xrd_{it}}{py_{it}}\right) = \frac{\psi^x}{\psi^x - 1}\ln\left(\mathscr{P}(\phi_i)\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i,\lambda_{ij})}{r-g+\tau(\phi_i)}\right]\right) + \left(\frac{\sigma}{\psi^x - 1} - 1\right)\ln n_{it} + \ln\left(\frac{w_t\eta^x}{Y_t}(\eta^x\psi^x w_t)^{\frac{\psi^x}{1-\psi^x}}\right), \quad \text{(A.8)}$$

The right-hand side of the equation contains three terms described in the main text. The first term captures the value of becoming the producer of an additional good, which is higher for firms with low intangible costs ϕ_i. Along the balanced growth path, the term is entirely captured by a firm fixed effect. The second term captures that innovation intensity falls in firm size. The final term is the time fixed effect. This motivates the reduced-form estimation equation (30).

A.7. Proof of Proposition 4

The proposition claims that firm-level fixed costs are identified by measured fixed costs $\hat{f}_{it}$:

$$\frac{\hat{f}_{it}}{p_{it}y_{it}} = \left(1 - \frac{1}{\hat{\mu}_{it}}\right) - \frac{\pi_{it}}{p_{it}y_{it}},$$

where notation follows the main text, as long as $\hat{\mu}_{it}$ is the harmonic average of product-level markups $\mu_{ijt} = p_{ijt}/mc_{ijt}$. Given observed profits $\pi_i = \sum_{j\in J_{it}}(p_{ijt}y_{ijt} - y_{ijt}mc_{ijt} - f_{ijt}) - \tilde{f}_{it}$, it follows that measured fixed costs $\hat{f}_{it}$ equal the firm's true fixed costs $\tilde{f}_{it} + \sum_{j\in J_{it}} f_{ijt}$ as long as:

$$\hat{\mu}_{it} \quad = \quad \frac{\sum_{j\in J_{it}} p_{ij}y_{ij}}{\sum_{j\in J_i} y_{ijt}mc_{ijt}} = \frac{\sum_{j\in J_{it}} p_{ij}y_{ij}}{\sum_{j\in J_i} y_{ijt}p_{ijt}\mu_{ijt}^{-1}}.$$

Proposition 4 is verified by inserting the Cobb Douglas demand function $p_{ijt}y_{ijt} = Y_t$:

$$\hat{\mu}_{it} = \left(n_i^{-1}\sum_{j\in J_i}\mu_{ijt}^{-1}\right)^{-1},$$

which is the harmonic average of the true product-level markups.

A.8. Derivation: Product and Firm-level Markups with the Hall (1988) equation

To derive the conditions under which measured firm-level markups from the Hall (1988) equation equal the harmonic average of true product-level markups, I first show that the Hall equation is valid in the model. Firms solve the following cost minimization problem for tangible inputs $z_{ijt,h}$:

$$\min_{z_{ijt,h}\forall j,t,h} \sum_{h=1}^{k} z_{ijt,h}w_{ht} \text{ s.t. } y_{ijt} = \tfrac{1}{s_{ijt}} z(z_{ijt,1}, z_{ijt,2}, .., z_{ijt,k})$$

The first-order condition for a particular tangible input $z_{ijt,h}$ is given by:

$$w_{ht} = \lambda_{ijt}\frac{1}{s_{ijt}}\frac{\partial z(z_{ijt,1}, z_{ijt,2}, .., z_{ijt,k})}{\partial z_{ijt,h}},$$

where λ_{ijt} is the Lagrange multiplier of the cost-minimization problem, which measures marginal costs. Dividing both sides by output y_{ijt} and prices p_{ijt}, and multiplying both sides by $z_{ijt,h}$ gives:

$$\frac{z_{ijt,h}w_{ht}}{y_{ijt}p_{ijt}} = \frac{\lambda_{ijt}}{p_{ijt}}\frac{1}{s_{ijt}}\frac{\partial z(z_{ijt,1}, z_{ijt,2}, .., z_{ijt,k})}{\partial z_{ijt,h}}\frac{1}{y_{ijt}},$$

where the first term on the right-hand side is the inverse of the product's true markup. Rewriting gives:

$$\mu_{ijt} = \alpha_{ijt}\left(\frac{y_{ijt}p_{ijt}}{z_{ijt,h}w_{ht}}\right), \tag{A.9}$$

where α_{ijt} is the elasticity of y_{ijt} with respect to $z_{ijt,h}$. This is the Hall (1988) equation.

In practice, with lack of data on product-level input and revenue data, markups are calculated at the firm level. In the empirical analysis, I measure firm-level markups as follows:

$$\widehat{\mu}_{it} = \widehat{\alpha}_{it}\left(\frac{py_{it}}{w_{ht}z_{it,h}}\right),$$

where $\widehat{\alpha}_{it}$ is an estimate of the elasticity of output with respect to tangible input $z_{it,h} = \sum_{j\in J_{it}} z_{ijt,h}$. Expressed in terms of product-level revenue and input spending, firm-level markups are measured as:

$$\widehat{\mu}_{it} = \widehat{\alpha}_{it}\left(\frac{\sum_{j\in J_{it}} p_{ijt}y_{ijt}}{\sum_{j\in J_{it}} w_{ht}z_{ijt,h}}\right),$$

From (A.9), product-level spending on materials can be substituted with product-level markups:

$$\widehat{\mu}_{it} = \widehat{\alpha}_{it}\left(\frac{\sum_{j\in J_{it}} p_{ijt}y_{ijt}}{\sum_{j\in J_{it}} \alpha_{ijt}p_{ijt}y_{ijt}\mu_{ijt}^{-1}}\right) = \left(\frac{1}{n_i}\sum_{j\in J_{it}}\frac{\alpha_{ijt}}{\widehat{\alpha}_{it}}\mu_{ijt}^{-1}\right)^{-1}, \tag{A.10}$$

where the final step uses the Cobb-Douglas demand function.

It follows that firm-level markups from the Hall (1988) equation measure the harmonic average of the product-level markups, as required for Proposition 4, in two cases. For multi-product firms, (A.10) shows that measured firm markups indeed equal the harmonic average of true product levels as long as $\alpha_{ijt} = \widehat{\alpha}_{it}$, so that:

$$\widehat{\mu}_{it} = \left(\frac{1}{n_i}\sum_{j\in J_{it}}\mu_{ijt}^{-1}\right)^{-1},$$

as required. In other words, as long as product-level output elasticities are the same across a firm's products and the elasticity estimate is correct, the firm-level markups can be used to identify the firm's total fixed costs. This is the case, for example, for the production function in Section I, where $\alpha_{ijt} = 1$ and where the sole input that satisfies the assumptions on $z_{ijt,h}$ is production labor l_{ijt}.

For single-product firms, the markup is correctly measured as long as the output elasticity of input $z_{ijt,h}$ is correctly estimated. To assure this, I build on the production function estimation literature, as detailed in Appendix C. Both in the data and in the calibrated model, the majority of firms produces a single product, and estimates of fixed costs for these firms are consistent.

A.9. Derivation of Average Fixed Costs as a Function of homogeneous ϕ

I next show that the average ratio of fixed costs over total costs across firms in a calibration where firms have a homogeneous $\phi_i = \phi$ declines in ϕ. A homogeneous rise in ϕ is different from an idiosyncratic decline in ϕ_i because the homogeneous rise will affect the labor share, which enters in the first-order condition for intangibles. The average ratio of fixed-costs over total costs is:

$$\int_0^1 \frac{f(s_j^*, \phi_i)}{tc_j} dj = \int_0^1 \frac{1}{1-\varepsilon_{f(s_j^*,\phi),s_j^*}} dj.$$

where firm subscripts are omitted as all firms have equal intangible cost parameters. To understand how the average ratio of fixed costs over total costs changes in ϕ, consider the following total derivative:

$$\frac{d\int_0^1 \frac{f(s_j^*,\phi_i)}{tc_j} dj}{d\phi} = \int_0^1 (1-\varepsilon_{f(s_j^*,\phi),s_j^*})^{-2} \frac{d\varepsilon_{f(s_j^*,\phi),s_j^*}}{d\phi}$$

Hence it suffices to show that the final term on the right-hand side is negative to show that the ratio of fixed costs over total costs declines in ϕ. This is the case if:

$$\begin{aligned} \int_0^1 \frac{d\varepsilon_{f(s_{ij}^*,\phi_i),s_{ij}^*}}{d\phi} dj &= -\theta \int_0^1 \left(\frac{d {s_{ij}^*}^{-\theta} / ({s_{ij}^*}^{-\theta} - 1)}{d\phi_i} \right) dj, \\ &= -\theta \int_0^1 \left(\theta \frac{{s_{ij}^*}^{-\theta-1}}{({s_{ij}^*}^{-\theta}-1)^2} \frac{ds_{ij}^*}{d\phi} \right) dj, \\ &< 0. \end{aligned} \tag{A.11}$$

It follows that the right-hand side must be positive for average fixed costs to rise in ϕ. To see that this is the case, define $\tilde{s} \equiv \left[\frac{w}{Y}\theta\phi\right]^{\frac{1}{\theta+1}}$ such that $s_j = \tilde{s}\lambda_j^{\frac{1}{\theta+1}}$, where firm subscripts have been omitted due to the homogeneity in ϕ. This yields the following expression for the labor share:

$$\frac{w}{Y} = \frac{\tilde{s}}{L^p} \int \lambda_j^{-\frac{\theta}{\theta+1}} dj.$$

which is the ratio of (23) and (24). Inserting this into the first-order condition for intangibles 7 gives

$$\tilde{s}\lambda_j^{\frac{1}{\theta+1}} = \left(\left(1 + \phi - \phi \tilde{s}_j^{-\theta} \int_0^1 \lambda_h^{-\frac{\theta}{\theta+1}} dh - L^e - L^r \right)^{-1} \tilde{s} \int \lambda_h^{-\frac{\theta}{\theta+1}} dh \theta\phi \right)^{\frac{1}{\theta+1}} \lambda_j^{\frac{1}{\theta+1}}$$

Isolating $\tilde{s}$ yields the following closed-form expression:

$$\tilde{s} = \left[\left(\frac{1-L^e-L^r}{\phi} + 1 \right) \frac{1}{1+\theta\Lambda} \right]^{-\frac{1}{\theta}}.$$

where $\Lambda \equiv \int \lambda_h^{-\frac{\theta}{\theta+1}} dh$ is a constant that does not depend on ϕ. It then follows from $s_j = \tilde{s}\lambda_j^{\frac{1}{\theta+1}}$ that for a given L^e and L^r, we have $\frac{ds_{ij}^*}{d\phi} > 0$, as required.[1]

[1] The fraction of employment that is dedicated to research, L^r, is constant as the estimation targets R&D spending. While lower values of ϕ may raise R&D spending, the calibration of the R&D cost parameter offsets any effect on L^r by raising the R&D cost scalar. Employment to create new firms, L^e, falls in ϕ. The estimation targets the entry rate, so that lower incentives to enter when ϕ rises are offset by a higher cost scalar η^e. As employment of entrants is the product of wages (which do not depend on ϕ, the entry rate, and η^e, this strengthens the negative relationship between the average ratio of fixed costs over total costs and ϕ.

Appendix B. Data

B.1. Construction of the Datasets

B.1.1. Compustat Data

Data on the income statement and balance sheet for U.S. listed firms is obtained from S&P's Compustat. The panel to estimate markups comes Burstein et al. (2019). It is cleaned by dropping firms with sales, costs of good sold, operating costs and physical assets that are missing, negative, of less than 1000 dollars in value. Following Baqaee and Farhi (2020), I also drop firms with ratios of sales to the cost of goods sold or of sales to selling, general, and administrative expenses outside of the 2.5-97.5 percentile range. I restrict the sample to firms outside of finance, insurance and real estate and start the sample in 1979 to match the start of the Business Dynamics Statistics.

I merge the Compustat data with IT data from the CiTDB for a subset of years. For 2003 to 2009, the data contains ticker symbols for most listed firms that can be matched to Compustat. I use these codes to obtain an initial match for the 1997 to 2009 years. For 2010 to 2015 I match the datasets based on the name of the parent company of a site. I first standardize the names by removing spaces and capitalization, as well as an extensive list of common words in firm names, such as 'Inc', 'Company' or 'Ltd'. The code to perform the standardization was kindly provided by Hazell et al. (2021). I then perform a precise merge on firm names, which is successful for 64% of firms in the Compustat data. I also use the name matching to complement the ticker-based matching for the 1997-2009 years.

B.1.2. French Administrative Data

Balance Sheet and Income Statement The main firm-level datasets are FICUS from 1994 to 2007 and FARE from 2008 to 2016. I obtained access to the merged FICUS and FARE panel from Burstein et al. (2019). They developed the merge of FICUS and FARE, with code that was partly provided by Isabelle Mejean. I thank them for their help in obtaining data access and for permission to use the data for this project. They append FICUS with FARE using a firm identifier (the *siren code*) that consistently tracks firms over time. I keep all firms in legal category 5, which means all non-profit firms and private contractors are excluded from the sample. I also drop firms with operating subsidies in excess of 10% of revenues. Firms in financial industries and firms with missing or negative sales, assets, materials or employment are also excluded. From 2004, INSEE starts to group firms that are owned by the same company in single *siren* codes. This treatment has been gradually extended over time, which means that data on groups in later years of the data contain more consolidated firms. The unit of observation is a legal entity (*unité légale*), although subsidiaries of the largest companies are grouped as a single entity. From 2009 onwards, data is provided separately for the underlying firms (legal entities) and for the group. To have a consistent panel (and prevent an artificial increase in firm concentration), Burstein et al. (2019) group firms along the pre-2009 definitions and extend that treatment backwards and forwards.

Software and IT Data on software comes from the Annual Enterprise Survey (*Enquête Annuelle d'Entreprises*, EAE), which is a survey of around 12,000 firms between 1994 and 2007. There are separate surveys for major industries (agriculture, construction, manufacturing, services, transportation) which differ in variables and coverage. The survey is comprehensive for firms with at least 20 employees, and smaller firms are sampled for all sectors except manufacturing. The survey is merged to FARE-FICUS using the SIREN firm identifier. The level of observation is the legal unit, for firms that are aggregated prior to 2009 by INSEE as discussed in the main text. From 2008 onwards I use data from the E-Commerce Survey (*Enquête sur les Technologies de l'Information de la Communication* - TIC). This survey contains questions on the use of IT systems annually from 2008 to 2016. This dataset contains dummies on the adoption of specific IT systems such as Enterprise Resource Planning and Customer Resource Management.

Research and Development Data on R&D comes from the Community Innovation Survey (*Enquête Communautaire sur L'Innovation* - CIS). The CIS is carried out by national statistical offices throughout the European Union, and is coordinated by Eurostat. The survey is voluntary, but sample weights are adjusted for non-response to create nationally representative data. The French survey is carried out by INSEE, and contains consistent variables on research and development expenditures in 1996, 2000, 2004, 2006, 2008, 2010, 2012, 2014 and 2016.
Product Count The number of products by firm comes from the Annual Production Survey (*Enquête Annuelle de Production*, EAP). This survey is used for annual data on industrial production for the EU's PRODCOM statistics. The survey is available for manufacturing only, from 2009 to 2016. I count the number of unique products each year by firm, excluding products on which the firm acts as outsourcer, or was only involved in product design (M1 and M5).

B.2. Variable Definitions

B.2.1. Compustat Data

Revenue is total sales. The Compustat Fundamentals variable is SALE.
Cost of goods sold involves all direct costs involved with producing a good. This includes the cost of materials and other intermediate inputs, as well as the labor directly used to produce a good. It is observed on the income statement. The Compustat variable is COGS.
Selling, general and administrative expense are all direct and indirect selling, general and administrative expenses. They include overhead costs and costs such as advertisement or packaging and distribution. It is observed on the income statement. The Compustat variable is XSGA.
Operating expenses are the sum of cost of goods sold and selling, general, and administrative expenses. The Compustat variable is XOPR.
Capital stock The firm's production capital is defined as the contemporaneous balance sheet value of gross property, plants and equipment (tangible fixed assets). The Compustat variable is PPEGT.
Operating profits are measured as income before extraordinary items. I add expenditures on research and development because these are expensed in the American data yet not in the French data. This furthermore prevents a spuriously positive correlation between the fixed cost measure (which declines in profits) and research and development. The Compustat variable is IB.
Research and development expenditures include all the costs incurred for the development of new products and services. They also include R&D activities undertaken by others for which the firm paid. They are observed on the income statement. The Compustat variable is XRD.
Product count is obtained from the Compustat Historical Segments File. I count the number of products that firms produce as the number of unique primary 6-digit NAICS codes of business segments that firms report. In the adjusted count I assign a count of 1 for firms absent in the segments file.
IT usage is obtained from the CiTDB. I derive two variables from the dataset. The first is the firm's number of personal computers and laptops per employee. Although it does not directly speak to software, this variable is the most common variable derived from the database in prior work. It is the only variable that the dataset consistently collects over time, and is available for 1997 to 2015, except for 2011. This measure is frequently used as a measure IT intensity, and examples of prior papers that rely on this measure include Bloom et al. (2012), Bloom et al. (2016) and Hershbein and Kahn (2018). I calculate the firm-level value of this variable by taking the sum of PCs and employees across all sites that are linked to a firm, and then take the ratio of both. The second variable is the firm's software budget. I take budgets from the data on the firm's headquarter, because coverage of sites varies over time. This variable has been used by, e.g., He et al. (2021). The variable speaks directly to spending on intangibles in the data, but it has two shortcomings. First, the variable is only available for 2010, 2012, 2013, 2014 and 2015.

Second, the budget is estimated based on a combination of a survey and a site's characteristics, such as the type of information technologies that the site has installed. The data provider validates the data extensively, but details of the exercise are not provided. I therefore test robustness of results based on software spending with results from IT intensity.

B.2.2. French Administrative Data

Revenue is total sales, including exports. In FICUS years this is CATOTAL, in FARE years this is REDI_R310. In regressions, firm-size is controlled for by a third degree polynomial of log revenue.

Employment is the full-time equivalent of the number of directly employed workers by the firm averaged over each accounting quarter. In FICUS, the data is based on tax records for small firms, and on a combination of survey and tax data for large firms (variable name: EFFSALM). In FARE the variable is REDI_E200, which is based on the administrative DADS dataset.

Wage bill is defined as the sum of wage payments (SALTRAI in FICUS, REDI_R216 in FARE) and social security contributions (CHARSOC in FICUS, REDI_R217 in FARE).

Direct production inputs are calculated as the sum of merchandise purchases (goods intended for resale) and the purchase of raw materials, corrected for fluctuations in inventory. In FICUS, the respective variables are ACHAMAR, ACHAMPR, VARSTMA, and VARSTMP. The corresponding variables in FARE are REDI_R210, REDI_R212,REDI_R211, and REDI_213.

Other purchases are defined as purchases of services form other firms. This includes outsourcing costs, lease payments, rental charges for equipment and furniture, maintenance expenses, insurance premiums, and costs for external market research, advertising, transportation, and external consultants (AUTACHA in FICUS, REDI_R214 in FARE).

Operating profits is defined as revenue minus the wage bill, expenditure on direct production inputs, other purchases, import duties and similar taxes (IMPOTAX in FICUS, REDI_R215 in FARE) capital depreciation (DOTAMOR in FICUS), provisions (DOTPROV in FICUS), and other charges (AUTCHEX in FICUS). The sum of the wage bill, material input expenses, capital depreciation, provisions, and other charges is REDI_R201 in FARE.

Capital stock is measured as fixed tangible assets. This includes land, buildings, machinery, and other installations. The associated variable is IMMOCOR in FICUS, and IMMO_CORP in FARE. Capital is not calculated using the perpetual inventory method because investment is missing for 2008.

Industry codes are converted to NACE Rev. 2 codes using official nomenclatures. Firms that are observed before and after changes to industry classifications are assigned their NACE Rev. 2 code for all years, while other firms are assigned a code from official nomenclatures. Industries without a 1-to-1 match in nomenclatures are assigned the NACE Rev. 2 that is observed most frequently for firms with their industry codes. Firms that switch codes are assigned their modal code for all years.

Research and Development are measured as all innovative expenditures by firms as reported in the CIS. Subcategories of expenditures fluctuate with each version of the survey, but total expenditures seems consistently defined.

Software Investments The variable for software investments closely follows the definition in Lashkari et al. (2022). The underlying variables are observed from 1994 to 2007 in the EAE.[2] The main variable for software is I460. This variable contains all software investments and is available for all sectors. Because missing observations are coded as 0, I drop these firm-years when analysing software. An additional sub-division into externally purchased and internally developed software is available for a subset of firms (I461, I462, I463, I464, I465). Where available, I use this to clean cases where I460 is smaller than I461-I465, and verify that summary statistics match Lashkari et al. (2022).

[2] As coverage is consistent from 1995 onwards, all analysis with software investments starts in that year.

B.3. Data Citations

Data sets obtained from the U.S. Bureau of Economic Analysis:
BEA (2019a), BEA (2019b), BEA (2019c), BEA (2019d), BEA (2019e), BEA (2020).

Data sets obtained from the U.S. Census Bureau:
Bureau (2016), Bureau (2019).

Data sets obtained from the French statistical office (Insee):
Insee (2009a), Insee (2010b), Insee (2011a), Insee (2012b), Insee (2013a), Insee (2014b), Insee (2015a), Insee (2016b), Insee (2017), Insee (1996), Insee (2000), Insee (2004), Insee (2006), Insee (2008a), Insee (2010a), Insee (2012a), Insee (2014a), Insee (2016a), Insee (2008b), Insee (2009b), Insee (2010c), Insee (2011b), Insee (2012c), Insee (2013b), Insee (2014c), Insee (2015b), Insee (2016c), Insee (2009c).

References for data sets obtained from previous papers:
Burstein et al. (2019), Caballero et al. (2017), The Conference Board (2018), Kogan et al. (2016), Saibene (2017).

References for productivity data:
Feenstra et al. (2015), Fernald (2014).

Reference for U.S. firm-level data:
Harte-Hanks (2017), Standard and Poor's (2019), CRSP (2021).

Appendix C. Markup and Fixed Costs Estimation

This appendix summarizes the iterative GMM approach by De Loecker and Warzynski (2012) that is used to estimate the output elasticity of a variable input, in order to calculate fixed costs. I first outline the production function estimation procedure for both France and the U.S., and subsequent discuss the robustness of the resulting series for fixed costs. I also discuss the implication of recent criticisms on the method that I use to calculate markups from the production function.

C.1. Estimation Procedure

I follow the literature that estimates production functions to measure markups along De Loecker and Warzynski (2012). The model in Section II assumes that output is a function of tangible inputs (through $z(z_{ijt,1}, z_{ijt,2}, .., z_{ijt,k})$) and intangible inputs, which collapse to s_{ijt}. Tangible inputs are assumed to be flexible, while intangible inputs are assumed to be fixed within periods. In practice datasets contain limited information on individual inputs at the firm-level.

Besides the firm-level aggregation problem discussed in A, this yields two complications. First, the production function can only be estimated with the broad categories as inputs. To maximize the flexibility of the estimation, I approximate this general production function by estimating a translog function that contains the (squared) log of all observed inputs. Second, the broad categories usually are as broad as labor, capital and intermediate inputs. Most of these categories contain a combination of tangible and intangible inputs in the context of the model. In order for estimated markups to obey equation (A.9), the input h may only consist of a flexibly set tangible input. Both in Compustat and in the French data I use h that is most commonly assumed to be a flexible input in the literature.

De Loecker and Warzynski (2012)'s estimation then identifies the translog production function. The procedure is designed to deal with two empirical complications that are not in the model. First, output may be observed with error, causing attenuation bias in the estimates. Second, if firms have different unobserved idiosyncratic total factor productivities, inputs and outputs may correlate through productivity, again causing bias. Both problems are addressed in a separate stage in the estimation. In the first step, output is non-parametrically regressed on all observed inputs in the production function. The fitted value of this regression is output cleaned of measurement error, which serves as the dependent variable in the remainder of the analysis. In the second stage, the production function is identified under the assumption that the idiosyncratic productivity follows an AR(1) process. The production function is identified under the assumption that flexible inputs and lagged fixed inputs (set before the shock occurs), are respectively orthogonal to the lagged and current productivity shock.[3]

Below I detail the exact estimation of the output elasticity of h for both datasets.

C.1.1. Implementation: France

The production function estimation for France come from Burstein et al. (2019) who analyse the cyclical properties of French markups, and I thank the authors for permission to use their estimates for this project. In line with their work, I use markups based on the estimated elasticity of output with respect to materials m, which are least likely to contain intangbile inputs in the context of the model. The main

[3] An algorithm then iterates over the parameters of the production function. For each iteration, productivity is calculated as the difference between cleaned output and the product of inputs and the assumed production function parameters. These estimates are then auto-regressed to obtain AR(1) productivity shocks, which are then correlated with the inputs. The iteration continues until the correlation between the productivity shock and the current fixed variables and lagged flexible variables is zero.

estimates of the production function use data on capital k_{it}, labor l_{it} and materials m_{it} and estimate the production function for each 2-digit industry with at least 12 firms in the data, along:

$$y_{it} = \beta^l l_{it} + \beta^{ll} l_{it}^2 + \beta^k k_{it} + \beta^{kk} k_{it}^2 + \beta^m m_{it} + \beta^{mm} m_{it}^2 + \omega_{it} + \epsilon_t \tag{A.12}$$

where cross-terms are omitted to prevent measurement error in one of the inputs to directly affect the estimated elasticity of other inputs.[4] Capital is measured through fixed tangible assets, labor is the number of employees and materials equal firm purchases. In contrast to (i.e.) U.S. Census data, data on materials is available annually for firms in all industries. I instrument k_{it} with its current value, I assume that firms cannot increase capital in response to a contemporaneous productivity shock. By instrumenting l_{it} and m_{it} by their lagged value I assume that they may depend on contemporaneous productivity shocks, but require autocorrelation in factor prices.[5]

The three-factor production function is commonly used in the literature and is therefore the basis of estimates in the main text. To assess the robustness of these estimates, I also estimate a more extensive production function with four production factors. The FARE-FICUS dataset allows materials to be divided into direct production inputs v_{it} (intermediate goods for resale and expenses on primary commodities) and other purchases o_{it}, which include the purchase of external services like advertising. An output elasticity can only be used to estimate markups when the factor is freely set each period, which seems most likely to hold for v_{it}. Direct production inputs v_{it} are most likely to only be tangible, as they only include expenses on intermediate goods for resale or expenses on primary commodities. That is why I use the elasticity of output with respect to v_{it} to measure markups from the four-factor production function. The production function reads:

$$y_{it} = \beta^l l_{it} + \beta^{ll} l_{it}^2 + \beta^k k_{it} + \beta^{kk} k_{it}^2 + \beta^v v_{it} + \beta^{vv} v_{it}^2 + \beta^o o_{it} + \beta^{oo} o_{it}^2 + \omega_{it} + \epsilon_t \tag{A.13}$$

Because of the large number of firms in the data, I estimate this more extensive production function separately for each 4-digit industry.

Gross output in the production function is measured through sales, which has been criticized in a number of recent papers. While a review of the debate goes beyond the scope of this paper, a particularly relevant critique is presented in Bond et al. (2021). They show that when markups are measured by multiplying the inverse of a factor's share in revenue with the revenue function elasticity rather than the production function elasticity, the resulting markup is biased to an average value of 1.

In practice, markups estimated with the De Loecker and Warzynski (2012) methodology do not measure the revenue elasticity as revenue is purged from factors unrelated to inputs in the first stage and output is deflated with sector deflators (see, e.g., De Loecker 2021). The French data furthermore allows for a comparison of markups obtained from data on revenue versus data on quantities, because the French product-level data on manufacturing (the *EAP*) contains price data from 2009. Using this data, Burstein et al. (2019) show that markups based on quantity data have a 0.83 correlation coefficient with markups based on revenue data. Note, furthermore, that the model only relies on fixed costs in order to calibrate the initial level of intangible efficiency. Bias in markup estimates therefore only affect the initial calibration of ϕ_i. Appendix C.4 furthermore shows that the average fixed costs derived from the De Loecker and Warzynski 2012 markups align with averages from alternative measures.

C.1.2. Implementation: United States

Markups and production function elasticities for U.S. publicly listed firms come from De Loecker et al. (2020). I estimate the elasticities using code created for Burstein et al. (2019) that replicates De Loecker

[4] This follows De Loecker et al. (2020) in their treatment of capital.

[5] For France it is reasonable to assume that labor is, in fact, not set freely and could therefore be instrumented by contemporaneously. This turns out to have no significant effect on the estimated production function.

Table A1: Summary Statistics on Estimated Markups

	Mean	Std. Dev.	Median	10th Pct.	90th Pct.	Observations
France						
Basic production function	1.38	0.46	1.26	0.96	1.91	9,913,058
Extended production function	1.47	5.17	1.01	0.53	2.59	8,477,467
United States						
COGS production function	1.50	.58	1.33	1.01	2.25	127,682
COGS and SG&A production function	1.30	.51	1.15	0.87	1.94	127,682

et al. (2020), and merge the elasticities with my data to calculate markups and fixed costs. A constraint of the analysis of markups for these firms is that data on materials and the wage bill is not available from the income statement. Instead, De Loecker et al. (2020) use a broad category of operating expenses (cost of goods sold) that captures all expenditures that are directly related to the cost of production. Results in the main text use these markup estimates.

One critique on using a production function estimation with capital and cost of goods sold is that it does not account for selling, general, and administrative expenses (SG&A), which have become more important over time. Adding SG&A to cost of goods sold to form a single input in a production function is evenly problematic because 1) a large part of SG&A are fixed overhead costs as well as expenditures on intangible inputs,[6] and 2) it assumes all operating expenses are perfect substitutes. Instead, I test the robustness the main results by adding SG&A as an input in a production function along (A.12).

C.2. Robustness of Fixed Cost Trends

C.2.1. France

The results in the main text are robust to using the more extensive four-factor production function. After estimating the industry-level production function coefficients, I calculate the firm-level markup as the product of the input elasticity and the inverse of the input's revenue share. I then calculate the fixed cost share in line with (27). Markups at the firm-level are summarized in Table A1. The table shows that the extensive production function estimates a very similar average markup to the markup from the standard three-factor production function. The variance of markups, however, is significantly greater when using the four-factor production function. This is likely due to the additional parameters that need to be estimated at the 4-digit level, or because firms have some flexibility in what costs fall under direct production inputs v versus other purchases o.

The trends of aggregate fixed costs are plotted in Figure A1. The solid-blue line is replicated from the main text and is for the three-factor standard production function, while the squared-green line uses the four-factor extensive production function. Both figures show that the sales-weighted average fixed cost share has increased strongly over the 1994 to 2016 sample, with the largest increase occurring between 1994 and 2010, after which the increase moderates.

C.2.2. United States

Markups from the two-factor and three-factor production functions are highly correlated. The bottom panel of TableA1 presents summary statistics for both and shows that they mainly differ in terms of their their level. When adding SG&A, over 30% of all firms have markups below 1 and the median markup is 1.15. Though the 2-factor admits markups around 15 percentage points above that at most percentiles, both series co-move strongly. The firm-level correlation is 0.92. While the correlation of the markup

[6] Heterogeneity in fixed costs across firms will then cause an underestimation of the input elasticities and markups.

series is close, the difference in levels between the series have a large effect on the predicted level of fixed costs. The right plot in Figure A1 shows that the 3-factor production function predicts *negative* average fixed costs as a percentage of total costs between 1980 and 2004. This is likely to be driven by an underestimation of the markup; of the firms with a 3-factor markup below unity, 63% report positive profits. The predicted increase in fixed costs over the sample is 13 percentage points, which is similar to the predicted increase in the main text. Figure A1 does raise concerns about the correct calibration target for the initial level of fixed costs. Appendix C.4 shows that fixed costs from alternative measures imply a similar average level to the level in the main text.

C.3. Fixed Costs and Capital Costs

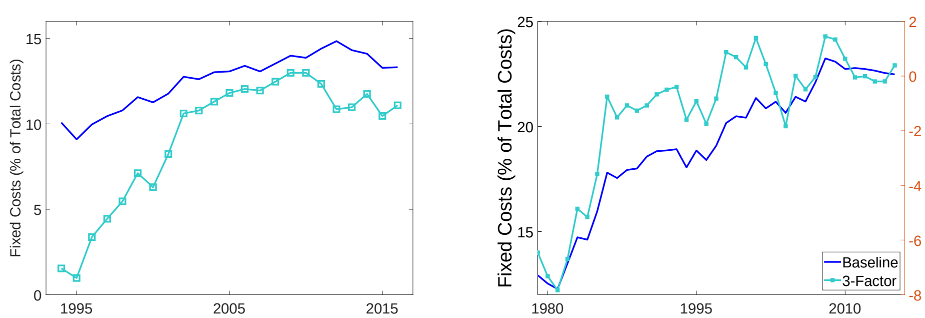
Fixed costs in the main text are calculated using data on estimated markups and measured operating profits from the income statement. Operating profits account for capital costs in the form of depreciation, but they do not account for other rental costs of capital that the firm owns. An exact estimate of these costs is difficult because these rental costs must account for the fair risk premium of purchasing capital, which is not directly observed.

In this appendix I show that the trends and cross-sectional distribution of fixed costs is robust to various controls for rental costs. To resolve the lack of directly observable measures of capital rental costs, I rely on various estimates of risk-free interest rates and capital risk premia in Caballero et al. (2017), each with slight differences in underlying assumptions.[7] I use four alternative approaches to calculate r_t^K. Caballero et al. (2017) calculate rental rates of capital either under the assumption that there has not been capital-biased technological change over the sample, or that markups have remained flat. Under the first assumption, they calculate capital required returns for three cases, corresponding to different elasticities of substitution between capital and other factors (1.25, 1.00 and 0.80 in cases 1, 2, and 3, respectively). I focus on these series, given the role of markups in the paper.[8] While neither is perfectly in line with the model, they can be used to assess whether accounting for capital costs with common methods affects the empirical results.

I calculate profits along:

$$\pi_{it} = \tilde{\pi}_{it} - r_t^K K_{it}$$

Figure A1. Robustness of Trends in Aggregate Fixed Cost Share

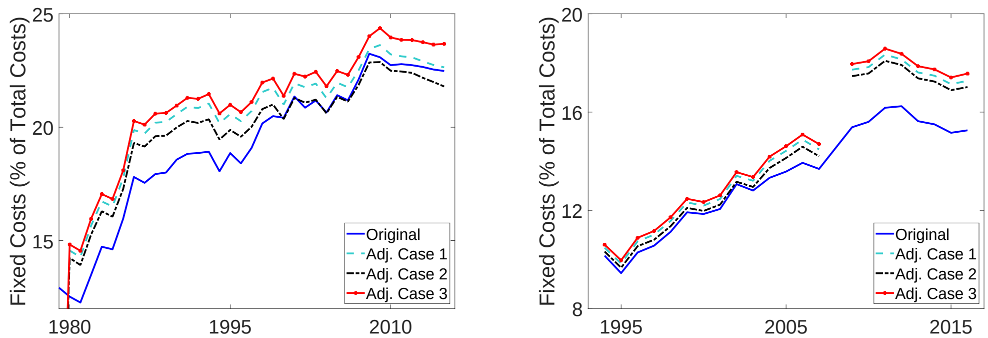


(a) France (b) United States

[7] Caballero et al. (2017) provide these estimates for years at the beginning, middle and end of my sample. I interpolate the variables linearly to obtain annual estimates for the risk-free interest rates and capital risk premia.

[8] Results from the series under assumption (b) are very similar with firm-level correlations of fixed costs exceeding 0.99.

Figure A2. Comparison of Fixed Cost Series Accounting for Capital Costs



(a) United States (b) France

Notes: Sales-weighted average of fixed costs as a percentage of total costs, U.S. listed firms (left) and universe of French firms (right). Fixed costs are inferred from the difference between profits as a percentage of sales and the marginal cost markup. A discontinuity in the French investment and depreciation data prevents the calculation of capital-adjusted fixed costs for 2008.

where $\tilde{\pi}_{it}$ is the original series of operating profits excluding depreciation and K_{it} is the firm's stock of property plants and equipment. The same series is used for France and the US. I then calculate fixed costs with the alternative profit rates and compare results.

Results in the paper are robust to the alternative definition of operating profits. Figure A2 compares trends in the weighted average of various fixed cost series. The original series is blue-solid, other series denote the various cases described above. The series generally show that, accounting for the required rate of return, fixed costs are higher. This is expected because accounting for capital costs lowers profits.[9] The series become slightly closer over the sample because of the decline in the risk-free rate, although the decline is moderated by a rise in capital risk premia. The French data has a discontinuity around 2008, when investment and depreciation data change definitions. At the firm-level, Table A2 presents correlations of fixed costs across the specifications. The original fixed costs estimates have a firm-level correlation with the alternative estimates of at least 0.98 for all of the specifications.

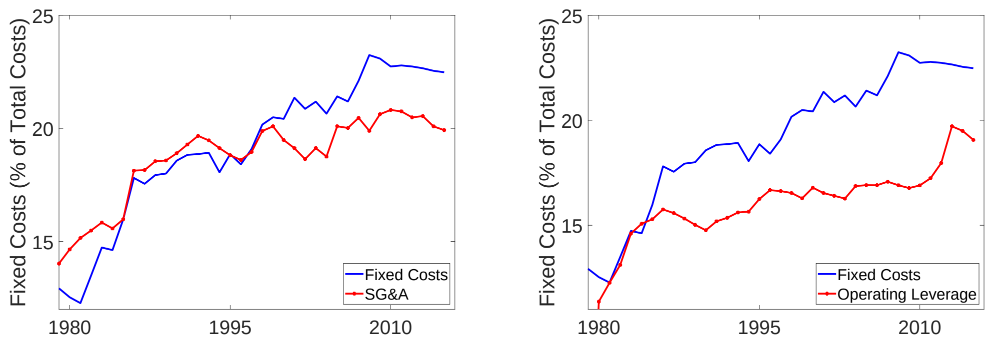
Table A2: Correlations Between Alternative Fixed Cost Series

	Original	Adj. Case 1	Adj. Case 2	Adj. Case 3
United States				
Original	1.00	0.98	0.98	0.98
Adjusted Case 1	0.98	1.00	>0.99	>0.99
Adjusted Case 2	0.98	>0.99	1.00	>0.99
Adjusted Case 3	0.98	>0.99	>0.99	1.00
France				
Original	1.00	0.99	0.99	0.99
Adjusted Case 1	0.99	1.00	>0.99	>0.99
Adjusted Case 2	0.99	>0.99	1.00	>0.99
Adjusted Case 3	0.99	>0.99	>0.99	1.00

Notes: Correlations between the main fixed cost series (Original; accounting for capital costs through depreciation) and the rental-cost adjusted series using estimates from Caballero et al. (2017). See text for differences between adjusted series.

[9] The original (blue-solid) series does account for depreciation costs as reported by the firm, whereas the adjusted cases assume a fixed depreciation rate of 7.3% in line with Caballero et al. (2017). This explains why there are some years in which the adjusted case 2 (black-dashed) is lower than the original series.

Figure A3. Alternative Measures for the Ratio of Fixed Costs to Total Costs



(a) Fixed Costs versus SG&A

(b) Fixed Costs versus Operating Leverage

Notes: Sales-weighted average of fixed costs as a percentage of total costs for U.S. listed firms, versus SG&A over total costs (left) and average operating leverage based on the sensitivity of firm-costs to firm-sales from Saibene (2017) (right). Fixed costs in the solid-blue lines are the main series from the paper.

C.4. Alternative Approaches to Fixed Cost Calculation

There are two alternative common approaches to the calculation of fixed costs in the literature. The first is to assign particular costs on the profits and loss (P&L) statement to either fixed costs or variable costs. This approach is used for U.S. firms in De Loecker et al. (2020), who assume that SG&A represents fixed costs. The advantage of this approach is that the fixed cost estimates are firm-specific and time-varying, and that they can be obtained without uncertainty as long as one believes the classification. Figure A3a in this letter plots the trend in fixed costs as measured as the ratio of SG&A over total costs. It shows that the average SG&A-ratio was slightly higher than the measure of fixed costs in this paper (14.6%), and that the increase over time was smaller (5.7 percentage points). This could be explained by the presence of some variable costs in SG&A, which are predicted to fall in my paper's model. Broadly, however, fixed costs according to the SG&A-ratio are in line with the main measure of fixed costs in the paper. The firm-level correlation between the measures is 0.66. A second practice is to measure fixed costs from the responsiveness of costs to changes in sales. This approach originates from the empirical corporate finance literature, where the ratio of fixed costs over total costs is also known as *operating leverage*. García-Feijóo and Jorgensen (2010) summarize this literature, and Saibene (2017) provides estimates for Compustat firms. He estimates operating leverage from a firm-specific regression of costs on sales and plots the annual average of this across Compustat firms using all firms operating in that year. A limitation of this approach is that the elasticity cannot be estimated by firm-year, which complicates a panel analysis like in Section 2. The series are reproduced here as the red-marked line in Figure A3b.

D4. Within versus Between Sector Changes in Rise of Fixed Costs

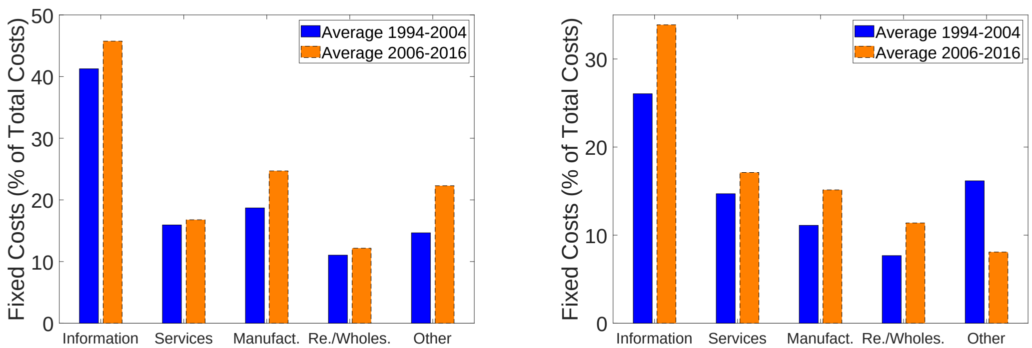
Figure A4 illustrates the sectoral composition of fixed costs. It shows that fixed costs as a fraction of total costs are especially high in the information sector (NAICS industry 51 for the U.S. and NACE industry JB and JC for France). The distribution of fixed costs across sectors is similar for the U.S. and France and the majority of sectors have seen an increase in their average ratio of fixed- to variable costs. The latter suggests that fixed costs have increased at the aggregate level because of an increase in the importance of fixed costs within sectors and not because high-fixed costs sectors have become larger over time. To formally show that the aggregate rise of fixed costs is driven by within-sector reallocation, I perform the following within-between decomposition:

$$\Delta \frac{\tilde{F}_t}{TC_t} = \sum_{j\in J} s_{jt-1} \Delta \frac{\tilde{F}_{jt}}{TC_{jt}} + \sum_{j\in J} \Delta s_{jt} \frac{\tilde{F}_{jt-1}}{TC_{jt-1}} + \sum_{j\in J} \Delta s_{jt-1} \Delta \frac{\tilde{F}_{jt}}{TC_{jt}}$$

where $\tilde{F}_t / TC_t$ is the aggregate fixed cost share, $\tilde{F}_{jt} / TC_{jt}$ the sector-level counterpart, and s_j the fraction of sales by sector j. The first term captures changes due to increases in fixed costs within sectors. The second term captures the 'between' share: changes because of changes in the relative size of sectors. The last term is the interaction of both. I perform the decomposition annually and regress each term on the change in the aggregate fixed cost share.

The coefficients are presented in Table A3. Figure A5 illustrates the contribution of within and between shares over time, by plotting the development of fixed costs holding other contributors constant. The results show that within-sector reallocation was largely responsible for the rise of fixed costs.

Figure A4. Weighted-Average Ratio of Fixed Costs to Total Costs across Sectors

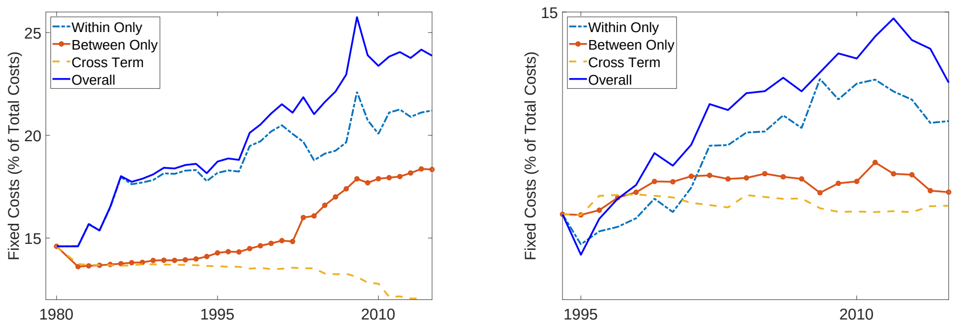


(a) United States

(b) France

Notes: Sales-weighted average of fixed costs fraction by sector for U.S. listed firms (left) and the universe of French firms (right). Sectors are ordered by the average fixed-cost share in the last ten years of the French sample. Industry definitions for the United States (NAICS): 51 for information, 64 and above for services, 31, 32 for manufacturing, and 42, 44, 45 for wholesale and retail; for France (NACE/ISIC): JB, JC for information, I, M, N for services, B, C, D, E for manufacturing, and G for wholesale and retail.

Figure A5. Within-Between Decomposition of the Rise of Fixed Costs



(a) United States (b) France

Notes: Within-between decomposition of the rise of fixed costs for U.S. listed firms (left) and the universe of French firms (right).

Table A3: Decomposition of Changes in Aggregate Fixed Cost Share

	Within Sectors	Between Sectors	Cross Term	Total
United States	0.88	0.11	0.01	1
	(0.06)	(0.05)	(0.03)	
France	0.80	0.17	0.03	1
	(0.004)	(0.003)	(0.003)	

Notes: Standard errors in brackets.

Appendix D. Macroeconomic Trends in France

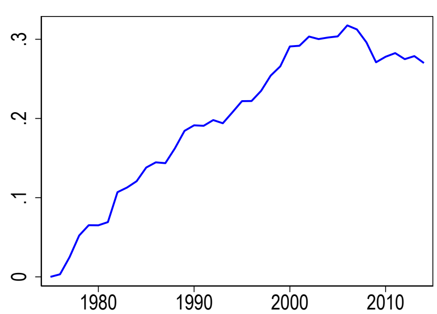
The introduction summarizes three recent trends: the slowdown of productivity growth, the fall in business dynamism and the rise of corporate profits. This appendix gives an overview of the macroeconomic trends for France.[10]

The slowdown of productivity growth is depicted in Figure A6. It plots an index of the log of TFP at constant prices, standardized to 0 in 1975. The figure shows that TFP was growing at a steady rate for most years between 1975 and 2000. There was a significant slowdown in the early 2000s, and productivity growth over the 2005-2020 era has been slightly negative.

The decline in business dynamism is summarized with three statistics, following the literature. The first is the reallocation rate in Figure A7a, which is the sum of job destruction and creation rates. I calculate the reallocation rate across French firms using the FARE-FICUS dataset for 1994-2016. Because this sample coincides with the Great Recession, which brought a strong transitory increase in reallocation due to job destruction, I plot the HP trend. The second fact is the decline of entry of new firms. Figure A7b captures this trend by plotting the fraction of employees that work for a firm that enters the FARE-FICUS dataset in a given year. Note that this may include firms that have undergone significant organizational changes that have caused their firm identifier to change. The figure shows that employment by entrants has declined by almost half within the 1994-2016 sample. The third fact is the decline of skewness of the firm growth distribution. As discussed by Decker et al. (2017), small (young) high-growth firms have historically been an important contributor to productivity growth. They infer the decline in skewness of the growth distribution from the decline between the 90th and 10th, and between the 90th and 50th percentile of the growth distribution. Figure A8 shows that both have declined by around 40% between 1994-2016. The difference between the 50th and 10th percentile has remained flat, in line with U.S. evidence.

The rise of corporate profits is measured through the marginal cost markup. This is a measure of marginal rather than average profits, a distinction that is key in Section II. Figure A9a plots the average sales-weighted markups for French firms between 1994 and 2016. The markups has increased modestly, in line with previous evidence (e.g. IMF 2019). Though not directly measuring market power, concentration also displays a modestly positive trend over the sample. This is shown in Figure A9b, which depicts

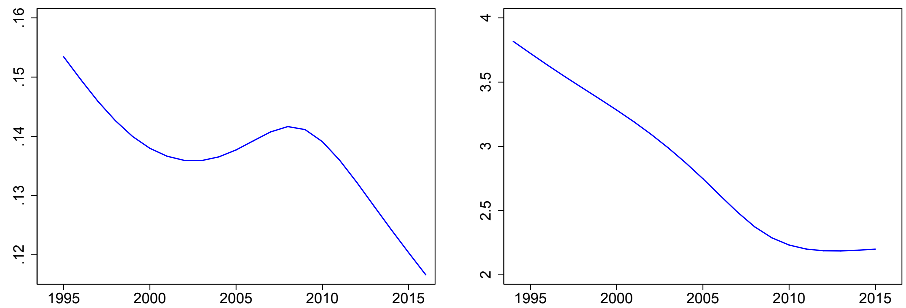
Figure A6. Total Factor Productivity in France



Notes: Log TFP at constant prices, 1975=0. Data: Penn World Tables.

[10] Whether market power is increasing across advanced economies remains a subject of debate. The slowdown of productivity growth and the decline of start-ups have been widely documented (e.g. Adler et al. 2017 and Calvino et al. 2016), while the rise of market power and firm concentration seems to be larger in the U.S. Döttling et al. (2017) and Cavalleri et al. (2019) find no increase in industry concentration in Europe between 2000 and 2013, using Orbis data. Bajgar et al. (2019) document a rise in concentration in most of Europe when accounting for ownership structures and the coverage of small firms in Orbis. Aquilante et al. (2019) also find an increase in U.K. industry concentration between 1998 and 2016.

Figure A7. Business Dynamism in France



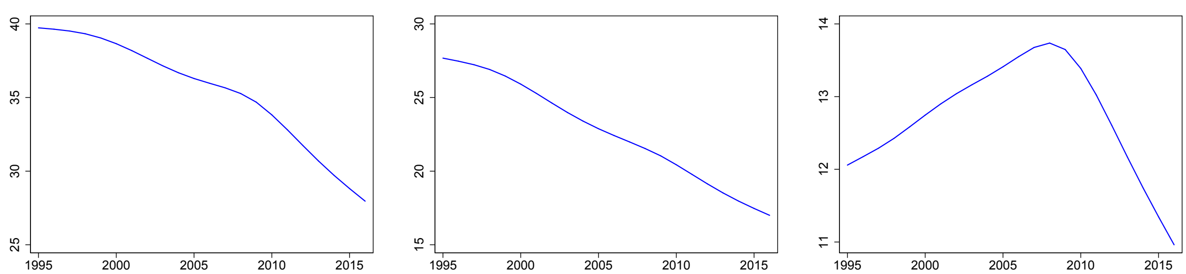
(a) Reallocation Rate

(b) Entrants

Notes: Both figures plot HP trends. Left figure: sum of job creation and job destruction rates across companies. Right figure: Percentage of employment by new firms (≤ 1yr) in private sector employment. HP trend.

the average Herfindahl Index across 5-digit industries. The rise of concentration has been linked to the decline in the labor share by Autor et al. (2020) through the reallocation of activity to firms with low labor shares. This result has been replicated for France for 1994-2007 by Lashkari et al. (2022). Note that the increase in concentration depends on measurement. The graph below presents an average of the Herfindahl across sectors. Weighing sectors by value added gives an increase in the Herfindahl index from 2008 from 0.087 in 1994 up to 0.122 in 2008, but a modest decline to 0.117 afterwards.

Figure A8. Skewness of the Employment-Growth Distribution



(a) 90-10 Difference

(b) 90-50 Difference

(c) 50-10 Difference

Notes: Difference (perc. point) in growth between percentiles of the employment-growth distribution. HP trend.

Appendix E. Computational Algorithm

The balanced growth path equilibrium along definition 1 is found by solving the system of detrended equilibrium equations as a fixed point. The algorithm works as follows:

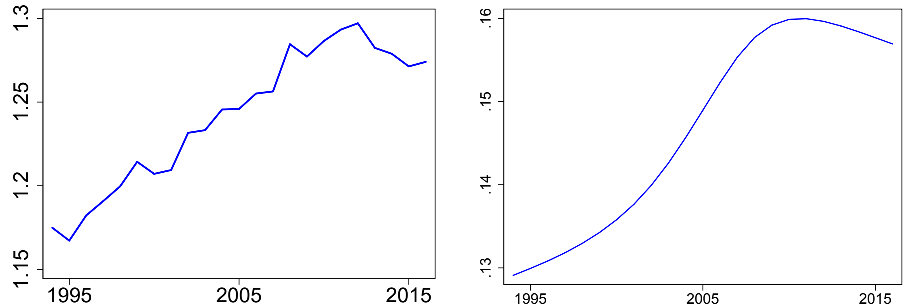
1. Solve the fixed point:

   (a) Guess a level of Y/Q, w/Q, $\tau(\phi)$, and $K(\phi)$.

   (b) Collect choke prices by solving:

$$\left(p^c(\phi_i) - w[1 - s^*(\phi_i)]\right) Y - w\phi_i \left([1 - s(\phi_i)]^{-\theta} - 1\right) = 0 \text{ where } \phi_i \in \Phi$$

Figure A9. Markups and Firm Concentration in France



(a) Markups (b) Concentration

Notes: Left figure: sales-weighted marginal cost markups using the Hall (1988) equation with production function elasticities estimated with iterative GMM as in De Loecker and Warzynski (2012). Details in Appendix C . Right figure: average Herfindahl index across 5-digit NACE industries. HP trend.

(c) Given the vector of choke prices and the guess for $K(\phi)$, calculate the following objects:
   - a $|\Phi| \times |\Phi|$ matrix $\mathbf{P}$ with probabilities that a firm of type $\phi_i \in \Phi$ successfully innovates when facing $\phi_{-i} \in \Phi$ as in (15) and a vector with the weighted average over this probability $\sum_{\phi_{-i}\in\Phi} K(\phi_{-i})\mathbf{P}(\phi_i, \phi_{-i})$ with the probabilities that a type's innovation is successful in general.
   - the set of distributions of $\lambda_{ij} \sim H(\lambda)$ for each combination of $\phi_i \in \Phi$ and $\phi_{-i} \in \Phi$ truncated at $p^c(\phi_i)/p^c(\phi_{-i})$.
   - the expectation of markups (8) given the truncated distributions and the guess for $K(\phi)$.
   - the optimal innovation efforts by incumbents and entrants given markups, $\mathbf{P}$, Y, w, $\tau(\phi)$, and $K(\phi)$.

(d) Calculate Y from (24) and w from (23). Use the innovation effort by incumbents and entrants to calculate $\tau(\phi)$ using (14) and (20), (21) and (22) to find $K(\phi)$.

(e) Repeat from step (b) until the model has converged.

2. Perform the firm simulation, building computationally on Akcigit and Kerr (2018) and Acemoglu et al. (2018):

   (a) Collect the equilibrium Y, w, $\tau(\phi)$, $K(\phi)$, $x_{n_i}(\phi_i)$, e for all n and all $\phi_i \in \Phi$.

   (b) Discretize time by introducing a sufficiently large number of instances per year so that $x_{n_i}(\phi_i) < 1$ and $e < 1$.

   (c) Initialize the firm-size distribution using (20) and (21).

   (d) Simulate firms until the markup distribution has converged, then collect moments.

3. In the structural estimation, the resulting moments are then compared to the targets using the penalty function described in the main text. The parameters are updated along either a genetic algorithm or particle swarm algorithm – to optimize fit – until the penalty function is minimized.

The transitional dynamics are numerically solved using the following algorithm:

1. Create a fine grid with a T-year horizon, allowing each year to consist of $\tilde{T}$ instances.

2. Guess an initial value function of innovation activities $V()$ equal to the new steady-state level for each type in $\phi_i \in \Phi$ at each point of the grid. Similarly guess the paths of wages w/Q and output Y/Q at their new steady-state level.

3. Initialize the firm-size and type distribution $K(\phi)$ and $M_n(\phi)$ to their original steady state.

4. Iterate over the path of the value function as follows:

   (a) Solve the static optimization problem and the dynamic innovation decisions for incumbents and entrants for each point on the grid using the initial guess for $V()$.

   (b) Given the innovation and static decisions, simulate the development for a large (N) number of products and track the innovation step-sizes λ in $N \times (T\tilde{T})$ matrix Λ and similarly a matrix of ownership types using a forward loop over the grid.[11]

   (c) Update the value function using the new sequences for Y, w, the firm-type and -size distribution, and distributions for markups and λs implied by Λ. This involves calculating:

      i. the expectation of profits $\pi_{kt}(\phi_i, \lambda_{ij})$ at each instance t on the grid $t = 1, .., T$ separately for each cohort of patents k.

      ii. the value of obtaining the patent to produce an additional product for incumbents of type ϕ at time k as follows:

$$V^k(\phi_i) = \mathbb{E}^k_{\phi_i}\left[\sum_{t=k+1}^{\epsilon T}\prod_{h=k+1}^{t}\left(\frac{1-\tau_h(\phi_i)}{1+\rho}\right)\pi_{kt}(\phi_i, \lambda_{ij})\right]$$

      which is a discretization of the original value function, where ϵ is set so that the present value of profits in instances exceeding ϵT approaches zero.[12]

   (d) Use the resulting value for each type on each point of the grid as the guess for $V()$ in step (a) in the next iteration. Continue until the path of the value function converges.

   (e) Smooth the transition paths for productivity growth to remove noise induced by simulating software growth for a finite number of firm-products.

[11] This simulation is needed because the changing composition of firm types means the distribution of realized λs has no analytical representation. I then use the resulting distribution of markups to calculate the efficiency wedge as in (24), as well as a path for Y and w. These serve as the basis for the algorithm's next iteration.

[12] I set $T = 4000$ (corresponding to 80 years), $\epsilon = 11$ (a profit horizon of 600 years), and set $N = 10000$.

Appendix F. Additional Figures and Tables

Table A4: Relationship between Technology Adoption and Fixed-Cost Share (France)

	Software Adoption					
Fixed-Cost Share (log)	ERP	CRM	SCM	CAD	RFID	Spec.
Adoption Dummy	0.060	0.030	0.022	0.080	0.141	0.230
	(0.013)	(0.014)	(0.019)	(0.030)	(0.032)	(0.020)
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Industry fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Revenue (product-count) control	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.252	0.252	0.257	0.248	0.286	0.265
Observations	54,709	59,190	39,033	17,139	14,656	41,619

Notes: Explanatory variable is a dummy for the adoption of the technology specified in the column header. Industry-fixed effects at the 5-digit NACE level in lieu of firm fixed effects, as firms are randomly sampled. Observations are weighted by sample weights. Firm-clustered standard errors in parentheses. Observation counts differ, as not every measure was included in each survey year.

Table A5: Relationship between PC Intensity and Fixed-Cost Share (United States)

Fixed-Cost Share (log)	I	II	III	IV	V	VI
PC Intensity (log)	0.163	0.135	0.107	0.097	0.017	0.003
	(0.017)	(0.016)	(0.015)	(0.017)	(0.007)	(0.007)
R^2	0.023	0.047	0.137	0.139	0.005	0.027
Observations	16,806	16,806	16,806	16,806	16,806	16,806
Year fixed effects	No	No	No	Yes	No	Yes
Firm fixed effects	No	No	No	No	Yes	Yes
Industry fixed effects	No	No	Yes	Yes	No	No
Revenue (product-count) control	No	Yes	Yes	Yes	Yes	Yes

Notes: Dependent variable is fixed costs over total costs (log). Explanatory variable is IT intensity, measured through the number of personal computers per employee. Revenue is deflated with the sector-specific gross output deflator from EU-KLEMS. Variables are winsorized at 1% tails. Firm-clustered standard errors are given in parentheses. Revenue is in logs, as it is proportional to product count in the model. Sector fixed effects are at the 2-digit level.

Table A6: Relationship between Software Budget and Research & Development (United States)

R&D intensity (log)	I	II	III	IV	V	VI
Software budget over total costs (log)	0.266	0.243	0.215	0.219	0.019	0.006
	(0.016)	(0.019)	(0.019)	(0.019)	(0.008)	(0.008)
R^2	0.163	0.166	0.285	0.286	0.035	0.064
Observations	3,787	3,787	3,787	3,787	3,787	3,787
Year fixed effects	No	No	No	Yes	No	Yes
Firm fixed effects	No	No	No	No	Yes	Yes
Industry fixed effects	No	No	Yes	Yes	No	No
Revenue (product-count) control	No	Yes	Yes	Yes	Yes	Yes

Notes: Dependent variable is R&D intensity (log). Explanatory variable is software budget as a percentage of total costs (log). Revenue is deflated with the sector-specific gross output deflator, software with the input deflator from EU-KLEMS. Variables are winsorized at 1% tails. Firm-clustered standard errors are given in parentheses. Revenue is in logs, as it is proportional to product count in the model. Sector fixed effects are at the 2-digit level.

Table A7: Comparison of Theory and Data for Untargeted Moments

		United States			France		
	Quartile	**Model**	**Data**	**St. Dev.**	**Model**	**Data**	**St. Dev.**
Size and Age	1st (Age)	1.20	2.04	(1.04)	1.22	1.98	(1.01)
	2nd (Age)	1.20	2.33	(1.05)	1.22	2.39	(1.06)
	3rd (Age)	1.20	2.52	(1.09)	1.94	2.67	(1.08)
	4th (Age)	2.04	2.88	(1.08)	2.13	3.02	(1.04)
Exit Rate and Age	1st (Age)	.152	.110	(.318)	.143	.087	(.282)
	2nd (Age)	.152	.113	(.317)	.143	.056	(.231)
	3rd (Age)	.152	.105	(.306)	.103	.041	(.197)
	4th (Age)	.113	.060	(.265)	.092	.039	(.194)
Exit Rate and Size	1st (Size)	.162	.121	(.333)	.158	.122	(.327)
	2nd (Size)	.162	.101	(.312)	.158	.055	(.229)
	3rd (Size)	.162	.083	(.287)	.028	.038	(.191)
	4th (Size)	.024	.063	(.251)	.003	.030	(.171)
Product Loss Probability and Age	1st (Age)	.173	.038	(.190)	.174	.058	(.234)
	2nd (Age)	.173	.054	(.226)	.174	.077	(.266)
	3rd (Age)	.173	.057	(.232)	.234	.096	(.295)
	4th (Age)	.228	.063	(.244)	.252	.120	(.325)

Notes: U.S. data is from Compustat data (1980 to 2016). French data is from the full FICUS-FARE dataset (1994-2016). Size is measured as sector-deflated sales, age as the number of years since creation or Compustat entry. Exit is a dummy equal to 1 if a firm no longer appears in Compustat/FICUS-FARE in subsequent years. Product loss is a dummy equal to 1 if a firm produces fewer goods the subsequent year in the segment/EAP data. Items under 'model' and 'data' are the mean of the variable within the quartile considered.

Table A8: Data vs Model: Elasticity of Fixed-over-Total Costs Ratio with respect to Fixed Costs

	I			II		
Fixed costs over total costs (log)	Initial S.S.	Final S.S.	Data	Initial S.S.	Final S.S.	Data
United States						
Fixed costs (log)	0.49	0.74	0.95	0.49	0.46	0.88
	(.001)	(.002)	(.002)	(.001)	(.001)	(.003)
R^2	0.99+	0.89	0.96	0.99+	0.98	0.94
France						
	0.63	0.75	0.92	0.63	0.66	0.93
Fixed costs (log)	(.002)	(.019)	(.001)	(.002)	(.030)	(.000)
R^2	0.99	0.95	0.95	0.99	0.97	0.97
Firm Fixed Effects	No	No	No	Yes	Yes	Yes

Notes: Dependent variable is the log of the ratio of fixed costs over total costs. Explanatory variable is the log of fixed costs. Firm-clustered standard errors in parentheses. Columns headed 'Initial S.S.' are from simulated data for the initial steady state where ϕ_i is homogeneous across firms, while columns headed 'Final S.S.' are for the final simulation. The U.S. regression uses 115,564 observations from Compustat (1980-2015), the French regression uses 7,648,443 observations (1994-2015). Data regressions control for size through the log of revenue and time fixed effects, while regressions on the simulated data directly control for the log of product count.

Table A9: Data vs Model: Relationship between Firm Size and Fixed Cost Intangibles

	I			II		
Fixed costs (log)	Initial S.S.	Final S.S.	Data	Initial S.S.	Final S.S.	Data
United States						
Product count / Revenue (log)	1.00	1.07	0.90	1.00	1.00	0.90
	(.001)	(.001)	(.004)	(.001)	(.000)	(.007)
R^2	0.94	0.98	0.82	0.95	0.97	0.63
France						
Product count / Revenue (log)	1.01	1.08	0.80	1.01	1.00	0.52
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
R^2	0.92	0.96	0.54	0.93	0.93	0.16
Firm Fixed Effects	No	No	No	Yes	Yes	Yes

Notes: Dependent variable is the log of fixed costs. Explanatory variable is firm-size, measured through the log of product count in model columns and the log of revenue in data columns. Firm-clustered standard errors in parentheses. Columns headed 'Initial S.S.' are from simulated data for the initial steady state where ϕ_i is homogeneous across firms, while columns headed 'Final S.S.' are for the final simulation. The U.S. regression uses 115,564 observations from Compustat (1980-2015), the French regression uses 7,648,443 observations (1994-2015). Data regressions control for time fixed effects.

Table A10: Data vs Model: Relationship between Innovation and Fixed Cost Intangibles

	I			II		
R&D Intensity (log)	Initial S.S.	Final S.S.	Data	Initial S.S.	Final S.S.	Data
United States						
Fixed costs over total costs (log)	0.00	12.8	0.58	0.00	0.00	0.16
	(.000)	(.062)	(.019)	(.000)	(.001)	(.012)
R^2	1.00	0.81	0.25	1.00	1.00	0.04
France						
Fixed costs over total costs (log)	0.00	3.17	0.34	0.00	0.00	0.05
	(.000)	(.455)	(.017)	(.000)	(.000)	(.003)
R^2	1.00	0.49	0.11	1.00	1.00	0.03
Firm Fixed Effects	No	No	No	Yes	Yes	Yes

Notes: Dependent variable is the log of ratio of R&D spending over sales. Explanatory variable is the log of fixed costs over total costs. Firm-clustered standard errors in parentheses. Columns headed 'Initial S.S.' are from simulated data for the initial steady state where ϕ_i is homogeneous across firms, while columns headed 'Final S.S.' are for the final simulation. The U.S. regression uses 58,246 observations from Compustat (1980-2015), the French regression uses 20,666 observations (1994-2015). Data regressions control for firm size through log of revenue, model regressions through the log of product count.

Table A11: Data vs Model: Relationship between Markups and Fixed Cost Intangibles

	I			II		
Markup (log)	Initial S.S.	Final S.S.	Data	Initial S.S.	Final S.S.	Data
United States						
Fixed costs over total costs (log)	-0.83	0.10	0.44	-0.83	-0.91	-2.42
	(.004)	(.004)	(.036)	(.004)	(.013)	(9.59)
R^2	0.98	0.22	N.A.	0.99	0.93	N.A.
France						
Fixed costs over total costs (log)	-0.44	-0.11	0.31	-0.43	-0.36	0.25
	(.005)	(.010)	(.007)	(.006)	(.067)	(.04)
R^2	0.95	0.20	N.A.	0.94	0.77	N.A.
Firm Fixed Effects	No	No	No	Yes	Yes	Yes

Notes: Dependent variable is the log of the firm's markup. Explanatory variable is the log of fixed costs. Firm-clustered standard errors in parentheses. Columns headed 'Initial S.S.' are from simulated data for the initial steady state where ϕ_i is homogeneous across firms, while columns headed 'Final S.S.' are for the final simulation. Data regressions are based on Table 4 columns III and IV.

Table A12: Relationship between Fixed Costs and Variable Costs (Estimation of θ)

	United States		France	
Variable Costs over Purged Sales	I	II	III	IV
Fixed Costs over Purged Sales (θ)	1.07	0.86	1.34	1.12
	(0.015)	(0.029)	(0.002)	(0.002)
Value in main calibration [robustness]	2.00 [0.86]	2.00 [0.86]	2.00 [0.86]	2.00 [0.86]
R^2	0.94	0.92	0.91	0.91
Observations	115,673	115,673	7,648,443	7,648,443
Year Fixed Effects & ϕ control	Yes	Yes	Yes	Yes
Firm fixed effect	No	Yes	No	Yes

Notes: Dependent variable is the ratio of variable costs over sales, purged for time effects. Explanatory variable is fixed costs over purged sales. Firm-clustered standard errors in parentheses. All regressions include a third-degree polynomial of the ratio of fixed costs over total costs, interacted with time fixed effects, to control for ϕ_i and w_t. Columns II and IV additionally include firm fixed effects.

Table A13: Balanced Growth Path Comparison - Robustness Checks for θ

	United States				France					
	Δ Model ($\theta = 2$) Shock A	**Δ Model** ($\theta = 0.86$) Shock B	**Δ Model** ($\theta = 0.86$) Shock A	**Δ Data**	**Δ Model** ($\theta = 2$) Shock A	**Δ Model** ($\theta = 0.86$) Shock B	**Δ Model** ($\theta = 0.86$) Shock A	**Δ Model** ($\theta = 1.12$) Shock B	**Δ Model** ($\theta = 1.12$) Shock A	**Δ Data**
Cost Structure										
Fixed cost (%)	10.6 pp	10.7 pp	16.1 pp	10.6 pp	4.5 pp	4.5 pp	12.2 pp	4.5 pp	6.5 pp	4.5 pp
Productivity										
Prod. Growth	-0.32 pp	-0.14 pp	- 0.47 pp	-0.9 pp	-0.08 pp	-0.04 pp	-0.32 pp	-0.03 pp	-0.08 pp	-1.3 pp
R&D/v.a.	34.5%	25.4%	48.5%	64.5%	20.1%	8.5%	27.4%	20.9%	30.0%	5.6%
Business Dyn.										
Entry rate	-4.6 pp	-1.6 pp	-6.6%	-5.8 pp	-1.1 pp	-0.5 pp	-3.4 pp	-0.7 pp	-1.3 pp	-3.8 pp
Realloc. Rate	-36.3%	-24.2 %	-42.5%	-23%	-17.6%	-13.2 %	-36.3 %	-13.2%	-19.3 %	-23%
Model Objects										
Labor Wedge	6.6 pt	5.5	10.7	N.A.	3.2 pt	1.8 pt	6.4 pt	3.6 pt	5.2 pt	N.A.
Efficiency Wedge	0.04 pt	0.04 pt	0.03 pt	N.A.	0.036 pt	0.023 pt	0.051 pt	0.056 pt	0.08 pt	N.A.

Notes: This table contains a robustness check for the balanced growth path results in Table 7. Rather than estimating the model with $\theta = 2$, the model is estimated with $\theta = 0.86$. This achieves a pass-through of marginal cost shocks to markups of -35% rather than -25%, in line with the main results in Amiti et al. (2019). For France there is an additional column with $\theta = 1.12$, in line with the relationship between fixed costs and variable costs in Table A12. Data columns present the empirical moments, while model columns present the theoretical moments. Columns headed Shock A provide the change in the steady state variables when the same shock is applied to firms as in the main calibration. Shock B presents results where the shock is re-estimated for each new calibration. They are included separately, as the re-calibrated shock assigns the higher intangible efficiency to a larger fraction of entrants than in the baseline calibration, such that the model predicts significantly smaller declines in entry than the baseline calibration and the data. For shock B the respective calibrations (in order of columns) for $[\overline{\phi}/\phi, G(\overline{\phi})]$ is $[0.85, 0.251]$, $[0.92, 0.121]$, and $[0.86, 0.083]$. Shock A is $[0.74, 0.084]$ for the U.S., $[0.83, 0.054]$ for France.

Table A14: Structural Estimation - Alternative Value Function Specification

Par.	Moment	United States Par. Value (Old/New)	Data (Main)	Model (Full)	Model Target	France Par. Value (Old/New)	Data (Main)	Model (Full)	Model Target
$\overline{\lambda}$	Productivity Gr.	0.06/0.06	1.4%	1.3%	1.3%	0.06/0.06	1.4%	1.3%	1.3%
ϕ	Fixed Costs (%)	0.215/0.215	12.9%	12.9%	12.9%	0.279/0.279	9.4%	9.5%	9.5%
σ	Gibrat's Law	0.521/0.563	-0.036	-0.035	-0.035	0.623/0.671	-0.035	-0.035	-0.035
η^e	Entry Rate	2.47/2.47	14.0%	13.2%	13.8%	1.73/1.73	11.2%	10%	10.0%
η^x	R&D Intensity	3.36/3.36/	2.7%	2.5%	2.5%	2.29/2.29	2.8%	2.4%	3.2%

Table A15: Comparison of Steady States - Constant Markup

	United States			France		
	Δ Model (Var. μ)	**Δ Model** (Fixed $\overline{\mu}$)	**Δ Data**	**Δ Model** (Var. μ)	**Δ Model** (Fixed $\overline{\mu}$)	**Δ Data**
Cost Structure						
Average Fixed-Cost Share	10.6 pp	12.1 pp	10.6 pp	4.5 pp	2.3 pp	4.5 pp
Slowdown of Productivity Growth						
Productivity Growth Rate	-0.3 pp	-0.5 pp	-0.9 pp	-0.08 pp	-0.1 pp	-1.3 pp
Aggregate R&D over Value Added	34.5%	-29.5%	64.5%	20.1%	-8.5%	5.6%
Decline of Business Dynamism						
Entry rate	-4.6 pp	-4.7 pp	-5.8 pp	-1.1 pp	-0.4 pp	-3.8 pp
Reallocation Rate	-36.3%	-51.0%	-23%	-17.0%	-13.1%	-23%

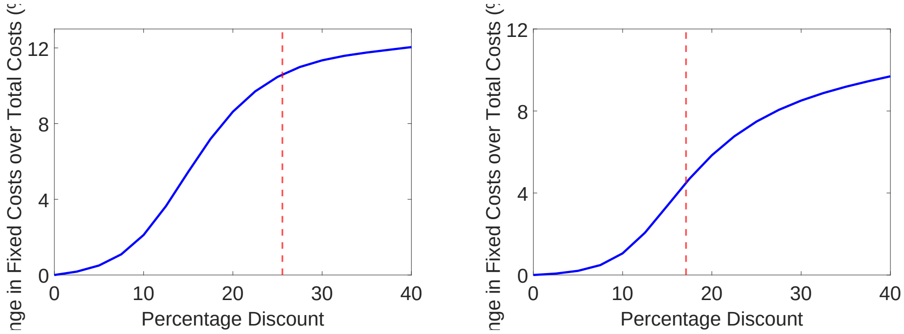
Notes: Data columns present the empirical moments, while model columns present the theoretical moments. Model - $\bar{\mu}$ columns present theoretical moments where markups are exogenous and homogeneous across firms in both steady states. The change in productivity growth is the difference between growth from 1969-1994 (France) or 1969-1979 (U.S.) to growth post-2005. Other French moments equal the difference between values in 1994 and in 2016. Other U.S. moments equal the difference between 1980 and 2016.

Table A16: Comparison of Steady States - Alternative Value Function Specification

	United States			France		
	Δ Model (Main)	**Δ Model** (Full Val.)	**Δ Data**	**Δ Model** (Main)	**Δ Model** (Full Val.)	**Δ Data**
Cost Structure						
Average Fixed-Cost Share	10.6 pp	10.5 pp	10.6 pp	4.5pp	4.9 pp	4.5 pp
Slowdown of Productivity Growth						
Productivity Growth Rate	-0.32 pp	-0.34 pp	-0.9 pp	-0.1 pp	-0.1 pp	-1.3 pp
Aggregate R&D over Value Added	34.8%	48.2%	64.5%	20.1%	34.5%	5.6%
Decline of Business Dynamism						
Entry rate	-4.6 pp	-5.5 pp	-5.8 pp	-1.1 pp	-1.6 pp	-3.8 pp
Reallocation Rate	-36.3%	-36.7%	-23%	-17.6%	-16.1%	-23%
Rise of Market Power						
Average Markup	14.7 pt	13.9pt	30 pt	5.6pt	5.6 pt	11 pt
Model Objects						
Labor Wedge	6.6 pt	6.6 pt	N.A.	3.2 pt	3.2 pt	N.A.
Efficiency Wedge	.04 pt	.04 pt	N.A.	.04 pt	.04 pt	N.A.

Notes: Data columns present the empirical moments, while Model - Main columns present the theoretical moments from the model in the main analysis. Model - Full Val. columns present moments where the value function includes the R&D option value. The change in productivity growth is the difference between growth from 1969-1994 (France) or 1969-1979 (U.S.) to growth post 2005. Other French moments equal the difference between values in 1994 and in 2016. Other U.S. moments equal the difference between 1980 and 2016.

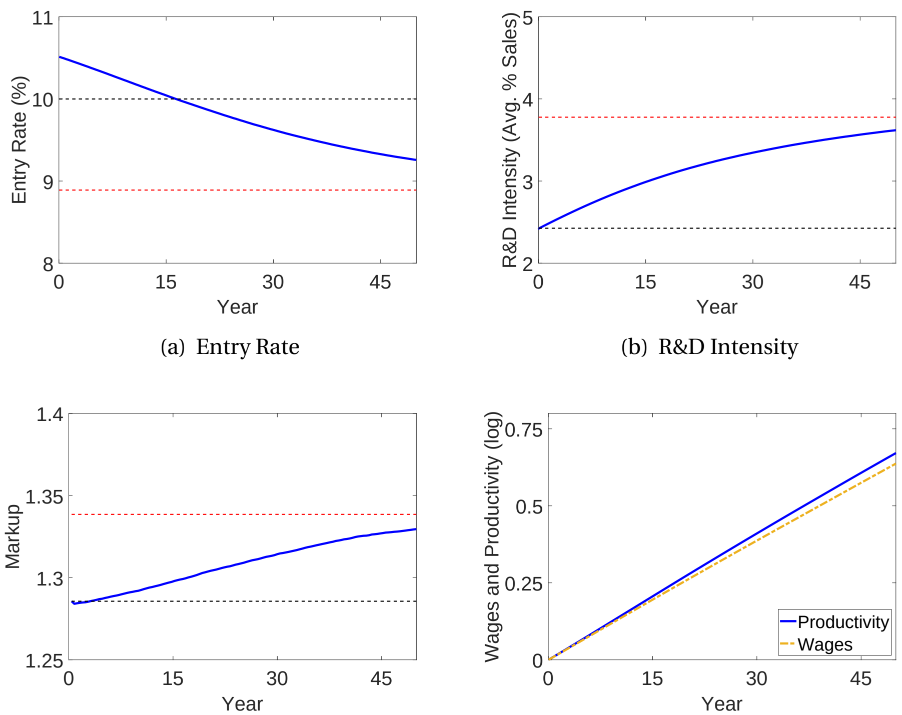
Figure A10. Relationship between Average Fixed Costs and Intangible Costs $\overline{\phi}$



(a) United States (b) France

Notes: The figure presents a comparative static, plotting the effect of the intangibles cost parameter $\overline{\phi}$ on the average ratio of fixed costs over total costs along the balanced growth path. The horizontal axis plots $(1-\overline{\phi}/\phi)\times 100\%$, that is, it plots the percentage discount on intangible costs that the low-cost firms receive. Vertical-red lines plot the percentage discount in the calibration of the final steady state. The vertical axis measures average fixed costs across products, which is the model-consistent counterpart of the revenue-weighted firm-level fixed costs (plotted in Figure A3) as revenue is proportional to a firm's product count in the model.

Figure A11. Transition Path for Various Variables (France)

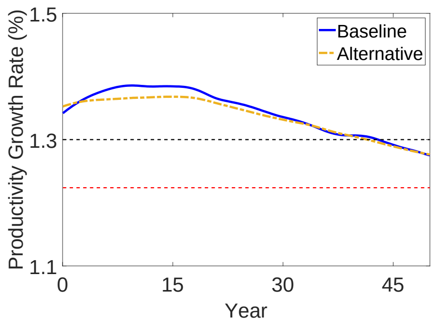


(a) Entry Rate (b) R&D Intensity

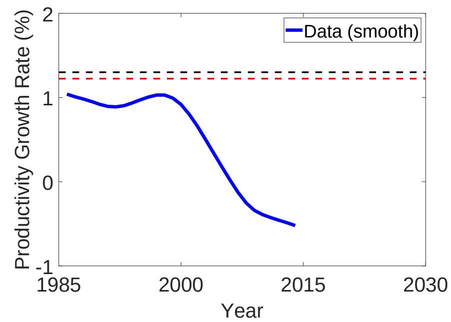
(c) Markups (d) Wages and Productivity

Notes: Black and red dashed lines (respectively) indicate the original and the new steady state. Figure (a) presents the entry rate, (b) presents R&D intensity (the average ratio of R&D over sales), (c) presents the average markup, (d) presents the path of wages (which tracks quality) and productivity (which tracks quality and intangibles).

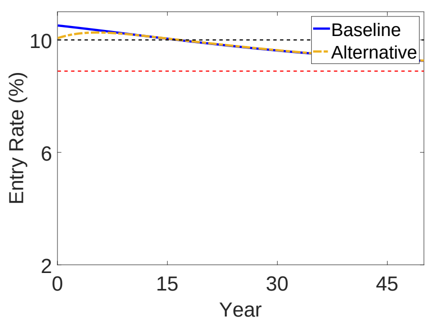
Figure A12. Transition Path: Model Predictions versus Data (France)



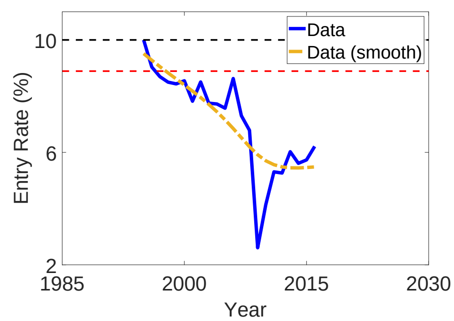
(a) Productivity Growth* (Model)



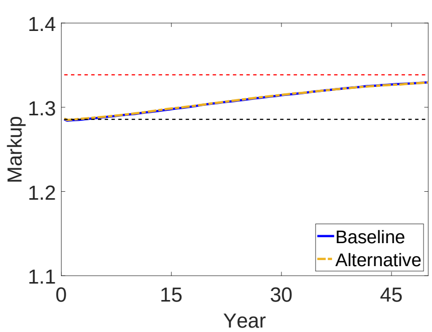
(b) Productivity Growth (Data)



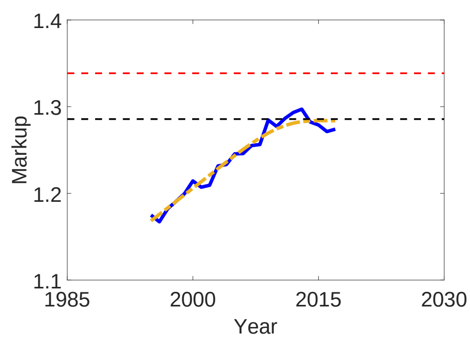
(c) Entry Rate (Model)



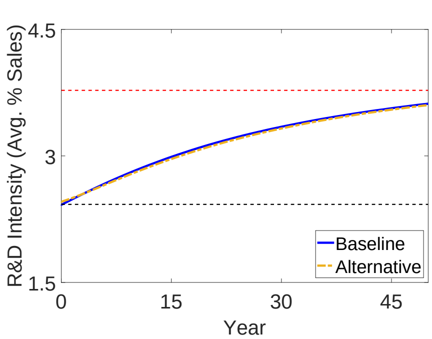
(d) Entry Rate (Data)



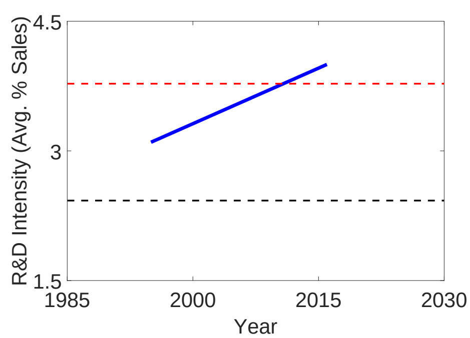
(e) Markup (Model)



(f) Markup (Data)



(g) R&D Intensity (Model)



(h) R&D Intensity (Data)*

Notes: Black and red dashed lines (respectively) indicate the original and the new steady state. Calibration is for France. Productivity growth in Figure A11b only includes smoothed data as productivity growth is highly volatile. HP-filter smoothing parameter is 100. Data sources: productivity growth from Penn World Tables, R&D from CIS (1996, 2016), entry (imputed using trend in entrant employment weights) and markups from FICUS-FARE. Note that productivity growth is not plotted on the same scale between model and data.

Appendix G. Alternative Calibration: Markup Target

The model in the main analysis predicts average markups of 1.45 for the U.S. calibration and 1.32 for the French calibration. This overstates the actual markups at the beginning of the sample. Markups are overestimated because the ratio of fixed costs over total costs is used to calibrate initial intangible cost ϕ_i. As markups are given by the ratio of innovation steps λ_{ij} and the fraction of marginal costs retained s_{ij}, lower intangible costs yield higher markups. No separate parameter disciplines markups.

In this robustness check I recalibrate the model to match average markups. Markups replace fixed-over-total costs as the moment related to ϕ_i. I target initial markups of 1.27 and 1.17 in the U.S. and French calibration, respectively. Table A17 presents the estimation results. The new calibrations have higher intangible cost parameters, so that firms chose a higher s_{ij} and therefore have lower markups than in the main calibration. Because the lower markups reduce the incentive to innovate, the new calibrations have lower innovation cost scalars to preserve innovation and growth rates. Table A18 presents the model's performance on the main targets. The model matches aggregate growth and the relationship between firm size and firm growth perfectly, while it understates U.S.entry and French R&D intensity. Average markups are matched with precision in both calibrations. In exchange for the well-matched markups, the model now underpredicts fixed costs as a percentage of total costs. One interpretation is that fixed costs in the data comprise of intangibles-induced fixed costs, which raise markups, and other fixed costs. Other fixed costs are not present in the model and therefore create a tradeoff in simultaneously matching markups and fixed costs.

The main experiment, in which a group of high-intangible firms is introduced in the economy, is rerun in Table A19. The experiment now targets changes in entry rates and in average markups. The table presents changes in the balanced growth path values of the variables of interest. Columns headed 'Main' are reproductions of Table A16 in the main text while columns headed 'Markup Calib.' contain results for the new calibration. Overall, results for the new calibration are in line with the original results. Compared to the main calibration, the model performs better at matching the (targeted) rise of markups, as well as the (untargeted) decline of the reallocation rate. The model is still able to explain around one third of the slowdown of productivity growth, although the predicted increase in R&D now vastly exceeds the increase in the data. The rise of fixed costs is well-matched for the U.S., although it is overstated in the French calibration. Overall, the rise of intangibles has a similar qualitative effect in the model: it reduces productivity growth and dynamism, and raises market power and R&D.

Table A17: Overview of Parameters - Markup Target

Parameter	**Description**	**Main** Calib. (U.S.)	**Markup** Calib. (U.S.)	**Main** Calib. (Fr.)	**Markup** Calib. (Fr.)
η^x	Cost scalar innov. (incumbents)	3.36	2.05	1.73	1.56
η^e	Cost scalar innov. (entrants)	2.44	2.58	2.29	1.78
$\overline{\lambda}$	Average innovation step size	.060	.067	.061	.067
σ	OLS reg. firm-size and growth	.521	.574	.623	.552
ϕ	Intangible costs	.215	.292	.279	.369

Notes: This table presents values for the structurally estimated parameters. Columns headed 'Main' contain parameters in the main analysis and reproduce the entries in Table A17. Columns headed Markup Calib. contain the calibration that uses the markups target.

Table A18: Comparison of Theory and Data for Targeted Moments: Calibration with Markup Target

			United States		France	
Parameter	**Moment**	**Weight Ω**	**Model**	**Target**	**Model**	**Target**
$\bar{\lambda}$	Long-term growth rate of productivity	2	1.3%	1.3%	1.3%	1.3%
ϕ	Average markup	2	1.27	1.27	1.17	1.17
ϕ	Fixed costs as a fraction of total costs	0	9.2%	12.9%	5.4%	9.5%
σ	Relation between firm growth and size	1	-.035	-.035	-.035	-.035
η^e	Entry rate (fraction of firms age 1 or less)	1	9.4%	13.8%	9.9%	10.0%
η^x	Ratio of research and development to sales	1	2.3%	2.5%	1.6%	3.2%

Table A19: Comparison of Steady States - Calibration with Markup Target

	United States			France		
	Δ Model (Main)	**Δ Model** (Markup Calib.)	**Δ Data**	**Δ Model** (Main)	**Δ Model** (Markup Calib.)	**Δ Data**
Cost Structure						
Average Fixed-Cost Share	10.6 pp	10.4 pp	10.6 pp	4.5 pp	6.8 pp	4.5 pp
Slowdown of Productivity Growth						
Productivity Growth Rate	-0.3 pp	-0.3 pp	-0.9 pp	-0.08 pp	-0.04 pp	-1.3 pp
Aggregate R&D over Value Added	34.5%	91.7%	64.5%	20.1%	47.1%	5.6%
Decline of Business Dynamism						
Entry rate	-4.6pp	-5.6 pp	-5.8 pp	-1.1 pp	-1.2 pp	-3.8 pp
Reallocation Rate	-36.3%	-27.8%	-23%	-17.6%	-17.2%	-23%
Rise of Market Power						
Average Markup	14.6 pt	27.8 pt	30 pt	6.4 pt	11 pt	11 pt
Model Objects						
Labor Wedge	6.6 pt	14.2 pt	N.A.	3.2 pt	7.3 pt	N.A.
Efficiency Wedge	0.04 pt	0.02 pt	N.A.	0.04 pt	0.12 pt	N.A.

Notes: Data columns present the empirical moments, while Model - Main columns present the theoretical moments from the model in the main analysis. Model - Markup Calib. columns present moments when the calibration uses the markup moment for intangible use. The change in productivity growth is the difference between growth from 1969-1994 (France) or 1969-1979 (U.S.) to growth post 2005. Other French moments equal the difference between values in 1994 and in 2016. Other U.S. moments: difference between 1980 and 2016.

Appendix H. Optimal Allocation of Researchers

Growth in experiment (1)
The growth rate of total factor productivity is the product of the measure of patents that researchers generate and the average improvements to quality that these patents embody:

$$\tilde{g} = \overline{\lambda}\left(e + \textstyle\sum_{\phi_i \in \Phi} \sum_{n=1}^{\infty} M_n(\phi_i) x_n(\phi_i)\right),$$

where the innovation by entrants and incumbents is multiplied by the average innovation step size $\overline{\lambda}$ because all innovation is successful.

Growth in experiment (2)
To calculate the efficient growth rate, I first calculate the number of researchers $\overline{L}^{rd}$ that incumbents employ in the final steady state:

$$\overline{L}^{rd} = \textstyle\sum_{\phi_i \in \Phi} \sum_{n=1}^{\infty} M_{\phi_i, n} \eta^x x_n(\phi_i)^{\psi^x} n^{-\sigma},$$

I then allocate them across firms of different sizes so that the marginal research productivity of researchers is equalized:

$$\widetilde{rd}_n^x = \overline{L}^{rd}\left(n^{\frac{\sigma}{\psi^x - 1}}\left[\textstyle\sum_{n=1}^{\infty} M_n n^{\frac{\sigma}{\psi^x - 1}}\right]^{-1}\right), \tag{A.14}$$

where $M_n = \sum_{\phi \in \Phi} M_n(\phi)$ is the measure of size n firms in the final steady state and where $\widetilde{rd}_n^x$ is the optimal measure of researchers assigned to firms of size n. The growth rate of productivity is:

$$\tilde{g} = \overline{\lambda}\left(e + \sum_{n=1}^{\infty} M_n \left(\widetilde{rd}_n^x n^{\sigma}/\eta^x\right)^{1/\psi^x}\right). \tag{A.15}$$

Growth in experiment (3)
The total number of researchers available is $\overline{L}^{rd} + \overline{L}^{e}$, where the second term measures the number of researchers employed by entrants, $\overline{L}^{e} = \eta^e e^{\psi^e}$. Given $\psi^x = \psi^e$, the optimal allocation to incumbents is:

$$\widetilde{rd}_n^x = \left(\overline{L}^{rd} + \overline{L}^{e}\right)\left((\eta^x/\eta^e)^{\frac{1}{\psi^x - 1}} + \textstyle\sum_{n=1}^{\infty} M_n n^{\frac{\sigma}{\psi^x - 1}}\right)^{-1} n^{\frac{\sigma}{\theta - 1}}, \tag{A.16}$$

while the optimal allocation to entrants is:

$$\widetilde{rd}^e = \left(\overline{L}^{rd} + \overline{L}^{e}\right)\left((\eta^x/\eta^e)^{\frac{1}{\psi^x - 1}} + \textstyle\sum_{n=1}^{\infty} M_n n^{\frac{\sigma}{\psi^x - 1}}\right)^{-1} (\eta^x/\eta^e)^{\frac{1}{\psi^x - 1}}. \tag{A.17}$$

Hence the growth rate of productivity is:

$$\tilde{g} = \overline{\lambda}\left(\left(\widetilde{rd}^e/\eta^e\right)^{1/\psi^e} + \textstyle\sum_{n=1}^{\infty} M_n \left(\widetilde{rd}_n^x n^{\sigma}/\eta^x\right)^{1/\psi^x}\right). \tag{A.18}$$

Research subsidy to resolve misallocation
In the remainder of this appendix, I quantify the wedge between the private returns to innovation across entrants and incumbents with different intangible efficiencies. To do so, I calculate the factor by which a targeted research subsidy, Ξ_i^1, would have to multiply the private value of research at high-ϕ and entrants, relative to the subsidy (or tax) imposed on low-ϕ firms. I also introduce a homogeneous subsidy (or tax) on R&D, Ξ^2, which controls the aggregate fraction of resources that is devoted to R&D.

My focus is on the Ξ_i^1 that equates the marginal research productivity across researchers, to quantify the difference in private rates of return to R&D across firms. I abstract from optimizing the homogeneous Ξ^2, as this is the tool through which the social planner would address the usual Schumpeterian innovation distortions, which the model has in common with the rest of the literature.[13] Taxes enter an incumbent's first-order condition as follows:

$$\left(x_i/n_i^{\frac{\sigma}{\psi^x-1}}\right)=\left(\mathscr{P}(\phi_i)\Xi_i^1\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i,\lambda_{ij})}{r-g+\tau(\phi_i)}\right](\eta^x\psi^x w\Xi^2)^{-1}\right)^{\frac{1}{\psi^x-1}}, \tag{A.19}$$

where I standardize the equation by product-count because the wedge between the firm's private return from research and its research productivity does not depend on n_i. The Ξ_i^1 that implements the statically optimal allocation for experiment 2 (A.14) and experiment 3 (A.16) are given by:

$$\Xi_i^1=\overline{mpr}\left(\mathscr{P}(\phi_i)\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i,\lambda_{ij})}{r-g+\tau(\phi_i)}\right]\right)^{-1}$$

in both cases, where $\overline{mpr}$ denotes the (common) marginal research product. In experiment 2, this is

$$\overline{mpr}=\left[\overline{L}^{rd}\left(\left[\sum_{n=1}^{\infty}M_n n^{\frac{\sigma}{\psi^x-1}}\right]^{-1}\right)\right]^{\frac{1-\psi^x}{\psi^x}}(\eta^x)^{\frac{1}{\psi^x}}(\psi^x)^{-1},$$

where $\overline{L}^{rd}$ is pinned down by Ξ^2. Conversely, when reallocating all researchers (experiment 3), $\overline{mpr}$ is

$$\overline{mpr}=\left[\left(\overline{L}^{rd}+\overline{L}^{e}\right)\left((\eta^x/\eta^e)^{\frac{1}{\psi^x-1}}+\sum_{n=1}^{\infty}M_n n^{\frac{\sigma}{\psi^x-1}}\right)^{-1}\right]^{\frac{1-\psi^x}{\psi^x}}(\eta^x)^{\frac{1}{\psi^x}}(\psi^x)^{-1}, \tag{A.20}$$

The first-order condition for entry in the presence of the research subsidies (or taxes) is given by

$$e=\left(\sum_{\phi_e\in\Phi}G(\phi_e)\mathscr{P}\left(\phi_e\right)\Xi_e^1\mathbb{E}_{\phi_h}\left[\frac{\pi(\phi_e,\lambda_{ej})}{r-g+\tau(\phi_e)}\right](\eta^e\psi^e w\Xi^2)^{-1}\right)^{\frac{1}{\psi^e-1}}. \tag{A.21}$$

The marginal research product equals that of incumbents in (A.20). Given that $\psi^e=\psi^x$, the subsidy that equates the marginal research product of entrants to that of incumbents is given by

$$\Xi_e^1=\overline{mpr}\left(\sum_{\phi_e\in\Phi}G(\phi_e)\mathscr{P}(\phi_i)\mathbb{E}_{\phi_i}\left[\frac{\pi_t(\phi_i,\lambda_{ij})}{r-g+\tau(\phi_i)}\right]\right)^{-1}\eta^e/\eta^x.$$

The results are presented in Table A20. It expresses the optimal Ξ_i^1 for entrants and high-ϕ incumbents relative to Ξ_i^1 for low-ϕ incumbents. By normalizing the Ξ_i^1 this way, the table quantifies the wedge between the private value of research across firms with different intangible efficiencies for a given Ξ^2.

Table A20: Required Relative R&D Subsidy to Offset Researcher Misallocation

	United States		France	
	Experiment 2 (reallocate inc. R&D)	Experiment 3 (reallocate all R&D)	Experiment 2 (reallocate inc. R&D)	Experiment 3 (reallocate all R&D)
Incumbents, high ϕ	4.73	4.73	2.20	2.20
Entrants	N.A.	3.65	N.A.	2.09

Notes: The table gives the R&D subsidy Ξ_i relative to Ξ_i for low-ϕ firms that equate marginal research products across researchers.

[13] The model also features a single inelastically supplied input, labor, so that the tradeoffs of a usual optimal taxation and subsidy exercise are not present. I therefore focus on Ξ_i^1, which addresses the new distortions in the model.

Appendix I. Alternative Production Function: Firm-Level Intangibles

I.1. Framework

I.1.1. Production and Intangibles

The framework follows the setup in Section I, except where discussed in this appendix. There are two main changes. The first is that firms reduce the marginal costs across all their products in exchange for a fixed cost levied at the firm level.[14] Total costs for firm i that produces portfolio J_i are therefore:

$$tc_i = \sum_{j \in J_i} w s_i y_{ij} + \phi_i w \left(s_i^{-\theta} - 1 \right),$$

where the first term denotes the firm's total variable costs and the second term denotes the firm's total fixed costs. The firm-level s_i identifies the share of marginal costs that the firm keeps. The optimal s_i now depends on both firm size n_i and the firm's intangible costs ϕ_i. As a second change, I assume that a firm's optimal markup is $\mu_i = \overline{\lambda}/s_i$, where $\overline{\lambda}$ is (say) the average innovation step size. The simplified pricing rule is needed, as the markup would depend on the use of intangibles by the firm's competitors in the rigorous Bertrand-Nash equilibrium.[15] The cost-minimizing firm sets intangibles so that

$$s_i = \min \left[\left(\left[\sum_{j \in J_i}^{n_i} y_{ij} \right]^{-1} \theta \phi_i \right)^{\frac{1}{\theta+1}}, 1 \right]. \tag{A.22}$$

The marginal cost of producing any of the firm's goods is ws_i, so that output under the optimal markup is given by $y_{ij} = Y/(w\overline{\lambda})$. Inserting this into the first-order condition above implies that s_i can be written in terms of a firm's number of products n_i and its intangibles cost parameter ϕ_i:

$$s_{n_i}(\phi_i) = \min \left[\left(n_i^{-1} w Y^{-1} \overline{\lambda} \theta \phi_i \right)^{\frac{1}{\theta+1}}, 1 \right],$$

where n_i is the cardinality of J_i, which implies that firms with more products choose higher fixed costs, as they benefit from lower marginal costs across a greater number of products.

I.1.2. Choke Prices and Creative Destruction

The rate of creative destruction declines in the number of products that a firm produces. High-n_i firms are able to produce at lower average costs, and are therefore able to sell at lower prices. Unless an innovator draws a sufficiently large innovation step, a higher-n_i incumbent can therefore keep producing the good. Recall that the rate of creative destruction is the product of the arrival rate of innovations by other firms on goods that the firm currently produces, and the probability that these firms have a sufficiently low choke price. If incumbent i faces an innovator of type ϕ_h that currently produces n_h goods and that develops a higher-quality version of good j, the probability that firm h successfully takes over production is given by:

$$\text{Prob}\left(\lambda_{hj} \geq \frac{p^c_{n_h+1}(\phi_h)}{p^c_{n_i}(\phi_i)} \right) = \min \left[\left(\frac{p^c_{n_i}(\phi_i)}{p^c_{n_h+1}(\phi_h)} \right)^{\frac{\overline{\lambda}}{\overline{\lambda}-1}}, 1 \right], \tag{A.23}$$

[14] Complementarities across products that create positive returns to scale on a firm's demand side are explored in the Klette and Kortum (2004)-framework in a recent paper by Feijoo-Moreira (2021).

[15] Competing firms no longer set intangibles to zero in the alternative model, because they use intangibles across their products. The competitors' own use of intangibles, furthermore, changes over time if they start or cease to produce goods, and depends on the use of intangibles by the firms with which they, in turn, compete. Because the model is characterized by a continuum of products and firms, it is unfeasible to track the resulting infinite-dimensional object.

which uses the Pareto distribution of innovation steps with mean step size $\overline{\lambda}$. The choke price $p^c_{n_i}(\phi_i)$ is the price at which a type-ϕ_i firm is indifferent between producing n_i goods, including some good j, or $n_i - 1$ goods. The rate of creative destruction follows from taking the product of the probability (A.23) with the flow of innovative patents from firms of each size n_h, and entrants, summed across firm types. All firms face equal flows of innovations to each of their products, but the size-dependent probability of success yields that creative destruction rates strictly decline in n_i. Using notation from the main text, we have:

$$\tau_{n_i}(\phi_i) = \sum_{\phi_k \in \Phi} \left[\sum_{n_h=1}^{n_i} M_{n_h}(\phi_k) x_{n_h}(\phi_k) \left(\frac{P^c_{n_i}(\phi_i)}{P^c_{n_h+1}(\phi_k)} \right)^{\frac{\lambda}{\overline{\lambda}-1}} + \sum_{n_h=n_i+1}^{\infty} M_{n_h}(\phi_k) x_{n_h}(\phi_k) + G(\phi_k) e \left(\frac{P^c_{n_i}(\phi_i)}{P^c_1(\phi_k)} \right)^{\frac{\lambda}{\overline{\lambda}-1}} \right].$$

The creative destruction rate therefore hinges on the relative choke price across firms. To find the choke price, I compare profits for a type-ϕ_i firm that produces $n_i - 1$ goods under cost minimization and optimal markups

$$\pi_{n_i-1}(\phi_i) = (n_i - 1)(1 - s_{n_i-1}(\phi_i)\lambda^{-1})Y - \phi_i w (s_{n_i-1}(\phi_i)^{-\theta} - 1),$$

to profits for a firm i that produces some additional good j and sells it at price p_{ij},

$$\tilde{\pi}_i = \Big([n_i - 1][1 - \tilde{s}_i \lambda^{-1}] Y + (1 - p_{ij}^{-1} w \tilde{s}_i) Y \Big) - \phi_i w (\tilde{s}_i^{-\theta} - 1),$$

where optimal intangibles depend on p_{ij} because output of good j affects the firm's overall output, in line with first-order condition (A.22). The choke price for j sets $\tilde{\pi}_i = \tilde{\pi}_{n-1}$, and therefore solves

$$\tilde{s}_i^{\theta} s_{n_i-1}(\phi_i)^{\theta} - (n_i - 1)\lambda^{-1} \tilde{s}_i^{\theta} s_{n_i-1}(\phi_i)^{\theta} \left(\tilde{s}_i - s_{n_i-1}(\phi_i) \right) - \phi_i \frac{w}{Y} (s_{n_i-1}(\phi_i)^{\theta} - \tilde{s}_i^{\theta}) - w \tilde{s}_i^{\theta+1} s_{n_i-1}(\phi_i)^{\theta} p_{ij}^{-1} = 0,$$

which yields that for a given parameterization, wage and aggregate output, the choke price is only a function of n_i and ϕ_i. There is no analytical solution for the choke price as it appears with various powers through the intangibles first-order condition, although it is straightforward to see that $\lim_{n_i \to \infty} p^c_{n_i}(\phi_i) = 0$; as n_i becomes large, $s_{n_i-1}(\phi_i)$ converges to zero and optimal marginal costs eventually approach zero. The choke price therefore converges to zero. From the diminishing choke price in n_i it follows that $\lim_{n_i \to \infty} \tau_{n_i}(\phi_i) = 0$, that is, firms become unbeatable as their size increases.

I.1.3. Innovation

Firms maximize their value by choosing the flow rate x_i at which they receive a patent to produce goods that they do not currently produce. As in the main model, firms hire rd^x_i researchers for research and development as a function of x_i along

$$rd^x_{n_i}(x_i) = \eta^x x_i^{\psi^x} n_i^{-\sigma}. \tag{A.24}$$

The associated value function, with notation from the main text, reads as

$$rV_{tn_i}(\phi_i) - \dot{V}_{tn_i}(\phi_i) = \max_{x_i} \left\{ \begin{array}{c} \sum_{j \in J_i} \pi_{tn_i}(\phi_i) + \tau_{n_i}(\phi_i) \left[V_{tn_i-1}(\phi_i) - V_{tn_i}(\phi_i) \right] \\ + x_i \text{Prob} \left(\lambda_{ij} \geq \frac{p^c_{n_i+1}(\phi_i)}{p^c_{n_{-i}}(\phi_i)} \right) \left[V_{tn_i+1}(\phi_i) - V_{tn_i}(\phi_i) \right] - w_t \eta^x (x_i)^{\psi^x} n_i^{-\sigma}) \end{array} \right\},$$

where the main difference with the main text is that the value function, profits per product and the rate of creative destruction are now also a function of n_i. The solution to the value function is similar to the solution to the extended value function in Section V, as the following proposition makes clear.

Proposition I.1. *The value function of a firm that produces a portfolio of goods J_i with cardinality n_i grows at rate g along the balanced growth path and is given by*

$$V_{n_i}(\phi_i) = n_i \Upsilon^1_{n_i}(\phi_i) + \Upsilon^2_{n_i}(\phi_i),$$

where $\Upsilon^1_{n_i}$ is the present value of the per-product, size-dependent profit stream for a firm that produces n_i goods and where time-subscripts are omitted for readability:

$$\Upsilon^1_{n_i}(\phi_i) = \frac{\pi_{n_i}(\phi_i)}{r - g + \tau_{n_i}(\phi_i)}.$$

while $\Upsilon^2_{n_i}$ is the option value of research and development, which evolves along the sequence

$$\begin{aligned}\Upsilon^2_{n_i+1}(\phi_i) \quad = \quad & \frac{(r-g)\Upsilon^2_{n_i}(\phi_i) - n_i \tau_{n_i}(\phi_i)\left(\Upsilon^2_{n_i-1}(\phi_i) + \Upsilon^2_{n_i-1}(\phi_i)\cdot(n_i-1) - \Upsilon^2_{n_i}(\phi_i)\right)}{x_{n_i}(\phi_i)\cdot(1-\psi^x)\cdot \text{Prob}\left(\lambda_{ij} \geq \frac{p^c_{n_i+1}(\phi_i)}{p^c_{n_{-i}}(\phi_{-i})}\right)} \\ & + \Upsilon^2_{n_i}(\phi_i) + n_i \Upsilon^1_{n_i}(\phi_i) - \Upsilon^1_{n_i+1}(\phi_i)\cdot(n_i+1),\end{aligned}$$

where $x_{n_i}(\phi_i)$ is the value-maximizing rate of innovation. The dynamic first-order conditions are

$$x_{n_i}(\phi_i) = \left(\text{Prob}\left(\lambda_{ij} \geq \frac{p^c_{n_i+1}(\phi_i)}{p^c_{n_{-i}}(\phi_i)}\right) \frac{\left[(n_i+1)\Upsilon^1_{n_i+1}(\phi_i) - n_i \Upsilon^1_{n_i}(\phi_i) + \Upsilon^2_{n_i+1}(\phi_i) - \Upsilon^2_{n_i}(\phi_i)\right]}{\eta^x \psi^x w_t}\right)^{\frac{1}{\psi^x - 1}} n_i^{\frac{\sigma}{\psi^x - 1}}.$$

$$e = \left(\sum_{\phi_e \in \Phi} \text{Prob}\left(\lambda_{ej} \geq \frac{p^c_1(\phi_e)}{p^c_{n_{-i}}(\phi_{-i})}\right) \frac{\left[\Upsilon^1_1(\phi_e) + \Upsilon^2_1(\phi_e)\right]}{\eta^e \psi^e w}\right)^{\frac{1}{\psi^e - 1}}.$$

Proof: Closely follows proof for Proposition A.1 in Appendix A.

It follows that innovation rate $x_{n_i}(\phi_i)$ depends on firm-size through three channels. First, profits increase in n_i because large firms have lower average costs and higher markups because they deploy more intangibles. The rate at which profits from acquiring a product are discounted is also lower for high-n_i firms because of their lower rates of creative destruction. The third channel is through innovation costs: the inclusion of $n_i^{-\sigma}$ in (A.24) implies that the number of researchers that firms hire for a given innovation rate depends on n_i. In the main model, $\sigma > 0$ so that large firms have lower innovation costs. This in line with Akcigit and Kerr (2018), and assures that the model matches empirical evidence on the firm-size, firm-growth relationship. As will be clear from the next section, the model with firm-level intangibles requires $\sigma < 0$ in order to have a solution.[16]

I.1.4. Firm-size distribution

Finally, consider the implication of firm-level intangibles for the firm-size distribution. The firm-size distribution is stationary along the balanced growth path, if the model admits one. To find the stationary distributions, consider the law of motion for the measure of firms with more than one product:

$$\begin{aligned}\dot{M}_{n_i}(\phi_i) \quad = \quad & M_{n_i-1}(\phi_i) x_{n_i-1}(\phi_i) \text{Prob}\left(\lambda_{ij} \geq \frac{p^c_{n_i}(\phi_i)}{p^c_{n_{-i}}(\phi_{-i})}\right) - M_{n_i}(\phi_i) x_{n_i}(\phi_i) \text{Prob}\left(\lambda_{ij} \geq \frac{p^c_{n_i+1}(\phi_i)}{p^c_{n_{-i}}(\phi_{-i})}\right) \\ & + \left(M_{n_i+1}(\phi_i)\tau_{n_i+1}(\phi_i)\cdot(n_i+1) - n_i M_{n_i}(\phi_i)\tau_{n_i}(\phi_i)\right).\end{aligned}$$

[16] A fourth channel is that the change in the innovation option value depends on n_i, but the direction of this effect depends on the relative magnitude of the other three channels.

Table A21: Overview of Estimated Parameters: Model with Firm-Level Intangibles

Parameter	Description	Method	Value (U.S.)	Value (France)
$\overline{\lambda}$	Average innovation step size	Indirect inference	.138	.129
ϕ	Intangible costs	Indirect inference	.579	.834
σ	Relationship firm-size and firm-growth	Indirect inference	-7.82	-4.54
η^e	Cost scalar of innovation (entrants)	Indirect inference	2.69	8.82
η^x	Cost scalar of innovation (incumbents)	Indirect inference	6.91	4.42

The first term captures entry and exit out of the measure of firms with n_i products through innovation by firms with $n_i - 1$ and n_i products, respectively. The second term captures entry and exit of firms with $n_i + 1$ and n_i products that ceased producing one of their products through creative destruction. For the measure of single-product firms, the law of motion reads as

$$\dot{M}_1(\phi_i) = \left(\text{Prob}\left(\lambda_{ij} \geq \frac{p_1^c(\phi_i)}{p_{n_{-i}}^c(\phi_i)}\right)e - x_1(\phi_i)M_1(\phi_i)\text{Prob}\left(\lambda_{ij} \geq \frac{p_2^c(\phi_i)}{p_{n_{-i}}^c(\phi_i)}\right)\right) + \left(2M_2(\phi_i)\tau_2(\phi_i) - M_1(\phi_i)\tau_1(\phi_i)\right),$$

where e is the entry rate. The stationary firm-size distribution follows from setting both equations to zero for each n_i. The model features a stationary firm-size distribution as long as expected firm growth remains negative as $n_i \to \infty$. The expected growth rate $\text{E}(g_i)$ of a firm that produces n_i products and has intangible costs ϕ_i is given by

$$\text{E}(g_i) = \frac{x_{n_i}(\phi_i)}{n_i} - \tau_{n_i}(\phi_i),$$

where the value-maximizing per-product innovation rate is given by

$$\frac{x_{n_i}(\phi_i)}{n_i} = \left(\text{Prob}\left(\lambda_{ij} \geq \frac{p_{n_i+1}^c(\phi_i)}{p_{n_{-i}}^c(\phi_i)}\right)\frac{\left[(n_i+1)\Upsilon_{n_i+1}^1(\phi_i) - n_i\Upsilon_{n_i}^1(\phi_i) + \Upsilon_{n_i+1}^2(\phi_i) - \Upsilon_{n_i}^2(\phi_i)\right]}{\eta^x\psi^x w_t}\right)^{\frac{1}{\psi^x-1}} n_i^{\frac{\sigma}{\psi^x-1}-1}.$$

Given that profits and the success probability increase in firm-size, it follows that a sufficiently low σ is needed to guarantee that expected firm-growth does not rise above zero when firm-size increases, to offset the decline of the creative destruction rate.

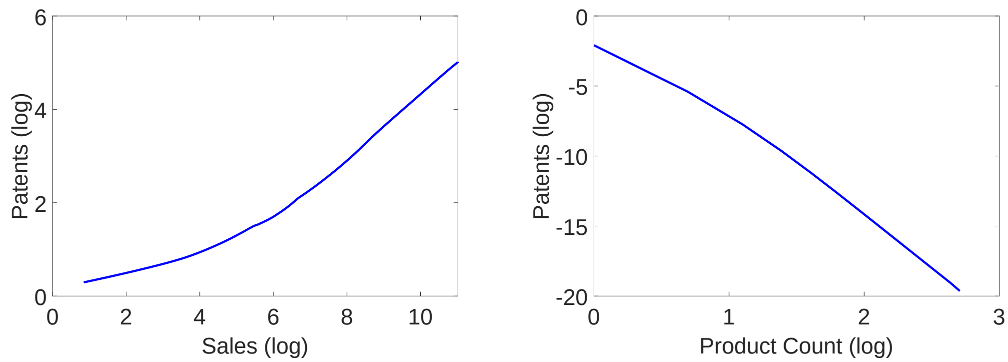
I.2. Quantification

Because firm-level intangibles change the incentive to reduce marginal costs and the relationship between firm-size and firm-growth, I perform a new structural estimation for the initial steady state. As in Section V.A and V.B, I leave the percentage reduction in intangible costs and the fraction of entrants that receive the lower costs is the same as in the main calibration. This facilitates a direct comparison of the effect of introducing a high-intangible group of firms in the models with product- and firm-level intangibles. The moments and the algorithm in the structural estimation are also unchanged.

The parameter estimates are summarized in Table A21. The calibration has changed in two main ways from the main model in Table 5. First, there is a substantial increase in the initial (homogeneous) intangible costs ϕ, which offsets the additional incentive to reduce marginal costs because intangibles now apply across goods that firms produce. Second, the parameter σ, which governs the relationship between firm-size and innovation costs, is now large and negative. At -4.50 in the U.S. calibration, the parameter yields that the same number of researchers in a firm that produces $n_i = 2$ goods will on average create 82% fewer patents than the same number of researchers in a firm that produces $n_i = 1$ goods.[17] This contrasts with other Klette and Kortum (2004) models, which assume that the same num-

[17] This follows from the fact that the expected number of patents created by $\overline{rd}$ researchers according to the innovation-cost function is $\overline{x}_{n_i} = (\eta^x)^{-1/\psi^x} n^{\sigma/\theta}(\overline{rd})^{1/\psi^x}$, where $\psi^x = 2$ and $\sigma = -4.5$.

Figure A13. Relationship between Firm Size and Innovation: Data versus Model



(a) Data (b) Model

Notes: Left-hand figure plots the relationship between log of patents expenditures (vertical axis) and log sales from a lowess regression. Data is from the Compustat sample from the manuscript. Right-hand figure plots log optimal patents ($x_{n_i}(\phi_i)$) in the model, against the log number of products that firms produce.

ber of researchers produce more patents in larger firms. The model therefore predicts a steep decline in innovation productivity with size.

The negative effect of size on research productivity creates strong counterfactual predictions on the relationship between size and innovative activity. The model predicts, for instance, that the flow rate of patents to incumbents, $x_{n_i}(\phi_i)$, falls rapidly with size. It is straightforward to contrast this with empirical evidence from U.S. patents.[18] I obtain patent data from Stoffman et al. (2019), who extend the dataset of Kogan et al. (2017) until 2015, and merge the patents to the Compustat sample using CRSP identifiers. Figure A13a plots this relationship with (log) sales on the horizontal axis and the (log) number of patents that firms produce, both winsorized at 1% sales. The solid-blue line presents estimates from a lowess regression between both, which shows a strong and near-linear positive relationship for all values of sales. An OLS regression yields a coefficient of 0.46 with a standard error of 0.01. Figure A13b plots the model's counterpart in the form of the relationship between the log-optimal innovation rate $x_{n_i}(\phi_i)$ and the log-number of goods that the firm produces.[19] The figure confirms that, with the negative σ required to have a non-degenerate firm-size distribution, the model features a clear counterfactual prediction for the relationship between innovation and size.

The model's ability to match its targeted moments is displayed in Table A22. The table shows that the model with firm-level intangibles is well able to predict firms' average fixed costs, the steady-state growth rate of total factor productivity along the balanced growth path, as well as the relationship between firm-growth and firm-size. Compared to the main model, the model with firm-level intangibles struggles to match the empirical entry rate and the average ratio of research and development to sales in both the U.S. and the French structural estimation. Overall, the model's ability to match empirical moments is slightly worse, which is likely due to the interrelationship between parameters: because profitability and creative destruction rates now strongly depend on firm size, the parameters governing innovation costs, firm growth, and aggregate growth are now closely connected.

[18] At the time of writing, there exists no publicly available link of French patents to firms in the FARE-FICUS dataset.
[19] Sales is the appropriate measure of size, as it is proportional to the number of products that firms sell in the model.

I.3. Rise in Intangibles

This section presents the effect of a rise in intangibles, and shows that the mechanisms and effects are approximately in line with the product-level intangibles model. I start with the experiment in the main text where a fraction of entrants are awarded a lower intangibles cost parameter ϕ_i. In line with the main calibration, low-cost firms have a 33% discount on their intangible costs and comprise 12% of potential entrants in the U.S. calibration. In the French calibration, low-cost firms have a 28% discount and form 6% of entrants.

The effect of the introduction of high-intangible firms is presented in Table A23. Results from the original model are presented under columns headed ‘Main’, the new results are presented under ‘Firm-Intan’. The table shows that the model with firm-level intangibles predicts the same qualitative effects of the rise of a group of high-intangible firms. The model predicts a decline in productivity growth despite an increase in aggregate research and development, a fall in entry and reallocation rates, and a rise in average markups. As expected when significantly altering the structure of production in the model, the quantitative results do differ from the main model. For the same introduction of high-intangible firms, the new model predicts overall increases in fixed costs and intangible shares well above the actual increases in the data. The additional increase is driven by a denominator effect: while the effect of a reduction in ϕ_i on s_i is not amplified in the new model, the same increase in fixed costs now reduces marginal costs across a greater number of products, reducing overall variable costs and raising the fixed-to-total cost ratio further. An illustration is provided in Figure A14 for the U.S. calibration.[20] The figure plots the relationship between size and either profits, marginal costs, creative destruction in the original (solid-blue) and final steady state (red-circled and yellow-dashed). Sub-figure (b) plots the relationship between n_i and s_i. It shows that the low-ϕ_i firms indeed choose lower marginal costs, but that the effect does not depend strongly on the firms’ size. Table A23 also shows that the new model features a greater decline in productivity growth than the initial model. This is because the R&D investments by low-ϕ_i now have an additional long-term effect: the investments cause a rise in firm concentration and average firm-size that further reduces the ability of other firms to innovate on their products. This lowers the rate of creative destruction for firms of all types, as shown in sub-figure (c). The new model predicts a smaller absolute decline in entry: the rate falls by 1.6 percentage points instead of 5.8 percentage points, but this is largely driven by the lower initial entry rate.[21]

The results above are conditional on a sufficiently small fraction of firms receiving the lower ϕ_i. As in the main model, a reduction in ϕ_i across all firms increases the profitability of all firms (Figure A14a) and would therefore raise the incentive for all firms to invest in R&D. I conclude that the model with firm-level intangibles features broadly the same mechanisms as the main model.

Table A22: Comparison of Theory and Data for Targeted Moments

			United States		France	
Parameter	**Moment**	**Weight Ω**	**Model**	**Target**	**Model**	**Target**
$\overline{\lambda}$	Long-term growth rate of productivity	2	2.1%	1.3%	1.3%	1.3%
ϕ	Fixed costs as a fraction of total costs	2	13.0%	12.9%	9.2%	9.5%
σ	Relation between firm growth and size	1	-.036	-.035	-.037	-.035
η^e	Entry rate (fraction of firms age 1 or less)	1	10.7%	13.8%	5.9%	10%
η^x	Ratio of research and development to sales	1	5.4%	2.5%	4.0%	3.2%

[20] The plot for the French calibration is qualitatively similar and available upon request.

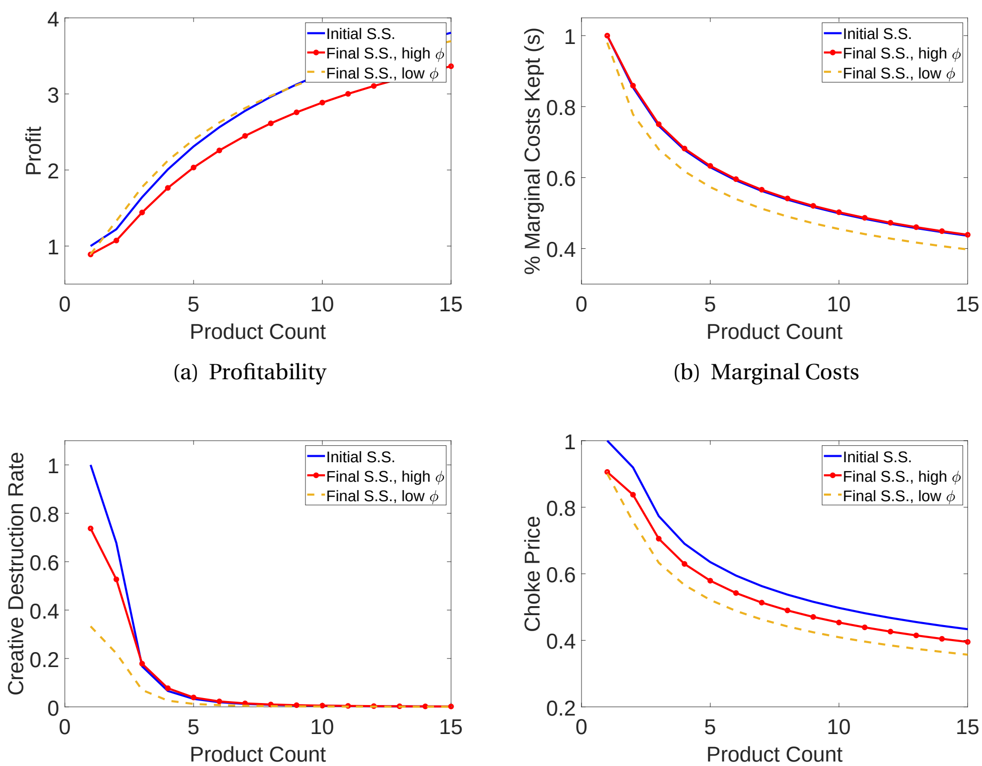
[21] The initial entry rate is 13.7% in the main model, so that the fall in entry yields a 42% decline. The new model has an initial entry rate of 6%, so that the decline remains economically significant at 27%.

Table A23: Comparison of Steady States, Firm-Level Intangibles

	United States			France		
	Δ Model (Main)	**Δ Model** (Firm-Intan)	**Δ Data**	**Δ Model** (Main)	**Δ Model** (Firm-Intan)	**Δ Data**
Cost Structure						
Average Fixed-Cost Share	10.6 pp	11.9 pp	10.6 pp	4.5 pp	10.4 pp	4.5 pp
Slowdown of Productivity Growth						
Productivity Growth Rate	-0.3 pp	-0.8 pp	-0.9 pp	-0.1 pp	-0.4 pp	-1.3 pp
Aggregate R&D over Value Added	34.8%	75.9%	64.5%	22.1%	37.5%	5.6%
Decline of Business Dynamism						
Entry rate	-4.5 pp	-2.7pp	-5.8 pp	-1.0 pp	-0.5 pp	-3.8 pp
Reallocation Rate	-35.9%	-54.4%	-23%	-17.0%	-39.1%	-23.0%
Rise of Market Power						
Average Markup	14.7pt	12.9 pt	29.7 pt	6.4 pt	8.9 pt	11 pt

Notes: Data columns present the empirical moments, while Model - Main columns present the theoretical moments from the model in the main analysis. Firm-Intan. columns present moments for the model with firm-level intangibles. The change in productivity growth is the difference between growth from 1969-1994 (France) or 1969-1979 (U.S.) to growth post 2005. Other French moments equal the difference between values in 1994 and in 2016. Other U.S. moments are the difference between 1980 and 2016.

Figure A14. Relationship between Size and Relevant Variables (U.S. Calibration)



(a) Profitability

(b) Marginal Costs

(c) Creative Destruction

(d) Choke Price

Notes: Plots depict the relationship between n_i (horizontal axis) and the respective variable. All plots are standardized with the value of a single-product firm in the initial steady state. Solid-blue lines plots are for the initial steady state where all firms draw the high ϕ_i. Dashed-yellow (circled-red) lines are for the low-ϕ_i (high-ϕ_i) firms in the final steady state.

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