Marking to Market versus Taking to Market

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Online appendix

E.1. Proof of Proposition 1

Step 1. Design of the optimal contract

Bids below or equal to l are uninformative and thus there cannot be a benefit from selling below l (and there will be a loss if the sale is at a discount). Let b denote the set of genuine bids strictly above l received by the firm (equal to $\emptyset$ if the firm receives no such bid). The same offer can be made by several bidders, in which case each is counted as an element of the set. Let $\hat{b}$ denote the set of bids strictly above l announced by the agent. That the agent can issue fake bids implies $b \subseteq \hat{b}$. Let $x_k(s, \hat{b})$ denote the probability that the principal asks for a resale at the kth bid $\hat{b}_k$ of $\hat{b}$, and

$$\text{(E.1)} \qquad x(s, \hat{b}) = \sum_k x_k(s, \hat{b}) \in [0, 1],$$

with $x(s, \emptyset) = 0$.

The revelation principle implies that we can focus on direct incentive-compatible mechanisms that elicit truth-telling: $\hat{b} = b$. If a resale fails to go through because the bid is fake, it is clearly optimal to let the agent receive zero utility. More generally, let us denote by $u(s, b)$ the utility that a given direct mechanism grants to the agent when the signal realization is s and the set of legitimate bids strictly above l received by the firm is b. For $y \in \{l; h\}$ and every signal realization s, we also let

$$\text{(E.2)} \qquad U_y(s) = E_b[u(s, b) \mid s, y].$$

Truth-telling implies that

$$\text{(E.3)} \qquad U_l(s) = u(s, \emptyset).$$

Finally, we denote $c_h(s)$ the expected cost of resales associated with this mechanism given s and $y = h$:

$$\text{(E.4)} \qquad c_h(s) = E_b\left[\sum_k x_k(s, b)(h - b_k) \mid s, h\right].$$

Let $u_0(s, b)$ denote the payoff to the agent if there is no resale (the agent receives this payoff with probability $1 - x(s, b)$). Truth-telling implies:

(E.5) $$u(s, \emptyset) \geq (1 - x(s, b))u_0(s, b) \text{ for all } b,$$

otherwise the agent would issue some fake bids b if the signal is s and the firm receives no bid above l. Furthermore, for all b,

(E.6) $$u(s, b) \leq x(s, b) + (1 - x(s, b))u_0(s, b) \leq x(s, b) + u(s, \emptyset),$$

which together with $u(s, b) \leq 1$ implies

(E.7) $$u(s, b) \leq \min\{1, x(s, b) + u(s, \emptyset)\} = u(s, \emptyset) + \min\{1 - u(s, \emptyset), x(s, b)\},$$

and in turn

(E.8) $$U_h(s) \leq u(s, \emptyset) + E_b[\min\{1 - u(s, \emptyset), x(s, b)\} \mid s, h].$$

The optimal contract must then solve:

(E.9) $$\min_{\{U_h(.), u(.,\emptyset), x(.,.)\}} \left\{ \int \left[pf_h(s)\left[U_h(s) + c_h(s)\right] + (1 - p)f_l(s)u(s, \emptyset) \right] ds \right\}$$

subject to the ex-ante incentive constraint

(E.10) $$\int \left[f_h(s)U_h(s) - f_l(s)u(s, \emptyset) \right] ds \geq \beta,$$

and, for all s, the ex-post truthtelling constraint

(E.11) $$U_h(s) \leq u(s, \emptyset) + E_b[\min\{1 - u(s, \emptyset), x(s, b)\} \mid s, h].$$

Condition (E.10) is the ex-ante incentive-compatibility constraint ensuring that the agent exerts effort, and condition (E.11) is the incentive constraint for truth telling about bids (plus feasibility $x(s, b) \leq 1$).[1] Note that the incentive constraints depend only on $x(s, b)$ and not on its allocation among the $x_k(s, b)$. So the minimization of c_h requires choosing the highest bid in b (or no bid at all). And so for $b \neq \emptyset$, $c_h(s) = E_b[x(s, b)(h - \max_{\{b_k \in b\}} b_k)|s, h]$. The optimal policy therefore depends only on the distribution $H(t)$ of the highest bid for asset h (there is no point picking $x(s, b) \neq x(s, b')$ for $b \neq b'$ but $\max_{\{b_k \in b\}} b_k = \max_{\{b_k \in b'\}} b_k$, since both have the same informational content concerning the agent's performance).

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[1] As is standard, we ignore other ex-post constraints when solving this program. It will be evident that the solution satisfies them.

Furthermore, a simple inspection of the program shows that at the optimal contract, (E.10) and (E.11) must be binding. Substituting (E.11) into (E.10), and $\int [pf_h(s)U_h(s) - pf_l(s)u(s,\emptyset)]\,ds$ with $p\beta$ in (E.9) yields a Lagrangian:

(E.12)

$$
\mathcal{L} = -\int [f_l(s)u(s,\emptyset) + pc_h(s)f_h(s)]\,ds - p\beta
$$

$$
+ \mu\left[\int [(f_h(s) - f_l(s))u(s,\emptyset) + f_h(s)E_b[\min\{1 - u(s,\emptyset), x(s,b)\} \mid s, h]]\,ds - \beta\right]
$$

$$
= \int \left[\begin{array}{l} \left[\mu - (\mu+1)\frac{f_l(s)}{f_h(s)}\right]u(s,\emptyset) \\ + \mu E_b[\min\{1 - u(s,\emptyset), x(s,b)\} \mid s, h] - pc_h(s) \end{array} \right] f_h(s)\,ds
$$

(E.13) $- \mu\beta - p\beta$,

where μ is the shadow price of (E.10). Letting $\hat{x}(s,t) \equiv x(s,b)$ for $t = \max_{b_k \in b}\{b_k\}$, the Lagrangian can be rewritten

$$
\mathcal{L} = \int \left[\begin{array}{l} \left[\mu - (\mu+1)\frac{f_l(s)}{f_h(s)}\right]u(s,\emptyset) \\ + \int [\mu\min\{1 - u(s,\emptyset), \hat{x}(s,t)\} - p(h-t)\hat{x}(s,t)]\,dH(t) \end{array} \right] f_h(s)\,ds
$$

(E.14) $- \mu\beta - p\beta$.

It is optimal to set $\hat{x}(s,t) = 1 - u(s,\emptyset)$ for all t such that $p(h-t) \le \mu$ and $\hat{x}(s,t) = 0$ otherwise. Accordingly, we define

(E.15) $r = \inf\{t \in [r_-, h] \mid p(h-t) \le \mu\}$,

and

(E.16) $\hat{x}(s,t) = \mathbb{1}_{\{t \ge r\}}(1 - u(s,\emptyset))$.

This yields

$$
\mathcal{L} = \int \left[\begin{array}{l} \left[\mu H(r) + p\int_r^h (h-t)dH(t) - (\mu+1)\frac{f_l(s)}{f_h(s)}\right]u(s,\emptyset) \\ + \int_r^h \left[\mu - p(h-t)\right]dH(t) \end{array} \right] f_h(s)\,ds
$$

(E.17) $- \mu\beta - p\beta$,

and the monotonicity of f_l/f_h implies that there exists σ such that $u(s,\emptyset) = \mathbb{1}_{\{s \ge \sigma\}}$.

Overall, the optimal contract rewards the agent if the signal is above σ or if it is not and a resale is executed above r.

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Step 2. Characterization of r and σ

As a result, the optimal contract corresponds to a pair (σ, r) that solves

(E.18)
$$\min_{\{\sigma, r\}} \left\{ p\beta + 1 - F_l(\sigma) + pF_h(\sigma) \int_r^h (h - t)dH(t) \right\}$$

s.t.

(E.19)
$$F_l(\sigma) - H(r)F_h(\sigma) = \beta,$$

(E.20)
$$r_- \leq r \leq r_+.$$

If there exists no σ such that
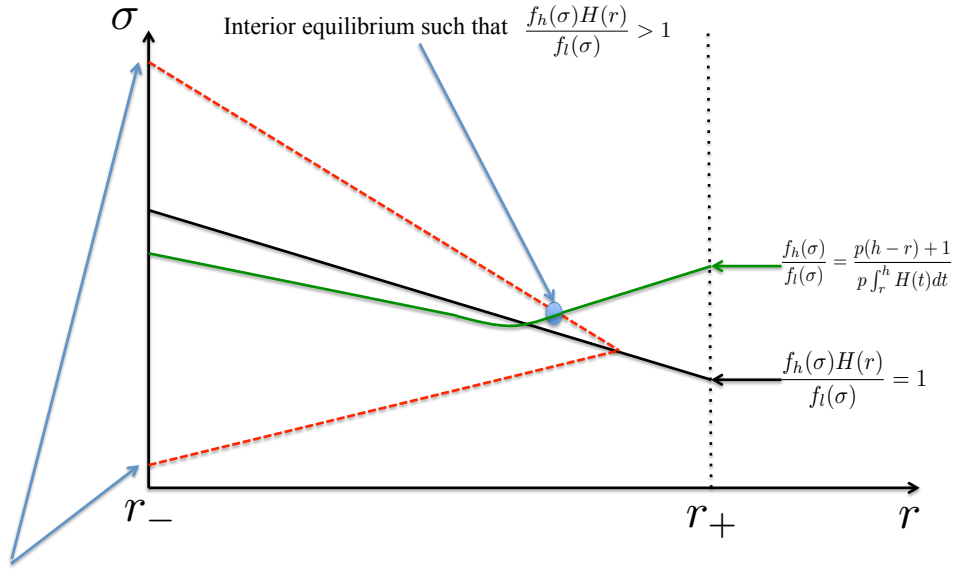
(E.21)
$$F_l(\sigma) - q_0 F_h(\sigma) = \beta,$$
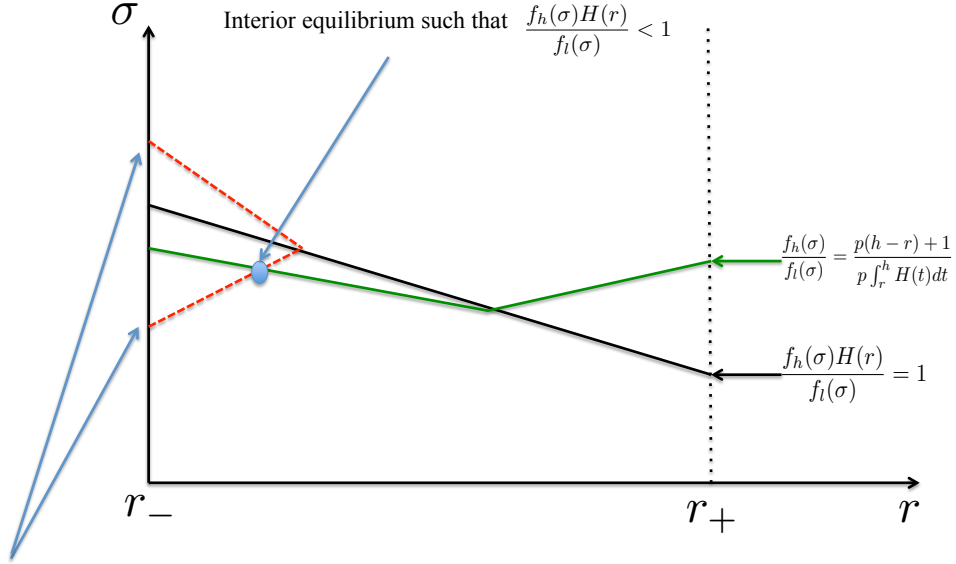
then there exists no contract that elicits high effort.

Looking for an interior solution (that is, ignoring (E.20)), the first-order condition reads:

(E.22)
$$\frac{f_h(\sigma)}{f_l(\sigma)} = \frac{p(h - r) + 1}{p \int_r^h H(t)dt} \equiv T(r).$$

The system of equations $\{(\text{E.19});(\text{E.22})\}$ has at most one solution (σ, r). It is best seen graphically on Figure 3.

Figure 3. First-order condition and incentive-compatibility constraint in the (r,σ) plane. Each graph shows a possible configuration for the IC constraints. The upper one corresponds to a low β, the lower one to a high β.

The equation $[f_h(\sigma)/f_l(\sigma)]H(r) = 1$ defines a decreasing frontier in the plane (r, σ) over $r \in [r_-, r_+]$. The first-order condition (E.22) implicitly defines σ as first decreasing in r below this frontier and then increasing above it over $[r_-, r_+]$. The incentive-compatibility condition defines implicitly two thresholds σ as functions of $r \in [r_-, r_+]$, one that is increasing and lies below the frontier and one that is decreasing and lies above it. These two graphs intersect at the frontier at the maximum value of r for which there exists at least one σ such that the incentive-compatibility constraint is satisfied.

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From Figure 3, it is easy to see that there is at most one (σ, r) that solves {(E.19);(E.22)}. If there is no solution, then the contract is a corner solution that can be of three types:

1. The contract is (σ, r_+), where σ is the largest solution to $F_l(\sigma) - F_h(\sigma) = \beta$, such that $f_h(\sigma)/f_l(\sigma) > T(r_+)$.

2. The contract is (σ, r_-), where σ is the largest solution to $F_l(\sigma) - q_0 F_h(\sigma) = \beta$, such that $f_h(\sigma)/f_l(\sigma) < T(r_-)$.

3. The contract is (σ, r_-), where σ is the smallest solution to $F_l(\sigma) - q_0 F_h(\sigma) = \beta$, such that $f_h(\sigma)/f_l(\sigma) > T(r_-)$.

Intuitively, type 1 corresponds to the situation in which the bids are so low that the firm relies exclusively on the signal and never resells its project. Type 2 corresponds to the case in which bids are sufficiently close to h that the firm finds it optimal to rely as much as possible on resales, but $1 - q_0 < \beta$ forces it to rely on the signal as well. Type 3 is the situation in which resales are very expensive but the firm is very constrained and needs to use them a lot. In this case it seeks to set the signal cutoff at the lowest possible incentive-compatible value so as to minimize the ex-ante frequency of resales.

Graphically, these three cases correspond to three configurations in which the graph corresponding to the first-order condition does not intersect with any of the two dotted curves associated with the incentive-compatibility constraint in Figure 3, either because it is in between them (type 1), above them (type 2), or below them (type 3).

E.2. Proof of Lemma 1

Consider a high-winner-curse equilibrium.

Step 1: Contracts and informed bidding strategies

Step 1 in the proof of Proposition 1, establishing that the optimal contract must be of the form (σ, r), applies to any distribution of the highest bid H provided informed bids are above l and do not depend on s. Any contract in a high-winner-curse equilibrium must therefore be of this form. We show that this implies that bidding strategies are either degenerate or must have a differentiable c.d.f. (For brevity we omit the proof that bidding strategies are symmetric conditionally on the bidder's type.) If a bidding strategy has h in its support then it must be a Dirac delta at h since bidders must be indifferent between all bids. Suppose a bidding strategy does not include h in its

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support. (We tackle equilibria with all bids equal to h at the end of this proof.) If it had an atom at some point of its support (necessarily strictly smaller than h), bids in a right neighborhood of this point would strictly dominate a bid at this point, which cannot be the case. So H is continuous.

We then show that the lower bound of the support of S, r_-, is equal to the reservation price r anticipated by bidders. It must be that $r_- \geq r$ because bids below r are not accepted whereas bids above are accepted with a strictly positive probability and thus yield a strictly positive profit. Suppose $r_- > r$. We know from the above that S is atomless at r_-. This implies that bidding r strictly dominates bidding r_-, which cannot be the case. Thus $r = r_-$.

We then compute π, the expected profit of an informed buyer. We have just seen that the distribution of bids S is atomless with support of the form $[r, r_+] \subset [r, h]$. Thus the ex-ante (before matching) expected profit π of a bidder satisfies:

(E.23)
$$\text{For all } t \in [r, r_+],\ pF_h(\sigma) \sum_{k \geq 1} \frac{q_k}{1 - q_0}(h - t)S^{k-1}(t) = \pi$$

which for $t = r$ yields

(E.24)
$$\pi = \frac{pF_h(\sigma)q_1(h - r)}{1 - q_0}.$$

Furthermore, expression (E.23) implies that S and therefore H are differentiable, and so the first-order condition characterizing the optimal contract in Step 2 in the proof of Proposition 1 applies.

Step 2: Equilibrium incentive-compatibility constraint and first-order condition in an interior equilibrium

There are λ buyers per firm on average so the expected resale cost for a firm is

(E.25)
$$pF_h(\sigma) \int_r^h (h - t)dH(t) = \lambda\pi.$$

The incentive-compatibility constraint (17) is derived in the body of the paper, and the first-order condition (18) stems from

(E.26)
$$\frac{f_h(\sigma)}{f_l(\sigma)} = \frac{p(h - r) + 1}{pH(r)(h - r) + p\int_r^h (h - t)dH(t)},$$

(E.27)
$$H(r) = q_0,$$

(E.28)
$$p\int_r^h (h - t)dH(t) = \frac{\lambda p q_1(h - r)}{1 - q_0}.$$

Step 3: Elimination of corner equilibria

Suppose a high-winner-curse equilibrium features corner contracts ((15) is slack). We know that $r = r_-$ and so type-1 corner equilibria in which $r = r_+$ are ruled out. We focus on corner equilibria of types 2 and 3. We generalize the standard forward induction argument to the case of multiple informed agents by considering a deviation by one bidder, fixing other bidders' equilibrium strategies. So the "sender" is a given bidder and the "receiver" the firm. Conditional on a given public signal s (the dependence on which we will omit for notational simplicity), the sender's action is a bid t. The receiver's action is a state-and-bid-contingent acceptance of a bid—that of the sender or of other bidders—or of no bid at all. The sender's utility is an expected payoff given his expectations over bids by other bidders and the firm's decision. The sender can be of three types—informed of an h-payoff, informed of an l-payoff, or uninformed. The latter two types make zero profit in equilibrium, while all types make zero profit when deviating to $t \in (l, r)$ as this offer is rejected.

To apply the Intuitive Criterion, we need to define the receiver's best response to arbitrary bids and arbitrary beliefs about the sender's type. If the largest bid received by the firm is larger than r, then rejecting $t \in (l, r)$ is the unique element of any best response. If all other bids are below or at l (the other possibility given presumed equilibrium behavior for the other bidders), then any probability of executing t can be optimal for some beliefs about the bidder's type provided (15) is slack and t is sufficiently close to r.

If t is sufficiently close to (but below) r, an uninformed bidder cannot obtain a profit that exceeds the equilibrium level (equal to 0) from a receiver's best response, as this would contradict the assumption of a high-winner-curse equilibrium (and a fortiori nor can an informed bidder knowing l). By contrast, a sender informed about an h-payoff increases his profit relative to the equilibrium profit provided the receiver puts all the weight on the sender's knowing that $y = h$.

From the Intuitive Criterion, the bid t should then be interpreted as coming from a bidder who is informed of an h-payoff, and therefore the receiver should choose to accept it when there is no higher offer. Thus the Intuitive Criterion rules out equilibria in which condition (15) is slack.

Step 4: Equilibria are high-winner-curse ones when l is sufficiently small

Steps 1, 2 and 3 show that any high-winner-curse equilibrium with bids strictly below h must be such that (σ, r) satisfies (17) and (18). We now show that for l sufficiently small, all the equilibria with nondegenerate bids are exactly the high-winner-curse equilibria defined by the (zero, one, or two) (σ, r) that solve $\{(17); (18)\}$.

Note first that the reasoning leading to condition (E.14) in the proof of Proposition 1 holds

even when H depends on s. Condition (E.14) shows that $\sigma = \inf\{s \mid u(s, \emptyset) = 1\}$ is finite. It also shows that firms never accept bids below $r = h - \frac{\mu}{p}$, where μ and thus r do not depend on l because (E.14) is the Lagrangian of a program that does not. Thus, a sufficient condition for uninformed bidders to make a strict loss when bidding at or above r is that l satisfies:

$$(\text{E.29}) \qquad p f_h(\sigma) h + (1 - p) f_l(\sigma) l < [p f_h(\sigma) + (1 - p) f_l(\sigma)] r.$$

For such an l, informed bids such that $r_-(s) = r(s) > r$ can be eliminated using the Intuitive Criterion as in Step 3 and thus in equilibrium it must be that $r_-(s) = r$. The indifference argument in (E.23) that leads to the determination of S then implies that if the reserve price is non-contingent on s, then so is S and therefore H.

This establishes that under (E.29) an equilibrium must be a high-winner-curse one. Finally, noting that there are at most two solutions (σ, r) to $\{(17); (18)\}$, one can choose l sufficiently small that they both correspond to a high-winner-curse equilibrium.

Of course, condition (E.29), which presumes that the uninformed bidder wins whenever $y = h$ (i.e. that he faces no competition from the informed) is simple but stronger than needed. More generally, a necessary and sufficient condition for the uninformed not to want to bid above l when bidders use symmetric bidding strategies $S(.)$ is that for all $t \geq r$

$$(\text{E.30}) \qquad p f_h(s)[q_0(h - t) + q_1 S(t)(h - r)] \leq (1 - p) f_l(s)(t - l).$$

See Section E.10 for a proof of the more general version of (E.30) when informed bids are state-contingent. Surprisingly, the binding constraint may not be the possibility that the uninformed bid r, but rather that they bid a t greater than r. Section E.10 shows, though, that a sufficient condition for (E.30) to reflect only the fact that the uninformed may bid r:

$$(\text{E.31}) \qquad p f_h(s) q_0(h - r) \leq (1 - p) f_l(s)(r - l).$$

is $q_0 q_2 \geq (1 - q_0)^2$. Section E.10 also demonstrates the existence of, and characterizes equilibria with state-contingent reservation prices when (E.30) is violated.

Equilibria with degenerate bids

If $\beta \leq 1 - q_0$, then the second-best in which $\sigma = +\infty$ and the good asset is sold at price h with probability $\beta/(1 - q_0)$ is clearly an equilibrium. This second-best is not an equilibrium if $\beta > 1 - q_0$ because an infinite σ cannot be incentive-compatible, and an equilibrium with a finite σ and $r = h$ would not survive our refinement.

E.3. Proof of Proposition 4

It is easy to see that an optimal σ'-contract consists in rewarding the agent based on a signal larger than $\sigma^R \geq \sigma'$ and allowing him to sell the asset above some reserve price otherwise, where σ^R may be infinite. That the incentive-compatibility constraint must be binding, and that the equilibrium probability of resale is q_0 whenever resale is allowed, imply that for every $\sigma' \in (\underline{\sigma}, \overline{\sigma}]$, the only equilibrium with σ'-contracts is the one with the contract $(\overline{\sigma}, \overline{r})$. If $\sigma' > \overline{\sigma}$, there is no equilibrium with incentive-compatible σ'-contracts.

E.4. Proof of Proposition 5

In equilibrium, each firm $i \in [0, 1]$ takes as given the profit k that buyers expect from matching with other firms, and offers a contract (σ', r') rationally anticipating that this will attract a Poisson intensity λ' such that

$$(\text{E.32}) \qquad pF_h(\sigma')\frac{q_1(\lambda')(h - r')}{1 - q_0(\lambda')} = k,$$

where $q_k(\lambda)$ is defined in (10). Firm i also expects all bids to be above the reserve price, so that the equilibrium probability of a failure to resale is $q_0(\lambda')$. It proves convenient to write firm i's program with the control variables (σ, λ) rather than (σ, r):

$$(\text{E.33}) \qquad \min_{\{\sigma', \lambda'\}} \left\{1 - F_l(\sigma') + \lambda'k\right\}$$

s.t.

$$(\text{E.34}) \qquad F_l(\sigma') - q_0(\lambda')F_h(\sigma') = \beta.$$

An equilibrium is then characterized by (σ, r) such that the solution to this program is attained at (σ, λ) when $k = pF_h(\sigma)q_1(\lambda)(h - r)/(1 - q_0(\lambda))$.

The first-order condition to the firm's program reads:[2]

$$(\text{E.35}) \qquad \frac{f_h(\sigma')q_0(\lambda')}{f_l(\sigma')} = 1 - \frac{q_0'(\lambda')F_h(\sigma')}{k} > 1.$$

Suppose first that the incentive-compatibility constraint (17) admits two solutions, and recall that we denote the largest by $\overline{\sigma}$. Then the solution (σ', λ') to the firm's contracting problem, given

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[2]We also checked that the second-order condition for a minimum is globally satisfied.

by $\{(E.34); (E.35)\}$, must be equal to $(\overline{\sigma}, \lambda)$ in equilibrium. Injecting the equilibrium value of k in (E.35), this implies that firms post an equilibrium reserve price r_P implicitly defined as

$$(E.36) \qquad \frac{f_h(\overline{\sigma})q_0(\lambda)}{f_l(\overline{\sigma})} = 1 + \frac{1}{\frac{pq_1(\lambda)(h - r^P)}{q_0(\lambda)(1 - q_0(\lambda))}}$$

Comparing (E.36) and (18) at the same value of σ shows that $r^P < \overline{r}$ for a given $\overline{\sigma}$.

Finally, suppose that the incentive-compatibility constraint (17) admits only one solution. Then the first-order condition is slack and the unique equilibrium yields the second-best outcome.

E.5. Proof of Proposition 6

The proof that the optimal contract contract must be of the form $(r(\lambda), \sigma(\lambda))$ when λ is revealed only at date 1 is very similar to that of Step 1 in the proof of Proposition 1 and we skip it. The optimal contract $(r(\lambda), \sigma(\lambda))$ solves

$$(E.37) \qquad \max_{(r(.), \sigma(.))} \left\{ \int \left[1 - F_l(\sigma(\lambda)) + p F_h(\sigma(\lambda)) \int_{r(\lambda)}^h (h - t) \frac{\partial H(t, \lambda)}{\partial t} dt \right] d\Lambda(\lambda) \right\}$$

$$s.t.$$

$$(E.38) \qquad \int \left[F_l(\sigma(\lambda)) - H(r(\lambda), \lambda) F_h(\sigma(\lambda)) \right] d\Lambda(\lambda) = \beta$$

The first-order condition yields

$$(E.39) \qquad r(\lambda) = r,$$

$$(E.40) \qquad \frac{f_h(\sigma(\lambda))}{f_l(\sigma(\lambda))} = \frac{p(h - r) + 1}{p \int_r^h H(t, \lambda) dt}.$$

It only remains to show that $H(t, \lambda)$ increases strictly in λ in the sense of first-order stochastic dominance $(\partial H / \partial \lambda < 0)$. Note first that the distribution of bids $S(., \lambda)$ satisfies

$$(E.41) \qquad \sum_{k \geq 1} \frac{q_k(\lambda)}{q_1(\lambda)} (h - t) S^{k-1}(t, \lambda) = h - r.$$

That q_k / q_1 strictly increases in λ implies that $S(t, \lambda)$ must strictly decrease w.r.t. λ for all t. Second,

$$(E.42) \qquad H = \sum_{k \geq 0} q_k S^k$$

implies that so does H.

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E.6. Proof of Proposition 7

Uninformed bidders have a perfectly elastic demand at any price below l. For ϵ sufficiently small, for each λ such that $(1 - q_0(\lambda)) \in [\beta - \epsilon, \beta)$, there exists a unique σ such that (24) holds and $f_h q_0 / f_l > 1$ for (λ, σ). Continuity implies that the set of σ defined this way is an interval $[\sigma_1, +\infty)$. Further, differentiating the incentive-compatibility (24) constraint yields:

$$(E.43) \qquad \frac{\partial \sigma}{\partial \lambda} = \frac{\frac{\partial q_0}{\partial \lambda} F_h + q_0 \frac{\partial F_h}{\partial \lambda} - \frac{\partial F_l}{\partial \lambda}}{f_l - q_0 f_h} > 0$$

because

$$(E.44) \qquad q_0 \frac{\partial F_h}{\partial \lambda} - \frac{\partial F_l}{\partial \lambda} \leq \frac{\partial F_h}{\partial \lambda} \left(q_0 - \frac{f_l}{f_h} \right) \leq 0.$$

from conditions (21) and (22).

For any $\sigma \geq \sigma_1$, one can then uniquely define $\lambda(\sigma)$ and $r(\sigma)$ such that (24) and (25) are satisfied for $(\sigma, r(\sigma), \lambda(\sigma))$. Define then $\Sigma(\sigma)$ as the solution to (23) for such $(r(\sigma), \lambda(\sigma))$. For κ sufficiently small, $r(\sigma)$ takes values in a compact set that is sufficiently close to h and thus $\Sigma(\sigma) \geq \sigma_1$ for all $\sigma \geq \sigma_1$. Also, Σ is bounded from above because $\lambda(\sigma)$ and $r(\sigma)$ take values in compact sets. Denoting σ_2 this upper bound, Σ is an (obviously continuous) mapping over $[\sigma_1, \sigma_2]$ and thus admits a fixed point. This fixed point σ and the associated value of λ given by (24) and r from (25) form a stable equilibrium by construction.

E.7. Proof of Proposition 8

We first prove that the public interventions mentioned in the proposition have the claimed impact, and then we show that this impact reduces firms' agency costs.

Liquidity support measures. If information acquisition is subsidized by an amount x, then the equilibrium is characterized by the conditions $\{(23); (24); (25)\}$ up to replacing κ with $\kappa - x$.

Consider an unregulated ($x = 0$) high-winner-curse stable equilibrium (σ, r, λ). For x sufficiently small, one can apply the same reasoning as that in the proof of Proposition 8 and arrive at a mapping Σ_x that has a fixed point σ' arbitrarily close to σ. This fixed point σ' and the associated value of λ' given by (24) and r' from (25) form a (stable) regulated equilibrium. Let us show that $(\sigma', r', \lambda') > (\sigma, r, \lambda)$. We first show that $\Sigma_x > \Sigma$. This stems from the fact that for given σ, λ, (25) generates a value of r that increases in x, and then the right-hand side of (23) is increasing in r. That $\Sigma_x > \Sigma$ implies $\sigma' > \sigma$, and from (E.43) this implies in turn that $\lambda' > \lambda$. To see that

$r' > r$, one can rewrite (25) as:

$$\text{(E.45)} \qquad p[F_l(\sigma, \lambda) - \beta]\frac{q_1(\lambda)(h - r)}{q_0(\lambda)(1 - q_0(\lambda))} = \kappa - x.$$

This shows that an increase in σ and λ while satisfying (24) also yields an increase in r in (25) since $q_1/q_0(1 - q_0)$ increases in λ, and (22) imply:

$$\text{(E.46)} \qquad f_l + \frac{\partial F_l}{\partial \lambda}\frac{\partial \lambda}{\partial \sigma} \geq \frac{\partial \lambda}{\partial \sigma}\frac{\frac{\partial q_0}{\partial \lambda}F_h}{\left(1 - \frac{q_0 f_h}{f_l}\right)} > 0.$$

In addition, r clearly increases w.r.t. x in (25) holding σ and λ constant.

Regulating accounting conservatism. To a high-winner-curse stable equilibrium corresponds the degree of accounting conservatism defined in (9):

$$\text{(E.47)} \qquad \alpha = \frac{pf_h(\sigma)[1 - q_0(\lambda)]}{pf_h(\sigma) + (1 - p)f_l(\sigma)}.$$

Since (24) implicitly defines σ as increasing in λ from (E.43) whereas (E.47) defines σ as decreasing in λ, imposing a degree of conservatism α' strictly larger than but sufficiently close to α leads to a regulated equilibrium in which firms are forced to use a signal cut-off $\sigma' > \sigma$ and this must lead to a liquidity level $\lambda' > \lambda$. The proof that this in turn yield $r' > r$ from (25) is identical to the one above using expression (E.45).

Stable equilibria are inefficient. Consider a high-winner-curse stable equilibrium (σ, r, λ). Given λ, the equilibrium contract (σ, r) must coincide with the solution in (σ', r') of firms' contracting problem:

$$\text{(E.48)} \qquad \min_{(\sigma', r')} V(\sigma', r'; \lambda, r) = 1 - F_l(\sigma', \lambda) + pF_h(\sigma', \lambda)\int_{r'}^{h}(h - t)\frac{\partial H(t, \lambda, r)}{\partial t}dt$$

s.t.

$$\text{(E.49)} \qquad F_l(\sigma', \lambda) - F_h(\sigma', \lambda)H(r', \lambda, r) = \beta.$$

The Lagrangian of this program is

$$\text{(E.50)} \qquad \mathfrak{L} = -V(\sigma', r'; \lambda, r) + \mu[F_l(\sigma', \lambda) - F_h(\sigma', \lambda)H(r', \lambda, r) - \beta].$$

The first-order condition w.r.t. r' yields $\mu = p(h - r')$. The envelope theorem then yields that at the equilibrium values $(\sigma', r') = (\sigma, r)$

$$\frac{\partial \mathfrak{L}}{\partial \lambda} = -\left[[1 + p(h - r)]\frac{\partial F_l(\sigma, \lambda)}{\partial \lambda} - \frac{\partial F_h(\sigma, \lambda)}{\partial \lambda}\left[p\int_{r}^{h}H(t, \lambda, r)dt\right]\right]$$
$$+ pF_h(\sigma, \lambda)\int_{r}^{h}\frac{\partial H(t, \lambda, r)}{\partial \lambda}dt,$$

$$\frac{\partial \mathfrak{L}}{\partial r} = pF_h(\sigma, \lambda)\int_{r}^{h}\frac{\partial H(t, \lambda, r)}{\partial r}dt.$$

The first-order condition (6) and condition (22) imply that

$$[1 + p(h-r)]\frac{\partial F_l(\sigma,\lambda)}{\partial\lambda} - \frac{\partial F_h(\sigma,\lambda)}{\partial\lambda}\left[p\int_r^h H(t,\lambda,r)dt\right]$$

is positive in equilibrium.

Also, the equilibrium distribution of an informed bid, $S(.,\lambda,r)$, is given by:

(E.51)
$$\sum_{k\geq 1}\frac{q_k(\lambda)}{q_1(\lambda)}(h-t)S^{k-1}(t,\lambda,r) = h-r,$$

That q_k/q_1 increases w.r.t. λ for all $k\geq 1$ implies that $S(t,\lambda,r)$ must decrease w.r.t. λ for all r, t. It is also transparent from (E.51) that $S(t,\lambda,r)$ decreases in r for all λ, t. Also,

(E.52)
$$H = \sum_{k\geq 0}q_k S^k$$

implies that H increases in the sense of first-order stochastic dominance when S and $\{q_k\}$ do. Overall this implies $\partial V/\partial\lambda < 0$, $\partial V/\partial r < 0$.

As a result, any regulation leading to small increases in σ, r, and λ reduces firms' agency costs because an increase in σ has no first-order impact on firms' agency costs from the envelope theorem whereas increases in λ and r reduce them from the above.

E.8. Proof of Proposition 9

An equilibrium with posted contract and free entry is a triplet (σ,r,λ) such that (σ,λ) is the solution to

(E.53)
$$\min_{\{\sigma',\lambda'\}}\{1 - F_l(\sigma',\lambda) + \lambda'\kappa\}$$

s.t.

(E.54)
$$F_l(\sigma',\lambda) - q_0(\lambda')F_h(\sigma',\lambda) \geq \beta,$$

and the triplet satisfies

(E.55)
$$pF_h(\sigma,\lambda)\frac{q_1(\lambda)(h-r)}{1-q_0(\lambda)} = \kappa.$$

The equilibrium is thus characterized by:

(E.56)
$$\frac{f_h(\sigma,\lambda)q_0(\lambda)}{f_l(\sigma,\lambda)} = 1 + \frac{1}{\frac{pq_1(\lambda)(h-r)}{q_0(\lambda)(1-q_0(\lambda))}},$$

(E.57)
$$F_l(\sigma,\lambda) - q_0(\lambda)F_h(\sigma,\lambda) = \beta,$$

(E.58)
$$pF_h(\sigma,\lambda)\frac{q_1(\lambda)(h-r)}{1-q_0(\lambda)} = \kappa,$$

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where (E.56) is the first-order condition from firms' program. A solution to this system can be constructed as a fixed point the very same way as in the proof of Proposition 7.

The Lagrangian of $\{(E.53); (E.54)\}$ is

(E.59) $$\mathfrak{L} = F_l(\sigma', \lambda) - \lambda'\kappa + \mu(F_l(\sigma', \lambda) - q_0(\lambda')F_h(\sigma', \lambda) - \beta).$$

The envelope theorem then yields that at the equilibrium values $(\sigma', \lambda') = (\sigma, \lambda)$

$$\frac{\partial\mathfrak{L}}{\partial\lambda} = (1 + \mu)\left(\frac{\partial F_l(\sigma, \lambda)}{\partial\lambda} - \frac{\partial F_h(\sigma, \lambda)}{\partial\lambda}\frac{f_l(\sigma, \lambda)}{f_h(\sigma, \lambda)}\right),$$

which yields the result.

The rest of the appendix states and proves various results that extend the results discussed throughout the paper.

E.9. Second-order condition for program $\{(3); (4)\}$

The Lagrangian of the program can be written

(E.60) $$L(\sigma, r, \nu) = -f(\sigma, r) + \nu g(\sigma, r)$$

where

(E.61) $$f(\sigma, r) = p\beta + 1 - F_l(\sigma) + pF_h(\sigma)\int_r^h (h - t)dH(t),$$

(E.62) $$g(\sigma, r) = F_l(\sigma) - H(r)F_h(\sigma).$$

We need to compute the determinant of the bordered Hessian matrix

(E.63) $$\begin{bmatrix} 0 & -g_\sigma & -g_r \\ -g_\sigma & L_{\sigma\sigma} & L_{\sigma r} \\ -g_r & L_{r\sigma} & L_{rr} \end{bmatrix}.$$

We have

(E.64)
$$L_\sigma = f_l - pf_h \int_r^h (h-t)dH(t) + \nu(f_l - H(r)f_h) = 0,$$

(E.65)
$$L_r = p(h-r)F_h \frac{dH(r)}{dr} - \nu F_h \frac{dH(r)}{dr} = 0 \rightarrow \nu = p(h-r),$$

(E.66)
$$L_{\sigma r} = p(h-r)f_h \frac{dH(r)}{dr} - \nu f_h \frac{dH(r)}{dr} = 0,$$

(E.67)
$$L_{r\sigma} = p(h-r)f_h \frac{dH(r)}{dr} - \nu f_h \frac{dH(r)}{dr} = 0,$$

(E.68)
$$L_{rr} = -pF_h \frac{dH(r)}{dr},$$

(E.69)
$$L_{\sigma\sigma} = f_l' - pf_h' \int_r^h (h-t)dH(t) + \nu(f_l' - H(r)f_h') = f_l(1 + p(h-r))(f_l'/f_l - f_h'/f_h),$$

(E.70)
$$g_\sigma = f_l - H(r)f_h,$$

(E.71)
$$g_r = -\frac{dH(r)}{dr}F_h.$$

This yields a determinant

(E.72) $\quad (H(r)f_h - f_l)^2 pF_h \dfrac{dH(r)}{dr} + F_h^2 f_l[1 + p(h-r)] \left(\dfrac{dH(r)}{dr}\right)^2 (f_h'/f_h - f_l'/f_l) > 0.$

E.10. Low-winner-curse equilibria in Section II

Suppose a solution (σ, r) to $\{(17); (18)\}$ is such that uninformed bidders would find bidding above r profitable. We construct a low-winner-curse equilibrium such that firms' contracts consist in the signal cut-off σ and reserve prices $r(s)$ that weakly increase with respect to the public signal s.

Step 1. We define for every realization of the public signal s the minimum reserve price for

which uninformed bidders find bidding l optimal. For every $\rho \in (l, h]$, we first define $S(., \rho)$ as

(E.73)
$$\begin{cases} S(t, \rho) = 0 & \text{if } t \leq \rho, \\ \sum_{k \geq 1} q_k(h - t)S^{k-1}(t, \rho) = q_1(h - \rho) & \text{for all } t \in \left[\rho, h - \frac{q_1(h-\rho)}{1-q_0}\right], \\ S(t, \rho) = 1 & \text{if } t \geq h - \frac{q_1(h-\rho)}{1-q_0}. \end{cases}$$

$S(., \rho)$, a decreasing function of ρ, is the c.d.f. of an informed bid if the bidder believes the reserve price is ρ and uninformed bidders bid below l. An uninformed bidder observing a public signal $s \leq \sigma$ does not find it profitable to bid $t \geq \rho$ if and only if

(E.74)
$$p f_h(s) H(t, \rho)(h - t) \leq (1 - p) f_l(s)(t - l).$$

This inequality holds for all $s \leq \sigma$, $t \geq \rho$ if and only if it holds at $s = \sigma$ for all t from the monotonicity of f_h/f_l. We have

(E.75)
$$H(t, \rho) = \sum_{k \geq 0} q_k S(t, \rho)^k = q_0 + (1 - q_0)S(t, \rho) \sum_{k \geq 1} \frac{q_k}{1 - q_0} S^{k-1}(t, \rho)$$

(E.76)
$$= q_0 + q_1 S(t, \rho)\frac{h - \rho}{h - t}.$$

from (E.73). This implies that bidding t is not profitable if and only if:

(E.77)
$$p f_h(\sigma)[q_0(h - t) + q_1 S(t, \rho)(h - \rho)] \leq (1 - p) f_l(\sigma)(t - l).$$

Define then for all public signal s

(E.78)
$$\rho(s) = \inf\{\rho \mid \textit{For all } t \geq \rho,\ p f_h(s)[q_0(h - t) + q_1 S(t, \rho)(h - \rho)] \leq (1 - p) f_l(s)(t - l)\}.$$

This set is not empty because (E.77) is satisfied for ρ sufficiently close to h, and is bounded below because it is not satisfied for ρ sufficiently close to l. Thus $\rho(s)$, the smallest reserve price at which uninformed bidders find bidding l optimal, is well defined for all s. The monotonicity of f_h/f_l implies that $\rho(s)$ is increasing as the infimum of a set that decreases with s.

Step 2. Suppose that a firm expects a distribution of informed bids $H(., s)$ that is contingent on the public signal s, and such that $H(\rho(s), s) = q_0$ for all s. With such contingent informed bids, the firm's program reads:

(E.79)
$$\min_{\{\sigma, r(s)\}} \left\{ p\beta + 1 - F_l(\sigma) + \int_{-\infty}^{\sigma} p f_h(s) \int_{r(s)}^{h} (h - t) dH(t, s) ds \right\}$$

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s.t.

(E.80) $$F_l(\sigma) - \int_{-\infty}^{\sigma} H(r(s), s) f_h(s) ds = \beta,$$

(E.81) $$\rho(s) \le r(s).$$

Ignoring the feasibility constraint, the first-order condition with respect to $r(s)$ yields that $h - r(s)$ must be constant. At the optimal contract, the reserve price is therefore either a constant r', or such that the firm accepts all bids strictly above $\rho(s)$ when $\rho(s) > r'$.

Step 3. Expecting such contracts, uninformed bidders find bidding below l optimal by construction of $\rho(s)$ and informed bidders mix their bids above $\inf\{r'; \rho(s)\}$. The monotonicity of $\rho(s)$ therefore implies that their minimum bid is r' below a signal cut-off $\sigma' < \sigma$ and $\rho(s)$ for $s \in [\sigma', \sigma]$. Thus, the value of r' is pinned down by the first-order conditions w.r.t. r' ($\mu = p(h - r')$) and σ:

(E.82) $$\frac{f_h(\sigma)}{f_l(\sigma)} = \frac{p(h - r') + 1}{p(h - r')q_0 + p \int_{\rho(\sigma)}^{h} (h - t) dH(t, \sigma)},$$

(E.83) $$= \frac{p(h - r') + 1}{p(h - r')q_0 + \frac{\lambda p q_1 (h - \rho(\sigma))}{(1 - q_0)}}.$$

If (E.83) is not satisfied for any $r' > l$, then if σ is such that $f_h(\sigma)q_0/f_l(\sigma) \ge 1$, the equilibrium is such that the reserve price is $\rho(s)$ for all value of s (that is, $\sigma' = -\infty$), and

(E.84) $$\frac{f_h(\sigma)}{f_l(\sigma)} \le \frac{p(h - l) + 1}{p(h - l)q_0 + \frac{\lambda p q_1 (h - \rho(\sigma))}{(1 - q_0)}}.$$

If $f_h(\sigma)q_0/f_l(\sigma) < 1$, then there exists no low-winner-curse equilibrium associated with σ.

This low-winner-curse-equilibrium survives the Intuitive Criterion in the case in which uninformed bidders are indifferent between bidding l and $\rho(s)$ for all $s \in [\sigma', \sigma]$. In this case, uninformed bidders would benefit from bidding $\rho(s) - \epsilon$ for ϵ sufficiently small if this offer is accepted with strictly positive probability, and so the Intuitive Criterion does not rule out firms' beliefs that such a bid may stem from an uninformed bidder.

A sufficient condition ensuring that the no-mimicking constraint binds at $\rho(s)$ when it binds is $q_0 q_2 \ge (1 - q_0)^2$. This holds indeed if for all $t \in (\rho(s), r_+(s))$,

(E.85) $$\frac{\partial}{\partial t} [p f_h(s) [q_0(h - t) + q_1 S(t, \rho)(h - \rho(s))] - (1 - p) f_l(s)(t - l)] \le 0,$$

which in turn holds if

(E.86) $$\frac{\partial S}{\partial t} \le \frac{q_0}{q_1(h - \rho(s))}.$$

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From (E.73),

(E.87)
$$q_2 S \le q_1 \frac{t - \rho(s)}{h - t},$$

and

(E.88)
$$(t - \rho(s))\frac{\partial S}{\partial t} \le \frac{h - \rho(s)}{h - t} S.$$

Combining both expressions and noticing that $t \le r_+(s) = h - q_1(h - \rho(s))/(1 - q_0)$ yields the result.

E.11. The agent values date-2 consumption

Throughout the paper, we take the agent's reward date as fixed ($t = 1$) for expositional simplicity. In this section, we more generally follow the literature on incentives provision for agents with liquidity needs[3] and assume that delaying compensation to date 2 involves a social cost. Suppose that the agent derives utility at both dates 1 and 2 and has preferences

(E.89)
$$u_0 + u_1 + \delta u_2,$$

where $\delta, u_1, u_2 \in [0, 1]$. The principal can still provide the agent with utility u_t at cost u_t for $t \in \{1; 2\}$. The baseline model corresponds to the case in which $\delta = 0$. The second-best case without measurement frictions corresponds to $\delta = 1$. It is straightforward to extend the analysis of the optimal contract to the case in which $\delta \in (0, 1)$.

In case of a date-1 resale, the price reveals the project's payoff, and thus the principal knows it at date 2. If the firm holds on to the asset until date 2, the payoff is revealed at this date. This implies that either way, the principal knows the payoff at date 2. The information conveyed by the date-1 signal and resale (if any) is thus immaterial at this date. Also, the cost of providing utility at date 2 is independent of any utility already provided at date 1. Thus, one can without loss of generality consider only contracts whereby the date-2 compensation does not depend on the contracting history, only on the realized payoff. Denote (σ, r, τ) such a contract. It is such that the agent is rewarded at date 1 if the signal is above σ, or if it is below σ and he manages to sell the project at a price above r. He may also be rewarded at date 2 with probability τ if the project pays off h. Ignoring feasibility constraints, an optimal contract solves the counterpart of $\{(3),(4)\}$

(E.90)
$$\min_{\{\sigma, r, \tau\}} \left\{ 1 - F_l(\sigma) + p F_h(\sigma) \int_r^h (h - t)dH(t) + p(1 - \delta)\tau \right\}$$

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[3] See e.g. Aghion et al (2004), or Faure-Grimaud-Gromb (2004).

s.t.

(E.91) $$F_l(\sigma) - H(r)F_h(\sigma) + \delta\tau = \beta.$$

For brevity, we discuss only the case in which the optimal contract corresponds to an interior solution of this program. The interior solution is then characterized by (E.91) and two first-order conditions:

(E.92) $$\frac{f_h(\sigma)}{f_l(\sigma)} = \frac{p(h-r)+1}{p\int_r^h H(t)dt},$$

(E.93) $$h - r = \frac{1-\delta}{\delta}.$$

It is then obvious that an increase in δ, starting from a contract such that $f_h H/f_l > 1$, yields an increase in σ, r, and τ.

This extension of the model bears interesting relationship to the accounting standard IFRS 9 issued in 2014. In this standard, the business model used by an entity for managing an asset affects the measurement of this asset. The "hold and collect" business model, whereby firms acquire assets to collect their cash flows until maturity, is the one that corresponds to a lower use of marking to market. In line with this, this simple extension predicts that firms with more patient agents (a higher δ other things being equal) rely more on the "hold to collect" model and, at the same time, rely less on market data because (σ, r, y) increases in δ. Thus we rationalize this connection between "business model" and measurement regime.

E.12. Alternative welfare criteria

Here we suppose that the agent has preferences over dates 0 and 1 consumptions, c_0 and c_1,

(E.94) $$c_0 + \delta \min\{c_1, 1\},$$

where $\delta \in (0, 1]$. The private benefit from shirking, B, is enjoyed at date 0, and so becomes B/δ from the point of view of the principal. Transferring expected date-1 utility u to the agent costs u/δ to the principal. We consider both the situation in which the principal owns the project and that in which the agent owns it and faces a competitive financial market. We normalize the reservation utility of the principal to 0 and suppose that that of the agent is always below the rewards that the optimal contract grants him (for example, because it is also equal to 0). The agent is cashless at date 0.

A. The principal is the owner of the project

Let c denote the expected resale costs, w the expected date-1 reward of the agent, and i the investment cost. Let $T \geq 0$ denote a date-0 lump-sum transfer from the principal to the agent.[4] The owner receives $ph + (1-p)l - i - w - c - T$ and the agent $\delta w + T$. And so, for social welfare weights α_P and α_A, the social welfare function is (up to a constant)

(E.95) $\qquad W = \alpha_P(-w - c - T) + \alpha_A(\delta w + T) = -\alpha_P c - (\alpha_P - \delta\alpha_A)w + (\alpha_A - \alpha_P)T,$

and the principal's individual rationality constaint is

(E.96) $\qquad ph + (1-p)l - i - c - w \geq T.$

If (E.96) is slack at the optimum then it must be that $\alpha_P \geq \alpha_A$, otherwise welfare could be increased by increasing T. If $\alpha_P < \alpha_A$ then (E.96) is binding and the objective is (up to a multiplicative constant α_A)

(E.97) $$-[(1-\delta)w + c].$$

In the case in which the social planner cannot implement lump-sum transfers, the social welfare function is simply

(E.98) $$-\alpha_P c - (\alpha_P - \delta\alpha_A)w.$$

This implies overall:

Proposition E.1. *(Alternative welfare criteria) If the planner can implement lump-sum transfers, or if he cannot but uses weights such that $\alpha_P \geq \delta\alpha_A$, then the optimal contract minimizes a convex combination of expected resale costs and expected rewards to the agent that puts weakly more weight on expected resale costs. The case of equal weights is the one studied in the paper.*

B. The agent is the owner of the project

Under a competitive capital market, the agent solves $\max\{\delta w + T\}$ subject to the investors' break-even constraint: $ph + (1-p)l - i - w - c - T \geq 0$. This constraint clearly optimally binds and so the agent seeks to minimize again $(1-\delta)w + c$.

In sum, under all these alternative social objectives, the objective in the optimal contracting problem can always be expressed with weights $\omega \in [0, 1/2]$ on pure date-1 utility transfers and

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[4]Ignoring date-1 lump-sum transfers is without loss of generality since date-0 transfers weakly dominate them from $\delta \leq 1$.

$1 - \omega$ on resale costs:

(E.99)
$$\min_{\{\sigma,r\}} \left\{ \omega(p\beta' + 1 - F_l(\sigma)) + (1-\omega)pF_h(\sigma)\int_r^h (h-t)dH(t) \right\}$$

s.t.

(E.100) $F_l(\sigma) - H(r)F_h(\sigma) = \beta',$

(E.101) $r_- \leq r \leq r_+,$

where $\beta' = \beta/\delta$.

This program is identical to $\{(3),(4)\}$, up to putting different weights on rewards for luck and on resale costs in the objective and to scaling β. The first-order condition (which may be slack if the feasibility contraint (E.101) binds) is simply:

(E.102)
$$\frac{f_h(\sigma)}{f_l(\sigma)} = \frac{p(h-r) + 1 - \frac{1-2\omega}{1-\omega}}{p\int_r^h H(t)dt}.$$

It is easy to see from (E.100) and (E.102) that the analysis conducted in the paper when $\omega = 1/2$ carries over:

Optimal contracting problem in Section I. One can see from Figure 3 how this different objective affects the analysis. Replacing condition (6) with (E.102) amounts to shifting downwards the curve associated with (6) in the plane (r,σ), all the more so because ω is small. This downwards shift implies a higher reliance on market data (a lower σ). As in the case studied in the paper, the optimal contract may either be below or above the frontier $[f_h(\sigma)/f_l(\sigma)]H(r) = 1$ depending on the parameters.

Equilibria with exogenous λ in Section II. The equilibra cut-off σ do not depend on ω as they correspond to solutions to (E.100). Furthermore, (E.102) implies that the reserve price r associated with a given σ decreases with respect to ω. In other words, as the objective puts a smaller penalty on rewards for luck, bidders understand that firms are more reluctant to rely on resales, thereby setting more aggressive reserve prices. Accordingly, they bid more aggressively.

Endogenous λ in Section III. Here again, it is easy to see that the analysis of stable high-winner-curse equilibria carries over. Firms fail to internalize the same liquidity externalities as in the case $\omega = 1/2$.

E.13. Informed buyers make false negative/positive errors

Suppose that in Section II, all informed buyers matched to a firm with an h-payoff project receive the same incorrect signal that the payoff is l with probability ϵ_h (false negatives). Similarly, all

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buyers matched to a firm with an l-payoff project receive the same incorrect signal that the payoff is h with probability ϵ_l (false positives). We suppose $\epsilon_h + \epsilon_l < 1$. For brevity, we suppose that parameters are such that uninformed bidders optimally bid l in the relevant range.

Whereas this is immaterial when buyers are perfectly informed, we explicitly assume here that informed buyers observe the public signal received by the firm. As a result, an informed buyer who receives a low-payoff private signal and observes a public signal s expects a project's payoff

$$(E.103) \qquad l(s) = \frac{p\epsilon_h f_h(s)h + (1-p)(1-\epsilon_l)f_l(s)l}{p\epsilon_h f_h(s) + (1-p)(1-\epsilon_l)f_l(s)},$$

whereas a buyer who receives a high-payoff private signal and observes a public signal s updates to:

$$(E.104) \qquad h(s) = \frac{p(1-\epsilon_h)f_h(s)h + (1-p)\epsilon_l f_l(s)l}{p(1-\epsilon_h)f_h(s) + (1-p)\epsilon_l f_l(s)} > l(s).$$

Bidding game. All informed bidders for a given project share the same information about the payoff so that as in the perfect-information case, there is no uncertainty about other bidders' valuations.

Optimal contracts. We suppose that for each signal realization s, bids have the same properties as in the baseline model replacing h and l by $h(s)$ and $l(s)$, and we denote by $H(.,s)$ the distribution of the highest bid for a high-payoff project. (As in the baseline model, these properties are satisfied in equilibrium.) The same reasoning as in Step 1 in the proof of Proposition 1 shows that firms' optimal contract is of the form $(\sigma, r(s))$. The agent is rewarded for a signal above σ, or for a resale above $r(s)$ when $s \leq \sigma$. One can show that the optimal contract solves (ignoring feasibility constraints)

$$(E.105) \qquad \min_{\{\sigma, r(s)\}} \left\{ \begin{array}{l} p\beta + 1 - F_l(\sigma) + \epsilon_l \int_{-\infty}^{\sigma} f_l(s)(1 - H(r(s), s))ds \\ + \int_{-\infty}^{\sigma} \left[[p(1-\epsilon_h)f_h(s) + (1-p)\epsilon_l f_l(s)] \int_{r(s)}^{h(s)} (h(s) - t)dH(t,s) \right] ds \end{array} \right\}$$

s.t.

$$(E.106) \qquad (1-\epsilon_l)F_l(\sigma) - \epsilon_h F_h(\sigma) - \int_{-\infty}^{\sigma} H(r(s), s)[(1-\epsilon_h)f_h(s) - \epsilon_l f_l(s)]ds = \beta,$$

Comparing with the baseline model with perfect information ($\epsilon_h = \epsilon_l = 0$) the additional terms admit straightforward interpretations. Regarding the objective (E.105), false positives introduce additional rewards for luck due to the fact that an l-asset may be successfully resold (term $\epsilon_l \int f_l(1 - H)$). Unlike rewards for luck induced by high signals on l-projects, these also come at resale costs. On the other hand, expected resale costs of h-projects are reduced by false negatives for a given signal and reserve price.

The incentive-compatibility constraint is best interpreted in equilibrium when all bids are above the reserve price $r(s)$ for all signal s. In this case, (E.106) reads:

$$(E.107) \qquad F_l(\sigma) - q_0 F_h(\sigma) - (1 - q_0)(\epsilon_l F_l(\sigma) + \epsilon_h F_h(\sigma)) = \beta$$

Imperfect buyers' information reduces the incentives by $(1 - q_0)(\epsilon_l F_l + \epsilon_h F_h)$, representing that a signal below σ may lead to reselling an l-asset because of a false positive and to missing the resale of an h-asset because of a false negative. Note that this latter reduction in incentives is identical to an increase in the probability of not receiving bids for h-projects to $q_0 + (1 - q_0)\epsilon_h$ from q_0.

Finally, the first-order condition with respect to $r(s)$ is instructive. Denoting by μ the multiplier of the incentive-compatibility constraint and ignoring the feasibility constraint, one obtains:

$$(\text{E.108}) \qquad h(s) - r(s) = \frac{\mu}{p} - \frac{\mu + p}{p(1 - p)}\phi(s),$$

where

$$(\text{E.109}) \qquad \phi(s) = \frac{(1 - p)\epsilon_l f_l(s)}{p(1 - \epsilon_h)f_h(s) + (1 - p)\epsilon_l f_l(s)}$$

is the probability of a low payoff given a public signal s and a positive buyers' signal. It is decreasing in s, and so the left-hand side is increasing in s.

Note first that if there are only false negatives ($\epsilon_l = 0$) then the reserve price is constant as in the baseline model because $h(s) = h$ and $\phi(s) = 0$. Otherwise, expression (E.109) shows that there are two forces leading to opposite variations of $r(s)$ with respect to s. The right-hand side is increasing in s, reflecting that the rewards for luck from selling an l-project decrease with respect to s. This entails that the marginal discount on a resale $h(s) - r(s)$ should increase in s: Resales are higher-powered incentives for higher signals for which the risk of rewards for luck is lower. On the other hand, $h(s)$ increases in s, so that an increasing marginal discount $h(s) - r(s)$ may still result in an increasing $r(s)$. Noting that

$$(\text{E.110}) \qquad h(s) = (1 - \phi(s))h + \phi(s)l,$$

one can see that $r(s)$ is monotonic. It is increasing if and only if

$$(\text{E.111}) \qquad p(1 - p)(h - l) \geq \mu + p.$$

The following Proposition summarizes this discussion:

Proposition E.2. *(Imperfectly informed buyers)*

- *If buyers make only false-negative mistakes ($\epsilon_l = 0$), then the model is identical to that with perfectly informed buyers up to an increase in the probability of not receiving bids for h-projects to $q_0 + (1 - q_0)\epsilon_h$ from q_0.*

- *Otherwise, the optimal contract consists in a cut-off σ and a reserve price that is monotonic in s. The distribution of bids is therefore also (stochastically) monotonic in s.*

Proof. Discussion above. ∎

E.14. Microfoundations for condition $\{(21),(22)\}$

We interpret the signal s received by a firm as the price fetched by comparable assets sold by other firms. If, as implied by the optimal contract, only h-payoff assets are sold and the principal in a firm is able to relate the project selected by the agent to those sold on the market (there is no misclassification error), actual transactions are perfectly informative and reveal that the agent has selected a high-payoff project. So two routes to noisy market measurement can be taken. The first involves misclassification. The second posits that assets may trade for other reasons than the provision of incentives. We formalize each microfoundation in turn.

We suppose that date-0 private signals are conditionally independent across firms, and so a deterministic fraction p of firms select an h-payoff project in equilibrium. We endogenize firms' date-1 signal using rational expectations equilibrium as our equilibrium concept (see, e.g., Grossman 1981 or Grossman-Stiglitz 1980). Namely, we suppose that all asset resales take place at date 1, but that each firm can condition its own resale decision on the observation of transactions by other firms.[5]

Misclassification

A firm perfectly observes the transaction prices of resold assets (which are from the equilibrium contract only h-payoff assets). It however cannot ascertain perfectly how similar the resold assets are to its own asset. The accuracy of its classification is denoted a and has a differentiable increasing density $g(a)$ over $[0, 1]$ such that $g(0) = 0$ and $g > 0$ over $(0, 1]$.[6] When endowed with an asset of type $k \in \{1; 2\}$, a firm assigns a fraction a of any sample of assets in category k to category k, and misleadingly, a fraction $1 - a$ to the other category. The realizations of a are independent across firms, and firms do not observe their own a. A firm's signal s is then the fraction of resold assets to which it assigns the same type as its own asset and has conditional densities:

(E.112)
$$f_h(s) = f_l(1 - s) = g(s),$$

and

(E.113)
$$\frac{f_h(s)}{f_l(s)} = \frac{g(s)}{g(1 - s)}$$

is increasing in s because so is g. The distribution of the signal does not depend on λ, and thus condition $\{(21),(22)\}$ is satisfied.

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[5]This is similar to REE in Walrasian environments where agents condition their demand schedules on contemporaneous prices.

[6]For example, $g(a) = (1 + \chi)a^\chi$ for $\chi > 0$.

Proposition E.3. *(Misclassification risk) Under misclassification risk, Proposition 10 applies.*

Proof. Discussion above. ∎

With such misclassification risk, the signal does not depend on λ and so condition (22) is binding. This implies that imposing a higher degree of conservatism on firms reduces their agency costs only through the channel of a lower cost of resales due to more aggressive bids. The informativeness of market signals is unaffected by an increase in λ. We now develop an alternative microfoundation in which inequality (22) will be strict: An increase in λ will affect both the costs of taking to market and that of marking to market in this case.

Idiosyncratic risk

We now suppose that firms perfectly identify asset types when observing transactions by other firms, but that payoffs across assets of the same type differ along an idiosyncratic component. This corresponds to assets that are heterogenous in nature (over-the-counter derivatives, real estate,...). As mentioned in Section C, this can also stand for changes in an asset fundamental between measurement and disclosure dates. Such idiosyncratic noise per se does not generate noisy inference if only h-payoff assets are resold in equilibrium. So we also add a reason why l-payoff assets are occasionally resold. We describe in turn each ingredient and explain how it affects the equilibrium characterized in Section III.

First, we suppose that the date-2 payoff of each project is equal to $y + z$, where $y \in \{l; h\}$ is identical across projects of the same type. The new terms z are independently and identically distributed across firms with a c.d.f. Ψ that admits a differentiable log-concave density with full support over the real line. We suppose that for all $y \neq 0$, $\Psi(x+y)/\Psi(x)$ spans $(0, +\infty)$ as x spans $\mathbb{R}$.[7] Each firm (principal and agent) and all the buyers who are matched with it (informed or not) observe the realization of z for that firm at date 1, whereas other agents do not.

Second, we suppose that in addition to receiving a signal and bids, a firm (principal and agent) privately observes the value of its project's payoff at date 1 with probability $\gamma < \beta$ (so far we had $\gamma = 0$). This affects the provision of incentives to agents as follows. In the event of such an early payoff discovery, the agent receives utility 1 if the payoff is h and 0 if it is l. In the absence of early discovery, the agent is as before rewarded if the signal is above a cut-off σ, or if he successfully resells the asset at a price above $r + z$, where z is the firm's idiosyncratic shock. The pair $\{\sigma, r\}$ solves (3) subject to constraints (4) and (5), with the only change that β is replaced by $\beta' = (\beta - \gamma)/(1 - \gamma) < \beta$ in these equations.

Third, we also assume that a principal, when indifferent between reselling the firm's asset or

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[7]We could impose positive payoffs at the cost of some additional complexity.

not, always chooses to do so. This is for expositional simplicity: In the proof of Proposition E.4, we show that such a preference for trading arises endogenously from small gains from trades between principals and potential buyers. This affects equilibrium transactions as follows. We keep assuming an arbitrarily large mass of uninformed buyers, so that each firm always faces competitive uninformed buyers. Firms that discover a l-payoff at date 1 sell their asset to uninformed buyers at the price $l + z$, and do not reward the agent upon such a sale. Whereas l-payoff assets are sold if discovered early by a firm, h-payoff assets are sold only for measurement purposes in the absence of early discovery and when the firm receives a signal below σ and at least one bid above $r + z$. Indeed, indifference between bids implies that informed bids for h-assets are bounded away from h, the principal's valuation of the asset.

Finally, for tractability, we preserve the information structure assumed thus far with signals that are univariate and identically distributed across firms by assuming that each firm observes the price fetched by one asset of the same type as its own one before making its resale decision.[8] This observed transaction price therefore plays the role of the exogenous date-1 signal assumed thus far. We have:

Proposition E.4. *(Idiosyncratic risk) For κ and l sufficiently small, there exists a stable equilibrium with endogenous signals such that Proposition 8 applies and inequality (22) is strict. This implies that imposing a higher degree of conservatism reduces firms' agency costs by making both marking to market and taking to market strictly more efficient.*

Proof:

Step 1. Optimal contracting in the presence of early payoff discovery

We first revisit the contracting problem of Section I in the case in which, with a probability $\gamma < \beta$, the firm discovers the project value at date 1. It is clearly optimal that, in the event of such an early discovery, the agent receives utility 1 if the payoff is h and 0 if it is l. It is easy to see that the optimal course of action in the absence of early discovery solves the same problem as that in the case $\gamma = 0$ up to a replacement of the parameter β with $\beta' = (\beta - \gamma)/(1 - \gamma) < \beta$.

Step 2. Optimal contracting and bidding in the presence of early payoff discovery

This implies that in the setting of Section III with exogenous signals and endogenous number of informed bidders, Proposition 7 applies in the presence of such early discovery: For κ, l sufficiently small, a stable equilibrium exists and solves (23), (24), and (25) where the parameters β and κ are replaced by $\beta' = (\beta - \gamma)/(1 - \gamma)$ and $\kappa' = \kappa/(1 - \gamma)$ respectively.

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[8]Each firm could more generally observe any statistic from a finite sample of transactions.

Step 3. Optimal contracting and bidding in the presence of early payoff discovery and gains from trade

We now study how the presence of gains from trade between firms' principals and potential buyers affects such a stable equilibrium with possible early discovery described in Steps 1 and 2. We suppose that potential buyers value a payoff $y + z$ at $y + z + \epsilon$, where $\epsilon > 0$. We show that in the limiting case in which $\epsilon \to 0$ (infinitesimal gains from trade), these gains from trade induce the preference for trading that is directly assumed in the above. For ϵ sufficiently small, firms that discover an h-payoff and firms that receive a bid larger than r and a signal larger than σ never sell as their valuation of the project, h, exceeds that of the highest possible bid. This is because the condition that informed buyers be indifferent between bids for h-projects implies that their bids are bounded away from h. Sales that take place above r are therefore only for incentive purposes. Firms that discover an l-payoff always sell as uninformed bids are competitive and thus weakly larger than $l + z + \epsilon$. Firms that do not discover the early payoff but receive a sufficiently low public signal may also sell their project to uninformed buyers, although the signal below which they do so tends to $-\infty$ as $\epsilon \to 0$ because of adverse selection (the firm may be selling because it has discovered an l-payoff). Thus, in the limiting case $\epsilon \to 0$, gains from trade induce only sales of l-payoff projects discovered by firms.

Note that the resales meant to reap gains from trade do not affect informed bidding strategies nor informed bidders' expected profits. Thus the equilibrium values $\{\sigma; r; \lambda\}$ are as stated in Step 2.

Step 4. Endogenous signals

Step 3 shows that there exists a stable equilibrium in the extension of Section III to early payoff discovery and arbitrarily small gains from trade. It remains to show that this applies to the case in which the signals are given by:

(E.114) $$F_l(s) = \Psi(s - l),$$

(E.115) $$F_h(s, \lambda, r) = \Psi * H_1(s, \lambda, r),$$

where Ψ is the c.d.f. of z and

(E.116) $$H_1(s, \lambda, r) = \sum_{k \geq 1} \frac{q_k(\lambda)}{1 - q_0(\lambda)} S^k(s, \lambda, r),$$

where S is implicitly defined by (E.51). Such F_h, F_l satisfy (22) because $\partial F_l / \partial \lambda = 0$, $\partial F_h / \partial \lambda \leq 0$.

These signals depart from the assumptions of Section III to the extent that F_h also depends on r with $\partial F_h/\partial r \leq 0$. We leave it to the reader to check that the proofs of Propositions 7 and 8 can be simply adapted to this case. ■

Interestingly, this microfoundation is also suggestive of contagion phenomenons. If many firms observe the resale price generated by the same transaction by a firm with an h-project but a negative idiosyncratic shock, then this firm's resale sends low market signals, thereby inducing a large number of ex-post inefficient resales by firms with h-assets.

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