Online Appendix for "Taxes and Turnout"

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A Proofs for Section 1

A.1 Derivation of equation (4)

The total number of votes for party 1 is a random variable equal to

$$\tilde{V}^1 = \mathbb{E}[\sigma^1 \tilde{q}^1(\omega) B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)] =: \sigma^1 \tilde{\mathbf{B}}^1(p^1,p^2). \tag{21}$$

Analogously, the total number of votes for party 2 equals

$$\tilde{V}^2 = \mathbb{E}[\sigma^2 \tilde{q}^2(\omega)(1 - B(u(p^1,\omega) - u(p^2,\omega) \mid \omega))] =: \sigma^2 \tilde{\mathbf{B}}^2(p^1,p^2). \tag{22}$$

Given two party proposals p^1 and p^2, and given the turnout for party 2, σ^2, the best response problem of the group-rule-utilitarian supporters of party 1 is to choose σ^1 so as to maximize the expected value of the following expression

$$\begin{aligned}
&\mathbb{I}\{\tilde{V}^1 \geq \tilde{V}^2\}\mathbb{E}\left[B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)\; u(p^1,\omega)\right] \\
&+ \left(1 - \mathbb{I}\{\tilde{V}^1 \geq \tilde{V}^2\}\right) \times \\
&\mathbb{E}\left[B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)\; u(p^2,\omega) + \int_{-\infty}^{u(p^1,\omega)-u(p^2,\omega)} \varepsilon\, b(\varepsilon \mid \omega) d\varepsilon\right] \\
&- k(\sigma^1)\, \mathbb{E}[\tilde{q}^1(\omega) B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)].
\end{aligned}$$

In this expression, $\mathbb{I}$ is an indicator function and the product

$$\mathbb{I}\{\tilde{V}^1 \geq \tilde{V}^2\}\mathbb{E}\left[B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)\; u(p^1,\omega)\right]$$

is utilitarian welfare realized by the supporters of party 1 in the event that their party wins. Analogously,

$$(1-\mathbb{I}\{\cdot\})\,\mathbb{E}\left[B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)\;u(p^2,\omega)+\int\limits_{-\infty}^{u(p^1,\omega)-u(p^2,\omega)}\varepsilon\, b(\varepsilon\mid\omega)d\varepsilon\right]$$

is utilitarian welfare realized by the supporters of party 1 in the event that party 2 wins, where the integral term in this expression is the sum of the gains (or losses) that the supporters of party 1 realize because of their idiosyncratic party preference.

Upon exploiting the linearity of the expectations operator and dropping terms that do not depend on σ^1, we can equivalently write this optimization problem as follows: choose $\sigma^1 \in [0,1]$ to maximize

$$\pi^1(p^1,p^2,\sigma^1,\sigma^2)W^1(p^1,p^2)-k(\sigma^1)\mathbf{B}^1(p^1,p^2), \tag{23}$$

where $\pi^1(p^1,p^2,\sigma^1,\sigma^2)$ is the probability that $\tilde{V}^1 \geq \tilde{V}^2$, $W^1(p^1,p^2)$ is defined by (2) and captures the welfare gain that is realized by the supporters of party 1 if their party wins, and $\mathbf{B}^1(p^1,p^2)$ is defined by (1) and captures the expected value of the mass of the ethical supporters of party 1.

A.2 Proof of Proposition 1

Proof of Footnote 14. The arguments in the derivation of equation (5) imply that, for given p^1 and p^2, party 1's probability of winning the election is given by

$$\pi^1(p^1,p^2,\sigma^1,\sigma^2)=F_\eta\left(\frac{\sigma^1(p^1,p^2)}{\sigma^2(p^1,p^2)}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right).$$

Given σ^2, the best response problem of party 1's ethical voters in (4) can therefore be written as follows: choose σ^1 to maximize

$$F_\eta\left(\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right)W^1(p^1,p^2)-\kappa(\sigma^1)\,\mathbf{B}^1(p^1,p^2)$$

If the Inada conditions on the cost function hold, the derivative of the objective with respect to σ is strictly positive at $\sigma^1=0$ and strictly negative at $\sigma^1=1$. Thus, the best response is interior and characterized by a first-order condition. Given the concavity of F_η and the convexity of the cost function the solution is moreover unique. The same argument applies to the best response problem of party 2.

Equilibrium relative turnout. Using equations (21) and (22), the probability that party 1 wins the election is equal to the probability of the event

$$\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)} \geq \frac{\eta^2}{\eta^1}.$$

Denote by F_η the c.d.f. and by f_η the density of the random variable $\frac{\eta^2}{\eta^1}$. Thus,

$$\pi^1(p^1,p^2,\sigma^1,\sigma^2) = F_\eta\left(\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right). \tag{24}$$

We take the party platforms p^1 and p^2 as given and characterize equilibrium turnout. We say that the turnout game has an interior equilibrium if $0 < \sigma^{1*}(p^1,p^2) < 1$ and $0 < \sigma^{2*}(p^1,p^2) < 1$. An interior equilibrium is characterized by the first-order conditions

$$\pi^1_{\sigma^1}(\cdot)W^1(p^1,p^2) - \frac{\chi}{\lambda}\left(\sigma^1\right)^{1/\lambda-1}\mathbf{B}^1(p^1,p^2) = 0, \tag{25}$$

and

$$-\pi^1_{\sigma^2}(\cdot)W^2(p^1,p^2) - \frac{\chi}{\lambda}\left(\sigma^2\right)^{1/\lambda-1}\mathbf{B}^2(p^1,p^2) = 0. \tag{26}$$

Using equation (24), these first order conditions can also be written as

$$f_\eta\left(\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right)\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\frac{1}{\sigma^1}W^1(p^1,p^2) - \frac{\chi}{\lambda}\left(\sigma^1\right)^{1/\lambda-1}\mathbf{B}^1(p^1,p^2) = 0, \tag{27}$$

and

$$f_\eta\left(\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right)\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\frac{1}{\sigma^2}W^2(p^1,p^2) - \frac{\chi}{\lambda}\left(\sigma^2\right)^{1/\lambda-1}\mathbf{B}^2(p^1,p^2) = 0. \tag{28}$$

Equations (27) and (28) allow us to pin down the equilibrium value of relative turnout,

$$\frac{\sigma^{1*}(p^1,p^2)}{\sigma^{2*}(p^1,p^2)} \quad = \quad \left[\frac{W^1(p^1,p^2)\,/\,\mathbf{B}^1(p^1,p^2)}{W^2(p^1,p^2)\,/\,\mathbf{B}^2(p^1,p^2)}\right]^\lambda. \tag{29}$$

The left-hand side of this equation is a measure of party 1's turnout advantage: the larger σ_1^*/σ_2^*, the larger the number of ethical supporters who turn out to vote for party 1, relative to the number of supporters who turn out to vote for party 2. The right-hand side is a ratio of the welfare gains per capita, $W^j/\mathbf{B}^j$, that the supporters of both parties can realize in case of winning the election. Thus, according to equation (29), the relative turnout for party 1 is increasing in the relative amounts that its supporters and those of the competing party have at stake.

Derivation of equation (5). Under Assumption 1,

$$\tilde{\mathbf{B}}^1(p^1,p^2) = \eta^1\mathbf{B}^1(p^1,p^2) \quad \text{and} \quad \tilde{\mathbf{B}}^2(p^1,p^2) = \eta^2\mathbf{B}^2(p^1,p^2).$$

The probability that party 1 wins the election is therefore equal to the probability of the event

$$\sigma^1\eta^1\mathbf{B}^1(p^1,p^2) \geq \sigma^2\eta^2\mathbf{B}^2(p^1,p^2)$$

or, equivalently,

$$\frac{\sigma^1}{\sigma^2}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)} \geq \frac{\eta^2}{\eta^1}.$$

Let F_η be the c.d.f. of the random variable η^2/η^1. Then this probability can be written as

$$\Pi^1(p^1,p^2) = F_\eta\left(\frac{\sigma^1(p^1,p^2)}{\sigma^2(p^1,p^2)}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right).$$

Thus, the probability that party 1 wins the election is a non-decreasing function of

$$\frac{\sigma^1(p^1,p^2)}{\sigma^2(p^1,p^2)}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}.$$

Therefore, party 1's objective is to maximize this expression and party 2's objective is to minimize it.

Proof of Proposition 1. Party 1 seeks to maximize

$$\frac{\sigma^1(p^1,p^2)}{\sigma^2(p^1,p^2)}\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}$$

and party 2 seeks so minimize this term. Using equation (29) to substitute for $\frac{\sigma^1(p^1,p^2)}{\sigma^2(p^1,p^2)}$ yields

$$\left[\frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}\right]^{\lambda}\left[\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right]^{1-\lambda}$$

as the objective. We may as well assume that party 1 seeks to maximize a monotone transformation of this expression, whereas party 2 seeks to minimize it. Using the logarithm function as the monotone transformation yields Proposition 1.

A.3 Proof of Proposition 2

We first prove Proposition 2 under the assumption that $\mathcal{P}$ is a one-dimensional policy space. We then generalize the argument to higher dimensional policy spaces. We also use the following shorthands $Q := \mathbb{E}[\bar{q}(\omega)]$, $H_B(x) = \ln(\frac{x}{Q-x})$, $H_S(x) = \ln x$, Finally, we denote the derivatives of the functions H_B and H_S by h_b and h_s, respectively.

A.3.1 One-dimensional policy space

Best responses. Suppose that $\mathcal{P} = [\underline{p}, \overline{p}] \subset \mathbb{R}$. Fix p_2. The derivative of $\Pi^1(p^1,p^2)$ with respect to the first argument, henceforth denoted by Π^1_1, equals

$$\begin{aligned}\Pi^1_1(p^1,p^2) \quad &= \quad (1-\lambda)\, h_B(\mathbf{B^1}(p^1,p^2))\, \mathbb{E}[\bar{q}(\omega)\; b(u(p^1,\omega)-u(p^2,\omega)\mid\omega)\; u_1(p^1,\omega)] \\ &\quad +\lambda\; h_S(W^1(p^1,p^2))\; \mathbb{E}[B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)\; u_1(p^1,\omega)] \\ &\quad +\lambda\; h_S(W^2(p^1,p^2))\; \mathbb{E}[(1-B(u(p^1,\omega)-u(p^2,\omega)\mid\omega))\; u_1(p^1,\omega)]\;.\end{aligned}$$

After a rearrangement of terms and upon denoting

$$\gamma_B(p^1,p^2,\omega) = h_B(\mathbf{B^1}(p^1,p^2))\bar{q}(\omega)\; b(u(p^1,\omega)-u(p^2,\omega)\mid\omega)$$

and

$$\begin{aligned}\gamma_S(p^1,p^2,\omega) \quad &= \quad h_S(W^1(p^1,p^2))B(u(p^1,\omega)-u(p^2,\omega)\mid\omega) \\ &\quad +h_S(W^2(p^1,p^2))\,[1-B\,(u(p^1,\omega)-u(p^2,\omega)\mid\omega)]\;,\end{aligned}$$

we can also write this derivative as

$$\Pi_1^1(p^1,p^2) = \mathbb{E}\Big[\,\big\{(1-\lambda)\,\gamma_B(p^1,p^2,\omega) + \lambda\,\gamma_S(p^1,p^2,\omega)\big\}\,u_1(p^1,\omega)\Big]\,.$$

For later reference, recall that

$$\gamma_S(p^1,p^2,\omega) = \gamma_S^*(\omega) \quad \text{and} \quad \gamma_B(p^1,p^2,\omega) = \gamma_B^*(\omega)$$

whenever $p^1 = p^2$. By a symmetric argument, the derivative of $\Pi^1(p^1,p^2)$ with respect to the second argument p^2 equals

$$\Pi_2^1(p^1,p^2) = -\mathbb{E}\left[\big\{(1-\lambda)\,\gamma_B(p^1,p^2,\omega) + \lambda\,\gamma_S(p^1,p^2,\omega)\big\}\,u_1(p^2,\omega)\right]\,.$$

Under the regularity assumptions made in the text, p^1 is a best response to p^2 if and only if

$$\Pi_1^1(p^1,p^2) = 0 \quad \Leftrightarrow \quad \mathbb{E}\left[\big\{(1-\lambda)\,\gamma_B(p^1,p^2,\omega) + \lambda\,\gamma_S(p^1,p^2,\omega)\big\}\,u_1(p^1,\omega)\right] = 0\,. \tag{30}$$

Likewise, p^2 is a best response to p^1 if and only if

$$\Pi_2^1(p^1,p^2) = 0 \quad \Leftrightarrow \quad \mathbb{E}\left[\big\{(1-\lambda)\,\gamma_B(p^1,p^2,\omega) + \lambda\,\gamma_S(p^1,p^2,\omega)\big\}\,u_1(p^2,\omega)\right] = 0\,. \tag{31}$$

Existence of a symmetric equilibrium. Consider the policy p^* which solves

$$\mathbb{E}\left[\{(1-\lambda)\,\gamma_B^*(\omega) + \lambda\,\gamma_S^*(\omega)]\}\,u_1(p^1,\omega)\right] = 0\,.$$

This policy maximizes

$$\mathbb{E}\left[\{(1-\lambda)\,\gamma_B^*(\omega) + \lambda\,\gamma_S^*(\omega)\}\,u(p,\omega)\right]$$

over the set $\mathcal{P}$. Moreover, the pair of policies $(p^1,p^2) = (p^*,p^*)$ satisfies the first order conditions of both parties' best response problems in (30) and (31), respectively, and is hence an equilibrium.

Uniqueness. It remains to be shown that there is no other equilibrium. Suppose, to the contrary, that there is an equilibrium (p^1,p^2) with $p^1 \neq p^*$ or $p^2 \neq p^*$. In the following, we assume without loss of generality that $p^1 \neq p^*$. Since the game under study is zero-sum, this implies that also (p^1, p^*) is a Nash equilibria, see e.g. Osborne and Rubinstein (1994). This contradicts the assumption that party 1 has a unique best response to any policy $p^2 \in \mathcal{P}$. Thus, the assumption that there is an alternative equilibrium leads to a contradiction and must be false.

A.3.2 Multi-dimensional policy space

Suppose that $\mathcal{P}$ is a compact set. Let p^1 be an interior policy and let $h \in \mathcal{P}$ be a conceivable direction in which party 1 can deviate from p^1. We assume that such a deviation takes the form

$$p^1 + \mu\, h \ ,$$

where μ is a non-negative scalar that measures the size of the deviation from p^1. We denote by

$$\delta u(p^1, h, \omega)$$

the (functional) derivative of $u(p^1, \omega)$ in direction h at p^1. Equipped with this notation, we can now generalize the arguments for the one-dimensional policy space in a straightforward way.

Best responses. Given p^2, a best response for party 1 is a policy so that, for any admissible direction h,

$$\mathbb{E}\left[\left\{(1-\lambda)\,\gamma_B(p^1, p^2, \omega) + \lambda\,\gamma_S(p^1, p^2, \omega)\right\}\delta u(p^1, h, \omega)\right] = 0 \ . \tag{32}$$

Likewise, p^2 is a best response to p^1 if and only if, for any admissible direction h,

$$\mathbb{E}\left[\left\{(1-\lambda)\,\gamma_B(p^1, p^2, \omega) + \lambda\,\gamma_S(p^1, p^2, \omega)\right\}\delta u(p^2, h, \omega)\right] = 0 \ . \tag{33}$$

Existence of a symmetric equilibrium. Consider the policy p^* which solves, for any admissible direction h,

$$\mathbb{E}\left[\{(1-\lambda)\,\gamma_B^*(\omega) + \lambda\,\gamma_S^*(\omega)\}\,\delta u(p^*, h, \omega)\right] = 0 \ .$$

This policy maximizes

$$\mathbb{E}\left[\{(1-\lambda)\,\gamma_B^*(\omega) + \lambda\,\gamma_S^*(\omega)\}\,u(p, \omega)\right]$$

over the set $\mathcal{P}$. Moreover, the pair of policies $(p^1, p^2) = (p^*, p^*)$ satisfies the first order conditions of both parties' best response problems in (32) and (33), respectively, and is hence an equilibrium.

Uniqueness. Uniqueness follows from the same argument as above.

B Proofs for Section 2

B.1 Affine income taxes

Concave policy preferences. Consider affine income taxation with quasi-linear in consumption utility and isoelastic effort costs. Then

$$y^*(\tau, \omega) = (1-\tau)^\varepsilon \, \omega^{1+\varepsilon}$$

We can solve for tax revenue as a function of τ. This yields

$$r(\tau) = \tau(1-\tau)^\varepsilon \, \mathbb{E}\left[\omega^{1+\varepsilon}\right] \ ,$$

i.e.,

$$r'(\tau) = \left(1 - \frac{\tau}{1-\tau}\,\varepsilon\right)(1-\tau)^\varepsilon \, \mathbb{E}\left[\omega^{1+\varepsilon}\right]$$

and

$$r''(\tau) = -\varepsilon\,(1-\tau)^{\varepsilon-1}\left(2 + \frac{\tau}{1-\tau}(\varepsilon - 1)\right)\mathbb{E}\left[\omega^{1+\varepsilon}\right] \ .$$

Policy preferences are captured by the indirect utility function $u(\tau, \omega)$. By the envelope theorem,

$$u_1(\tau, \omega) = r'(\tau) - y^*(\tau, \omega)$$

The ideal policy for type ω, $\tau^*(\omega)$ solves

$$r'(\tau) - y^*(\tau, \omega) = 0 \ ,$$

or, equivalently,

$$\frac{\tau^*(\omega)}{1-\tau^*(\omega)} = \frac{1}{\varepsilon}\left(1 - \frac{\omega^{1+\varepsilon}}{\mathbb{E}\left[\omega^{1+\varepsilon}\right]}\right) \ .$$

Obviously, $\tau^* : \omega \mapsto \tau^*(\omega)$ is a strictly decreasing, continuously differentiable function. The second derivative of u with respect to τ is given by

$$u_{11}(\tau, \omega) = r''(\tau) - y_1^*(\tau, \omega) \ .$$

Note that, since $y_1^*(\tau, \omega) < 0$, concavity of α is not enough to ensure that policy preferences are concave.

Lemma 1. *Consider the affine income taxation setting with quasi-linear in consumption utility and isoelastic effort costs. Consider the policy space* $\mathcal{T} = [\tau^*(\underline{\omega}), \tau^*(\overline{\omega})]$. *Then* $\varepsilon \leq \frac{1}{2}$ *implies that, for all* $\tau \in \mathcal{T}$ *and all* $\omega \in [\underline{\omega}, \overline{\omega}]$, $u_{11}(\tau, \omega) \leq 0$.

Proof. Straightforward computations yield

$$u_{11}(\tau, \omega) = -\varepsilon \, (1-\tau)^{\varepsilon - 1} \left(\left(2 + \frac{\tau}{1-\tau}(\varepsilon - 1) \right) \mathbb{E}\left[\omega^{1+\varepsilon}\right] - \omega^{1+\varepsilon} \right) .$$

We seek to show that, for all $\tau \in \mathcal{T}$ and all $\omega \in [\underline{\omega}, \overline{\omega}]$,

$$2 + \frac{\tau}{1-\tau}(\varepsilon - 1) \geq \frac{\omega^{1+\varepsilon}}{\mathbb{E}\left[\omega^{1+\varepsilon}\right]} \ .$$

A (necessary and) sufficient condition is that

$$2 + \frac{\tau}{1-\tau}(\varepsilon - 1) \geq \frac{\bar{\omega}^{1+\varepsilon}}{\mathbb{E}\left[\omega^{1+\varepsilon}\right]} \ ,$$

or using the first order condition characterizing $\tau^*(\overline{\omega})$,

$$2 + \frac{\tau}{1-\tau}(\varepsilon - 1) \geq 1 - \frac{\tau^*(\overline{\omega})}{1 - \tau^*(\overline{\omega})}\varepsilon \ .$$

Since $\tau^* : \omega \mapsto \tau^*(\omega)$ is a strictly decreasing, a (necessary and) sufficient condition is that

$$2 + \frac{\tau^*(\overline{\omega})}{1-\tau^*(\overline{\omega})}(\varepsilon - 1) \geq 1 - \frac{\tau^*(\overline{\omega})}{1 - \tau^*(\overline{\omega})}\varepsilon \ .$$

Equivalently,

$$\frac{\tau^*(\overline{\omega})}{1-\tau^*(\overline{\omega})} \, (2\varepsilon - 1) \geq -1 \ .$$

Since

$$\frac{\tau^*(\overline{\omega})}{1-\tau^*(\overline{\omega})} < 0 \ ,$$

this inequality holds if $2\varepsilon - 1 \leq 0$, or, equivalently, if $\varepsilon \leq \frac{1}{2}$. □

Derivation of equation (11). It follows from Proposition 2 and equation (10) that τ^* maximizes

$$\mathbb{E}[\gamma^*(\omega)]r(\tau) + \mathbb{E}\left[\gamma^*(\omega)\left((1-\tau)y^*(\tau,\omega) - k(y^*(\tau,\omega))\right)\right] \ .$$

Using the envelope theorem, the first order condition can be written as

$$r'(\tau) - \mathbb{E}\left[\frac{\gamma^*(\omega)}{\mathbb{E}[\gamma^*(\omega)]}y^*(\tau,\omega)\right] = 0 \ ,$$

where

$$r'(\tau) = \mathbb{E}\left[y^*(\tau,\omega)\right] + \tau\mathbb{E}\left[y_1^*(\tau,\omega)\right] \ .$$

Rearranging terms yields

$$\begin{aligned} \tau\mathbb{E}\left[\frac{y_1^*(\tau,\omega)}{\mathbb{E}[y^*(\tau,\omega)]}\right] &= \mathbb{E}\left[\left(\frac{\gamma^*(\omega)}{\mathbb{E}[\gamma^*(\omega)]} - 1\right)\frac{y^*(\tau,\omega)}{\mathbb{E}[y^*(\tau,\omega)]}\right] \\ &= Cov\left(\frac{\gamma^*(\omega)}{\mathbb{E}[\gamma^*(\omega)]}, \frac{y^*(\tau,\omega)}{\mathbb{E}[y^*(\tau,\omega)]}\right) \ . \end{aligned} \tag{34}$$

With isoelastic effort costs and quasi-linearity in consumption, the first-order condition of individual utility maximization, $1-\tau = k_1(y,\omega)$, yields $y^*(\tau,\omega) = (1-\tau)^e\omega^{1+e}$ and $y_1^*(\tau,\omega) = -e\frac{1}{1-\tau}y^*(\tau,\omega)$. Substituting these expressions into (34) yields (11).

B.2 *CRP* taxes

Concave policy preferences. Policy preferences captured by the indirect utility function

$$u(\tau,\omega) = \ln r(\tau) + (1-\tau)\ln y^*(\tau,\omega) - k(y^*(\tau,\omega),\omega) \ .$$

By the envelope theorem,

$$u_1(\tau,\omega) = \frac{r'(\tau)}{r(\tau)} - \ln y^*(\tau,\omega) \ .$$

With isoleastic effort costs, straightforward computations yield

$$u_1(\tau,\omega) = -\frac{\tau}{1-\tau}\frac{1}{1+\frac{1}{\varepsilon}} + \ln\left(\mathbb{E}\left[(\ln\omega)\omega^{1-\tau}\right]\right) - \ln\omega \ .$$

and

$$u_{11}(\tau,\omega) = -\frac{1}{1+\frac{1}{\varepsilon}}(1-\tau)^{-2} - \mathbb{E}[(\ln\omega)^2\,\omega^{1-\tau}]\ .$$

Clearly, for $\tau \in (0,1)$, $u_{11}(\tau,\omega) < 0$, for all ω.

Derivation of equation (13).

Individual utility-maximization. Under a CRP schedule, an individual with earnings y has a consumption level of $c = ry^{1-\tau}$. With log consumption utility and iso-elastic effort costs, an individual of type ω solves the following utility-maximization problem

$$\max_y \quad \ln r + (1-\tau)\ln y - \frac{1}{1+1/e}\left(\frac{y}{\omega}\right)^{1+1/e}\ .$$

Utility-maximizing earnings are hence given by $y^*(\tau,\omega) = (1-\tau)^{\frac{e}{1+e}}\omega$.

Tax revenue. We use the government budget constraint, $\mathbb{E}[T(y^*(\tau,\omega))] = 0$, to solve for r as a function of τ, which yields $r(\tau) = \frac{\mathbb{E}[y^*(\tau,\omega)]}{\mathbb{E}[y^*(\tau,\omega)^{1-\tau}]}$, or, equivalently,

$$r(\tau) = (1-\tau)^{\tau\frac{e}{1+e}}\ \frac{\mathbb{E}[\omega]}{\mathbb{E}[\omega^{1-\tau}]}$$

Policy preferences. Policy preferences are therefore captured by the indirect utility function $u(\tau,\omega) = \ln r(\tau) + (1-\tau)\ln y^*(\tau,\omega) - \frac{1}{1+1/e}\left(\frac{y^*(\tau,\omega)}{\omega}\right)^{1+1/e}$, or, equivalently,

$$u(\tau,\omega) = \frac{e}{1+e}\ln(1-\tau) + \ln\left(\frac{\mathbb{E}[\omega]}{\mathbb{E}[\omega^{1-\tau}]}\right) + (1-\tau)\ln\omega - \frac{e}{1+e}(1-\tau)\ . \qquad (35)$$

The assumption that $\ln\omega$ is normally distributed with mean μ_ω and variance σ^2_ω, can be shown to imply that

$$\mathbb{E}[\omega] = \exp\left(\mu_\omega + \frac{1}{2}\sigma^2_\omega\right) \quad \text{and} \quad \mathbb{E}[\omega^{1-\tau}] = \exp\left((1-\tau)\mu_\omega + \frac{1}{2}(1-\tau)^2\sigma^2_\omega\right)\ .$$

Thus,

$$\begin{aligned} u(\tau,\omega) &= \tfrac{e}{1+e}\ln(1-\tau) - \tfrac{e}{1+e}(1-\tau) \\ &\quad + \Big(1-(1-\tau)\Big)\mu_\omega + \tfrac{1}{2}\Big(1-(1-\tau)^2\Big)\sigma_\omega^2 \qquad (36) \\ &\quad + (1-\tau)\ln\omega \;. \end{aligned}$$

Equilibrium policy. By Proposition 2, the political equilibrium tax policy maximizes $\mathbb{E}[\gamma^*(\omega)u(\tau,\omega)]$, or, equivalently,

$$\begin{aligned} \mathbb{E}[\bar{\gamma}^*(\omega)u(\tau,\omega)] &= \tfrac{e}{1+e}\ln(1-\tau) - \tfrac{e}{1+e}(1-\tau) \\ &\quad + \Big(1-(1-\tau)\Big)\mu_\omega + \tfrac{1}{2}\Big(1-(1-\tau)^2\Big)\sigma_\omega^2 \\ &\quad + (1-\tau)\mathbb{E}[\bar{\gamma}^*(\omega)\ln\omega] \;, \end{aligned}$$

where $\bar{\gamma}^*(\omega) = \frac{\gamma^*(\omega)}{\mathbb{E}[\gamma^*(\omega)]}$. It is convenient to think of this objective as a function of $(1-\tau)$ rather than τ. The first order condition characterizing the equilibrium value of $1-\tau$ is

$$\frac{e}{1+e}\frac{\tau}{1-\tau} - \mu_\omega - (1-\tau)\sigma_\omega^2 + \mathbb{E}[\bar{\gamma}^*(\omega)\ln\omega] = 0 \;.$$

Rewriting this equation, using that $\mathbb{E}[\bar{\gamma}^*(\omega)\ln\omega] - \mu_\omega = Cov\left(\frac{\gamma^*(\omega)}{\mathbb{E}[\gamma^*(\omega)]}, \ln\omega\right)$, yields equation (13) in the main text.

B.3 Non-linear income taxes

Suppose that the preferences of a type ω individual over (c,y)-pairs are represented by a quasi-linear in consumption utility function $c - k(y,\omega)$, with $k_1 > 0$, $k_{11} > 0$, $k_2 < 0$ and $k_{12} < 0$.

In the following we sketch the argument for why any tax system T can be represented by a non-decreasing earnings function $\mathbf{y} : \Omega \to \mathbb{R}_+$. Specifically, by the taxation principle, see e.g. Hammond (1979); Guesnerie (1995), an allocation $(\mathbf{c},\mathbf{y})$ consisting of a consumption schedule $\mathbf{c} : \Omega \to \mathbb{R}_+$ and an earnings schedule $\mathbf{y} : \Omega \to \mathbb{R}_+$ can be induced by an income tax if and only if it satisfies the resource constraint,

$$\mathbb{E}[\mathbf{y}(\omega)] \geq \mathbb{E}[\mathbf{c}(\omega)] \qquad (37)$$

and incentive compatibility constraints: for all ω and ω',

$$u(\omega) \geq \mathbf{c}(\omega') - k(\mathbf{y}(\omega'), \omega) \ , \tag{38}$$

where

$$u(\omega) := \mathbf{c}(\omega) - k(\mathbf{y}(\omega), \omega) \tag{39}$$

gives the utility that a type ω individual realizes under allocation $(\mathbf{c}, \mathbf{y})$.

It is also well-known how to obtain a characterization of incentive-compatible allocations in models with quasilinear preferences, see e.g. Myerson (1981). The utility realized by any one type-ω individual can be written as a sum of two terms, the minimal level of utility that is realized by the "poorest" type and the extra utility realized by higher types. More formally, an application of the envelope theorem makes it possible to show that incentive compatibility holds if and only if two conditions are satisfied. First, for all ω,

$$u(\omega) = \underline{u} + \rho(\mathbf{y}, \omega) \quad \text{where} \quad \rho(\mathbf{y}, \omega) = -\int_{\underline{\omega}}^{\omega} k_2(\mathbf{y}(z), z) \ dz \ , \tag{40}$$

and $\underline{u} := u(\underline{\omega})$ is a shorthand for the lowest type's utility and $-\int_{\underline{\omega}}^{\omega} k_2(\mathbf{y}(z), z) \ dz$ is the *information rent* realized by a higher type $\omega > \underline{\omega}$ in the presence of incentive compatibility constraints.[44] Second, $\mathbf{y}$ is a non-decreasing function, i.e., individuals with higher productive abilities must not earn less than individuals with lower productive abilities.

We can use these insights to derive a representation of preferences over tax polices in a reduced form that only depends on the income function $\mathbf{y}$ and no longer involves a reference to the consumption function $\mathbf{c}$. This will enable us to represent a tax policy design problem as a problem that no longer involve resource and incentive constraints. Suppose that $(\mathbf{c}, \mathbf{y})$ is incentive compatible, then using (39), (40) and an integration by parts we obtain

$$\mathbb{E}[c(\omega)] = \underline{u} + \mathbb{E}\left[k(\mathbf{y}(\omega), \omega) - \frac{1 - F(\omega)}{f(\omega)} \ k_2(\mathbf{y}(\omega), \omega)\right] \ .$$

Plugging this expression into the public sector budget constraint $\mathbb{E}[\mathbf{y}(\omega)] - \mathbb{E}[\mathbf{c}(\omega)] = 0$

[44]This terminology reflects that private information on types is the impediment to first-best redistribution.

yields an expression for $\underline{u}$; it is equal to the *virtual surplus* that is associated with an earnings function y:

$$\underline{u} := s_v(\mathbf{y}) := \mathbb{E}\left[y(\omega) - k(\mathbf{y}(\omega), \omega) + \frac{1 - F(\omega)}{f(\omega)}\, k_2(\mathbf{y}(\omega), \omega)\right] . \tag{41}$$

The virtual surplus is a surplus measure that takes account of the information rents that tax-payers realize and which reduces what is available for the lowest type. To arrive at the virtual surplus, the (non-virtual) surplus of aggregate output over costs of effort

$$s(\mathbf{y}) := \mathbb{E}\left[\mathbf{y}(\omega) - k(\mathbf{y}(\omega), \omega)\right]$$

is reduced by the aggregate information rent

$$-\mathbb{E}\left[\int_{\underline{\omega}}^{\omega} k_2(\mathbf{y}(z), z)\; dz\right] = -\mathbb{E}\left[\frac{1 - F(\omega)}{f(\omega)}\, k_2(\mathbf{y}(\omega), \omega)\right] ,$$

where the equality follows from an integration by parts. Thus,

$$\underline{u} = s_v(\mathbf{y}) = \mathbb{E}\left[\mathbf{y}(\omega) - k(\mathbf{y}(\omega), \omega) + \frac{1 - F(\omega)}{f(\omega)}\, k_2(\mathbf{y}(\omega), \omega)\right] . \tag{42}$$

Indirect utility induced by an incentive compatible allocation can now be written as a sum of virtual surplus and information rents

$$u(\omega) := s_v(\mathbf{y}) + \rho(\mathbf{y}, \omega) \tag{43}$$

With this characterization, the utility realized by a type ω individual depends on the whole earnings schedule $\mathbf{y} : \Omega \to \mathbb{R}_+$ but no longer on the consumption schedule $\mathbf{c} : \Omega \to \mathbb{R}_+$.

For the representation of policy preferences, we make the dependence of u on the earnings function explicit and write $u(\mathbf{y}, \omega)$ rather than simply $u(\omega)$.

To summarize, for non-linear income taxation, the policy space is the set of all non-decreasing earnings functions. Any such function generates a payoff profile that is characterized by equations (40) and (42).

Derivation of equation (14). In part D.3 of the Online-Appendix we show that, in a symmetric pure strategy equilibrium,

$$\frac{T'(\mathbf{y}^*(\omega))}{1-T'(\mathbf{y}^*(\omega))} \quad = \quad -\frac{1-F(\omega)}{f(\omega)}\left(1-\Gamma^*(\omega)\right)\ \frac{k_{21}(\mathbf{y}^*(\omega),\omega))}{k_1(\mathbf{y}^*(\omega),\omega)}\ ,$$

see equation (71). With an isoelastic effort cost function we can substitute $-\left(1+\frac{1}{e}\right)\frac{1}{\omega}$ for $\frac{k_{21}(\mathbf{y}^*(\omega),\omega))}{k_1(\mathbf{y}^*(\omega),\omega)}$, which yields (14).

B.4 Comparative statics

B.4.1 Using political equilibrium weights to order equilibrium tax systems

Consider two specifications of the model's primitives giving rise to two different weighting functions that are respectively denoted by $\gamma_0^* : \omega \mapsto \gamma_0^*(\omega)$ and $\gamma_1^* : \omega \mapsto \gamma_1^*(\omega)$. Suppose that there is a decreasing function $\delta : \omega \mapsto \delta(\omega)$ with $\mathbb{E}[\delta(\omega)] = 0$ so that

$$\frac{\gamma_1^*(\omega)}{\mathbb{E}[\gamma_1^*(\omega)]} = \frac{\gamma_0^*(\omega)}{\mathbb{E}[\gamma_0^*(\omega)]} + \delta(\omega)\ . \tag{44}$$

For ease of notation, let $\bar{\gamma}_1^*(\omega) := \frac{\gamma_1^*(\omega)}{\mathbb{E}[\gamma_1^*(\omega)]}$ and $\bar{\gamma}_0^*(\omega) := \frac{\gamma_0^*(\omega)}{\mathbb{E}[\gamma_0^*(\omega)]}$, so that $\mathbb{E}[\bar{\gamma}_0^*(\omega)] = \mathbb{E}[\bar{\gamma}_1^*(\omega)] = 1$. In the following we show that, for all models of redistributive taxation that we consider, the equilibrium tax system associated with γ_1^* is more redistributive than the one associated with γ_0^*. First note that, for any increasing function $z(\cdot)$, we have

$$\mathbb{E}_\omega[\bar{\gamma}_1^*(\omega)\, z(\omega)] = \mathbb{E}_\omega[\bar{\gamma}_0^*(\omega)\, z(\omega)] + \mathbb{E}_\omega\left[\delta(\omega)\, z(\omega)\right],$$

where $\mathbb{E}_\omega\left[\delta(\omega)\, z(\omega)\right] = Cov\left(\delta(\omega), z(\omega)\right) < 0$. Therefore, we obtain

$$\mathbb{E}[\bar{\gamma}_1^*(\omega)\, z(\omega)] < \mathbb{E}[\bar{\gamma}_0^*(\omega)\, z(\omega)].$$

For $z(\omega) = \omega^{1+e}$, this inequality implies that

$$Cov(\bar{\gamma}_1^*(\omega), \omega^{1+e}) < Cov(\bar{\gamma}_0^*(\omega), \omega^{1+e})\ .$$

Thus, the equilibrium marginal tax rate in the affine taxation setting satisfies $\tau_1^* > \tau_0^*$. Analogously, applying this inequality to the function $z(\omega) = \ln \omega$ implies

$$Cov(\bar{\gamma}_1^*(\omega), \ln \omega) < Cov(\bar{\gamma}_0^*(\omega), \ln \omega) \ .$$

Thus, the equilibrium rate of progressivity in the CRP setting satisfies $\tau_1^* > \tau_0^*$. Finally, for unrestricted non-linear taxation, define the functions

$$\Gamma_0^* : \omega \mapsto \Gamma_0^*(\omega) = \mathbb{E}\left[\bar{\gamma}_0^*(s) \mid s \geq \omega\right] \ ,$$

and

$$\Gamma_1^* : \omega \mapsto \Gamma_1^*(\omega) = \mathbb{E}\left[\bar{\gamma}_1^*(s) \mid s \geq \omega\right] \ ,$$

and note that

$$\Gamma_0^*(\underline{\omega}) = \Gamma_1^*(\underline{\omega}) = 1 \ .$$

Moreover, for any ω,

$$\Gamma_1^*(\omega) = \Gamma_0^*(\omega) + \Delta(\omega) \ , \quad \text{where} \quad \Delta(\omega) := \mathbb{E}\left[\delta(s) \mid s \geq \omega\right] \ .$$

Note that $\mathbb{E}[\delta(\omega)] = 0$ implies that $\Delta(\underline{\omega}) = 0$, so that, since δ is decreasing, $\Delta(\omega) < 0$, for all $\omega > \underline{\omega}$. Thus,

$$\Gamma_1^*(\omega) < \Gamma_0^*(\omega) \ ,$$

for all $\omega > \underline{\omega}$. Equation (14) therefore implies that marginal tax rates for all types $\omega > \underline{\omega}$ are higher when political equilibrium weights are given by $\gamma_1^* : \omega \mapsto \gamma_1^*(\omega)$, as compared to the case where they are given by $\gamma_0^* : \omega \mapsto \gamma_0^*(\omega)$.

B.4.2 Increasing W^{1s}/W^{2s}: proof of Proposition 3

Consider a shift in idiosyncratic party preferences so that ratio $\frac{W^{1s}}{W^{2s}}$ increases from an initial value $a_0 \geq 1$ to a new value a_1. Also assume that this change does not affect the size of the parties' bases, $\mathbf{B}^{1s}$ and $\mathbf{B}^{2s}$, nor the within-base income distributions captured by $B^{1s} : \omega \mapsto B^{1s}(\omega)$.

The implications for relative turnout follows from equation (29) and Proposition 2. Together with the assumption of linear voting costs, $\lambda = 1$, they imply that

$$\frac{\sigma^{1*}}{\sigma^{2*}} = \frac{W^{1s} \,/\, \mathbf{B}^{1s}}{W^{2s} \,/\, \mathbf{B}^{2s}} \tag{45}$$

Thus, $a_1 > a_0$ implies that $\left(\frac{\sigma^{1*}}{\sigma^{2*}}\right)_1 > \left(\frac{\sigma^{1*}}{\sigma^{2*}}\right)_0$.

To see that the equilibrium tax system becomes more redistributive, note that

$$\begin{aligned}\bar{\gamma}_0^*(\omega) &:= \tfrac{\gamma_0^*(\omega)}{\mathbb{E}[\gamma_0^*(\omega)]} \\ &= \tfrac{B^{1s}(\omega)+a_0(1-B^{1s}(\omega))}{E[B^{1s}(\omega)+a_0(1-B^{1s}(\omega))]} \\ &= \tfrac{a_0-(a_0-1)B^{1s}(\omega)}{a_0-(a_0-1)\mathbb{E}_\omega[B^{1s}(\omega)]} ,\end{aligned}$$

and that

$$\begin{aligned}\bar{\gamma}_1^*(\omega) &:= \tfrac{\gamma_1^*(\omega)}{\mathbb{E}[\gamma_1^*(\omega)]} \\ &= \tfrac{a_1-(a_1-1)B^{1s}(\omega)}{a_1-(a_1-1)\mathbb{E}_\omega[B^{1s}(\omega)]} .\end{aligned}$$

With $a_1 > a_0 \geq 1$ and $B^{1s} : \omega \mapsto B^{1s}(\omega)$ non-decreasing, both $\bar{\gamma}_0^*$ and $\bar{\gamma}_1^*$ are non-increasing functions. Moreover, $\bar{\gamma}_0^*(\underline{\omega}) = \bar{\gamma}_1^*(\underline{\omega}) = 1$, and, since $a_1 > a_1$,

$$| \frac{\partial}{\partial \omega} \, \bar{\gamma}_1^*(\omega) \, | \quad > \quad | \, \frac{\partial}{\partial \omega} \, \bar{\gamma}_0^*(\omega) \, | \, ,$$

for all $\omega > \underline{\omega}$. Hence,

$$\bar{\gamma}_1^*(\omega) \quad < \quad \bar{\gamma}_0^*(\omega) \, ,$$

for all $\omega > \underline{\omega}$.

These observations imply that there exists a decreasing function $\delta\,(\omega)$ with mean $\mathbb{E}\,[\delta\,(\omega)] = 0$ satisfying (15). Thus, the tax system associated with weighting function γ_1^* is more redistributive than the one associated with weighting function γ_0^*.

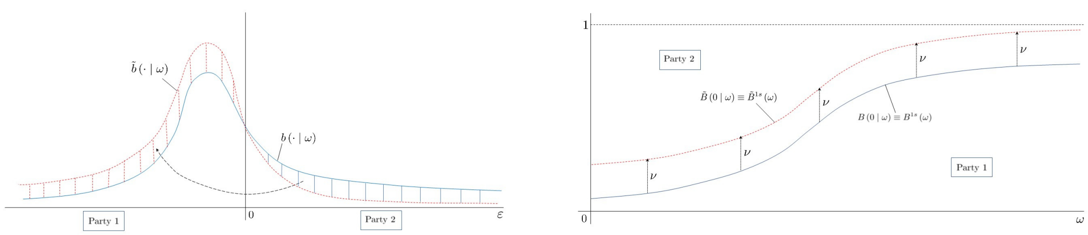
B.4.3 Increasing the base of party 1

Suppose that the base of party 1 rises uniformly: there is $\nu > 0$ such that, for all $\omega \in \Omega$, $B_1^{1s}(\omega) = B_0^{1s}(\omega) + \nu$, where the functions B_0^{1s} and B_1^{1s} characterize party 1's base before and after the preference shift. The fact that the shift is uniform implies that it does not affect the density functions that describe the distribution of idiosyncratic party biases, see Figure 5, panel (b). We will now show that, in response to such a shift, the equilibrium tax schedule becomes more redistributive.

First note that this shift leads, mechanically, to an increase of the ratio $\frac{W^{1s}}{W^{2s}}$. The sum of the stakes of party 1's supporters goes up as more supporters are added. For

Figure 5: Comparative statics: uniform rise in political support

(a) Shift in the density $b(\varepsilon \mid \omega)$ for a fixed ω (b) Shift in the cdf $B(0 \mid \omega)$ as a function of ω



the analogous reason, the sum of the stakes of the supporters of party 2 goes down. As shown in the previous section B.4.2, this effect in isolation makes the equilibrium tax schedule more redistributive.

For the thought experiment considered here, the proof in section B.4.2 has to be adapted, though, to accommodate the change from B_0^{1s} to B_1^{1s}. This adjustment is straightforward, however, because a uniform shift is without consequence for the slope of the weighting functions $\bar{\gamma}_0^*$ and $\bar{\gamma}_1^*$. Thus, the adaptation is line-by-line. We therefore omit the details.

B.4.4 Making party 1 more pro-market

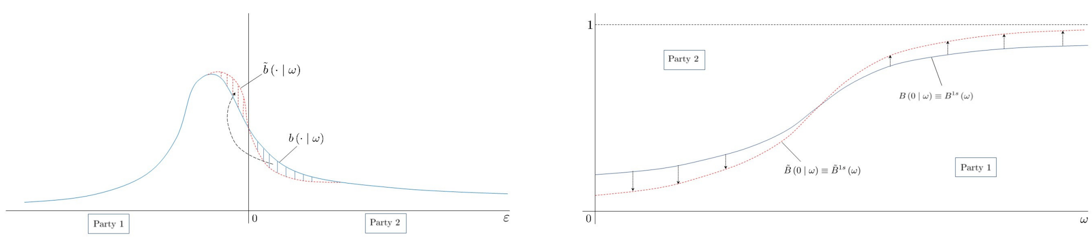
Consider a marginal shift in political biases such that $B_1^{1s}(\omega) = B_0^{1s}(\omega) + \nu\beta(\omega)$, where $\beta(\cdot)$ is an increasing function with mean $\mathbb{E}[\beta(\omega)] = 0$. We, moreover, assume that the shift is concentrated on swing voters, i.e., on voters with party preferences ε close to zero.

To explain the nature of the thought experiment, fix ω so that $\beta(\omega) > 0$. We let voters with initial party preferences $\varepsilon_0 \in [0, \nu \cdot \bar{\varepsilon}(\omega)]$ in favor of party 2, swing to preferences $\varepsilon_1 \in [\nu \cdot \underline{\varepsilon}(\omega), 0]$ that favor party 1. Figure 6 provides an illustration. Panel (a) focuses on a high level of income ω: within this income group, party 1 is dominant, both before and (even more so) after the political preference shift. Panel (b) shows the resulting shift in party 1's base as a function of ω: both parties become stronger in the income groups where they were already strong.

In the following, to evaluate the consequences of such a shift, we look into the marginal changes of endogenous variables as $\nu \to 0$. Ultimately, we show that the equilibrium tax system becomes, at the margin, more redistributive. To this end, we

Figure 6: Comparative statics: rise in polarization

(a) Shift in the density $b(\varepsilon \mid \omega)$ for a large ω (b) Shift in the cdf $B(0 \mid \omega)$ as a function of ω



first show that, at $\nu = 0$, the marginal effect on the size of the parties' bases and the relative stakes of their supporters vanishes.

To see this, fix some ω so that $\beta(\omega) > 0$. The stakes of the supporters of party 2 go down by $\int_0^{\nu \cdot \bar{\varepsilon}(\omega)} \varepsilon b(\varepsilon \mid \omega)\, d\varepsilon$, an expression that is bounded from above by

$$\nu \cdot \bar{\varepsilon}(\omega) \int_0^{\nu \cdot \bar{\varepsilon}(\omega)} b(\varepsilon \mid \omega)\, d\varepsilon = \nu\, \bar{\varepsilon}(\omega) \left(B(\nu\, \bar{\varepsilon}(\omega) \mid \omega) - B(0 \mid \omega) \right),$$

implying that the marginal effect of a change in ν vanishes at $\nu = 0$. Since the same reasoning holds for any ω, the relative intensity of preferences W^{1s}/W^{2s} is not affected by the shift in political preferences. Moreover, $\mathbb{E}[B_1^{1s}(\omega)] = \mathbb{E}[B_0^{1s}(\omega)] =: \mathbb{E}[B^{1s}(\omega)]$ since the shift in political preferences satisfies $\mathbb{E}[\beta(\omega)] = 0$.

It remains to be shown that the tax system becomes more redistributive. To this end, we adapt the arguments in section B.4.2. Let $a = W^{1s}/W^{2s}$. The political equilibrium weights prior to the shift of preferences are given by

$$\begin{aligned} \bar{\gamma}_0^*(\omega) &:= \tfrac{\gamma_0^*(\omega)}{\mathbb{E}_\omega[\gamma_0^*(\omega)]} \\ &= \tfrac{a-(a-1)B_0^{1s}(\omega)}{a-(a-1)\mathbb{E}_\omega[B^{1s}(\omega)]}\ . \end{aligned}$$

After the shift they are equal to

$$\begin{aligned} \bar{\gamma}_1^*(\omega) &:= \tfrac{\gamma_1^*(\omega)}{\mathbb{E}_\omega[\gamma_1^*(\omega)]} \\ &= \tfrac{a-(a-1)B_1^{1s}(\omega)}{a-(a-1)\mathbb{E}_\omega[B^{1s}(\omega)]}\ . \end{aligned}$$

With $B_1^{1s}(\omega) = B_0^{1s}(\omega) + \nu\beta(\omega)$, and β increasing, we have, for all $\nu > 0$,

$$\frac{\partial}{\partial \omega} B_1^{1s}(\omega) \quad > \quad \frac{\partial}{\partial \omega} B_0^{1s}(\omega) \, . \tag{46}$$

To complete the argument note that $\bar{\gamma}_0^*$ and $\bar{\gamma}_1^*$ are non-increasing functions as both B_1^{1s} and B_0^{1s} are increasing by assumption. Moreover, $\bar{\gamma}_0^*(\underline{\omega}) = \bar{\gamma}_1^*(\underline{\omega}) = 1$, and, by (46),

$$\mid \frac{\partial}{\partial \omega} \bar{\gamma}_1^*(\omega) \mid \quad > \quad \mid \frac{\partial}{\partial \omega} \bar{\gamma}_0^*(\omega) \mid ,$$

for all $\omega > \underline{\omega}$. Hence,

$$\bar{\gamma}_1^*(\omega) \quad < \quad \bar{\gamma}_0^*(\omega) \, ,$$

for all $\omega > \underline{\omega}$.

These observations imply that there exists a decreasing function $\delta(\omega)$ with mean $\mathbb{E}[\delta(\omega)] = 0$ satisfying (15). Thus, the tax system associated with weighting function γ_1^* is more redistributive than the one associated with weighting function γ_0^*.

C Proofs of Section 3

C.1 Proof of Proposition 5

C.1.1 Preliminaries

Let $\mathcal{P} = [\underline{p}, \overline{p}] \subset \mathbb{R}$. Let $p^*(\omega) = \text{argmax}_{p \in \mathcal{P}} \, u(p, \omega)$ be the ideal policy for voter type ω. The voters' ideal policies lie in the interior of $\mathcal{P}$ and satisfy the first order condition $u_1(p^*(\omega), \omega) = 0$. The single-crossing condition implies that $p^* : \omega \mapsto p^*(\omega)$ is non-increasing. Thus, for some $\varepsilon, \delta > 0$, $\underline{p} = p^*(\overline{\omega}) - \varepsilon$ and $\overline{p} = p^*(\underline{\omega}) + \delta$. The single-crossing property also implies that all types ω strictly prefer $p^*(\underline{\omega})$ over $\overline{p}$ and $p^*(\overline{\omega})$ over $\underline{p}$. Thus, $[p^*(\overline{\omega}), p^*(\underline{\omega})] \subset \mathcal{P}$ is the set of Pareto-efficient policies.

Assuming for simplicity that $\lambda = 1$, the objective of party 1 is to maximize

$$\Pi^1(p^1, p^2) = \frac{W^1(p^1, p^2)}{W^2(p^1, p^2)}$$

and the objective of party 2 is to minimize this expression. Focusing on this case simplifies the exposition, but as we clarify below the argument does not depend on it and extends to any value of λ. Henceforth, we denote by Π_1^1 and Π_2^1 the partial derivatives of Π^1 with respect to p^1 and p^2, respectively.

Be reminded that $\Delta u(p^1, p^2, \omega) = u(p^1, \omega) - u(p^2, \omega)$, $W^1(p^1, p^2) = \mathbb{E}\left[G^1_W(\Delta u(\cdot) \mid \omega)\right]$ and $W^2(p^1, p^2) = \mathbb{E}\left[G^2_W(\Delta u(\cdot) \mid \omega)\right]$, where

$$G^1_W(x \mid \omega) := \int_{-\infty}^{x} (x - \varepsilon)\, b(\varepsilon \mid \omega) d\varepsilon \ ,$$

and

$$G^2_W(x \mid \omega) := \int_{x}^{\infty} (\varepsilon - x)\, b(\varepsilon \mid \omega) d\varepsilon \ .$$

The derivatives of the functions $G^1_W(\cdot \mid \omega)$ and $G^2_W(\cdot \mid \omega)$ are respectively given by

$$g^1_W(x \mid \omega) := B(x \mid \omega) \quad \text{and} \quad g^2_W(x \mid \omega) := -\left(1 - B(x \mid \omega)\right) \ .$$

Lemma 2 (**Best responses exist and are interior**). *For any $p^2 \in \mathcal{P}$, there is a best response of party 1. Any best response of party 1 lies in the interior of $\mathcal{P}$ and satisfies the first order condition $\Pi^1_1(p^1, p^2) = 0$. Analogously, for any $p^2 \in \mathcal{P}$ there is a best response of party 2. Any best response of party 2 is interior and satisfies the first order condition $\Pi^1_2(p^1, p^2) = 0$.*

Proof. We only prove the statements referring to the best responses of party 1. For any p^2, the function $\Pi^1(\cdot, p^2)$ is continuous in p^1 and therefore attains a maximum on the compact policy space $\mathcal{P} = [\underline{p}, \overline{p}]$. The function $\Pi^1(\cdot, p^2)$ is, moreover, differentiable. To prove that the maximum is interior and satisfies first-order conditions we show that, for any p^2,

$$\Pi^1_1(\underline{p}, p^2) > 0 \quad \text{and} \quad \Pi^1_1(\overline{p}, p^2) < 0 \ .$$

Given p^2, the derivative of $\Pi^1(\cdot, p^2)$ with respect to p^1 can be written as

$$\begin{aligned}
\Pi^1_1(p^1, p^2) &= \frac{1}{W^2(p^1, p^2)} \mathbb{E}\left[g^1_W(\Delta u(\cdot) \mid \omega) u_1(p^1, \omega)\right] \\
&\quad - \frac{W^1(p^1, p^2)}{\left(W^2(p^1, p^2)\right)^2} \mathbb{E}\left[g^2_W(\Delta u(\cdot) \mid \omega) u_1(p^1, \omega)\right] \\
&= \frac{1 + \Pi^1(p^1, p^2)}{W^2(p^1, p^2)}\ \mathbb{E}\left[\gamma^1(\omega \mid p^1, p^2)\ u_1(p^1, \omega)\right]
\end{aligned}$$

where

$$\gamma^1(\omega \mid p^1, p^2) = \frac{1}{1 + \Pi^1(p^1, p^2)}\ B(\Delta u(\cdot) \mid \omega) + \frac{\Pi^1(p^1, p^2)}{1 + \Pi^1(p^1, p^2)}\ \left(1 - B(\Delta u(\cdot) \mid \omega)\right).$$

Let $p^1 = \underline{p}$, then $u_1(p^1, \omega) > 0$ for all ω and hence $\Pi_1^1(p^1, p^2) > 0$. Analogously, if $p^1 = \overline{p}$, then $u_1(p^1, \omega) < 0$ for all ω and hence $\Pi_1^1(p^1, p^2) < 0$. □

Lemma 3 (**Best responses are continuous**)**.** *For given p^2, let $p^{1*}(p^2)$ be a solution to the first order condition $\Pi_1^1(p^1, p^2) = 0$. Then p^{1*} is a continuous function.*

Proof. The implicit function theorem can be applied to the first-order condition for party 1 and implies that p^{1*} is a differentiable, hence continuous, function of p^2. □

Lemma 4 (**Existence of a fixed point**)**.** *The function p^{1*} has a fixed point.*

Proof. The function p^{1*} is a continuous function from $\mathcal{P}$ to $\mathcal{P}$, where $\mathcal{P}$ is a non-empty, compact and convex set. Therefore, it has a fixed point by Brouwer's fixed point theorem. □

Lemma 5 (**Uniqueness of the fixed point**)**.** *If utility functions are concave in p, $u_{11}(p, \omega) < 0$ for all p and ω, the function p^{1*} has only one fixed point.*

Proof. Let (p^1, p^2) be such a fixed point of the best response function p^{1*}. Such a fixed point satisfies $\Pi_1^1(p^1, p^2) = 0$, i.e.,

$$\mathbb{E}\left[\gamma^1(\omega \mid p^1, p^2)\; u_1(p^1, \omega)\right] = 0 \ ,$$

and

$$p^1 = p^2 \ .$$

These two equations uniquely pin down p^1. To see this, note first that $p^1 = p^2$ implies $\Delta u(p^1, p^2, \omega) = 0$ for all ω and that $\gamma^1(\omega \mid p^1, p^2)$ depends on p^1 and p^2 only via $\Delta u(p^1, p^2, \omega)$. Let $\gamma(\omega)$ be the corresponding value of $\gamma^1(\omega \mid p^1, p^2)$, i.e.,

$$\gamma(\omega) := \frac{1}{1 + \Pi_*^1} B(0 \mid \omega) + \frac{\Pi_*^1}{1 + \Pi_*^1}\left(1 - B(0 \mid \omega)\right), \quad \text{with} \quad \Pi_*^1 := \frac{\mathbb{E}\left[G_W^1(0 \mid \omega)\right]}{\mathbb{E}\left[G_W^2(0 \mid \omega)\right]} \ .$$

Then p^1 solves

$$\mathcal{A}(p^1) := \mathbb{E}\left[\gamma(\omega)\; u_1(p^1, \omega)\right] = 0 \ .$$

To see that this equation has a unique solution, note that the auxiliary function $\mathcal{A}(p^1)$ is differentiable, and decreasing as $\mathcal{A}'(p^1) = \mathbb{E}\left[\gamma(\omega)\; u_{11}(p^1, \omega)\right] < 0$. Moreover, $\mathcal{A}(\underline{p}) > 0$ and $\mathcal{A}(\overline{p}) < 0$. Thus, there is one and only one solution to the equation $\mathcal{A}(p^1) = 0$. □

C.1.2 Proof of Claim 1.

Suppose that

$$\begin{aligned} \Pi^1(p,p') &= a + b\,\Pi^2(p',p) \\ & \; a + b\,(1-\Pi^2(p',p))\ . \end{aligned}$$

Hence,

$$\Pi^1_1(p,p') = -b\,\Pi^1_2(p',p)\ .$$

Now suppose that $(\hat{p},\hat{p})$ is a fixed point of party 1's best response problem. Then,

$$\Pi^1(\hat{p},\hat{p}) \geq \Pi^1(p,\hat{p})\ ,$$

for all $p \in \mathcal{P}$. Further note that

$$\begin{aligned} \Pi^1(\hat{p},\hat{p}) - \Pi^1(p,\hat{p}) &= \int_p^{\hat{p}} \Pi^1_1(s,\hat{p})\,ds \\ &= -b\,\int_p^{\hat{p}} \Pi^1_2(\hat{p},s)\,ds \\ &= -b\,\Big(\Pi^1(\hat{p},\hat{p}) - \Pi^1(\hat{p},p)\Big) \\ &= b\Big(\Pi^1(\hat{p},p) - \Pi^1(\hat{p},\hat{p})\Big)\ . \end{aligned}$$

Hence,

$$\Pi^1(\hat{p},\hat{p}) \leq \Pi^1(\hat{p},p)\ ,$$

for all $p \in \mathcal{P}$, implying that $(\hat{p},\hat{p})$ is a saddle point of Π^1, and hence an equilibrium.

C.1.3 Proof of Claim 2.

Together Lemma 2 and the premise of Claim 2 imply that, for every p_2, there is a unique value of p^1 so that

$$\Pi^1_1(p^1,p^2) = 0 \quad \text{and} \quad \Pi^1_{11}(p^1,p^2) < 0\ .$$

This value of p^1 is the unique best response of party 1 to policy p^2. Mutatis mutandis, the same holds true for party 2. Under these conditions the following Lemma holds true.

Lemma 6 (**Identical fixed points**). *Suppose that utility functions are concave in p, $u_{11}(p,\omega) < 0$ for all p and ω. Then, the best response functions p^{1*} and p^{2*} have the same fixed point.*

Proof. As argued above, if (p^1, p^2) is a fixed point of p^{1*} it satisfies

$$\mathbb{E}\left[\gamma^1(\omega \mid p^1, p^2)\ u_1(p^1, \omega)\right] = 0 \tag{47}$$

and

$$p^1 = p^2 \ . \tag{48}$$

By Lemma 5, there is only one solution to this system of equations.

Analogously, given p^1, the best responses of party 2, $p^{2*}(p^1)$ is obtained as the solution to

$$\min_{p^2 \in \mathcal{P}} \quad \Pi^1(p^1, p^2)$$

and solves the first-order condition

$$\Pi^1_2(p^1, p^2) = -\frac{1 + \Pi^1(p^1, p^2)}{W^2(p^1, p^2)}\ \mathbb{E}\left[\gamma^1(\omega \mid p^1, p^2)\ u_1(p^2, \omega)\right] = 0\ .$$

Thus, a fixed point (p^1, p^2) of p^{2*} satisfies

$$\mathbb{E}\left[\gamma^1(\omega \mid p^1, p^2)\ u_1(p^2, \omega)\right] = 0\ , \tag{49}$$

and

$$p^1 = p^2 \ . \tag{50}$$

Hence, a fixed point of p^{2*} solves the same system of equations as a fixed point of p^{1*}. Thus, the two functions have the same fixed point. $\square$

Therefore, if $\hat{p}$ is a fixed point of p^{1*}, then $(\hat{p}, \hat{p})$ is a symmetric equilibrium in pure strategies. By Lemma 4 such a fixed point exists.

It remains to be shown that there can be no other equilibrium. Suppose to the contrary that there is an alternative Nash equilibrium (p^1, p^2) with $p^1 \neq \hat{p}$ or $p^2 \neq \hat{p}$. Since the game under study is zero-sum, this implies that $(p^1, \hat{p})$ and $\hat{p}, p^2$ are also Nash equilibria, see e.g. Osborne and Rubinstein (1994). Suppose without loss of generality that $p^1 \neq \hat{p}$. Then, this implies that, for party 1, both $\hat{p}$ and p^1 are best responses to $\hat{p}^2$. This contradicts the assumption that party 1 has a unique best response to any policy $p^2 \in \mathcal{P}$. Thus, the assumption that there is an alternative equilibrium leads to a contradiction and must be false.

C.1.4 General Objective, $0 < \lambda < 1$.

The preceding argument uses that $W^1(\cdot)$ and $W^2(\cdot)$ depend on p^1 and p^2 only via $\Delta u(p^1, p^2, \omega) = u(p^1, \omega) - u(p^2, \omega)$ and the derivatives of the objective function can be written as a weighted sum of the different types' marginal utilities, where the weights are all positive. These properties remain intact with a more general objective function of the form

$$\Pi^1(p^1, p^2) = (1-\lambda) \ln \left(\frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)} \right) + \lambda \ln \left(\frac{W^1(p^1, p^2)}{W^2(p^1, p^2)} \right) .$$

For instance, the best response condition $\Pi_1^1(p^1, p^2) = 0$ for party 1 can then be written as

$$\mathbb{E}\left[\gamma^{1,\lambda}(\omega \mid p^1, p^2)\; u_1(p^1, \omega)\right] = 0 ,$$

where

$$\gamma^{1,\lambda}(\omega \mid p^1, p^2) \quad = \quad (1-\lambda)\, \bar{q}(\omega)\; b(\cdot \mid \omega) + \lambda \left(\tfrac{1}{\mathbf{B}^1(\cdot)} + \tfrac{1}{\mathbf{B}^2(\cdot)} \right)^{-1} \gamma^1(\omega \mid p^1, p^2).$$

C.2 Derivation of inequalities (19) and (20)

The objective of party 1 is to maximize

$$\Pi^1(p^1, p^2) = \frac{W^1(p^1, p^2)}{W^2(p^1, p^2)}$$

and the objective of party 2 is to minimize this expression. For later reference, note that $W^1(p^1, p^2) = \mathbb{E}\left[G_W^1(\Delta u(\cdot) \mid \omega)\right]$ and $W^2(p^1, p^2) = \mathbb{E}\left[G_W^2(\Delta u(\cdot) \mid \omega)\right]$ where

$$G_W^1(x \mid \omega) := \int_{-\infty}^{x} (x - \varepsilon)\, b(\varepsilon \mid \omega) d\varepsilon ,$$

and

$$G_W^2(x \mid \omega) := \int_{x}^{\infty} (\varepsilon - x)\, b(\varepsilon \mid \omega) d\varepsilon .$$

The derivatives of the functions $G^1_W(\cdot \mid \omega)$ and $G^2_W(\cdot \mid \omega)$ are respectively given by

$$g^1_W(x \mid \omega) := B(x \mid \omega) \quad \text{and} \quad g^2_W(x \mid \omega) := -\left(1 - B(x \mid \omega)\right) \ .$$

Given a policy p^2, the first order condition of party 1's best response problem is

$$\Pi^1_1(\cdot) = \frac{1}{W^2(\cdot)^2} \left\{ W^1_1(\cdot) W^2(\cdot) - W^2_1(\cdot) W^1(\cdot) \right\} = 0 \ . \tag{51}$$

The second derivative Π^1_{11}, evaluated at a policy that satisfies this first order condition, equals

$$\Pi^1_{11}(\cdot) = \frac{1}{W^2(\cdot)^2} \{ W^1_{11}(\cdot) W^2(\cdot) - W^2_{11}(\cdot) W^1(\cdot) \} \ . \tag{52}$$

Thus, $\Pi^1_{11}(\cdot) < 0$ holds provided that

$$W^1_{11}(\cdot) = \mathbb{E}[b(\Delta u(\cdot) \mid \omega) u_1(p^1, \omega) + B(\Delta u(\cdot) \mid \omega) u_{11}(p^1, \omega)] \quad < \quad 0 \ , \tag{53}$$

and

$$W^2_{11}(\cdot) = \mathbb{E}[b(\Delta u(\cdot) \mid \omega) u_1(p^1, \omega) - (1 - B(\Delta u(\cdot) \mid \omega)) u_{11}(p^1, \omega)] \quad > \quad 0 \ . \tag{54}$$

It is now straightforward to verify that the inequalities (19) and (20) stated in the main text imply that the inequalities (53) and (54) hold.

C.3 Existence of mixed-strategy equilibria

Glicksberg's existence theorem implies the existence of a mixed strategy equilibrium for a zero-sum game under the following conditions:

- *Pure strategy spaces are compact.*
- *The payoff function Π^1 is continuous in (p_1, p_2) for $(p_1, p_2) \in \mathcal{P}^2$.*

In the following, we introduce notions of compactness and continuity that can be applied to a policy space of non-negative, bounded and monotonic functions. Verifying that these properties indeed hold is then straightforward. The existence of a mixed strategy equilibrium then follows from Glicksberg's existence theorem.

Compactness. Let $\Omega = [\underline{\omega}, \overline{\omega}]$. Let the set of feasible earnings levels also be a compact subset of the reals and denote it by $\mathcal{Y} = [0, \overline{y}]$. An earnings function $\mathbf{y}$:

$\Omega \to \mathcal{Y}$ can be viewed as an element of a compact set $\Omega \times \mathcal{Y}$. Now consider a sequence of earnings functions $\{\mathbf{y}^k\}_{k=1}^{\infty}$ that converges to a limit function $\bar{\mathbf{y}}$ in the sense that, for every $\omega \in \Omega$, $\{\mathbf{y}^k(\omega)\}$ is a sequence in $\mathcal{Y}$ that converges to a limit point $\bar{y}(\omega)$.

The domain of all functions in the sequence is constant and equal to $\Omega = [\underline{\omega}, \overline{\omega}]$, which is also the domain of the limit function. Thus, we only need to worry about the convergence in the image of these functions. The image is an element of $\mathcal{Y}^{\#\Omega}$, a cartesian product of compact sets. By Tychonoff's theorem, a cartesian product of compact sets is a compact set. By assumption, the sequence $\{\mathbf{y}^k(\Omega)\}_{k=1}^{\infty}$, $\mathbf{y}^k(\Omega) \in \mathcal{Y}^{\#\Omega}$, converges to a limit $\bar{\mathbf{y}}$. Since $\mathcal{Y}^{\#\Omega}$ is a compact set it follows that $\bar{\mathbf{y}} \in \mathcal{Y}^{\#\Omega}$.

Continuity. A sketch of the argument suffices. What enters the parties' objective function Π^1 are averages, by type ω, of continuous functions (c.d.f.'s of party preferences) of utility differentials implied by policy differences. By hypothesis, $p_a \to p_b$, implies $u(p_a, \omega) \to u(p_b, \omega)$, for all $\omega \in \Omega$. Thus if the difference between two policies p_a and p_b vanishes in the sense of uniform convergence, then, for every type ω, the contribution to the objective under p_a converges to the contribution under p_b. This property survives continuous transformations and integration.

D Proof of Proposition 4

D.1 Regularity conditions

Optimal tax problems. We impose regularity conditions that are familiar from the literature on optimal taxation. As will become clear in the subsequent paragraph, these regularity conditions also facilitate the analysis of the parties' best responses.

As outlined in part B.3 of the Online-Appendix, a non-linear tax schedule can be represented by a non-negative, bounded and monotonic earnings function $\mathbf{y} : \Omega \to \mathbb{R}_+$. The policy preferences of a type ω individual are then represented by $u(\mathbf{y}, \omega)$. Social welfare S induced by an earnings function $\mathbf{y}$ is

$$S(\mathbf{y}) = \mathbb{E}\left[g(\omega)\; u(\mathbf{y}, \omega)\right] \; .$$

where $g : \Omega \to \mathbb{R}_+$ specifies the weights of different types in the welfare function. Without loss of generality we let $\mathbb{E}[g(\omega)] = 1$. The *full* optimal tax problem is to choose the earnings function $\mathbf{y}$ that maximizes this welfare objective over the set of non-decreasing functions. The *relaxed* problem is to choose the earnings function that maximizes this welfare objective over the set of all functions.

Assumption 3. *Suppose that the following conditions hold:*

1. *For any weighting function $g : \Omega \to \mathbb{R}_+$ with $\mathbb{E}[g(\omega)] = 1$, the above problem of welfare-maximization has a unique solution. Let $\mathbf{y}_g$ be the earnings function that solves this problem.*

2. *For any weighting function $g : \Omega \to \mathbb{R}_+$ with $\mathbb{E}[g(\omega)] = 1$, the relaxed problem of welfare-maximization has a unique solution. Let $\mathbf{y}_g^r$ be the earnings function that solves the this relaxed problem.*

3. *The function $\mathbf{y}_g^r$ satisfies Diamond's formula; i.e. for all ω,*

$$1 - k_1(\mathbf{y}(\omega), \omega) = -\frac{1 - F(\omega)}{f(\omega)} \; (1 - \mathcal{G}_g(\omega)) \; k_{21}(\mathbf{y}(\omega), \omega) \; ,$$

   where $\mathcal{G}_g(\omega) := \mathbb{E}[g(z) \mid z \geq \omega]$. Moreover, $\mathbf{y}_g^r$ is the only function that satisfies Diamond's formula.

4. *For ω so that the monotonicity constraint on $\mathbf{y}_g$ is not binding, $\mathbf{y}_g^r(\omega) = \mathbf{y}_g(\omega)$.*

Assumption 3 is routinely invoked in models of optimal income taxation. The assumption can be justified. With an appropriate choice of the primitives, the solutions to the relaxed and the full problem of welfare-maximization can be shown to satisfy properties 1.– 4. We simply impose Assumption 3 as a shortcut.

To get from an earnings function $\mathbf{y}$ to the associated tax schedule T, one can use the first-order condition of the utility-maximization problem that individuals face in the presence of this tax system. If tax system T induces an incentive-compatible allocation $(\mathbf{c}, \mathbf{y})$, then

$$1 - T'(\mathbf{y}(\omega)) = k_1(\mathbf{y}(\omega), \omega) \; .$$

Hence, $1 - k_1(y(\omega), \omega)$ is interpreted as the marginal tax rate that type-ω agents face.

Best response problems. For ease of exposition, we focus on relaxed best response problems. Thus, given an earnings function $\mathbf{y}^2$ proposed by party 2, the problem of party 1 is to choose $\mathbf{y}^1$ so as to maximize $\Pi^1(\mathbf{y}^1, \mathbf{y}^2)$. This problem differs from the full best response problem that incorporates the constraint that $\mathbf{y}^1$ must be a non-decreasing function. Obviously, if the solution to the relaxed problem is non-decreasing then it is also a solution to the full problem. Otherwise, the solution of the full problem will give rise to bunching. While it is well-known how the analysis would have to be modified if bunching is an issue, see e.g. Hellwig (2007); Brett and Weymark (2016), the trade-offs that shape best responses are more easily exposed when focusing on the relaxed problem.

The equilibrium analysis that follows involves a characterization of best responses. As will become clear, the focus on relaxed best response problems in conjunction with Assumption 3 then implies that best responses are characterized by a version of Diamond's rule, albeit with a different weighting function.

D.2 Best responses: necessary conditions

For ease of exposition, we assume that $\lambda = 1$ so that

$$\Pi^1(\mathbf{y}^1, \mathbf{y}^2) = \frac{W^1(\mathbf{y}^1, \mathbf{y}^2)}{W^2(\mathbf{y}^1, \mathbf{y}^2)} \ .$$

The best response of party 1 can be viewed as a compromise between maximizing the expression in the numerator and minimizing the expression in the denominator. We first look at each of these auxiliary objectives in isolation and then turn to the compromise.

Maximizing $W^1(\mathbf{y}^1, \mathbf{y}^2)$. Remember that $W^1(y^1, y^2) = \mathbb{E}[G^1_W(\Delta u(\cdot))]$ and that the derivative of $G^1_W(\cdot \mid \omega)$ equals $g^1_W(\cdot \mid \omega) = B\left(\Delta u(\cdot) \mid \omega\right)$. We write

$$\begin{aligned} \bar{g}^1_W(\omega \mid \mathbf{y}^1, \mathbf{y}^2) \ &:= \ \mathbb{E}[B(u(\mathbf{y}^1, \omega') - u(\mathbf{y}^2, \omega') \mid \omega') \mid \omega' \geq \omega] \\ &= \ \textstyle\int_{\omega}^{\bar{\omega}} B(u(\mathbf{y}^1, \omega') - u(\mathbf{y}^2, \omega') \mid \omega') \frac{f(\omega')}{1-F(\omega)} d\omega' \end{aligned}$$

for the average value of $B(u(\mathbf{y}^1, \omega') - u(\mathbf{y}^2, \omega') \mid \omega')$ among individuals with a type ω' above some cutoff ω. To interpret these expressions, suppose that party 1 offers slightly more utility to type ω' individuals. Then $B(u(\mathbf{y}^1, \omega') - u(\mathbf{y}^2, \omega') \mid \omega')$ measures the extra gain that type ω'-supporters of party 1 realize in the event that party 1 wins rather party 2. Therefore, $\bar{g}^1_W(\omega \mid \mathbf{y}^1, \mathbf{y}^2)$ is the gain that party 1 can generate by offering all agents with types above ω slightly more utility. The gain that party 1 can generate by slightly raising everybody's utility is given by $\bar{g}^1_W(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)$ and the ratio

$$\mathcal{G}_W^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2) := \frac{\bar{g}_W^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2)}{\bar{g}_W^1(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)}$$

relates the gain from making everybody with a type above ω better off to the gain from making everybody better off.

Lemma 7. *Given $\mathbf{y}^2$, the solution to $\max_{\mathbf{y}^1} W^1(\mathbf{y}^1, \mathbf{y}^2)$ is such that, for all ω,*

$$\frac{T'(\mathbf{y}^1(\omega))}{1 - T'(\mathbf{y}^1(\omega))} \quad = \quad -\frac{1 - F(\omega)}{f(\omega)} \, \left(1 - \mathcal{G}_W^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2)\right) \, \frac{k_{21}(\mathbf{y}^1(\omega), \omega))}{k_1(\mathbf{y}^1(\omega), \omega)} \, . \tag{55}$$

Proof. We begin by stating party 1's best response problem in a way that enables an analysis using a Gateaux differential. Let $\mathbf{y}^1 = \mathbf{y}^{1*} + \mu \, h^1$, be a perturbed version of party 1's best response $\mathbf{y}^{1*}$, in which μ is a scalar and $h^1 : \Omega \to \mathbb{R}$ is a function. If $\mathbf{y}^{1*}$ is a best response, then, for any perturbation (μ, h^1),

$$\mathbb{E}\left[G_W^1 \left(u(\mathbf{y}^{1*}, \omega) - u(\mathbf{y}^2, \omega) \mid \omega\right)\right] \geq \mathbb{E}\left[G_W^1 \left(u(\mathbf{y}^{1*} + \mu \, h^1, \omega) - u(\mathbf{y}^2 \omega) \mid \omega\right)\right] \, . \tag{56}$$

Equivalently, using the characterization of policy preferences in part B.3 of the Online-Appendix, for any function h^1, $\mu = 0$ must be a maximizer of the auxiliary function

$$A(\mu \mid \mathbf{y}^{1*}, \mathbf{y}^2) = \mathbb{E}\left[G_W^1 \left(s_v(\mathbf{y}^{1*} + \mu \, h^1) + \rho(\mathbf{y}^{1*} + \mu \, h^1, \omega) - u(\mathbf{y}^2, \omega) \mid \omega\right)\right] \, .$$

In the following, we will characterize $\mathbf{y}^{1*}$ by analyzing the implications of the requirement that the derivative of this expression with respect to μ, evaluated at $\mu = 0$, is equal to zero. We express this condition as

$$A_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) = 0 \, , \tag{57}$$

for all functions h^1.[45] Using the characterization of information rents in part B.3 of the Online-Appendix, $\rho(\mathbf{y}, \omega) = -\int_{\underline{\omega}}^{\omega} k_2(\mathbf{y}(z), z) \, dz$, as well as the definition of the virtual surplus, $s_v(\mathbf{y}) = \mathbb{E}\left[\mathbf{y}(\omega) - k(\mathbf{y}(\omega), \omega) + \frac{1-F(\omega)}{f(\omega)} \, k_2(\mathbf{y}(\omega), \omega)\right]$, we note that

$$A_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) = \mathbb{E}\left[g_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2) \, \left(s_{v,h^1}(\mathbf{y}^{1*}) - \int_{\underline{\omega}}^{\omega} h^1(z) \, k_{21}(\mathbf{y}^{1*}(z), z) \, dz\right)\right] \, ,$$

[45]Formally, $A_{h^1}(y^{1*}, y^2)$ is the Gateaux differential of $\mathbb{E}\left[G_W^1 \left(s_v(\mathbf{y}^1) + \rho(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid \omega\right)\right]$ in direction h^1 evaluated at $\mathbf{y}^1 = \mathbf{y}^{1*}$.

where

$$g_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2) := b\left(s_v(\mathbf{y}^{1*}) - \int_{\underline{\omega}}^{\omega} k_2(\mathbf{y}^{1*}(z), z)\, dz - u(\mathbf{y}^2, \omega) \mid \omega\right)$$

and

$$s_{v,h^1}(\mathbf{y}^{1*}) \quad := \quad \mathbb{E}\left[h^1(\omega)\left(1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)} k_{21}(\mathbf{y}^{1*}(\omega), \omega)\right)\right]$$

is the Gateaux differential of the virtual surplus $s_v(\mathbf{y}^1)$ in direction h^1 evaluated at $\mathbf{y}^1 = \mathbf{y}^{1*}$. Thus, $A_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2)$ can now be rewritten as

$$\begin{aligned} A_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) \quad &= \quad \bar{g}_W^1(\underline{\omega} \mid y^{1*}, y^2)\mathbb{E}\left[h^1(\omega)\left(1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)} k_{21}(\mathbf{y}^{1*}(\omega), \omega)\right)\right] \\ &\qquad -\mathbb{E}\left[g_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\ \textstyle\int_{\underline{\omega}}^{\omega} h^1(z)\ k_{21}(\mathbf{y}^{1*}(z), z)\ dz\right]\ , \end{aligned}$$

where, for any $\omega \in \Omega$, $\bar{g}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2) := \mathbb{E}[g_W^1(\omega' \mid \mathbf{y}^{1*}, \mathbf{y}^2) \mid \omega' \geq \omega]$. Moreover, an integration by parts shows that

$$\begin{aligned} &\mathbb{E}\left[g_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\ \int_{\underline{\omega}}^{\omega} h^1(z)\ k_{21}(\mathbf{y}^{1*}(z), z)\ dz\right] \\ = \quad &\mathbb{E}\left[h^1(\omega)\ \bar{g}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\ \frac{1-F(\omega)}{f(\omega)} k_{21}(\mathbf{y}^{1*}(\omega), \omega)\right] \end{aligned} \tag{58}$$

so that condition (57) can equivalently be written as the requirement that, for all functions h^1,

$$\mathbb{E}\left[h^1(\omega)\left(1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + (1 - \mathcal{G}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2))\frac{1-F(\omega)}{f(\omega)} k_{21}(\mathbf{y}^{1*}(\omega), \omega)\right)\right] = 0\ , \tag{59}$$

where $\mathcal{G}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2) = \frac{\bar{g}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)}{\bar{g}_W^1(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)}$. Condition (59) can hold only if, for all ω,

$$1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + (1 - \mathcal{G}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2))\frac{1-F(\omega)}{f(\omega)} k_{21}(\mathbf{y}^{1*}(\omega), \omega) = 0\ , \tag{60}$$

or, equivalently, if

$$\frac{1 - k_1(\mathbf{y}^{1*}(\omega), \omega)}{k_1(\mathbf{y}^{1*}(\omega), \omega)} = -(1 - \mathcal{G}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2))\ \frac{1-F(\omega)}{f(\omega)}\ \frac{k_{21}(\mathbf{y}^{1*}(\omega), \omega)}{k_1(\mathbf{y}^{1*}(\omega), \omega)}\ . \tag{61}$$

Using $T'(\mathbf{y}^{1*}(\omega)) = 1 - k_1(\mathbf{y}^{1*}(\omega), \omega)$ we can rewrite this equation as

$$\frac{T'(\mathbf{y}^{1*}(\omega))}{1 - T'(\mathbf{y}^{1*}(\omega))} = -(1 - \mathcal{G}_W^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)) \; \frac{1 - F(\omega)}{f(\omega)} \; \frac{k_{21}(\mathbf{y}^{1*}(\omega), \omega)}{k_1(\mathbf{y}^{1*}(\omega), \omega)} \,, \tag{62}$$

which is what had to be shown. □

Minimizing $W^2(\mathbf{y}^1, \mathbf{y}^2)$. The following Lemma describes the solution to another auxiliary problem for party 1, namely the problem to choose policy with the objective to minimize what is at stake for the supporters of party 2. We omit a proof and discussion of the Lemma as it would involve only a straightforward adjustment to those of Lemma 7. The Lemma involves a weighting function $\mathcal{G}_W^2$ for information rents that is derived from $W^2(\mathbf{y}^1, \mathbf{y}^2) = \mathbb{E}[G_W^2(u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid \omega)]$ along the same lines as $\mathcal{G}_W^1$ is derived from $W^1(\mathbf{y}^1, \mathbf{y}^2)$, where we now have:

$$g_W^2(\cdot \mid \omega) \equiv 1 - B\left(u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid \omega\right) .$$

Lemma 8. *Given $\mathbf{y}^2$, the solution to $\min_{\mathbf{y}^1} W^2(\mathbf{y}^1, \mathbf{y}^2)$ is such that, for all ω,*

$$\frac{T'(\mathbf{y}^1(\omega))}{1 - T'(\mathbf{y}^1(\omega))} \quad = \quad -\frac{1 - F(\omega)}{f(\omega)} \; \left(1 - \mathcal{G}_W^2(\omega \mid \mathbf{y}^1, \mathbf{y}^2)\right) \; \frac{k_{21}(\mathbf{y}^1(\omega), \omega))}{k_1(\mathbf{y}^1(\omega), \omega)} \,. \tag{63}$$

Party 1's best response. We introduce notation for a weighted average of $\mathcal{G}_W^1$ and $\mathcal{G}_W^2$. Let

$$\begin{aligned}
\gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2) \;\; &:= \;\; \frac{1}{1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^2)} \; g_W^1(u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid y^1, y^2) \\
&\qquad + \frac{\Pi^1(\mathbf{y}^1, \mathbf{y}^2)}{1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^2)} \; g_W^2(u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid y^1, y^2) \\
&= \;\; \frac{1}{1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^2)} \; B\left(u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid \omega\right) \\
&\qquad + \frac{\Pi^1(\mathbf{y}^1, \mathbf{y}^2)}{1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^2)} \; \left(1 - B\left(u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) \mid \omega\right)\right)
\end{aligned}$$

and

$$\bar{\gamma}^1(\omega \mid y^1, y^2) \;\; := \;\; \mathbb{E}[\gamma^1(\omega' \mid y^1, y^2) \mid \omega' \geq \omega] \,.$$

Also define

$$\Gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2) := \frac{\bar{\gamma}^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2)}{\bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)}.$$

Lemma 9. *Given $\mathbf{y}^2$, if $\mathbf{y}^1$ is a maximizer of $\Pi^1(\mathbf{y}^1, \mathbf{y}^2)$ then, for all ω,*

$$\frac{T'(\mathbf{y}^1(\omega))}{1 - T'(\mathbf{y}^1(\omega))} \quad = \quad -\frac{1 - F(\omega)}{f(\omega)} \, \left(1 - \Gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2)\right) \, \frac{k_{21}(\mathbf{y}^1(\omega), \omega))}{k_1(\mathbf{y}^1(\omega), \omega)} \, . \tag{64}$$

Proof. Given $\mathbf{y}^2$ we look at the problem to choose $\mathbf{y}^1$ with the objective to maximize

$$\Pi^1(\mathbf{y}^1, \mathbf{y}^2) = \frac{W^1(\mathbf{y}^1, \mathbf{y}^2)}{W^2(\mathbf{y}^1, \mathbf{y}^2)} \, .$$

Suppose that $\mathbf{y}^{1*}$ is a solution to that problem. Then, it must also be that case that $\mu = 0$ solves the problem to choose a scalar μ with the objective to maximize

$$\Pi^1(\mathbf{y}^{1*} + \mu \, h^1, \mathbf{y}^2) = \frac{W^1(\mathbf{y}^{1*} + \mu \, h^1, \mathbf{y}^2)}{W^2(\mathbf{y}^{1*} + \mu \, h^1, \mathbf{y}^2)} \, .$$

for any given but arbitrary function h^1. That is, we can characterize $\mathbf{y}^{1*}$ be the requirement that

$$\left. \frac{\partial \Pi^1(\mathbf{y}^{1*} + \mu \, h^1, \mathbf{y}^2)}{\partial \mu} \right|_{\mu=0} = 0 \, ,$$

or, equivalently, that

$$W^1_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) \, W^2(\mathbf{y}^{1*}, \mathbf{y}^2) - W^1(\mathbf{y}^{1*}, \mathbf{y}^2) \, W^2_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) = 0 \, ,$$

where $W^j_{h^1}$ is the Gateaux differential of W^j in direction h^1. The following equations provide a characterization of $W^1_{h^1}$ and $W^2_{h^1}$ The equation follow from a straightforward adaptation of the arguments in the proof of Lemma 7. The Gateaux differential of W^1 in the direction h^1 evaluated at $(\mathbf{y}^{1*}, \mathbf{y}^2)$ equals

$$\begin{aligned} &W^1_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) = \bar{g}^1_W(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2) \times \\ &\mathbb{E}\left[h^1(\omega) \left\{1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)} \left(1 - \mathcal{G}^1_W(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\right) k_{21}(\mathbf{y}^{1*}(\omega), \omega) \right\}\right] . \end{aligned} \tag{65}$$

The Gateaux differential of W^2 in the direction h^1 evaluated at (y^{1*}, y^2) equals

$$W^2_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) = -\bar{g}^2_W(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)\times$$
$$\mathbb{E}\left[h^1(\omega)\left\{1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)}\left(1 - \mathcal{G}^2_W(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\right) k_{21}(\mathbf{y}^{1*}(\omega), \omega)\right\}\right]. \tag{66}$$

Straightforward algebra now yields the observation that the Gateaux differential of $\Pi^1 = W^1/W^2$ in the direction h^1 evaluated at $(\mathbf{y}^{1*}, \mathbf{y}^2)$ has the same sign as

$$\begin{aligned}
& \frac{W^1_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2)\, W^2(\mathbf{y}^{1*}, \mathbf{y}^2) - W^1(\mathbf{y}^{1*}, \mathbf{y}^2)\, W^2_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2)}{\bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)(1 + \Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2))W^2(\mathbf{y}^{1*}, \mathbf{y}^2)} \\
= \; & \frac{1}{\bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)}\left\{\frac{1}{1 + \Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)} W^1_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2) - \frac{\Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)}{1 + \Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)} W^2_{h^1}(\mathbf{y}^{1*}, \mathbf{y}^2)\right\}
\end{aligned}$$

Also notice that

$$\begin{aligned}
& \frac{1}{1 + \Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)} \bar{g}^1_W(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)\left(1 - \mathcal{G}^1_W(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\right) \\
& + \frac{\Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)}{1 + \Pi^1(\mathbf{y}^{1*}, \mathbf{y}^2)} \bar{g}^2_W(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)\left(1 - \mathcal{G}^2_W(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\right) \\
= \; & \bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^{1*}, \mathbf{y}^2)\left(1 - \Gamma^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\right).
\end{aligned}$$

Therefore, expressions (65) and (66) imply that the Gateaux differential of Π^1 is given by

$$\mathbb{E}\left[h^1(\omega)\left\{1 - k_1(\mathbf{y}^{1*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)}\left(1 - \Gamma^1(\omega \mid \mathbf{y}^{1*}, \mathbf{y}^2)\right) k_{21}(\mathbf{y}^{1*}(\omega), \omega)\right\}\right]. \tag{67}$$

□

For later reference, we state the analogues to the expressions in the proof of Lemma 9 for party 2's best response problem. The Gateaux differential of W^1 in the direction h^2 evaluated at $(\mathbf{y}^1, \mathbf{y}^{2*})$ equals

$$W^1_{h^2}(\mathbf{y}^1, \mathbf{y}^{2*}) = -\bar{g}^1_W(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^{2*})\times$$
$$\mathbb{E}\left[h^2(\omega)\left\{1 - k_1(\mathbf{y}^{2*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)}\left(1 - \mathcal{G}^1_W(\omega \mid \mathbf{y}^1, \mathbf{y}^{2*})\right) k_{21}(\mathbf{y}^{2*}(\omega), \omega)\right\}\right]. \tag{68}$$

The Gateaux differential of W^2 in the direction h^2 evaluated at (y^1, y^{2*}) equals

$$W^2_{h^2}(\mathbf{y}^1, \mathbf{y}^{2*}) = \bar{g}^2_W(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^{2*}) \times$$
$$\mathbb{E}\left[h^2(\omega)\left\{1 - k_1(\mathbf{y}^{2*}(\omega), \omega) + \tfrac{1-F(\omega)}{f(\omega)}\left(1 - \mathcal{G}^2_W(\omega \mid \mathbf{y}^1, \mathbf{y}^{2*})\right) k_{21}(\mathbf{y}^{2*}(\omega), \omega)\right\}\right] . \tag{69}$$

The Gateaux differential of Π^1 in the direction h^2 evaluated at $(\mathbf{y}^1, \mathbf{y}^{2*})$ then has the same sign as

$$\begin{aligned} & \frac{W^1_{h^2}(\mathbf{y}^1, \mathbf{y}^{2*})\, W^2(\mathbf{y}^1, \mathbf{y}^{2*}) - W^1(\mathbf{y}^1, \mathbf{y}^{2*})\, W^2_{h^2}(\mathbf{y}^1, \mathbf{y}^{2*})}{\bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^{2*})(1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^{2*}))W^2(\mathbf{y}^1, \mathbf{y}^{2*})} \\ = \;& -\mathbb{E}\left[h^2(\omega)\left\{1 - k_1(\mathbf{y}^{2*}(\omega), \omega) + \frac{1-F(\omega)}{f(\omega)}\left(1 - \Gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^{2*})\right) k_{21}(\mathbf{y}^{2*}(\omega), \omega)\right\}\right] . \end{aligned}$$

Thus, we obtain an analogous characterization of party 2's best responses.

Lemma 10. *Given $\mathbf{y}^1$, if $\mathbf{y}^2$ is a minimizer of $\Pi^1(\mathbf{y}^1, \mathbf{y}^2)$ then, for all ω,*

$$\frac{T'(\mathbf{y}^2(\omega))}{1 - T'(\mathbf{y}^2(\omega))} = -\frac{1-F(\omega)}{f(\omega)}\left(1 - \Gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2)\right)\frac{k_{21}(\mathbf{y}^2(\omega), \omega))}{k_1(\mathbf{y}^2(\omega), \omega)} . \tag{70}$$

D.3 An equilibrium candidate

We hypothesize the existence of a symmetric equilibrium, $\mathbf{y}^1 = \mathbf{y}^2$. When both parties propose the same policies, $u(\mathbf{y}^1, \omega) - u(\mathbf{y}^2, \omega) = 0$, for all ω. Henceforth, we make use of the following shorthands: whenever $\mathbf{y}^1 = \mathbf{y}^2$ we write $\gamma^*(\omega)$ rather than $\gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2)$ and $\Gamma^*(\omega)$ rather than $\Gamma^1(\omega \mid \mathbf{y}^1, \mathbf{y}^2)$.

If $\mathbf{y}^*$ is a symmetric equilibrium policy then, by Lemmas 9 and 10 it has to be such that

$$\frac{T'(\mathbf{y}^*(\omega))}{1 - T'(\mathbf{y}^*(\omega))} = -\frac{1-F(\omega)}{f(\omega)}\left(1 - \Gamma^*(\omega)\right)\frac{k_{21}(\mathbf{y}^*(\omega), \omega))}{k_1(\mathbf{y}^*(\omega), \omega)} . \tag{71}$$

Also note that, by Assumption 3, the function $\mathbf{y}^*$ is the unique candidate for a symmetric equilibrium.

The function $\mathbf{y}^*$ satisfies necessary conditions of both parties' best response problems: Given $\mathbf{y}^2 = \mathbf{y}^*$, $\mathbf{y}^1 = \mathbf{y}^*$ is a local extremum of the functional $\Pi^1(\cdot, \mathbf{y}^*) : \mathbf{y}^1 \to \Pi^1(\mathbf{y}^1, \mathbf{y}^*)$. Likewise, given $\mathbf{y}^1 = \mathbf{y}^*$, $\mathbf{y}^2 = \mathbf{y}^*$ is a local extremum of the functional $\Pi^1(\mathbf{y}^*, \cdot) : \mathbf{y}^2 \to \Pi^1(\mathbf{y}^*, \mathbf{y}^2)$.

D.4 Second order conditions / Saddle point

We now show that, under the premises of Proposition 4, the hypothetical equilibrium $(\mathbf{y}^1, \mathbf{y}^2) = (\mathbf{y}^*, \mathbf{y}^*)$ is a local saddle point of the function Π^1.

Lemma 11. *Suppose that the the premises of Proposition 4 are satisfied. Then, a pair of policies that satisfies (71) is a saddle point of the function* Π^1.

Proof. We seek to show that $(\mathbf{y}^*, \mathbf{y}^*)$ is a saddle point of the function

$$\Pi^1(\mathbf{y}^1, \mathbf{y}^2) = \frac{W^1(\Pi^1(\mathbf{y}^1, \mathbf{y}^2))}{W^2(\Pi^1(\mathbf{y}^1, \mathbf{y}^2))} \ .$$

We now state this saddle point condition in a way that enables an analysis using functional derivatives. Let $\mathbf{y}^1 = \mathbf{y}^{1*} + \mu^1 \ h^1$, be a perturbed version of $\mathbf{y}^{1*}$, in which μ^1 is a scalar and $h^1 : \Omega \to \mathbb{R}$ is a function. Analogously, let $\mathbf{y}^2 = \mathbf{y}^{2*} + \mu^2 \ h^2$, be a perturbed version of $\mathbf{y}^2$. The local saddle point condition according to which, for all $(\mathbf{y}^1, \mathbf{y}^2)$ in neighborhood of $(\mathbf{y}^*, \mathbf{y}^*)$,

$$\Pi^1(\mathbf{y}^1, \mathbf{y}^*) \leq \Pi^1(\mathbf{y}^*, \mathbf{y}^*) \leq \Pi^1(\mathbf{y}^*, \mathbf{y}^2)$$

can therefore be written as: for any pair of perturbations (μ^1, h^1) and (μ^2, h^2),

$$\Pi^1(\mathbf{y}^* + \mu^1 \ h^1, \mathbf{y}^*) \leq \Pi^1(\mathbf{y}^*, \mathbf{y}^*) \leq \Pi^1(\mathbf{y}^*, \mathbf{y}^* + \mu^2 \ h^2) \ . \tag{72}$$

Equivalently, for all functions (h^1, h^2), the point $(\mu^1, \mu^2) = (0, 0)$ must be a saddle-point of

$$\Pi^1(\mathbf{y}^* + \mu^1 \ h^1, \mathbf{y}^* + \mu^1 \ h^2) = \frac{W^1(\mathbf{y}^1 + \mu^1 \ h^1, \mathbf{y}^2 + \mu^2 \ h^2)}{W^2(\mathbf{y}^* + \mu^1 \ h^1, \mathbf{y}^2 + \mu^2 \ h^2)} \ .$$

In the following, we use subscripts to indicate derivatives with respect to μ^1 and μ^2, respectively.

Having a saddle point requires that all entries of the Jacobi-matrix

$$J_\Pi(y^*, y^*) = \begin{pmatrix} \Pi^1_{\mu^1}(\mathbf{y}^*, \mathbf{y}^*) \\ \Pi^1_{\mu^2}(\mathbf{y}^*, \mathbf{y}^*) \end{pmatrix}$$

are equal to zero and that the Hessian

$$H_\Pi(y^*, y^*) = \begin{pmatrix} \Pi^1_{\mu^1,\mu^1}(\mathbf{y}^*, \mathbf{y}^*) & \Pi^1_{\mu^1,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) \\ \Pi^1_{\mu^1,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) & \Pi^1_{\mu^2,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) \end{pmatrix}$$

is indefinite. As an implication of the necessary condition (71), all entries of the Jacobi-matrix are equal to zero. Hence, what remains to be shown is that $H_\Pi(\mathbf{y}^*, \mathbf{y}^*)$ is indefinite. To this end, it suffices to show that $\Pi^1_{\mu^1,\mu^1}(\mathbf{y}^*, \mathbf{y}^*) < 0$, and $\Pi^1_{\mu^2,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) > 0$. These two inequalities can be shown to hold provided that

$$\frac{\partial}{\partial \mu^1} \left\{ W^1_{\mu^1}(\mathbf{y}^*, \mathbf{y}^*)\; W^2(\mathbf{y}^*, \mathbf{y}^*) - W^1(\mathbf{y}^*, \mathbf{y}^*)\; W^2_{\mu^1}(\mathbf{y}^*, \mathbf{y}^*) \right\} < 0 \,,$$

and

$$\frac{\partial}{\partial \mu^2} \left\{ W^1_{\mu^2}(\mathbf{y}^*, \mathbf{y}^*)\; W^2(\mathbf{y}^*, \mathbf{y}^*) - W^1(\mathbf{y}^*, \mathbf{y}^*)\; W^2_{\mu^2}(\mathbf{y}^*, \mathbf{y}^*) \right\} > 0 \,,$$

or, equivalently, if

$$W^1_{\mu^1,\mu^1}(\mathbf{y}^*, \mathbf{y}^*)\; W^2(\mathbf{y}^*, \mathbf{y}^*) - W^1(\mathbf{y}^*, \mathbf{y}^*)\; W^2_{\mu^1,\mu^1}(\mathbf{y}^*, \mathbf{y}^*) < 0 \,, \tag{73}$$

and

$$W^1_{\mu^2,\mu^2}(\mathbf{y}^*, \mathbf{y}^*)\; W^2(\mathbf{y}^*, \mathbf{y}^*) - W^1(\mathbf{y}^*, \mathbf{y}^*)\; W^2_{\mu^2,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) > 0 \,. \tag{74}$$

Since $W^1(\mathbf{y}^*, \mathbf{y}^*) > 0$ and $W^2(\mathbf{y}^*, \mathbf{y}^*) > 0$ sufficient conditions for the validity of (73) and (74) are

$$W^1_{\mu^1,\mu^1}(\mathbf{y}^*, \mathbf{y}^*) < 0 \quad \text{and} \quad W^2_{\mu^1,\mu^1}(\mathbf{y}^*, \mathbf{y}^*) > 0 \,, \tag{75}$$

and

$$W^1_{\mu^2,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) > 0 \quad \text{and} \quad W^2_{\mu^2,\mu^2}(\mathbf{y}^*, \mathbf{y}^*) < 0 \,. \tag{76}$$

We can now use the expressions for $W^1_{\mu^1}$, $W^2_{\mu^1}$, $W^1_{\mu^2}$ and $W^2_{\mu^2}$ (or, equivalently, the Gateaux differentials $W^1_{h^1}$, $W^2_{h^1}$, $W^1_{h^2}$ and $W^2_{h^2}$) derived above – see equations (65) (66), (68), and (69) – to compute $W^1_{\mu^1,\mu^1}$, $W^2_{\mu^1,\mu^1}$, $W^1_{\mu^2,\mu^2}$ and $W^2_{\mu^2,\mu^2}$.

Exploiting the assumption of unform party biases, so that $B\left(\Delta u(\cdot) \mid \omega\right) = \alpha\left(\omega\right) + \beta\left(\omega\right)\; \Delta u(\cdot)$, evaluating the resulting expressions in the limit case $\beta(\omega)$ close to zero and $\alpha(\omega)$ close to $\frac{1}{2}$, for all ω, one can verify that (75) and (76) indeed hold. For instance, we then find that

$$\begin{aligned}
&W^1_{\mu^1,\mu^1}(\mathbf{y}^*,\mathbf{y}^*)\\
= \;& \bar{\alpha}(\underline{\omega})\mathbb{E}\left[h^1(\omega)^2\left(-k_{11}(\mathbf{y}^*(\omega),\omega)+\frac{1-F(\omega)}{f(\omega)}\left(1-\frac{\bar{\alpha}(\omega)}{\bar{\alpha}(\underline{\omega})}\right)k_{211}(\mathbf{y}^*(\omega),\omega)\right)\right],
\end{aligned}$$

where $\bar{\alpha}(\omega) := \mathbb{E}\left[\alpha(\omega') \mid \omega' \geq \omega\right]$. With $\alpha(\omega) = \frac{1}{2}$, for all ω, it follows that $1-\frac{\bar{\alpha}(\omega)}{\bar{\alpha}(\underline{\omega})} = 0$, so that

$$W^1_{\mu^1,\mu^1}(\mathbf{y}^*,\mathbf{y}^*) = \bar{\alpha}(\underline{\omega})\mathbb{E}\left[h^1(\omega)^2\left(-k_{11}(\mathbf{y}^*(\omega),\omega)\right)\right] < 0\,.$$

□

D.5 Existence and uniqueness of equilibrium

We now show that the local saddle point characterized in the previous proof is indeed an equilibrium point. To this end, we need to show that it is a best response for party 1 to play the hypothetical equilibrium strategy – on the assumption that party 2 also plays this strategy. The results stated so far only imply that playing the hypothetical equilibrium strategy is a local best response for party 1. What remains to be shown is that this local best response is also the global best response and that there is no other global best response.

In the following, we will use the contraction mapping theorem to show that this is indeed the case. A symmetric argument then implies that it is a best response for party 2 to play the hypothetical equilibrium strategy provided that party 1 plays accordingly. To simplify the exposition, suppose moreover that the effort cost function is iso-elastic,

$$k(y,\omega) = \frac{1}{1+1/e}\left(\frac{y}{\omega}\right)^{1+1/e}\,.$$

The condition characterizing party 1's best response $\mathbf{y}^{1*}$ to the hypothetical equilibrium strategy $\mathbf{y}^*$ then simplifies,

$$\frac{1-k_1(\mathbf{y}^{1*}(\omega),\omega)}{k_1(\mathbf{y}^{1*}(\omega),\omega)} = \left(1+\frac{1}{e}\right)\frac{1-F(\omega)}{f(\omega)\,\omega}\left(1-\Gamma^1(\omega \mid \mathbf{y}^{1*},\mathbf{y}^*)\right)\,. \tag{77}$$

or, equivalently,

$$\omega^{1+\frac{1}{e}}\mathbf{y}^{1*}(\omega)^{-\frac{1}{e}} - 1 = \left(1+\frac{1}{e}\right)\frac{1-F(\omega)}{f(\omega)\,\omega}\left(1-\Gamma^1(\omega \mid \mathbf{y}^{1*},\mathbf{y}^*)\right)\,.$$

Now, for an arbitrary earnings function $\mathbf{y}$ define $A(\mathbf{y}) = \{A(\omega, \mathbf{y})\}_{\omega\in\Omega}$ with

$$A(\omega, \mathbf{y}) = \left(1 + \left(1 + \frac{1}{e}\right) \frac{1 - F(\omega)}{f(\omega)\,\omega} \left(1 - \Gamma^1(\omega \mid \mathbf{y}, \mathbf{y}^*)\right)\right)^{-e} \omega^{1+e} \,.$$

Armed with this notation, we rewrite the previous equation one more time as

$$\mathbf{y}^{1*}(\omega) = A(\omega, \mathbf{y}^{1*}) \;,$$

for all ω. We also know from the previous arguments that this equation is satisfied for $\mathbf{y}^{1*} = \mathbf{y}^*$.

It proves useful to interpret this equation as characterizing a fixed point in a functional space. Thus, given an arbitrary earnings function $\mathbf{y}$, first interpret $A(\cdot)$ as a functional of the earnings function $\mathbf{y}$ and then define by $\mathbf{y}^{A*}(A(\mathbf{y}))$ the earnings function that satisfies,

$$\mathbf{y}^{A*}(A(\mathbf{y}), \omega) = A(\omega, \mathbf{y}) \;,$$

for all ω. By interpreting $\mathbf{y}^{A*}$ also as a function of $\mathbf{y}$, we can say that a fixed point of $\mathbf{y}^{A*}$ is an earnings function $\mathbf{y}^{fix}$ with the property that $\mathbf{y}^{A*}(A(\mathbf{y}^{fix})) = \mathbf{y}^{fix}$. By the previous arguments, we also know that $\mathbf{y}^*$ is such a fixed point.

Now, if $\mathbf{y}^*$ is not the best response of party 1, this implies that there must be another solution $\mathbf{y}^{fix} \neq \mathbf{y}^*$ to this fixed point equation. In the following we will rule out this possibility, by showing that, under the conditions of Proposition 4, $\mathbf{y}^{A*}$ is a contraction mapping and therefore has one and only one fixed point.

Consider a metric space of earnings functions equipped with the *sup* metric, i.e. for two earnings functions $\mathbf{y}_a$ and $\mathbf{y}_b$,

$$d\left(\mathbf{y}_a, \mathbf{y}_b\right) := \sup_{\omega\in\Omega} \mid \mathbf{y}_a(\omega) - \mathbf{y}_b(\omega) \mid \;.$$

To establish that $\mathbf{y}^{A*}(\cdot)$ is a contraction mapping, we need to show that, for any pair $(\mathbf{y}_a, \mathbf{y}_b)$,

$$d\left(\mathbf{y}^{A*}(A(\mathbf{y}_a)), \mathbf{y}^{A*}(A(\mathbf{y}_b))\right) \leq \delta \; d\left(\mathbf{y}_a, \mathbf{y}_b\right) \;, \tag{78}$$

for some $\delta \in (0, 1)$.

Remember that the analysis proceeds under the assumption that $\alpha(\omega) \in [\frac{1}{2} - \bar{\alpha}, \frac{1}{2} + \bar{\alpha}]$ and $\beta(\omega) \leq \bar{\beta}$, for all ω. In the following, we show that an appropriate choice of $\bar{\alpha}$ and $\bar{\beta}$ ensures that, for any ω, $\mid \mathbf{y}^{A*}(\omega, A(\mathbf{y}_a)) - \mathbf{y}^{A*}(\omega, A(\mathbf{y}_b)) \mid$ becomes arbitrarily small. Note that

$$
\begin{aligned}
&\mid \mathbf{y}^{A*}(\omega, A(\mathbf{y}_a)) - \mathbf{y}^{A*}(\omega, A(\mathbf{y}_b)) \mid \\
&= \omega^{1+e} \mid \left(1 + \tfrac{1-F(\omega)}{f(\omega)\,\omega}\,(1-\Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*))\,\tfrac{1}{e}\right)^{-e} \\
&\qquad\qquad\qquad - \left(1 + \tfrac{1-F(\omega)}{f(\omega)\,\omega}\,(1-\Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*))\,\tfrac{1}{e}\right)^{-e} \mid \ .
\end{aligned}
$$

Moreover, by continuity,

$$
\Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*) \to \Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*)
$$

implies

$$
\begin{aligned}
&\mid \left(1 + \tfrac{1-F(\omega)}{f(\omega)\,\omega}\,(1-\Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*))\,\tfrac{1}{e}\right)^{-e} \\
&\quad - \left(1 + \tfrac{1-F(\omega)}{f(\omega)\,\omega}\,(1-\Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*))\,\tfrac{1}{e}\right)^{-e} \mid \ \to \ 0\ .
\end{aligned}
$$

Thus, it suffices to show that $\Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*)$ is arbitrarily close to $\Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*)$ for an appropriate choice of $\bar{\alpha}$ and $\bar{\beta}$.

Let $\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) = u(\mathbf{y}_a, \omega) - u(\mathbf{y}^*, \omega)$. Also let

$$
\overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*) = \max_{\omega \in \Omega}\ \Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega)\ ,
$$

and

$$
\underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*) = \min_{\omega \in \Omega}\ \Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega)\ .
$$

It is without loss of generality to assume that $\mathbf{y}_a$ is a Pareto-efficient earnings function which implies that

$$
\overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*) > 0 > \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)\ .
$$

Using the notation introduced in Section D.2 above, we can write

$$
\Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*) \;\;=\;\; \lambda^1(\mathbf{y}_a, \mathbf{y}^*)\, \mathcal{G}^1_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*) + \lambda^2(\mathbf{y}_a, \mathbf{y}^*))\, \mathcal{G}^2_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*)\ , \tag{79}
$$

where

$$
\mathcal{G}^1_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*) = \frac{\int_{\omega}^{\overline{\omega}} B(\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \mid \omega)\, d\omega}{\int_{\underline{\omega}}^{\overline{\omega}} B(\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \mid \omega)\, d\omega} = \frac{\int_{\omega}^{\overline{\omega}} \{\alpha(\omega) + \beta(\omega)\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega)\} d\omega}{\int_{\underline{\omega}}^{\overline{\omega}} \{\alpha(\omega) + \beta(\omega)\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega)\} d\omega}
$$

and

$$
\begin{aligned}
\mathcal{G}_W^2(\omega \mid \mathbf{y}_a, \mathbf{y}^*) &= \frac{\int_\omega^{\overline{\omega}} (1 - B(\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \mid \omega))\, d\omega}{\int_{\underline{\omega}}^{\overline{\omega}} (1 - B(\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \mid \omega))\, d\omega} \\
&= \frac{\int_\omega^{\overline{\omega}} \{1 - \alpha(\omega) - \beta(\omega)\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega)\} d\omega}{\int_{\underline{\omega}}^{\overline{\omega}} \{1 - \alpha(\omega) - \beta(\omega)\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega)\} d\omega}
\end{aligned}
$$

and, for any pair of earnings functions $(\mathbf{y}^1, \mathbf{y}^2)$

$$
\begin{aligned}
\lambda^1(\mathbf{y}^1, \mathbf{y}^2) &:= \frac{1}{1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^2)} \frac{\bar{g}_W^1(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)}{\bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)} \\
\lambda^2(\mathbf{y}^1, \mathbf{y}^2) &:= \frac{\Pi^1(\mathbf{y}^1, \mathbf{y}^2)}{1 + \Pi^1(\mathbf{y}^1, \mathbf{y}^2)} \frac{\bar{g}_W^2(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)}{\bar{\gamma}^1(\underline{\omega} \mid \mathbf{y}^1, \mathbf{y}^2)}
\end{aligned}
$$

so that

$$
\lambda^1(\mathbf{y}^1, \mathbf{y}^2) + \lambda^2(\mathbf{y}^1, \mathbf{y}^2) = 1.
$$

The assumptions that $\alpha(\omega) \in [\frac{1}{2} - \bar{\alpha}, \frac{1}{2} + \bar{\alpha}]$ and $\beta(\omega) \leq \bar{\beta}$, for all ω, imply that, for all ω,

$$
\begin{aligned}
\tfrac{1}{2} + \bar{\alpha} + \bar{\beta}\, \overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*) &\geq \alpha(\omega) + \beta(\omega)\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \\
&\geq \tfrac{1}{2} - \bar{\alpha} + \bar{\beta}\, \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)\ ,
\end{aligned}
$$

and

$$
\begin{aligned}
\tfrac{1}{2} + \bar{\alpha} - \bar{\beta}\, \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*) &\geq 1 - \alpha(\omega) - \beta(\omega)\Delta u(\mathbf{y}_a, \mathbf{y}^*, \omega) \\
&\geq \tfrac{1}{2} - \bar{\alpha} - \bar{\beta}\, \overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)\ .
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\frac{(1 - F(\omega))(\frac{1}{2} + \bar{\alpha} + \bar{\beta}\, \overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*))}{\frac{1}{2} - \bar{\alpha} + \bar{\beta}\, \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)} &\geq \mathcal{G}_W^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*) \\
&\geq \frac{(1 - F(\omega))(\frac{1}{2} - \bar{\alpha} + \bar{\beta}\, \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*))}{\frac{1}{2} + \bar{\alpha} + \bar{\beta}\, \overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)}\ ,
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{(1 - F(\omega))(\frac{1}{2} + \bar{\alpha} - \bar{\beta}\, \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*))}{\frac{1}{2} - \bar{\alpha} - \bar{\beta}\, \overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)} &\geq \mathcal{G}_W^2(\omega \mid \mathbf{y}_a, \mathbf{y}^*) \\
&\geq \frac{(1 - F(\omega))(\frac{1}{2} - \bar{\alpha} - \bar{\beta}\, \overline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*))}{\frac{1}{2} - \bar{\alpha} - \bar{\beta}\, \underline{\Delta u}(\mathbf{y}_a, \mathbf{y}^*)}\ ,
\end{aligned}
$$

which implies that

$$\lim_{\bar{\alpha},\bar{\beta}\to 0} \mathcal{G}^1_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*) = \lim_{\bar{\alpha},\bar{\beta}\to 0} \mathcal{G}^2_W(\omega \mid \mathbf{y}_a, \mathbf{y}^*) = 1 - F(\omega) \ . \tag{80}$$

Hence, for all ω,

$$\lim_{\bar{\alpha},\bar{\beta}\to 0} \Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*) = 1 - F(\omega) \ , \tag{81}$$

and, by a symmetric argument,

$$\lim_{\bar{\alpha},\bar{\beta}\to 0} \Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*) = 1 - F(\omega) \ , \tag{82}$$

for all ω. Equations (79)-(82) imply that

$$\lim\nolimits_{\bar{\alpha},\bar{\beta}\to 0} \Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*) - \Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*) = 0 \ ,$$

for all ω. Thus, for any pair of functions $(\mathbf{y}_a, \mathbf{y}_b)$, $\bar{\alpha}, \bar{\beta} \to 0$ implies

$$\Gamma^1(\omega \mid \mathbf{y}_a, \mathbf{y}^*) \to \Gamma^1(\omega \mid \mathbf{y}_b, \mathbf{y}^*)$$

Thus, for $\bar{\alpha}, \bar{\beta}$ sufficiently close to zero, $\mathbf{y}^{A*}$ is a contraction mapping.

E Case Study: Asymmetric Demobilization in the era of Angela Merkel

We use the theoretical framework in the body of the text for an analysis of German politics between 2005 and 2017. Merkel became the leader of the Christian democrats (CDU, center-right) in 2000 and successfully ran for the chancellory in 2005, 2009, 2013 and 2017. Her main competitor was the Socialdemocratic Party (SPD, center-left).

Policy space $\mathcal{P}$. We consider a policy space of linear income taxes and let the variable ω index an individual's position in the income distribution. One can interpret a redistributive policy platform τ^j of party j either narrowly or broadly. First, as in Meltzer and Richard (1981), the tax rate τ^j can be interpreted as the "size of the government" or of the welfare state, which includes income taxes and monetary transfers but also social insurance, public education, etc. Second, τ^j can be interpreted more broadly as an index of the party's position on the "left-right" axis, a higher value of τ^j corresponding to a more leftist platform. In addition to the previous variables, this broad index would also account for, e.g., the party's stance on the minimum wage, gay marriage, or nuclear energy. These policies also played an important role in Merkel's campaign strategy which is discussed in more detail below.

Party preferences and policy preferences. By convention and without loss of generality, we interpret smaller values of ε as more "conservative" preferences, and larger values of ε as more "liberal" preferences. Thus, we identify party 1 with the CDU, and party 2 with the SPD: given identical policy platforms $\tau^1 = \tau^2$, party 1 (resp., party 2) is overly supported by voters with conservative preferences $\varepsilon < 0$ (resp., liberal preferences $\varepsilon > 0$). In practice, these party preferences may be shaped by party identities, e.g., roots in the worker's movement or the Christian churches, or the cultural milieu from which parties recruit their members. Party preferences may also reflect fixed party positions that are not adjusted in the political campaign. For instance, a salient issue in German politics in the Merkel era was whether families should be supported by direct transfers, as advocated by the CDU, or by publicly-provided childcare, as preferred by the SPD.

E.1 The status quo ante

We assume that potential voters of the SPD have stronger preferences for redistributive policies than potential voters of the CDU.[46] Therefore, we suppose that the electorate of party 1 (CDU) is over-represented among the rich (i.e., high ω), while party 2 (SPD) is over-represented among the poor (i.e., low ω). Thus, for the analysis that follows, we take as a starting point policies (τ^1, τ^2) such that:

(i) The function $B(u(\tau^1, \omega) - u(\tau^2, \omega) \mid \omega)$ is increasing in ω.

[46] For instance, the election outcomes in 2009 and 2013 show the following pattern: the vote shares of SPD and CDU among public servants and white collar workers were, by and large, in line with the parties' overall vote shares, see Jung et al. (2010, 2015). Hence, in absolute numbers, the CDU got more votes from these groups than the SPD. In relative terms, the CDU was stronger among the self-employed and the SPD among workers. The CDU voters also tend to be older and more formally educated. Thus, SPD voters benefit to a larger extent from redistributive policies.

Status quo policy τ^1. The 2005 election was an early election called by Merkel's predecessor from the SPD, Gerhard Schröder. After an adoption of controversial labor market reforms, the SPD had lost various state elections. When the 2005 election was called, the CDU had a strong 21 percent lead over the SPD in opinion polls, and was expected to become by far the strongest party. The CDU decided to run on a pro-market platform, emphasizing the need for deregulation and lower taxes. Over the course of the election campaign, however, the SPD recovered and in the end the CDU won only by a tiny margin of victory: it was only 1 percent ahead of the SPD. Notice that this outcome is consistent with the implications of Proposition 2 discussed in the main text: because it was the clear front runner, the CDU would have maximized its chances of victory by focusing on demobilizing the opposition, rather than mobilizing its own electorate – i.e., by running on a redistributive platform more favorable to the SPD's core supporters rather than taking a fiscally conservative stance to benefit its own base. Therefore, the CDU's status quo policy τ^1 ahead of the 2009 election was "too far" to the right, i.e.:

(ii) $\tau^1 < \tau^{1*}$, where τ^{1*} denotes party 1's best response to τ^2, i.e. to the policy prosed by party 2.

One can show that this condition is satisfied if τ^1 is small enough, or if it is below and close enough to the best response $\tau^{1*}(\tau^2)$ to party 2's policy.

Odds of winning $\bar{\pi}^1(\tau^1, \tau^2)$. In 2009, the CDU was clearly headed for reelection. The polls estimated that the CDU would get 35 percent of the votes, against 25 percent for the SPD and less than 15 percent for all the other parties. A week before election day, Merkel traded at 1.08 (1/12) in the "next Chancellor" market on Betfair – i.e., party 2 was given a chance of winning $1-\bar{\pi}^1(\tau^1, \tau^2)$ of 8 percent. This motivates the following assumption:

(iii) Party 1 is the likely winner of the election, i.e., the probability that party 1 wins is $\bar{\pi}^1(\tau^1, \tau^2) > 1/2$.

E.2 CDU's *Asymmetric Demobilization* strategy

After the federal election in 2005 the CDU adopted the strategy of *asymmetric demobilization*. What defines this strategy is an avoidance of controversial positions or even an adoption of the rival's position in an attempt to lower the turnout of its potential voters. This strategy was successful and continued during the 2013 and 2017 campaigns. The clearest illustration is given by the 2013 official CDU program, which included many policies traditionally advocated by the SPD including the creation of a minimum wage, rent control in tight city areas, a financial transactions tax, a floor on pensions, or tax credits for families and single mothers. In addition, in 2011 Merkel had announced a plan to shut down all nuclear reactors by 2022, a measure traditionally favored by the left-leaning Green party. In 2017, the CDU avoided controversial topics on economic and social policy, and Merkel initiated a parliamentary decision on the question of gay marriage that her SPD opponent had made a central campaign issue – at the cost of alienating her own base. Narrative records of this strategy abound in the national and international press. While such journalistic documentation of the CDU's asymmetric demobilization strategy is overwhelming, it is also apparent in systematic quantitative analyses of party positions by political scientists, as we now describe.

Data sources. The *Manifesto Project*, see Volkens et al. (2018), provides a quantitative text analysis of party manifestos. The text is split into quasi-sentences, units of text that contain one political statement. Quasi-sentences are then assigned to categories such as *Free Market Economy*, *Market Regulation*, *Welfare State Expansion* or *Welfare State Limitation*.[47] Following our discussion of the policy space $\mathcal{P}$ above, we focus on two such indices. See Volkens et al. (2018) for a detailed description of the data set and the methodology.[48]

First, we use the *Welfare State* index, which corresponds to our narrower interpretation of a policy platform τ^j. This index aggregates all of the favourable mentions of the "need to introduce, maintain or expand any public social service or social security scheme ... for example: government funding of health care, child care, elder care and pensions, social housing"; and of "equality: concept of social justice and the need for fair treatment of all people". Second, we use the *Right-Left* index, which corresponds to our broader interpretation of a policy platform τ^j. This index positions a party

[47] The overall analysis is not restricted to economic policy dimensions, but also contains categories for positions on foreign policy, migration, political corruption and others.

[48] An alternative data source is the *Chapel Hill Expert Survey*, see Polk et al. (2017); Bakker et al. (2015). It also provides an analysis of party positions in various dimensions, including a left-versus-right positioning for economic policy issues. It differs from the *Manifesto Project* in that it is based on a survey of expert opinions as opposed to the text of party manifestos. This data set does not yet cover the most recent federal election in Germany in 2017. For the elections between 2002 and 2013 it shows the same pattern as the Party Manifesto data.

manifesto on a one-dimensional policy space by taking the share of quasi-sentences that are indicative of rightist positions (e.g., favorable mentions of military, freedom and human rights, constitutionalism, political authority, free market economy, incentives, economic orthodoxy, welfare state limitation, national way of life, traditional morality, law and order, civic mindedness) and substracts the share of quasi-sentences that are indicative of leftist positions (e.g., favorable mentions of anti-imperialism, peace, internationalism, democracy, economic planning, protectionism, nationalization, welfare state expansion, education expansion, labor groups).

Results. The table below describes how the positions of the CDU and the SPD evolved according to the two indices from the Manifesto Project for the federal elections since 2002. Both indices are normalized to 1 for the SPD in 2002. Larger (resp., smaller) values of the "Welfare State" index mean that the party's manifesto puts stronger (resp., weaker) emphasis on the expansion of the welfare state. Larger (resp., smaller) values of the "Right-Left" index mean that the party's manifesto is located further to the right (resp., left). This table shows clearly that the party positions diverged between the 2002 and 2005 elections. While the SPD reinforced its emphasis on welfare state expansion (the Welfare State index increased from 1 to 1.49) and overall moved further to the left (the Right-Left index decreased from 1 to -0.53), instead the CDU advocated a smaller welfare state (the corresponding index decreased from 0.85 to 0.58) and overall moved further to the right (the corresponding index increased from 5.06 to 6.25). From 2009 onwards, instead, the CDU moved to the left according to both indices: the welfare state index increased continuously from 0.58 in 2005 to 1.08 in 2017, and the right-left index decreased from 6.26 in 2005 to 0.67 in 2017. The two parties moved in parallel: when the SPD moved to the left, so did the CDU. Notice that according to both indices, the CDU was substantially more left-leaning in 2017 than the SPD was in 2002.

	Welfare State		**Right-Left**	
	SPD	**CDU**	**SPD**	**CDU**
2002	1	0.85	1	5.06
2005	1.49	0.58	-0.53	6.25
2009	1.76	0.74	-4.46	2.13
2013	2.14	0.83	-5.75	0.63
2017	1.83	1.08	-5.23	0.67

E.3 Analysis of turnout and election outcomes

In this section we analyze the impact of the CDU's asymmetric demobilization strategy on turnout rates and election results. Our goal is to confront the comparative statics predictions of our model with the outcomes of German elections from 2009 to 2017.

Comparative statics predictions. A major insight of our theoretical analysis is that a party that is leading in the polls has an incentive to adopt a platform that is appealing to the core supporters of its competitor. Thereby the potential voters of the competitor are demobilized. Proposition 6 below is an adaptation of this finding to the German context described above. For convenience, we invoke additional functional form assumptions.

Assumption 4.

a) *Voting costs are linear:* $\lambda = 1$.

b) *Party biases* ε *follow a uniform distribution at each income level: for any* $\omega \in \Omega$ *there exist* $\alpha(\omega) \in (0,1)$ *and* $\beta(\omega) > 0$ *such that* $B(x \mid \omega) = \alpha(\omega) + \beta(\omega)\, x$. *Moreover, the distributions have a wide support and are close to symmetric:*

   There exists $\bar{\beta}$ *close to zero so that* $0 < \beta(\omega) \leq \bar{\beta}$, *for all* ω.

   There exists $\bar{\alpha}$ *close to zero so that,* $\alpha(\omega) \in [\frac{1}{2} - \bar{\alpha}, \frac{1}{2} + \bar{\alpha}]$.

c) *The random variables* η^1 *and* η^2*, defined in Assumption 1, are uniformly distributed on an interval* $[1-\delta, 1+\delta]$ *with* $\delta > 0$.

d) *Party 1 is more right-leaning than party 2:* $\tau^1 \leq \tau^2$ *implies that* $B(u(\tau^1,\omega) - u(\tau^2,\omega) \mid \omega)$ *is increasing in* ω*, and that* $B(u(\tau^1,\underline{\omega}) - u(\tau^2,\underline{\omega}) \mid \underline{\omega}) < \frac{1}{2}$*, i.e., among the very poor there is more support for party 2 than for party 1.*

As the proof below makes clear, Assumption 4 is sufficient, but by no means necessary, to obtain our next result.

Proposition 6. *Suppose that Assumption 4 holds. Consider a one-dimensional policy space and suppose that policy preferences satisfy the single crossing property and are concave. Suppose that* $\mathcal{P} = [\underline{\tau}, \overline{\tau}]$ *is a set of Pareto-efficient tax systems. Consider* $\tau^1, \tau^2 \in (\underline{\tau}, \overline{\tau})$ *with* $\tau^1 < \tau^2$, $\bar{\pi}^1(\tau^1, \tau^2) > \frac{1}{2}$*, and* $\tau^1 < \mathit{argmax}_\tau\ \bar{\pi}^1(\tau, \tau^2)$. *Then a marginal increase of party 1's tax rate has the following implications:*

1. *Party 1's probability of winning increases.*

2. *Party 1's expected vote share increases.*

3. *Overall turnout decreases.*

4. *The demobilization is asymmetric: σ^{1*}/σ^{2*} increases.*

A proof of Proposition 6 can be found in Section E.4 below. In the remainder of this section we confront the theoretical predictions in Proposition 6 with the election outcomes in Germany.

Empirical election outcomes. As we discussed above, the strategy of asymmetric demobilization was adopted in the 2009, 2013 and 2017 elections in response to the 2005 experience, in which Merkel learned that running on a platform that appeals to the core voters of her own party could jeopardize an almost sure victory. This strategy paid off: despite a similar lead in the polls in 2009, her margin of victory over the SPD increased from 1 percent in 2005 to more than 10 percent. Overall turnout (70.8 percent) went down by 6.9 percentage points compared to the 2005 election, and was at an all-time low. Crucially, turnout was lower among potential SPD voters than among potential CDU voters: 52 percent of the potential SPD voters indeed voted for the SPD, whereas 62 percent of the potential CDU voters voted for the CDU, see Jung et al. (2010); Forschungsgruppe Wahlen (2013b,a).[49]

In 2013, the CDU moved further left in parallel with the SPD. The election outcome was again a great success for the CDU: it gained 41.5 percent of the votes, was close to an absolute majority in parliament, and was 16 percent ahead of the SPD. Again, mobilization was asymmetric: turnout was 51 percent among the potential SPD voters and 69 percent among the potential CDU voters, see Forschungsgruppe Wahlen (2015). In 2017, the rise of a right-wing populist party implied large losses for the CDU relative to the 2013 election. The SPD also lost, however, and so the CDU stayed more than 12 percent ahead of the SPD. Moreover, it defended its dominant

[49]These numbers are obtained in the following way. The research institute Forschungsgruppe Wahlen runs a monthly survey with a representative sample of voters. The study is known as the *Politbarometer*. Shortly before an election it includes questions on prospective voting behavior. A person who plans to vote SPD or who includes the SPD in the set of conceivable parties is considered a potential SPD voter. Likewise for the CDU. The ratio of actual to potential voters then gives the numbers of 62 percent for the CDU and of 52 percent of the SPD. As a caveat, note that the Politbarometer is not a panel; i.e., it is not tracking the actual voting behavior of the participants in the survey.

position in the German party system: as the only party with more than 30 percent of the votes, every realistic option for government formation had the CDU in the leading role with Merkel as the chancellor. Again, turnout of potential CDU voters (60 percent) was much higher than the turnout of the potential SPD voters (44 percent), see Forschungsgruppe Wahlen (2018). Overall turnout was slightly higher in 2013 and somewhat higher in 2017 than it was in 2009, at 76 percent, but still lower than any turnout ratio observed prior to 2009.

These outcomes are all consistent with our theoretical comparative statics predictions of Proposition 6.

E.4 Proof of Proposition 6

Party 1's probability of winning. If policy preferences are concave, and if part b) of Assumption 4 is satisfied, then one can easily show that the function

$$\Pi^1(\cdot,\tau^2):\tau^1\mapsto\Pi^1(\tau^1,\tau^2):=\frac{W^1(\tau^1,\tau^2)}{W^2(\tau^1,\tau^2)}=\frac{\sigma^{1*}(\tau^1,\tau^2)\mathbf{B}^1(\tau^1,\tau^2)}{\sigma^{2*}(\tau^1,\tau^2)\mathbf{B}^2(\tau^1,\tau^2)}$$

is globally concave for every value of τ^2. Moreover, recall that party 1's probability of winning the election is an increasing function of $\Pi^1(\tau^1,\tau^2)$. Thus, for every value of τ^2, there is a unique best response and moving closer to that best response unambiguously increases the winning probability.

Party 1's expected vote share. The total number of votes for party j is equal to $\tilde{V}^j(\tau^1,\tau^2)=\sigma^{j*}(\tau^1,\tau^2)\tilde{\mathbf{B}}^j(\tau^1,\tau^2)$. Hence party 1's expected vote share is equal to

$$\begin{aligned}\mathbb{E}_\eta\left[\frac{\sigma^{1*}(\tau^1,\tau^2)\tilde{\mathbf{B}}^1(\tau^1,\tau^2)}{\sigma^{1*}(\tau^1,\tau^2)\tilde{\mathbf{B}}^1(\tau^1,\tau^2)+\sigma^{2*}(\tau^1,\tau^2)\tilde{\mathbf{B}}^2(\tau^1,\tau^2)}\right] &= \mathbb{E}_\eta\left[\left(1+\frac{\sigma^{2*}(\tau^1,\tau^2)}{\sigma^{1*}(\tau^1,\tau^2)}\frac{\eta^2\mathbf{B}^2(\tau^1,\tau^2)}{\eta^1\mathbf{B}^1(\tau^1,\tau^2)}\right)^{-1}\right]\\ &= \mathbb{E}_\eta\left[\left(1+\frac{W^2(\tau^1,\tau^2)}{W^1(\tau^1,\tau^2)}\frac{\eta^2}{\eta^1}\right)^{-1}\right]\\ &= \mathbb{E}_\eta\left[\left(1+\frac{1}{\Pi^1(\tau^1,\tau^2)}\frac{\eta^2}{\eta^1}\right)^{-1}\right],\end{aligned}$$

where expectations are taken with respect to the distribution of $\frac{\eta^2}{\eta^1}$. With $\frac{\partial\Pi^1(\tau^1,\tau^2)}{\partial\tau^1}>0$, this expression increases in τ^1.

Overall turnout. Expected overall turnout is equal to

$$\begin{aligned}
\Sigma &:= \mathbb{E}_\eta\left[\frac{\sigma^{1*}(\tau^1,\tau^2)\tilde{\mathbf{B}}^1(\tau^1,\tau^2)+\sigma^{2*}(\tau^1,\tau^2)\tilde{\mathbf{B}}^2(\tau^1,\tau^2)}{\tilde{\mathbf{B}}^1(\tau^1,\tau^2)+\tilde{\mathbf{B}}^2(\tau^1,\tau^2)}\right] \\
&= \frac{\sigma^{1*}(\tau^1,\tau^2)\mathbf{B}^1(\tau^1,\tau^2)+\sigma^{2*}(\tau^1,\tau^2)\mathbf{B}^2(\tau^1,\tau^2)}{\mathbb{E}\left[\bar{q}(\omega)\right]} \\
&= \frac{\sigma^{2*}(\tau^1,\tau^2)\mathbf{B}^2(\tau^1,\tau^2)}{\mathbb{E}\left[\bar{q}(\omega)\right]}\left(1+\frac{W^1(\tau^1,\tau^2)}{W^2(\tau^1,\tau^2)}\right) \\
&= \frac{f_\eta\left(\Pi^1(\tau^1,\tau^2)\right)\Pi^1(\tau^1,\tau^2)W^2(p^1,p^2)}{\kappa\mathbb{E}\left[\bar{q}(\omega)\right]}\left(1+\Pi^1(\tau^1,\tau^2)\right),
\end{aligned}$$

where f_η is the density of the random variable $\frac{\eta^2}{\eta^1}$ and the last equality follows from (28). Denoting by g_η the density of the random variable $\frac{\eta^1}{\eta^2}$, a change of variables implies that $f_\eta(x)\,dx = -g_\eta(\frac{1}{x})d(\frac{1}{x})$, that is, $f_\eta(x) = \frac{1}{x^2}g_\eta(\frac{1}{x})$. Therefore, we can rewrite the previous expression as

$$\Sigma := \frac{W^2(p^1,p^2)}{\kappa\mathbb{E}\left[\bar{q}(\omega)\right]}g_\eta\left(\frac{1}{\Pi^1(\tau^1,\tau^2)}\right)\left(1+\frac{1}{\Pi^1(\tau^1,\tau^2)}\right).$$

We now show that $\partial\Sigma/\partial\tau^1 < 0$. Since all the terms in the expression for Σ are positive, and since $\frac{\partial\Pi^1(\tau^1,\tau^2)}{\partial\tau^1} > 0$ as shown above, the result follows if both $g_\eta\left(1/\Pi^1(\tau^1,\tau^2)\right)$ and $W^2(\tau^1,\tau^2)$ are decreasing in τ^1.

We first show that $g_\eta\left(1/\Pi^1(\tau^1,\tau^2)\right)$ is decreasing in τ^1. Part (c) of Assumption 4 can be shown to imply that[50]

$$g_\eta(x) = \frac{1}{(\bar{\eta}-\underline{\eta})^2}\int \eta^2\mathbb{I}_{\{\eta^2\in[\underline{\eta},\bar{\eta}]\}}\mathbb{I}_{\{\eta^2\in[\underline{\eta}/x,\bar{\eta}/x]\}}d\eta^2,$$

where $\underline{\eta} := 1-\delta$, $\bar{\eta} := 1+\delta$ and $\mathbb{I}$ is the indicator function. Note that if $x > 1$, we have $\underline{\eta}/x < \underline{\eta}$ and $\bar{\eta}/x < \bar{\eta}$, so that $[\underline{\eta}/x,\bar{\eta}/x]\cap[\underline{\eta},\bar{\eta}] = [\underline{\eta},\bar{\eta}/x]$. Conversely, if $x < 1$,

[50] To see this, note that

$$\begin{aligned}
G_\eta(x) &= \text{prob}(\tfrac{\eta^1}{\eta^2}\le x) \\
&= \textstyle\int \text{prob}(\eta^1 \le x\eta^2 \mid \eta^2)\mu^2(\eta^2)d\eta^2 \\
&= \textstyle\int \frac{x\eta^2-\underline{\eta}}{\bar{\eta}-\underline{\eta}}\mathbb{I}_{\{\eta^2\in[\underline{\eta}/x,\bar{\eta}/x]\}}\mu^2(\eta^2)d\eta^2
\end{aligned}$$

where $\mu^2(\eta^2) = \frac{\mathbb{I}_{\{\eta^2\in[\underline{\eta},\bar{\eta}]\}}}{\bar{\eta}-\underline{\eta}}$ is the density of η^2. Computing the derivative with respect to x yields the expression for $g_\eta(x)$ in the text.

we have $[\underline{\eta}/x, \bar{\eta}/x] \cap [\underline{\eta}, \bar{\eta}] = [\underline{\eta}/x, \bar{\eta}]$. Therefore,

$$g_\eta(x) = \frac{1}{(\bar{\eta}-\underline{\eta})^2} \begin{cases} \int_{\underline{\eta}/x}^{\bar{\eta}} \eta^2 d\eta^2 & \text{if } x \leq 1, \\ \int_{\underline{\eta}}^{\bar{\eta}/x} \eta^2 d\eta^2 & \text{if } x > 1. \end{cases}$$

We easily obtain that, for any $x \leq 1$,

$$g_\eta(x) = \frac{\bar{\eta}^2 - \underline{\eta}^2/x^2}{2(\bar{\eta}-\underline{\eta})^2},$$

which is increasing in x. It is, moreover, straightforward to verify that $G_\eta(1) = 1/2$. Now recall that party 1's probability of winning increases in response to the deviation considered in this proof, so that $\partial[1/\Pi^1(\tau^1,\tau^2)]/\partial\tau^1 < 0$. But party 2's probability of winning, which is the probability of the event $\frac{\sigma^2}{\sigma^1}\frac{\mathbf{B}^2(p^1,p^2)}{\mathbf{B}^1(p^1,p^2)} \geq \frac{\eta^1}{\eta^2}$, is equal to $\bar{\pi}^2(\tau^1,\tau^2) = G_\eta\left(1/\Pi^1(\tau^1,\tau^2)\right)$. But $\bar{\pi}^2(\tau^1,\tau^2) = 1 - \bar{\pi}^1(\tau^1,\tau^2) < 1/2$ by assumption. Thus, we must have $1/\Pi^1(\tau^1,\tau^2) < 1$. Therefore, $g_\eta(1/\Pi^1(\tau^1,\tau^2))$ is locally decreasing in τ^1, and hence $\partial g_\eta(1/\Pi^1(\tau^1,\tau^2))/\partial\tau^1 < 0$ in response to party 1's deviation.

We now show that $W^2(\tau^1,\tau^2)$ is decreasing. First, recall that $\Pi^1(\tau^1,\tau^2) = \frac{W^1(\tau^1,\tau^2)}{W^2(\tau^1,\tau^2)}$, or, equivalently,

$$H_s(W^1(\tau^1,\tau^2)) - H_s(W^2(\tau^1,\tau^2))$$

increases in τ^1. Further note that

$$\frac{\partial W^1(\tau^1,\tau^2)}{\partial \tau^1} = \mathbb{E}\left[B\left(u\left(\tau^1,\omega\right) - u\left(\tau^2,\omega\right) \mid \omega\right) u_1(\tau^1,\omega)\right]$$

and

$$\frac{\partial W^2(\tau^1,\tau^2)}{\partial \tau^1} = -\mathbb{E}\left[\left(1 - B\left(u\left(\tau^1,\omega\right) - u\left(\tau^2,\omega\right) \mid \omega\right)\right) u_1(\tau^1,\omega)\right]$$

Thus, $\Pi^1(\tau^1,\tau^2)$ increasing in τ^1 is equivalent to

$$\mathbb{E}\left[\left\{B(\Delta u(\omega) \mid \omega) + \frac{h_s(W^2)}{h_s(W^1)}(1 - B(\Delta u(\omega) \mid \omega))\right\} u_1(\tau^1,\omega)\right] > 0\,,$$

or, once more, equivalently,

$$\mathbb{E}\left[\left\{(1-\lambda')B(\Delta u(\omega) \mid \omega) + \lambda'(1-B(\Delta u(\omega) \mid \omega))\right\} u_1(\tau^1,\omega)\right] > 0\ , \tag{83}$$

where $\Delta u(\omega)$ is a shorthand for $u(\tau^1,\omega) - u(\tau^2,\omega)$, and λ' for $\frac{h_s(W^2)}{h_s(W^1)}\left(1+\frac{h_s(W^2)}{h_s(W^1)}\right)^{-1}$. By the single crossing property $u_1(\tau^1,\omega)$ is decreasing in ω. Also, since τ^1 is, by assumption, an interior policy, $u_1(\tau^1,\omega)$ is positive for small values of ω and negative for large values of ω.

We now proceed by contradiction. Suppose, contrary to what we seek to show, that $\frac{\partial W^2(\tau^1,\tau^2)}{\partial \tau^1} \geq 0$, or, equivalently, that

$$\mathbb{E}\left[(1-B(\Delta u(\omega) \mid \omega))\ \ u_1(\tau^1,\omega)\right] \leq 0\ . \tag{84}$$

Now compare the weighting functions

$$\gamma'(\omega) := (1-\lambda')B(\Delta u(\omega) \mid \omega) + \lambda'(1-B(\Delta u(\omega) \mid \omega)$$

and

$$\gamma(\omega) := 1 - B(\Delta u(\omega) \mid \omega)\ .$$

Since by hypothesis $B(\Delta u(\omega) \mid \omega)$ is increasing in ω, γ' puts less weight on low values of ω, corresponding to positive values of $u_1(\tau^1,\omega)$, and more weight on high values of ω, where $u_1(\tau^1,\omega)$ takes negative values. Therefore, (84) implies that

$$\mathbb{E}\left[\left\{(1-\lambda')B(\Delta u(\omega) \mid \omega) + \lambda'(1-B(\Delta u(\omega) \mid \omega))\right\} u_1(\tau^1,\omega)\right] < 0\ ,$$

a contradiction (83). Thus, the assumption that $\frac{\partial W^2(\tau^1,\tau^2)}{\partial \tau^1} \geq 0$ has led to a contradiction and must be false.

Relative turnout. Finally, we show that the relative turnout σ^{1*}/σ^{2*} increases. We have

$$\frac{\sigma^{1*}(\tau^1,\tau^2)}{\sigma^{2*}(\tau^1,\tau^2)} = \Pi^1(\tau^1,\tau^2)\frac{\mathbb{E}\left[\bar{q}\,(\omega)\right] - \mathbf{B}^1(\tau^1,\tau^2)}{\mathbf{B}^1(\tau^1,\tau^2)}.$$

In response to party 2's deviation, we have

$$\frac{\partial \mathbf{B}^1(\tau^1,\tau^2)}{\partial \tau^1} = \mathbb{E}\left[\bar{q}(\omega)\, b\left(u\left(\tau^1,\omega\right) - u\left(\tau^2,\omega\right) \mid \omega\right) \frac{\partial u\left(\tau^1,\omega\right)}{\partial \tau^1}\right]$$

$$= \mathbb{E}\left[\bar{q}(\omega)\, b\left(u\left(\tau^1,\omega\right) - u\left(\tau^2,\omega\right) \mid \omega\right) \left(-y^1(\omega) + \mathbb{E}y^1 - \frac{\tau^1}{1-\tau^1} e\mathbb{E}\left[y^1(\omega)\right]\right)\right].$$

Part b) of Assumption 4 implies that this derivative is close to zero. Hence, the result follows immediately from the fact that $\Pi^1(\tau^1,\tau^2)$ increases in τ^1.

F Alternative Settings

F.1 A model that includes ethical voters who always vote

We now assume that the base of each party is split into three groups: a group that always votes, a group that always abstains, and a group of voters whose voting decision follows from a rule-utilitarian calculation. We denote by $\tilde{q}^{jv}(\omega)$ the fraction of definite voters among the type ω supporters of party j, by $\tilde{q}^{ja}(\omega)$ the fraction of definite abstainers and by $\tilde{q}^{ju}(\omega)$ the fraction of rule-utilitarian or ethical supporters, with $\tilde{q}^{jv}(\omega) + \tilde{q}^{ja}(\omega) + \tilde{q}^{ju}(\omega) = 1$. We assume that these are random quantities both from the perspective of parties when choosing platforms and from the perspective of voters when choosing whether or not to vote. We write $\tilde{q}^j = \{\tilde{q}^{jv}(\omega), \tilde{q}^{ja}(\omega), \tilde{q}^{ju}(\omega)\}_{\omega\in\Omega}$ for the collection of random variables that refer to party j. We denote the expected value of the random variable $\tilde{q}^{ju}(\omega)$ by $\bar{q}^{ju}(\omega)$. The total number of votes for party 1 is then a random variable equal to

$$\tilde{V}^1(p^1,p^2,\sigma^1,\tilde{q}^1) = \mathbb{E}[(\tilde{q}^{1v}(\omega) + \sigma^1\ \tilde{q}^{1u}(\omega))B(u(p^1,\omega) - u(p^2,\omega) \mid \omega)]\ .$$

Analogously, the total number of votes for party 2 equals

$$\tilde{V}^2(p^1,p^2,\sigma^2,\tilde{q}^2) = \mathbb{E}[(\tilde{q}^{2v}(\omega) + \sigma^2\ \tilde{q}^{2u}(\omega))(1 - B(u(p^1,\omega)\ - u(p^2,\omega) \mid \omega))]\ .$$

We assume throughout that $\mu \to \infty$, so that the per capital cost of voting is equal to $\kappa\sigma^j$.

Assume furthermore that the random variables $\tilde{q}^1$ and $\tilde{q}^2$ are driven by aggregate shocks that affect the shares of definite and rule-utilitarian voters one the one hand and of definite abstainers on the other so that the following two properties are satisfied. First, the ratio of definite and rule-utilitarian voters is not subject to randomness; i.e., shocks affect the ratio of potential voters to definite abstainers without affecting the internal composition of the set of potential voters. Second, among the supporters of party j, the ratio of definite to rule-utilitarian voters is the same for all types.

Assumption 5. *There is a pair of independent random variables, η_1 and η_2, so that, for all ω,*

$$\tilde{q}^{1v}(\omega) = \bar{q}^{1v}(\omega)\,\eta_1 \quad \textit{and} \quad \tilde{q}^{1u}(\omega) = \bar{q}^{1u}(\omega)\,\eta_1$$

and

$$\tilde{q}^{2v}(\omega) = \bar{q}^{2v}(\omega)\,\eta_2 \quad \textit{and} \quad \tilde{q}^{2u}(\omega) = \bar{q}^{2u}(\omega)\,\eta_2.$$

In addition, there are numbers q^{1v}, q^{1u}, q^{2v} and q^{2u} so that, for all ω

$$\bar{q}^{1v}(\omega) = q^{1v} \quad \textit{and} \quad \bar{q}^{1u}(\omega) = q^{1u}$$

and

$$\bar{q}^{2v}(\omega) = q^{2v} \quad \textit{and} \quad \bar{q}^{2u}(\omega) = q^{2u}\ .$$

Under Assumption 5 the total number of votes for party 1 can be written as

$$\tilde{V}^1(p^1, p^2, \sigma^1, \tilde{q}^1) = \eta^1\ V^1(p^1, p^2, \sigma^1)$$

where $V^1(p^1, p^2, \sigma^1) := m^1(\sigma^1)\ \mathbf{B}^1(p^1, p^2)$ and $m^1(\sigma^1) := q^{1v} + \sigma^1\ q^{1u}$ is a multiplier that determines how party 1's base $\mathbf{B}^1(p^1, p^2)$ is transformed into actual votes. Analogously, the votes for party 2 are given by $\tilde{V}^2(p^1, p^2, \sigma^2, \tilde{q}^2) = \eta^2\ V^2(p^1, p^2, \sigma^2)$, where $V^2(p^1, p^2, \sigma^2) := m^2(\sigma^2)\ \mathbf{B}^2(p^1, p^2)$ and $m^2(\sigma^2) = q^{2v} + \sigma^2\ q^{2u}$. Armed with this notation, we can express the probability that party 1 wins as

$$\pi^1(p^1, p^2, \sigma^1, \sigma^2) = P\left(\frac{V^1(p^1, p^2, \sigma^1)}{V^2(p^1, p^2, \sigma^2)}\right) = P\left(\frac{m^1(\sigma^1)}{m^2(\sigma^2)}\frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)}\right)\ , \tag{85}$$

where P is the *cdf* of the random variable η^2/η^1. Its density function is denoted by ρ. Note that imposing Assumption 5 implies a multiplicative separability between the term

$$R^\sigma(p^1, p^2) = \frac{m^1(\sigma^1(p^1, p^2))}{m^2(\sigma^2(p^1, p^2))}\ , \tag{86}$$

that is shaped by the rule-utilitarian voter's participation thresholds and the ratio of their bases

$$R^{\mathbf{B}}(p^1,p^2)=\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}$$

so that we can write

$$\pi^1(p^1,p^2,\sigma^1,\sigma^2)=P\left(R^{\sigma}(p^1,p^2)\ R^{\mathbf{B}}(p^1,p^2)\right)\ . \tag{87}$$

Turnout. For now, we take the party platforms p^1 and p^2 as given and characterize the parties' equilibrium turnout. We say that the turnout game has an interior equilibrium if $0<\sigma^{1*}(p^1,p^2)<1$ and $0<\sigma^{2*}(p^1,p^2)<1$. If the function P is continuously differentiable then an interior equilibrium is characterized by the first order conditions

$$\pi^1_{\sigma^1}(\cdot)\ W^1-\kappa\ q^{1u}\ \mathbf{B}^1=0\ , \tag{88}$$

and

$$-\ \pi^1_{\sigma^2}(\cdot)\ W^2-\kappa\ q^{2u}\ \mathbf{B}^2=0\ . \tag{89}$$

Using Assumption 5 we can rewrite these conditions as

$$\frac{\rho(\cdot)R^{\sigma}(p^1,p^2)}{q^{1v}+\sigma^1\ q^{1u}}\ W^1-\kappa\ \mathbf{B}^1=0\ , \tag{90}$$

and

$$\frac{\rho(\cdot)R^{\sigma}(p^1,p^2)}{q^{2v}+\sigma^2\ q^{2u}}\ W^2-\kappa\ \mathbf{B}^2=0\ . \tag{91}$$

Equations (90) and (91) imply that

$$R^{\sigma}(p^1,p^2)=\frac{W^1/\kappa\ \mathbf{B}^1}{W^2/\kappa\ \mathbf{B}^2}=\frac{W^1/\mathbf{B}^1}{W^2/\ \mathbf{B}^2}\ , \tag{92}$$

which is the same expression as (29) in the body of the text.

Probability of winning. Suppose first that parties seek to maximize the probability of winning, i.e.,

$$P\left(R^{\sigma}(p^1,p^2)\ \frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right)$$

and party 2 seeks to minimize this expression. As P is a non-decreasing function we can as well assume that party 1 maximizes

$$R^{\sigma}(p^1,p^2)\,\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}$$

or any monotone transformation of it such as, e.g.,

$$\ln\left(R^{\sigma}(p^1,p^2)\right)+\ln\left(\mathbf{B}^1(p^1,p^2)\right)-\ln\left(\mathbf{B}^2(p^1,p^2)\right)\ . \tag{93}$$

Remark 1. The "conventional" probabilistic voting model can be viewed as a special case of this that is defined by two properties. First, since turnout is exogenous and universal, $R^{\sigma}(p^1,p^2)=1$ for all (p^1,p^2) and hence $\ln\left(R^{\sigma}(p^1,p^2)\right)=0$. Second, and again for the reason that turnout is exogenous and universal, $V^1=\mathbf{B}^1(p^1,p^2)$ and $V^2=\mathbf{B}^2(p^1,p^2)=1-\mathbf{B}^1(p^1,p^2)$. In the probabilistic voting model, the objective of party 1 can therefore be taken to be $\ln\left(\mathbf{B}^1(p^1,p^2)\right)-\ln\left(1-\mathbf{B}^1(p^1,p^2)\right)$ or simply $V^1=\mathbf{B}^1(p^1,p^2)$. I.e., maximizing the probability of winning is the same as maximizing the number of votes.

Remark 2. With Nash equilibrium rather than subgame perfect equilibrium as the solution concept (or, equivalently, with $\mu=0$), the parties view $R^{\sigma}(p^1,p^2)$ as exogenously fixed, albeit at the level that is induced by the equilibrium policies. Party 1 then seeks to maximize

$$\ln\left(\mathbf{B}^1(p^1,p^2)\right)-\ln\left(\mathbf{B}^2(p^1,p^2)\right)$$

and party 2 seeks to minimize this expression. Since $\mathbf{B}^2(p^1,p^2)=1-\mathbf{B}^1(p^1,p^2)$, party 1's objective can as well simply taken to be $\mathbf{B}^1(p^1,p^2)$ and $\mathbf{B}^2(p^1,p^2)$ can be taken to be the objective of party 2. Nash equilibrium then requires that p^1 solves $\max_{\hat{p}^1\in P}\ \mathbf{B}^1(\hat{p}^1,p^2)$ and that p^2 solves $\max_{\hat{p}^2\in P}\ \mathbf{B}^2(p^1,\hat{p}^2)$. Note that these equilibrium are also the equilibrium conditions in the "conventional" probabilistic voting model. Thus, equilibrium existence in the "conventional" probabilistic voting model implies the existence of a Nash equilibrium in the given setup.

If the turnout subgame has an interior equilibrium, then the probability of winning for party 1 can be written in a reduced form that no longer involves an explicit reference to the participation thresholds σ^1 and σ^2. Specifically, equation (92) implies that the winning probability in (87) becomes

$$\bar{\pi}^1(p^1,p^2) = P\left(\Pi^1(p^1,p^2)\right) \quad \text{for} \quad \Pi^1(p^1,p^2) := \frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}\,. \tag{94}$$

Thus, as in the main body of the text (Proposition 1), under Assumption 5, if (p^1,p^2) is a pair of interior subgame perfect equilibrium policies, then it it is a saddle point of the function Π.

F.2 Public goods

Our framework for studying endogenous turnout and endogenous platforms in political competition is developed for a generic policy domain. We have emphasized that the set of non-linear income tax systems is a policy domain of particular interest. That said, our framework can also be applied to study the implications of endogenous turnout for political competition over other policy domains. In this section, we briefly summarize the results from such an analysis. Specifically, we report on the implications of our framework for public goods provision.

Individuals have preferences over public goods that are given by $u(\omega,p) = \omega\, p - k(p)$, where $p \in \mathbb{R}_+$ denotes the quantity of the public good, $\omega \in \Omega$ is an individual's public goods preference and the cost function k captures the per capita cost of public goods provision.[51] We begin with a characterization of the public good provision level that party 1 would choose if its sole objective was to mobilize its supporters. In this case, it would choose q^1 with the objective to maximize

$$W^1(p^1,p^2) = \mathbb{E}[G^1_W(\omega\, p^1 - k(p^1) - u(p^2,\omega) \mid \omega)]\,,$$

where we denote by

$$\begin{aligned} G^1_W(x \mid \omega) &:= \int_{-\infty}^{x} (x-\varepsilon)\, b(\varepsilon \mid \omega) d\varepsilon \\ G^2_W(x \mid \omega) &:= \int_{x}^{\infty} (\varepsilon - x)\, b(\varepsilon \mid \omega) d\varepsilon. \end{aligned}$$

Note that the derivatives of the functions $G^1_W(\cdot \mid \omega)$ and $G^2_W(\cdot \mid \omega)$ are respectively

[51] In an economy with a continuum of individuals and private information on public goods preferences, equal cost sharing is the only way of satisfying robust incentive compatibility, budget balance and anonymity, see Bierbrauer and Hellwig (2016).

given by

$$\begin{aligned} g_W^1(x \mid \omega) &:= B(x \mid \omega) \\ g_W^2(x \mid \omega) &:= -(1 - B(x \mid \omega)) . \end{aligned}$$

Given p^2, the first order condition characterizing the optimal choice of p^1 is

$$\mathbb{E}\left[\mathcal{G}_W^1(\omega \mid p^1, p^2)\, \omega\right] = k'(p^1)$$

where

$$\mathcal{G}_W^1(\omega \mid p^1, p^2) = \frac{g_W^1(u(p^1, \omega) - u(p^2, \omega) \mid \omega)}{\mathbb{E}[g_W^1(u(p^1, \omega) - u(p^2, \omega) \mid \omega)]} .$$

This first order condition is a political economy analogue to the Samuelson rule for first-best public good provision. For the given setup, the Samuelson rule stipulates that $\mathbb{E}[\omega] = k'(p)$, i.e., it requires equal weights for all public goods preferences. For the purpose of mobilizing its supporters, party 1 does not apply equal weights. Instead the public good preferences of different individuals are weighted according to the function $\mathcal{G}_W^1$. The public good provision level that party 1 would choose if it only wanted only to demobilize the supporters of party 2 is such that

$$\mathbb{E}\left[\mathcal{G}_W^2(\omega \mid p^1, p^2)\, \omega\right] = k'(p^1) ,$$

and the policy that maximizes $\frac{W^1(p^1 p^2)}{W^2(p^1, p^2)}$ satisfies

$$\mathbb{E}[\mathcal{G}_{SP}^1(\omega \mid p^1, p^2)\, \omega] = k'(p^1)$$

where

$$\mathcal{G}_{SP}^1(\omega \mid p^1, p^2) := \lambda^1(p^1, p^2)\, \mathcal{G}_W^1(\omega \mid p^1, p^2) + (1 - \lambda^1(p^1, p^2))\, \mathcal{G}_W^2(\omega \mid p^1, p^2)$$

and $\lambda^1(p^1, p^2)$ is defined by

$$\lambda^1(p^1, p^2) \;:=\; \frac{1}{1 + \Pi^1(p^1, p^2)} \frac{\bar{g}_W^1(\underline{\omega} \mid p^1, p^2)}{\mathbb{E}[\gamma^1(\omega \mid p^1, p^2)]}$$

with

$$\begin{aligned}\gamma^1(\omega \mid p^1, p^2) \;:=\; & \frac{1}{1+\Pi^1(p^1,p^2)} g_W^1(u^1(\omega) - u^2(\omega) \mid p^1, p^2) \\ & + \frac{\Pi^1(p^1,p^2)}{1+\Pi^1(p^1,p^2)} g_W^2(u^1(\omega) - u^2(\omega) \mid p^1, p^2).\end{aligned}$$

Again, the party compromises between mobilizing its own supporters and demobilizing the supporters of the other party – with the weight on the own supporters being smaller if the party is more likely to win.

F.3 Alternative modelling choices for ethical voting

The ethical voter models by Feddersen and Sandroni (2006), on the one hand, and by Coate and Conlin (2004), on the other differ, in some aspects. For instance, Feddersen and Sandroni (2006) assume that the population consists of ethical voters and of non-ethical voters. Moreover, the fraction of ethical voters is a priori uncertain. Uncertainty over election outcomes in Feddersen and Sandroni (2006) is entirely due to this uncertainty about the fraction of ethical voters. Coate and Conlin (2004), by contrast, assume that all voters are ethical voters. Uncertainty over election outcomes in their framework is driven by uncertainty over the policy preferences of these ethical voters.

In this section of the Online-Appendix we show that these modelling choices are not essential for our main results. We could go either way. In the main text, we present an analysis that adopts the framework of Feddersen and Sandroni (2006). We show that we could as well work with the model of Coate and Conlin (2004) in Section F.3.2.

For tractability, our adaptation of Feddersen and Sandroni makes use of an assumption which implies that the parties' bases add up to a constant. An advantage is that it becomes transparent that the standard probabilistic voting model is nested as a special case of our analysis. In Section F.3.3 we present an extension that does not rest on this assumption. The extension shows that the parties' trade-off between attracting swing voters, mobilizing their own core voters and demobilizing the opponent's core voters is at the heart of our analysis, with or without the assumption that the parties' bases add up to a constant.

The bottom line is that what is really driving our results is the combination of probabilistic and ethical voting. We use the probabilistic voting model to determine preferences over policies and parties. We use a model of ethical voting to determine turnout. How exactly we model ethical voting is of secondary importance. Our analysis is robust to alternative specifications of ethical voting.

F.3.1 A general framework

We begin with a general framework for political competition that connects probabilistic and ethical voting. As will become clear, the general framework contains as special cases

- an environment where all voters are ethical voters and with uncertainty about policy preferences as in Coate and Conlin (2004),
- an environment where the population share of ethical voters is a random quantity as in Feddersen and Sandroni (2006).

Party and policy preferences. Consider a pair of policies p^1 and p^2 proposed by parties 1 and 2, respectively. As in the body of the text, a type ω-individual preferes a victory of party 1 if

$$u(p^1, \omega) - u(p^2, \omega) \geq \varepsilon \ ,$$

where the utility function u captures policy preferences and the variable ε captures idiosyncratic party preferences. We assume that, conditional on ω, ε is a random variable with a distribution that can be represented by a cumulative distribution function $\tilde{B}(\cdot \mid \omega, \eta)$. Thus,

$$\tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta)$$

is the fraction of type ω-voters who are better off if party 1 wins. The complement

$$1 - \tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta)$$

is the fraction of type ω-voters who are better off if party 2 wins.

The formulation differs from the one in the main text in that we allow these distributions to be random objects themselves. The distributions of idiosyncratic party preferences now depend on the realization of a random variable η. Thus, we allow for uncertainty in policy preferences as in Coate and Conlin (2004).

Example: Aggregate uncertainty on preferences. At this stage there is no need to introduce more specific assumptions about η. Still, the following example might be helpful to get an idea of what the randomness in party and policy preferences might look like: For any type ω, there is a set of feasible distributions $\Phi(\omega)$, with generic entry $\tilde{B}(\cdot \mid \omega, \eta)$. Distributions in this set can be ordered according to first order stochastic dominance. Let this order be represented by a mapping from the unit interval to the set of feasible distributions. Also suppose that there is a random variable η_ω taking values in the unit interval, indicating which of these distributions materializes. Finally, let there be one such a random variable for each type ω. Then the random variable η that governs the *state of the system* is a stochastic process that can be written as $\eta = \{\eta_\omega\}_{\omega \in \Omega}$.

In Feddersen and Sadroni (2006), by contrast, party and policy preference are deterministically fixed once the alternatives p^1 and p^2 are given. The following assumption contains a more formal version of this statement.

Assumption 6 (**Feddersen and Sandroni: No aggregate uncertainty on preferences**). *For every type ω, there exists a cumulative distribution function $B(\cdot \mid \omega)$, so that, for all p^1 and p^2 and all possible realizations of η,*

$$\tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta) = B(u(p^1, \omega) - u(p^2, \omega) \mid \omega) \ .$$

Ethical and non-ethical voters. A complete description of the state of the system also requires to specify, for each type, the fraction of ethical voters. Formally, let $\tilde{q}^1(\omega, \eta)$ be the fraction of ethical voters among those type ω-individuals who are better off if party 1 wins. Likewise denote by $\tilde{q}^2(\omega, \eta)$ be the fraction of ethical type ω-individuals who are better off if party 2 wins. In the approach of Feddersen and Sandroni these are random objects. Here, we capture this again, through the dependence on an aggregate shock, or, more specifically, the random variable η. By contrast, in the model of Coate and Conlin, $\tilde{q}^1$ and $\tilde{q}^1$ are set equal to one. For ease of reference, we also highlight this assumption.

Assumption 7 (**Coate and Conlin: All voters are ethical voters**). *For any ω, $\tilde{q}^1(\omega, \eta)$ and $\tilde{q}^2(\omega, \eta)$ are degenerate random variables so that*

$$\tilde{q}^1(\omega, \eta) = \tilde{q}^2(\omega, \eta) = 1$$

for all realizations of η.

The parties' bases. The potential voters of party 1 are those who vote for party 1 in case of turning out to vote. This mass of these voters is a random variable

$$\tilde{\mathbf{B}}^1(p^1,p^2,\eta) := \mathbb{E}\left[\tilde{q}^1(\omega,\eta)\ \tilde{B}(u(p^1,\omega)-u(p^2,\omega)\mid\omega,\eta)\right]$$

Analogously, the mass party 2's potential voters is given by

$$\tilde{\mathbf{B}}^2(p^1,p^2,\eta) := \mathbb{E}\left[\tilde{q}^2(\omega,\eta)(1-\tilde{B}(u(p^1,\omega)-u(p^2,\omega)\mid\omega,\eta))\right]$$

We denote, respectively, by

$$\mathbf{B}^1(p^1,p^2) = \int \tilde{\mathbf{B}}^1(p^1,p^2,\eta)\ dP(\eta)$$

and

$$\mathbf{B}^2(p^1,p^2) = \int \tilde{\mathbf{B}}^2(p^1,p^2,\eta)\ dP(\eta)$$

the expected values of $\tilde{\mathbf{B}}^1(p^1,p^2,\eta)$ and $\tilde{\mathbf{B}}^2(p^1,p^2,\eta)$, where P is the cumulative distribution function of the random variable η. For brevity, we also refer to $\mathbf{B}^1(p^1,p^2)$ and $\mathbf{B}^2(p^1,p^2)$ as the parties' bases.

The turnout subgame. As in the main text, the ethical voters of party 1 choose σ^1 to maximize

$$\pi^1(\sigma^1,\sigma^2,p^1,p^2)\ W^1(p^1,p^2)-k(\sigma^1)\ \mathbf{B}^1(p^1,p^2)$$

and the ethical voters of party 2 choose σ^2 to maximize

$$(1-\pi^1(\sigma^1,\sigma^2,p^1,p^2))\ W^2(p^1,p^2)-k(\sigma^2)\ \mathbf{B}^2(p^1,p^2)\ .$$

We have to adjust, however, our definitions of $W^1(p^1,p^2)$ and $W^2(p^1,p^2)$ so that they are consistent with the more general setup that we are currently exploring. We now let

$$\tilde{W}^1(p^1,p^2,\eta) = \mathbb{E}\left[\int_{\mathbb{R}} \max\left\{\ u(p^1,\omega)-[u(p^2,\omega)+\varepsilon]\ ,\ 0\right\}\ \tilde{b}(\varepsilon\mid\omega,\eta)\,\mathrm{d}\varepsilon\right]. \qquad (95)$$

denote the stakes for the ethical voters of party 1 in state η and let $W^1(p^1,p^2)$ be the expectation of $\tilde{W}^1(p^1,p^2,\eta)$, conditional on party 1 winning the election. We denote by $\tilde{b}(\cdot \mid \omega,\eta)$ the derivative of $\tilde{B}(\cdot \mid \omega,\eta)$, i.e. $\tilde{b}(\cdot \mid \omega,\eta)$ is the density of ε, conditional on type ω and state η. We define $\tilde{W}^2(p^1,p^2,\eta)$ and $W^2(p^1,p^2)$ analogously. Party 1 wins the election if

$$\sigma^1\, \tilde{\mathbf{B}}^1(p^1,p^2,\eta) \geq \sigma^2\, \tilde{\mathbf{B}}^2(p^1,p^2,\eta)\ ,$$

where σ^1 and σ^2 are the turnout rates of the potential voters of party 1 and party 2, respectively. Equivalently, party 1 wins if

$$\frac{\sigma^1}{\sigma^2} \times \frac{\tilde{\mathbf{B}}^1(p^1,p^2,\eta)}{\tilde{\mathbf{B}}^2(p^1,p^2,\eta)} \geq 1\ .$$

The probability that party 1 wins the election is therefore given by

$$\pi^1(\sigma^1,\sigma^2) = \text{prob}\left(\frac{\sigma^1}{\sigma^2} \times \frac{\tilde{\mathbf{B}}^1(p^1,p^2,\eta)}{\tilde{\mathbf{B}}^2(p^1,p^2,\eta)} \geq 1\right)\ .$$

For later reference, note that we can also write this winning probability as an average winning probability over the different states η, i.e. so that

$$\bar{\pi}^1(p^1,p^2) = \int \text{prob}\left(\frac{\sigma^1}{\sigma^2} \times \frac{\tilde{\mathbf{B}}^1(p^1,p^2,\eta)}{\tilde{\mathbf{B}}^2(p^1,p^2,\eta)} \geq 1 \mid \eta\right) dP(\eta)\ . \tag{96}$$

Note that the turnout rates enter this expression only via the ratio $\frac{\sigma^1}{\sigma^2}$. This implies that our analysis of the turnout subgame – for given policies p^1 ans p^2 – does not depend on wether we adopt the Feddersen-Sandroni or the Coate-Conlin formulation. As a consequence, Lemma 1 in the main text goes through. Thus, irrespectively of whether Assumption 7 or Assumption 6 is imposed, in an equilibrium of the turnout subgame

$$\frac{\sigma^{1*}(p^1,p^2)}{\sigma^{2*}(p^1,p^2)} = \left[\frac{W^1(p^1,p^2)\,/\,\mathbf{B}^1(p^1,p^2)}{W^2(p^1,p^2)\,/\,\mathbf{B}^2(p^1,p^2)}\right]^{\lambda}\ . \tag{97}$$

F.3.2 Adopting the approach of Coate and Conlin: Only ethical voters

We now impose Assumption 7, i.e. the Assumption made by Coate and Conlin (2004) that there are only ethical voters. Thus, to have non-trivial winning probabilities, we must not at the same time impose Assumption 6. Put differently, we suppose that policy preferences are subject to aggregate shocks. We will establish two findings: First, our Proposition 1 in the main text rests on a simplifying assumption on the nature of aggregate uncertainty. The same assumption can be made in the Coate and Conlin version of our model and has the same effect. Proposition 1 is therefore robust to the way in which we model ethical voting. Second, the parties' bases add up to a constant. A model of ethical voting that does not share this property therefore requires to relax Assumption 7.

Recall equations (96) and (97), i.e. that taking the endogeneity of turnout into account, the probability of winning can be written as

$$\bar{\pi}^1(p^1, p^2) = \int \text{prob}\left(\frac{\sigma^1}{\sigma^2} \times \frac{\tilde{\mathbf{B}}^1(p^1, p^2, \eta)}{\tilde{\mathbf{B}}^2(p^1, p^2, \eta)} \geq 1 \mid \eta\right) dP(\eta) \ ,$$

where

$$\frac{\sigma^1}{\sigma^2} = \left[\frac{W^1(p^1, p^2) \,/\, \mathbf{B}^1(p^1, p^2)}{W^2(p^1, p^2) \,/\, \mathbf{B}^2(p^1, p^2)}\right]^\lambda \ .$$

In principle, there is no problem to working directly with this objective, it gives raise to the same tradeoffs as those highlighted in our manuscript. For a tractable comparative statics analysis, we would, however, have to impose (possibly non-parametric) assumptions on how different realizations of the random variable η shift the distributions $\tilde{B}(\cdot)$. Our Assumption 1 in the main text is one conceivable way of doing this, a way that has the advantage of simplicity. The main text focuses on the Feddersen-Sandroni version of ethical voting and Assumption 1 is imposed in this context. As we will now explain, we can get to same conclusions also with a Coate-Conlin approach. To see this, consider the following assumption.

Assumption 8 (Multiplicative shocks I). *Suppose that $\eta = (\eta^1, \eta^2)$ is a pair of two random variables η^1 and η^2 so that*

$$\tilde{\mathbf{B}}^1(p^1, p^2, \eta) = \eta^1 \ \mathbf{B}^1(p^1, p^2) \tag{98}$$

and

$$\tilde{\mathbf{B}}^2(p^1, p^2, \eta) = \eta^2 \ \mathbf{B}^2(p^1, p^2) \ . \tag{99}$$

Under Assumption 8 the aggregate shocks η^1 and η^2 can be interpreted as percentage deviations of the random variables $\tilde{\mathbf{B}}^1(p^1, p^2, \eta)$ and $\tilde{\mathbf{B}}^2(p^1, p^2, \eta)$ from their respective means. To see this, suppose that the means of both η^1 and η^2 are equal to

1 and rewrite (98) and (99) as

$$\frac{\tilde{\mathbf{B}}^1(p^1,p^2,\eta)-\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^1(p^1,p^2)}=\eta^1-1$$

and

$$\frac{\tilde{\mathbf{B}}^2(p^1,p^2,\eta)-\mathbf{B}^2(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}=\eta^2-1$$

The left hand sides of these equations give the percentage deviation of the random variables $\tilde{\mathbf{B}}^1(p^1,p^2,\eta)$ and $\tilde{\mathbf{B}}^2(p^1,p^2,\eta)$ from their respective means. The right-hand sides give the deviations of η^1 and η^2 from their means.

Under Assumption 8 the expression for the probability of winning in (96) becomes

$$\bar{\pi}^1(p^1,p^2)=\text{prob}\left(\left[\frac{W^1(p^1,p^2)\,/\,\mathbf{B}^1(p^1,p^2)}{W^2(p^1,p^2)\,/\,\mathbf{B}^2(p^1,p^2)}\right]^{\lambda}\times\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\geq\frac{\eta^2}{\eta^1}\right) \tag{100}$$

Upon letting $\eta:=\frac{\eta^2}{\eta^1}$, we can write this as

$$\bar{\pi}^1(p^1,p^2)=P\left(\left[\frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}\right]^{\lambda}\times\left[\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right]^{1-\lambda}\right)\,. \tag{101}$$

P is a cumulative distribution function and hence a monotonic function. Maximizing (minimizing) $\bar{\pi}^1(p^1,p^2)$ is therefore equivalent to maximizing (minimizing) the argument of P,

$$\left[\frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}\right]^{\lambda}\times\left[\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\right]^{1-\lambda}$$

or any monotone transformation of it such as

$$(1-\lambda)\,\log\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\;+\;\lambda\,\log\frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}\,.$$

We summarize these observations in the following Lemma.

Lemma 12. *Suppose that Assumptions 7 and 8 hold. Then party 1's objective is to maximize*

$$\Pi^1(p^1,p^2)\;\;:=\;\;(1-\lambda)\,\log\frac{\mathbf{B}^1(p^1,p^2)}{\mathbf{B}^2(p^1,p^2)}\;+\;\lambda\,\log\frac{W^1(p^1,p^2)}{W^2(p^1,p^2)}, \tag{102}$$

and party 2's objective is to minimize it. Thus, if (p^{1*}, p^{2*}) is a pair of interior subgame perfect equilibrium policies, then it is a saddle point of the function $\Pi^1(p^1, p^2)$.

Note that the Lemma gives exactly the same conclusion as Proposition 1 in the body of the text. This shows that – even though Assumptions (98) and (99) may have a more plausible microfoundation in the Feddersen-Sadroni-model – our approach is essentially agnostic on the question how to best model ethical voting. We can work both with the Coate-Conlin formulation and with the Feddersen-Sandroni formulation.

The parties' bases add up to a constant. The following Lemma shows that the Coate-Conlin specification of ethical voting shares one property of the model that we present in the main text, namely that the parties bases add up to a constant. Thus, a change of policies that increases the base for, say, party 1 translates one-for-one into a decrease of the base of party 2.

Lemma 13. *Suppose that Assumption 7 holds. Then*

$$\mathbf{B}^1(p^1, p^2) + \mathbf{B}^2(p^1, p^2) = 1 \ .$$

Proof. If $\tilde{q}^1(\omega, \eta) = \tilde{q}^2(\omega, \eta) = 1$ for all realizations of η, we have

$$\begin{aligned}
\tilde{\mathbf{B}}^1(p^1, p^2, \eta) &:= \mathbb{E}\left[\tilde{q}^1(\omega, \eta)\ \tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta)\right] \\
&= \mathbb{E}\left[\tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta)\right] \\
&= 1 - \mathbb{E}\left[1 - \tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta)\right] \\
&= 1 - \mathbb{E}\left[\tilde{q}^2(\omega, \eta)(1 - \tilde{B}(u(p^1, \omega) - u(p^2, \omega) \mid \omega, \eta))\right] \\
&= 1 - \tilde{\mathbf{B}}^2(p^1, p^2, \eta) \ .
\end{aligned}$$

Hence, also

$$\begin{aligned}\mathbf{B}^1(p^1,p^2) &:= \textstyle\int \tilde{\mathbf{B}}^1(p^1,p^2,\eta)\; dP(\eta)\\ &= 1-\textstyle\int \tilde{\mathbf{B}}^2(p^1,p^2,\eta)\; dP(\eta)\\ &= 1-\mathbf{B}^2(p^1,p^2)\;.\end{aligned}$$

□

F.3.3 An alternative version of the Feddersen-Sandroni model in which the parties' bases do not add up to a constant

In the following, we consider an extension of our model in which the parties' bases do not add up to a constant. It follows from Lemma 13 that we cannot employ Assumption 7 according to which the electorate consists, in all states, entirely of ethical voters. For ease of exposition, we impose instead Assumption 6, due to Feddersen and Sandroni, so that there is no aggregate uncertainty in policy preferences. This has the expositional advantage that all the aggregate uncertainty in the model is due to the randomness of the share of ethical voters.

We seek to show that the parties' tradeoffs between attracting swing voters, catering to their own core voters in an attempt to mobilize them and catering to the rival's core voters with the intention to demobilize them does not rest on the assumption that the parties's bases add up to a constant. Recall that, in the main text, this property is implied by the assumption, that, for any type ω, the random variables $\tilde{q}^1(\omega,\eta)$ and $\tilde{q}^2(\omega,\eta)$ have the same mean

$$\bar{q}(\omega) \quad := \quad \int \tilde{q}^1(\omega,\eta)\; dP(\eta) \quad = \quad \int \tilde{q}^1(\omega,\eta)\; dP(\eta)\;.$$

Therefore,

$$\begin{aligned}
\mathbf{B}^1(p^1,p^2) &= \int \tilde{\mathbf{B}}^1(p^1,p^2,\eta)\, dP(\eta) r \\
&= \int \mathbb{E}[\tilde{q}^1(\omega,\eta)\; B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)]\, dP(\eta) \\
&= \mathbb{E}\left[\left(\int \tilde{q}^1(\omega,\eta)\; dP(\eta)\right) B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)\right] \\
&= \mathbb{E}\left[\bar{q}(\omega) B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)\right]\ ,
\end{aligned} \tag{103}$$

and, by the same logic,

$$\mathbf{B}^2(p^1,p^2) \;=\; \mathbb{E}\left[\bar{q}(\omega)(1-B(u(p^1,\omega)-u(p^2,\omega)\mid\omega))\right]\ . \tag{104}$$

Obviously, equations (103) and (104) imply that

$$\mathbf{B}^1(p^1,p^2)+\mathbf{B}^2(p^1,p^2)=\mathbb{E}[\bar{q}(\omega)]$$

so that the two bases add up to an exogenous constant $\mathbb{E}[\bar{q}(\omega)]$, i.e. a term that does not depend on the policies that are proposed.

Example. As a simple case that avoids the property that the parties bases add up to a constant consider the following Assumption.

Assumption 9 (Party specific means). *There are numbers $\bar{q}^1$ and $\bar{q}^2$ so that, for all ω,*

$$\bar{q}^1 \;=\; \int \tilde{q}^1(\omega,\eta)\; dP(\eta) \quad \textit{and} \quad \bar{q}^2 \;=\; \int \tilde{q}^2(\omega,\eta)\; dP(\eta)\ .$$

The assumption says, all supporters of party 1 are, irrespective of their type ω, equally likely to be of the ethical type: For any supporter of party 1, this probability is equal to $\bar{q}^1$. Likewise, all supporters of party 2 are of the ethical type with probability $\bar{q}^2$.

An implication of this Assumption is that

$$\mathbf{B}^1(p^1,p^2)=\bar{q}^1\;\mathbb{E}[B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)]$$

and

$$\mathbf{B}^2(p^1,p^2)=\bar{q}^2\;\mathbb{E}[1-B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)]\ .$$

Hence,

$$\mathbf{B}^1(p^1,p^2)+\mathbf{B}^2(p^1,p^2)=\bar{q}^2+\left(\bar{q}^1-\bar{q}^2\right)\mathbb{E}[B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)]\ ,$$

which implies that the bases add up to a quantity that depends on p^1 and p^2. The overall mass of potential voters therefore does depend on the policies that the parties. Also note that

$$\begin{aligned}\mathbf{B}^2(p^1,p^2) &= \bar{q}^2-\bar{q}^2\mathbb{E}[B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)] \\ &= \bar{q}^2-\tfrac{\bar{q}^2}{\bar{q}^1}\,\bar{q}^1\,\mathbb{E}[B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)] \\ &= \bar{q}^2-\tfrac{\bar{q}^2}{\bar{q}^1}\,\mathbf{B}^1(p^1,p^2).\end{aligned}$$

Note that it is still the case that an increase of party 1's base implies a decrease of party 2's base – even though no longer one-by-one.

Henceforth and in parallel to our previous analysis we impose an assumption of multiplicative shocks. This assumption of multiplicative shocks is consistent with Assumption 9, i.e. both assumptions can hold simultaneously, but does not require it. That is, we can have multiplicative shocks without party specific means.

Assumption 10 (Multiplicative shocks II). *Let $\bar{q}^1(\omega):=\int\tilde{q}^1(\omega,\eta)dP(\eta)$ be the expected value of the random variable $\tilde{q}^1(\omega,\eta)$ for any ω. Analogously, let $\bar{q}^2(\omega):=\int\tilde{q}^2(\omega,\eta)dP(\eta)$ be the expected value of the random variable $\tilde{q}^2(\omega,\eta)$. Suppose that $\eta=(\eta^1,\eta^2)$ is a pair of two random variables η^1 and η^2 so that, for all ω,*

$$\tilde{q}^1(\omega,\eta)=\eta^1\ \bar{q}^1(\omega) \tag{105}$$

and

$$\tilde{q}^2(\omega,\eta)=\eta^2\ \bar{q}^2(\omega)\ . \tag{106}$$

Note the following implications of this Assumption:

$$\begin{aligned}\tilde{\mathbf{B}}^1(p^1,p^2,\eta) &= \mathbb{E}[\tilde{q}^1(\omega,\eta)B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)] \\ &= \eta^1\mathbb{E}[\bar{q}^1(\omega)B(u(p^1,\omega)-u(p^2,\omega)\mid\omega)] \\ &= \eta^1\ \mathbf{B}^1(p^1,p^2)\end{aligned}$$

and, analogously,

$$\tilde{\mathbf{B}}^2(p^1, p^2, \eta) \;\; = \;\; \eta^2\, \mathbf{B}^2(p^1, p^2)\ .$$

This shows that equations (98) and (99) – imposed previously in our analysis of the Coate and Conlin model – also hold in the given context. An immediate implication is that Lemma 12 also extends to the given setup. This observation yields the following Corollary.

Corollary 1. *Suppose that Assumption 10 holds. Then party 1's objective is to maximize*

$$\Pi^1(p^1, p^2) \;\; := \;\; (1-\lambda)\, \log \frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)} \; + \; \lambda\, \log \frac{W^1(p^1, p^2)}{W^2(p^1, p^2)}, \tag{107}$$

and party 2's objective is to minimize it. Thus, if (p^{1*}, p^{2*}) is a pair of interior subgame perfect equilibrium policies, then it is a saddle point of the function $\Pi^1(p^1, p^2)$.

The significance of the Corollary is to show that Proposition 1 in our main text also extends to a model in which the parties' bases do not add up to a constant. Thus, the tradeoffs that we highlight in our main text also extend to the given setup, albeit with some modifications. To understand these modifications, it is again instructive to look first at the polar cases $\mu = \infty$ and $\mu = 0$.

For $\mu = \infty$, the parties' bases do not matter at all for the probability of winning the election. The analysis therefore has exactly the same logic as the one presented in the body of the text: Party 1 focuses on maximizing

$$\frac{W^1(p^1, p^2)}{W^2(p^1, p^2)}$$

and party 2 seeks to minimize this expression. From the perspective of party 1, the numerator $W^1(p^1, p^2)$ points to the political returns from increasing the stakes for its own core voters, the denominator points to the political returns from decreasing the stakes for party 2's core voters. Moreover, how these motives balance depends on the equilibrium value of $W^1(p^1, p^2)/W^2(p^1, p^2)$. The larger this quantity, the larger is party 1's equilibrium probability of winning and the more it has an incentive to focus on the demobilization of the potential voters of party 2.[52]

[52] Recall from the analysis in the main text that any equilibrium is symmetric and that this observation makes it possible to pin down the equilibrium value of $W^1(p^1, p^2)/W^2(p^1, p^2)$.

The case $\mu = 0$ is the exact mirror image. The stakes for the parties' core voters play no role, and all that matters is the ratio of the parties bases. Party 1 now seeks to maximize

$$\frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)}$$

while party 2 minimizes this expression. This problem of party 1 problem is – contrary to our analysis in the main text – not generally equivalent to maximizing $\mathbf{B}^1(p^1, p^2)$.

Remark 3. This equivalence holds, however, if we impose, in addition, Assumption 9. To see this, note that in this case,

$$\frac{\mathbf{B}^1(p^1, p^2)}{\mathbf{B}^2(p^1, p^2)} = \frac{\mathbf{B}^1(p^1, p^2)}{\bar{q}^2 - \frac{\bar{q}^2}{\bar{q}^1}\,\mathbf{B}^1(p^1, p^2)} \ ,$$

which is an expression that is increasing in $\mathbf{B}^1(p^1, p^2)$.

If the equivalence does not hold, party 1 faces a tradeoff between maximizing $\mathbf{B}^1(p^1, p^2)$ and minimizing $\mathbf{B}^2(p^1, p^2)$. Maximizing $\mathbf{B}^1(p^1, p^2)$ would mean to cater primarily to those voters who are likely to swing into the base of party 1. Minimizing $\mathbf{B}^2(p^1, p^2)$ would give priority to those voters who swing out of the base of party 2 if party 1 offers a better deal. Since the bases do not add up to a constant, those who swing out of the base of party 2 do not automatically swing into the base of party 1. Thus, there is again a tradeoff between doing something that is good for the own vote share and doing something that is bad for the rival's vote share. How this tradeoff is resolved depends, again, on the equilibrium value of $\mathbf{B}^1(p^1, p^2)/\mathbf{B}^2(p^1, p^2)$. The larger this value the larger the weight on the minimization of the rival's base.

Obviously, for values of μ that are interior, $\mu \in (0, \infty)$ both forces are at play, and the parties consider the implications of their platforms choices both for their relative base advantage, as measured by $\mathbf{B}^1(p^1, p^2)/\mathbf{B}^2(p^1, p^2)$, and for their relative stake advantage, as measured by $W^1(p^1, p^2)/W^2(p^1, p^2)$.

To summarize this discussion we highlight two observations: First, a more general model in which the parties bases do not add up to a constant gives rise to the same tradeoffs as our analysis in the main text, but possibly with some modifications in the relevant formulas. Second, if we impose an additional assumption, Assumption 9, then no such modifications are needed and the analysis in the main text literally extends – even though the parties' bases do not add up to a constant.

References

Bakker, Ryan, Catherine de Vries, Erica Edwards, Liesbet Hooghe, Seth Jolly, Gary Marks, Jonathan Polk, Jan Rovny, Marco Steenbergen, and Milada Vachudova, "Measuring party positions in Europe: The Chapel Hill expert survey trend file, 1999-2010.," *Party Politics*, 2015, (21.1), 143–152.

Bierbrauer, Felix J and Martin F Hellwig, "Robustly coalition-proof incentive mechanisms for public good provision are voting mechanisms and vice versa," *The Review of Economic Studies*, 2016, *83* (4), 1440–1464.

Brett, C. and J.A. Weymark, "Voting over selfishly optimal nonlinear income tax schedules with a minimum-utility constraint," *Journal of Mathematical Economics*, 2016, *67*, 18–31.

Forschungsgruppe Wahlen, Mannheim, "Politbarometer Ost 2009 (Cumulated Data Set, incl. Flash)," 2013.

_ , "Politbarometer West 2009 (Cumulated Data Set, incl. Flash)," 2013.

_ , "Politbarometer 2013 (Cumulated Data Set, incl. Flash)," 2015.

_ , "Politbarometer 2017 (Cumulated Data Set)," 2018.

Guesnerie, R., *A Contribution to the Pure Theory of Taxation*, Cambridge University Press, 1995.

Hammond, P., "Straightforward Individual Incentive Compatibility in Large Economies," *Review of Economic Studies*, 1979, *46*, 263–282.

Hellwig, M.F., "A Contribution to the Theory of Optimal Utilitarian Income Taxation," *Journal of Public Economics*, 2007, *91*, 1449–1477.

Jung, Matthias, Yvonne Schroth, and Andrea Wolf, "Wählerverhalten und Wahlergebnis," in Karl-Rodolf Korte, ed., *Die Bundestagswahl 2009*, Springer, 2010.

_ , _ , and _ , "Wählerverhalten und Wahlergebnis: Angela Merkels Sieg in der Mitte," in Karl-Rodolf Korte, ed., *Die Bundestagswahl 2013*, Springer, 2015.

Meltzer, A. and S. Richard, "A Rational Theory of the Size of Government," *Journal of Political Economy*, 1981, *89*, 914–927.

Myerson, Roger B., "Optimal Auction Design," *Mathematics of Operations Research*, 1981, *6* (1), 58–73.

Osborne, M. and A. Rubinstein, *A course in Game Theory*, MIT Press, Cambridge, MA., 1994.

Polk, Jonathan, Jan Rovny, Ryan Bakker, Erica Edwards, Liesbet Hooghe, Seth Jolly, Jelle Koedam, Filip Kostelka, Gary Marks, Gijs Schumacher, Marco Steenbergen, Milada Vachudova, and Marko Zilovic, "Explaining the salience of anti-elitism and reducing political corruption for political parties in Europe with the 2014 Chapel Hill Expert Survey data," *Research & Politics*, 2017, pp. 1–9.

Volkens, Andrea, Werner Krause, Pola Lehmann, Theres Matthieß, Nicolas Merz, Sven Regel, and Bernhard Weßels, "The Manifesto Data Collection. Manifesto Project (MRG/CMP/MARPOR)," 2018.