A political-economic analysis of free-trade agreements: Comment

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Abstract: In his paper in the American Economic Review, Levy (1997) develops a political economy model of free-trade agreements (FTAs). He emphasizes that the homotheticity restriction of the production function assumed for the differentiated product is crucial for his model. This comment shows that this homotheticity assumption is unnecessary and actually problematic. It is problematic in the sense that the model ends up not having a "well-defined" equilibrium. I fix this problem and rework the model using a different production function with fixed cost. This comment also points out that the necessity of the homotheticity restriction on the production function of differentiated goods is a common misunderstanding in trade literature. (*JEL*: F15)

Introduction

Philip Levy (1997) develops a median voter theory of free-trade agreements (FTAs) and demonstrates that bilateral FTAs can undermine political support for further multilateral trade liberalization. This influential paper has been widely cited in the trade literature. However, there is a problem arising from the homotheticity of the production function assumed for the differentiated product in the model. I fix the problem and rework the model using a different production function.

The Problem in Levy (1997)

In his model, Levy assumes that countries differ only in factor endowments (capital, *K* and labor, *L*) and in the distribution of factor ownership. Each agent *i* owns one unit of labor and k_i unit of capital. Hence the total capital is $K = \sum_{i=1}^{L} k_i$, and the income of agent *i* is $\mathrm{I_i} = rk_i + w,\ (\mathrm{i} = 1, 2, \ldots \mathrm{L})$, where r is interest rate and w is wage rate.

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There are two sectors of production. The homogeneous product *Y* (numeraire good) is produced under constant returns to scale with a production function defined as $y = \gamma_Y K_Y^{\mu} L_Y^{1-\mu}$. The differentiated products *X* are produced under increasing returns to scale (IRS). For each variety of *X*, Levy assumes a homothetic production function $x = \gamma_X K_X^{\xi\eta} L_X^{\xi(1-\eta)}$, where $\xi > 1$ is the increasing returns to scale (IRS) parameter. This product function, however, is incorrect as it leads to indeterminacy of the optimal production of *X*.

Given this production function, we can obtain the cost function of the differentiated goods by solving $\underset{L_X, K_X}{Min}\, C(x) = wL_X + rK_X$, s.t. $\text{x} = \gamma_X K_X^{\xi\eta} L_X^{\xi(1-\eta)}$

The resulting separable cost function is:

$$C_X(w,r) = f(w,r)x^{\frac{1}{\xi}}, \text{ where } f(w,r) = \gamma_X^{-1/\xi} w^{1-\eta} r^{\eta} \left(\frac{\eta}{1-\eta}\right)^{-\eta} \left(\frac{1}{1-\eta}\right)$$

Hence the average cost and marginal cost can be written as:

$$AC(x) = f(w,r)x^{\frac{1}{\xi}-1} \text{ and } MC(x) = f(w,r)\frac{1}{\xi}x^{\frac{1}{\xi}-1}$$

The profit maximization condition (*MC*=*MR*) and the free-entry condition (*p*=*AC*) are $\beta p = f(w,r)\frac{1}{\xi}x^{\frac{1}{\xi}-1}$ and $p = f(w,r)x^{\frac{1}{\xi}-1}$ respectively.

The above two conditions are different only by some parameters and solving them simultaneously yields no solution for x^*. Graphically we can show that the two curves parallel to each other and never cross to give a solution. It is likely that Levy makes a mistake on the free-entry condition. If he mistakenly writes the free-entry condition as $p = f(w,r)x^{\frac{1}{\xi}}$, he would end up with a solution $x^* = \frac{1}{\xi\beta} = \frac{\sigma}{\xi(\sigma-1)}$, as shown by Levy (1997, p. 515).

The Corrected Political Economy Model of FTAs

Assuming a fixed cost is the tradition of the monopolistic competition trade literature. To correct the problem in Levy's model, I assume a production function for the differentiated good as $x = \gamma_X K_X^{\eta} L_X^{1-\eta} - a$, where a is the fixed cost of production measured in the unit of *X*. The cost functions can be easily derived from the production functions of X and Y as $C_Y(w,r) = c_Y(w,r)y$ and $C_X(w,r) = c_X(w,r)(x+a)$, where $c_Y(w,r)$ is the unit cost of the goods Y and $c_X(w,r)$ is the marginal cost function of *X*. Both cost functions are separable.

On the consumption side, agents are assumed to have identical utility functions as $U = U_X^{\alpha} y^{1-\alpha}$, where U_X is the sub-utility function for consumptions in *X*, with a Dixit-Spence-Stiglitz type CES functional form.

$$U_X = \left(\sum_{j=1}^{n} D_j^{\beta}\right)^{\frac{1}{\beta}}, \ \beta = (1 - \frac{1}{\sigma}), \ \sigma > 1$$

where D_j is the consumption of variety x_j by an agent; n is the number of varieties; and σ is the elasticity of substitution between varieties. Following a two-stage budgeting process, agent *i*'s optimal consumption of Y and x_j are $y = I_i(1-\alpha)$ and $D_j = \frac{\alpha I_i}{np}$, where p is the relative price of *X* in terms of *Y*. Substituting these optimal consumptions into the utility function for agent i yields $U_i = I_i(1-\alpha)^{1-\alpha}\alpha^{\alpha}n^{\alpha/(\sigma-1)}p^{-\alpha}$. Therefore, agent i's utility under FTA relative to autarky can be written as:

$$\frac{U_i^{FTA}}{U_i^{AUT}} = \underbrace{\left(\frac{I_i^{FTA}}{I_i^{AUT}}\right)\left(\frac{p^{FTA}}{p^{AUT}}\right)^{-\alpha}}_{CAE} * \underbrace{\left(\frac{n^{FTA}}{n^{AUT}}\right)^{\alpha/(\sigma-1)}}_{VE}$$

Levy calls the first two terms on the right hand side "comparative advantage effect (*CAE*)" and the last term "variety effect (*VE*)". The magnitude of the relative utility determines the desirability of an FTA for the agent. The reduced forms of *CAE* and *VE* can be solved from the following general equilibrium system.

(i). *Production side:*

a). Free entry condition for *Y* and *X*: [i.e., $p_Y = 1 = AC(y)$; $p_X = p = AC(x)$]

(1). $$1 = c_Y(w,r) = \gamma_Y^{-1}w^{1-\mu}r^{\mu}\left(\frac{\mu}{1-\mu}\right)^{-\mu}\left(\frac{1}{1-\mu}\right)$$

(2). $$p = \frac{ac_X(w,r)}{x} + c_X(w,r), \text{ where } c_X(w,r) = \gamma_X^{-1}w^{1-\eta}r^{\eta}\left(\frac{\eta}{1-\eta}\right)^{-\eta}\left(\frac{1}{1-\eta}\right)$$

b). Profit maximization condition for *X*: [i.e., *MR*=*MC*]

(3). $$\beta p = c_X(w,r)$$

(ii). *Factor market: full employment conditions*

(4). $$y\frac{\partial c_Y}{\partial w} + nx\frac{\partial c_X}{\partial w} = L$$

(5). $$y\frac{\partial c_Y}{\partial r} + nx\frac{\partial c_X}{\partial r} = K$$

(iii). *Demand side: utility maximization conditions in the first stage budgeting*

(6). $$pnx = \alpha(wL + rK)$$

(7). $y = (1-\alpha)(wL + rK)$

The free-entry condition (2) and profit maximization condition (3) uniquely determine the optimal production for each variety of *X*: $x^* = a(\sigma - 1)$, which is different from Levy's result as shown on page 515 in Levy (1997).

In the following, I show only the major steps in solving the model:

(8). $$\left(\frac{w}{r}\right) = bk,\ \text{where b} = \frac{1-[\mu + \alpha\beta(\eta - \mu)]}{[\mu + \alpha\beta(\eta - \mu)]}$$

which says that the ratio of returns to labor and capital depends only on the overall capital-labor ratio and some parameters.

(9). $$n = f(.)K^{\eta}L^{1-\eta},\ \text{where } f(.) = \frac{\alpha}{a\sigma}\gamma_X\left(\frac{\eta}{1-\eta}\right)^{\eta}(1-\eta)[b^{\eta} + b^{\eta-1}]$$

which says that the number of varieties is a function of total *K* and *L*, the fixed cost and some parameters. Hence the variety effect can be shown as:

(10). $$VE = \left(\frac{n^{FTA}}{n^{AUT}}\right)^{\frac{\alpha}{\sigma-1}} = [(1+\lambda_K)^{\eta}(1+\lambda_L)^{1-\eta}]^{\frac{\alpha}{\sigma-1}}$$

where $\lambda_K = \dfrac{K^{FTA} - K^{AUT}}{K^{AUT}}$; $\lambda_L = \dfrac{L^{FTA} - L^{AUT}}{L^{AUT}}$ are the percentage increases in *K* and *L* when a country moves from autarky to an integrated economy resulting from an FTA. Equation (10) above is same as equation (12) on page 515 in Levy (1997).

(11). $$\frac{I_i^{FTA}}{I_i^{AUT}} = \left(\frac{w^{FTA}}{w^{AUT}}\right)\left(\frac{b + \rho_i/\varphi}{b + \rho_i}\right)$$

where $\rho_i = \frac{k_i}{k^{AUT}} = \frac{K_i / L_i}{K^{AUT} / L^{AUT}}$ and $\varphi = \frac{k^{FTA}}{k^{AUT}} = \frac{K^{FTA} / L^{FTA}}{K^{AUT} / L^{AUT}}$. $\rho_i > 1$ ($\rho_i < 1$) if agent i is relatively capital-rich (capital-poor) under autarky. $\varphi > 1$ ($\varphi < 1$) if partner country is more capital-abundant (labor-abundant).

(12). $$\frac{p^{FTA}}{p^{AUT}} = \left(\frac{w^{FTA}}{w^{AUT}} \right) \varphi^{-\eta}$$

(13). $$\frac{w^{FTA}}{w^{AUT}} = \varphi^{\mu}$$

Combining (11)-(13) yields the comparative advantage effect as:

(14). $$CAE = \left(\frac{I_i^{FTA}}{I_i^{AUT}} \right) \left(\frac{p^{FTA}}{p^{AUT}} \right)^{-\alpha} = \left(\frac{b + \rho_i / \varphi}{b + \rho_i} \right) \varphi^{[\alpha\eta + (1-\alpha)\mu]}$$

where $b = \frac{1 - [\mu + \alpha\beta(\eta - \mu)]}{[\mu + \alpha\beta(\eta - \mu)]}$ is a function of the parameters. Equation (14) above is similar to equation (15) on page 516 in Levy [1997]. The parameters b and θ, however, are different.

Conclusions

It is fortunate that the mistake Levy makes on the free-entry condition helps him go through the model. Otherwise, with the homothetic production function, the model would not have a "well-defined" equilibrium. This comment serves to correct the mistake and complete this widely cited political economy model of FTAs. It also shows that homotheticity assumption of the Cobb-Douglas production function for differentiated goods is unnecessary, and actually, problematic.

This correction points out a common misunderstanding on the monopolistic competition models in trade literature. As emphasized in footnote 13 on page 514 in Levy (1997), "*The results do not depend on the Cobb-Douglas form of the production functions. They do depend, however, on the assumption of homotheticity in production*." Levy (1997) is not alone in this regard. For example, even in the classic book by Dixit and Norman (1980), they note on page 285, "*This result [equation (55)] depends crucially on hometheticity in production*." Although this conclusion will hold for certain forms of homothetic production functions, as shown by Dixit and Norman (1980, p.281-287), it can not carry over to the Cobb-Douglas production. Given the popularity of Levy's paper and the importance of modeling differentiated goods under monopolistic competition, I believe that the correction of this problem is nontrivial.

Reference

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