Online Appendix for "Corporate Finance and Monetary Policy"

Guillaume Rocheteau, Randall Wright, Cathy Zhang

A1. The bargaining set

Consider the non-monetary economy first. In a match between an entrepreneur and a bank, the surpluses are $S^e = f(k) - k - \phi$ and $S^b = \phi$. If the pledgeability constraint is slack, the surplus is maximized at $f(k^*) - k^*$. Then the frontier is linear, $S^e + S^b = f(k^*) - k^*$, as in the right panel of Figure 1. The constraint is slack if $\phi \leq \chi_b f(k^*) - k^*$. Hence, the frontier has a linear portion iff $\chi_b \geq k^*/f(k^*)$, and is entirely linear if $f(k^*) - k^* \leq \chi_b f(k^*) - k^*$, which only occurs when $\chi_b = 1$.

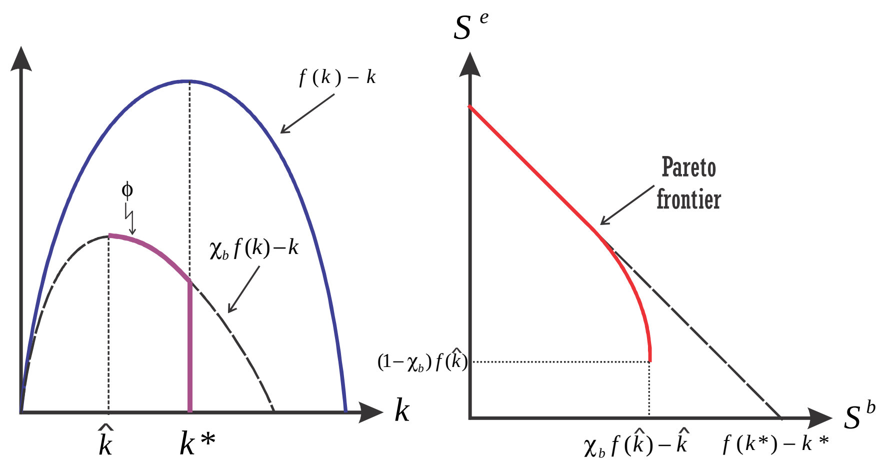

Figure 1: Pareto frontier for bank loans

If the pledgeability constraint binds, then $\phi = \chi_b f(k) - k$, as in the left panel of Figure 1. Take a pair (ϕ, k) below the curve $\chi_b f(k) - k$ such that $k < k^*$. By raising k, S^e increases. Moreover, $k \geq \hat{k} = \arg\max\left[\chi_b f(k) - k\right]$, since otherwise one could raise $\phi = \chi_b f(k) - k$ and increase both surpluses. Hence, the frontier when the constraint binds is

$$\left\{(S^e, S^b) \in \mathbb{R}^2_+ : S^e = (1-\chi_b) f(k),\, S^b = \chi_b f(k) - k,\, k \in \left[\hat{k}, \bar{k}\right]\right\},$$

where $\bar{k} = k^*$ if $\chi_b f(k^*) \geq k^*$, and $\bar{k}$ is the largest solution to $\chi_b f(\bar{k}) - \bar{k} = 0$ otherwise. It is easy to check the frontier is downward sloping, $\partial S^e/\partial S^b < 0$, and $\partial S^e/\partial S^b \to -\infty$ as

$k \to \hat{k}$. If $\chi_b f(k^*) \geq k^*$ then $\partial S^e/\partial S^b \to -1$ as $k \to k^*$. The bargaining set is not convex since the point on the frontier that maximizes S^b, $\chi_b f(\hat{k}) - \hat{k}$, is above the horizontal axis. Hence, the entrepreneur enjoys a positive surplus, $(1-\chi_b)f(\hat{k})$, due to limited pledgeability.

We now characterize the Pareto frontier of a pairwise meeting between a bank and an entrepreneur in the monetary economy. Suppose the entrepreneur holds a^e_m real balances. The equation of the Pareto frontier is determined by:

$$S^e = \max_{k,d,\phi} [f(k) - k - \phi - \Delta^m(a^e_m)] \quad \text{st} \quad \phi \geq S^b \quad \text{and} \quad k + \phi \leq \chi_b f(k) + a^e_m.$$

Suppose the borrowing constraint does not bind. Then, $k = k^*$ and

$$S^e + S^b = f(k^*) - k^* - \Delta^m(a^e_m).$$

The frontier is linear. The borrowing constraint does not bind if $S^b \leq \chi_b f(k^*) - k^* + a^e_m$. If it binds then $\phi = S^b$ and $k + \phi = \chi_b f(k) + a^e_m$ and hence the Pareto frontier is given by

$$\begin{aligned} S^e &= (1 - \chi_b)f(k) - a^e_m - \Delta^m(a^e_m) \\ S^b &= \chi_b f(k) - k + a^e_m, \end{aligned}$$

where $k \geq \hat{k}$, $S^e \geq 0$ and $S^b \geq 0$. For given S^e,

$$\begin{aligned} \frac{\partial S^b}{\partial a^e_m} &= \frac{\chi_b f'(k) - 1}{(1-\chi_b)f'(k)} [1 + \Delta'(a^e_m)] + 1 \\ &= \frac{f'(a^e_m)[\chi_b f'(k) - 1] + (1-\chi_b)f'(k)}{(1-\chi_b)f'(k)}. \end{aligned}$$

The Pareto frontier shifts inward as a^e_m increases if

$$\frac{(1-\chi_b)f'(k)}{1 - \chi_b f'(k)} \leq f'(a^e_m).$$

We show with the numerical examples below that this condition does not always hold, i.e., the Pareto frontier can shift outward as a^e_m increases because of the complementarity between money and credit when the borrowing constraint binds. As an example, see Figure 2, which sets $\chi_b = 0.2$ and plots the family of Pareto frontiers for different values of a^e_m.

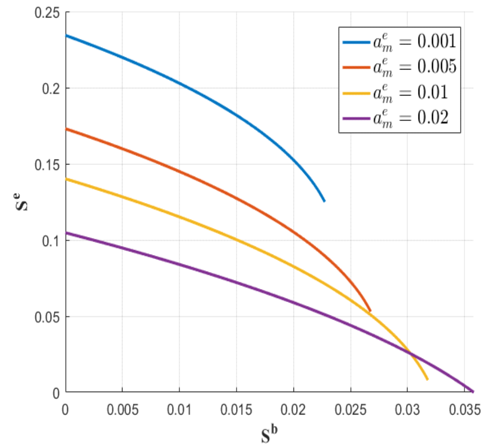

Figure 2: Pareto frontier for different real balances

A2. An alternative bargaining solution

As an alternative to the Nash solution, many recent models use Kalai's proportional bargaining solution, which in this context is given by:

$$\max_{\phi,k} S^b = \phi \text{ st } S^e \geq \frac{1-\theta}{\theta} S^b \text{ and } k + \phi \leq \chi_b f(k).$$

Thus, a bank chooses (ϕ, k) to maximize S^b subject to the entrepreneur getting at least a fraction $1-\theta$ of the total surplus. In fact, the strict proportional solution requires a strict equality in the first constraint; we use an inequality to guarantee existence despite nonconvexity of the bargaining set, which formally corresponds to the lexicographic proportional solution. Provided $\chi_b \geq \chi_b^*$, the pledgeability constraint is slack and Kalai coincides with Nash. If the constraint binds, k solves

$$(\chi_b - \theta) f(k) = (1-\theta)k \text{ if } \chi_b > \theta \text{ and } k \geq \hat{k};\ k = \hat{k} \text{ otherwise.}$$

Thus, the solution $k \geq \hat{k}$ splits the surplus so the bank gets a share θ of the surplus and satisfies the constraint. If $k < \hat{k}$, the solution is not Pareto optimal: by increasing k to $\hat{k}$, S^b reaches its maximum, while S^e increases. In that case, we select $k = \hat{k}$, in accordance with the lexicographic proportional solution. The lending rate when the constraint binds is

$$r_b = \frac{\theta(1-\chi_b)}{\chi_b - \theta} \text{ if } \theta \leq \hat{\theta} \equiv \frac{\chi_b f(\hat{k}) - \hat{k}}{f(\hat{k}) - \hat{k}};\ r_b = \frac{\hat{\theta}(1-\chi_b)}{\chi_b - \hat{\theta}} \text{ otherwise.}$$

Provided θ is not too large, r_b decreasing with χ_b. If $f(k) = zk^{\gamma}$, e.g., one can check r_b and k are given by:

$$r_b = \begin{array}{c} \frac{\theta(1-\gamma)}{\gamma} \\ \frac{\theta(1-\chi_b)}{\chi_b-\theta} \\ \frac{1-\gamma}{\gamma} \end{array} \text{ and } k = \begin{array}{c} (\gamma z)^{\frac{1}{1-\gamma}} \\ \left[\frac{(\chi_b-\theta)z}{1-\theta}\right]^{\frac{1}{1-\gamma}} \\ (\chi_b z\gamma)^{\frac{1}{1-\gamma}} \end{array} \text{ if } \chi_b \in \begin{array}{c} \geq (1-\theta)\gamma+\theta \\ \left[\frac{\theta}{1-\gamma(1-\theta)}, (1-\theta)\gamma+\theta\right) \\ < \frac{\theta}{1-\gamma(1-\theta)} \end{array}$$

For low χ_b, r_b is maximized and independent of χ_b and θ; in this case the constraint binds and k maximizes S^b. For intermediate χ_b, r_b is decreasing in χ_b and increasing in θ. For high χ_b, the constraint is slack, so k and r_b are independent of χ_b.

A3. Limited commitment

In the text, the entrepreneur's borrowing limit is a fraction χ_b of $f(k)$. This can be motivated by, instead of moral hazard, limited commitment. Assume banks can no longer seize output: entrepreneurs can abscond with it all and default on the loan. However, banks have a record of repayment histories, and can punish defaulters by exclusion from future credit. An endogenous debt constraint ensures entrepreneurs repay debts, which depends on $\bar{W}^e = W^e(0,0) = \beta\{\alpha\lambda[f(k) - k - \phi] + \bar{W}^e\}$. An entrepreneur in stage 2 with no wealth has an investment opportunity in the next period with probability $\alpha\lambda$, in which case he gets surplus $f(k) - k - \phi$. Solving for $\bar{W}^e$, we obtain

$$\bar{W}^e = \frac{\alpha\lambda\left[f(k) - k - \phi\right]}{\rho}. \tag{1}$$

Thus, the value of being an entrepreneur is the discounted sum of profits, net of fees. By defaulting, an entrepreneur is banished to autarky and loses $\bar{W}^e$, making the borrowing constraint $\psi + \phi \leq \bar{W}^e$.

Under Nash bargaining the loan contract solves

$$(k, \phi) \in \arg\max \left[f(k) - k - \phi\right]^{1-\theta} \phi^{\theta} \text{ st } k + \phi \leq \bar{W}^e. \tag{2}$$

The problem is convex, since $\bar{W}^e$ is independent of k. The frontier is

$$S^e + S^b = f(k^*) - k^* \text{ if } S^b \leq \bar{W}^e - k^*;\ \Delta^{-1}(S^e + S^b) + S^b = \bar{W}^e \text{ otherwise,}$$

where $\Delta(k) \equiv f(k) - k$ is the total surplus when the constraint binds. Relative to Figure 1, the frontier now intersects the horizontal axis at $S^e = 0$. Notice $k = k^*$ and $\phi = \theta\left[f(k^*) - k^*\right]$ if $\bar{W}^e \geq k^* + \phi$. Using this, the value of an entrepreneur who is not constrained is $\bar{W}^e = \alpha\lambda\left(1-\theta\right)\left[f(k^*) - k^*\right]/\rho$. Accordingly, entrepreneurs are not constrained if

$$\rho \leq \rho^* \equiv \frac{\alpha\lambda\left(1-\theta\right)\left[f(k^*) - k^*\right]}{(1-\theta)k^* + \theta f(k^*)}.$$

Next suppose the constraint binds. The solution to (2) is

$$\bar{W}^e = \frac{\theta f(k) + (1-\theta)f'(k)k}{(1-\theta)f'(k) + \theta}. \tag{3}$$

Now the borrowing limit $\bar{W}^e$ is a weighted average of $f(k)$ and the supplier's cost, k. In this case,

$$\bar{W}^e = \frac{\alpha\lambda}{\rho + \alpha\lambda} f(k). \tag{4}$$

The limit from (4) is analogous to the pledgeability constraint in Section 4 where $\chi_b = \alpha\lambda/(\rho+\alpha\lambda)$. Here pledgeability depends on ρ, λ and α. A difference however is that the RHS of (4) uses future output.

Substituting $\bar{W}^e$ from (4) into (3), k solves

$$\frac{k}{f(k)} = \frac{\alpha\lambda(1-\theta)f'(k) - \rho\theta}{\left(\rho+\alpha\lambda\right)\left(1-\theta\right)f'(k)}. \tag{5}$$

Notice $k = 0$ always solves (5), as is standard. In addition, there is solution $k > 0$ uniquely determined, since the LHS (5) is increasing in k while the RHS is decreasing for all k such that $\alpha\lambda(1-\theta)f'(k) > \rho\theta$. The positive solution increases with α and λ and decreases with ρ and θ. The lending rate is

$$r_b = \frac{\bar{W}^e - k}{k} = \frac{\alpha\lambda}{\rho+\alpha\lambda}\frac{f(k)}{k} - 1,$$

which increases with θ. Notice r_b depends on ρ, since the debt limit is determined by future surpluses, as well as λ and α.

Given $f(k) = zk^\gamma$, when the borrowing constraint is slack, $k^* = (\gamma z)^{\frac{1}{1-\gamma}}$ and $r_b = \theta\left(1-\gamma\right)/\theta$, identical to Section 4. When it binds,

$$k = \left[\frac{\chi_b(1-\theta)z\gamma}{(1-\theta)\gamma + (1-\chi_b)\theta}\right]^{\frac{1}{1-\gamma}} \text{ and } r_b = \frac{(1-\chi_b)\theta}{(1-\theta)\gamma},$$

where $\chi_b \equiv \alpha\lambda/(\rho+\alpha\lambda)$. Now k increases while r_b decreases with pledgeability.

A4. Strategic foundations for bargaining

While the strategic foundations of Nash bargaining are very well known, there are some nuances here, like commitment issues and nonconvexities; therefore, we provide the details. Consider a game with alternating offers between the entrepreneur and bank. There is no discounting, but an exogenous risk of breakdown. At the initial stage, the entrepreneur makes an offer (k^e, d^e, ϕ^e), and the bank can say either yes or no. If it says yes, the offer is implemented. If it say no, the game continues. With probability δ^e negotiations end with no loan; with probability $1 - \delta^e$ the bank makes an offer (k^b, d^b, ϕ^b), and the entrepreneur can either say yes or no. If he says yes, the offer is implemented. If he say no, the game continues. With probability δ^b negotiations end; with probability $1 - \delta^b$ the games continues as in the initial stage. See the game tree in Figure 3. A node with two players corresponds to a simultaneous move and the risk of breakdown is a move by Nature.

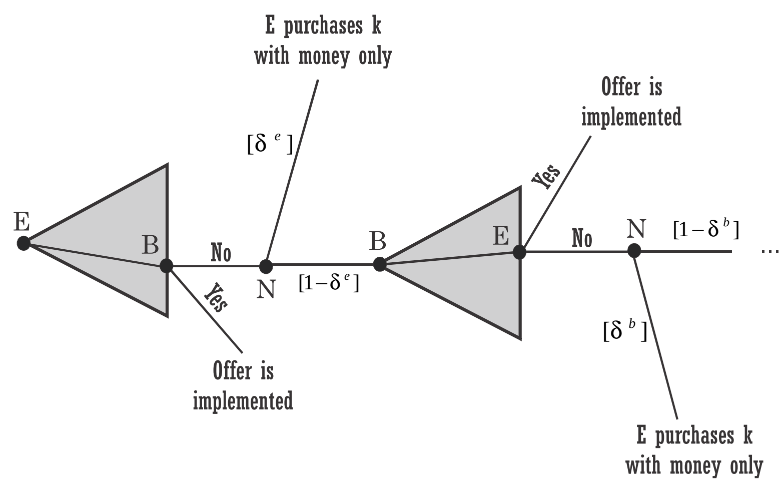

Figure 3: Game tree

Consider stationary equilibria with offers, (k^e, d^e, ϕ^e) and (k^b, d^b, ϕ^b). We restrict attention to acceptance rules in the form of reservation surpluses, R^e and R^b, that specify a minimum surplus required for an agent to accept. Entrepreneurs accept an offer if $f(k) - \psi - \phi \geq R^e$, and banks accept if $\phi \geq R^b$. When it is the entrepreneur turn to make an offer,

$$S^e(R^b) = \max_{k,\phi} \left\{ [f(k) - k - \phi] \, \mathbb{I}_{\{\phi \geq R^b\}} \right\} \text{ st } \; k + \phi \leq \chi_b f(k) + a_m^e,$$

where $\mathbb{I}_{\{\phi \geq R^b\}}$ is an indicator function that equals one if $\phi \geq R^b$ (we ignore the down payment

d, because the entrepreneur uses his real balances before requesting a loan). The solution is:

$$\begin{aligned} S^e(R^b) &= f(k^*) - k^* - R^b \ \text{ if } \ R^b \leq \chi_b f(k^*) - k^* + a_m^e && (6) \\ &= f(k) - k - R^b \quad \text{if} \ \ R^b \in (\chi_b f(k^*) - k^* + a_m^e, \chi_b f(\hat{k}) - \hat{k} + a_m^e] && (7) \end{aligned}$$

where k is the largest solution to $\chi_b f(k) - k = R^b - a_m^e$. If the reservation surplus of the bank is sufficiently low, the entrepreneur can finance k^* and $\phi = R^b$; if R^b is larger but not too large, the entrepreneur asks for the largest loan satisfying the liquidity constraint; if R^b is too large the entrepreneur cannot satisfy $\phi \geq R^b$ and get a surplus. It can be checked that $S^e(R^b)$ is decreasing and concave with $S^e(0) > 0$.

Similarly, the bank's surplus when it is his turn to make an offer is

$$S^b(R^e) = \max_{k,\phi} \left\{ \phi \mathbb{I}_{\{f(k)-k-\phi \geq R^e\}} \right\} \ \text{st} \ k + \phi \leq \chi_b f(k) + a_m^e.$$

The bank maximizes his payoff subject to the acceptance rule and liquidity constraint. The solution is

$$\begin{aligned} S^b(R^e) &= f(k^*) - k^* - R^e \ \text{ if } \ R^e \in [(1-\chi_b) f(k^*) - a_m^e, f(k^*) - k^*] && (8) \\ &= \chi_b f(\hat{k}) - \hat{k} + a_m^e \qquad \text{if} \qquad R^e \leq (1-\chi_b) f(\hat{k}) - a_m^e && (9) \\ &= f(k) - k - R^e \qquad \text{otherwise,} && (10) \end{aligned}$$

where k solves $(1-\chi_b) f(k) = R^e + a_m^e$. If the entrepreneur's reservation surplus is large but not so large the bank would not participate, the bank offers to finance k^*; if R^e is low, the bank asks for a payment such that the constraint binds; and below a threshold for R^e, k maximizes $\chi_b f(k) - k$. It can be checked that $S^b(R^e)$ is nondecreasing, concave, and $S^b(R^e) > 0$.

The endogenous reservations surpluses solve

$$\begin{aligned} R^e &= (1-\delta^b) S^e(R^b) + \delta^b \Delta^m(a_m^e) && (11) \\ R^b &= (1-\delta^e) S^b(R^e). && (12) \end{aligned}$$

Thus, R^e is the surplus that makes the entrepreneur indifferent between accepting or rejecting, and similarly for (12). Note that after a breakdown the bank receives no surplus.

Figure 4 shows (11) in blue and (12) in red; both are downward sloping and concave. To establish existence, let $\bar{R}^e > 0$ be the R^e such that $S^b(R^e) = 0$. By the duality of the entrepreneur and bank problems, $\bar{R}^e = S^e(0)$. Moreover, provided that $a^e_m < k^*$ then $\Delta^m(a^e_m) < S^e(0)$. Hence, the blue curve is below the red curve at $R^b = 0$. The red curve has a maximum $(1-\delta^e)S^b(0) < \chi_b f(\hat{k}) - \hat{k} + a^e_m$. So at $R^b = \chi_b f(\hat{k}) - \hat{k} + a^e_m$ the blue curve is to the right of the red curve. Hence, they intersect, so a solution exists. Uniqueness follows from concavity of the relationships and the fact that when they are linear, they have different slopes.

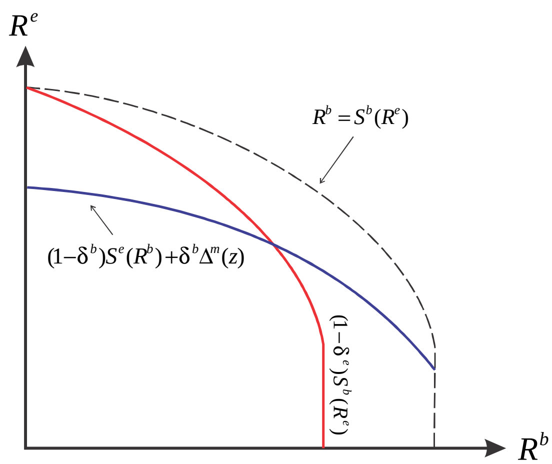

Figure 4: Determination of (R^b, R^e)

A stationary, subgame perfect equilibrium is composed of two offers, (k^e, d^e, ϕ^e) and (k^b, d^b, ϕ^b), and two reservation surpluses, R^e and R^b, solving the above conditions, as is completely standard. Existence and uniqueness here follow from the above discussion. Now consider letting the risk of breakdown get small by rewriting $\delta^e = \varepsilon\bar{\delta}^e$ and $\delta^b = \varepsilon\bar{\delta}^b$. As $\varepsilon \to 0$, $S^e(R^b) - R^e \to 0$ and $S^b(R^e) - R^b \to 0$ (i.e., when the breakdown risk gets small, first-mover advantage vanishes). Graphically, the reservation values at the intersection of the curves in Figure 4 converge to a point on the dashed curve.

Suppose first the borrowing constraint does not bind. Then $S^e(R^b) = f(k^*) - k^* - R^b$ and $S^b(R^e) = f(k^*) - k^* - R^e$. Thus, both the entrepreneur and bank offer k^*, and use ϕ to satisfy the acceptance rules. Taking the limit as $\varepsilon \to 0$,

$$R^e \to \frac{\bar{\delta}^e \left[f(k^*) - k^*\right] + \bar{\delta}^b \Delta^m(a_m^e)}{\bar{\delta}^e + \bar{\delta}^b} \tag{13}$$

$$R^b \to \frac{\bar{\delta}^b}{\bar{\delta}^e + \bar{\delta}^b} \left[f(k^*) - k^* - \Delta^m(a_m^e)\right]. \tag{14}$$

The banks' surplus approaches a fraction $\bar{\delta}^b/(\bar{\delta}^e + \bar{\delta}^b)$ of the total surplus, coinciding with the Nash solution with $\theta = \bar{\delta}^b / \left(\bar{\delta}^e + \bar{\delta}^b\right)$.

Now suppose the liquidity constraint binds. Then $S^e(R^b) = f(k^e) - k^e - R^b$ where k^e is the highest solution to $k^e + R^b = \chi_b f(k^e) + a_m^e$, and $S^b(R^e) = f(k^b) - k^b - R^e$ where k^b is the solution to $(1 - \chi_b) f(k^b) = R^e + a_m^e$. Now (k^e, k^b, R^e, R^b) solves

$$R^e = (1 - \bar{\delta}^b \varepsilon) \left[f(k^e) - k^e - R^b\right] + \bar{\delta}^b \varepsilon \Delta^m(a_m^e) \tag{15}$$

$$R^b = (1 - \bar{\delta}^e \varepsilon) \left[f(k^b) - k^b - R^e\right] \tag{16}$$

$$R^b = \chi_b f(k^e) - k^e + a_m^e \tag{17}$$

$$R^e = (1 - \chi_b) f(k^b) - a_m^e. \tag{18}$$

Rearranging (15)-(16) we obtain

$$R^e = \frac{(1 - \bar{\delta}^b \varepsilon) \left\{f(k^e) - k^e - (1 - \bar{\delta}^e \varepsilon) \left[f(k^b) - k^b\right]\right\} + \bar{\delta}^b \varepsilon \Delta^m(a_m^e)}{1 - (1 - \bar{\delta}^b \varepsilon)(1 - \bar{\delta}^e \varepsilon)}.$$

Letting $\varepsilon \to 0$ and using L'Hopital's rule, we get

$$R^e = \frac{\bar{\delta}^e \left[f(k) - k\right] + \left[f'(k) - 1\right] \left(\frac{dk^e}{d\varepsilon} - \frac{dk^b}{d\varepsilon}\right) + \bar{\delta}^b \Delta^m(a_m^e)}{\bar{\delta}^b + \bar{\delta}^e}. \tag{19}$$

The terms $dk^e/d\varepsilon$ and $dk^b/d\varepsilon$ are obtained by differentiating (15)-(18) in the neighborhood of $\varepsilon = 0$,

$$\frac{dk^e}{d\varepsilon} - \frac{dk^b}{d\varepsilon} = \frac{\bar{\delta}^e \left[f(k) - k - R^e\right]}{1 - \chi_b f'(k)}. \tag{20}$$

Substituting (20) into (19) and replacing R^e by $(1 - \chi_b) f(k) - a_m^e$, we get

$$\left(\frac{\bar{\delta}^b}{\bar{\delta}^e}\right) \frac{1 - \chi_b f'(k)}{(1 - \chi_b) f'(k)} = \frac{\chi_b f(k) - k + a_m^e}{(1 - \chi_b) f(k) - a_m^e - \Delta^m(a_m^e)}. \tag{21}$$

This corresponds to the FOC from Nash bargaining with $\theta = \bar{\delta}^b/(\bar{\delta}^e + \bar{\delta}^b)$. As usual, subgame perfect equilibrium in the game generates the same outcome as Nash bargaining.

A6. Introducing a corporate bonds market

In the presence of the corporate bonds market, the profits of the unbanked entrepreneurs from an investment opportunity are

$$\Delta^m = \max_{k \geq 0} \{f(k) - (1 + i_e)k\},$$

where i_e is the interest rate on a loan in the (corporate bonds) market where entrepreneurs can borrow and lend their cash. The solution is such that $(1 + i_e)k \leq f(k)$, hence we can omit the borrowing constraint. The cost of borrowing, i_e, coincides with the opportunity cost of financing k with a_m^e, which is the foregone interest from not lending a_m^e in the corporate bonds market. So we can think of the entrepreneur as both borrowing on the corporate bonds market to finance k and lending a_m^e at the same rate i_e. Importantly, the lending of a_m^e in the corporate bonds market happens whether the entrepreneur is banked or unbanked. Hence, the surplus from being banked is

$$f(k) - k - \phi - (k - \ell)i_e - \Delta^m.$$

It is independent of a_m^e. The third and fourth terms correspond to two types of interest payments: ϕ is the interest payment to the bank while $(k - \ell)i_e$ is the interest payment to other entrepreneurs in the corporate bonds market. The generalized Nash bargaining problem between the bank and the entrepreneur is

$$\max_{k,\ell,\phi} [f(k) - k - \phi - (k - \ell)i_e - \Delta^m]^{1-\theta} \phi^\theta,$$

where $\ell \leq k$ is the bank loan and $k - \ell$ is the amount borrowed in the corporate bond market. For all $i_e > 0$ it is jointly efficient to save the borrowing cost on the corporate bonds market, $k = \ell = k^*$ and $\phi = \theta [f(k^*) - k^* - \Delta^m]$.

The value of an entrepreneur in the first stage solves:

$$V^e(a_m) = (1 + i_e)a_m + \lambda\Delta^m + \lambda\alpha(1 - \theta) [f(k^*) - k^* - \Delta^m] + W^e(0, 0), \tag{22}$$

where we omit illiquid bonds. The entrepreneur lends his real balances in the corporate bonds market at the interest rate i_e. With probability λ, he receives an investment opportunity and can secure a surplus equal to Δ^m. With probability α, he meets a bank and raises his surplus by $(1-\theta)\left[f(k^*)-k^*-\Delta^m\right]$.

The entrepreneur's choice of real balances in stage 2 is the solution to

$$\max_{\hat{a}_m \geq 0}\left\{-\frac{\hat{a}_m}{1+r_m}+\beta V^e(\hat{a}_m)\right\}.$$

Substituting $V^e(\hat{a}_m)$ by its expression in (22), the choice of real balances reduces to

$$\max_{\hat{a}_m \geq 0}\left\{(i_e - i)\,\hat{a}_m\right\}.$$

So a positive solution exists if $i_e = i$. The interest rate in the corporate bonds market compensates entrepreneurs for the holding cost of cash. As a result, in equilibrium entrepreneurs are indifferent between bringing cash in the corporate bonds market or not. Market clearing in the corporate bonds market implies $A_m = \lambda(1-\alpha)k^m$, aggregate real balances are equal to the aggregate investment of unbanked entrepreneurs.

While the expression for ϕ is similar to the one in the economy without corporate bonds market, the expression for Δ^m differs. Without corporate bonds market:

$$\tilde{\Delta}^m = f(\tilde{k}) - \tilde{k} \;\text{ where } i = \lambda\left[1-\alpha(1-\theta)\right]\left[f'(\tilde{k})-1\right].$$

Using that $i_e = i$, it follows that $k^m > \tilde{k}$. Because money holdings can be allocated to unbanked entrepreneurs with an investment opportunity through the corporate bonds market, there are no idle money balances and hence investment increases. However, Δ^m includes the borrowing cost whereas $\tilde{\Delta}^m$ does not include the cost of holding money which is sunk at the time of the investment. Therefore, $\tilde{\Delta}^m$ can be larger than Δ^m. For instance, if $\lambda = 1$ and $\alpha \approx 0$, then $\tilde{k} \approx k^m$ but $\tilde{\Delta}^m > \Delta^m$. As a result, the intermediation fee is larger in the economy with the corporate bonds market. Intuitively, because the cost of internally financing the investment has been sunk in the economy without a corporate bonds market, the entrepreneur ends up being in a stronger position to bargain over the interest payment with the bank.

A7. Structure of interest rates

We extend our model to characterize the structure of rates of return across different assets. First, we assume that one-period government bonds are partially liquid in the following sense: an investor holding a_g^e units of bonds in stage 1 can trade a fraction $\chi_g \in [0,1]$ in exchange for k. Second, in order to make bank loans comparable to one-period bonds we assume that they are repaid after one period. More precisely, investment opportunities in stage 1 of period t generate output $f(k)$ in stage 2 of period $t+1$ and bank loans offered in t are repaid at the time when investment pays off. Banks who issue IOUs in period t can commit to redeem them in stage 2 of period t. Because there is now a mismatch between the maturity of banks' liabilities and the maturity of the loans we allow banks to produce the numéraire at a unit cost (alternatively, we could assume banks are large entities with a large number of loans and liabilities, or that banks' IOUs are repaid in $t+1$).

The surplus of an unbanked entrepreneur is

$$\Delta^m(a_m^e + \chi_g a_g^e) = \beta f(k^m) - k^m \text{ where } k^m = \min\{a_m^e + \chi_g a_g^e, k^*\},$$

where $f'(k^*) = \beta^{-1} = 1+\rho$. In contrast to the formulation in the main text, output is product with a one-period lag, hence the discounting. Note that the liquid assets of the entrepreneur are composed of real balances, a_m^e, and the pledgeable bonds, $\chi_g a_g^e$. The bargaining problem with the bank is

$$\max_{k,d,\phi} \left[\beta f(k) - d - \beta\Phi - \Delta^m(a_m^e + \chi_g a_g^e)\right]^{1-\theta} \left[-(k-d) + \beta\Phi\right]^{\theta}$$
$$\text{st } \Phi \leq \chi_b f(k) \text{ and } d \leq a_m^e + \chi_g a_g^e,$$

where $\Phi = k - d + \phi$ is the sum of the principal and interest payments paid by the entrepreneur to the bank in $t+1$. The surplus of the bank is the difference between the IOU it must repay in t, $k-d$, and the discounted value of the repayment by the entrepreneur in $t+1$, $\beta\Phi$. The down payment can now be composed of real balances and bonds and it cannot exceed the liquid wealth $a_m^e + \chi_g a_g^e$. If the liquidity constraint does not bind, $k^c = k^*$ and

$$\Phi = \frac{(k^* - k^m) + \theta\left[\beta f(k^*) - k^* - \Delta^m(a_m^e + \chi_g a_g^e)\right]}{\beta}.$$

The discounted payment to the bank, $\beta\Phi$, is the sum of the repayment of the loan and a fraction θ of the surplus generated by the bank loan. The liquidity constraint does not bind if

$$a_m^e + \chi_g a_g^e + \theta\Delta^m(a_m^e + \chi_g a_g^e) \geq (1-\theta)k^* + (\theta - \chi_b)\beta f(k^*). \tag{23}$$

Using the definition of Φ, interest payments are equal to

$$\phi = \rho(k^* - k^m) + \frac{\theta\left[\beta f(k^*) - k^* - \Delta^m(a_m^e + \chi_g a_g^e)\right]}{\beta}.$$

The first term is the interest payment if banks had no bargaining power. In that case, the real rate of return of the loan would be equal to the rate of time preference. The second term is a rent that the bank can extract given its bargaining power. Dividing by the loan size, $k^* - k^m$, the real lending rate is

$$r_b = \rho + \frac{\theta\left[\beta f(k^*) - k^* - \Delta^m(a_m^e + \chi_g a_g^e)\right]}{\beta(k^* - k^m)}. \tag{24}$$

So the real lending rate is the sum of the rate of time preference and an intermediation premium that depends on banks' bargaining power.

Suppose next that the liquidity constraint does bind. The solution to the bargaining problem is

$$\begin{aligned}
\frac{\chi_b \beta f(k) - k + a_m^e + \chi_g a_g^e}{(1-\chi_b)\beta f(k) - (a_m^e + \chi_g a_g^e) - \Delta^m(a_m^e + \chi_g a_g^e)} &= \frac{\theta}{1-\theta}\frac{1 - \chi_b \beta f'(k)}{(1-\chi_b)\beta f'(k)} \\
\Phi &= \chi_b f(k).
\end{aligned}$$

These equations are analogous to the ones in the main text where the production function is scaled by β and a_m^e is replaced with $a_m^e + \chi_g a_g^e$. In this case the real lending rate is

$$r_b = \frac{\chi_b f(k)}{k - (a_m^e + \chi_g a_g^e)} - 1.$$

From (2), the entrepreneur's choice of money balances and bond holdings solves

$$\max_{a_m^e, a_g^e \geq 0} \left\{ -i a_m^e - \left(\frac{i - i_g}{1 + i_g}\right) a_g^e + \lambda(1-\alpha)\Delta^m(a_m^e + \chi_g a_g^e) + \alpha\lambda\Delta^c(a_m^e + \chi_g a_g^e) \right\}, \tag{25}$$

where

$$\Delta^c(a_m^e) = \begin{cases} (1-\theta)\left[\beta f(k^*) - k^*\right] + \theta\Delta^m(a_m^e + \chi_g a_g^e) & \text{if } a_m^e + \chi_g a_g^e \geq a^* \\ (1-\chi_b)\beta f(k^c) - \left(a_m^e + \chi_g a_g^e\right) & \text{if } a_m^e + \chi_g a_g^e < a^*, \end{cases}$$

where a^* is the value of $a_m^e + \chi_g a_g^e$ such that (23) holds at equality.

In order to get closed form expressions, consider the regime where the liquidity constraint does not bind. Using a second-order approximation for $\Delta^m(a_m^e + \chi_g a_g^e)$,

$$\Delta^m(a_m^e + \chi_g a_g^e) \approx \beta f(k^*) - k^* + \beta f''(k^*)\frac{(k^m - k^*)^2}{2}.$$

Plug this expression into (24) to obtain

$$r_b \approx \rho - \frac{\theta f''(k^*)(k^* - k^m)}{2}. \tag{26}$$

Assuming interior solutions the FOC from (25) gives

$$\lambda \left[1 - \alpha(1-\theta)\right] \left[\beta f'(k^m) - 1\right] = i = \frac{i - i_g}{\chi_g \left(1 + i_g\right)}$$

In order to guarantee that the solution is interior we would have to check that $\chi_g a_g^e < k^m$. It follows that the spread between illiquid and liquid bonds is

$$\frac{i - i_g}{1 + i_g} = \frac{\rho - r_g}{1 + r_g} = \chi_g i.$$

Using a first-order approximation of the LHS,

$$k^m - k^* \approx \frac{i}{\beta f''(k^*)\lambda \left[1 - \alpha(1-\theta)\right]}. \tag{27}$$

Substitute $k^m - k^*$ from (27) into (26), the lending rate can be approximated as:

$$r_b \approx \rho + (1+\rho)\frac{\theta i}{2\lambda \left[1 - \alpha(1-\theta)\right]}. \tag{28}$$

If $\rho = 0$, the expression for r_b corresponds to the one in the text. Alternatively, the yield difference between a bank loan and a risk-free (illiquid) bond is

$$\frac{r_b - \rho}{1 + \rho} \approx \frac{\theta i}{2\lambda \left[1 - \alpha(1-\theta)\right]}. \tag{29}$$

A8. Long-lived investment projects

In the main text, investment projects are short-lived: investment opportunities in the first stage have a single pay-off in the second stage. In many macroeconomic applications investment opportunities have long-lasting payoffs, e.g., firms in the Pissarides or Melitz models or Lucas trees. Suppose that entrepreneurs can create long-lived assets (akin to Lucas trees) that generate a payoff $f(k)$ every period that depends on the initial investment, k. (The investment is putty-clay.) Those assets fully depreciate at the end of a period with probability δ. (One can think of it as the death rate of a firm/job.) The discounted sum of the output flows generated by this investment project is:

$$F(k) = \frac{f(k)}{1 - (1-\delta)\beta}.$$

The benchmark version of our model corresponds to the case $\delta = 1$.

For simplicity we set $\chi_s = 0$. We consider a lending contract composed of an investment size, k, an initial down payment, d, and a payment to the bank every period, Φ. The per-period payment to the bank is subject to the pledgeability constraint, $\Phi \leq \chi_b f(k)$. Equivalently, we can write the liquidity constraint as:

$$\frac{\Phi}{1-(1-\delta)\beta} \leq \chi_b F(k),$$

where the left side is the entrepreneur's debt expressed as the discounted sum of the payments to the bank and the right side is a fraction χ_b of the value of the investment project. The discounted sum of the banks' profits are:

$$-(k-d) + \frac{\Phi}{1-(1-\delta)\beta}.$$

The first term is the loan size, $k-d$. The second term is the discounted sum of the payments to the bank. If we denote $\phi = \Phi - [1-(1-\delta)\beta](k-d)$ the discounted sum of the bank's profits can be expressed as $\phi/[1-(1-\delta)\beta]$. The surplus of the entrepreneur from a bank loan is

$$F(k) - d - \frac{\Phi}{1-(1-\delta)\beta} - \Delta^m(a^e_m),$$

where $\Delta^m(a^e_m)$ is the surplus if the entrepreneur self-finances the investment. The first term is the value of the investment project, the second term is the down payment, and the third term corresponds to the interest payments to the bank. Using the definitions of ϕ and $F(k)$ we can reexpress this surplus as

$$\frac{f(k)-\phi}{1-(1-\delta)\beta} - k - \Delta^m(a^e_m).$$

The bargaining problem between the bank and entrepreneur becomes:

$$\max_{k,d,\phi} \left[\frac{f(k)-\phi}{1-(1-\delta)\beta} - k - \Delta^m(a^e_m)\right]^{1-\theta} \left[\frac{\phi}{1-(1-\delta)\beta}\right]^{\theta} \tag{30}$$

$$\text{s.t.} \quad \phi + [1-(1-\delta)\beta](k-d) \leq \chi_b f(k) \quad \text{and } d \leq a^e_m. \tag{31}$$

This bargaining problem coincides with the one in the main text when $\delta = 1$. The disagreement point, $\Delta^m(a^e_m)$, is computed as before where $f(k)$ is replaced with $F(k)$, i.e.,

$$\Delta^m(a^e_m) = F(k^m) - k^m \quad \text{where} \quad k^m = \min\{a^e_m, k^*\},$$

where k^* is such that $F'(k^*) = 1$. If the liquidity constraint does not bind, then the solution to (30)-(31) is

$$\begin{aligned} f'(k) &= 1-(1-\delta)\beta && (32) \\ \phi &= \theta\left\{f(k)-[1-(1-\delta)\beta]\left[k+\Delta^m(a_m^e)\right]\right\}. && (33) \end{aligned}$$

Equation (32) equalizes the marginal product of capital with the rate of time preference adjusted by the depreciation rate. Equation (33) gives the flow payment that splits the match surplus. If the liquidity constraint does bind then the bargaining problem (30)-(31) reduces to:

$$\max_{k\geq 0}\left[\frac{(1-\chi_b)f(k)}{1-(1-\delta)\beta}-a_m^e-\Delta^m(a_m^e)\right]^{1-\theta}\left[\frac{\chi_b f(k)}{1-(1-\delta)\beta}-(k-a_m^e)\right]^{\theta}.$$

The FOC is:

$$\begin{aligned} \frac{\chi_b f(k)-[1-(1-\delta)\beta]\,(k-a_m^e)}{(1-\chi_b)f(k)-[1-(1-\delta)\beta]\left[a_m^e+\Delta^m(a_m^e)\right]} &= \frac{\theta}{1-\theta}\frac{1-(1-\delta)\beta-\chi_b f'(k)}{(1-\chi_b)f'(k)} \\ \phi &= \chi_b f(k)-[1-(1-\delta)\beta]\,(k-a_m^e). \end{aligned}$$

The comparative statics are similar to the ones in the benchmark model when $\delta = 1$.

We compute the rate of return on the loan as

$$r_b = \frac{\Phi}{k-d}-\delta.$$

The first term corresponds to the interest payment as a frcation of the loan size, $k-d$. The second term is the probability at which the loan is terminated. It follows that

$$r_b = (1-\delta)\,(1-\beta)+\frac{\phi}{k-d}.$$

The second term on the RHS is the lending rate as computed in the main text. The first term on the RHS is the return necessary to compensate for the rate of time preference and the termination rate.

Assuming the pledgeability constraint does not bind, the entrepreneur's choice of money balances solves

$$\max_{a_m^e\geq 0}\left\{-ia_m^e+\lambda\,(1-\alpha)\,\Delta^m(a_m^e)+\alpha\lambda\Delta^c(a_m^e)\right\},$$

where $\Delta^c(a^e_m) = (1-\theta)\left[F(k^*) - k^*\right] + \theta\Delta^m(a^e_m)$. The FOC is

$$\frac{i}{\lambda\left[1-\alpha(1-\theta)\right]} = \frac{f'(k)}{1-(1-\delta)\beta} - 1.$$

Using the same approximations as in the main text,

$$r_b \approx \frac{(1-\delta)\,\rho}{1+\rho} + \frac{(\rho+\delta)\,\theta i}{2(1+\rho)\lambda\left[1-\alpha(1-\theta)\right]}.$$

The first term is the frictionless rate while the second term is the premium arising from frictions in the credit market.

A9. Money, trade credit, and bank credit

We now let entrepreneurs accumulate real balances and use trade credit. The surplus of an unbanked entrepreneur with a^e_m real balances is now

$$\Delta^m(a^e_m;\chi_s) = \max\left\{f(k^m) - k^m\right\} \text{ st } k^m \le a^e_m + \chi_s f(k^m).$$

It is easy to check that $k^m = k^*$ if $a^e_m \ge k^* - \chi_s f(k^*)$ and $k^m - \chi_s f(k^m) = a^e_m$ otherwise. Moreover, the marginal value of real balances is

$$\Delta^{m\prime}(a^e_m;\chi_s) = \frac{f'(k^m) - 1}{1-\chi_s f'(k^m)} \text{ if } a^e_m < k^* - \chi_s f(k^*),$$

where we used that $\partial k^m/\partial a^e_m = 1/\left[1-\chi_s f'(k^m)\right]$. Money has a multiplier effect on trade credit. An additional unit of real balances allows entrepreneurs to increase investment and hence pledgeable output, which in turn allows suppliers to offer bigger loans. Using a second-order Taylor series expansion for a^e_m close to $a^*_m = k^* - \chi_s f(k^*)$ so that k^m is close to k^*:

$$\begin{aligned}\Delta^m(a^e_m;\chi_s) &= \Delta^m(a^*_m;\chi_s) + \Delta^{m\prime\prime}(a^*_m;\chi_s)\frac{(a^*_m - a^e_m)^2}{2} \\ &= f(k^*) - k^* + \frac{f''(k^*)}{(1-\chi_s)^2}\frac{(a^*_m - a^e_m)^2}{2},\end{aligned}$$

where we used that $\Delta^{m\prime}(a^*_m;\chi_s) = 0$ in the first equality, i.e., a change in real balances only has a second-order effect on the entrepreneur's surplus when k^m is close to k^*. To obtain the second equality we used that

$$\begin{aligned}\Delta^{m\prime\prime}(a^e_m;\chi_s) &= \frac{f''(k^m)\left[1-\chi_s f'(k^m)\right] + \left[f'(k^m) - 1\right]\chi_s f''(k^m)}{\left[1-\chi_s f'(k^m)\right]^2}\frac{\partial k^m}{\partial a^e_m} \\ &= \frac{f''(k^*)}{(1-\chi_s)^2}.\end{aligned}$$

Consider next an entrepreneur in contact with a bank, where loan contracts now specify an investment level k, a down payment d, and the bank's fee ϕ. If the loan negotiations are unsuccessful, the entrepreneur can purchase k with cash and trade credit. So his surplus from the bank loan is $f(k) - k - \phi - \Delta^m(a_m^e; \chi_s)$. Then the bargaining problem is

$$\max_{k,d,\phi} \left[f(k) - k - \phi - \Delta^m(a_m^e; \chi_s)\right]^{1-\theta} \phi^\theta \text{ st } k - d + \phi \leq \chi_b f(k) \text{ and } d \leq a_m^e.$$

This problem is formally equivalent to the one studied earlier where the threat point, $\Delta^m(a_m^e; \chi_s)$, has been generalized. Notice that the contract does not specify if part of the loan, $k - d$, is provided by suppliers since it would not affect payoffs.

We consider equilibria where i is small so that the liquidity constraints, $k - d + \phi \leq \chi_b f(k)$ and $d \leq a_m^e$, do not bind. Assuming an interior solution, the entrepreneur's money demand is given by:

$$\frac{i}{\lambda \left[1 - \alpha(1 - \theta)\right]} = \frac{f'(k^m) - 1}{1 - \chi_s f'(k^m)}.$$

An interior solution exists if χ_s is less than some threshold. A first-order approximation of the RHS gives

$$\frac{i}{\lambda \left[1 - \alpha(1 - \theta)\right]} = \frac{f''(k^*)}{(1 - \chi_s)^2}(a_m^e - a_m^*). \tag{34}$$

The intermediation payment to the bank is

$$\phi = \theta \left[f(k^*) - k^* - \Delta^m(a_m^e; \chi_s)\right].$$

Using the approximations above,

$$\phi \approx \frac{-\theta \left[(1 - \chi_s)\, i\right]^2}{2\lambda^2 \left[1 - \alpha(1 - \theta)\right]^2 f''(k^*)}.$$

The availability of trade credit, $\chi_s > 0$, reduces the pass through from i to ϕ.

We consider two alternative assumptions for the distribution of loans to the entrepreneur and hence the definition of the lending rate. Suppose first that the bank is offering the full loan of size $k^* - a_m^e$. The lending rate is then defined as:

$$r_b = \frac{\theta \left[f(k^*) - k^* - \Delta^m(a_m^e; \chi_s)\right]}{k^* - a_m^e}.$$

From the approximation for $\Delta^m(a_m^e)$ above and the fact that $k^* - a_m^e \approx k^* - a_m^*$, a change in i only has a second-order effect on r_b.

Alternatively, suppose that the supplier borrows $a_m^* - a_m^e$ from the bank and $k^* - a_m^* = \chi_s f(k^*)$ from the supplier. The lending rate is now defined as

$$r_b' = \frac{\theta \left[f(k^*) - k^* - \Delta^m(a_m^e; \chi_s)\right]}{k^* - \chi_s f(k^*) - a_m^e}$$

Plugging the approximation for $\Delta^m(a_m^e)$ into the expression for r_b' gives

$$\begin{aligned} r_b' &= \frac{-\theta f''(k^*)}{(1-\chi_s)^2} \frac{(a_m^* - a_m^e)}{2} \\ &= \frac{\theta i}{2\lambda \left[1 - \alpha(1-\theta)\right]}. \end{aligned}$$

Better access to unintermediated credit, through a higher χ_s, does not affect the pass through from the policy rate to the lending rate.

A first-order approximation of $k^m - \chi_s f(k^m)$ in the neighborhood of k^* gives

$$k^m - \chi_s f(k^m) = a_m^* + (1-\chi_s)(k^m - k^*).$$

Hence, $a_m^e - a_m^* = (1-\chi_s)(k^m - k^*)$. Substituting this expression into (34),

$$k^m = \frac{(1-\chi_s)\, i}{\lambda \left[1 - \alpha(1-\theta)\right] f''(k^*)} + k^*.$$

Investment becomes less responsive to changes in the policy rate as χ_s increases.