Online Appendix: A Model of Safe Asset Determination

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This note contains three Appendices: ***Appendix A*** *is the Main Appendix to the paper providing proofs of the main results,* ***Appendix B*** *provides additional results pertaining to the uniqueness of the proposed equilibria, and* ***Appendix C*** *provides proofs in the common bond scenario to the simultaneous three assets global game considered as the robustness check for the sequential game presented in the paper.*

Appendix A: Main Appendix

A1. Joint-safety Equilibrium with non-monotone strategies and zero recovery

We now construct a joint safety equilibrium with non-monotone strategies and joint safety on the endogenously determined interval $[\delta_L, \delta_H]$. Given this equilibrium, we will compute the minimum value of $z = \underline{z}$ for which this equilibrium exists. The possibility of joint safety means that our equilibrium construction using threshold strategies is no longer possible. In a region where both countries are known to be safe (recall we consider the limit where $\sigma \to 0$), investors must be indifferent between the two countries, thus equalizing bond returns. Outside the joint safety interval, i.e., $\tilde{\delta} \in [-\bar{\delta}, \delta_L) \cup (\delta_H, \bar{\delta}]$, we are back to the case where the signal is so strong that only one country is safe.

We conjecture the following non-monotone strategy whereby investment in country 1 and in country 2 alternates on discrete intervals of length $k\sigma$ and $(2-k)\sigma$, with $k \in (0,2)$. The investor j's strategy given his private signal δ_j is $\phi(\delta_j) \in \{0,1\}$:

$$\phi(\delta_j) = \begin{cases} 0, & \delta_j < \delta_L \\ 1, & \delta_j \in [\delta_L, \delta_L + k\sigma] \cup [\delta_L + 2\sigma, \delta_L + (2+k)\sigma] \cup [\delta_L + 4\sigma, \delta_L + (4+k)\sigma] \cup ... \\ 0, & \delta_j \in [\delta_L + k\sigma, \delta_L + 2\sigma] \cup [\delta_L + (2+k)\sigma, \delta_L + 4\sigma] \cup [\delta_L + (4+k)\sigma, \delta_L + 6\sigma] \cup ... \\ 1, & \delta_j > \delta_H \end{cases} \tag{A.1}$$

As we will show shortly, the non-monotone oscillation occurs only when both countries are safe, where the equilibrium requires proportional investment in each safe country to equalize returns across two safe bonds. Clearly, k determines the fraction of agents in investing in country 1 when oscillation occurs, to which we turn next.

Fraction of agents in investing in country 1

Consider a region where all investors know that both countries are safe. In this case, the total investment in country 1 and 2 has to be $\frac{1+f}{1+s}$ and $\frac{s(1+f)}{1+s}$, respectively, to equalize returns. Take an agent with signal δ; introduce the function $\rho(\delta)$, which is the expected proportion of agents investing in country 1 given (own) signal δ. Then, given the assumed strategy for all agents and given that we are in the region where both countries are safe,

$$\rho(\delta) = \int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x} \frac{\phi(y)}{2\sigma} dy = \frac{k\sigma}{2\sigma}.$$

We choose k so that $\rho(\delta) = \frac{1}{1+s} \iff k = \frac{2}{1+s}$. This is because in equilibrium the proportion investing in country 1 must be constant and equal to $\frac{1}{1+s}$ to equalize returns.

Recall that x denotes the fraction of agents with signal realizations above the agent's private signal δ; and x follows a uniform distribution on $[0, 1]$. For any value of δ and x,

$$\rho(\delta, x) = \int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x} \frac{\phi(y)}{2\sigma} dy = \begin{cases} 0, & \delta + 2\sigma x < \delta_L \\ \frac{\delta+2\sigma x-\delta_L}{2\sigma}, & \delta + 2\sigma x \in (\delta_L, \delta_L + k\sigma) \\ \frac{1}{1+s}, & \delta_H - (2-k)\sigma > \delta > \delta_L + k\sigma \end{cases} \tag{A.2}$$

When we evaluate δ at the marginal agent with signal $\delta = \delta_L$, we have

$$\rho(\delta_L, x) = \begin{cases} 0, & x = 0 \\ x, & x \in \left(0, \frac{1}{1+s}\right) \\ \frac{1}{1+s}, & x > \frac{1}{1+s} \end{cases} \tag{A.3}$$

where we observe that $\rho(\delta_L, x)$ is less than or equal to $\frac{1}{1+s}$.

Lower boundary δ_L

In the completely safe region discussed above (for δ exceeding δ_L sufficiently), investors were indifferent between both strategies. This is not the case for agent with signals around the threshold signal δ_L: as the agent knows investors with signal below are always investing in country 2, country 1 is a perceived default risk. We now calculate the return of investing in either country, from the perspective of the boundary agent δ_L.

For the boundary agent δ_L, the return from investing only in country 2 (i.e. $\phi = 0$) is given by

$$\Pi_2(\delta_L) = \int_0^1 \frac{s}{(1+f)(1-\rho(\delta_L, x))} dx \tag{A.4}$$

where we integrate over all x as country 2 is safe regardless of x. We will show consistency of this assumption with the derived equilibrium later. Thus, plugging in, we have

$$\Pi_2(\delta_L) = \frac{s}{1+f}\left[\int_0^{\frac{1}{1+s}} \frac{1}{1-x} dx + \int_{\frac{1}{1+s}}^1 \frac{1}{\frac{s}{1+s}} dx\right] = \frac{s}{1+f}\left[\ln\frac{1+s}{s} + 1\right] < \frac{1+s}{1+f}. \tag{A.5}$$

where we used $s \ln \frac{1+s}{s} < 1$. Here, we see that payoff to investing in country 2 is lower than the expected payoff that would have realized if both countries were safe. This reflects the strategic substitution effect: because more people (in expectation) invest in the safe country 2, the return in country 2 is lower.

Now we turn to country 1. Since country 1 has default risk, we need to calculate the threshold $x = x_{min}$ so that country 1 becomes safe if there are $x > x_{min}$ measure of agents receiving better signals. To derive x_{min}, we first solve for $\rho_1^{min}(\delta)$, which is the minimum proportion of agents investing in country 1 that are needed to make country 1 safe given fundamental δ. We have

$$\theta_1(\delta) + (1+f)\rho_1^{min}(\delta) = 1 \iff \rho_1^{min}(\delta) = \frac{1-\theta_1(\delta)}{1+f}$$

Define x_{min} as the solution to $\rho(\delta_L, x) = \rho_1^{min}(\delta_L)$. Given equation (A.3), we have that,

$$x_{min} = \frac{1-\theta_1(\delta_L)}{1+f}. \tag{A.6}$$

The expected return of investing in country 1 given one's own signal δ_L and the conjectured strategies $\phi(\cdot)$ of everyone else is given by,

$$\begin{aligned} \Pi_1(\delta_L) &= \int_{x_{min}}^1 \frac{1}{(1+f)\rho(\delta_L, x)} dx = \frac{1}{1+f}\left[\int_{x_{\min}}^{\frac{1}{1+s}} \frac{1}{x} dx + \int_{\frac{1}{1+s}}^1 \frac{1}{1/(1+s)} dx\right] \\ &= \frac{1}{1+f}\left[\ln\frac{1}{1+s} - \ln x_{min} + s\right]. \end{aligned} \tag{A.7}$$

The boundary agent δ_L must be indifferent between investing in either country, i.e., $\Pi_2(\delta_L) = \Pi_1(\delta_L)$.

Plugging in (A.4) and (A.7), we have

$$\frac{s}{1+f}\left[\ln\frac{1+s}{s}+1\right] = \frac{1}{1+f}\left[\ln\frac{1}{1+s}-\ln x_{min}+s\right] \iff x_{min}=\frac{s^s}{(1+s)^{1+s}}. \tag{A.8}$$

We combine our two equations for x_{min}, (A.6) and (A.8), and use $1-\theta_1(\delta_L)=(1-\theta)\exp(-\delta_L)$, to obtain:

$$\frac{s^s}{(1+s)^{1+s}}=\frac{(1-\theta)\exp(-\delta_L)}{1+f}.$$

Recall $z=\ln\frac{1+f}{1-\theta}$; we have

$$\delta_L(z)=-z+(1+s)\ln(1+s)-s\ln s \tag{A.9}$$

Upper boundary δ_H

The derivation is symmetric to the above. We have

$$\rho(\delta,x)=\int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x}\frac{\phi(y)}{2\sigma}dy=\begin{cases}\frac{1}{1+s}, & \delta-2\sigma(1-x)<\delta_H-(2-k)\sigma\\ \frac{\delta+2\sigma x-\delta_H}{2\sigma} & \delta-2\sigma(1-x)\in(\delta_H-(2-k)\sigma,\delta_H)\\ 1, & \delta-2\sigma(1-x)>\delta_H\end{cases} \tag{A.10}$$

so that

$$\rho(\delta_H,x)=\begin{cases}\frac{1}{1+s}, & x<\frac{1}{1+s}\\ x, & x\in\left(\frac{1}{1+s},1\right)\\ 1, & x=1\end{cases} \tag{A.11}$$

which yields

$$\Pi_1(\delta_H)=\int_0^1\frac{1}{\left(1+\hat{f}\right)\rho(\delta_H,x)dy}dx=\frac{1}{1+f}\left[\ln(1+s)+1\right]<\frac{1+s}{1+f},$$

where we integrated over all x as country 1 is always safe in the vicinity of δ_H.

The default condition for country 2 is

$$s\theta_2(\delta_H)+(1+f)\left[1-\rho_2^{max}(\delta_H)\right]=s\iff 1-\rho_2^{max}(\delta_H)=s\frac{1-\theta_2(\delta_H)}{1+f}$$

where $\rho_2^{max}(\delta)$ is the maximum amount of agents investing in country 1 so that country 2 does not default. Assume, but later verify, that at δ_H we have $1-\rho_2^{max}(\delta_H)<\frac{s}{1+s}$, that is, country 2 would survive even if less than $\frac{s}{1+s}$ of investors invest in country 2. Define $x_{max}(\delta_H)$ as the solution to $\rho(\delta_H,x_{max})=\rho_2^{max}(\delta_H)$; (A.11) implies that

$$1-x_{max}(\delta_H)=s\frac{1-\theta_2(\delta_H)}{1+f}. \tag{A.12}$$

As a result, the return to country 2 is,

$$\begin{aligned}\Pi_2(\delta_H) &= \int_0^{x_{max}(\delta_H)}\frac{s}{(1+f)(1-\rho(\delta_H,x))dy}dx=\frac{s}{1+\hat{f}}\left[\int_0^{\frac{1}{1+s}}\frac{1}{1-\frac{1}{1+s}}dx+\int_{\frac{1}{1+s}}^{x_{max}(\delta_H)}\frac{1}{1-x}dx\right]\\ &= \frac{s}{1+f}\left[\frac{1}{s}+\ln\frac{s}{1+s}-\ln(1-x_{max}(\delta_H))\right]\end{aligned}$$

Indifference at the boundary agent δ_H requires $\Pi_1(\delta_H)=\Pi_2(\delta_H)$, which yields $1-x_{max}(\delta_H)=\frac{s}{(1+s)^{\frac{1+s}{s}}}$. Combining this result with (A.12) and $1-\theta_2(\delta_H)=(1-\theta)\exp(\delta_H)$, we solve,

$$\delta_H(z) = z-\frac{1+s}{s}\ln(1+s) \tag{A.13}$$

Verifying the equilibrium

We now verify the interior agents $\delta \in (\delta_L, \delta_H)$ have the appropriate incentives to play the conjectured strategy, and that our assumptions of country 1 (2) is always safe at δ_H (δ_L) are correct. As an investor with signal $\delta = \delta_L$ is indifferent, it is easy to show that agents with $\delta < \delta_L$ find it optimal to invest in country 2. Consider an investor with signal $\delta = \delta_L + k\sigma$ (i.e. let us consider the investors depicted by the black dot in Figure 3). Regardless of his relative position (as measured by x) in the signal distribution, this agent knows that a proportion $\frac{1}{1+s}$ of investors invest in country 1, thus making it safe for sure. Further, he knows that a proportion $\frac{s}{1+s}$ of investors invest in country 2, also making it safe. Therefore, this agent knows that (i) both countries are completely safe and that (ii) investment flows give arbitrage free prices. He is thus indifferent, and so is every investor with $\delta_L + k\sigma < \delta < \delta_H - (2-k)\,\sigma$.

Next, we consider an investor with $\delta \in (\delta_L, \delta_L + k\sigma)$. We know that country 2 will always survive, and thus we have

$$\Pi_2(\delta) = \int_0^1 \frac{s}{(1+f)\int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x} \frac{1-\phi(y)}{2\sigma}dy} dx.$$

Note that for any x with $x \geq -\frac{\delta-\delta_L-k\sigma}{2\sigma}$ we are in the oscillating region; for x below we are in the increasing part. Let $\varepsilon \equiv \frac{\delta-\delta_L}{2\sigma} \in \left(0, \frac{1}{1+s}\right)$ so that so that $\delta = \delta_L + 2\sigma\varepsilon$. Thus, we have

$$1-\rho(\delta,x) = \int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x} \frac{1-\phi(y)}{2\sigma}dy \quad = \quad \begin{cases} 1-\varepsilon-x, & x \in \left(0, \frac{1}{1+s}-\varepsilon\right), \\ \frac{s}{1+s}, & x \in \left(\frac{1}{1+s}-\varepsilon, 1\right). \end{cases} \tag{A.14}$$

Then, we have

$$\Pi_2(\delta) \quad = \quad \frac{s}{1+f}\left[\int_0^{\frac{1}{1+s}-\varepsilon} \frac{1}{1-\varepsilon-x}dx + \int_{\frac{1}{1+s}-\varepsilon}^{1} \frac{1}{\frac{s}{1+s}}dx\right] = \Pi_2(\delta_L) + \frac{s\left(\ln(1-\varepsilon) + \frac{1+s}{s}\varepsilon\right)}{1+f}$$

For investment in country 1, we know that, since $\delta > \delta_L$, we have $\rho_1^{min}(\delta) < \rho_1^{min}(\delta_L)$. First, note that

$$\rho(\delta,x) = \int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x} \frac{\phi(y)}{2\sigma}dy = \begin{cases} \varepsilon + x, & x \in \left(0, \frac{1}{1+s}-\varepsilon\right) \\ \frac{1}{1+s}, & x \in \left(\frac{1}{1+s}-\varepsilon, 1\right) \end{cases}$$

Let $x_{min}(\delta)$ be the measure of investors with higher signals than δ so that country 1 is safe. Since $\rho_1^{min}(\delta) = \frac{1-\theta_1(\delta)}{1+f}$, $x_{min}(\delta)$ is the lowest $x \in [0,1]$ such that

$$\rho(\delta,x) = \varepsilon + x \geq \rho_1^{min}(\delta).$$

Thus, we have

$$x_{min}(\delta) = x_{min}(\delta_L + 2\sigma\varepsilon) \quad = \quad \max\left\{\frac{1-\theta_1(\delta_L+2\sigma\varepsilon)}{1+f} - \varepsilon, 0\right\}. \tag{A.15}$$

The expected investment return from country 1 is

$$\begin{aligned} \Pi_1(\delta) \quad &= \quad \int_{x:\rho(\delta,x)\geq\rho_1^{min}(\delta)} \frac{1}{(1+f)\int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x}\frac{\phi(y)}{2\sigma}dy}dx \\ &= \quad \Pi_1(\delta_L) + \frac{1}{1+f}\left\{\ln x_{min}(\delta_L) - \ln\left[\varepsilon + x_{min}(\delta_L+2\sigma\varepsilon)\right] + (1+s)\,\varepsilon\right\} \end{aligned}$$

Thus, to show that $\Pi_1(\delta_L+2\sigma\varepsilon) \geq \Pi_2(\delta_L+2\sigma\varepsilon)$, we need to show that the following inequality holds for $\varepsilon \in \left(0, \frac{1}{1+s}\right)$:

$$g_L(\varepsilon) \equiv (1+f)(\Pi_1 - \Pi_2) = \ln x_{min}(\delta_L) - \ln\left[\varepsilon + x_{min}(\delta_L+2\sigma\varepsilon)\right] - s\ln(1-\varepsilon) \geq 0. \tag{A.16}$$

First, by using $\ln x_{min}(\delta_L) = s\ln s - (1+s)\ln(1+s)$ and $x_{min}\left(\delta_L + 2\sigma\frac{1}{1+s}\right) = 0$, we know the above inequality holds with equality at both end points $\varepsilon = 0$ and $\varepsilon = \frac{1}{1+s}$, i.e., $g_L(0) = g_L\left(\frac{1}{1+s}\right) = 0$.

Second, it is easy to show that there exists a unique ε^* such that $\frac{1-\theta_1(\delta_L+2\sigma\varepsilon^*)}{1+f}=\varepsilon^*$, at which point (A.15) binds at zero. We further note that at $\varepsilon=0$ we have $\frac{1-\theta_1(\delta_L)}{1+f}>0$. Thus, in (A.15) we have $\varepsilon^*>0$ and for $\varepsilon\in(0,\varepsilon^*)$ we have $x_{min}(\delta)=\frac{1-\theta_1(\delta_L+2\sigma\varepsilon)}{1+f}-\varepsilon>0$, and for $\varepsilon\in\left[\varepsilon^*,\frac{1}{1+s}\right]$ we have $x_{min}(\delta)=0$. Plugging in and taking derivative with respect to ε, we have

$$\frac{\partial}{\partial\varepsilon}\ln\left[\varepsilon+x_{min}\left(\delta_L+2\sigma\varepsilon\right)\right]=\begin{cases}\frac{-2\sigma\theta_1'(\delta_L+2\sigma\varepsilon)}{1-\theta_1(\delta_L+2\sigma\varepsilon)} & ,\varepsilon\in(0,\varepsilon^*)\\ \frac{1}{\varepsilon} & ,\varepsilon\in\left[\varepsilon^*,\frac{1}{1+s}\right]\end{cases}$$

Then, for (A.16), we have $g_L(\varepsilon)$ first rises and then drops:

$$g_L'(\varepsilon)=\begin{cases}\frac{2\sigma\theta_1'(\delta_L+2\sigma\varepsilon)}{1-\theta_1(\delta_L+2\sigma\varepsilon)}+\frac{s}{1-\varepsilon}>0 & ,\varepsilon\in(0,\varepsilon^*),\\ -\frac{1}{\varepsilon}+\frac{s}{1-\varepsilon}=\frac{(1+s)\varepsilon-1}{1-\varepsilon}<0 & ,\varepsilon\in\left[\varepsilon^*,\frac{1}{1+s}\right].\end{cases}$$

Combined with $g_L(0)=g_L\left(\frac{1}{1+s}\right)=0$ we know that $g_L(\varepsilon)>0,\forall\varepsilon\in\left(0,\frac{1}{1+s}\right)$, i.e., Thus, on $\varepsilon\in\left(0,\frac{1}{1+s}\right)$ the investors strictly want to invest in country 1.

We now consider the investors with $\delta\in(\delta_H-(2-k)\sigma,\delta_H)$. We know that country 1 will always survive, and thus we have

$$\Pi_1(\delta)=\int_0^1\frac{1}{(1+f)\int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x}\frac{\phi(y)}{2\sigma}dy}dx.$$

Let $\varepsilon\equiv\frac{\delta_H-\delta}{2\sigma}\in\left(0,\frac{s}{1+s}\right)$ so that so that $\delta=\delta_H-2\sigma\varepsilon$. Thus, we have

$$\rho(\delta,x)=\int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x}\frac{\phi(y)}{2\sigma}dy\quad=\quad\begin{cases}\frac{1}{1+s}, & x\in\left(0,\frac{1}{1+s}+\varepsilon\right),\\ x-\varepsilon, & x\in\left(\frac{1}{1+s}+\varepsilon,1\right).\end{cases}\tag{A.17}$$

Plugging in, we have

$$\Pi_1(\delta)=\frac{1}{1+f}\left[\int_0^{\frac{1}{1+s}+\varepsilon}\frac{1}{\frac{1}{1+s}}dx+\int_{\frac{1}{1+s}+\varepsilon}^1\frac{1}{x-\varepsilon}dx\right]=\frac{1}{1+f}\left[1+(1+s)\varepsilon+\ln(1-\varepsilon)+\ln(1+s)\right].$$

For investment in country 2, we know that, since $\delta<\delta_H$, we have $1-\rho_2^{max}(\delta)<1-\rho_2^{max}(\delta_L)\iff\rho_2^{max}(\delta_L)<\rho_2^{max}(\delta)$. First, note that

$$1-\rho(\delta,x)=\int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x}\frac{1-\phi(y)}{2\sigma}dy=\begin{cases}\frac{s}{1+s}, & x\in\left(0,\frac{1}{1+s}+\varepsilon\right),\\ 1+\varepsilon-x, & x\in\left(\frac{1}{1+s}+\varepsilon,1\right).\end{cases}$$

Let $x_{max}(\delta)$ be the measure of investors with higher signals than δ so that country 2 is safe. Since $1-\rho_2^{max}(\delta)=s\frac{1-\theta_2(\delta)}{1+f}$, $x_{max}(\delta)$ is the highest $x\in[0,1]$ such that

$$1-\rho(\delta,x)=1+\varepsilon-x\le 1-\rho_2^{max}(\delta).$$

Thus, we have

$$x_{max}(\delta)=x_{max}(\delta_H-2\sigma\varepsilon)\quad=\quad\min\left\{1+\varepsilon-s\frac{1-\theta_2(\delta_H-2\sigma\varepsilon)}{1+f},1\right\}.\tag{A.18}$$

The expected investment return from country 2 is

$$\begin{aligned}\Pi_2(\delta)\quad&=\quad\int_{x:\rho(\delta,x)\le\rho_2^{max}(\delta)}\frac{s}{(1+f)\int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x}\frac{1-\phi(y)}{2\sigma}dy}dx\\ &=\quad\frac{s}{1+f}\left[\frac{1+s}{s}\left(\frac{1}{1+s}+\varepsilon\right)-\ln\left[1+\varepsilon-x_{max}(\delta)\right]+\ln\left(\frac{s}{1+s}\right)\right]\end{aligned}$$

Differencing, we have

$$g_H(\varepsilon) = (1+f)\left[\Pi_1(\varepsilon) - \Pi_2(\varepsilon)\right] = \ln(1-\varepsilon) - s\ln s + (1+s)\ln(1+s) + s\ln\left[1+\varepsilon - x_{max}(\delta)\right]$$

with similar properties to $g_L(\varepsilon)$.

Finally, we need to pick σ appropriately so that there exists some natural number $N > 1$ so that $2N\sigma = \delta_H - \delta_L$. For this particular choice of $\sigma = \hat{\sigma}$, the limiting case of zero signal noise can be achieved when we take the sequence of $\sigma_n = \hat{\sigma}/n$ for $n = 1, 2, ...$

Equilibrium properties

First, with joint safety, the probability of survival for country 1 (or the probability of its bonds being the safe asset) is no longer one minus the probability of survival of country 2. Using $\tilde{\delta} \sim \mathbb{U}\left(-\overline{\delta}, \overline{\delta}\right)$, the probability of country 1 survival is

$$\Pr(\text{country 1 safe}) = \frac{\overline{\delta} - \delta_L}{2\overline{\delta}} = \frac{\overline{\delta} + z - (1+s)\ln(1+s) + s\ln s}{2\overline{\delta}}, \tag{A.19}$$

and the probability of country 2 survival is

$$\Pr(\text{country 2 safe}) = \frac{\delta_H + \overline{\delta}}{2\overline{\delta}} = \frac{\overline{\delta} + z - \frac{1+s}{s}\ln(1+s)}{2\overline{\delta}}.$$

As a result, the bonds issued by country 1 are more likely to be the safe assets than that issued by country 2 if the following condition holds:

$$s\ln s - (1+s)\ln(1+s) + \frac{1+s}{s}\ln(1+s) = s\ln s + \left(\frac{1}{s} - s\right)\ln(1+s) > 0. \tag{A.20}$$

This condition always holds: Define $F(s) \equiv s^2\ln s + \left(1-s^2\right)\ln(1+s)$, then $F(s) > 0$ holds for $s \in (0,1)$. It is clear that $F(0) = 0$ while $F(1) = 0$. Simple algebra shows that

$$F'(s) = 2s\ln s - 2s\ln(1+s) + 1, \frac{1}{2}F''(s) = \ln s - \ln(1+s) + 1 - \frac{s}{1+s} = \ln\left(\frac{s}{1+s}\right) + 1 - \frac{s}{1+s}.$$

Let $y = \frac{s}{1+s} \in (0,1)$; then because it is easy to show $\ln y + 1 - y < 0$ (due to concavity of $\ln y$), we know that $F''(s) < 0$. As a result, $F(s)$ is concave but $F(0) = F(1) = 0$. This immediately implies that $F(s) > 0$, which is our desired result. The condition is the same if we focus on sole survivals only instead of sole and joint survival, i.e., the bonds of country j are the only safe asset, the condition is exactly the same.

Country 1 has the highest likelihood of survival when $s \to 0$, which immediately follow from $-(1+s)\ln(1+s) + s\ln s$ is decreasing in s.

Obviously, the above equilibrium construction requires that $\delta_L(z) < \delta_H(z)$. Since $\delta_L(z)$ in (A.9) is decreasing in z while $\delta_H(z)$ in (A.13) is increasing in z, this condition $\delta_L(z) < \delta_H(z)$ holds if $z > \underline{z}$ so that $\delta_L(\underline{z}) = \delta_H(\underline{z})$ which gives $\underline{z}$:

$$-\underline{z} + (1+s)\ln(1+s) - s\ln s = \underline{z} - \frac{1+s}{s}\ln(1+s) \Rightarrow \underline{z} = \frac{1}{2}\left[\left(2+s+\frac{1}{s}\right)\ln(1+s) - s\ln s\right]$$

A2. Extension for a negative β asset

Suppose that θ, which proxies for the aggregate fundamental for both countries, is subject to shocks. For convenience, suppose that $\tilde{\theta}$ is drawn from the following uniform distribution $\tilde{\theta} \sim U\left[\underline{\theta}, \overline{\theta}\right]$, and recall $z\left(\tilde{\theta}\right) = \ln\frac{1+f}{1-\tilde{\theta}}$. Also, suppose that

$$l_i = l\tilde{\theta}, i \in \{1,2\}$$

where $l > 0$ is a positive constant, so that recovery is increasing in the fundamental shock. Using (16), we calculate the threshold $\delta^*(\theta)$ as a function of the realization of $\tilde{\theta} = \theta$, to be

$$\delta^*(\theta) = \frac{\left[(1-l\theta)s - (1-l\theta)\right]z(\theta) - (s+l\theta)\ln(s+l\theta) + (1+sl\theta)\ln(1+l\theta s) + l\theta\ln(l\theta) - sl\theta\ln(l\theta)}{(1-l\theta) + s(1-l\theta)}$$

Note that $\frac{d}{d\theta}\delta^*(\theta) < 0$; that is, a higher θ, by reducing rollover risk, makes country 1 safer.

In this exercise we consider a distribution so that the relative fundamental δ is almost surely, $\delta > \delta^*(\mathbb{E}[\theta])$. This implies that ex-ante country 1 bonds are more likely to be safe. Also, define $\hat{\theta}(\delta)$ so that $\delta^*\left(\hat{\theta}\right) = \delta$ holds; this is the critical value of fundamental $\theta = \hat{\theta}$ so that country 1's bonds lose safety. We choose δ so that $\hat{\theta} > \underline{\theta}$, which implies that with strictly positive probability, country 1 defaults given a sufficiently low fundamental.

We are interested in the β of the bond price of each country with respect to the θ shock, i.e.,

$$\beta_i(\delta) = \frac{Cov\left(p_i\left(\tilde{\theta};\delta\right),\tilde{\theta}\right)}{Var\left(\tilde{\theta}\right)} = \frac{\mathbb{E}\left[p_i\left(\tilde{\theta};\delta\right)\cdot\tilde{\theta}\right] - \mathbb{E}\left[\tilde{\theta}\right]\mathbb{E}\left[p_i\left(\tilde{\theta};\delta\right)\right]}{Var\left(\tilde{\theta}\right)}, \tag{A.21}$$

From equation (18), we know that

$$p_1(\theta;\delta) = \begin{cases} \frac{(1+f)l\theta}{s+l\theta} & \text{if } \theta¡\hat{\theta}(\delta) \text{ so country 1 defaults;} \\ \frac{1+f}{1+l\theta s} & \text{if } \theta \geq \hat{\theta}(\delta) \text{ so country 1 survives;} \end{cases}$$

and

$$p_2(\theta;\delta) = \begin{cases} \frac{1+f}{s+l\theta} & \text{if } \theta¡\hat{\theta}(\delta) \text{ so country 2 survives;} \\ \frac{(1+f)l\theta}{1+l\theta s} & \text{if } \theta \geq \hat{\theta}(\delta) \text{ so country 2 defaults.} \end{cases}$$

Given these pricing functions, it is straightforward to evaluate βs in (A.21). We vary country 1's relative strength δ and plot the βs for both bonds as a function of δ in Figure 4. We only plot the β for country 1's bonds, because $\beta_2 = -\beta_1/s$ in our model.[1]

A3. Single-survivor equilibrium with common bonds

In this appendix, we proof that $\delta^*(\alpha)$ is unique, $\delta^*(\alpha) \leq 0$, exists on $[0,\alpha^*]$, and has $\delta^*(\alpha^*) = 0$.

First, assume $s = 1$. Then, conjecture that $\delta^*(\alpha) = 0$ throughout by a simple symmetry argument. From (26), with $\theta_{def}(\delta^*(\alpha)) = \theta$, we then have

$$\alpha^* = \frac{1+s}{1+f}(1-\theta) = e^{-z}(1+s) \tag{A.22}$$

Next, assume $s < 1$ and $e^z > (1+s)$ so that $\delta^*(0) < 0$. Then, let us conjecture $\delta^*(\alpha) \leq 0$ for $\alpha \in (0,\alpha^*)$.

Setting $\Pi_1(\delta^*) = \Pi_2(\delta^*)$ from (21) after substituting in for $\hat{f}$ from (24), $\delta^*(\alpha)$ is implicitly defined via

$$0 = h(\delta^*,\alpha) \quad = \quad \ln\left[e^z\frac{1-\alpha}{e^{-\delta^*} - \frac{\alpha}{1+s}e^z}\right] - s\ln\left[\frac{e^z}{s}\frac{1-\alpha}{e^{\delta^*} - \frac{\alpha}{1+s}e^z}\right] \tag{A.23}$$

Then, consider $\tilde{\delta} = \delta^*(\alpha)_+$. At this point, country 1 just survives, even though the funding gap (scaled by size) of country 2 is the best among all defaulting countries. Then, for the monotone cutoff strategy to be consistent, we need the default condition

$$\alpha \leq \frac{1+s}{1+f}\left[1-\theta_2(\delta^*)\right] = \frac{1+s}{1+f}(1-\theta)e^{\delta^*} = e^{-z}(1+s)e^{\delta^*}$$

Suppose that the constraint is binding, which defines a loosest $\delta^*(\alpha)$ by

$$\hat{\delta}^*(\alpha) = z + \ln\left(\frac{\alpha}{1+s}\right) \iff e^{\hat{\delta}^*(\alpha)} = \frac{\alpha}{1+s}e^z \tag{A.24}$$

Assume that $\alpha < \alpha^* = \frac{1+s}{1+f}(1-\theta)$. Plugging in $\hat{\delta}^*(\alpha)$, we see that

$$h\left(\hat{\delta}^*(\alpha),\alpha\right) = \ln\left[e^z\frac{1-\alpha}{e^{-z}\frac{1+s}{\alpha} - \frac{\alpha}{1+s}e^z}\right] - s\ln\left[\frac{e^z}{s}\frac{1-\alpha}{e^z\frac{\alpha}{1+s} - \frac{\alpha}{1+s}e^z}\right] < 0 \tag{A.25}$$

[1]This is because cash-in-the-market-pricing implies that $p_1 + sp_2 = 1 + f$.

as the second term explodes, i.e. $\ln[\cdot] = \infty$. Thus, it must be that $0 > \delta^*(\alpha) > \hat{\delta}^*(\alpha)$—the first part by our assumption that $\delta^* < 0$ and the second by the construction. However, we note that $\hat{\delta}^*(\alpha^*) = 0$ so that $\delta^*(\alpha^*) = 0$. This is possible as $(\delta, \alpha) = (0, \alpha^*)$ is a root of h – both sides are exploding at this point. The restriction above also implies that $0 < \delta^*_\alpha(\alpha^*) < \hat{\delta}^*_\alpha(\alpha^*) = \frac{1}{\alpha^*}$ so that $\delta^*(\alpha)$ has a bounded and positive derivative at α^*.

We next show that for a fixed $\alpha \in [0, \alpha^*]$, there exists unique $\delta^*(\alpha)$ that solves $h(\delta^*, \alpha)$. Fix α. Then, consider $h(\delta^*, \alpha)$ as a function of δ^*. Differentiating w.r.t. δ^*, we have

$$\frac{\partial h(\delta^*, \alpha)}{\partial \delta^*} = \frac{e^{-\delta^*}\left(e^{\delta^*} - \frac{\alpha}{1+s}e^z\right) + se^{\delta^*}\left(e^{-\delta^*} - \frac{\alpha}{1+s}e^z\right)}{\left(e^{-\delta^*} - \frac{\alpha}{1+s}e^z\right)\left(e^{\delta^*} - \frac{\alpha}{1+s}e^z\right)}$$

Then, given that we have $\alpha < \alpha^*$ and $\hat{\delta}^*(\alpha) < \delta^* < 0$ by assumption, we have

$$\left(e^{-\delta^*} - \frac{\alpha}{1+s}e^z\right) > \left(e^{-\delta^*} - \frac{\alpha^*}{1+s}e^z\right) = e^{-\delta^*} - 1 > 0$$

by assumption on the sign of δ^*. Next, we have

$$\left(e^{\delta^*} - \frac{\alpha}{1+s}e^z\right) > \left(e^{\hat{\delta}^*(\alpha)} - \frac{\alpha}{1+s}e^z\right) = \frac{\alpha}{1+s}e^z - \frac{\alpha}{1+s}e^z = 0$$

by the assumption on $\delta^* \in \left(\hat{\delta}^*(\alpha), 0\right)$. Thus, we have $\frac{\partial h(\delta^*,\alpha)}{\partial \delta^*} > 0$. Finally, we know that $h\left(\hat{\delta}^*(\alpha), \alpha\right) < 0 < h(0, \alpha)$, so that a unique $\delta^*(\alpha) \in \left(\hat{\delta}^*(\alpha), 0\right)$ exists.

What remains to be shown is that $\delta^*(\alpha)$ does not cross 0 before α^*. Suppose it does. Then, there exists an $\hat{\alpha} > 0$ but $\hat{\alpha} \neq \alpha^*$ such that $\delta^*(\hat{\alpha}) = 0$. Then, we have

$$h(0, \hat{\alpha}) = \ln\left[e^z \frac{1-\hat{\alpha}}{1 - \frac{\hat{\alpha}}{1+s}e^z}\right] - s\ln\left[\frac{e^z}{s}\frac{1-\hat{\alpha}}{1 - \frac{\hat{\alpha}}{1+s}e^z}\right] = (1-s)\ln\left[e^z \frac{1-\hat{\alpha}}{1 - \frac{\hat{\alpha}}{1+s}e^z}\right] + s\ln s$$

Setting this equal to 0, we have

$$\ln\left[\frac{1-\hat{\alpha}}{1 - \frac{\hat{\alpha}}{1+s}e^z}\right] = \frac{-s\ln s}{1+s} - z \iff \frac{1 - e^{\left[\frac{-s\ln s}{1+s} - z\right]}}{\left[1 - \frac{1}{1+s}e^{\left[\frac{-s\ln s}{1+s}\right]}\right]} = \hat{\alpha}$$

Simplifying, we have

$$\hat{\alpha} = \frac{(1+s)\left(1 - s^{\frac{-s}{1+s}}e^{-z}\right)}{1 + s - s^{\frac{-s}{1+s}}}$$

Then, notice that $\hat{\alpha} > \alpha^* \iff \frac{(1+s)\left(1 - s^{\frac{-s}{1+s}}e^{-z}\right)}{1+s-s^{\frac{-s}{1+s}}} > e^{-z}(1+s)$, which simplifies to $1 > \alpha^*$. Thus, the function $\delta^*(\alpha)$ does not cross 0 before α^*.

A4. Joint safety equilibrium with common bonds

Let us conjecture a non-monotone oscillating strategy as in A.1.

Lower boundary δ_L.

The definitions of $\rho(\delta, x)$ and $\rho(\delta_L, x)$ are as in Appendix A.A1, and most of the result simply have $\hat{f}$ instead of f: as country 2 is safe to an agent with $\delta = \delta_L$, we have $\Pi_2(\delta_L) = \frac{s}{1+\hat{f}}\left[\ln\frac{1+s}{s} + 1\right] < \frac{1+s}{1+\hat{f}}$.

The common bonds change the safety condition for country 1 to

$$\theta_1(\delta) + \alpha p_c + \left(1 + \hat{f}\right)\rho_1^{min}(\delta) = 1 \iff \rho_1^{min}(\delta) = \frac{1 - \theta_1(\delta) - \alpha p_c}{1 + \hat{f}}$$

Define $x_{min}(\delta_L)$ as the solution to $\rho(\delta_L, x) = \rho_1^{min}(\delta_L)$. Given equation (A.3), we have that,

$$x_{min}(\delta_L) = \frac{1 - \theta_1(\delta_L) - \alpha p_c}{1 + \hat{f}} \tag{A.26}$$

Again, the expected return of investing in country 1 is given by $\Pi_1(\delta_L) = \frac{1}{1+\hat{f}}\left[\ln \frac{1}{1+s} - \ln x_{min}(\delta_L) + s\right]$. Indifference requires that $\Pi_2(\delta_L) = \Pi_1(\delta_L)$, which implies that

$$x_{min}(\delta_L) = \exp\left[s \ln s - (1+s)\ln(1+s)\right] \tag{A.27}$$

We combine the expressions for $x_{min}(\delta_L)$, (A.26) and (A.27), to solve for δ_L:

$$\delta_L = -\ln\left\{\frac{1}{1-\theta}\left[\left(1+\hat{f}\right)\frac{s^s}{(1+s)^{(1+s)}} + \alpha p_c\right]\right\}. \tag{A.28}$$

Upper boundary δ_H.

The derivation of $\rho(\delta, x)$ and $\rho(\delta_H, x)$ follow Appendix A.A1, , and most of the result simply have $\hat{f}$ instead of f. We have $\Pi_1(\delta_H) = \frac{\ln(1+s)+1}{1+\hat{f}}$ as country 1 is considered safe at $\delta_j = \delta_H$.

The default condition for country 2 is

$$s\theta_2(\delta) + s\alpha p_c + \left(1+\hat{f}\right)\left[1 - \rho_2^{max}(\delta)\right] = s \iff \left[1 - \rho_2^{max}(\delta)\right] = s\frac{1 - \theta_2(\delta) - \alpha p_c}{1+\hat{f}}$$

where $\rho_2^{max}(\delta)$ is the maximum amount of people investing in country 1 so that country 2 does not default. Define $x_{max}(\delta_H)$ as the solution to $\rho(\delta_H, x_{max}) = \rho_2^{max}(\delta_H)$. Given equation (A.11), we have that,

$$1 - x_{max}(\delta_H) = s\frac{1 - \theta_2(\delta_H) - \alpha p_c}{1+\hat{f}} \tag{A.29}$$

Then the return to investing in country 2 is again given by $\Pi_2(\delta_H) = \frac{s}{1+\hat{f}}\left[\frac{1}{s} + \ln\frac{s}{1+s} - \ln\left(1 - x_{max}(\delta_H)\right)\right]$.

Indifference requires $\Pi_1(\delta_H) = \Pi_2(\delta_H)$, which implies that

$$1 - x_{max}(\delta_H) = \frac{s}{(1+s)^{\frac{1+s}{s}}} \tag{A.30}$$

We combine the expressions for $x_{max}(\delta_H)$, (A.29) and (A.30), to solve for δ_H:

$$\delta_H = \ln\left\{\frac{1}{1-\theta}\left[\frac{1+\hat{f}}{(1+s)^{\frac{1+s}{s}}} + \alpha p_c\right]\right\} \tag{A.31}$$

The remainder of the proof, i.e., the verification argument, is exactly the same as in Appendix A.A1 and hence omitted here.

Cutoff $\alpha_{HL} < \alpha^*$.

First, the assumption $e^z > (1+s) \iff (1+f) > (1-\theta)(1+s)$ guarantees that there is *some* realizations of $\tilde{\delta}$ that would allow joint safety. Consider the total funding requirement,

$$total\left(\tilde{\delta}\right) = (1-\theta_1) + (1-\theta_2)s = (1-\theta)\left(e^{-\tilde{\delta}} + s \cdot e^{\tilde{\delta}}\right) \tag{A.32}$$

This is minimized at $\tilde{\delta} = -\frac{1}{2}\ln s \geq 0$ for a total funding requirement of $total\left(-\frac{1}{2}\ln s\right) = (1-\theta)2\sqrt{s}$. Next, note that $1 + s > 2\sqrt{s}$ so that $e^z > (1+s) > 2\sqrt{s}$.

Recall that $\alpha^* = e^{-z}(1+s)$. Then, assume that $z > \ln(1+s)$ so that $\alpha^* \in (0,1)$. Then, we have

$$\begin{aligned}\delta_H(\alpha^*) - \delta_L(\alpha^*) &= \ln\left\{\frac{e^z}{1+s}\left[\left(\frac{1}{1+s}\right)^{\frac{1}{s}}(1-\alpha^*)+\alpha^*\right]\right\} + \ln\left\{\frac{e^z}{1+s}\left[\left(\frac{s}{1+s}\right)^{s}(1-\alpha^*)+\alpha^*\right]\right\} \\ &= \ln\left[\left(\frac{1}{1+s}\right)^{\frac{1}{s}}\left(\frac{1}{\alpha^*}-1\right)+1\right] + \ln\left[\left(\frac{s}{1+s}\right)^{s}\left(\frac{1}{\alpha^*}-1\right)+1\right] > 0\end{aligned}$$

where we used $\left(\frac{1}{1+s}\right)^{\frac{1}{s}} < 1$ and $\left(\frac{s}{1+s}\right)^{s} < 1$ and $\frac{1}{\alpha^*} > 1$ in the last line. Thus, at α^* the oscillating equilibrium already exists. It is easy to show that the the joint safety region $[\delta_L(\alpha), \delta_H(\alpha)]$ is expanding uniformly in α, and thus that $\alpha_{HL} < \alpha^*$.

Finally, define α_{HL} as the solution to

$$\begin{aligned}0 &= \delta_H(\alpha_{HL}) - \delta_L(\alpha_{HL}) \\ &= 2[z - \ln(1+s)] + \ln\left[\left(\frac{1}{1+s}\right)^{\frac{1}{s}}(1-\alpha_{HL})+\alpha_{HL}\right] + \ln\left[\left(\frac{s}{1+s}\right)^{s}(1-\alpha_{HL})+\alpha_{HL}\right]\end{aligned}$$

Rearranging, we have

$$\left[\left(\frac{1}{1+s}\right)^{\frac{1}{s}}(1-\alpha_{HL})+\alpha_{HL}\right]\left[\left(\frac{s}{1+s}\right)^{s}(1-\alpha_{HL})+\alpha_{HL}\right] - e^{-2z}(1+s)^2 = 0$$

which is a quadratic equation in α_{HL}. We note that $e^{-2z}(1+s)^2 < 1 \iff 2[\ln(1+s) - z] < 0$, so that $\alpha_{HL} = 1$ makes the LHS positive. We also know that the LHS is increasing in α_{HL} for $\alpha_{HL} > 0$. Thus, there exists at most one positive root $\alpha_{HL} \in (0,1)$ under the assumption $z > \ln(1+s)$, and if not, both roots are negative. Solving for the larger root α_{HL}, and after some algebra, we can show that $\delta^*(\alpha_{HL}) = \delta_H(\alpha_{HL}) = \delta_L(\alpha_{HL})$.

Appendix B: Additional Results

B1. Additive Fundamental Structure

We have considered the specification of $1-\theta_i = (1-\theta)\exp\left((-1)^i \tilde{\delta}\right)$ for country i's fundamental. We now show that results are qualitatively similar with the alternative additive specification

$$\theta_1 = \theta + \tilde{\delta}, \text{ and } \theta_2 = \theta - \tilde{\delta}.$$

As $x = \Pr\left(\tilde{\delta} + \epsilon_j > \delta^*\right) = \frac{\tilde{\delta}+\sigma-\delta^*}{2\sigma} \Rightarrow \tilde{\delta} = \delta^* + (2x-1)\sigma$, we know that

$$\begin{aligned}\theta_1 &= \theta + \tilde{\delta} = \theta + \delta^* + (2x-1)\sigma \\ \theta_2 &= \theta - \tilde{\delta} = \theta - \delta^* - (2x-1)\sigma\end{aligned}$$

Given x, the large country 1 survives if and only if

$$p_1 - 1 + \theta_1 = (1+f)x - 1 + \theta + \delta^* + (2x-1)\sigma \geq 0 \Leftrightarrow x \geq \frac{1-\theta-\delta^*+\sigma}{1+f+2\sigma}$$

which implies the expected return from investing in country 1 is

$$\Pi_1 = \int_{\frac{1-\theta-\delta^*+\sigma}{1+f+2\sigma}}^{1} \frac{1}{(1+f)x}dx = \frac{1}{1+f}\ln\frac{1+f+2\sigma}{1-\theta-\delta^*+\sigma}.$$

For country 2, the bond is paid back if

$$\begin{aligned}(1+f)x' - s + s\theta_2 &= (1+f)x' - s + s\left[\theta - \delta^* - (2x-1)\sigma\right] \geq 0 \\ \Leftrightarrow x' &\geq \frac{s(1-\theta+\delta^*-\sigma)}{1+f+2s\sigma}\end{aligned}$$

which implies an expected return of

$$\Pi_2 = \int_{\frac{s(1-\theta+\delta^*-\sigma)}{1+f+2s\sigma}}^{1} \frac{s}{(1+f)x'}dx' = \frac{s}{1+f}\ln\frac{1+f+2s\sigma}{s(1-\theta+\delta^*+\sigma)}$$

As a result, the equilibrium threshold δ^* is pinned by by the indifference condition

$$\ln\frac{1+f+2\sigma}{1-\theta-\delta^*+\sigma} = s\ln\frac{1+f+2s\sigma}{s(1-\theta+\delta^*+\sigma)}.$$

Letting $\sigma \to 0$ we obtain

$$\ln\frac{1+f}{1-\theta-\delta^*} = s\ln\frac{1+f}{s(1-\theta+\delta^*)}. \tag{B.1}$$

We no longer have close-form solution for δ^* in (B.1), as δ^* shows up in both sides. However, the solution is unique because LHS (RHS) is increasing (decreasing) in δ^*. Finally, to ensure $\delta^* < 0$ so that the larger country 1 is relatively safer, we require the same sufficient condition of $z = \ln\frac{1+f}{1-\theta} > 1$ in this alternative specification.

B2. Uniqueness of the single-survivor equilibrium with threshold strategies within monotone strategies

First, let us define a few primitives. Let δ_j be a generic signal, and δ be the true state of the world. Further, let x denote the amount of pessimism of the investors, so that $x = 1$ is the most pessimistic agent (amongst all agents out there) and $x = 0$ is the least pessimistic agent. We then have $\delta(\delta_j, x) = \delta_j + 2\sigma\left(x - \frac{1}{2}\right)$. For most of the proofs, we assume wlog that the investor believes his signal to be the true signal, and thus all the action comes from movements in his relative position. As $\sigma \to 0$, fundamental uncertainty (that is movements in δ as a function of x) will vanish, whereas strategic uncertainty (relative ranking of investors as represented by x) remains.

Next, let us define $\phi(\delta_j)$ as the *proportion of funds* an investor with signal δ_j invests in country 1. Then define

$$\rho(\delta_j, x) = \frac{1}{2\sigma}\int_{\delta_j - 2\sigma(1-x)}^{\delta_j + 2\sigma x} \phi(y)\, dy$$

as the *aggregate proportion of investors in country 1* an investor with signal δ_j and level of pessimism x expects given the conjecture strategies $\phi(\cdot)$. Note that there is translation invariance

$$\rho(\delta_j, x) = \rho\left(\delta_j + \varepsilon, x - \frac{\varepsilon}{2\sigma}\right), \forall x \in \left(\frac{\varepsilon}{2\sigma}, 1\right)$$

Finally, define the (scaled by $1+f$) *difference in expected returns* as

$$\Delta(\delta_j) = \int_0^1 \mathbf{1}_{\{\rho(\delta,x)\geq\rho_{min}(\delta)\}} \frac{1}{\rho(\delta,x)}dx - \int_0^1 \mathbf{1}_{\{\rho(\delta,x)\leq\rho_{max}(\delta)\}} \frac{s}{1-\rho(\delta,x)}dx$$

Then, for any given conjectured difference function $\Delta(y)$, we must have

$$\phi(y) = \begin{cases} 1, & \Delta(y) > 0 \\ \in [0,1], & \Delta(y) = 0 \\ 0, & \Delta(y) < 0 \end{cases}$$

A *monotone strategy* is defined by $\phi'(y) \geq 0$ for all $y \in \left[-\bar{\delta}, \bar{\delta}\right]$, which implies that $\rho_\delta(\delta, x) \geq 0$ as well as $\rho_x(\delta, x) \geq 0$, i.e., $\rho(\delta, x)$ is monotone. This implies that we can write

$$\begin{aligned}
\Delta(\delta_j) &= \int_0^1 \mathbf{1}_{\{\rho(\delta_j,x)\geq\rho_{min}(\delta(\delta_j,x))\}} \frac{1}{\rho(\delta,x)}dx - \int_0^1 \mathbf{1}_{\{\rho(\delta_j,x)\leq\rho_{max}(\delta(\delta_j,x))\}} \frac{s}{1-\rho(\delta,x)}dx \\
&\approx \int_0^1 \mathbf{1}_{\{\rho(\delta_j,x)\geq\rho_{min}(\delta_j)\}} \frac{1}{\rho(\delta,x)}dx - \int_0^1 \mathbf{1}_{\{\rho(\delta_j,x)\leq\rho_{max}(\delta_j)\}} \frac{s}{1-\rho(\delta,x)}dx \\
&= \int_{x_{min}(\delta_j)}^1 \frac{1}{\rho(\delta_j,x)}dx - \int_0^{x_{max}(\delta_j)} \frac{s}{1-\rho(\delta_j,x)}dx
\end{aligned}$$

Country 1 survives if $\rho(\delta_j, x)$ is larger than $\rho_{min}(\delta(\delta_j, x))$. As the agent becomes more pessimistic relative to the other agents, i.e., x increases, the actual relative fundamental increases, and thus the threshold decreases:

$$\begin{aligned}
\partial_x \rho_{min}(\delta(\delta_j,x)) &= \partial_x e^{-z} e^{-\delta(\delta_j,x)} = -e^{-z}e^{-\delta(\delta_j,x)} 2\sigma < 0 \\
\partial_{\delta_j} \rho_{min}(\delta(\delta_j,x)) &= -e^{-z}e^{-\delta(\delta_j,x)} < 0
\end{aligned}$$

Thus, if $\rho(\delta, x)$ is monotone, there exists a unique threshold $x_{min}(\delta)$ above which country 1 is safe. Further, by the implicit function theorem, we have

$$\begin{aligned}
x'_{min}(\delta) &= -\frac{\rho_\delta(\delta,x) - \partial_\delta \rho_{min}\left(\tilde{\delta}(\delta,x)\right)}{\rho_x(\delta,x) - \partial_x \rho_{min}\left(\tilde{\delta}(\delta,x)\right)} \\
&= -\frac{\frac{\phi(\delta+2\sigma x)-\phi(\delta-2\sigma(1-x))}{2\sigma} + e^{-z}e^{-\tilde{\delta}(\delta,x)}}{\phi(\delta+2\sigma x) - \phi(\delta-2\sigma(1-x)) + e^{-z}e^{-\tilde{\delta}(\delta,x)}2\sigma} \\
&= -\frac{1}{2\sigma}
\end{aligned}$$

so that the pessimism threshold falls that makes country 1 safe. Similarly, we have

$$\begin{aligned}
x'_{max}(\delta) &= -\frac{\rho_\delta(\delta,x) - \partial_\delta \rho_{max}\left(\tilde{\delta}(\delta,x)\right)}{\partial_x(\delta,x) - \partial_x \rho_{max}\left(\tilde{\delta}(\delta,x)\right)} \\
&= -\frac{\frac{\phi(\delta+2\sigma x)-\phi(\delta-2\sigma(1-x))}{2\sigma} + se^{-z}e^{\tilde{\delta}(\delta,x)}}{\phi(\delta+2\sigma x) - \phi(\delta-2\sigma(1-x)) + se^{-z}e^{\tilde{\delta}(\delta,x)}2\sigma} \\
&= -\frac{1}{2\sigma}
\end{aligned}$$

We can thus approximate

$$x_{max}(\delta+\varepsilon) + \frac{\varepsilon}{2\sigma} \approx x_{max}(\delta) + x'_{max}(\delta)\varepsilon + \frac{\varepsilon}{2\sigma} = x_{max}(\delta) \quad \text{and} \quad x_{min}(\delta+\varepsilon) + \frac{\varepsilon}{2\sigma} \approx x_{min}(\delta)$$

Finally, suppose a δ exists for which the investor expects *joint safety*, i.e., both countries to be safe for sure. Then, we must have $\phi(\delta) = \frac{1}{1+s}$ by the no arbitrage condition. A single-survivor equilibrium with threshold strategies is defined by a single-crossing condition on $\Delta = \Pi_1 - \Pi_2$ and a *non-flat part at* 0, where $\Delta(\delta) > 0$ implies $\phi = 1$ and $\Delta(\delta) < 0$ implies $\phi = 0$. Consider any other equilibrium. By dominance regions, we know that for high δ, $\phi = 1$ will eventually be optimal, and for very low δ, $\phi = 0$ will eventually be optimal.

Thus, any other equilibrium is either characterized by (1) a flat part $\Delta(\delta) = 0$, (2) multiple crossings $\Delta(\delta) = 0$ or (3) a combination of the two. In our joint safety equilibrium supported by oscillating strategy, (3) is the case, with a flat part in the middle.

Monotonicity and uniqueness of threshold equilibrium

A monotone strategy $\phi(\delta)$ requires $\Delta(\delta)$ to change signs only once. Thus, $\Delta(\delta)$ either crosses zero at a single point, or approaches it from below, stays flat on an interval $[\delta_L, \delta_H]$, and then rises above zero. Thus, at any point δ s.t. $\Delta(\delta) = 0$ we must have $\Delta'(\delta) \geq 0$. As we want to show that a threshold equilibrium is the only equilibrium possible, we now rule out any flat parts of Δ at zero.

To this end, suppose an interval $[\delta_L, \delta_H]$ exists on which $\Delta(\delta) = 0$. Interior x_{min}, x_{max}. Suppose now that $x_{min}(\delta), x_{max}(\delta) \in (0,1)$. This means that both countries are at risk of default, so there is no possibility of joint safety across all possible $x \in [0,1]$ (it might exists for some x if $x_{min}(\delta) < x_{max}(\delta)$). Take $\varepsilon \in (0, \delta_H - \delta_L)$. Then, we write

$$\begin{aligned}
\Pi_1(\delta+\varepsilon) &= \int_{x_{min}(\delta+\varepsilon)}^{1} \frac{1}{\rho(\delta+\varepsilon,x)}dx \\
&= \int_{x_{min}(\delta+\varepsilon)+\frac{\varepsilon}{2\sigma}}^{1+\frac{\varepsilon}{2\sigma}} \frac{1}{\rho\left(\delta+\varepsilon, x-\frac{\varepsilon}{2\sigma}\right)}dx \\
&= \int_{x_{min}(\delta+\varepsilon)+\frac{\varepsilon}{2\sigma}}^{1} \frac{1}{\rho\left(\delta+\varepsilon, x-\frac{\varepsilon}{2\sigma}\right)}dx + \int_{1}^{1+\frac{\varepsilon}{2\sigma}} \frac{1}{\rho\left(\delta+\varepsilon, x-\frac{\varepsilon}{2\sigma}\right)}dx \\
&\approx \int_{x_{min}(\delta)}^{1} \frac{1}{\rho(\delta,x)}dx + \int_{1-\frac{\varepsilon}{2\sigma}}^{1} \frac{1}{\rho(\delta+\varepsilon,x)}dx \\
&= \Pi_1(\delta) + \underbrace{\int_{1-\frac{\varepsilon}{2\sigma}}^{1} \frac{1}{\rho(\delta+\varepsilon,x)}dx}_{\text{new pessimists}}
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
\Pi_2(\delta+\varepsilon) &= \int_0^{x_{max}(\delta+\varepsilon)} \frac{s}{1-\rho(\delta+\varepsilon,x)}dx \\
&= \int_{\frac{\varepsilon}{2\sigma}}^{x_{max}(\delta+\varepsilon)+\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho\left(\delta+\varepsilon,x-\frac{\varepsilon}{2\sigma}\right)}dx \\
&\approx \int_{\frac{\varepsilon}{2\sigma}}^{x_{max}(\delta)} \frac{s}{1-\rho(\delta,x)}dx + \int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta,x)}dx - \int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta,x)}dx \\
&= \int_0^{x_{max}(\delta)} \frac{s}{1-\rho(\delta,x)}dx - \int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta,x)}dx \\
&= \Pi_2(\delta) - \underbrace{\int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta,x)}dx}_{\text{old optimists}}
\end{aligned}$$

so that

$$\begin{aligned}
\Delta(\delta_L+\varepsilon) &= \Pi_1(\delta_L+\varepsilon) - \Pi_2(\delta_L+\varepsilon) \\
&= \Pi_1(\delta_L) + \int_{1-\frac{\varepsilon}{2\sigma}}^1 \frac{1}{\rho(\delta_L+\varepsilon,x)}dx - \left[\Pi_2(\delta_L) - \int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta_L,x)}dx\right] \\
&= \int_{1-\frac{\varepsilon}{2\sigma}}^1 \frac{1}{\rho(\delta_L+\varepsilon,x)}dx + \int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta_L,x)}dx > 0
\end{aligned}$$

But this implies that

$$\phi(\delta_L+\varepsilon) = 1$$

By monotonicity then, δ_L is the only point at which $\Delta(\delta) = 0$ and no flat parts can exist for $x_{min}, x_{max} \in (0,1)$. Cornered x_{min}, x_{max}. Next, suppose that at least one of the countries is going to survive regardless of x because of the assumed strategies. Wlog, let us focus on δ_L. First, let us rule out that $x_{min}(\delta_L) = 0$. Note that for any $\varepsilon > 0$, we have by the dominance boundaries $\Delta(\delta_L-\varepsilon) < 0$ and $\Delta(\delta_H+\varepsilon) > 0$, the highest and lowest point of the all flat parts. Further note that $x_{min}(\delta_L) = 0$ implies that country 1 always survives in the eyes of an investor with signal δ_L. By construction we have $\rho(\delta,0) = 0$— when the agent with signal δ_L is the most optimistic agent, he must believe by the conjecture on $\Delta(\delta)$ that everyone below him investors fully into country 2. But then this agent cannot believe that country 1 is safe regardless of x, as by assumption no country can survive without a minimum amount of investment.

Thus, at δ_L we must have $x_{max}(\delta_L) = 1$ and $x_{min}(\delta_H) = 0$—*country 2 always survives* given the strategies of the different agents. Then, we have the survival boundary of country 2 not changing, and thus again for $\varepsilon \in (0, \delta_H - \delta_L)$ we have

$$\begin{aligned}
\Pi_2(\delta+\varepsilon) &= \int_0^1 \frac{s}{1-\rho(\delta+\varepsilon,x)}dx \\
&= \int_{\frac{\varepsilon}{2\sigma}}^{1+\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho\left(\delta+\varepsilon,x-\frac{\varepsilon}{2\sigma}\right)}dx \\
&= \int_{\frac{\varepsilon}{2\sigma}}^1 \frac{s}{1-\rho(\delta,x)}dx + \int_{1-\frac{\varepsilon}{2\sigma}}^1 \frac{s}{1-\rho(\delta+\varepsilon,x)}dx \\
&= \int_0^1 \frac{s}{1-\rho(\delta,x)}dx + \int_{1-\frac{\varepsilon}{2\sigma}}^1 \frac{s}{1-\rho(\delta+\varepsilon,x)}dx - \int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta,x)}dx \\
&= \Pi_2(\delta) + \underbrace{\int_{1-\frac{\varepsilon}{2\sigma}}^1 \frac{s}{1-\rho(\delta+\varepsilon,x)}dx}_{\text{new pessimists}} - \underbrace{\int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta,x)}dx}_{\text{old optimists}}
\end{aligned}$$

Then, we have

$$\begin{aligned}
0=\Delta(\delta+\varepsilon) &= \Pi_1(\delta+\varepsilon)-\Pi_2(\delta+\varepsilon)\\
&= \Pi_1(\delta)+\int_{1-\frac{\varepsilon}{2\sigma}}^{1}\frac{1}{\rho(\delta+\varepsilon,x)}dx-\left[\Pi_2(\delta)+\int_{1-\frac{\varepsilon}{2\sigma}}^{1}\frac{s}{1-\rho(\delta+\varepsilon,x)}dx-\int_0^{\frac{\varepsilon}{2\sigma}}\frac{s}{1-\rho(\delta,x)}dx\right]\\
&= \int_{1-\frac{\varepsilon}{2\sigma}}^{1}\frac{1}{\rho(\delta+\varepsilon,x)}dx-\int_{1-\frac{\varepsilon}{2\sigma}}^{1}\frac{s}{1-\rho(\delta+\varepsilon,x)}dx+\int_0^{\frac{\varepsilon}{2\sigma}}\frac{s}{1-\rho(\delta,x)}dx\\
&= \underbrace{\int_{1-\frac{\varepsilon}{2\sigma}}^{1}\left[\frac{1}{\rho(\delta+\varepsilon,x)}-\frac{s}{1-\rho(\delta+\varepsilon,x)}\right]dx}_{\text{new pessimists}}+\underbrace{\int_0^{\frac{\varepsilon}{2\sigma}}\frac{s}{1-\rho(\delta,x)}dx}_{\text{old optimists}}
\end{aligned}$$

and there is now a *possibility* of a flat part. The intuition here is that we are balancing the returns that arise to the new most pessimistic investor (i.e. for high x) against the previous expected returns of the most optimistic investors (i.e. low x).

Taking derivatives around $\varepsilon=0$, we have

$$\begin{aligned}
\Delta(\delta+\varepsilon) &\approx \Delta(\delta)+\Delta'(\delta)\varepsilon\\
&= \frac{1}{2\sigma}\left[\frac{1}{\rho\left(\delta+\varepsilon,1-\frac{\varepsilon}{2\sigma}\right)}-\frac{s}{1-\rho\left(\delta+\varepsilon,1-\frac{\varepsilon}{2\sigma}\right)}\right]_{\varepsilon=0}\varepsilon\\
&\quad+\left[\int_{1-\frac{\varepsilon}{2\sigma}}^{1}\left[-\frac{\rho_\delta(\delta+\varepsilon,x)}{\rho(\delta+\varepsilon,x)^2}-\frac{s(-\rho_\delta(\delta+\varepsilon,x))}{[1-\rho(\delta+\varepsilon,x)]^2}\right]dx\right]_{\varepsilon=0}\varepsilon\\
&\quad+\frac{1}{2\sigma}\left[\frac{s}{1-\rho\left(\delta,\frac{\varepsilon}{2\sigma}\right)}\right]_{\varepsilon=0}\varepsilon\\
&= \frac{1}{2\sigma}\left[\frac{1}{\rho(\delta,1)}-\frac{s}{1-\rho(\delta,1)}+\frac{s}{1-\rho(\delta,0)}\right]\varepsilon
\end{aligned}$$

When $\delta=\delta_L$ we must have $\rho(\delta_L,0)=0$ by definition of δ_L. Then, the derivative $\Delta'(\delta_L)=0$ if

$$\rho(\delta_L,1)=\frac{-1+\sqrt{1+4s}}{2s}>\frac{1}{1+s}$$

which implies that least for some points on $(\delta_L,\delta_L+2\sigma)$ we must have $\phi(\delta)>\frac{1}{1+s}$.

By $x'_{min}(\delta)\le 0$ and $x'_{max}(\delta)\le 0$, as δ increases either we (i) move to a segment where $x_{min}(\delta),x_{max}(\delta)\in(0,1)$, an interior situation, or (ii) to a segment with $x_{min}(\delta)=0, x_{max}(\delta)=1$, a completely safe part.

But we know from the previous section that (i) immediately has $\Delta'(\delta)>0$, a violation of the premise that we are on a flat part for $\delta\in[\delta_L,\delta_H]$. Next, consider for (ii) any completely safe subset $J\subset(\delta_L,\delta_H)$ and $\delta\in J$. Then, we require $\rho(\delta,x)=\frac{1}{1+s},\forall x\in[0,1]$ by no arbitrage, which implies $\phi(\delta)=\frac{1}{1+s}$. But then we have a violation of monotonicity as $\rho(\delta_L,1)>\frac{1}{1+s}$. Thus, there cannot be any flat parts of $\Delta(\delta)$ at zero and the only equilibrium that survives is of the threshold form. By the construction in the paper, this threshold equilibrium is unique. Existence of threshold equilibrium. Consider our unique candidate equilibrium

$$\delta^*=-\frac{1-s}{1+s}z-\frac{s\ln s}{1+s}$$

derived in the main text. Consider now $\delta_j<\delta^*$. Then, we have

$$\Delta(\delta_j;\delta^*)=\int_{\rho(x)>\rho_{min}\left(\tilde{\delta}(x;\delta_j)\right)}\frac{1}{(1+f)\rho(x)}dx-s\int_{\rho(x)<\rho_{max}\left(\tilde{\delta}(x;\delta_j)\right)}\frac{1}{(1+f)(1-\rho(x))}dx$$

We know that $\Delta(\delta^*;\delta^*)=0$. But by our setup, we know that moving $\delta_j<\delta^*$ lowers both $\rho_{min}(\delta)$ and $\rho_{max}(\delta)$. Thus, we need to look at the difference between the parts we are adding (region in which country 1 survives) and parts we are subtracting (region in which country 2 survives):

$$\begin{aligned}\Delta_{\delta_j}(\delta_j;\delta^*) &= -\rho'_{min}(\delta_j)\frac{1}{(1+f)\,\rho_{min}(\delta_j)}+s\rho'_{max}(\delta_j)\frac{1}{(1+f)\,(1-\rho_{max}(\delta_j))}\\ &= \frac{1}{(1+f)}-s\frac{1}{(1+f)}=\frac{1-s}{1+f}>0\end{aligned}$$

where we used

$$\rho'_{min}(\delta_j)=-\rho_{min}(\delta_j)\quad\text{and}\quad\rho'_{max}(\delta_j)=-(1-\rho_{max}(\delta_j))$$

This is intuitive: as we increase δ_j, we are adding the most valuable states for country 1 (fixing $\rho(x)$) by evaluating at points set on which it will just survive, i.e., close to $\rho_{min}(\delta_j)$, and we are taking away the most valuable states for country 2 by evaluating at points set on which it will just default, i.e., close to $\rho_{max}(\delta_j)$.

B3. Single-survivor equilibrium with oscillating strategies because of positive recovery

Let us say that $s_1=1, s_2=s$ and l_is_i to be the recovery given default of country i, so that it returns $\frac{l_is_i}{y_i}$ per unit of dollar invested, where y_i is total investment in country i. Then if country 1 survives, to equalize return, we need

$$\frac{l_2s}{y_2}=\frac{1}{y_1},y_1+y_2=1+f\Rightarrow\frac{y_1}{y_2}=\frac{1}{l_2s}.$$

This gives prices equal to

$$\begin{aligned}p_1 &= y_1=\frac{(1+f)}{1+l_2s}\\ p_2 &= \frac{y_2}{s}=\frac{(1+f)\,l_2}{1+l_2s}\end{aligned}$$

Similarly, if country 2 survives, then

$$\frac{s}{y_2}=\frac{l_1}{y_1},y_1+y_2=1+f\Rightarrow\frac{y_1}{y_2}=\frac{l_1}{s}$$

which results in prices

$$\begin{aligned}p_1 &= y_1=\frac{(1+f)\,l_1}{l_1+s}\\ p_2 &= \frac{y_2}{s}=\frac{(1+f)}{l_1+s}\end{aligned}$$

Let

$$z=\ln\frac{1+f}{1-\theta}>0$$

and fiscal surplus is given by

$$\begin{aligned}\theta_1 &= 1-(1-\theta)\,e^{-\delta}=1-(1+f)\,e^{-z}e^{-\delta}\\ s\theta_2 &= s\left[1-(1-\theta)\,e^{\delta}\right]=s\left[1-(1+f)\,e^{-z}e^{\delta}\right]\end{aligned}$$

Define two constants $k_1>1$ and $k_2>1$ (which only occurs if $s<l_1$) so that

$$\begin{aligned}\frac{k_1}{2-k_1}=\frac{1}{l_2s} &\iff k_1=\frac{2}{1+l_2s}>1\\ \frac{k_2}{2-k_2}=\frac{s}{l_1} &\iff k_2=\frac{2s}{s+l_1}>1\end{aligned}$$

Then in the country-1-default region, $k_2\sigma$ measure of agents invest in country 2, i.e. play $\phi=0$, while $(2-k_2)\,\sigma$ measure of agents play $\phi=1$. Similarly in the country-2-default region, , $k_1\sigma$ measure of agents play $\phi=1$ while $(2-k_1)\,\sigma$ measure of agents play $\phi=0$.

Conjecture the following equilibrium strategy with cutoff δ^*

$$\phi(y)=\begin{cases}.... \\ 1, & y\in[\delta^*-2\sigma,\delta^*-k_2\sigma] \\ 0, & y\in[\delta^*-k_2\sigma,\delta^*] \\ 1, & y\in[\delta^*,\delta^*+k_1\sigma] \\ 0, & y\in[\delta^*+k_1\sigma,\delta^*+2\sigma] \\ 1, & y\in[\delta^*+2\sigma,\delta^*+2\sigma+k_1\sigma] \\ .... \end{cases}$$

In other words, two types of equilibria collide at δ^*. We conjecture that marginal investor at δ^* is indifferent, while the agents between $[\delta^*-k_2\sigma,\delta^*]$ strictly prefer $\phi=0$, and symmetrically the agents between $[\delta^*,\delta^*+k_1\sigma]$ strictly prefer $\phi=1$. Other agents in this economy are indifferent.

Let x denote the fraction of agents with signal realization above the agent's private signal δ_j, so that given x, the true fundamental is

$$\delta(x)=\delta_j-(1-2x)\sigma$$

Further, let $\rho(\delta_j,x)$ be the expected proportion agents investing in country 1 given x. Then, we have

$$\rho(\delta_j,x)=\begin{cases}1-\frac{k_2}{2}, & \delta+2\sigma x<\delta^*+(2-k_2)\sigma \\ x+cst, & else \\ \frac{k_1}{2} & \delta-2\sigma(1-x)>\delta^*-(2-k_1)\sigma\end{cases}$$

where cst is picked so that $\rho(\delta_j,x)$ is continuous in x. We note that the slope is generically x as we are replacing $\phi=0$ with $\phi=1$ marginally. At $\delta_j=\delta^*$, we have

$$\rho(\delta^*,x)=\begin{cases}1-\frac{k_2}{2}, & x<1-\frac{k_2}{2} \\ x, & else \\ \frac{k_1}{2} & x>\frac{k_1}{2}\end{cases}$$

and we need

$$1-\frac{k_2}{2}<\frac{k_1}{2}$$

Note that if we assume that $\rho_{min}(\delta),1-\rho_{max}(\delta)\in\left[1-\frac{k_2}{2},\frac{k_1}{2}\right]$ we have a 1-to-1 function between x and ρ that yields

$$x_{min}=\frac{1-\theta_1(\delta^*)}{1+f}=\frac{1-\theta}{1+f}e^{-\delta^*}\quad\Longleftrightarrow\quad\ln x_{min}=-z-\delta^*$$
$$1-x_{max}=s\frac{1-\theta_2(\delta^*)}{1+f}=s\frac{1-\theta}{1+f}e^{\delta^*}\quad\Longleftrightarrow\quad\ln(1-x_{max})=\ln s-z+\delta^*$$

Note here that we are ignoring fundamental uncertainty. Otherwise, we need to take account of the fact that in the mind of the agent,

$$\rho_{min}(\delta(x))=e^{-z}e^{-\delta(x)}=e^{-z}e^{-\left[\delta_j-(1-2x)\sigma\right]}$$

is the minimum investment in country 1 needed for it to survive conditional on x. For everything else below, we assume that $\rho_{min}(\delta(x))=\rho_{min}(\delta_j)$. Next, note that

$$x=\text{Fraction of people with signal above agent}$$

so that $x=1$ is the most pessimistic agent, and $x=0$ is the most optimistic. As $\rho(\delta,x)$ is increasing in x, we have

$$\begin{aligned} x<x_{min} &\quad\Longleftrightarrow\quad \text{Country 1 fails} \\ x>x_{min} &\quad\Longleftrightarrow\quad \text{Country 1 survives} \\ x<x_{max} &\quad\Longleftrightarrow\quad \text{Country 2 survives} \\ x>x_{max} &\quad\Longleftrightarrow\quad \text{Country 2 fails}\end{aligned}$$

Then, for the boundary agent, the expected return of investing in country 2 is given by

$$\begin{aligned}
\Pi_2(\delta^*) &= Return_2(survival) + Return_2(default) \\
&= \int_0^{x_{max}} \frac{s}{(1+f)(1-\rho(\delta^*,x))}dx + \int_{x_{max}}^1 \frac{l_2 s}{(1+f)(1-\rho(\delta^*,x))}dx \\
&= \int_0^{1-\frac{k_2}{2}} \frac{s}{(1+f)\left(1-\left(1-\frac{k_2}{2}\right)\right)}dx + \int_{1-\frac{k_2}{2}}^{x_{max}} \frac{s}{(1+f)(1-x)} \\
&\quad + \int_{x_{max}}^{\frac{k_1}{2}} \frac{l_2 s}{(1+f)(1-x)}dx + \int_{\frac{k_1}{2}}^1 \frac{l_2 s}{(1+f)\left(1-\frac{k_1}{2}\right)}dx \\
&= \left(1-\frac{k_2}{2}\right)\frac{s}{(1+f)\frac{k_2}{2}} + \frac{s}{1+f}\left[\ln\left(\frac{k_2}{2}\right) - \ln(1-x_{max})\right] \\
&\quad + \frac{l_2 s}{1+f}\left[\ln(1-x_{max}) - \ln\left(1-\frac{k_1}{2}\right)\right] + \left(1-\frac{k_1}{2}\right)\frac{l_2 s}{(1+f)\left(1-\frac{k_1}{2}\right)} \\
&= \frac{s}{(1+f)}\left\{\left(\frac{1-\frac{k_2}{2}}{\frac{k_2}{2}}\right) + \left[\ln\left(\frac{k_2}{2}\right) - \ln(1-x_{max})\right] + l_2 + l_2\left[\ln(1-x_{max}) - \ln\left(1-\frac{k_1}{2}\right)\right]\right\}
\end{aligned}$$

and the expected return of investing in country 1 is given by

$$\begin{aligned}
\Pi_1(\delta^*) &= \int_0^{x_{min}} \frac{l_1}{(1+f)\rho(\delta^*,x)}dx + \int_{x_{min}}^1 \frac{1}{(1+f)\rho(\delta^*,x)}dx \\
&= \int_0^{1-\frac{k_2}{2}} \frac{l_1}{(1+f)\left(1-\frac{k_2}{2}\right)}dx + \int_{1-\frac{k_2}{2}}^{x_{min}} \frac{l_1}{(1+f)x}dx \\
&\quad + \int_{x_{min}}^{\frac{k_1}{2}} \frac{1}{(1+f)x}dx + \int_{\frac{k_1}{2}}^1 \frac{1}{(1+f)\frac{k_1}{2}}dx \\
&= \left(1-\frac{k_2}{2}\right)\frac{l_1}{(1+f)\left(1-\frac{k_2}{2}\right)} + \frac{l_1}{1+f}\left[\ln(x_{min}) - \ln\left(1-\frac{k_2}{2}\right)\right] \\
&\quad + \frac{1}{1+f}\left[\ln\left(\frac{k_1}{2}\right) - \ln(x_{min})\right] + \left(1-\frac{k_1}{2}\right)\frac{1}{(1+f)\frac{k_1}{2}} \\
&= \frac{1}{1+f}\left\{l_1 + l_1\left[\ln(x_{min}) - \ln\left(1-\frac{k_2}{2}\right)\right] + \left[\ln\left(\frac{k_1}{2}\right) - \ln(x_{min})\right] + \left(\frac{1-\frac{k_1}{2}}{\frac{k_1}{2}}\right)\right\}
\end{aligned}$$

Note that

$$\begin{aligned}
\left(\frac{1-\frac{k_1}{2}}{\frac{k_1}{2}}\right) = \left(\frac{1}{\frac{k_1}{2}} - 1\right) &= 1 + sl_2 - 1 = sl_2 \\
\left(\frac{1-\frac{k_2}{2}}{\frac{k_2}{2}}\right) = \left(\frac{1}{\frac{k_2}{2}} - 1\right) &= \frac{s+l_1}{s} - \frac{s}{s} = \frac{l_1}{s}
\end{aligned}$$

Setting these equal, we have

$$\begin{aligned}
& s\left\{\frac{l_1}{s} + \left[\ln\left(\frac{k_2}{2}\right) - \ln(1-x_{max})\right] + l_2 + l_2\left[\ln(1-x_{max}) - \ln\left(1-\frac{k_1}{2}\right)\right]\right\} \\
=& \left\{l_1 + l_1\left[\ln(x_{min}) - \ln\left(1-\frac{k_2}{2}\right)\right] + \left[\ln\left(\frac{k_1}{2}\right) - \ln(x_{min})\right] + sl_2\right\}
\end{aligned}$$

Plugging in for k_1, k_2 and

$$\begin{aligned} \frac{k_1}{2} &= \frac{1}{1+l_2 s} \\ \frac{k_2}{2} &= \frac{s}{s+l_1} \\ 1-\frac{k_1}{2} &= \frac{l_2 s}{1+l_2 s} \\ 1-\frac{k_2}{2} &= \frac{l_1}{s+l_1} \\ \ln\left(x_{min}\right) &= -z-\delta^* \\ \ln\left(1-x_{max}\right) &= -z+\delta^*+\ln s \end{aligned}$$

Setting these equal, we have

$$\begin{aligned} & s\left\{\left[\ln\left(\frac{k_2}{2}\right)-\ln\left(1-x_{max}\right)\right]+l_2\left[\ln\left(1-x_{max}\right)-\ln\left(1-\frac{k_1}{2}\right)\right]\right\} \\ &= l_1\left[\ln\left(x_{min}\right)-\ln\left(1-\frac{k_2}{2}\right)\right]+\left[\ln\left(\frac{k_1}{2}\right)-\ln\left(x_{min}\right)\right] \\ \iff & s\left\{-\left(1-l_2\right)\ln\left(1-x_{max}\right)+\left[\ln\left(\frac{k_2}{2}\right)-l_2\ln\left(1-\frac{k_1}{2}\right)\right]\right\} \\ &= -\left(1-l_1\right)\ln\left(x_{min}\right)+\left[\ln\left(\frac{k_1}{2}\right)-l_1\ln\left(1-\frac{k_2}{2}\right)\right] \\ \iff & s\left\{\left(1-l_2\right)\left(z-\delta^*-\ln s\right)+\left[\ln\left(\frac{s}{s+l_1}\right)-l_2\ln\left(\frac{l_2 s}{1+l_2 s}\right)\right]\right\} \\ &= \left(1-l_1\right)\left(z+\delta^*\right)+\left[\ln\left(\frac{1}{1+l_2 s}\right)-l_1\ln\left(\frac{l_1}{s+l_1}\right)\right] \end{aligned}$$

Finally, solving for δ^*, we have

$$\begin{aligned} \delta^* &= \frac{s\left\{\left(1-l_2\right)\left(z-\ln s\right)+\left[\ln\left(\frac{s}{s+l_1}\right)-l_2\ln\left(\frac{l_2 s}{1+l_2 s}\right)\right]\right\}-\left(1-l_1\right)z-\left[\ln\left(\frac{1}{1+l_2 s}\right)-l_1\ln\left(\frac{l_1}{s+l_1}\right)\right]}{\left(1-l_1\right)+s\left(1-l_2\right)} \\ &= \frac{s\left\{\left(1-l_2\right)z-\left(1-l_2\right)\ln s+\ln s-\ln\left(s+l_1\right)-l_2\ln l_2-l_2\ln s+l_2\ln\left(1+l_2 s\right)\right\}}{\left(1-l_1\right)+s\left(1-l_2\right)} \\ &\quad+\frac{-\left(1-l_1\right)z+\ln\left(1+l_2 s\right)+l_1\ln\left(l_1\right)-l_1\ln\left(s+l_1\right)}{\left(1-l_1\right)+s\left(1-l_2\right)} \end{aligned}$$

so that finally

$$\delta^* = \frac{\left[\left(1-l_2\right)s-\left(1-l_1\right)\right]z-\left(s+l_1\right)\ln\left(s+l_1\right)+\left(1+sl_2\right)\ln\left(1+l_2 s\right)+l_1\ln l_1-sl_2\ln l_2}{\left(1-l_1\right)+s\left(1-l_2\right)} \tag{B.2}$$

Plugging in $l_1 = l_2 = 0$, we have

$$\delta^* = \frac{-\left(1-s\right)z-s\ln\left(s\right)}{1+s}$$

our benchmark result absent recovery. This is the only single-survivor equilibrium supported by threshold strategies.

We want to show that from the perspective of δ^*, for an x small enough so that $\rho\left(\delta^*, x\right) = 1-\frac{k_2}{2}$, does country 1 default? We know that $\rho_{min}(\delta^*) = e^{-z}e^{-\delta^*}$, so that

$$\begin{aligned} \rho_{min}(\delta^*) &> 1 - \frac{k_2}{2} \\ \iff \ln(\rho_{min}(\delta^*)) &> \ln\left(1 - \frac{k_2}{2}\right) \\ \iff -(\delta^* + z) &> \ln\left(\frac{l_1}{s + l_1}\right) \end{aligned}$$

which gives

$$\begin{aligned} & -[2(1-l_2)sz - (s+l_1)\ln(s+l_1) + (1+sl_2)\ln(1+l_2 s) + l_1 \ln l_1 - sl_2 \ln l_2] \\ > \quad & [(1-l_1) + s(1-l_2)][\ln l_1 - \ln(s+l_1)] \end{aligned}$$

and ultimately yields

$$F_1^*(l_1, l_2, s)] \equiv -2(1-l_2)sz - [1 + s(1-l_2)]\ln l_1 + sl_2 \ln l_2 + [1 + s(2-l_2)]\ln(s+l_1) - (1+l_2 s)\ln(1+l_2 s)$$

and the default condition is given by $F_1^*(l_1, l_2, s) \geq 0$. Assume $l_1 = l_2 = l$. Then, we have

$$F_1^*(l, l, s) = -2(1-l)sz - [1 - (1-2l)s]\ln l + [1 + s(2-l)]\ln(s+l) - (1+ls)\ln(1+ls)$$

We can show that $F_1^*(l, l, s)$ is always positive for small enough recovery l as the term $-[1-(1-2l)s]\ln l$ explodes, swamping any negative z effect.[2]

Next, we want to show that from the perspective of δ^*, for an x large enough so that $\rho(\delta^*, x) = \frac{k_1}{2}$, does country 2 default? We know that $1 - \rho_{max}(\delta^*) = se^{-z}e^{\delta^*}$, so that

$$\begin{aligned} 1 - \rho_{max}(\delta^*) &> 1 - \frac{k_1}{2} \\ \iff \ln(1 - \rho_{max}(\delta^*)) &> \ln\left(1 - \frac{k_1}{2}\right) \\ \iff \ln s - z + \delta^* &> \ln\left(\frac{l_2 s}{1 + l_2 s}\right) \end{aligned}$$

so that

$$\begin{aligned} & [(1-l_1) + s(1-l_2)]\ln s - 2(1-l_1)z - (s+l_1)\ln(s+l_1) + (1+sl_2)\ln(1+l_2 s) + l_1 \ln l_1 - sl_2 \ln l_2 \\ > \quad & [(1-l_1) + s(1-l_2)][\ln l_2 + \ln s - \ln(1+l_2 s)] \end{aligned}$$

Define

$$F_2^*(l_1, l_2, s) \equiv -2(1-l_1)z - (s+l_1)\ln(s+l_1) + (2-l_1+s)\ln(1+l_2 s) + l_1 \ln l_1 - [s + (1-l_1)]\ln l_2$$

and the default condition is given by $F_2^*(l_1, l_2, s) \geq 0$. Assuming equal recovery $l_1 = l_2 = l$, we have

$$F_2^*(l, l, s) = -2(1-l)z - (s+l)\ln(s+l) + (2-l+s)\ln(1+ls) - [s + (1-2l)]\ln l$$

We can show that $F_2^*(l, l, s)$ is always positive for small enough recovery l as the term $-[s+(1-2l)]\ln l$ explodes, swamping any negative z effect.

Let us consider an interior agent, i.e., $\delta \in [\delta^* - k_2\sigma, \delta^* + k_1\sigma]$. Let

$$\delta(\varepsilon) = \delta^* + 2\varepsilon\sigma$$

[2]Taking derivatives w.r.t. l and s, we have

$$\begin{aligned} \partial_l F_1^*(l, l, s) &= 2sz + s - \frac{(1+s)}{l} + \frac{1 + (2-l)s}{s+l} + 2s\ln l - s\ln(s+l) - s\ln(1+ls) \\ \partial_s F_1^*(l, l, s) &= \frac{1 + (2-l)s}{s+l} - l\ln(1+ls) + (2-l)\ln(s+l) - 2(1-l)z - l - (1-2l)\ln l \end{aligned}$$

with $\varepsilon \in \left[-\frac{k_2}{2}, \frac{k_1}{2}\right]$. Let us first consider investment in country 1. We have $\rho_{min}(\delta)$ as the default boundary, and actual investment is given by

$$\rho(\delta, x) = \begin{cases} 1 - \frac{k_2}{2}, & \delta^* + \varepsilon 2\sigma + 2\sigma x < \delta^* + (2 - k_2)\sigma \\ x + cst, & else \\ \frac{k_1}{2} & \delta^* + \varepsilon 2\sigma - 2\sigma(1 - x) > \delta^* - (2 - k_1)\sigma \end{cases} = \begin{cases} 1 - \frac{k_2}{2}, & 2\varepsilon\sigma + 2\sigma x < (2 - k_2)\sigma \\ x + cst, & else \\ \frac{k_1}{2} & 2\varepsilon\sigma - 2\sigma(1 - x) > -(2 - k_1)\sigma \end{cases}$$

which gives

$$\rho(\delta, x) = \begin{cases} 1 - \frac{k_2}{2}, & \varepsilon + x < 1 - \frac{k_2}{2} \\ x + \varepsilon, & else \\ \frac{k_1}{2} & \varepsilon + x > \frac{k_1}{2} \end{cases}$$

Note that we have $cst = \varepsilon$ by imposing continuity (which has to follow from $\rho(\delta, x)$ being an integral over strategies ϕ).

Let $x_{min}(\delta)$ be the lowest $x \in [0, 1]$ such that

$$\rho(\delta, x) = \varepsilon + x \geq \rho_{min}(\delta)$$

and we therefore have

$$x_{min}(\delta) = \max\{\rho_{min}(\delta) - \varepsilon, 0\}$$

Similarly, let $x_{max}(\delta)$ be the highest $x \in [0, 1]$ such that

$$1 - \rho(\delta, x) = 1 - \varepsilon - x \geq 1 - \rho_{max}(\delta)$$

and thus

$$1 - x_{max}(\delta) = \max\{1 - \rho_{max}(\delta) + \varepsilon, 0\}$$

The expected return of investing in country 1 is then given by

$$\begin{aligned}
\Pi_1(\delta) &= \int_{x:\rho(\delta,x)<\rho_{min}(x)} \frac{l_1}{(1+f)\,\rho(\delta,x)}dx + \int_{x:\rho(\delta,x)\geq\rho_{min}(x)} \frac{1}{(1+f)\,\rho(\delta,x)}dx \\
&= \int_0^{x_{min}(\delta)} \frac{l_1}{(1+f)\,\rho(\delta,x)}dx + \int_{x_{min}(\delta)}^1 \frac{1}{(1+f)\,\rho(\delta,x)}dx \\
&= \int_0^{1-\frac{k_2}{2}-\varepsilon} \frac{l_1}{(1+f)\left(1-\frac{k_2}{2}\right)}dx + \int_{1-\frac{k_2}{2}-\varepsilon}^{x_{min}(\delta)} \frac{l_1}{(1+f)\,(x+\varepsilon)}dx \\
&\quad + \int_{x_{min}(\delta)}^{\frac{k_1}{2}-\varepsilon} \frac{1}{(1+f)\,(x+\varepsilon)}dx + \int_{\frac{k_1}{2}-\varepsilon}^1 \frac{1}{(1+f)\,\frac{k_1}{2}}dx \\
&= \frac{l_1}{1+f}\left[\frac{1-\frac{k_2}{2}-\varepsilon}{1-\frac{k_2}{2}} + \ln\left(x_{min}(\delta)+\varepsilon\right) - \ln\left(1-\frac{k_2}{2}\right)\right] \\
&\quad + \frac{1}{1+f}\left[\ln\left(\frac{k_1}{2}\right) - \ln\left(x_{min}(\delta)+\varepsilon\right) + \frac{1-\frac{k_1}{2}+\varepsilon}{\frac{k_1}{2}}\right] \\
&= \frac{l_1}{1+f}\left[1-\frac{\varepsilon}{1-\frac{k_2}{2}} + \ln\left(x_{min}(\delta)+\varepsilon\right) - \ln\left(1-\frac{k_2}{2}\right)\right] \\
&\quad + \frac{1}{1+f}\left[\ln\left(\frac{k_1}{2}\right) - \ln\left(x_{min}(\delta)+\varepsilon\right) + \frac{1-\frac{k_1}{2}}{\frac{k_1}{2}} + \frac{\varepsilon}{\frac{k_1}{2}}\right] \\
&= \Pi_1(\delta^*) + \frac{l_1}{1+f}\left[-\frac{\varepsilon}{1-\frac{k_2}{2}} + \ln\left(x_{min}(\delta)+\varepsilon\right) - \ln x_{min}(\delta^*)\right] \\
&\quad + \frac{1}{1+f}\left[\ln x_{min}(\delta^*) - \ln\left(x_{min}(\delta)+\varepsilon\right) + \frac{\varepsilon}{\frac{k_1}{2}}\right] \\
&= \Pi_1(\delta^*) + \frac{1}{1+f}\left\{\varepsilon\left(\frac{1}{\frac{k_1}{2}} - \frac{l_1}{1-\frac{k_2}{2}}\right) - (1-l_1)\left[\ln\left(x_{min}(\delta)+\varepsilon\right) - \ln x_{min}(\delta^*)\right]\right\} \\
&= \Pi_1(\delta^*) + \frac{1}{1+f}\left\{\varepsilon\left[(1-l_1) - s\,(1-l_2)\right] - (1-l_1)\left[\ln\left(x_{min}(\delta)+\varepsilon\right) - \ln x_{min}(\delta^*)\right]\right\}
\end{aligned}$$

Similarly, investing in country 2 gives

$$\begin{aligned}
\Pi_2(\delta) &= \int_0^{x_{max}(\delta)} \frac{s}{(1+f)\,(1-\rho(\delta,x))}dx + \int_{x_{max}(\delta)}^1 \frac{l_2 s}{(1+f)\,(1-\rho(\delta,x))}dx \\
&= \int_0^{1-\frac{k_2}{2}-\varepsilon} \frac{s}{(1+f)\left(1-\left(1-\frac{k_2}{2}\right)\right)}dx + \int_{1-\frac{k_2}{2}-\varepsilon}^{x_{max}(\delta)} \frac{s}{(1+f)\,(1-x-\varepsilon)} \\
&\quad + \int_{x_{max}(\delta)}^{\frac{k_1}{2}-\varepsilon} \frac{l_2 s}{(1+f)\,(1-x-\varepsilon)}dx + \int_{\frac{k_1}{2}-\varepsilon}^1 \frac{l_2 s}{(1+f)\left(1-\frac{k_1}{2}\right)}dx \\
&= \frac{s}{1+f}\left[\frac{1-\frac{k_2}{2}-\varepsilon}{\frac{k_2}{2}} + \ln\left(\frac{k_2}{2}\right) - \ln\left(1-x_{max}(\delta)-\varepsilon\right)\right] \\
&\quad + \frac{sl_2}{1+f}\left[\ln\left(1-x_{max}(\delta)-\varepsilon\right) - \ln\left(1-\frac{k_1}{2}\right) + \frac{1-\frac{k_1}{2}+\varepsilon}{1-\frac{k_1}{2}}\right] \\
&= \Pi_2(\delta^*) + \frac{s}{1+f}\left\{\varepsilon\left(l_2\frac{1}{1-\frac{k_1}{2}} - \frac{1}{\frac{k_2}{2}}\right) + (1-l_2)\left[\ln\left(1-x_{max}(\delta^*)\right) - \ln\left(1-x_{max}(\delta)-\varepsilon\right)\right]\right\} \\
&= \Pi_2(\delta^*) + \frac{s}{1+f}\left\{\varepsilon\left[\frac{(1-l_1)-s\,(1-l_2)}{s}\right] + (1-l_2)\left[\ln\left(1-x_{max}(\delta^*)\right) - \ln\left(1-x_{max}(\delta)-\varepsilon\right)\right]\right\}
\end{aligned}$$

Let us define

$$\begin{aligned}
g(\varepsilon) &\equiv (1+f)\left[\Pi_1(\delta)-\Pi_2(\delta)\right] \\
&= \varepsilon\left[(1-l_1)-s(1-l_2)\right]-(1-l_1)\left[\ln\left(x_{min}(\delta)+\varepsilon\right)-\ln x_{min}(\delta^*)\right] \\
&\quad -s\left\{\varepsilon\left[\frac{(1-l_1)-s(1-l_2)}{s}\right]+(1-l_2)\left[\ln\left(1-x_{max}(\delta^*)\right)-\ln\left(1-x_{max}(\delta)-\varepsilon\right)\right]\right\} \\
&= -(1-l_1)\left[\ln\left(x_{min}(\delta^*+2\sigma\varepsilon)+\varepsilon\right)-\ln x_{min}(\delta^*)\right] \\
&\quad +s(1-l_2)\left[\ln\left(1-x_{max}(\delta^*+2\sigma\varepsilon)-\varepsilon\right)-\ln\left(1-x_{max}(\delta^*)\right)\right] \\
&\quad +\varepsilon\left\{\left[(1-l_1)-s(1-l_2)\right]-s\left[\frac{(1-l_1)-s(1-l_2)}{s}\right]\right\} \\
&= -(1-l_1)\left[\ln\left(x_{min}(\delta^*+2\sigma\varepsilon)+\varepsilon\right)-\ln x_{min}(\delta^*)\right] \\
&\quad +s(1-l_2)\left[\ln\left(1-x_{max}(\delta^*+2\sigma\varepsilon)-\varepsilon\right)-\ln\left(1-x_{max}(\delta^*)\right)\right]
\end{aligned}$$

Taking the derivative w.r.t. ε, we have many different cases. The issue is if x_{min} or x_{max} start binding first. Regardless, close to $\varepsilon=0$ we have neither x_{min} or x_{max} cornered, so that

$$\begin{aligned}
\ln\left(x_{min}(\delta^*+2\sigma\varepsilon)+\varepsilon\right) &= \ln\left(\rho_{min}(\delta(\varepsilon))\right)=-z-\delta(\varepsilon)=-z-(\delta^*+2\sigma\varepsilon) \\
\ln\left(1-x_{max}(\delta^*+2\sigma\varepsilon)-\varepsilon\right) &= \ln\left(1-\rho_{max}(\delta(\varepsilon))\right)=s\ln s-z+\delta(\varepsilon)=s\ln s-z+(\delta^*+2\sigma\varepsilon)
\end{aligned}$$

and thus for ε small we have

$$g'(\varepsilon)=-(1-l_1)(-)2\sigma+s(1-l_2)2\sigma=2\sigma\left[(1-l_1)+s(1-l_2)\right]>0$$

and indeed we have the incentives of the agents aligned with the conjectured strategies, at least around δ^*.

Next, we have to account for all the different cases – that is, we know that at some distance ε that x_{min}, x_{max} start binding at $0,1$, respectively.

Let ε_{min} be the point at which x_{min} becomes cornered, that is

$$\rho_{min}(\delta)=\varepsilon \iff e^{-z}e^{-(\delta^*+2\sigma\varepsilon)}=\varepsilon \iff 2\sigma\varepsilon+\ln\varepsilon=-z-\delta^*$$

Note that $\rho_{min}(\delta)>0$ so that there is no solution for $\varepsilon<0$.

Similarly, let ε_{max} be the point at which x_{max} becomes cornered, that is

$$1-\rho_{max}(\delta)=-\varepsilon \iff se^{-z}e^{\delta^*+2\sigma\varepsilon}=-\varepsilon \iff 2\sigma(-\varepsilon)+\ln(-\varepsilon)=\ln s-z+\delta^*$$

Note that $1-\rho_{max}(\delta)\geq 0$ so that there is no solution for $\varepsilon>0$. Positive ε. Consider positive ε. Thus, we only have to worry about x_{min} cornered. When x_{min} becomes cornered, then

$$\frac{\partial}{\partial\varepsilon}\ln\left(x_{min}(\delta^*+2\sigma\varepsilon)+\varepsilon\right)=\frac{1}{\varepsilon}$$

Then, we have

$$g'(\varepsilon)=-(1-l_1)\frac{1}{\varepsilon}+s(1-l_2)2\sigma$$

The derivative is increasing in ε, and is largest at $\varepsilon=\frac{k_1}{2}$ at a value of

$$g'\left(\frac{k_1}{2}\right)=-(1-l_1)(1+l_2s)+s(1-l_2)2\sigma$$

For small enough σ, this is always negative. Negative ε. Consider negative ε. Thus, we only have to worry about x_{max} cornered. When x_{max} becomes cornered, then

$$\frac{\partial}{\partial\varepsilon}\ln\left(1-x_{max}(\delta^*+2\sigma\varepsilon)-\varepsilon\right)=-\frac{1}{\varepsilon}$$

Then, we have

$$g'(\varepsilon)=(1-l_1)2\sigma+s(1-l_2)\left(-\frac{1}{\varepsilon}\right)$$

The derivative is again increasing in ε, and is largest at $\varepsilon = -\frac{k_2}{2}$ at a value of

$$g'\left(-\frac{k_2}{2}\right) = -\left(1 - l_2\right)\left(s + l_1\right) + \left(1 - l_1\right) 2\sigma$$

For small enough σ, this is always negative.

For $s = 1$ and $l_1 = l_2 = l$, we have symmetric conditions.

The last thing we need to do is to check that

$$g\left(-\frac{k_2}{2}\right) = g(0) = g\left(\frac{k_1}{2}\right) = 0$$

To this end, we can also proof that as $\sigma \to 0$, indeed one country (which one depending on on which side of δ^* the realization of δ falls) will always default. This is equivalent to the interior assumption for x_{max}, x_{min} we made. For this to hold, we need the following restrictions

$$1 - \frac{k_1}{2} \leq 1 - \rho_{max}\left(\delta^*\right) \leq \frac{k_2}{2} \tag{B.3}$$

$$1 - \frac{k_2}{2} \leq \rho_{min}\left(\delta^*\right) \leq \frac{k_1}{2} \tag{B.4}$$

The first line says that as $\sigma \to 0$, if $\delta < \delta^*$ then a proportion $\frac{k_2}{2}$ of investors invests in country 2, and it survives. However, if $\delta > \delta^*$, then only a proportion $1 - \frac{k_1}{2}$ of investors invests in country 2, and it defaults. Similar arguments hold for country 1, which is summarized by the second line.

This can be rewritten as

$$\begin{aligned} \ln\left(1 - \frac{k_1}{2}\right) &\leq \ln\left(1 - \rho_{max}\left(\delta^*\right)\right) \leq \ln\left(\frac{k_2}{2}\right) \\ \ln\left(1 - \frac{k_2}{2}\right) &\leq \ln \rho_{min}\left(\delta^*\right) \leq \ln\left(\frac{k_1}{2}\right) \end{aligned}$$

which gives

$$\begin{aligned} \ln\left(\frac{l_2 s}{1 + l_2 s}\right) &\leq \ln s - z + \delta^* \leq \ln\left(\frac{s}{s + l_1}\right) \\ \ln\left(\frac{l_1}{s + l_1}\right) &\leq -z - \delta^* \leq \ln\left(\frac{1}{1 + l_2 s}\right) \end{aligned}$$

equivalent to

$$\begin{aligned} \ln\left(\frac{l_2}{1 + l_2 s}\right) + z &\leq \delta^* \leq \ln\left(\frac{1}{s + l_1}\right) + z \\ \ln\left(\frac{l_1}{s + l_1}\right) + z &\leq -\delta^* \leq \ln\left(\frac{1}{1 + l_2 s}\right) + z \end{aligned}$$

equivalent to

$$\begin{aligned} \ln\left(l_2\right) - \ln\left(1 + l_2 s\right) + z &\leq \delta^* \leq -\ln\left(s + l_1\right) + z \\ -\ln\left(\frac{1}{1 + l_2 s}\right) - z &\leq \delta^* \leq -\ln\left(\frac{l_1}{s + l_1}\right) - z \end{aligned}$$

equivalent to

$$\begin{aligned} \ln\left(l_2\right) - \ln\left(1 + l_2 s\right) + z &\leq \delta^* \leq -\ln\left(s + l_1\right) + z \\ \ln\left(1 + l_2 s\right) - z &\leq \delta^* \leq \ln\left(s + l_1\right) - \ln\left(l_1\right) - z \end{aligned}$$

so that finally

$$\max\left[\ln\left(l_2\right) - \ln\left(1 + l_2 s\right) + z, \ln\left(1 + l_2 s\right) - z\right] \leq \delta^* \leq \min\left[-\ln\left(s + l_1\right) + z, \ln\left(s + l_1\right) - \ln\left(l_1\right) - z\right] \tag{B.5}$$

The first term is binding on the RHS for $z > \ln\left(1 + l_2 s\right) - \frac{1}{2}\ln\left(l_2\right)$, and the first term is binding on the left hand side for $z < \ln\left(s + l_1\right) - \frac{1}{2}\ln\left(l_1\right)$.

Appendix C: Robustness Common Bonds

Notational Convention: We will refer to Common Bonds (aka Eurobonds) as asset 0, their price per unit of face-value as p_0, and the proportion of investors investing in common bonds as ρ_0.

We maintain the main assumptions of the sequential setup: (i) there is an amount (face-value) $\alpha(1+s)$ of common bonds and an amount $(1-\alpha)s_i$ of individual bonds of country i available, (ii) each unit of common bonds (that is, per unit of face-value) is made up of $\frac{1}{1+s}$ units of country 1 bonds and $\frac{s}{1+s}$ units of country 2 bonds, and (iii) issuance proceeds of the common bonds accrue in proportions $\frac{1}{1+s}$ and $\frac{s}{1+s}$ to country 1 and 2, respectively.

We are looking for a simultaneous three asset equilibrium between assets 0,1, and 2 that has the *single-survivor* property, i.e., only one country survives. We will analyze the following oscillation strategy:

$$\underset{\text{proportion}}{\text{investment}}...\underset{\alpha}{|0|}\underset{(1-\alpha)}{2}\underset{\alpha}{|0|}\underset{(1-\alpha)}{2}\underset{h}{|\mathbf{0}|}\underset{(1-\alpha)}{1}\underset{\alpha}{|0|}\underset{(1-\alpha)}{1}\underset{\alpha}{|0|}...$$

We will sometimes refer to the central interval $\mathbf{0}$ as the central region, the changeover region or loosely the survival cutoff. The intuition of the strategy is as follows: when one country defaults for sure, the no arbitrage condition between the surviving country and the common bond requires investors to invests in proportions $(1-\alpha)$ and α into the surviving country's bonds and common bonds, respectively. Next, let us consider fundamentals close to the changeover region in which default risk of both countries appears. As the fundamental δ increases, country 2 becomes riskier and country 1 becomes safer. As a consequence, with common bonds being a portfolio of individual bonds, common bonds' value moves less than the individual country bonds. Thus, to achieve indifference, we would have to increase investment in common bonds to decrease common bond returns to a level on par with individual bonds around the central region when default risk starts affecting both countries. In particular, for any $\sigma > 0$, in such a region our strategy requires endogenous investment in the common bond on an interval $[\delta_L, \delta_H]$ of length $2\sigma \cdot h$, i.e., we have two degrees of freedom in the two points δ_L and δ_H, as described in more detail below. Importantly, for such a construction to be an equilibrium and still be tractable, we require that any such construction does not necessitate any further endogenous adjustment of the strategies away from the interval $[\delta_L, \delta_H]$. We term such a property *insulated* – an *insulated equilibrium* only depends on endogenous variables around the survival cutoff and does not require any further endogenous variables away from it.

Formally, let the (endogenous) width of the interval $\mathbf{0}$ be given by $2\sigma \cdot h$, while the intervals 1 and 2 have width $(1-\alpha)2\sigma$, and the intervals 0 have width $\alpha \cdot 2\sigma$. Further, let δ_L and δ_H denote the lower and upper end of interval $\mathbf{0}$, so that $h = \frac{\delta_H - \delta_L}{2\sigma}$. Second, we note that when we take $\sigma \to 0$, we have $\delta_L \to \delta^* \leftarrow \delta_H$ as long as h remains finite. Thus, in the limit, we transform the two degrees of freedom from (δ_L, δ_H) to (δ^*, h). For any strategy to yield an insulated equilibrium we require $h > \alpha$.[3] Lastly, we note that in the limit $\sigma \to 0$, we have

$$x_{min}(\delta_L) = x_{min}(\delta_H) + h$$
$$x_{max}(\delta_L) = x_{max}(\delta_H) + h$$

Suppose that country i is safe almost surely, and country $-i$ defaults almost surely. Then, no arbitrage between country i's bond (paying of 1 per unit of face-value) and the common bond 0 requires

$$\frac{1}{p_i} = \frac{\frac{s_i}{1+s}}{p_0}$$

[3]In case $h < \alpha$, we can still solve for δ^* and h, but realize that some of the payoffs $\Pi_i(\delta)$ away from δ_L and δ_H do not converge to indifference: at least for some $\delta < \delta_L$, we do not have indifference at oscillation widths $1-\alpha$ and α – this is easiest to see when we consider $\delta = \delta_L - (1-\alpha)2\sigma$; at this point there is still some influence of h as the no-arbitrage proportions, if indeed we assume play according to $1-\alpha$ and α away from δ_L, do not actually yield no arbitrage because of $h + (1-\alpha) < 1$ and so the proportions are off. Instead, we would need to build a sequence of intervals of endogenous width (similar to how we derived h) to make sure indifference holds at all δ's away from δ_L. But this any such equilibrium is not insulated anymore, as we now need to solve for an infinite number of endogenous intervals. Consequently, we are not succeeding at reducing the dimensionality of the problem significantly, and it remains intractable. If, however, the equilibrium fulfills $h > \alpha$, it is insulated, and the dimensionality reduces significantly to just (δ^*, h), making the model tractable. Some generalization of single-survivor equilibria can still be achieved in insulated form, but joint-safety equilibria immediately violate the insulated character of the equilibrium.

The supply of each bond is $(1-\alpha)\, s_i$ and $\alpha\,(1+s)$, respectively. Let ρ_i be the proportion of money flowing to bond i. Then, we must have

$$\begin{aligned}(1-\alpha)\, s_i p_i &= \rho_i\,(1+f)\\ \alpha\,(1+s)\, p_0 &= \rho_0\,(1+f) = (1-\rho_i)\,(1+f)\end{aligned}$$

where $\rho_0 = (1-\rho_i)$ and $\rho_{-i} = 0$. Plugging these into the no arbitrage condition, we have

$$(1+s)\, p_0 = s_i p_i \iff \frac{(1-\rho_i)\,(1+f)}{\alpha} = \frac{\rho_i\,(1+f)}{1-\alpha} \iff \rho_i = 1-\alpha$$

and $\rho_0 = (1-\rho_i) = \alpha$. Thus, regardless which country is considered "safe", as long as investors are certain of the safety of i they should invest their money in aggregate proportions $1-\alpha$ and α in the safe individual and common bonds, respectively. These no arbitrage investment proportions are incorporate via oscillation outside of the central interval $\mathbf{0}$ in proportions $\rho_i = 1-\alpha$ and $\rho_0 = \alpha$.

Finally, the default condition for country i is given by

$$\underbrace{\frac{s_i}{1+s}\,(1-\rho_1-\rho_2)}_{\text{Common bond revenue}} + \underbrace{\rho_i}_{\text{Individual bond revenue}} \geq s_i\,\frac{1-\theta_i}{1+f} = s_i e^{-z} e^{(-1)^i \delta}$$

Because the no-arbitrage proportions around the outside the central region are symmetric, we do not have separate cases for δ_L and δ_H. For δ_L, the cutoffs are $h, h+1-\alpha, \alpha, 1$, whereas for δ_H, the cutoffs are $0, 1-\alpha, \alpha-h, 1-h$. This abstractly leads to 5 different cases:

C1 $0 < h < \alpha < h+1-\alpha < 1$ equivalent to $0 < \alpha-h < 1-\alpha < 1-h$. We will ignore this case as we are concentrating on an insulated equilibrium with $h > \alpha$.

C2 $0 < h < h+1-\alpha < \alpha < 1$ equivalent to $0 < 1-\alpha < \alpha-h < 1-h$. We will ignore this case as we are concentrating on an insulated equilibrium with $h > \alpha$.

C3 $0 < \alpha < h < 1 < h+1-\alpha$ equivalent to $\alpha-h < 0 < 1-h < 1-\alpha$. This is a case consistent with an insulated equilibrium.

C4 $0 < \alpha < h < h+1-\alpha < 1$ equivalent to $\alpha-h < 0 < 1-\alpha < 1-h$. But this cases is impossible as $h+1-\alpha < 1 \iff h < \alpha$ which contradicts $\alpha < h$.

C5 $0 < h < \alpha < 1 < h+1-\alpha$ equivalent to $0 < \alpha-h < 1-h < 1-\alpha$. But this case is impossible as $1 < h+1-\alpha \iff \alpha < h$ which contradicts $h < \alpha$.

Thus, our analysis will focus solely on case C3. Lower boundary δ_L

C3 $0 < \alpha < h < 1 < h+1-\alpha$

$$\rho_1\left(\delta_L, x\right) = \begin{cases} 0 & (0,h)\\ x-h & (h,1)\end{cases}$$

$$\rho_2\left(\delta_L, x\right) = \begin{cases} 1-\alpha & (0,\alpha)\\ 1-x & (\alpha,1)\end{cases}$$

$$\rho_1\left(\delta_L, x\right) + \rho_2\left(\delta_L, x\right) = \begin{cases} 1-\alpha & (0,\alpha)\\ 1-x & (\alpha,h)\\ 1-h & (h,1)\end{cases}$$

For interior equilibria, we need $x_{min}\left(\delta_L\right) \in (h,1)$ and $x_{max}\left(\delta_L\right) \in (\alpha,1)$.

Upper boundary δ_H

C3 $0 < \alpha < h < 1 < h+1-\alpha$ equivalent to $\alpha-h < 0 < 1-h < 1-\alpha$

$$\rho_1\left(\delta_H, x\right) = \begin{cases} x & (0,1-\alpha)\\ 1-\alpha & (1-\alpha,1)\end{cases}$$

$$\rho_2\left(\delta_H, x\right) = \begin{cases} 1-x-h & (0,1-h)\\ 0 & (1-h,1)\end{cases}$$

$$\rho_1\left(\delta_H, x\right) + \rho_2\left(\delta_H, x\right) = \begin{cases} 1-h & (0,1-h)\\ x & (1-h,1-\alpha)\\ 1-\alpha & (1-\alpha,1)\end{cases}$$

For interior equilibria, we need $x_{min}(\delta_H) \in (0, 1-\alpha)$ and $x_{max}(\delta_H) \in (0, 1-h)$.

Simultaneous equations when $\sigma \to 0$

C3 $0 < \alpha < h < 1 < h + 1 - \alpha$ equivalent to $\alpha - h < 0 < 1 - h < 1 - \alpha$

$$\begin{aligned}
\Pi_1(\delta_L) &= (1-\alpha)\left[\int_{x_{min}(\delta_L)}^{1} \frac{1}{\rho_1(\delta_L, x)} dx\right] \\
&= (1-\alpha)\left[\int_{x_{min}(\delta_L)}^{1} \frac{1}{x-h} dx\right] \\
&= (1-\alpha)\left[\ln(1-h) - \ln(x_{min}(\delta_L) - h)\right] \\
\Pi_2(\delta_L) &= (1-\alpha)\, s \left[\int_{0}^{x_{max}(\delta_L)} \frac{1}{\rho_2(\delta_L, x)} dx\right] \\
&= (1-\alpha)\, s \left[\int_{0}^{\alpha} \frac{1}{1-\alpha} dx + \int_{\alpha}^{x_{max}(\delta_L)} \frac{1}{1-x} dx\right] \\
&= (1-\alpha)\, s \left[\frac{\alpha}{1-\alpha} + \ln(1-\alpha) - \ln(1 - x_{max}(\delta_L))\right]
\end{aligned}$$

$$\begin{aligned}
\Pi_1(\delta_H) &= (1-\alpha)\left[\int_{x_{min}(\delta_H)}^{1} \frac{1}{\rho_1(\delta_H, x)} dx\right] \\
&= (1-\alpha)\left[\int_{x_{min}(\delta_H)}^{1-\alpha} \frac{1}{x} dx + \int_{1-\alpha}^{1} \frac{1}{1-\alpha} dx\right] \\
&= (1-\alpha)\left[\ln(1-\alpha) - \ln(x_{min}(\delta_H)) + \frac{\alpha}{1-\alpha}\right] \\
\Pi_2(\delta_H) &= (1-\alpha)\, s \left[\int_{0}^{x_{max}(\delta_H)} \frac{1}{\rho_2(\delta_H, x)} dx\right] \\
&= (1-\alpha)\, s \left[\int_{0}^{x_{max}(\delta_H)} \frac{1}{1-x-h} dx\right] \\
&= (1-\alpha)\, s \left[\ln(1-h) - \ln(1 - x_{max}(\delta_H) - h)\right]
\end{aligned}$$

4 Possible cases: $x_{min}(\delta_L) \in (h, 1)$ and $x_{max}(\delta_L) \in (\alpha, h) \cup (h, 1)$, $x_{min}(\delta_H) \in (0, 1-h) \cup (1-h, 1-\alpha)$ and $x_{max}(\delta_H) \in (0, 1-h)$.

a) $x_{max}(\delta_L) \in (\alpha, h)$ (which implies $x_{max}(\delta_H) = 0$) and $x_{min}(\delta_H) \in (0, 1-h)$ (which implies $x_{min}(\delta_L) \in (h, 1)$)

$$\begin{aligned}
\frac{1}{1+s} h + x_{min}(\delta_L) - h = e^{-z} e^{-\delta_L} &\iff x_{min}(\delta_L) = e^{-z} e^{-\delta_L} + \frac{s}{1+s} h \\
\frac{s}{1+s} x_{max}(\delta_L) + 1 - x_{max}(\delta_L) = s \cdot e^{-z} e^{\delta_L} &\iff x_{max}(\delta_L) = (1+s)\left(1 - s \cdot e^{-z} e^{\delta_L}\right) \\
\frac{1}{1+s} h + x_{min}(\delta_H) = e^{-z} e^{-\delta_H} &\iff x_{min}(\delta_H) = e^{-z} e^{-\delta_H} - \frac{1}{1+s} h \\
x_{max}(\delta_H) = 0 &
\end{aligned}$$

$$\begin{aligned}
\Pi_0(\delta_L) =& \alpha \left[\int_{x_{min}(\delta_L)}^{1} \frac{1}{\rho_0(\delta_L, x)} dx + s \int_0^{x_{max}(\delta_L)} \frac{1}{\rho_0(\delta_L, x)} dx \right] \\
=& \alpha \left[\left(\int_{x_{min}(\delta_L)}^{1} \frac{1}{h} dx \right) + s \left(\int_0^{\alpha} \frac{1}{\alpha} dx + \int_{\alpha}^{x_{max}(\delta_L)} \frac{1}{x} dx \right) \right] \\
=& \alpha \left(\begin{array}{c} \left[\frac{1-x_{min}(\delta_L)}{h} \right] \\ +s\left[1+\ln\left(x_{max}(\delta_L)\right) - \ln(\alpha)\right] \end{array} \right) \\
\Pi_0(\delta_H) =& \alpha \left[\int_{x_{min}(\delta_H)}^{1} \frac{1}{\rho_0(\delta_H, x)} dx + s \int_0^{x_{max}(\delta_H)} \frac{1}{\rho_0(\delta_H, x)} dx \right] \\
=& \alpha \left[\left(\int_{x_{min}(\delta_H)}^{1-h} \frac{1}{h} dx + \int_{1-h}^{1-\alpha} \frac{1}{1-x} dx + \int_{1-\alpha}^{1} \frac{1}{\alpha} dx \right) + s \cdot 0 \right] \\
=& \alpha \left(\left[\frac{1-h-x_{min}(\delta_H)}{h} + \ln(h) - \ln(\alpha) + 1 \right] \right)
\end{aligned}$$

b) $x_{max}(\delta_L) \in (h, 1)$ (which implies $x_{max}(\delta_H) \in (0, 1-h)$) and $x_{min}(\delta_H) \in (0, 1-h)$ (which implies $x_{min}(\delta_L) \in (h, 1)$)

$$\begin{aligned}
\frac{1}{1+s}h + x_{min}(\delta_L) - h = e^{-z}e^{-\delta_L} &\iff x_{min}(\delta_L) = e^{-z}e^{-\delta_L} + \frac{s}{1+s}h \\
\frac{s}{1+s}h + 1 - x_{max}(\delta_L) = s \cdot e^{-z}e^{\delta_L} &\iff x_{max}(\delta_L) = 1 + \frac{s}{1+s}h - s \cdot e^{-z}e^{\delta_L} \\
\frac{1}{1+s}h + x_{min}(\delta_H) = e^{-z}e^{-\delta_H} &\iff x_{min}(\delta_H) = e^{-z}e^{-\delta_H} - \frac{1}{1+s}h \\
\frac{s}{1+s}h + 1 - x_{max}(\delta_H) - h = s \cdot e^{-z}e^{\delta_H} &\iff x_{max}(\delta_H) = 1 - \frac{1}{1+s}h - s \cdot e^{-z}e^{\delta_H}
\end{aligned}$$

$$\begin{aligned}
\Pi_0(\delta_L) =& \alpha \left[\int_{x_{min}(\delta_L)}^{1} \frac{1}{\rho_0(\delta_L, x)} dx + s \int_0^{x_{max}(\delta_L)} \frac{1}{\rho_0(\delta_L, x)} dx \right] \\
=& \alpha \left[\left(\int_{x_{min}(\delta_L)}^{1} \frac{1}{h} dx \right) + s \left(\int_0^{\alpha} \frac{1}{\alpha} dx + \int_{\alpha}^{h} \frac{1}{x} dx + \int_{h}^{x_{max}(\delta_L)} \frac{1}{h} dx \right) \right] \\
=& \alpha \left(\begin{array}{c} \left[\frac{1-x_{min}(\delta_L)}{h} \right] \\ +s\left[1+\ln(h) - \ln(\alpha) + \frac{x_{max}(\delta_L)-h}{h}\right] \end{array} \right) \\
\Pi_0(\delta_H) =& \alpha \left[\int_{x_{min}(\delta_H)}^{1} \frac{1}{\rho_0(\delta_H, x)} dx + s \int_0^{x_{max}(\delta_H)} \frac{1}{\rho_0(\delta_H, x)} dx \right] \\
=& \alpha \left[\left(\int_{x_{min}(\delta_H)}^{1-h} \frac{1}{h} dx + \int_{1-h}^{1-\alpha} \frac{1}{1-x} dx + \int_{1-\alpha}^{1} \frac{1}{\alpha} dx \right) + s \left(\int_0^{x_{max}(\delta_H)} \frac{1}{h} dx \right) \right] \\
=& \alpha \left(\begin{array}{c} \left[\frac{1-h-x_{min}(\delta_H)}{h} + \ln(h) - \ln(\alpha) + 1 \right] \\ +s\left[\frac{x_{max}(\delta_H)}{h}\right] \end{array} \right)
\end{aligned}$$

c) $x_{max}(\delta_L) \in (\alpha, h)$ (which implies $x_{max}(\delta_H) = 0$) and $x_{min}(\delta_H) \in (1-h, 1-\alpha)$ (which implies $x_{min}(\delta_L) = 1$)

$$\begin{aligned}
x_{min}(\delta_L) &= 1 \\
\frac{s}{1+s}x_{max}(\delta_L) + 1 - x_{max}(\delta_L) = s \cdot e^{-z}e^{\delta_L} &\iff x_{max}(\delta_L) = (1+s)\left(1 - s \cdot e^{-z}e^{\delta_L}\right) \\
\frac{1}{1+s}\left(1 - x_{min}(\delta_H)\right) + x_{min}(\delta_H) = e^{-z}e^{-\delta_H} &\iff x_{min}(\delta_H) = \frac{(1+s)e^{-z}e^{-\delta_H} - 1}{s} \\
x_{max}(\delta_H) &= 0
\end{aligned}$$

$$
\begin{aligned}
\Pi_0(\delta_L) =& \alpha\left[\int_{x_{min}(\delta_L)}^{1} \frac{1}{\rho_0(\delta_L,x)}dx + s\int_0^{x_{max}(\delta_L)} \frac{1}{\rho_0(\delta_L,x)}dx\right] \\
=& \alpha\left[0 + s\left(\int_0^{\alpha} \frac{1}{\alpha}dx + \int_{\alpha}^{x_{max}(\delta_L)} \frac{1}{x}dx\right)\right] \\
=& \alpha\left(s\left[1+\ln\left(x_{max}(\delta_L)\right) - \ln(\alpha)\right]\right) \\
\Pi_0(\delta_H) =& \alpha\left[\int_{x_{min}(\delta_H)}^{1} \frac{1}{\rho_0(\delta_H,x)}dx + s\int_0^{x_{max}(\delta_H)} \frac{1}{\rho_0(\delta_H,x)}dx\right] \\
=& \alpha\left[\left(\int_{x_{min}(\delta_H)}^{1-\alpha} \frac{1}{1-x}dx + \int_{1-\alpha}^{1} \frac{1}{\alpha}dx\right) + s\cdot 0\right] \\
=& \alpha\left(\left[\ln\left(1-x_{min}(\delta_H)\right) - \ln(\alpha) + 1\right]\right)
\end{aligned}
$$

d) $x_{max}(\delta_L) \in (h,1)$ (which implies $x_{max}(\delta_H) \in (0,1-h)$) and $x_{min}(\delta_H) \in (1-h,1-\alpha)$ (which implies $x_{min}(\delta_L) = 1$)

$$
\begin{aligned}
x_{min}(\delta_L) &= 1 \\
\frac{s}{1+s}h + 1 - x_{max}(\delta_L) = s\cdot e^{-z}e^{\delta_L} &\iff x_{max}(\delta_L) = 1 + \frac{s}{1+s}h - s\cdot e^{-z}e^{\delta_L} \\
\frac{1}{1+s}\left(1-x_{min}(\delta_H)\right) + x_{min}(\delta_H) = e^{-z}e^{-\delta_H} &\iff x_{min}(\delta_H) = \frac{(1+s)e^{-z}e^{-\delta_H} - 1}{s} \\
\frac{s}{1+s}h + 1 - x_{max}(\delta_H) - h = s\cdot e^{-z}e^{\delta_H} &\iff x_{max}(\delta_H) = 1 - \frac{1}{1+s}h - s\cdot e^{-z}e^{\delta_H}
\end{aligned}
$$

$$
\begin{aligned}
\Pi_0(\delta_L) =& \alpha\left[\int_{x_{min}(\delta_L)}^{1} \frac{1}{\rho_0(\delta_L,x)}dx + s\int_0^{x_{max}(\delta_L)} \frac{1}{\rho_0(\delta_L,x)}dx\right] \\
=& \alpha\left[0 + s\left(\int_0^{\alpha} \frac{1}{\alpha}dx + \int_{\alpha}^{h} \frac{1}{x}dx + \int_h^{x_{max}(\delta_L)} \frac{1}{h}dx\right)\right] \\
=& \alpha\left(s\left[1+\ln(h) - \ln(\alpha) + \frac{x_{max}(\delta_L) - h}{h}\right]\right) \\
\Pi_0(\delta_H) =& \alpha\left[\int_{x_{min}(\delta_H)}^{1} \frac{1}{\rho_0(\delta_H,x)}dx + s\int_0^{x_{max}(\delta_H)} \frac{1}{\rho_0(\delta_H,x)}dx\right] \\
=& \alpha\left[\left(\int_{x_{min}(\delta_H)}^{1-\alpha} \frac{1}{1-x}dx + \int_{1-\alpha}^{1} \frac{1}{\alpha}dx\right) + s\left(\int_0^{x_{max}(\delta_H)} \frac{1}{h}dx\right)\right] \\
=& \alpha\begin{pmatrix} \left[\ln\left(1-x_{min}(\delta_H)\right) - \ln(\alpha) + 1\right] \\ +s\left[\frac{x_{max}(\delta_H)}{h}\right] \end{pmatrix}
\end{aligned}
$$

Closed-form Approximations for $\alpha \approx 0$ Next, we approximate around $\alpha \approx 0$ to get some more analytical insights into the behavior of δ^* and h. To this end, we conjecture

$$
\begin{aligned}
h(\alpha) &= h_0 + h_1\alpha + \frac{h_2}{2}\alpha^2 \\
\delta^*(\alpha) &= \delta_0 + \delta_1\alpha
\end{aligned}
$$

As $\alpha \to 0$, to converge to the known solution of the two asset simultaneous game, we need

$$
\begin{aligned}
h_0 &= 0 \\
\delta_0 = \delta^* &= \frac{-(1-s)z - s\ln s}{(1+s)}
\end{aligned}
$$

Next, we take limits for each of the cases (except case CT2, which requires $\alpha \geq \frac{1}{2}$, so is not applicable), and impose $h_0 = 0$. First, note that $\lim_{\alpha\to 0}\Pi_0(\delta_L) = \lim_{\alpha\to 0}\Pi_0(\delta_H)$. Thus, we are looking for h_1, h_2

and δ_0, δ_1 that satisfy

$$\lim_{\alpha\to 0} \Pi_2(\delta_L) = \lim_{\alpha\to 0} \Pi_0(\delta_L) = \lim_{\alpha\to 0} \Pi_0(\delta_H) = \lim_{\alpha\to 0} \Pi_1(\delta_L)$$

Next, note that a local equilibrium requires $h(\alpha) \geq \alpha$, and thus for small α we require parameters such that $h_1 \geq 1$.

C3a We have

$$\begin{aligned}
\lim_{\alpha\to 0} \Pi_0(\delta_L) = \lim_{\alpha\to 0} \Pi_0(\delta_H) &= \frac{1+s-e^{-\delta_0-z}-e^{\delta_0-z}s^2}{h_1} \\
\lim_{\alpha\to 0} \Pi_1(\delta_L) = \lim_{\alpha\to 0} \Pi_1(\delta_H) &= -\ln\left[e^{-\delta_0-z}\right] \\
\lim_{\alpha\to 0} \Pi_2(\delta_L) = -s\ln\left[1-(1+s)\left(1-e^{\delta_0-z}s\right)\right] &\neq 0 = \lim_{\alpha\to 0} \Pi_2(\delta_H)
\end{aligned}$$

For consistency $\lim_{\alpha\to 0} \Pi_1(\delta_L) = \lim_{\alpha\to 0} \Pi_1(\delta_H)$, we require

$$1-(1+s)\left(1-e^{\delta_0-z}s\right) = 1 \iff e^{\delta_0-z}s = 1 \iff e^z = e^{\delta_0}s$$

The indifference condition is

$$-s\ln\left[1-(1+s)\left(1-e^{\delta_0-z}s\right)\right] = \frac{1-e^{-\delta_0-z}}{h_1} = -\ln\left[e^{-\delta_0-z}\right]$$

and equating the first and third term requires $e^z = e^{-\delta_0}$. These conditions can only hold for $z = -\frac{1}{2}\ln s$, and are violated for general parameters. Thus, case C3a is not possible in equilibrium for small α.[4]

C3b We have

$$\begin{aligned}
\lim_{\alpha\to 0} \Pi_0(\delta_L) = \lim_{\alpha\to 0} \Pi_0(\delta_H) &= \frac{1+s-e^{-\delta_0-z}-e^{\delta_0-z}s^2}{h_1} \\
\lim_{\alpha\to 0} \Pi_1(\delta_L) = \lim_{\alpha\to 0} \Pi_1(\delta_H) &= -\ln\left[e^{-\delta_0-z}\right] \\
\lim_{\alpha\to 0} \Pi_2(\delta_L) = \lim_{\alpha\to 0} \Pi_2(\delta_H) &= -s\ln\left[e^{\delta_0-z}s\right]
\end{aligned}$$

and the indifference condition is

$$-s\ln\left[e^{\delta_0-z}s\right] = \frac{1+s-e^{-\delta_0-z}-e^{\delta_0-z}s^2}{h_1(1+s)} = -\ln\left[e^{-\delta_0-z}\right] \iff \delta_0 = \frac{-(1-s)z-s\ln s}{1+s} = \delta^*$$

[4]A more direct proof: C3a requires $0<\alpha<h<1<h+1-\alpha$ and $x_{max}(\delta_L) \in (\alpha, h)$ (which implies $x_{max}(\delta_H) = 0$) and $x_{min}(\delta_H) \in (0, 1-h)$ (which implies $x_{min}(\delta_L) \in (h, 1)$)

$$\begin{aligned}
\frac{1}{1+s}h + x_{min}(\delta_L) - h = e^{-z}e^{-\delta_L} &\iff x_{min}(\delta_L) = e^{-z}e^{-\delta_L} + \frac{s}{1+s}h \\
\frac{s}{1+s}x_{max}(\delta_L) + 1 - x_{max}(\delta_L) = s\cdot e^{-z}e^{\delta_L} &\iff x_{max}(\delta_L) = (1+s)\left(1-s\cdot e^{-z}e^{\delta_L}\right) \\
\frac{1}{1+s}h + x_{min}(\delta_H) = e^{-z}e^{-\delta_H} &\iff x_{min}(\delta_H) = e^{-z}e^{-\delta_H} - \frac{1}{1+s}h \\
x_{max}(\delta_H) = 0 &
\end{aligned}$$

Note that $x_{max}(\delta_L) \to (1+s)\left(1-s\cdot e^{-z}e^{\delta^*}\right) = 0$, so that $e^{\delta^*} = s^{-1}e^z$; further, note that $x_{min} \to e^{-z}e^{-\delta^*} \in (0,1)$; plugging in, we have $e^{-2z}\cdot s \in (0,1)$, which is not a contradiction, but when inspecting the indifference condition for investment yields a contradiction.

Next, we have

$$h_1 = \frac{1+s-e^{-\delta_0-z}-e^{\delta_0-z}s^2}{(\delta_0+z)} = \frac{1+s-e^{-\left(\frac{2s\cdot z-s\ln s}{1+s}\right)}-e^{\frac{-2z-s\ln s}{1+s}}s^2}{\left(\frac{2s\cdot z-s\ln s}{1+s}\right)}$$

where we used $\delta_0+z=\frac{2s\cdot z-s\ln s}{1+s}$ and $\delta_0-z=\frac{-2z-s\ln s}{1+s}$. The insulated equilibrium as constructed exists around $\alpha\approx 0$ if $h_1>1$.

C3c We have

$$\lim_{\alpha\to 0}\Pi_0(\delta_L)=\lim_{\alpha\to 0}\Pi_0(\delta_H)=0$$

$$\lim_{\alpha\to 0}\Pi_1(\delta_L)=0\neq -\ln\left[\frac{(1+s)e^{-\delta_0-z}-1}{s}\right]=\lim_{\alpha\to 0}\Pi_1(\delta_H)$$

$$\lim_{\alpha\to 0}\Pi_2(\delta_L)=-s\ln\left[1-(1+s)\left(1-e^{\delta_0-z}s\right)\right]\neq 0=\lim_{\alpha\to 0}\Pi_2(\delta_H)$$

For consistency $\lim_{\alpha\to 0}\Pi_1(\delta_L)=\lim_{\alpha\to 0}\Pi_1(\delta_H)$, we require

$$(1+s)e^{-\delta_0-z}-1=s \iff e^z=e^{-\delta_0}$$

and for consistency $\lim_{\alpha\to 0}\Pi_1(\delta_L)=\lim_{\alpha\to 0}\Pi_1(\delta_H)$, we require

$$1-(1+s)\left(1-e^{\delta_0-z}s\right)=1 \iff e^{\delta_0-z}s=1 \iff e^z=e^{\delta_0}s$$

These two conditions can only hold for $z=-\frac{1}{2}\ln s$, and are violated for general parameters. Thus, case C3c is not possible in equilibrium for small α.[5]

C3d We have

$$\lim_{\alpha\to 0}\Pi_0(\delta_L)=\lim_{\alpha\to 0}\Pi_0(\delta_H)=\frac{s-e^{\delta_0-z}s^2}{h_1}$$

$$\lim_{\alpha\to 0}\Pi_1(\delta_L)=0\neq -\ln\left[\frac{(1+s)e^{-\delta_0-z}-1}{s}\right]=\lim_{\alpha\to 0}\Pi_1(\delta_H)$$

$$\lim_{\alpha\to 0}\Pi_2(\delta_L)=\lim_{\alpha\to 0}\Pi_2(\delta_H)=-s\ln\left[e^{\delta_0-z}s\right]$$

For consistency $\lim_{\alpha\to 0}\Pi_1(\delta_L)=\lim_{\alpha\to 0}\Pi_1(\delta_H)$, we require

$$(1+s)e^{-\delta_0-z}-1=s \iff e^z=e^{-\delta_0}$$

But then for indifference we require

$$-s\ln\left[e^{\delta_0-z}s\right]=\frac{s-e^{-\delta_0-z}s^2}{h_1}=-\ln\left[\frac{(1+s)e^{-\delta_0-z}-1}{s}\right]$$

[5]A more direct proof: C3c requires $0<\alpha<h<1<h+1-\alpha$ and $x_{max}(\delta_L)\in(\alpha,h)$ (which implies $x_{max}(\delta_H)=0$) and $x_{min}(\delta_H)\in(1-h,1-\alpha)$ (which implies $x_{min}(\delta_L)=1$)

$$x_{min}(\delta_L)=1$$

$$\frac{s}{1+s}x_{max}(\delta_L)+1-x_{max}(\delta_L)=s\cdot e^{-z}e^{\delta_L} \iff x_{max}(\delta_L)=(1+s)\left(1-s\cdot e^{-z}e^{\delta_L}\right)$$

$$\frac{1}{1+s}(1-x_{min}(\delta_H))+x_{min}(\delta_H)=e^{-z}e^{-\delta_H} \iff x_{min}(\delta_H)=\frac{(1+s)e^{-z}e^{-\delta_H}-1}{s}$$

$$x_{max}(\delta_H)=0$$

Thus, we have $x_{max}(\delta_L)\to(1+s)\left(1-s\cdot e^{-z}e^{\delta_L}\right)=0$ and $x_{min}(\delta_H)=\frac{(1+s)e^{-z}e^{-\delta_H}-1}{s}=1$. But as $\delta_L\to\delta^*\leftarrow\delta_H$, so we require $e^{\delta^*}=s^{-1}e^z=e^{-z}$, which in turn requires the specific parameter restriction $z=-\frac{1}{2}\ln s$.

But we know the third term is equal to 0, so the first term requires $e^{\delta_0 - z} s = 1 \iff e^{\delta_0} s = e^z$ which can only hold for $z = -\frac{1}{2} \ln s$, and are violated for general parameters. Thus, case C3d is not possible in equilibrium for small α.[6]

Thus, we are left with only case C3b for small α, which fulfills the insulated equilibrium criterion for points (s, z) such that

$$\left\{ (s,z) : h_1(s,z) = \frac{1 + s - e^{-\left(\frac{2s \cdot z - s \ln s}{1+s}\right)} - e^{\frac{-2z - s \ln s}{1+s}} s^2}{\left(\frac{2s \cdot z - s \ln s}{1+s}\right)} \geq 1 \right\}$$

Figure C.1 maps the set of points (s, z) for which the insulated criterion is fulfilled.

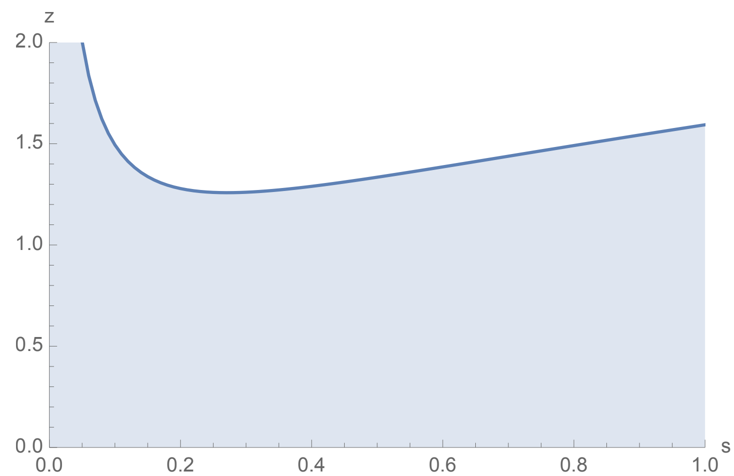

FIGURE C.1. **Existence of insulated simultaneous single-survivor common bond equilibrium for small α:** SET OF POINTS (s, z) FOR WHICH AN INSULATED SINGLE-SURVIVOR EQUILIBRIUM EXISTS IN THE COMMON BONDS CASE FOR $\alpha \approx 0$, I.E., $\{(s,z) : h_1(s,z) \geq 1\}$.

Verifying the equilibrium. Note that, away from $\alpha = 0$, we have the expected returns at either end-point not equal, even as $\sigma \to 0$, because strategic uncertainty does not vanish:

$$\lim_{\sigma \to 0} \Pi_i(\delta_L) \neq \lim_{\sigma \to 0} \Pi_i(\delta_H)$$

[6] A more direct proof: C3d requires $0 < \alpha < h < 1 < h + 1 - \alpha$ and $x_{max}(\delta_L) \in (h, 1)$ (which implies $x_{max}(\delta_H) \in (0, 1-h)$) and $x_{min}(\delta_H) \in (1-h, 1-\alpha)$ (which implies $x_{min}(\delta_L) = 1$)

$$x_{min}(\delta_L) = 1$$

$$\frac{s}{1+s} h + 1 - x_{max}(\delta_L) = s \cdot e^{-z} e^{\delta_L} \iff x_{max}(\delta_L) = 1 + \frac{s}{1+s} h - s \cdot e^{-z} e^{\delta_L}$$

$$\frac{1}{1+s}(1 - x_{min}(\delta_H)) + x_{min}(\delta_H) = e^{-z} e^{-\delta_H} \iff x_{min}(\delta_H) = \frac{(1+s) e^{-z} e^{-\delta_H} - 1}{s}$$

$$\frac{s}{1+s} h + 1 - x_{max}(\delta_H) - h = s \cdot e^{-z} e^{\delta_H} \iff x_{max}(\delta_H) = 1 - \frac{1}{1+s} h - s \cdot e^{-z} e^{\delta_H}$$

Thus, we have $x_{min}(\delta_H) \to \frac{(1+s)e^{-z}e^{-\delta_H} - 1}{s} = 1$, which requires $e^{\delta^*} = e^{-z}$; similarly, we have $x_{max} = 1 - s \cdot e^{-z} e^{\delta^*} \in (0,1)$; plugging in, we have $1 - s \cdot e^{-2z} \in (0,1)$ which does not give a contradiction, but when inspecting the indifference condition for investment yields a contradiction.

To verify the equilibrium, we need to check that for any $\delta \in [\delta_L, \delta_H]$, indeed common bonds are the most attractive asset, for $\delta < \delta_L$, bond 2 is the most attractive asset, and for $\delta > \delta_H$, bond 1 is the most attractive asset. For a given δ_L, δ_H, let

$$\delta \equiv \delta_L + 2\sigma\varepsilon$$

with $\varepsilon \in (0, h)$, so that $\varepsilon = 0$ yields δ_L and $\varepsilon = h$ yields δ_H. Then, for $\varepsilon \in [0, h]$, we have

$$\rho_1(\delta, x) = \begin{cases} 0 & (0, h-\varepsilon) \\ x+\varepsilon-h & (h-\varepsilon, h-\varepsilon+1-\alpha) \\ 1-\alpha & (h-\varepsilon+1-\alpha, 1) \end{cases}$$

$$\rho_2(\delta, x) = \begin{cases} 1-\alpha & (0, \alpha-\varepsilon) \\ 1-(x+\varepsilon) & (\alpha-\varepsilon, 1-\varepsilon) \\ 0 & (1-\varepsilon, 1) \end{cases}$$

where of course if for example as in C3 we have $\alpha < h$, then some intervals are empty (i.e., $(0, \alpha - \varepsilon) = \emptyset$ for $\varepsilon \in (\alpha, h)$). For interior equilibria, we need $x_{min}(\delta) \in (h-\varepsilon, h-\varepsilon+1-\alpha)$ and $x_{max}(\delta) \in (\alpha-\varepsilon, 1-\varepsilon)$.

C3 $0 < \alpha < h < 1 < h + 1 - \alpha$

$$\rho_1(\delta, x) + \rho_2(\delta, x) = \begin{cases} 1-\alpha & (0, \alpha-\varepsilon) \\ 1-(x+\varepsilon) & (\alpha-\varepsilon, h-\varepsilon) \\ 1-h & (h-\varepsilon, 1-\varepsilon) \\ x+\varepsilon-h & (1-\varepsilon, h-\varepsilon+1-\alpha) \\ 1-\alpha & (h-\varepsilon+1-\alpha, 1) \end{cases}$$

Let us calculate expected returns as a function of ε.[7] To calculate expected returns, we have to conjecture a position of $x_{min}(\delta)$ and $x_{max}(\delta)$. For $\alpha \approx 0$, we can only be in case C3b, and our numerical results for our benchmark cases show that this case is applicable even when α increases. Thus, we only show the expected returns for this case:

C3b $0 < \alpha < h < 1 < h + 1 - \alpha$
$x_{min} \in (h-\varepsilon, 1-\varepsilon)$ and $x_{max}(\delta) \in (h-\varepsilon, 1-\varepsilon)$. Now the position of ε in relation to α and $h-\alpha$ matters, i.e., three intervals matter: $\varepsilon < \min\{h-\alpha, \alpha\}$, $\varepsilon \in (\min\{h-\alpha, \alpha\}, \max\{h-\alpha, \alpha\})$, and $\varepsilon > \max\{h-\alpha, \alpha\}$. Two sub-cases arise, which essentially define the relation of $h-\alpha$ to α:

a) $\min\{h-\alpha, \alpha\} = h-\alpha \iff \alpha < h < 2\alpha$ (this is the applicable case for our benchmark cases $(s, z) = \left(\frac{1}{4}, 1\right)$ and $(s, z) = \left(\frac{1}{2}, 1\right)$, as numerically h is very close to α). Thus, the three intervals are $\varepsilon < h-\alpha$, $\varepsilon \in (h-\alpha, \alpha)$, and $\varepsilon > \alpha$. Note that $\varepsilon = \frac{h}{2}$ gives the midpoint δ_M, and the midpoint is part of interval $\frac{h}{2} \in (h-\alpha, \alpha)$.[8]

[7]Note that $\varepsilon = \frac{1}{2}h$ gives the central interval **0** midpoint

$$\delta_M = \frac{\delta_H + \delta_L}{2} = \frac{\delta_H - \delta_L + 2\delta_L}{2} = \delta_L + \sigma \cdot h.$$

[8]Consider $\frac{h}{2} < h - \alpha \iff \alpha < \frac{h}{2} \iff 2\alpha < h$, which violates the assumptions. Next, consider $\frac{h}{2} > \alpha \iff h > 2\alpha$, which also violates the assumptions. Thus, only $\frac{h}{2} \in (h-\alpha, \alpha)$ is consistent with $\alpha < h < 2\alpha$.

For $\varepsilon < h-\alpha = \min\{h-\alpha, \alpha\}$ so that $h-\varepsilon+1-\alpha > 1$ as well as $\alpha-\varepsilon > 0$, we have

$$\begin{aligned}
\Pi_0(\delta) &= \alpha\left[\int_{x_{min}(\delta)}^{1} \frac{1}{\rho_0(\delta,x)}dx + s\int_0^{x_{max}(\delta)} \frac{1}{\rho_0(\delta,x)}dx\right] \\
&= \alpha\left[\int_{x_{min}(\delta)}^{1-\varepsilon} \frac{1}{1-(1-h)}dx + \int_{1-\varepsilon}^{1} \frac{1}{1-(x+\varepsilon-h)}dx\right] \\
&+ \alpha\cdot s\left[\int_0^{\alpha-\varepsilon} \frac{1}{1-(1-\alpha)}dx + \int_{\alpha-\varepsilon}^{h-\varepsilon} \frac{1}{1-[1-(x+\varepsilon)]}dx + \int_{h-\varepsilon}^{x_{max}(\delta)} \frac{1}{1-(1-h)}dx\right] \\
&= \alpha\left[\frac{1-\varepsilon-x_{min}(\delta)}{h} + \ln(h) - \ln(h-\varepsilon)\right] \\
&+ \alpha\cdot s\left[\frac{\alpha-\varepsilon}{\alpha} + \ln(h) - \ln(\alpha) + \frac{x_{max}(\delta)+\varepsilon-h}{h}\right]
\end{aligned}$$

$$\begin{aligned}
\Pi_1(\delta) &= (1-\alpha)\left[\int_{x_{min}(\delta)}^{1} \frac{1}{\rho_1(\delta,x)}dx\right] \\
&= (1-\alpha)\left[\int_{x_{min}(\delta)}^{1} \frac{1}{x+\varepsilon-h}dx\right] \\
&= (1-\alpha)\left[\ln(1+\varepsilon-h) - \ln(x_{min}(\delta)+\varepsilon-h)\right] \\
\Pi_2(\delta) &= (1-\alpha)\,s\left[\int_0^{x_{max}(\delta)} \frac{1}{\rho_2(\delta,x)}dx\right] \\
&= (1-\alpha)\,s\left[\int_0^{\alpha-\varepsilon} \frac{1}{1-\alpha}dx + \int_{\alpha-\varepsilon}^{x_{max}(\delta)} \frac{1}{1-(x+\varepsilon)}dx\right] \\
&= (1-\alpha)\,s\left[\frac{\alpha-\varepsilon}{1-\alpha} + \ln(1-\alpha) - \ln(1-(x_{max}(\delta)+\varepsilon))\right]
\end{aligned}$$

For $\varepsilon \in (h-\alpha, \alpha)$ so that $h-\varepsilon+1-\alpha < 1$ and $\alpha-\varepsilon > 0$, we have

$$\begin{aligned}
\Pi_0(\delta) &= \alpha\left[\int_{x_{min}(\delta)}^{1} \frac{1}{\rho_0(\delta,x)}dx + s\int_0^{x_{max}(\delta)} \frac{1}{\rho_0(\delta,x)}dx\right] \\
&= \alpha\left[\int_{x_{min}(\delta)}^{1-\varepsilon} \frac{1}{1-(1-h)}dx + \int_{1-\varepsilon}^{h-\varepsilon+1-\alpha} \frac{1}{1-(x+\varepsilon-h)}dx + \int_{h-\varepsilon+1-\alpha}^{1} \frac{1}{1-(1-\alpha)}dx\right] \\
&+ \alpha\cdot s\left[\int_0^{\alpha-\varepsilon} \frac{1}{1-(1-\alpha)}dx + \int_{\alpha-\varepsilon}^{h-\varepsilon} \frac{1}{1-[1-(x+\varepsilon)]}dx + \int_{h-\varepsilon}^{x_{max}(\delta)} \frac{1}{1-(1-h)}dx\right] \\
&= \alpha\left[\frac{1-\varepsilon-x_{min}(\delta)}{h} + \ln(h) - \ln(\alpha) + \frac{\varepsilon+\alpha-h}{\alpha}\right] \\
&+ \alpha\cdot s\left[\frac{\alpha-\varepsilon}{\alpha} + \ln(h) - \ln(\alpha) + \frac{x_{max}(\delta)+\varepsilon-h}{h}\right]
\end{aligned}$$

$$\begin{aligned}\Pi_1(\delta) &= (1-\alpha)\left[\int_{x_{min}(\delta)}^{1}\frac{1}{\rho_1(\delta,x)}dx\right]\\ &= (1-\alpha)\left[\int_{x_{min}(\delta)}^{h-\varepsilon+1-\alpha}\frac{1}{x+\varepsilon-h}dx+\int_{h-\varepsilon+1-\alpha}^{1}\frac{1}{1-\alpha}dx\right]\\ &= (1-\alpha)\left[\ln(1-\alpha)-\ln(x_{min}(\delta)+\varepsilon-h)+\frac{\varepsilon+\alpha-h}{1-\alpha}\right]\\ \Pi_2(\delta) &= (1-\alpha)s\left[\int_0^{x_{max}(\delta)}\frac{1}{\rho_2(\delta,x)}dx\right]\\ &= (1-\alpha)s\left[\int_0^{\alpha-\varepsilon}\frac{1}{1-\alpha}dx+\int_{\alpha-\varepsilon}^{x_{max}(\delta)}\frac{1}{1-(x+\varepsilon)}dx\right]\\ &= (1-\alpha)s\left[\frac{\alpha-\varepsilon}{1-\alpha}+\ln(1-\alpha)-\ln(1-(x_{max}(\delta)+\varepsilon))\right]\end{aligned}$$

For $\varepsilon > \alpha = \max\{h-\alpha,\alpha\}$ so that $h-\varepsilon+1-\alpha<1$ as well as $\alpha-\varepsilon<0$, we have

$$\begin{aligned}\Pi_0(\delta) &= \alpha\left[\int_{x_{min}(\delta)}^{1}\frac{1}{\rho_0(\delta,x)}dx+s\int_0^{x_{max}(\delta)}\frac{1}{\rho_0(\delta,x)}dx\right]\\ &= \alpha\left[\int_{x_{min}(\delta)}^{1-\varepsilon}\frac{1}{1-(1-h)}dx+\int_{1-\varepsilon}^{h-\varepsilon+1-\alpha}\frac{1}{1-(x+\varepsilon-h)}dx+\int_{h-\varepsilon+1-\alpha}^{1}\frac{1}{1-(1-\alpha)}dx\right]\\ &+\alpha\cdot s\left[\int_0^{h-\varepsilon}\frac{1}{1-[1-(x+\varepsilon)]}dx+\int_{h-\varepsilon}^{x_{max}(\delta)}\frac{1}{1-(1-h)}dx\right]\\ &= \alpha\left[\frac{1-\varepsilon-x_{min}(\delta)}{h}+\ln(h)-\ln(\alpha)+\frac{\varepsilon+\alpha-h}{\alpha}\right]\\ &+\alpha\cdot s\left[\ln(h)-\ln(\varepsilon)+\frac{x_{max}(\delta)+\varepsilon-h}{h}\right]\end{aligned}$$

$$\begin{aligned}\Pi_1(\delta) &= (1-\alpha)\left[\int_{x_{min}(\delta)}^{1}\frac{1}{\rho_1(\delta,x)}dx\right]\\ &= (1-\alpha)\left[\int_{x_{min}(\delta)}^{h-\varepsilon+1-\alpha}\frac{1}{x+\varepsilon-h}dx+\int_{h-\varepsilon+1-\alpha}^{1}\frac{1}{1-\alpha}dx\right]\\ &= (1-\alpha)\left[\ln(1-\alpha)-\ln(x_{min}(\delta)+\varepsilon-h)+\frac{\varepsilon+\alpha-h}{1-\alpha}\right]\\ \Pi_2(\delta) &= (1-\alpha)s\left[\int_0^{x_{max}(\delta)}\frac{1}{\rho_2(\delta,x)}dx\right]\\ &= (1-\alpha)s\left[\int_0^{x_{max}(\delta)}\frac{1}{1-(x+\varepsilon)}dx\right]\\ &= (1-\alpha)s\left[\ln(1-\varepsilon)-\ln(1-(x_{max}(\delta)+\varepsilon))\right]\end{aligned}$$

Next, we numerically check $\Pi_0(\delta^*;\varepsilon) > \max\{\Pi_1(\delta^*;\varepsilon),\Pi_2(\delta^*;\varepsilon)\}$ for candidate equilibria (h,δ^*) for any $\varepsilon\in[0,h]$. This holds for all numerically solved for candidate equilibria.

b) $h-\alpha>\alpha \iff h>2\alpha>\alpha$ would be the other case, but we do not observe numerically any h that are twice the size of α. Calculations for this case, as well as for cases C3a C3c and C3d are available upon request.

The numerical results (h,δ^*_{sim}) as well as the comparison δ^*_{seq} for cases $(s=.25,z=1)$ and $(s=.5,z=1)$ are presented in Figure C.2. The left Panels show the equilibrium h as the solid blue line in comparison to the 45 degree line as the dashed yellow line, thus visualizing the insulated requirement $h>\alpha$. We restrict the graph to levels of α for which this condition holds. The right Panels then show the equilibrium δ^*_{sim} as the solid blue line in comparison to their sequential counterpart δ^*_{seq} as the dashed yellow line.

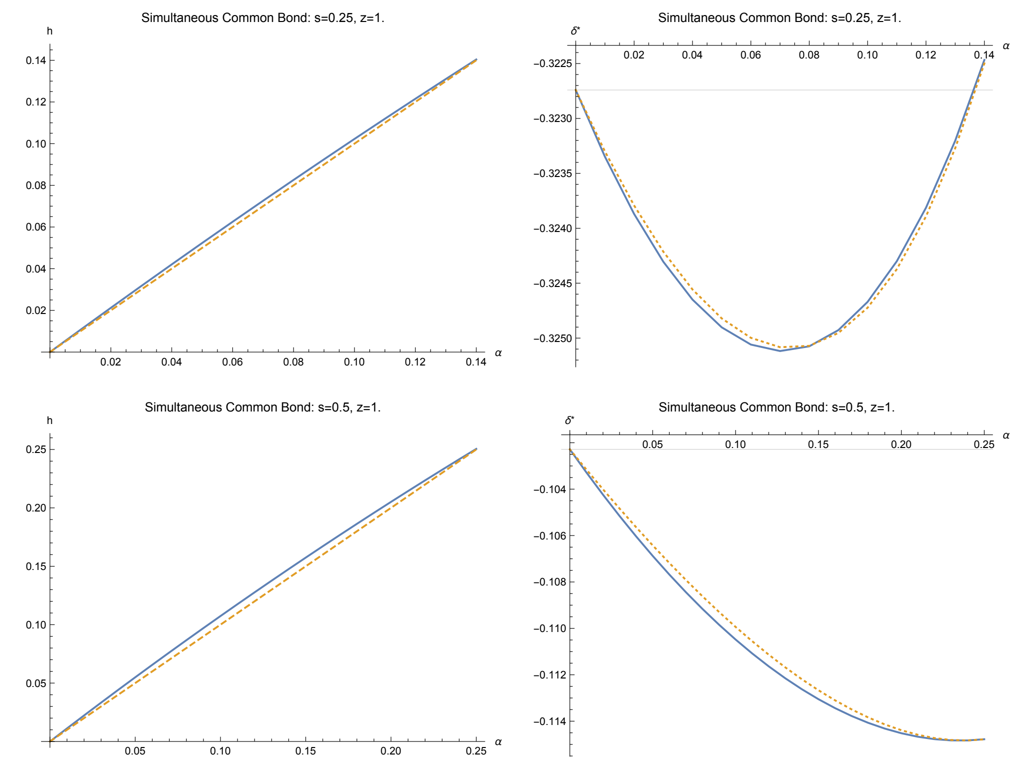

FIGURE C.2. **Robustness of single-survivor common bond equilibrium to sequential timing assumption:** SIMULTANEOUS EQUILIBRIUM CENTRAL INTERVAL WIDTH h (SOLID BLUE LINE) IN COMPARISON TO 45 DEGREE LINE (DASHED YELLOW LINE) (LEFT PANELS); SIMULTANEOUS EQUILIBRIUM THRESHOLD δ^*_{sim} (SOLID BLUE LINE) IN COMPARISON TO THE SEQUENTIAL EQUILIBRIUM THRESHOLD δ^*_{seq} (DASHED YELLOW LINE) (RIGHT PANELS).