How Efficient is Dynamic Competition? The Case of Price as Investment
— Online Appendix —

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May 22, 2019

The Online Appendix contains additional details and results pertaining to the indicated sections of the main paper.

OA1 Model

OA1.1 Firms' decisions

We use $V_n(\mathbf{e})$ to denote the expected NPV of future cash flows to firm n in state $\mathbf{e}$ at the beginning of the period and $U_n(\mathbf{e}')$ to denote the expected NPV of future cash flows to firm n in state $\mathbf{e}'$ *after* pricing decisions but *before* exit and entry decisions are made. The price-setting phase determines the value function $\mathbf{V}_n$ along with the policy function $\mathbf{p}_n$ with typical element $V_n(\mathbf{e})$, respectively, $p_n(\mathbf{e})$; the exit-entry phase determines the value function $\mathbf{U}_n$ along with the policy function $\boldsymbol{\phi}_n$ with typical element $U_n(\mathbf{e}')$, respectively, $\phi_n(\mathbf{e}')$.

If firm n is a potential entrant, then we set its price to infinity so that $D_n(\mathbf{p}) = 0$. To facilitate the analysis of the remaining decisions, we focus on firm 1; the derivations for firm 2 are analogous.

Exit decision of incumbent firm. If incumbent firm 1 exits the industry, it receives the scrap value X_1 in the current period and perishes. If it does not exit, its expected NPV is

$$\widehat{X}_1(\mathbf{e}') = \beta \left[V_1(\mathbf{e}')(1-\phi_2(\mathbf{e}')) + V_1(e_1', 0)\phi_2(\mathbf{e}') \right].$$

The probability of incumbent firm 1 exiting the industry in state $\mathbf{e}'$ is therefore $\phi_1(\mathbf{e}') = E_X \left[1 \left[X_1 \geq \widehat{X}_1(\mathbf{e}') \right] \right] = 1 - F_X(\widehat{X}_1(\mathbf{e}'))$, where $1\left[\cdot\right]$ is the indicator function and $\widehat{X}_1(\mathbf{e}')$ is the critical level of the scrap value above which exit occurs. Specifically,

$$\phi_1(\mathbf{e}') \quad = \quad 1 - F_X(\widehat{X}_1(\mathbf{e}'))$$

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$$= \begin{cases} 1 & \text{if} \quad \widehat{X}_1(\mathbf{e}') < \overline{X} - \Delta_X, \\ \frac{1}{2} - \frac{\left[\widehat{X}_1(\mathbf{e}') - \overline{X}\right]}{2\Delta_X} & \text{if} \quad \widehat{X}_1(\mathbf{e}') \in [\overline{X} - \Delta_X, \overline{X} + \Delta_X], \\ 0 & \text{if} \quad \widehat{X}_1(\mathbf{e}') > \overline{X} + \Delta_X. \end{cases}$$

Moreover, the expected NPV of incumbent firm 1 in the exit-entry phase is given by the Bellman equation

$$U_1(\mathbf{e}') = E_X \left[\max\left\{\widehat{X}_1(\mathbf{e}'), X_1\right\}\right]$$
$$= (1 - \phi_1(\mathbf{e}'))\beta \left[V_1(\mathbf{e}')(1 - \phi_2(\mathbf{e}')) + V_1(e_1', 0)\phi_2(\mathbf{e}')\right] + \phi_1(\mathbf{e}') E_X \left[X_1 | X_1 \geq \widehat{X}_1(\mathbf{e}')\right], \tag{OA1}$$

where $E_X \left[X_1 | X_1 \geq \widehat{X}_1(\mathbf{e}')\right]$ is the expectation of the scrap value conditional on exiting the industry. Specifically,

$$\begin{aligned} E_X \left[X_1 | X_1 \geq \widehat{X}_1(\mathbf{e}')\right] &= \frac{\int_{F_X^{-1}(1-\phi_1(\mathbf{e}'))}^{\overline{X}+\Delta_X} X_1 dF_X(X_1)}{\phi_1(\mathbf{e}')} \\ &= \frac{1}{\phi_1(\mathbf{e}')} \left[Z_X(0) - Z_X\left(1 - \phi_1(\mathbf{e}')\right)\right], \end{aligned}$$

where

$$Z_X(1-\phi) = \frac{1}{\Delta_X^2} \begin{cases} -\frac{1}{6}\left(\overline{X} - \Delta_X\right)^3 & \text{if} \quad 1 - \phi \leq 0, \\ \frac{1}{2}\left(\Delta_X - \overline{X}\right)\left(F_X^{-1}(1-\phi)\right)^2 + \frac{1}{3}\left(F_X^{-1}(1-\phi)\right)^3 & \text{if} \quad 1 - \phi \in \left[0, \frac{1}{2}\right], \\ \frac{1}{2}\left(\Delta_X + \overline{X}\right)\left(F_X^{-1}(1-\phi)\right)^2 - \frac{1}{3}\left(F_X^{-1}(1-\phi)\right)^3 - \frac{1}{3}\overline{X}^3 & \text{if} \quad 1 - \phi \in \left[\frac{1}{2}, 1\right], \\ \frac{1}{6}\left(\overline{X} + \Delta_X\right)^3 - \frac{1}{3}\overline{X}^3 & \text{if} \quad 1 - \phi \geq 1 \end{cases}$$

and

$$F_X^{-1}(1-\phi) = \overline{X} + \Delta_X \begin{cases} -1 & \text{if} \quad 1 - \phi \leq 0, \\ -1 + \sqrt{2(1-\phi)} & \text{if} \quad 1 - \phi \in \left[0, \frac{1}{2}\right], \\ 1 - \sqrt{2\phi} & \text{if} \quad 1 - \phi \in \left[\frac{1}{2}, 1\right], \\ 1 & \text{if} \quad 1 - \phi \geq 1. \end{cases}$$

Entry decision of potential entrant. There is a large queue of potential entrants. Depending on the number of incumbent firms, up to two potential entrants can enter the industry in each period. If a potential entrant does not enter, it perishes. If it enters, it becomes an incumbent firm without prior experience in the subsequent period. Hence, upon entry, the expected NPV of potential entrant 1 is

$$\widehat{S}_1(\mathbf{e}') = \beta \left[V_1(1, e_2')(1 - \phi_2(\mathbf{e}')) + V_1(1, 0)\phi_2(\mathbf{e}')\right].$$

In addition, potential entrant 1 incurs the setup cost S_1 in the current period. The probability of potential entrant 1 *not* entering the industry in state $\mathbf{e}'$ is therefore $\phi_1(\mathbf{e}') = E_S\left[1\left[S_1 \geq \widehat{S}_1(\mathbf{e}')\right]\right] = 1 - F_S(\widehat{S}_1(\mathbf{e}'))$, where $\widehat{S}_1(\mathbf{e}')$ is the critical level of the setup cost below which entry occurs. Specifically,

$$\phi_1(\mathbf{e}') = 1 - F_S(\widehat{S}_1(\mathbf{e}'))$$

$$= \begin{cases} 1 & \text{if} \quad \widehat{S}_1(\mathbf{e}') < \overline{S} - \Delta_S, \\ \frac{1}{2} - \frac{\left[\widehat{S}_1(\mathbf{e}') - \overline{S}\right]}{2\Delta_S} & \text{if} \quad \widehat{S}_1(\mathbf{e}') \in [\overline{S} - \Delta_S, \overline{S} + \Delta_S], \\ 0 & \text{if} \quad \widehat{S}_1(\mathbf{e}') > \overline{S} + \Delta_S. \end{cases}$$

Moreover, the expected NPV of potential entrant 1 in the exit-entry phase is given by the Bellman equation

$$U_1(\mathbf{e}') = E_S\left[\max\left\{\widehat{S}_1(\mathbf{e}') - S_1, 0\right\}\right]$$
$$= (1 - \phi_1(\mathbf{e}'))\left\{\beta[V_1(1, e_2')(1 - \phi_2(\mathbf{e}')) + V_1(1,0)\phi_2(\mathbf{e}')] - E_S\left[S_1 | S_1 \leq \widehat{S}_1(\mathbf{e}')\right]\right\}, \quad \text{(OA2)}$$

where $E_S\left[S_1 | S_1 \leq \widehat{S}_1(\mathbf{e}')\right]$ is the expectation of the setup cost conditional on entering the industry. Specifically,

$$\begin{aligned} E_S\left[S_1 | S_1 \leq \widehat{S}_1(\mathbf{e}')\right] &= \frac{\int_{\overline{S}-\Delta_S}^{F_S^{-1}(1-\phi_1(\mathbf{e}'))} S_1 dF_S(S_1)}{(1 - \phi_1(\mathbf{e}'))} \\ &= \frac{1}{\phi_1(\mathbf{e}')}\left[Z_S\left(1 - \phi_1(\mathbf{e}')\right) - Z_S(1)\right], \end{aligned}$$

where

$$Z_S(1-\phi) = \frac{1}{\Delta_S^2}\begin{cases} -\frac{1}{6}\left(\overline{S} - \Delta_S\right)^3 & \text{if} \quad 1 - \phi \leq 0, \\ \frac{1}{2}\left(\Delta_S - \overline{S}\right)\left(F_S^{-1}(1-\phi)\right)^2 + \frac{1}{3}\left(F_S^{-1}(1-\phi)\right)^3 & \text{if} \quad 1 - \phi \in \left[0, \frac{1}{2}\right], \\ \frac{1}{2}\left(\Delta_S + \overline{S}\right)\left(F_S^{-1}(1-\phi)\right)^2 - \frac{1}{3}\left(F_S^{-1}(1-\phi)\right)^3 - \frac{1}{3}\overline{S}^3 & \text{if} \quad 1 - \phi \in \left[\frac{1}{2}, 1\right], \\ \frac{1}{6}\left(\overline{S} + \Delta_S\right)^3 - \frac{1}{3}\overline{S}^3 & \text{if} \quad 1 - \phi \geq 1 \end{cases}$$

and

$$F_S^{-1}(1-\phi) = \overline{S} + \Delta_S\begin{cases} -1 & \text{if} \quad 1 - \phi \leq 0, \\ -1 + \sqrt{2(1-\phi)} & \text{if} \quad 1 - \phi \in \left[0, \frac{1}{2}\right], \\ 1 - \sqrt{2\phi} & \text{if} \quad 1 - \phi \in \left[\frac{1}{2}, 1\right], \\ 1 & \text{if} \quad 1 - \phi \geq 1. \end{cases}$$

Pricing decision of incumbent firm. In the price-setting phase, the expected NPV of incumbent firm 1 is

$$V_1(\mathbf{e}) = \max_{p_1} D_1(p_1, p_2(\mathbf{e}))(p_1 - c(e_1)) + \sum_{n=0}^{2} D_n(p_1, p_2(\mathbf{e}))U_1\left(\mathbf{e}^{n+}\right)$$
$$= \max_{p_1} D_1(p_1, p_2(\mathbf{e}))(p_1 - c(e_1)) + U_1(\mathbf{e}) + \sum_{n=1}^{2} D_n(p_1, p_2(\mathbf{e}))\left[U_1\left(\mathbf{e}^{n+}\right) - U_1(\mathbf{e})\right], \quad \text{(OA3)}$$

where we let $\mathbf{e}^{0+} = \mathbf{e}$ and use the fact that $\sum_{n=0}^{2} D_n(\mathbf{p}) = 1$. Because the maximand on the right-hand side of Bellman equation (OA3) is strictly quasiconcave in p_1 (given $p_2(\mathbf{e})$), the pricing decision $p_1(\mathbf{e})$ of incumbent firm 1 in state $\mathbf{e}$ is uniquely determined by the first-order condition

$$p_1(\mathbf{e}) - \frac{\sigma}{1 - D_1(\mathbf{p}(\mathbf{e}))} - c(e_1) + \left[U_1\left(\mathbf{e}^{1+}\right) - U_1(\mathbf{e})\right] + \Upsilon(p_2(\mathbf{e}))\left[U_1(\mathbf{e}) - U_1\left(\mathbf{e}^{2+}\right)\right] = 0 \quad \text{(OA4)}$$

(equation (2) in the main paper) and the probability of firm 2 making a sale conditional on firm 1 not making a sale is

$$\Upsilon(p_2(\mathbf{e})) = \frac{D_2(\mathbf{p}(\mathbf{e}))}{1 - D_1(\mathbf{p}(\mathbf{e}))} = \frac{\exp\left(-\frac{p_2(\mathbf{e})}{\sigma}\right)}{\exp\left(-\frac{p_0}{\sigma}\right) + \exp\left(-\frac{p_2(\mathbf{e})}{\sigma}\right)}.$$

OA1.2 Equivalence to a model that switches the price-setting and exit-entry phases

We show that our model is equivalent to a model that switches the order of the price-setting and exit-entry phases. During the exit-entry phase, the state changes from $\mathbf{e}$ to $\mathbf{e}'$; during the price-setting phase, the state changes from $\mathbf{e}'$ to $\mathbf{e}''$. Discounting occurs after the price-setting phase. The exit-entry phase determines the value function $\widehat{\mathbf{U}}_n$ along with the policy function $\widehat{\boldsymbol{\phi}}_n$ with typical element $\widehat{U}_n(\mathbf{e}')$, respectively, $\widehat{\phi}_n(\mathbf{e}')$; the price-setting phase determines the value function $\widehat{\mathbf{V}}_n$ along with the policy function $\widehat{\mathbf{p}}_n$ with typical element $\widehat{V}_n(\mathbf{e})$, respectively, $\widehat{p}_n(\mathbf{e})$.

We work backwards from the price-setting phase to the exit-entry phase.

Pricing decision of incumbent firm. In the price-setting phase, the expected NPV of incumbent firm 1 is

$$\widehat{V}_1(\mathbf{e}') = \max_{p_1} D_1(p_1, \widehat{p}_2(\mathbf{e}'))(p_1 - c(e_1')) + \beta\widehat{U}_1(\mathbf{e}') + \sum_{n=1}^{2} D_n(p_1, \widehat{p}_2(\mathbf{e}'))\beta\left[\widehat{U}_1\left(\mathbf{e}'^{n+}\right) - \widehat{U}_1(\mathbf{e}')\right]. \tag{OA5}$$

The pricing decision $\widehat{p}_1(\mathbf{e}')$ is uniquely determined by the first-order condition

$$\widehat{p}_1(\mathbf{e}') - \frac{\sigma}{1 - D_1(\widehat{\mathbf{p}}(\mathbf{e}'))} - c(e_1') + \beta\left[\widehat{U}_1\left(\mathbf{e}'^{1+}\right) - \widehat{U}_1(\mathbf{e}')\right] + \Upsilon(\widehat{p}_2(\mathbf{e}'))\beta\left[\widehat{U}_1(\mathbf{e}') - \widehat{U}_1\left(\mathbf{e}'^{2+}\right)\right] = 0. \tag{OA6}$$

Exit decision of incumbent firm. In the exit-entry phase, if incumbent firm 1 exits the industry, it receives the scrap value X_1 in the current period and perishes. If it does not exit, its expected NPV is

$$\widehat{\widehat{X}}_1(\mathbf{e}) = \left[\widehat{V}_1(\mathbf{e})(1 - \widehat{\phi}_2(\mathbf{e})) + \widehat{V}_1(e_1, 0)\widehat{\phi}_2(\mathbf{e})\right].$$

The probability of incumbent firm 1 exiting the industry in state $\mathbf{e}$ is therefore $\widehat{\phi}_1(\mathbf{e}) = 1 - F_X(\widehat{\widehat{X}}_1(\mathbf{e}))$ and the expected NPV of incumbent firm 1 in the exit-entry phase is given by the Bellman equation

$$\widehat{U}_1(\mathbf{e}) = (1 - \widehat{\phi}_1(\mathbf{e}))\left[\widehat{V}_1(\mathbf{e})(1 - \widehat{\phi}_2(\mathbf{e})) + \widehat{V}_1(e_1, 0)\widehat{\phi}_2(\mathbf{e})\right] + \widehat{\phi}_1(\mathbf{e})E_X\left[X_1 | X_1 \geq \widehat{\widehat{X}}_1(\mathbf{e})\right]. \tag{OA7}$$

Entry decision of potential entrant. If a potential entrant does not enter, it perishes. If it enters, it becomes an incumbent firm without prior experience in the subsequent period. Hence, upon entry, the expected NPV of potential entrant 1 is

$$\widehat{\widehat{S}}_1(\mathbf{e}) = \left[\widehat{V}_1(1, e_2)(1 - \widehat{\phi}_2(\mathbf{e})) + \widehat{V}_1(1, 0)\widehat{\phi}_2(\mathbf{e})\right].$$

The probability of potential entrant 1 *not* entering the industry in state $\mathbf{e}$ is therefore $\widehat{\phi}_1(\mathbf{e}) = 1 - F_S(\widehat{\widehat{S}}_1(\mathbf{e}))$ and the expected NPV of potential entrant 1 in the exit-entry phase is given by the Bellman equation

$$\widehat{U}_1(\mathbf{e}) = (1 - \widehat{\phi}_1(\mathbf{e}))\left\{[\widehat{V}_1(1, e_2)(1 - \widehat{\phi}_2(\mathbf{e})) + \widehat{V}_1(1, 0)\widehat{\phi}_2(\mathbf{e})] - E_S\left[S_1 | S_1 \leq \widehat{\widehat{S}}_1(\mathbf{e})\right]\right\}. \quad \text{(OA8)}$$

Equivalence. Let $\mathbf{V}_1$, $\mathbf{U}_1$, $\mathbf{p}_1$, and $\boldsymbol{\phi}_1$ solve equations (OA1), (OA2), (OA3), and (OA4). Define

$$\widehat{\mathbf{V}}_1 = \beta \mathbf{V}_1,$$
$$\widehat{\mathbf{U}}_1 = \frac{1}{\beta}\mathbf{U}_1,$$
$$\widehat{\mathbf{p}}_1 = \mathbf{p}_1,$$
$$\widehat{\boldsymbol{\phi}}_1 = \boldsymbol{\phi}_1.$$

It is straightforward to verify that $\widehat{\mathbf{V}}_1$, $\widehat{\mathbf{U}}_1$, $\widehat{\mathbf{p}}_1$, and $\widehat{\boldsymbol{\phi}}_1$ solve equations (OA5), (OA6), (OA7), and (OA8). This establishes the equivalence between the models.

OA1.3 Equivalence to a model with per-period, avoidable fixed costs

We show that our model with scrap values is equivalent to a model with per-period, avoidable fixed costs but without scrap values. For simplicity, we focus on the special case of mixed exit and entry strategies ($\Delta_X = \Delta_S = 0$).[1]

Incumbent firm. First consider incumbent firm 1. In the exit-entry phase in state $\mathbf{e}'$ with $e'_1 > 0$, the Bellman equation (OA1) becomes

$$U_1(\mathbf{e}') = \max\left\{\widehat{X}_1(\mathbf{e}'), \overline{X}\right\}, \quad \text{(OA9)}$$

where

$$\widehat{X}_1(\mathbf{e}') = \beta\left[V_1(\mathbf{e}')(1 - \phi_2(\mathbf{e}')) + V_1(e'_1, 0)\phi_2(\mathbf{e}')\right].$$

In the price-setting phase in state $\mathbf{e}$ with $e_1 > 0$, the Bellman equation (OA3) becomes

$$V_1(\mathbf{e}) = \max_{p_1} D_1(p_1, p_2(\mathbf{e}))(p_1 - c(e_1)) - \overline{F} + U_1(\mathbf{e}) + \sum_{n=1}^{2} D_n(p_1, p_2(\mathbf{e}))\left[U_1\left(\mathbf{e}^{n+}\right) - U_1(\mathbf{e})\right], \quad \text{(OA10)}$$

[1]As $\Delta_X \to 0$ and $\Delta_S \to 0$, the scrap value is fixed at $\overline{X}$ and the setup cost at $\overline{S}$ and we revert to mixed exit and entry strategies (Ulrich Doraszelski & Mark Satterthwaite 2010, Ulrich Doraszelski & Juan F. Escobar 2010).

where $\overline{F} \geq 0$ is per-period, avoidable fixed costs. Note that incumbent firm 1 can avoid the fixed costs for the subsequent period by deciding to exit the industry in the current period.

Potential entrant. Next consider potential entrant 1. In the exit-entry phase in state $\mathbf{e}'$ with $e_1' = 0$, the Bellman equation (OA2) becomes

$$U_1(\mathbf{e}') = \max\left\{\widehat{S}_1(\mathbf{e}') - \overline{S}, 0\right\}, \tag{OA11}$$

where

$$\widehat{S}_1(\mathbf{e}') = \beta[V_1(1, e_2')(1 - \phi_2(\mathbf{e}')) + V_1(1, 0)\phi_2(\mathbf{e}')].$$

Equilibrium. Let $\mathbf{V}_1^{(\overline{X},\overline{S},\overline{F})}$, $\mathbf{U}_1^{(\overline{X},\overline{S},\overline{F})}$, $\mathbf{p}_1^{(\overline{X},\overline{S},\overline{F})}$, and $\boldsymbol{\phi}_1^{(\overline{X},\overline{S},\overline{F})}$ denote the value and policy functions of firm 1 in a symmetric equilibrium for given values of $\left(\overline{X}, \overline{S}, \overline{F}\right)$; these solve the Bellman equations (OA9), (OA10), and (OA11) along with the corresponding optimality conditions.

Equivalence. Our model sets $\overline{X} \geq 0$ and $\overline{F} = 0$. We show that our model is equivalent to an alternative model that sets $\overline{X}' = 0$ and $\overline{F}' \geq 0$. To this end, we show that if $\mathbf{V}_1^{(\overline{X},\overline{S},0)}$, $\mathbf{U}_1^{(\overline{X},\overline{S},0)}$, $\mathbf{p}_1^{(\overline{X},\overline{S},0)}$, and $\boldsymbol{\phi}_1^{(\overline{X},\overline{S},0)}$ solve the Bellman equations (OA9), (OA10), and (OA11) given $\left(\overline{X}, \overline{S}, 0\right)$, then

$$\mathbf{V}_1^{(0,\overline{S}',\overline{F}')} = \mathbf{V}_1^{(\overline{X},\overline{S},0)} - \frac{\overline{X}}{\beta},$$

$$\mathbf{U}_1^{(0,\overline{S}',\overline{F}')}(\mathbf{e}) = \begin{cases} \mathbf{U}_1^{(\overline{X},\overline{S},0)}(\mathbf{e}) & \text{if} \quad e_1 = 0, \\ \mathbf{U}_1^{(\overline{X},\overline{S},0)}(\mathbf{e}) - \overline{X} & \text{if} \quad e_1 > 0, \end{cases}$$

$$\mathbf{p}_1^{(0,\overline{S}',\overline{F}')} = \mathbf{p}_1^{(\overline{X},\overline{S},0)},$$

$$\boldsymbol{\phi}_1^{(0,\overline{S}',\overline{F}')} = \boldsymbol{\phi}_1^{(\overline{X},\overline{S},0)}$$

solve these equations given $\left(0, \overline{S}' = \overline{S} - \overline{X}, \overline{F}' = \frac{(1-\beta)\overline{X}}{\beta}\right)$.

Starting with incumbent firm 1, plugging in the Bellman equations (OA9) and (OA10) given $\left(0, \overline{S}' = \overline{S} - \overline{X}, \overline{F}' = \frac{(1-\beta)\overline{X}}{\beta}\right)$ reduce to those under $\left(\overline{X}, \overline{S}, 0\right)$. Turning to potential entrant 1, the Bellman equation (OA11) given $\left(0, \overline{S}' = \overline{S} - \overline{X}, \overline{F}' = \frac{(1-\beta)\overline{X}}{\beta}\right)$ similarly reduces to that under $\left(\overline{X}, \overline{S}, 0\right)$.

OA2 First-best planner, welfare, and deadweight loss

OA2.1 First-best planner

We use $V^{FB}(\mathbf{e})$ to denote the expected NPV of total surplus in state $\mathbf{e}$ at the beginning of the period and $U^{FB}(\mathbf{e}')$ the expected NPV of total surplus in state $\mathbf{e}'$ *after* pricing decisions but *before* exit and entry decisions are made. The price-setting phase determines the value function $\mathbf{V}^{FB}$ along with the policy functions $\mathbf{p}_n^{FB}$ for $n \in \{1,2\}$; the exit-entry phase determines the value function $\mathbf{U}^{FB}$ along with the policy functions $\boldsymbol{\psi}_{\boldsymbol{\iota}}^{FB}$ for $\boldsymbol{\iota} \in \{0,1\}^2$. We refer to $\boldsymbol{\iota} = (\iota_1, \iota_2)$ as the operating decisions of the first-best planner and let $\psi_{1,1}^{FB}(\mathbf{e}')$ denote the probability that the planner in state $\mathbf{e}'$ decides to operate both firms in the subsequent period, $\psi_{1,0}^{FB}(\mathbf{e}')$ the probability that the planner decides to operate only firm 1, $\psi_{0,1}^{FB}(\mathbf{e}')$ the probability that the planner decides to operate only firm 2, and $\psi_{0,0}^{FB}(\mathbf{e}')$ the probability that the planner decides to operate neither firm. Note that $\sum_{\boldsymbol{\iota}\in\{0,1\}^2} \psi_{\boldsymbol{\iota}}^{FB}(\mathbf{e}') = 1$ and that the probability that firm 1 does *not* operate in state $\mathbf{e}'$ is $\phi_1^{FB}(\mathbf{e}') = \sum_{\iota_2=0}^{1} \psi_{0,\iota_2}^{FB}(\mathbf{e}')$. Note further that, by construction, $TS_\beta^{FB} = V^{FB}(0,0)$.

Operating decisions. Define

$$U_{\boldsymbol{\iota}}^{FB}(\mathbf{e}', \mathbf{X}, \mathbf{S}) = \begin{cases} \beta V^{FB}(e_1'\iota_1, e_2'\iota_2) + X_1(1-\iota_1) + X_2(1-\iota_2) & \text{if} \quad e_1' \neq 0, e_2' > 0, \\ \beta V^{FB}(\iota_1, e_2'\iota_2) - S_1\iota_1 + X_2(1-\iota_2) & \text{if} \quad e_1' = 0, e_2' > 0, \\ \beta V^{FB}(e_1'\iota_1, \iota_2) + X_1(1-\iota_1) - S_2\iota_2 & \text{if} \quad e_1' > 0, e_2' = 0, \\ \beta V^{FB}(\iota_1, \iota_2) - S_1\iota_1 - S_2\iota_2 & \text{if} \quad e_1' = 0, e_2' = 0 \end{cases} \tag{OA12}$$

to be the expected NPV of total surplus in state $\mathbf{e}'$ given operating decisions $\boldsymbol{\iota} \in \{0,1\}^2$, scrap values $\mathbf{X} = (X_1, X_2)$, and setup costs $\mathbf{S} = (S_1, S_2)$. Equation (OA12) distinguishes between firm n actively producing in the current period ($e_n' > 0$) and it being inactive ($e_n' = 0$). If firm n is active, then the first-best planner receives the scrap value X_n upon deciding *not* to operate it in the subsequent period ($\iota_n = 0$); if firm n is inactive, then the planner incurs the setup cost S_n upon deciding to operate it ($\iota_n = 1$). The optimal operating decisions are

$$U^{FB}\left(\mathbf{e}', \mathbf{X}, \mathbf{S}\right) = \max_{\boldsymbol{\iota}\in\{0,1\}^2} U_{\boldsymbol{\iota}}^{FB}(\mathbf{e}', \mathbf{X}, \mathbf{S}),$$

with associated operating probabilities

$$\psi_{\boldsymbol{\iota}}^{FB}\left(\mathbf{e}'\right) = E_{\mathbf{X},\mathbf{S}}\left[1\left[U^{FB}\left(\mathbf{e}', \mathbf{X}, \mathbf{S}\right) = U_{\boldsymbol{\iota}}^{FB}(\mathbf{e}', \mathbf{X}, \mathbf{S})\right]\right] \tag{OA13}$$

for $\boldsymbol{\iota} \in \{0,1\}^2$. Finally, the Bellman equation in the exit-entry phase is

$$U^{FB}(\mathbf{e}') = E_{\mathbf{X},\mathbf{S}}\left[U^{FB}\left(\mathbf{e}', \mathbf{X}, \mathbf{S}\right)\right]. \tag{OA14}$$

Pricing decisions. In the price-setting phase, the expected NPV of total surplus is

$$V^{FB}(\mathbf{e}) = \max_{\mathbf{p}} CS(\mathbf{p}) + \sum_{n=1}^{2} D_n(\mathbf{p})(p_n - c(e_n)) + \sum_{n=0}^{2} D_n(\mathbf{p}) U^{FB}\left(\mathbf{e}^{n+}\right), \tag{OA15}$$

where the first term is consumer surplus and the second term is the static profit of incumbent firms.[2] Because the outside good is priced at cost, its profit is zero.

[2]If firm n is inactive, then we again set its price to infinity so that $D_n(\mathbf{p}) = 0$ and its contribution to $CS(\mathbf{p})$ is zero.

The solution to the maximization problem on the right-hand side of Bellman equation (OA15) can be shown to exist and to be unique, and it is given by

$$p_n^{FB}(\mathbf{e}) = c(e_n) - \left[U^{FB}\left(\mathbf{e}^{n+}\right) - U^{FB}(\mathbf{e})\right]$$

for $n \in \{1,2\}$. The pricing decision $p_n^{FB}(\mathbf{e})$ reflects the marginal cost of production $c(e_n)$ of incumbent firm n net of the marginal benefit to society of moving the firm down its learning curve.

Solution. Without loss of generality, we take the first-best planner solution to be symmetric in that $V^{FB}(\mathbf{e}) = V^{FB}(e_2, e_1)$, $U^{FB}(\mathbf{e}) = U^{FB}(e_2, e_1)$, $p_1^{FB}(\mathbf{e}) = p_2^{FB}(e_2, e_1)$, and $\psi_{\boldsymbol{\iota}}^{FB}(\mathbf{e}) = \psi_{\iota_2,\iota_1}^{FB}(e_2, e_1)$ for $\boldsymbol{\iota} \in \{0,1\}^2$.

We solve the first-best planner problem using value function iteration combined with quasi-Monte Carlo integration (Halton sequences of length $10,000$) to evaluate the operating probabilities in equation (OA13) and the Bellman equation (OA14).

OA2.2 Welfare and deadweight loss

Under centralized exit and entry, producer surplus in state $\mathbf{e}$ is

$$\begin{aligned}
PS^{FB}(\mathbf{e}) = \sum_{n=1}^{2} D_n\left(\mathbf{p}^{FB}(\mathbf{e})\right)\left(p_n^{FB}(\mathbf{e}) - c(e_n)\right) + \sum_{n=0}^{2} D_n\left(\mathbf{p}^{FB}(\mathbf{e})\right) \\
\times\Big\{\psi_{1,1}^{FB}\left(\mathbf{e}^{n+}\right) E_{\mathbf{X},\mathbf{S}}\left[-1\left[e_1=0\right]S_1 - 1\left[e_2=0\right]S_2 | U^{FB}\left(\mathbf{e}^{n+},\mathbf{X},\mathbf{S}\right) = U_{1,1}^{FB}\left(\mathbf{e}^{n+},\mathbf{X},\mathbf{S}\right)\right] \\
+\psi_{1,0}^{FB}\left(\mathbf{e}^{n+}\right) E_{\mathbf{X},\mathbf{S}}\left[-1\left[e_1=0\right]S_1 + 1\left[e_2>0\right]X_2 | U^{FB}\left(\mathbf{e}^{n+},\mathbf{X},\mathbf{S}\right) = U_{1,0}^{FB}\left(\mathbf{e}^{n+},\mathbf{X},\mathbf{S}\right)\right] \\
+\psi_{0,1}^{FB}\left(\mathbf{e}^{n+}\right) E_{\mathbf{X},\mathbf{S}}\left[1\left[e_1>0\right]X_1 - 1\left[e_2=0\right]S_2 | U^{FB}\left(\mathbf{e}^{n+},\mathbf{X},\mathbf{S}\right) = U_{0,1}^{FB}\left(\mathbf{e}^{n+},\mathbf{X},\mathbf{S}\right)\right] \\
+\psi_{0,0}^{FB}\left(\mathbf{e}^{n+}\right) E_{\mathbf{X},\mathbf{S}}\left[1\left[e_1>0\right]X_1 + 1\left[e_2>0\right]X_2 | U^{FB}\left(\mathbf{e}^{n+},\mathbf{X},\mathbf{S}\right) = U_{0,0}^{FB}\left(\mathbf{e}^{n+},\mathbf{X},\mathbf{S}\right)\right]\Big\},
\end{aligned}$$

where $\mathbf{p}^{FB}(\mathbf{e}) = \left(p_1^{FB}(\mathbf{e}), p_2^{FB}(\mathbf{e})\right)$.

OA3 Is dynamic competition necessarily fully efficient?

In contrast to rent-seeking models, firms in our learning-by-doing model jostle for competitive advantage by pricing aggressively rather than by engaging in socially wasteful activities. To the extent that rents can be efficiently transferred from firms to consumers, one may thus conjecture that dynamic competition is necessarily fully efficient. This conjecture, however, overlooks that dynamic competition extends beyond pricing into exit and entry. We highlight distortions in exit and entry and demonstrate that dynamic competition is not necessarily fully efficient in an analytically tractable special case of our model with a two-step learning curve, homogeneous goods, and mixed exit and entry strategies:

Assumption 1 (Two-step learning curve)

1. *$M = m = 2$;*

2. $\sigma = 0$;

3. $\Delta_X = \Delta_S = 0$.

Because goods are homogeneous by part (2) of Assumption 1, the firm that sets the lowest price makes the sale.[3] Moreover, aggregate demand for the inside goods is inelastic at prices below p_0. There are therefore no distortions in pricing.

To rule out uninteresting scenarios we further assume:

Assumption 2 (Parameter restrictions)

1. $p_0 \geq \kappa$;

2. $\overline{S} > \overline{X} \geq 0$;

3. $\beta \left(p_0 - \kappa + \frac{\beta}{1-\beta} (p_0 - \rho\kappa) \right) > \overline{S}$.

By part (1) of Assumption 2, the marginal cost of the outside good $p_0 = c_0$ is at least as high as the marginal cost $c(1) = \kappa$ of an incumbent firm at the top of its learning curve. By part (2), the setup cost is positive and partially sunk and the scrap value is nonnegative. By part (3), operating a single firm forever is socially beneficial.

First-best planner solution. The first-best planner solution is straightforward:

Proposition 1 (First-best planner solution) *Under Assumptions 1 and 2, there exists the first-best planner solution shown in Table OA1.*[4]

We prove Proposition 1 below after first describing its implications.

Because goods are homogeneous and product variety is not socially beneficial, the planner operates the industry as a natural monopoly. In state $(0, 0)$ in period 0, the planner decides to operate a single firm (say firm 1) in the subsequent period. In state $(1, 0)$ in period 1, firm 1 charges just below p_0, makes the sale, and moves down its learning curve. The industry remains in state $(2, 0)$ in period $t \geq 2$ and firm 1 again makes the sale. The expected NPV of total surplus is thus[5]

$$TS_{\beta}^{FB} = v - p_0 + \beta (v - \kappa) + \frac{\beta^2}{1-\beta} (v - \rho\kappa) - \overline{S} = \frac{v - p_0}{1-\beta} + \beta \left(p_0 - \kappa + \frac{\beta}{1-\beta} (p_0 - \rho\kappa) \right) - \overline{S},$$

and the maximum value added by the industry is

$$VA_{\beta} = \beta \left(p_0 - \kappa + \frac{\beta}{1-\beta} (p_0 - \rho\kappa) \right) - \overline{S}.$$

[3] If there is more than one such firm, each of them makes the sale with equal probability.

[4] Note that while there exist asymmetric solutions, we focus on the symmetric solution. In particular, we set $\psi_{1,0}^{FB}(\mathbf{e}) = \psi_{0,1}^{FB}(\mathbf{e}) = \frac{1}{2}$ in state $\mathbf{e} = (e, e)$. Note also that while firm n may charge any price below p_0, we arbitrarily set $p_n(\mathbf{e}) = p_0^-$ in state $\mathbf{e} \geq (0, 0)$.

[5] The term $v - p_0$ arises because the consumer purchases the outside good in state $(0, 0)$.

$\mathbf{e}$	$p_1^{FB}(\mathbf{e})$	$p_2^{FB}(\mathbf{e})$	$\psi_{0,0}^{FB}(\mathbf{e})$	$\psi_{1,0}^{FB}(\mathbf{e})$	$\psi_{0,1}^{FB}(\mathbf{e})$	$\psi_{1,1}^{FB}(\mathbf{e})$	$V^{FB}(\mathbf{e})$	$U^{FB}(\mathbf{e})$
$(0,0)$	∞	∞	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$v - p_0 + \beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa) - \overline{S}$	$\beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa) - \overline{S}$
$(0,1)$	∞	p_0^-	0	0	1	0	$v - \kappa + \frac{\beta}{1-\beta}(v-\rho\kappa)$	$\beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa)$
$(0,2)$	∞	p_0^-	0	0	1	0	$\frac{1}{1-\beta}(v-\rho\kappa)$	$\frac{\beta}{1-\beta}(v-\rho\kappa)$
$(1,0)$	p_0^-	∞	0	1	0	0	$v - \kappa + \frac{\beta}{1-\beta}(v-\rho\kappa)$	$\beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa)$
$(1,1)$	p_0^-	p_0^-	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$v - \kappa + \frac{\beta}{1-\beta}(v-\rho\kappa) + \overline{X}$	$\beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa) + \overline{X}$
$(1,2)$	p_0	p_0^-	0	0	1	0	$\frac{1}{1-\beta}(v-\rho\kappa) + \overline{X}$	$\frac{\beta}{1-\beta}(v-\rho\kappa) + \overline{X}$
$(2,0)$	p_0^-	∞	0	1	0	0	$\frac{1}{1-\beta}(v-\rho\kappa)$	$\frac{\beta}{1-\beta}(v-\rho\kappa)$
$(2,1)$	p_0^-	p_0	0	1	0	0	$\frac{1}{1-\beta}(v-\rho\kappa) + \overline{X}$	$\frac{\beta}{1-\beta}(v-\rho\kappa) + \overline{X}$
$(2,2)$	p_0^-	p_0^-	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{1-\beta}(v-\rho\kappa) + \overline{X}$	$\frac{\beta}{1-\beta}(v-\rho\kappa) + \overline{X}$

Table OA1: First-best planner solution. Two-step learning curve. In columns labelled $p_n^{FB}(\mathbf{e})$, superscript $-$ indicates that firm n charges just below the price stated.

Proof of Proposition 1. The proof proceeds in two steps. First, we show that given the policy functions, the value functions solve the Bellman equations (OA14) and (OA15). Second, we show that there is no profitable one-shot deviation in any state of the industry.

Plugging in the policy functions, the Bellman equations (OA14) and (OA15) become:

$$\begin{aligned}
U^{FB}(0,0) &= -\overline{S} + \beta\left(\frac{1}{2}V^{FB}(0,1) + \frac{1}{2}V^{FB}(1,0)\right),\\
U^{FB}(0,1) &= \beta V^{FB}(0,1),\\
U^{FB}(0,2) &= \beta V^{FB}(0,2),\\
U^{FB}(1,0) &= \beta V^{FB}(1,0),\\
U^{FB}(1,1) &= \overline{X} + \beta\left(\frac{1}{2}V^{FB}(0,1) + \frac{1}{2}V^{FB}(1,0)\right),\\
U^{FB}(1,2) &= \overline{X} + \beta V^{FB}(0,2),\\
U^{FB}(2,0) &= \beta V^{FB}(2,0),\\
U^{FB}(2,1) &= \overline{X} + \beta V^{FB}(2,0),\\
U^{FB}(2,2) &= \overline{X} + \beta\left(\frac{1}{2}V^{FB}(0,2) + \frac{1}{2}V^{FB}(2,0)\right),\\
V^{FB}(0,0) &= v - p_0 + U^{FB}(0,0),\\
V^{FB}(0,1) &= v - \kappa + U^{FB}(0,2),\\
V^{FB}(0,2) &= v - \rho\kappa + U^{FB}(0,2),\\
V^{FB}(1,0) &= v - \kappa + U^{FB}(2,0),\\
V^{FB}(1,1) &= v - \kappa + \frac{1}{2}U^{FB}(1,2) + \frac{1}{2}U^{FB}(2,1),\\
V^{FB}(1,2) &= v - \rho\kappa + U^{FB}(1,2),\\
V^{FB}(2,0) &= v - \rho\kappa + U^{FB}(2,0),\\
V^{FB}(2,1) &= v - \rho\kappa + U^{FB}(2,1),\\
V^{FB}(2,2) &= v - \rho\kappa + U^{FB}(2,2).
\end{aligned}$$

It is easy but tedious to show that the value functions solve the Bellman equations.

We proceed state-by-state to show that there is no profitable one-shot deviation. It suffices to consider deviations in pure strategies.

1. Exit-entry phase in state $\mathbf{e} = (0,0)$: Deviating to $\psi_{0,0}^{FB}(\mathbf{e}) = 1$ yields $\beta V^{FB}(\mathbf{e}) < U^{FB}(\mathbf{e})$ by part (iii) of Assumption 2 because

$$\begin{aligned}
\beta\left(v - p_0 + \beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa) - \overline{S}\right) &< \beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa) - \overline{S}\\
\Leftrightarrow (1-\beta)\overline{S} &< (1-\beta)\beta(p_0-\kappa) + \beta^2(p_0-\rho\kappa)\\
\Leftrightarrow \overline{S} &< \beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0-\rho\kappa)\right).
\end{aligned}$$

Deviating to $\psi_{1,0}^{FB}(\mathbf{e}) = 1$ yields $-\overline{S} + \beta V^{FB}(1,0) = U^{FB}(\mathbf{e})$. Deviating to $\psi_{0,1}^{FB}(\mathbf{e}) = 1$ yields $-\overline{S} + \beta V^{FB}(0,1) = U^{FB}(\mathbf{e})$. Deviating to $\psi_{1,1}^{FB}(\mathbf{e}) = 1$ yields $-2\overline{S} + \beta V^{FB}(1,1) < U^{FB}(\mathbf{e})$ by part (ii) of Assumption 2 because

$$-2\overline{S} + \beta\left(v - \kappa + \frac{\beta}{1-\beta}(v - \rho\kappa) + \overline{X}\right) < \beta(v - \kappa) + \frac{\beta^2}{1-\beta}(v - \rho\kappa) - \overline{S}$$
$$\Leftrightarrow \beta\overline{X} < \overline{S}.$$

2. Exit-entry phase in state $\mathbf{e} = (0,1)$: Deviating to $\psi_{0,0}^{FB}(\mathbf{e}) = 1$ yields $\overline{X} + \beta V^{FB}(0,0) < U^{FB}(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2. Deviating to $\psi_{1,0}^{FB}(\mathbf{e}) = 1$ yields $\overline{X} - \overline{S} + \beta V^{FB}(1,0) < U^{FB}(\mathbf{e})$ by part (ii) of Assumption 2. Deviating to $\psi_{1,1}^{FB}(\mathbf{e}) = 1$ yields $-\overline{S} + \beta V^{FB}(1,1) < U^{FB}(\mathbf{e})$ by part (ii) of Assumption 2.

3. Exit-entry phase in state $\mathbf{e} = (0,2)$: Deviating to $\psi_{0,0}^{FB}(\mathbf{e}) = 1$ yields $\overline{X} + \beta V^{FB}(0,0) < U^{FB}(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2. Deviating to $\psi_{1,0}^{FB}(\mathbf{e}) = 1$ yields $\overline{X} - \overline{S} + \beta V^{FB}(1,0) < U^{FB}(\mathbf{e})$ by part (ii) of Assumption 2. Deviating to $\psi_{1,1}^{FB}(\mathbf{e}) = 1$ yields $-\overline{S} + \beta V^{FB}(1,2) < U^{FB}(\mathbf{e})$ by part (ii) of Assumption 2.

4. Exit-entry phase in state $\mathbf{e} = (1,0)$: Analogous to exit-entry phase in state $\mathbf{e} = (0,1)$.

5. Exit-entry phase in state $\mathbf{e} = (1,1)$: Deviating to $\psi_{0,0}^{FB}(\mathbf{e}) = 1$ yields $2\overline{X} + \beta V^{FB}(0,0) < U^{FB}(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2. Deviating to $\psi_{1,0}^{FB}(\mathbf{e}) = 1$ yields $\overline{X} + \beta V^{FB}(1,0) = U^{FB}(\mathbf{e})$. Deviating to $\psi_{0,1}^{FB}(\mathbf{e}) = 1$ yields $\overline{X} + \beta V^{FB}(0,1) = U^{FB}(\mathbf{e})$. Deviating to $\psi_{1,1}^{FB}(\mathbf{e}) = 1$ yields $\beta V^{FB}(\mathbf{e}) < U^{FB}(\mathbf{e})$.

6. Exit-entry phase in state $\mathbf{e} = (1,2)$: Deviating to $\psi_{0,0}^{FB}(\mathbf{e}) = 1$ yields $2\overline{X} + \beta V^{FB}(0,0) < U^{FB}(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2. Deviating to $\psi_{1,0}^{FB}(\mathbf{e}) = 1$ yields $\overline{X} + \beta V^{FB}(1,0) < U^{FB}(\mathbf{e})$. Deviating to $\psi_{1,1}^{FB}(\mathbf{e}) = 1$ yields $\beta V^{FB}(1,2) < U^{FB}(\mathbf{e})$.

7. Exit-entry phase in state $\mathbf{e} = (2,0)$: Analogous to exit-entry phase in state $\mathbf{e} = (0,2)$.

8. Exit-entry phase in state $\mathbf{e} = (2,1)$: Analogous to exit-entry phase in state $\mathbf{e} = (1,2)$.

9. Exit-entry phase in state $\mathbf{e} = (2,2)$: Deviating to $\psi_{0,0}^{FB}(\mathbf{e}) = 1$ yields $2\overline{X} + \beta V^{FB}(0,0) < U^{FB}(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2. Deviating to $\psi_{1,0}^{FB}(\mathbf{e}) = 1$ yields $\overline{X} + \beta V^{FB}(2,0) = U^{FB}(\mathbf{e})$. Deviating to $\psi_{0,1}^{FB}(\mathbf{e}) = 1$ yields $\overline{X} + \beta V^{FB}(0,2) = U^{FB}(\mathbf{e})$. Deviating to $\psi_{1,1}^{FB}(\mathbf{e}) = 1$ yields $\beta V^{FB}(\mathbf{e}) < U^{FB}(\mathbf{e})$.

10. Price-setting phase in state $\mathbf{e} = (0,0)$: By default.

11. Price-setting phase in state $\mathbf{e} = (0,1)$: Deviating to firm 2 matching the outside good ($p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(v - p_0) + \frac{1}{2}(v - \kappa) + \frac{1}{2}U^{FB}(\mathbf{e}) + \frac{1}{2}U^{FB}(0,2) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 2 being undercut by the outside good ($p_2^{FB}(\mathbf{e}) > p_0$) yields $v - p_0 + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$ by part (i) of Assumption 2.

12. Price-setting phase in state $\mathbf{e} = (0, 2)$: Deviating to firm 2 matching the outside good ($p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(v - p_0) + \frac{1}{2}(v - \rho\kappa) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 2 being undercut by the outside good ($p_2^{FB}(\mathbf{e}) > p_0$) yields $v - p_0 + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$ by part (i) of Assumption 2.

13. Price-setting phase in state $\mathbf{e} = (1, 0)$: Analogous to price-setting phase in state $\mathbf{e} = (0, 1)$.

14. Price-setting phase in state $\mathbf{e} = (1, 1)$: Deviating to firm 1, say, matching the outside good and firm 2 undercutting the outside good ($p_1^{FB}(\mathbf{e}) = p_0$ and $p_2^{FB}(\mathbf{e}) = p_0^-$) yields $v - \kappa + U^{FB}(1, 2) = V^{FB}(\mathbf{e})$. Deviating to firm 1, say, being undercut by the outside good and firm 2 undercutting the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) = p_0^-$) yields $v - \kappa + U^{FB}(1, 2) = V^{FB}(\mathbf{e})$. Deviating to firm 1 matching the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) = p_0$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{3}(v - p_0) + \frac{2}{3}(v - \kappa) + \frac{1}{3}U^{FB}(\mathbf{e}) + \frac{1}{3}U^{FB}(2, 1) + \frac{1}{3}U^{FB}(1, 2) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1, say, being undercut by the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(v - p_0) + \frac{1}{2}(v - \kappa) + \frac{1}{2}U^{FB}(\mathbf{e}) + \frac{1}{2}U^{FB}(1, 2) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1 being undercut by the outside good and firm 2 being undercut by the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) > p_0$) yields $(v - p_0) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$ by part (i) of Assumption 2.

15. Price-setting phase in state $\mathbf{e} = (1, 2)$: Deviating to firm 1 undercutting the outside good and firm 2 undercutting the outside good ($p_1^{FB}(\mathbf{e}) = p_0^-$ and $p_2^{FB}(\mathbf{e}) = p_0^-$) yields

$$\frac{1}{2}(v - \kappa) + \frac{1}{2}(v - \rho\kappa) + \frac{1}{2}U^{FB}(2, 2) + \frac{1}{2}U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e}).$$

Deviating to firm 1 being undercut by the outside good and firm 2 undercutting the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) = p_0^-$) yields $v - \rho\kappa + U^{FB}(\mathbf{e}) = V^{FB}(\mathbf{e})$. Deviating to firm 1 undercutting the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) = p_0^-$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields $v - \kappa + U^{FB}(2, 2) \leq V^{FB}(\mathbf{e})$. Deviating to firm 1 matching the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) = p_0$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{3}(v - p_0) + \frac{1}{3}(v - \kappa) + \frac{1}{3}(v - \rho\kappa) + \frac{2}{3}U^{FB}(\mathbf{e}) + \frac{1}{3}U^{FB}(2, 2) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1 being undercut by the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(v - p_0) + \frac{1}{2}(v - \rho\kappa) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1 undercutting the outside good and firm 2 being undercut by the outside good ($p_1^{FB}(\mathbf{e}) = p_0^-$ and $p_2^{FB}(\mathbf{e}) > p_0$) yields $v - \kappa + U^{FB}(2,2) \leq V^{FB}(\mathbf{e})$. Deviating to firm 1 matching the outside good and firm 2 being undercut by the outside good ($p_1^{FB}(\mathbf{e}) = p_0$ and $p_2^{FB}(\mathbf{e}) > p_0$) yields

$$\frac{1}{2}(v - p_0) + \frac{1}{2}(v - \kappa) + \frac{1}{2}U^{FB}(\mathbf{e}) + \frac{1}{2}U^{FB}(2,2) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1 being undercut by the outside good and firm 2 being undercut by the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) > p_0$) yields $(v - p_0) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$ by part (i) of Assumption 2.

16. Price-setting phase in state $\mathbf{e} = (2,0)$: Analogous to price-setting phase in state $\mathbf{e} = (0,2)$.

17. Price-setting phase in state $\mathbf{e} = (2,1)$: Analogous to price-setting phase in state $\mathbf{e} = (1,2)$.

18. Price-setting phase in state $\mathbf{e} = (2,2)$: Deviating to firm 1, say, matching the outside good and firm 2 undercutting the outside good ($p_1^{FB}(\mathbf{e}) = p_0$ and $p_2^{FB}(\mathbf{e}) = p_0^-$) yields $v - \rho\kappa + U^{FB}(\mathbf{e}) = V^{FB}(\mathbf{e})$. Deviating to firm 1, say, being undercut by the outside good and firm 2 undercutting the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) = p_0^-$) yields $v - \rho\kappa + U^{FB}(\mathbf{e}) = V^{FB}(\mathbf{e})$. Deviating to firm 1 matching the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) = p_0$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{3}(v - p_0) + \frac{2}{3}(v - \rho\kappa) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1, say, being undercut by the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(v - p_0) + \frac{1}{2}(v - \rho\kappa) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1 being undercut by the outside good and firm 2 being undercut by the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) > p_0$) yields $(v - p_0) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$ by part (i) of Assumption 2.

Equilibrium. Even if pricing is efficient, exit and entry may not be:

Proposition 2 (Equilibrium.) *Under Assumptions 1 and 2, there exists the equilibrium shown in Table OA2. The deadweight loss is*

$$DWL_\beta = \frac{\phi_1(0,0)(1-\beta)}{1 - \beta\phi_1(0,0)^2} VA_\beta + \frac{(1 - \phi_1(0,0))^2}{1 - \beta\phi_1(0,0)^2}\left(\overline{S} - \beta\overline{X}\right) \tag{OA16}$$

and the relative deadweight loss is

$$\frac{DWL_\beta}{VA_\beta} = \frac{\phi_1(0,0) - \beta\phi_1(0,0)^2}{1 - \beta\phi_1(0,0)^2}. \tag{OA17}$$

Moreover, $\frac{d\left(1-\phi_1(0,0)^2\right)}{d\rho} < 0$ *and* $\frac{d(DWL_\beta/VA_\beta)}{d\rho} > 0$*: as learning economies strengthen, the probability* $1-\phi_1(0,0)^2$ *that the industry "takes off" increases and the relative deadweight loss* $\frac{DWL_\beta}{VA_\beta}$ *decreases.*

We prove Proposition 2 below.

The deadweight loss arises because the entry process is decentralized and uncoordinated. The industry can therefore suffer from over-entry and under-entry. To illustrate, we sketch out the evolution of the industry in the equilibrium shown in Table OA2. In state $(0,0)$ in period 0, a single firm enters the industry with probability $2(1-\phi_1(0,0))\phi_1(0,0)$, both firms enter with probability $(1-\phi_1(0,0))^2$, and no firms enter with probability $\phi_1(0,0)^2$. The industry continues to evolve as follows:

- *Case 1.* If a single firm (say firm 1) enters, then in state $(1,0)$ in period 1 it charges a price just below the price of the outside good p_0, makes the sale, and moves down its learning curve. In state $(2,0)$ firm 1 remains in the industry ($\phi_1(2,0)=0$) and firm 2 does not enter ($\phi_1(0,2)=1$). The industry remains in state $(2,0)$ in period $t \geq 2$, and firm 1 again makes the sale.

- *Case 2: Over-entry.* If both firms enter, then in state $(1,1)$ in period 1 they charge a price less than static marginal cost κ. One of the firms (say firm 1) makes the sale and moves down its learning curve. In state $(2,1)$, the leader (firm 1) remains in the industry ($\phi_1(2,1)=0$) and the follower (firm 2) exits ($\phi_1(1,2)=1$). The industry moves to—and remains in—state $(2,0)$ in period $t \geq 2$. Note that pricing in state $(1,1)$ is so aggressive that both firms incur a loss of $-\left(\frac{\beta}{1-\beta}(p_0-\rho\kappa)-\overline{X}\right)$ that fully dissipates any future gains from monopolizing the industry.

- *Case 3: Under-entry.* If no firm enters, then the above process repeats itself in state $(0,0)$ in period 1.

In short, the intuition that dynamic competition is necessarily fully efficient is incomplete. In the equilibrium shown in Table OA2, while the industry evolves towards the monopolistic structure that the first-best planner operates, this may happen slowly over time due to either over-entry or under-entry.[6] Wasteful duplication and delay (Patrick Bolton & Joseph Farrell 1990) are both integral parts of the equilibrium.

The equilibrium shown in Table OA2 further entails a war of attrition (J. Maynard Smith 1974, Jean Tirole 1988, Jeremy Bulow & Paul Klemperer 1999) in state $(2,2)$, although state $(2,2)$ is off the equilibrium path starting from state $(0,0)$. The war of attrition arises because a firm is better off staying in the industry if its rival exits but worse off if its rival stays. As a firm hopes to outlast its rival, it clings to the industry. The resulting non-operating probability is $\phi_1(2,2) = \frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0-\rho\kappa)-\beta\overline{X}} \in (0,1)$, whereas the first-best planner

[6] The first term in equation (OA16) is due to under-entry and the "discount factor" $\frac{\phi_1(0,0)(1-\beta)}{1-\beta\phi_1(0,0)^2}$ captures the stochastic length of time over which under-entry may occur; the second term is due to over-entry and the "discount factor" $\frac{(1-\phi_1(0,0))^2}{1-\beta\phi_1(0,0)^2}$ captures the stochastic length of time over which over-entry can occur after potentially many periods of under-entry.

$\mathbf{e}$	$p_1(\mathbf{e})$	$\phi_1(\mathbf{e})$	$V_1(\mathbf{e})$	$U_1(\mathbf{e})$
$(0,0)$	∞	$\frac{\overline{S}-\beta\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	$-$	0
$(0,1)$	∞	1	$-$	0
$(0,2)$	∞	1	$-$	0
$(1,0)$	p_0^-	0	$p_0-\kappa+\frac{\beta}{1-\beta}\left(p_0-\rho\kappa\right)$	$\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}\left(p_0-\rho\kappa\right)\right)$
$(1,1)$	$\kappa-\left(\frac{\beta}{1-\beta}\left(p_0-\rho\kappa\right)-\overline{X}\right)$	$\frac{(1-\beta)\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$
$(1,2)$	κ	1	$\overline{X}$	$\overline{X}$
$(2,0)$	p_0^-	0	$\frac{p_0-\rho\kappa}{1-\beta}$	$\frac{\beta}{1-\beta}\left(p_0-\rho\kappa\right)$
$(2,1)$	κ^-	0	$(1-\rho)\kappa+\frac{\beta}{1-\beta}\left(p_0-\rho\kappa\right)$	$\frac{\beta}{1-\beta}\left(p_0-\rho\kappa\right)$
$(2,2)$	$\rho\kappa$	$\frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0-\rho\kappa)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$

Table OA2: Equilibrium. Two-step learning curve. In column labelled $p_1(\mathbf{e})$, superscript $-$ indicates that firm 1 charges just below the price stated.

ceases to operate one of the two firms in state $(2,2)$. Because the exit process is decentralized and uncoordinated, the industry can suffer not only from over-exit but also from under-exit.[7]

Proof of Proposition 2. The proof proceeds in two steps. First, we show that given the policy functions, the value functions solve the Bellman equations (OA1), (OA2), and (OA3). Second, we show that there is no profitable one-shot deviation in any state of the industry.

Plugging in the policy functions, the Bellman equations (OA1), (OA2), and (OA3) become:

$$
\begin{aligned}
U_1(0,0) &= \left(1 - \frac{\overline{S} - \beta\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\right) \\
&\times \left(-\overline{S} + \frac{\overline{S} - \beta\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\beta V_1(1,0) + \left(1 - \frac{\overline{S} - \beta\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\right)\beta V_1(1,1)\right), \\
U_1(0,1) &= 0, \\
U_1(0,2) &= 0, \\
U_1(1,0) &= \beta V_1(1,0), \\
U_1(1,1) &= \frac{(1-\beta)\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\overline{X} + \left(1 - \frac{(1-\beta)\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\right) \\
&\times \left(\frac{(1-\beta)\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\beta V_1(1,0) + \left(1 - \frac{(1-\beta)\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\right)\beta V_1(1,1)\right), \\
U_1(1,2) &= \overline{X}, \\
U_1(2,0) &= \beta V_1(2,0), \\
U_1(2,1) &= \beta V_1(2,0), \\
U_1(2,2) &= \frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \beta\overline{X}}\overline{X} + \left(1 - \frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \beta\overline{X}}\right) \\
&\times \left(\frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \beta\overline{X}}\beta V_1(2,0) + \left(1 - \frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \beta\overline{X}}\right)\beta V_1(2,2)\right), \\
V_1(1,0) &= p_0 - \kappa + U_1(2,0), \\
V_1(1,1) &= -\frac{1}{2}\left(\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \overline{X}\right) + \frac{1}{2}U_1(1,2) + \frac{1}{2}U_1(2,1), \\
V_1(1,2) &= U_1(1,2), \\
V_1(2,0) &= p_0 - \rho\kappa + U_1(2,0), \\
V_1(2,1) &= \kappa(1-\rho) + U_1(2,1),
\end{aligned}
$$

[7] Coordination failures in exit and entry may be exacerbated if there are more than two firms (Luis M.B. Cabral 1993, Nikolaos Vettas 1998). Intuitively, the support of the binomial distribution becomes more spread out with more draws.

$$V_1(2,2) = U_1(2,2),$$

where we omit the Bellman equation (OA3) for state $\mathbf{e}$ if $e_1 = 0$. Recall that the firm that sets the lowest price makes the sale for sure and that, if there is more than one such firm, each of them makes the sale with equal probability. It is easy but tedious to show that the value functions solve the Bellman equations.

We proceed state-by-state to show that there is no profitable one-shot deviation. It suffices to consider deviations in pure strategies.[8]

1. Exit-entry phase in state $\mathbf{e} = (0,0)$: Deviating to $\phi_1(\mathbf{e}) = 0$ yields

$$-\overline{S} + \frac{\overline{S} - \beta\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\beta V_1(1,0) + \left(1 - \frac{\overline{S} - \beta\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\right)\beta V_1(1,1)$$
$$= 0 = U_1(\mathbf{e}).$$

   Deviating to $\phi_1(\mathbf{e}) = 1$ yields $0 = U_1(\mathbf{e})$.

2. Exit-entry phase in state $\mathbf{e} = (0,1)$: Deviating to $\phi_1(\mathbf{e}) = 0$ yields $-\overline{S} + \beta V_1(1,1) < U_1(\mathbf{e})$ by part (ii) of Assumption 2.

3. Exit-entry phase in state $\mathbf{e} = (0,2)$: Deviating to $\phi_1(\mathbf{e}) = 0$ yields $-\overline{S} + \beta V_1(1,2) < U_1(\mathbf{e})$ by part (ii) of Assumption 2.

4. Exit-entry phase in state $\mathbf{e} = (1,0)$: Deviating to $\phi_1(\mathbf{e}) = 1$ yields $\overline{X} < U_1(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2.

5. Exit-entry phase in state $\mathbf{e} = (1,1)$: Deviating to $\phi_1(\mathbf{e}) = 0$ yields

$$\frac{(1-\beta)\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\beta V_1(1,0) + \left(1 - \frac{(1-\beta)\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\right)\beta V_1(1,1)$$
$$= \overline{X} = U_1(\mathbf{e}).$$

   Deviating to $\phi_1(\mathbf{e}) = 1$ yields $\overline{X} = U_1(\mathbf{e})$.

6. Exit-entry phase in state $\mathbf{e} = (1,2)$: Deviating to $\phi_1(\mathbf{e}) = 0$ yields $\beta V_1(\mathbf{e}) = \beta\overline{X} < \overline{X} = U_1(\mathbf{e})$.

7. Exit-entry phase in state $\mathbf{e} = (2,0)$: Deviating to $\phi_1(\mathbf{e}) = 1$ yields $\overline{X} < U_1(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2.

8. Exit-entry phase in state $\mathbf{e} = (2,1)$: Deviating to $\phi_1(\mathbf{e}) = 1$ yields $\overline{X} < U_1(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2.

[8] Note that in the price-setting phase in state $\mathbf{e} > (0,0)$, the outside good remains priced out of the market even after a deviation by parts (i), (ii), and (iii) of Assumption 2.

9. Exit-entry phase in state $\mathbf{e} = (2,2)$: Deviating to $\phi_1(\mathbf{e}) = 0$ yields

$$\frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \beta\overline{X}}\beta V_1(2,0) + \left(1 - \frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \beta\overline{X}}\right)\beta V_1(2,2)$$
$$= \overline{X} = U_1(\mathbf{e}).$$

Deviating to $\phi_1(\mathbf{e}) = 1$ yields $\overline{X} = U_1(\mathbf{e})$.

10. Price-setting phase in state $\mathbf{e} = (0,0)$: By default.

11. Price-setting phase in state $\mathbf{e} = (0,1)$: By default.

12. Price-setting phase in state $\mathbf{e} = (0,2)$: By default.

13. Price-setting phase in state $\mathbf{e} = (1,0)$: Deviating to match the outside good ($p_1(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(p_0 - \kappa) + \frac{1}{2}U_1(2,0) + \frac{1}{2}U_1(\mathbf{e}) < V_1(\mathbf{e}).$$

Deviating to be undercut by the outside good ($p_1(\mathbf{e}) > p_0$) yields $U_1(\mathbf{e}) < V_1(\mathbf{e})$.

14. Price-setting phase in state $\mathbf{e} = (1,1)$: Deviating to undercut firm 2 ($p_1(\mathbf{e}) = \left(\kappa - \left(\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \overline{X}\right)\right)^-$ yields

$$-\left(\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \overline{X}\right) + U_1(2,1) = V_1(\mathbf{e}).$$

Deviating to be undercut by firm 2 ($p_1(\mathbf{e}) > \kappa - \left(\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \overline{X}\right)$) yields $U_1(1,2) = V_1(\mathbf{e})$.

15. Price-setting phase in state $\mathbf{e} = (1,2)$: Deviating to match firm 2 ($p_1(\mathbf{e}) = \kappa^-$) yields

$$\frac{1}{2}U_1(\mathbf{e}) + \frac{1}{2}U_1(2,2) = V_1(\mathbf{e}).$$

Deviating to undercut firm 2 ($p_1(\mathbf{e}) = \kappa^{--}$, where κ^{--} is the price just below κ^-) yields $U_1(2,2) = V_1(\mathbf{e})$.

16. Price-setting phase in state $\mathbf{e} = (2,0)$: Deviating to match the outside good ($p_1(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(p_0 - \rho\kappa) + U_1(2,0) < V_1(\mathbf{e}).$$

Deviating to be undercut by the outside good ($p_1(\mathbf{e}) > p_0$) yields $U_1(\mathbf{e}) < V_1(\mathbf{e})$.

17. Price-setting phase in state $\mathbf{e} = (2,1)$: Deviating to match firm 2 ($p_1(\mathbf{e}) = \kappa$) yields

$$\frac{1}{2}(1-\rho)\kappa + \frac{1}{2}U_1(\mathbf{e}) + \frac{1}{2}U_1(2,2) < V_1(\mathbf{e})$$

by parts (ii) and (iii) of Assumption 2. Deviating to be undercut by firm 2 ($p_1(\mathbf{e}) > \kappa$) yields $U_1(2,2) < V_1(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2.

18. Price-setting phase in state $\mathbf{e} = (2,2)$: Deviating to undercut firm 2 ($p_1(\mathbf{e}) = \rho\kappa^-$) yields $U_1(\mathbf{e}) = V_1(\mathbf{e})$. Deviating to be undercut by firm 2 ($p_1(\mathbf{e}) > \rho\kappa$) yields $U_1(\mathbf{e}) = V_1(\mathbf{e})$.

Additional equilibria. Under Assumptions 1 and 2, there exist two other equilibria, shown in Tables OA3 and OA4, in addition to the one in Table OA2.[9] Even in this special case of our model, multiple equilibria are endemic. These equilibria differ from the one in Table OA2 only in the exit-entry phase in state $(1,0)$. In the first equilibrium, the incumbent firm exits the industry and the potential entrant enters ($\phi_1(1,0) = 1$ and $\phi_1(0,1) = 0$); in the second equilibrium, the incumbent firm and the potential entrant play mixed strategies ($\phi_1(1,0) = \frac{\overline{S}-\beta\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}} \in (0,1)$ and $\phi_1(0,1) = \frac{(1-\beta)\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}} \in (0,1)$). Because the exit-entry phase in state $(1,0)$ is off the equilibrium path starting from state $(0,0)$, however, these equilibria give rise to the deadweight loss in equations (OA16) and (OA17).

Equilibrium with cost-inefficient exit. Under the additional assumption that $\overline{X} \geq \frac{\beta}{1-\beta}\kappa(1-\rho)$, there exists the equilibrium, shown in Table OA5, with cost-inefficient exit.[10] In state $(2,1)$, the incumbent firms play mixed strategies. Hence, the lower-cost firm may exit the industry while the higher-cost firm does not. Note that this equilibrium entails cost-inefficient exit not only in an *ex post* sense but also in an *ex ante* sense as the lower-cost firm exits the industry with higher probability than the higher-cost firm ($\phi_1(2,1) > \phi_1(1,2)$).

OA4 Numerical analysis and equilibrium

OA4.1 Parameterization and computation

Figure OA1 shows the number of equilibria that we have computed for six two-dimensional slices through the equilibrium correspondence along (ρ,σ), (ρ,p_0), $(\rho,\overline{X})$, (σ,p_0), $(\sigma,\overline{X})$, and $(p_0,\overline{X})$. White indicates a unique equilibrium at a parameterization and darker shades of blue indicate larger numbers of equilibria. A red cross indicates a paramaterization where we have been unable to compute an equilibrium.

Figure OA2 shows the probability $1-\phi_1(0,0)^2$ that the industry "takes off", averaged across equilibria within parameterizations, for the six slices through the equilibrium correspondence. A blue circle indicates that at the parameterization under consideration there exists an equilibrium with $1-\phi_1(0,0)^2 < 0.01$ that we exclude from the subsequent analysis. Darker shades of blue indicate higher probabilities.

OA4.2 Equilibrium and first-best planner solution

Aggressive and accommodative equilibria. The upper panels of Figure OA3 show the aggressive equilibrium at the baseline parameterization in Table 1 in the main paper and the lower panels the accommodative equilibrium. The left panels show the pricing decision

[9] The proof is similar to that of Proposition 2 and therefore omitted.
[10] The proof is similar to that of Proposition 2 and therefore omitted.

$\mathbf{e}$	$p_1(\mathbf{e})$	$\phi_1(\mathbf{e})$	$V_1(\mathbf{e})$	$U_1(\mathbf{e})$
$(0,0)$	∞	$\frac{\overline{S}-\beta\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	–	0
$(0,1)$	∞	0	–	$\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\overline{S}$
$(0,2)$	∞	1	–	0
$(1,0)$	p_0^-	$\frac{\overline{S}-\beta\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	$p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)$	$\overline{X}$
$(1,1)$	$\kappa-\left(\frac{\beta}{1-\beta}(p_0-\rho\kappa)-\overline{X}\right)$	$\frac{(1-\beta)\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$
$(1,2)$	κ	1	$\overline{X}$	$\overline{X}$
$(2,0)$	p_0^-	0	$\frac{p_0-\rho\kappa}{1-\beta}$	$\frac{\beta}{1-\beta}(p_0-\rho\kappa)$
$(2,1)$	κ^-	0	$(1-\rho)\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)$	$\frac{\beta}{1-\beta}(p_0-\rho\kappa)$
$(2,2)$	$\rho\kappa$	$\frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0-\rho\kappa)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$

Table OA3: Additional equilibrium 1. Two-step learning curve. In column labelled $p_1(\mathbf{e})$, superscript $-$ indicates that firm 1 charges just below the price stated.

$\mathbf{e}$	$p_1(\mathbf{e})$	$\phi_1(\mathbf{e})$	$V_1(\mathbf{e})$	$U_1(\mathbf{e})$
$(0,0)$	∞	$\frac{\overline{S}-\beta\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	–	0
$(0,1)$	∞	$\frac{(1-\beta)\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	–	0
$(0,2)$	∞	1	–	0
$(1,0)$	p_0^-	$\frac{\overline{S}-\beta\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	$p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)$	$\overline{X}$
$(1,1)$	$\kappa-\left(\frac{\beta}{1-\beta}(p_0-\rho\kappa)-\overline{X}\right)$	$\frac{(1-\beta)\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$
$(1,2)$	κ	1	$\overline{X}$	$\overline{X}$
$(2,0)$	p_0^-	0	$\frac{p_0-\rho\kappa}{1-\beta}$	$\frac{\beta}{1-\beta}(p_0-\rho\kappa)$
$(2,1)$	κ^-	0	$(1-\rho)\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)$	$\frac{\beta}{1-\beta}(p_0-\rho\kappa)$
$(2,2)$	$\rho\kappa$	$\frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0-\rho\kappa)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$

Table OA4: Additional equilibrium 2. Two-step learning curve. In column labelled $p_1(\mathbf{e})$, superscript $-$ indicates that firm 1 charges just below the price stated.

$\mathbf{e}$	$p_1(\mathbf{e})$	$\phi_1(\mathbf{e})$	$V_1(\mathbf{e})$	$U_1(\mathbf{e})$
$(0,0)$	∞	$\frac{\overline{S}-\beta\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	–	0
$(0,1)$	∞	1	–	0
$(0,2)$	∞	1	–	0
$(1,0)$	p_0^-	0	$p_0-\kappa+\frac{\beta}{1-\beta}\left(p_0-\rho\kappa\right)$	$\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}\left(p_0-\rho\kappa\right)\right)$
$(1,1)$	κ	$\frac{(1-\beta)\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$
$(1,2)$	κ	$\frac{(1-\beta)\overline{X}-\beta(1-\rho)\kappa}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$
$(2,0)$	p_0^-	0	$\frac{p_0-\rho\kappa}{1-\beta}$	$\frac{\beta}{1-\beta}\left(p_0-\rho\kappa\right)$
$(2,1)$	κ^-	$\frac{(1-\beta)\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	$(1-\rho)\kappa+\overline{X}$	$\overline{X}$
$(2,2)$	$\rho\kappa$	$\frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0-\rho\kappa)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$

Table OA5: Equilibrium with cost-inefficient exit. Two-step learning curve. In column labelled $p_1(\mathbf{e})$, superscript $-$ indicates that firm 1 charges just below the price stated.

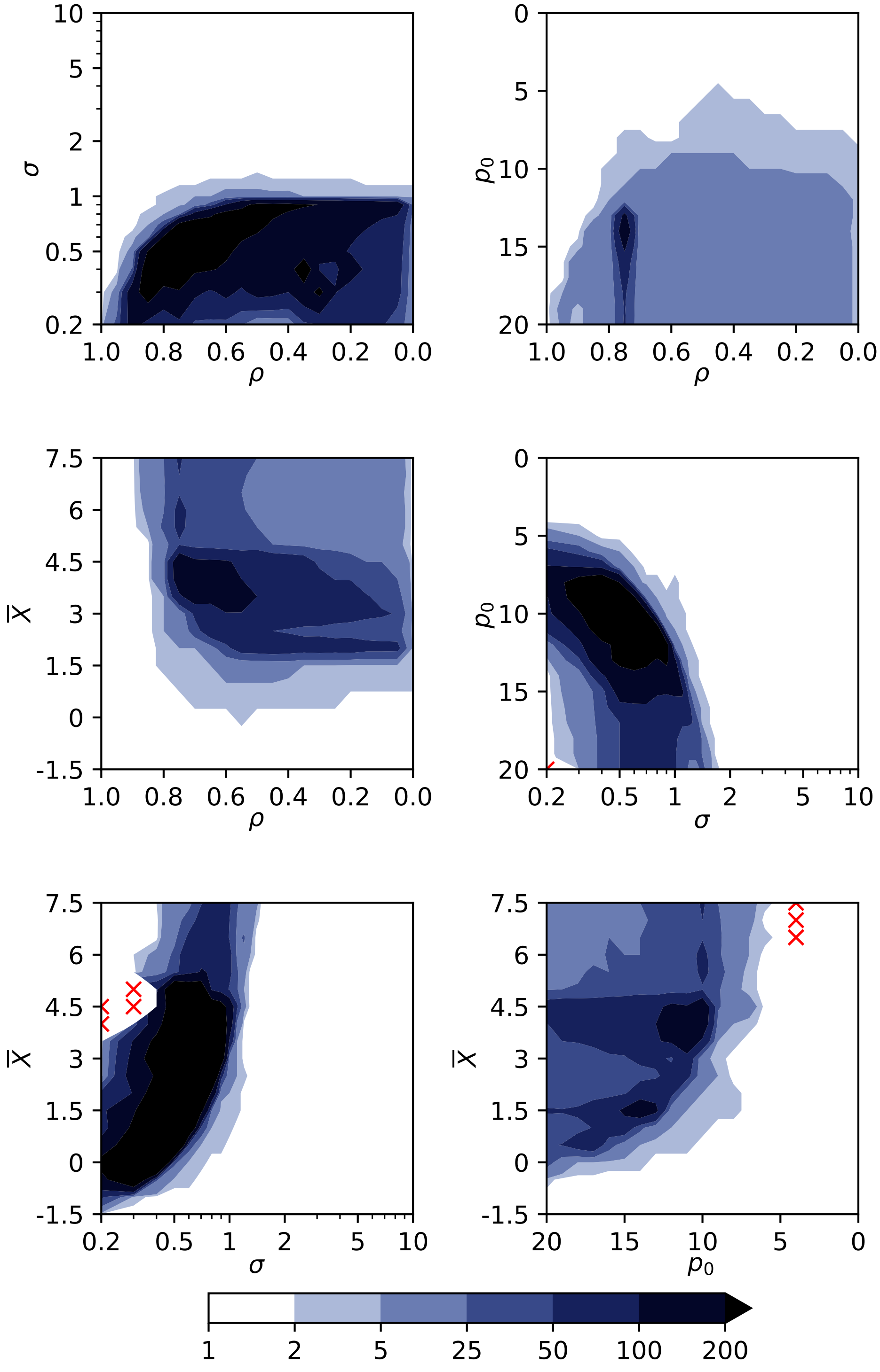

Figure OA1: Number of equilibria. Slices through equilibrium correspondence. Red cross indicates computation failure.

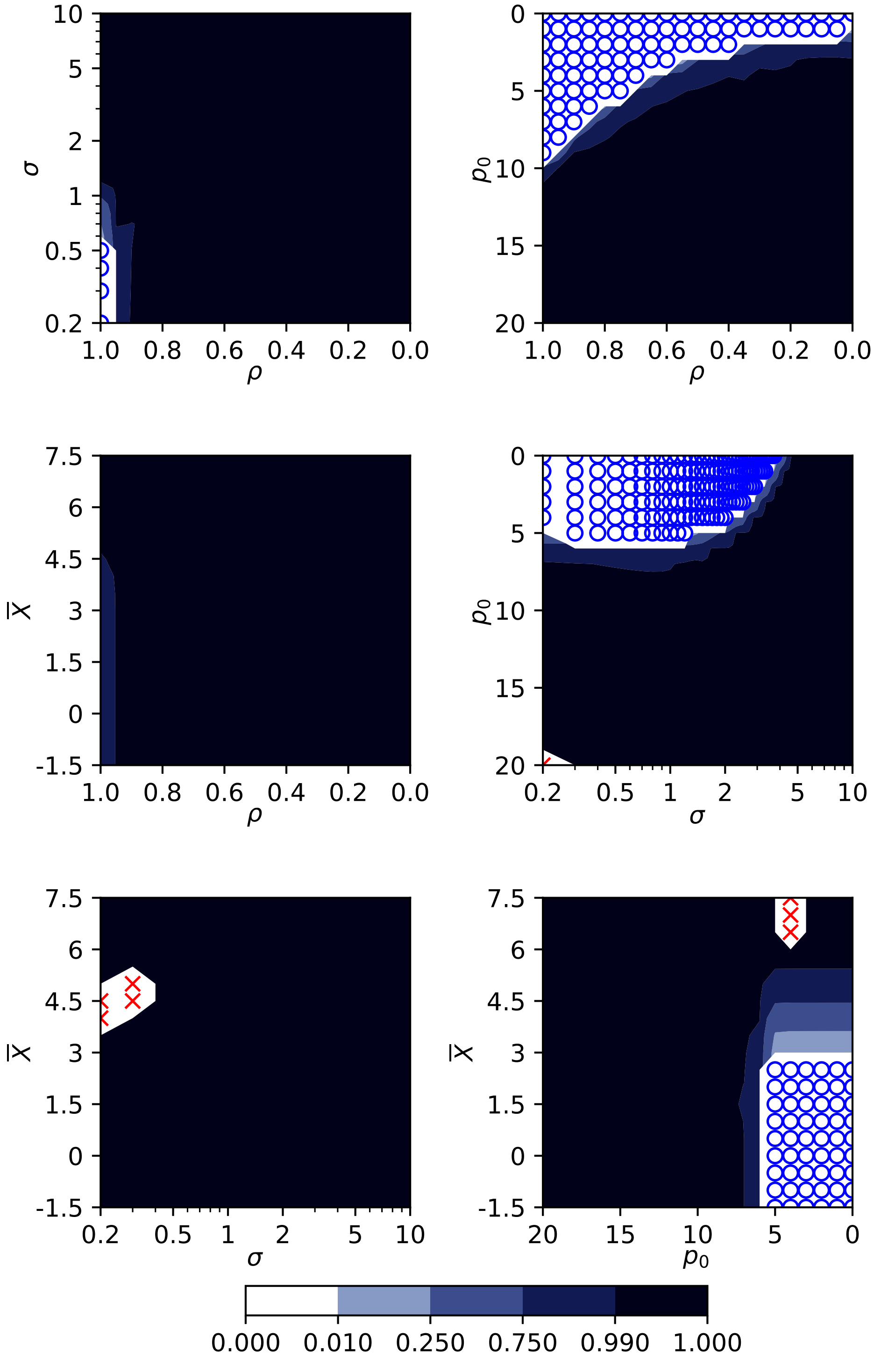

Figure OA2: Probability $1-\phi_1(0,0)^2$ that industry "takes off". Equilibria within parameterizations weighted equally. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

of firm 1, the middle panels the non-operating probability of firm 2, and the right panels the time path of the probability distribution over industry structures (empty, monopoly, and duopoly).

First-best planner solution. Figure OA4 shows the first-best planner solution at the baseline parameterization in Table 1 in the main paper; it is analogous to Figure OA3.

Industry structure, conduct, and performance metrics. The expected short-run and long-run number of firms is

$$N_1 = \sum_{\mathbf{e}} \mu_1(\mathbf{e}) N(\mathbf{e}), \quad N_\infty = \sum_{\mathbf{e}} \mu_\infty(\mathbf{e}) N(\mathbf{e}),$$

where number of firms in state $\mathbf{e}$ is

$$N(\mathbf{e}) = \sum_{n=1}^{2} 1\left[e_n > 0\right].$$

The expected short-run and long-run average price is

$$\overline{p}_1 = \sum_{\mathbf{e}\geq(0,0)} \frac{\mu_1(\mathbf{e})}{1-\mu_1(0,0)} \overline{p}(\mathbf{e}), \quad \overline{p}_\infty = \sum_{\mathbf{e}\geq(0,0)} \frac{\mu_\infty(\mathbf{e})}{1-\mu_\infty(0,0)} \overline{p}(\mathbf{e}),$$

where (share-weighted) average price in state $\mathbf{e}$ is

$$\overline{p}(\mathbf{e}) = \sum_{n=1}^{2} \frac{D_n(p_1(\mathbf{e}), p_2(\mathbf{e}))}{1 - D_0(p_1(\mathbf{e}), p_2(\mathbf{e}))} p_n(\mathbf{e}).$$

The expected time to maturity is

$$T^m = E\left[\min\{t \geq 0 | \mathbf{e}_t \in \Omega\} \middle| \mathbf{e}_0 = (0,0)\right],$$

where $\mathbf{e}_t$ is the state of the industry in period t and

$$\Omega = \{(m,0), \ldots, (M,0), (0,m), \ldots, (0,M), (m,m), \ldots, (M,M)\}$$

is the set of states in which the industry is either a mature monopoly or a mature duopoly. $\min\{t \geq 0 | \mathbf{e}_t \in \Omega\}$ is the so-called first passage time into the set of states Ω. It can be shown that T^m is the solution to a system of linear equations (Vidyadhar G. Kulkarni 1995, equation (4.72)).

The expected NPV of consumer surplus CS_β is defined analogously to the expected NPV of total surplus TS_β in equation (3) in the main paper.

OA5 Does dynamic competition lead to low deadweight loss?

Complementing Figure 1 and Result 1 in the main paper, Figure OA5 shows the relative deadweight loss $\frac{DWL_\beta}{VA_\beta}$, averaged across equilibria within parameterizations, for the six slices through the equilibrium correspondence. Darker shades of blue indicate larger losses.

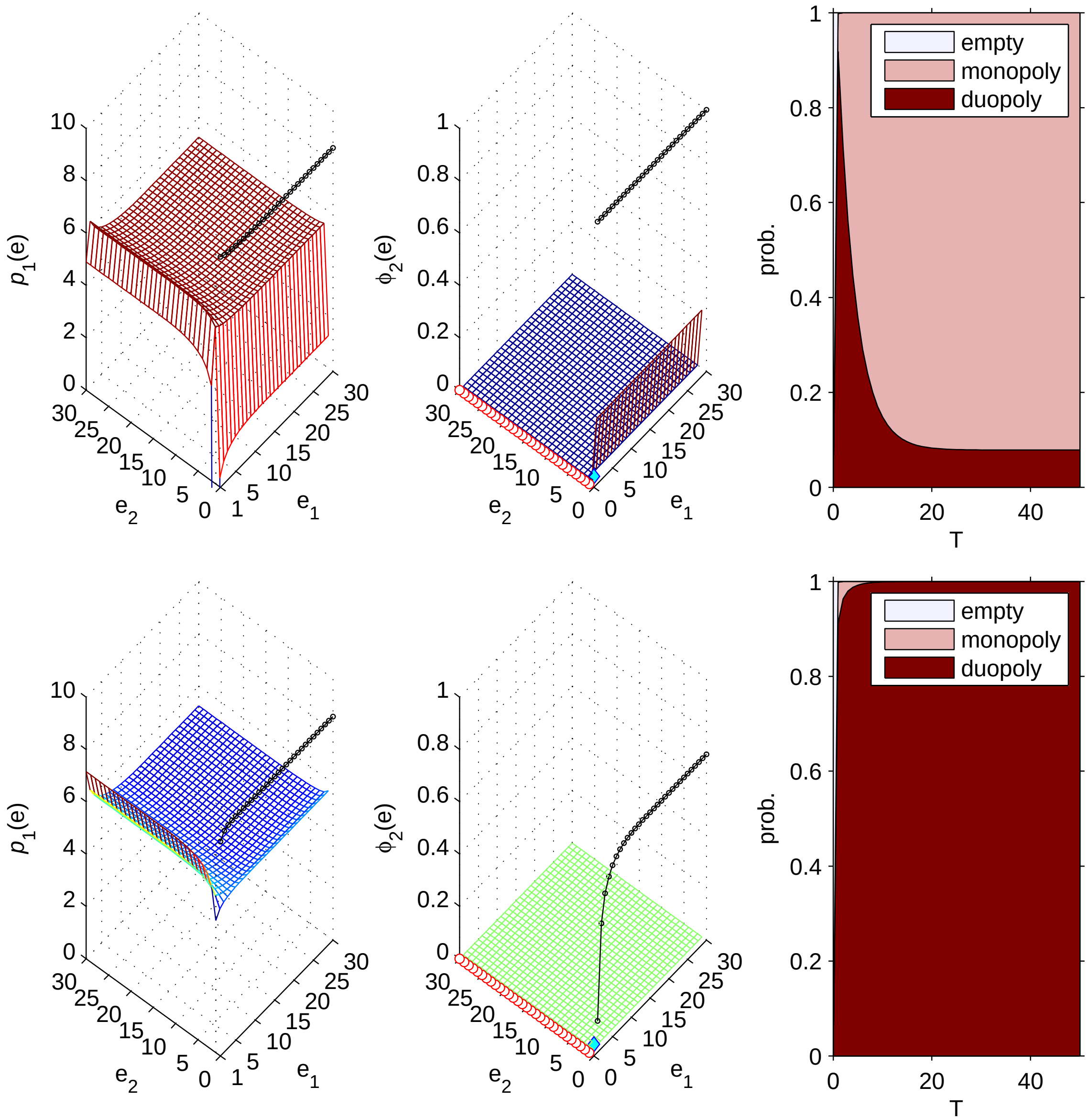

Figure OA3: Aggressive (upper panels) and accommodative (lower panels) equilibrium. Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures (right panels). Dots above the surface in left panels are $p_1(e_1, 0)$ for $e_1 > 0$ and dots in middle panels are $\phi_2(0, e_2)$ for $e_2 > 0$ and $\phi_2(e_1, 0)$ for $e_1 \geq 0$. Baseline parameterization.

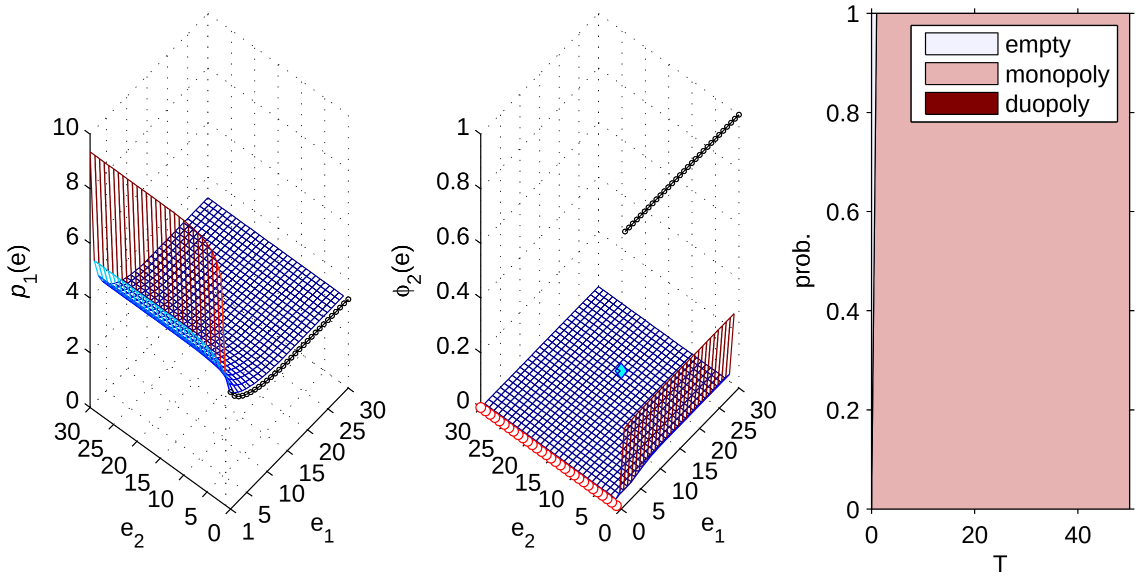

Figure OA4: First-best planner solution. Pricing decision of firm 1 (left panel), non-operating probability of firm 2 (middle panel), and time path of probability distribution over industry structures (right panel). Dots beside the surface in left panel are $p_1(e_1, 0)$ for $e_1 > 0$ and dots in middle panel are $\phi_2(0, e_2)$ for $e_2 > 0$ and $\phi_2(e_1, 0)$ for $e_1 \geq 0$. Baseline parameterization.

OA5.1 Deadweight loss in perspective: static non-cooperative pricing counterfactual and collusive solution

Static non-cooperative pricing counterfactual. In the price-setting phase, incumbent firm 1 maximizes static profit

$$\max_{p_1} D_1(p_1, p_2^{SN}(\mathbf{e}))(p_1 - c(e_1)),$$

and the pricing decision $p_1^{SN}(\mathbf{e})$ is uniquely determined by the first-order condition

$$p_1^{SN}(\mathbf{e}) = c(e_1) + \frac{\sigma}{1 - D_1(\mathbf{p}^{SN}(\mathbf{e}))},$$

where $\mathbf{p}^{SN}(\mathbf{e}) = \left(p_1^{SN}(\mathbf{e}), p_2^{SN}(\mathbf{e})\right)$. The expected NPV of incumbent firm 1 is

$$V_1^{SN}(\mathbf{e}) = D_1(\mathbf{p}^{SN}(\mathbf{e}))(p_1^{SN}(\mathbf{e}) - c(e_1))$$
$$+U_1^{SN}(\mathbf{e}) + \sum_{n=1}^{2} D_n(\mathbf{p}^{SN}(\mathbf{e})) \left[U_1^{SN}\left(\mathbf{e}^{n+}\right) - U_1^{SN}(\mathbf{e})\right]$$

and, in contrast to the pricing decision, accounts for the impact of a sale on the value of continued play. The exit-entry phase is as described in Section OA1.1.[11] Our computations always led to a unique solution.

[11] Our static non-cooperative pricing counterfactual loosely corresponds to the version of the war of attrition presented in Tirole (1988), with the addition of learning-by-doing and product differentiation.

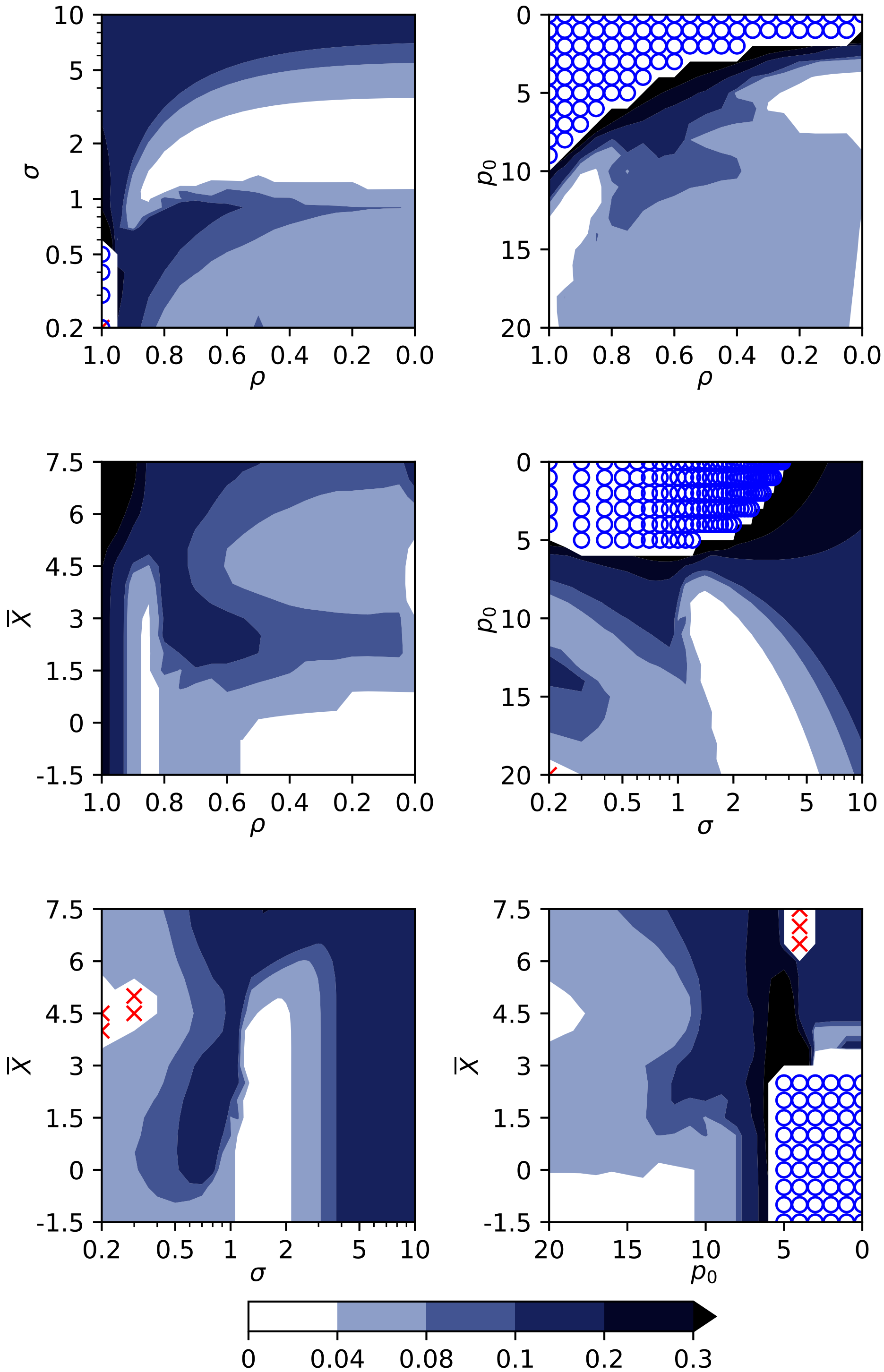

Figure OA5: Relative deadweight loss $\frac{DWL_\beta}{VA_\beta}$. Equilibria within parameterizations weighted equally. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

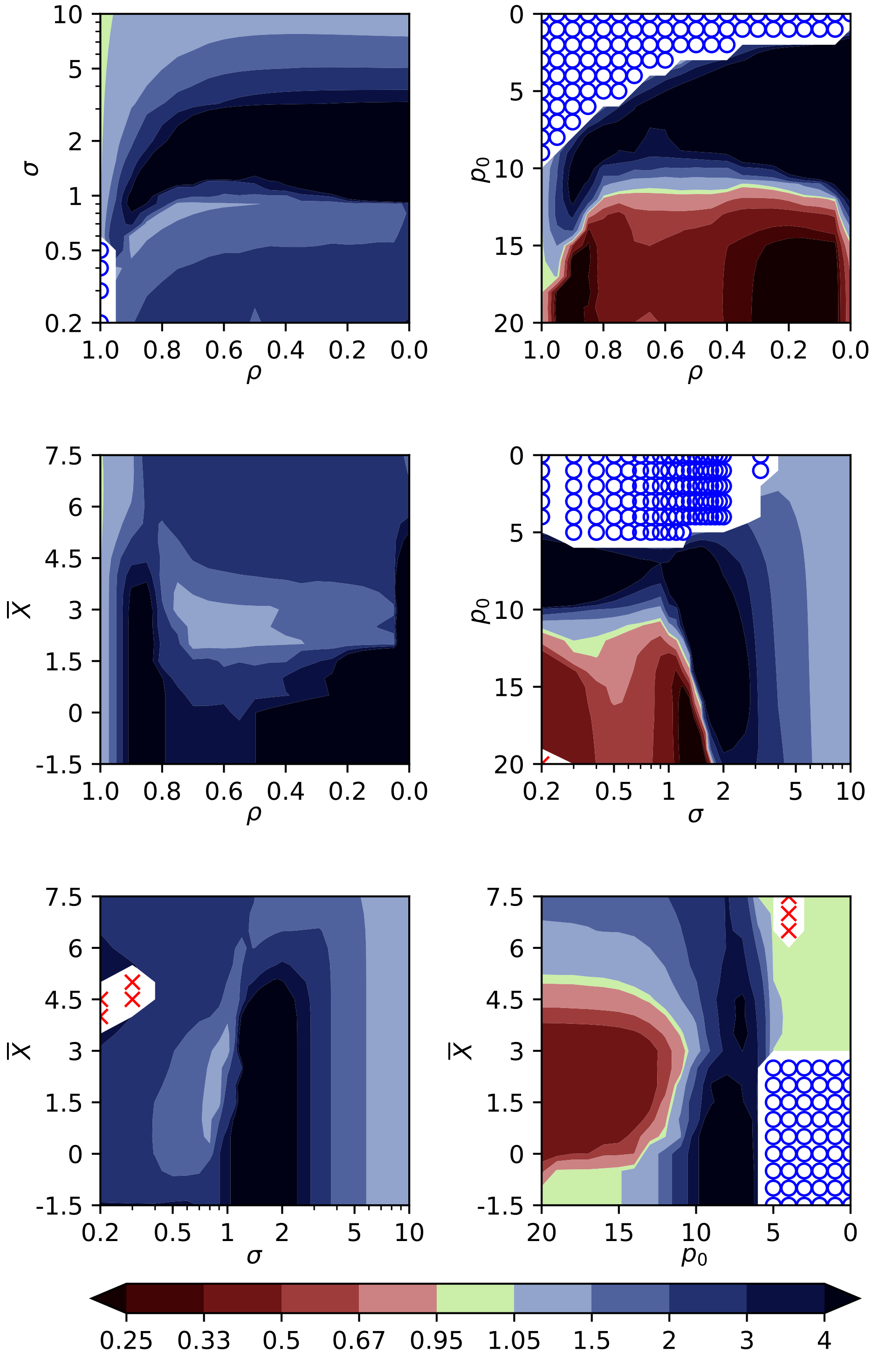

Figure OA6: Deadweight loss ratio $\frac{DWL^{SN}_{\beta}}{DWL_{\beta}}$. Static noncooperative pricing counterfactual. Equilibria within parameterizations weighted equally. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

Complementing Figure 2 and Result 3 in the main paper, Figure OA6 shows the deadweight loss ratio $\frac{DWL_\beta^{SN}}{DWL_\beta}$, averaged across equilibria within parameterizations, for the six slices through the equilibrium. Red, green, and blue indicates negative, zero, and positive values, respectively, and darker shades indicate larger values (in absolute value).

We note that DWL_β^{SN} is smaller than DWL_β in a number of parameterizations that mostly involve an unattractive outside good ($p_0 \geq 15$). The outside good constrains pricing decisions and profitability much more in a monopolistic than in a duopolistic industry. A less attractive outside good lifts this constraint and sharpens the incentive to monopolize the industry in equilibrium. But if firms ignore the investment role of price in the static non-cooperative pricing counterfactual, then a duopolistic industry with a lower deadweight loss emerges.

Collusive solution. In the price-setting phase, the expected NPV of producer surplus is

$$V^{CO}(\mathbf{e}) = \max_{\mathbf{p}} \sum_{n=1}^{2} D_n(\mathbf{p})(p_n - c(e_n)) + \sum_{n=0}^{2} D_n(\mathbf{p}) U^{CO}\left(\mathbf{e}^{n+}\right)$$

and the pricing decisions $\mathbf{p}^{CO}(\mathbf{e}) = \left(p_1^{CO}(\mathbf{e}), p_2^{CO}(\mathbf{e})\right)$ are uniquely determined by the first-order conditions

$$p_n^{CO}(\mathbf{e}) - \frac{\sigma}{1 - D_n(\mathbf{p}^{CO}(\mathbf{e}))} - c(e_n) + \left[U^{CO}\left(\mathbf{e}^{n+}\right) - U^{CO}(\mathbf{e})\right] = 0$$

for $n \in \{1, 2\}$. The exit-entry phase is as described in Section OA2.1. The collusive solution exists and is unique by the contraction mapping theorem; without loss of generality, we take it to be symmetric.

Complementing Figure 2 and Result 4 in the main paper, Figure OA7 shows the deadweight loss ratio $\frac{DWL_\beta^{CO}}{DWL_\beta}$, averaged across equilibria within parameterizations, for the six slices through the equilibrium. Red, green, and blue indicates negative, zero, and positive values, respectively, and darker shades indicate larger values (in absolute value).

OA5.2 Differences between equilibria and first-best planner solution

There are typically substantial differences between the equilibria and the first-best planner solution. We first compare the expected short-run and long-run number of firms between the equilibria and the first-best planner solution. Recall that the first-best planner does not necessarily operate the industry as a natural monopoly, in particular if the degree of product differentiation—and thus the social benefit of product variety—is sufficiently large.

Figure OA8 shows the expected short-run number of firms N_1, averaged across equilibria within parameterizations, for the six slices through the equilibrium correspondence. Darker shades of blue indicate larger numbers. Figure OA9 analogously shows the expected short-run number of firms N_1^{FB} under the first-best planner solution.

Figure OA10 shows the distribution of $N_1 - N_1^{FB}$ as a solid line and breaks out the best equilibrium as a dotted line and the worst equilibrium as a dashed line. Result OA1 highlights some findings:

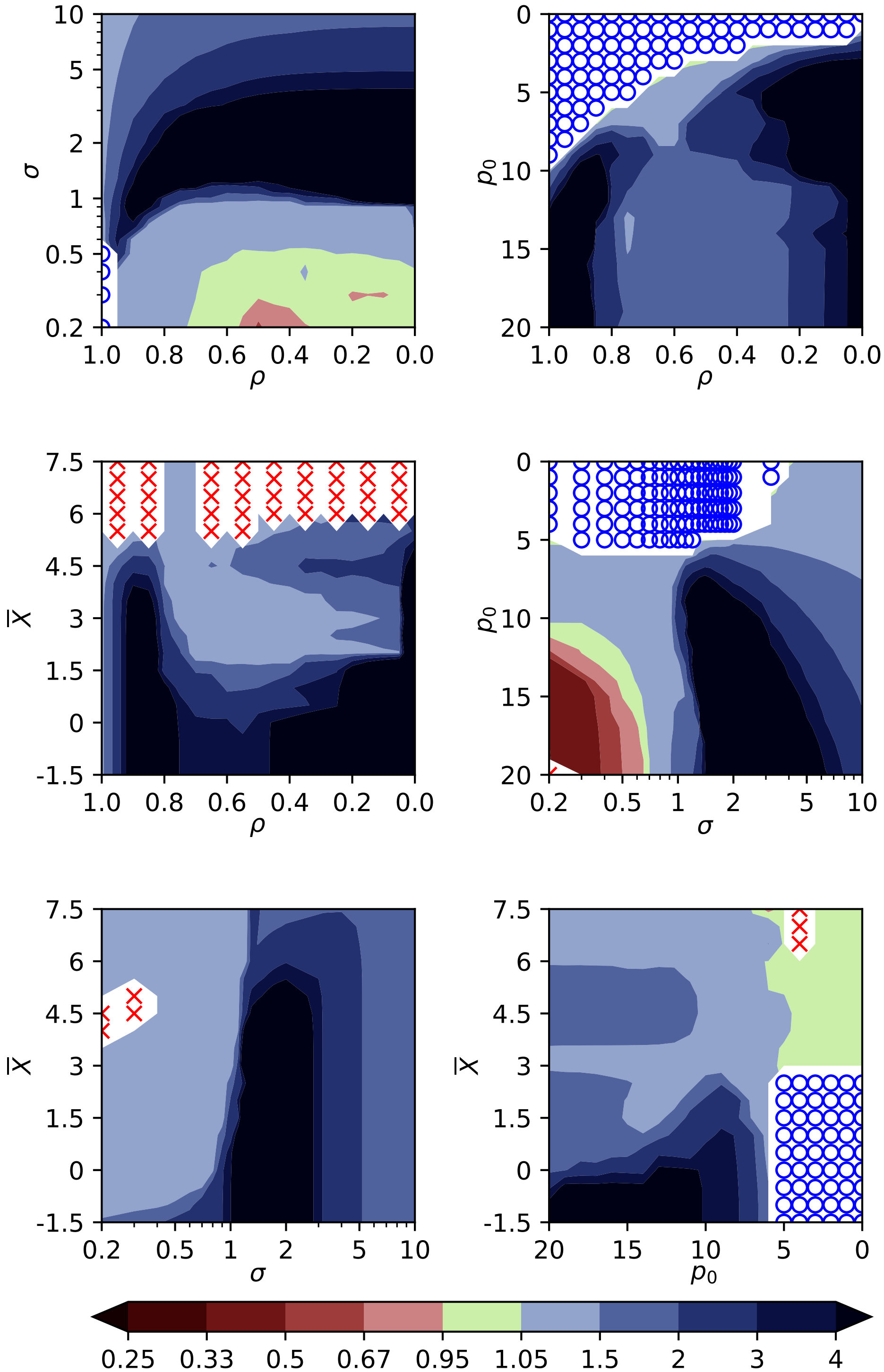

Figure OA7: Deadweight loss ratio $\frac{DWL_{\beta}^{CO}}{DWL_{\beta}}$. Collusive solution. Equilibria within parameterizations weighted equally. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

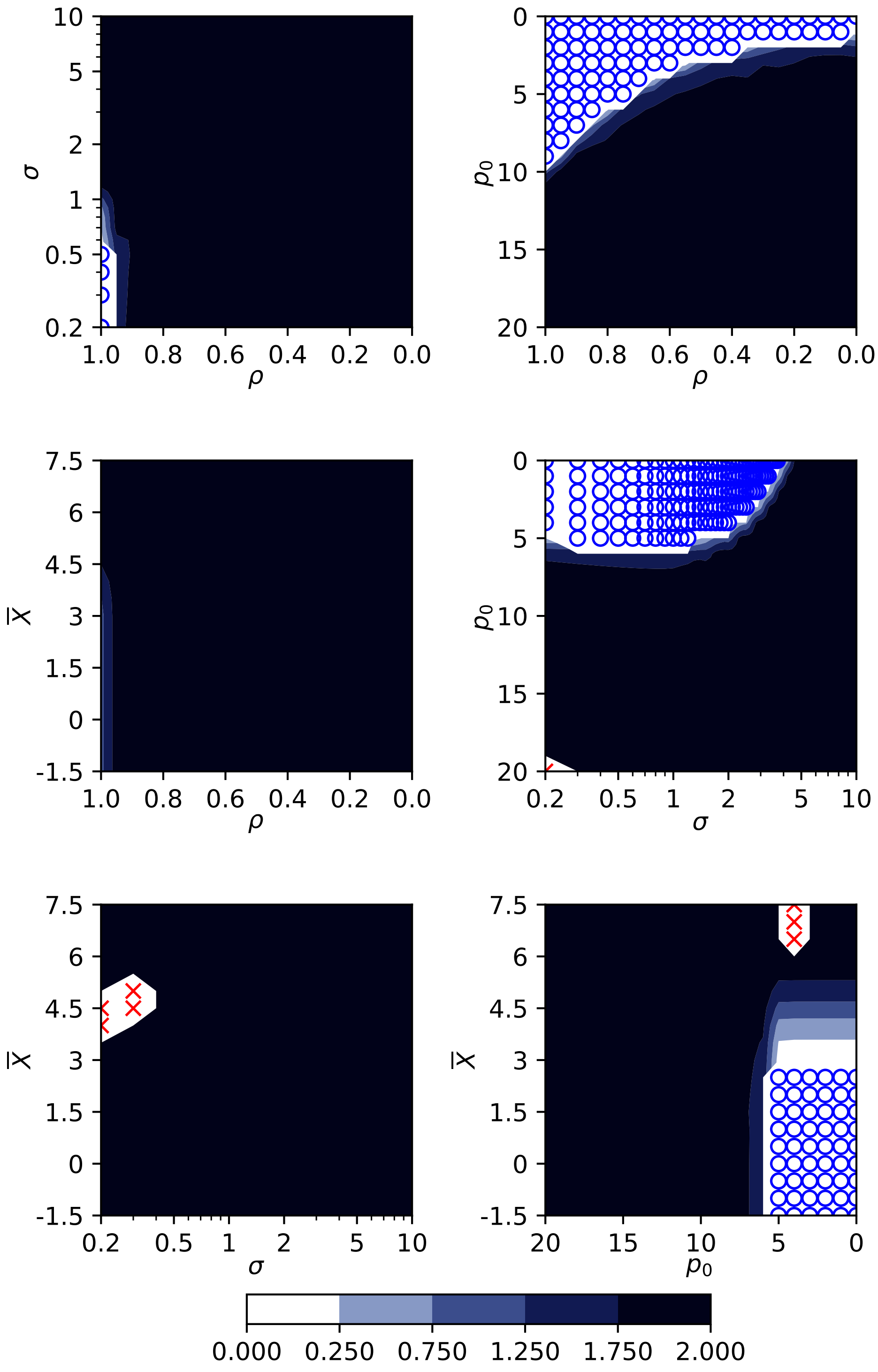

Figure OA8: Expected short-run number of firms N_1. Equilibria within parameterizations weighted equally. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

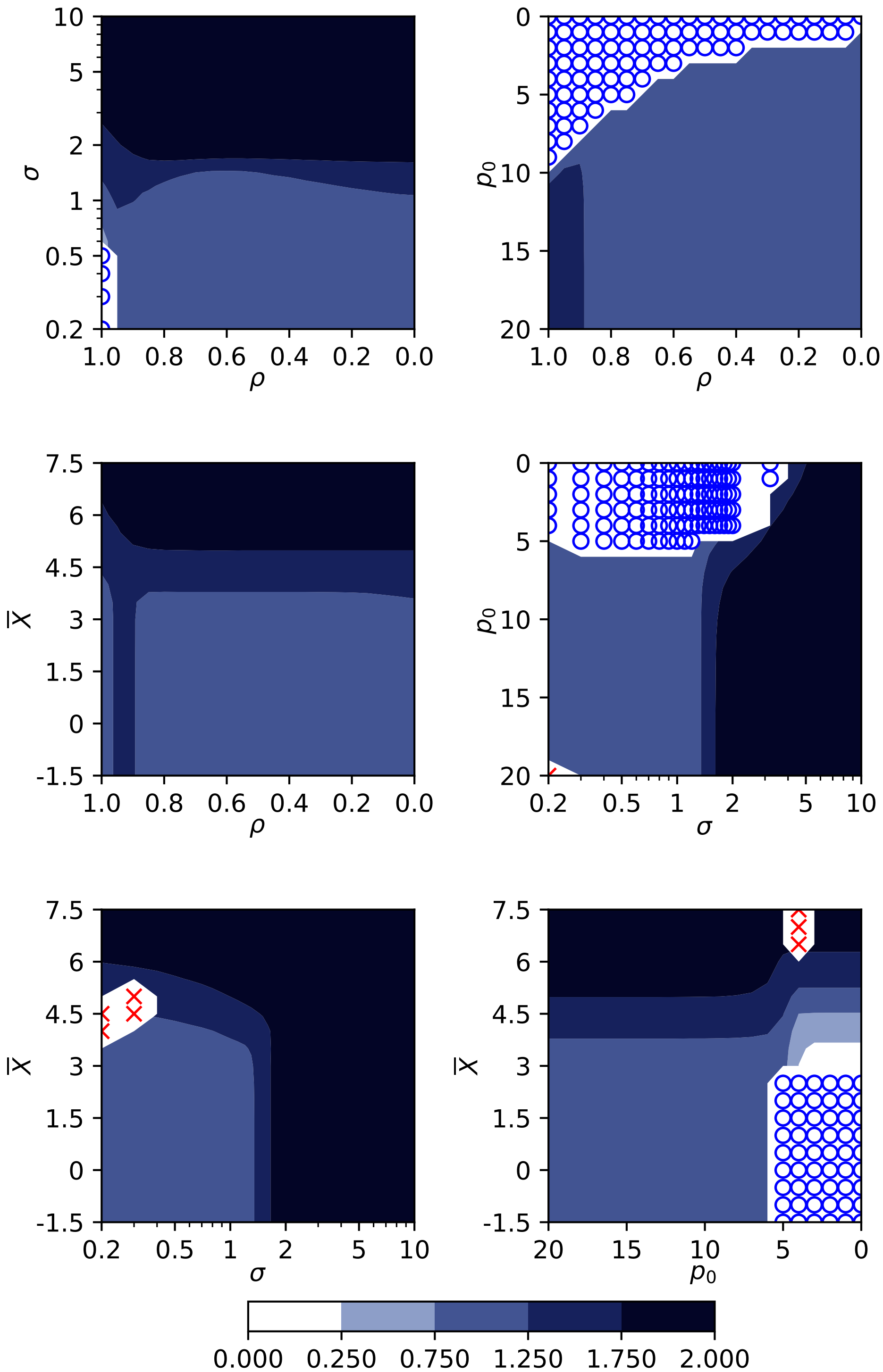

Figure OA9: Expected short-run number of firms N_1^{FB}. First-best planner solution. Slices through solution correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

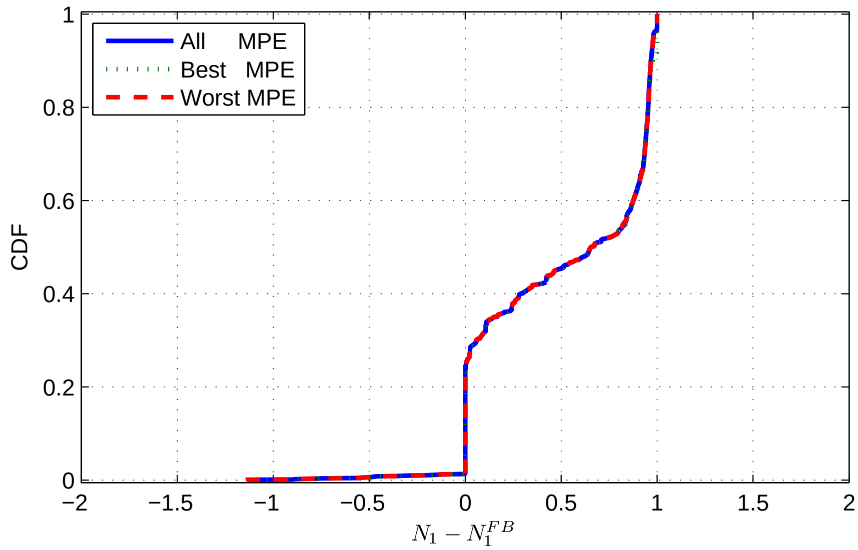

Figure OA10: Distribution of $N_1 - N_1^{FB}$. All equilibria (solid line), best equilibrium (dotted line), and worst equilibrium (dashed line). Parameterizations and equilibria within parameterizations weighted equally.

Result OA1 *N_1 is larger than N_1^{FB} in 76.15% of parameterizations and smaller than N_1^{FB} in 1.33% of parameterizations. The median of $N_1 - N_1^{FB}$ is 0.6567.*[12]

Thus, the equilibria typically have too many firms in the short run, consistent with overentry. They very rarely have too few firms in the short run. Figure OA10 also breaks out the best equilibrium as a dotted line and the worst equilibrium as a dashed line. Similar to our examples in Section 4.2 in the main paper, there is no discernible difference between the best and the worst equilibrium.

Figure OA11 shows the expected long-run number of firms N_∞, averaged across equilibria within parameterizations, for the six slices through the equilibrium correspondence. Darker shades of blue indicate larger numbers. Figure OA12 analogously shows the expected long-run number of firms N_∞^{FB} under the first-best planner solution.

Figure OA13 shows the distribution of $N_\infty - N_\infty^{FB}$ as a solid line and breaks out the best equilibrium as a dotted line and the worst equilibrium as a dashed line. Result OA2 summarizes:

Result OA2 *(1) N_∞ is larger than N_∞^{FB} in 53.77% of parameterizations and smaller than N_∞^{FB} in 5.07% of parameterizations. The median of $N_\infty - N_\infty^{FB}$ is 0.0038. (2) For the best equilibrium, N_∞ is larger than N_∞^{FB} in 59.21% of parameterizations and smaller than N_∞^{FB} in 0.94% of parameterization. The median of $N_\infty - N_\infty^{FB}$ is 0.1327. (3) For the worst equilibrium, N_∞ is larger than N_∞^{FB} in 58.77% of parameterizations and smaller than N_∞^{FB} in 6.27% of parameterizations. The median of $N_\infty - N_\infty^{FB}$ is 0.0167.*

[12]In stating Result OA1, we take N_1 to be equal to N_1^{FB} if $\left|N_1 - N_1^{FB}\right| < 0.0001$ to account for the limited precision of our computations. We proceed analogously in stating Result OA2.

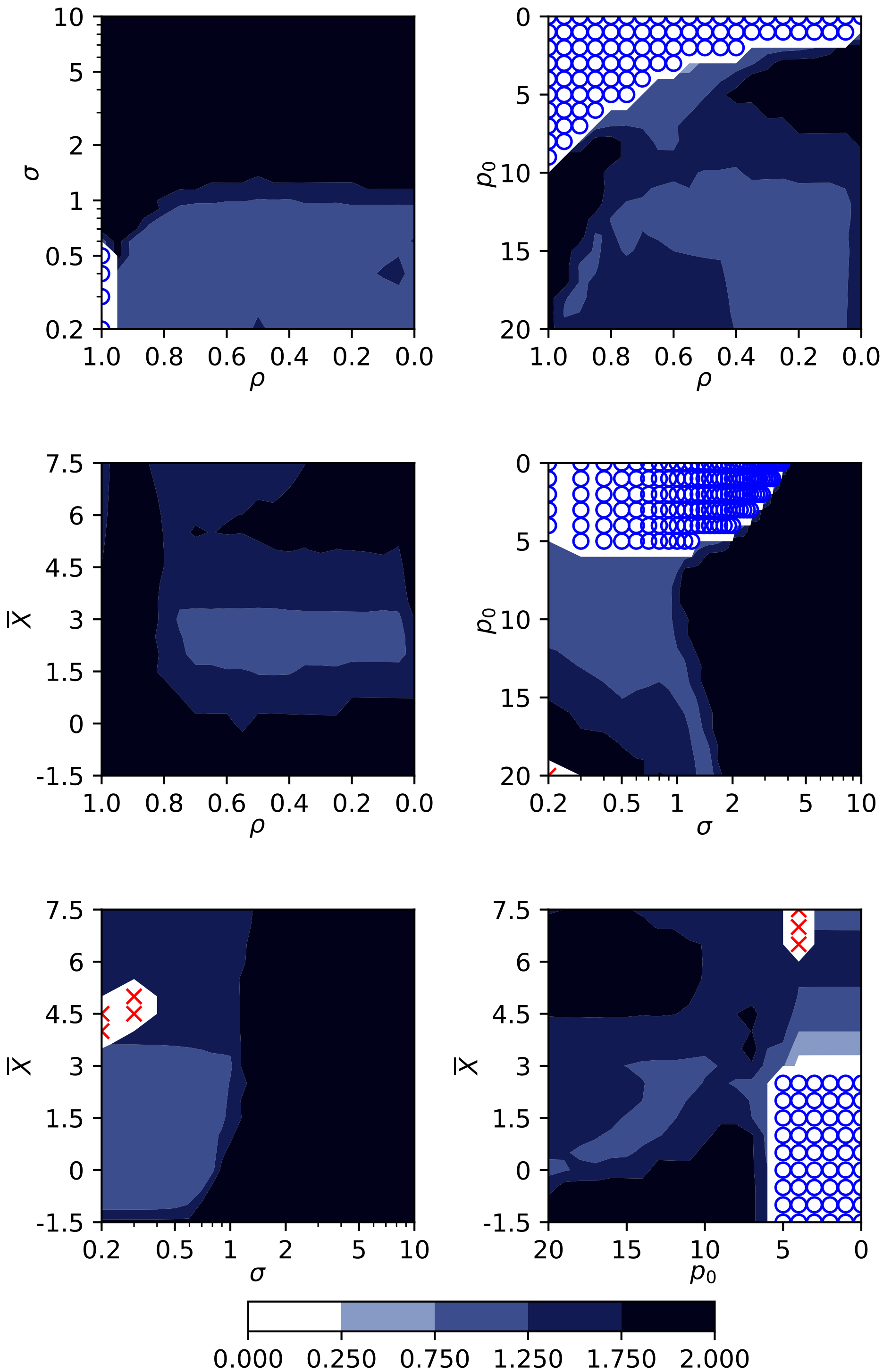

Figure OA11: Expected long-run number of firms N_∞. Equilibria within parameterizations weighted equally. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

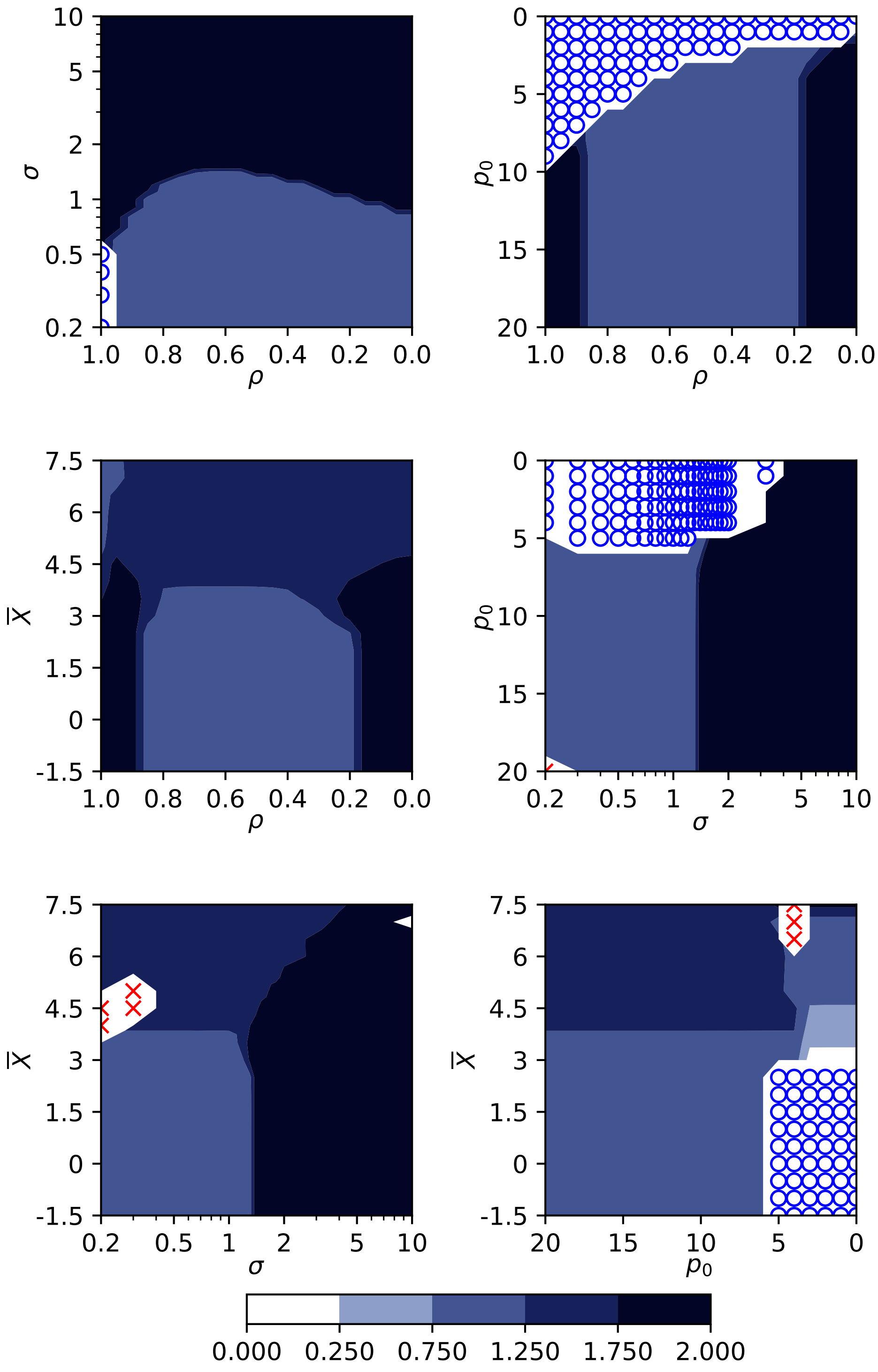

Figure OA12: Expected long-run number of firms N_∞^{FB}. First-best planner solution. Slices through solution correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

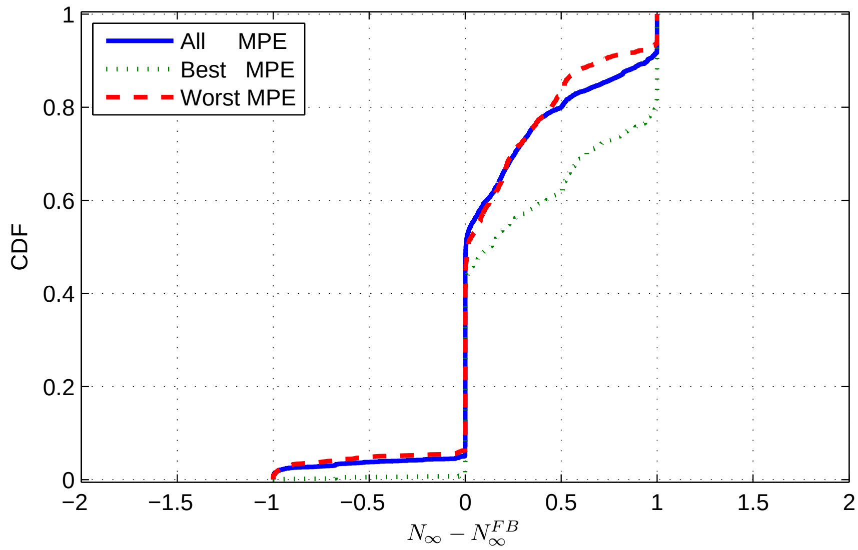

Figure OA13: Distribution $N_\infty - N_\infty^{FB}$. All equilibria (solid line), best equilibrium (dotted line), and worst equilibrium (dashed line). Parameterizations and equilibria within parameterizations weighted equally.

Thus, the equilibria regularly have too many firms in the long run, consistent with under-exit. This tendency is exacerbated in the best equilibrium. The equilibria very rarely have too few firms in the long run.

We next turn to the speed at which firms move down their learning curves. Recall that the expected time to maturity T^m depends on both the number of incumbent firms and their pricing decisions. Figure OA14 shows the expected time to maturity T^m, averaged across equilibria within parameterizations, for the six slices through the equilibrium correspondence. Darker shades of blue indicate larger values. Figure OA15 analogously shows the expected time to maturity $T^{m,FB}$ under the first-best planner solution.

Figure OA16 shows the distribution of $T^m - T^{m,FB}$ as a solid line and breaks out the best equilibrium as a dotted line and the worst equilibrium as a dashed line. Result OA3 summarizes:

Result OA3 *(1) T^m is larger than $T^{m,FB}$ in 90.69% of parameterizations and smaller than $T^{m,FB}$ in 8.35% of parameterizations.*[13] *The median of $T^m - T^{m,FB}$ is 5.2502. (2) For the best equilibrium, T^m is larger than $T^{m,FB}$ in 91.97% of parameterizations and smaller than $T^{m,FB}$ in 6.46% of parameterization. The median of $T^m - T^{m,FB}$ is 11.3581. (3) For the worst equilibrium, T^m is larger than $T^{m,FB}$ in 90.49% of parameterizations and smaller than $T^{m,FB}$ in 8.69% of parameterizations. The median of $T^m - T^{m,FB}$ is 6.6216.*

The speed of learning in the equilibria is generally too slow. Moreover, the best equilibrium exhausts learning economies even more slowly than the worst equilibrium. This is because pricing is initially less aggressive and more firms split sales in an accommodative equilibrium than in an aggressive equilibrium.

[13] In stating Result OA3, we take T^m to be equal to $T^{m,FB}$ if $\left|T^m - T^{m,FB}\right| < 0.1$.

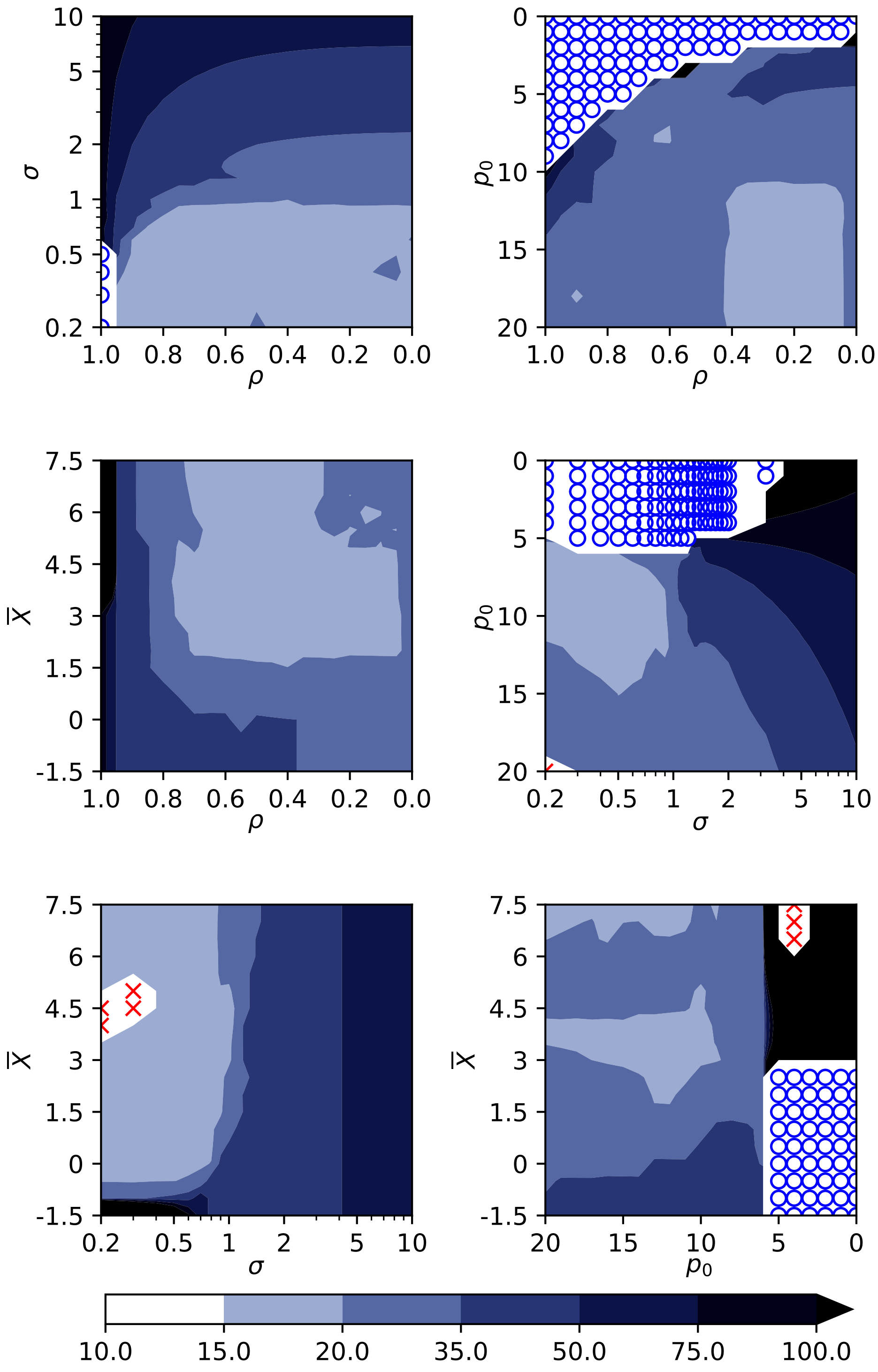

Figure OA14: Expected time to maturity T^m. Equilibria within parameterizations weighted equally. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

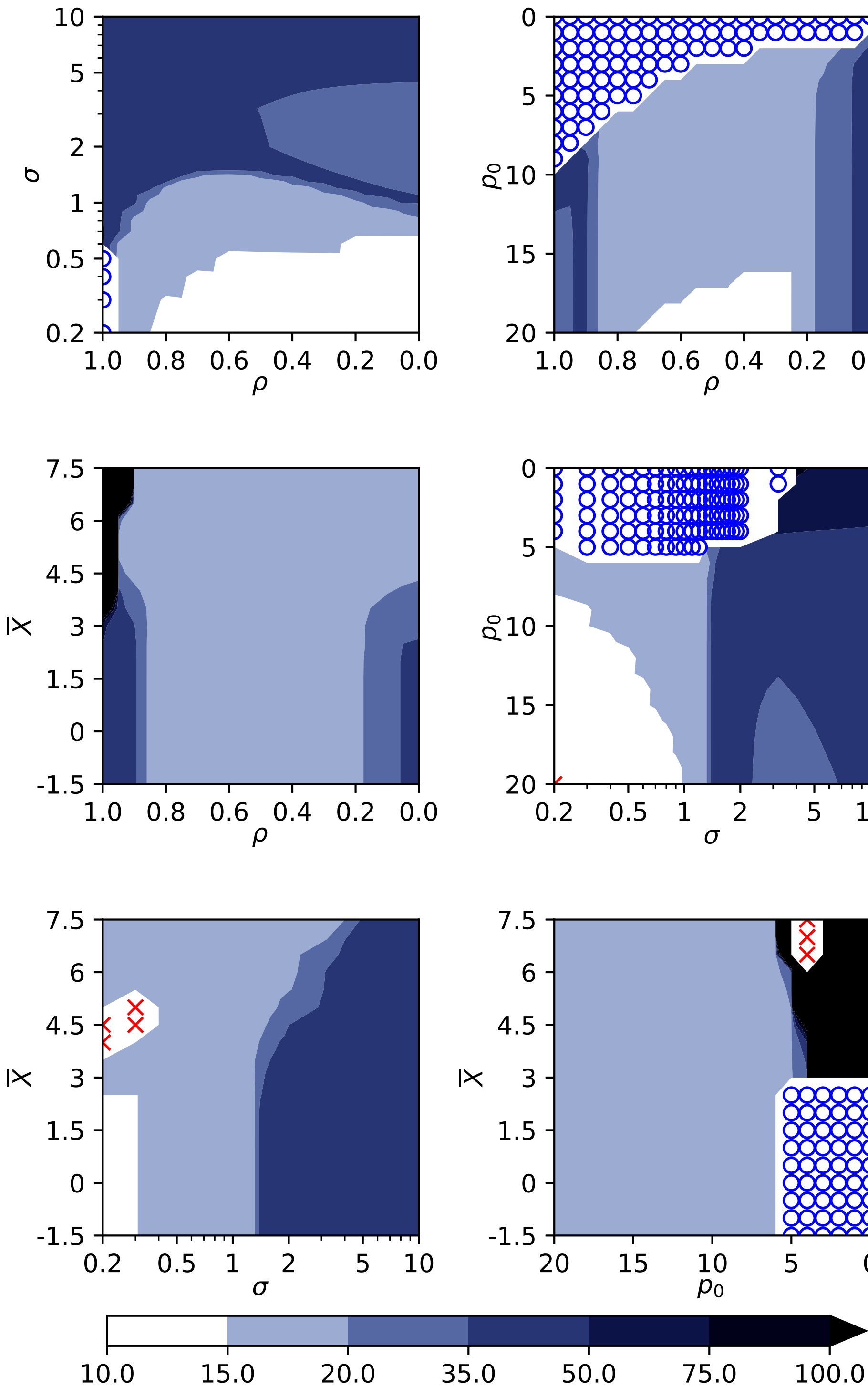

Figure OA15: Expected time to maturity $T^{m,FB}$. First-best planner solution. Slices through solution correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

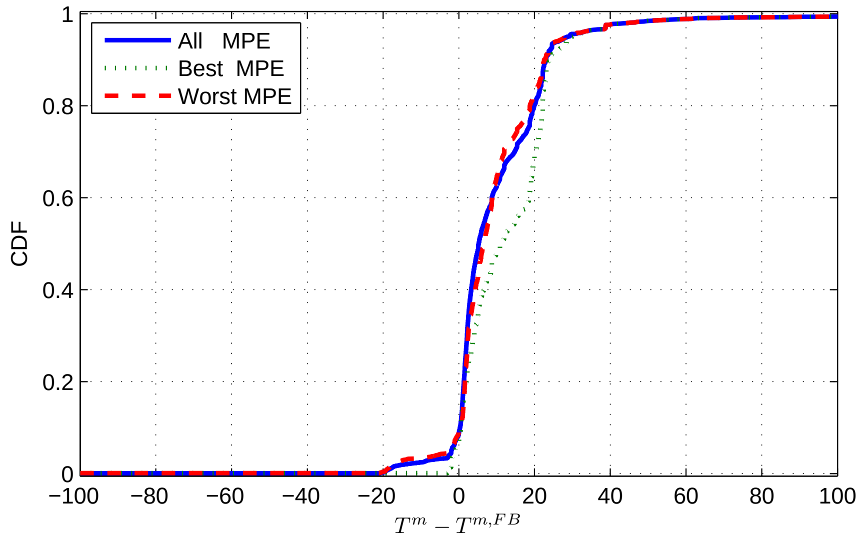

Figure OA16: Distribution of $T^m - T^{m,FB}$. All equilibria (solid line), best equilibrium (dotted line), and worst equilibrium (dashed line). Parameterizations and equilibria within parameterizations weighted equally.

OA5.3 Discounting

Comparative statics. As $\beta \to 0$ and firms become myopic, wells and trenches vanish. In the limit of $\beta = 0$, equation (OA6) for the equivalent model that switches the price-setting and exit-entry phases implies that the equilibrium entails static non-cooperative pricing. Conversely, as $\beta \to 1$, wells and trenches deepen. More patient firms have a stronger incentive to cut prices in the present so as to seize the leadership position in the future. We refer the reader to Section 8.1 of David Besanko, Ulrich Doraszelski, Yaroslav Kryukov & Mark Satterthwaite (2010) and the accompanying Online Appendix for further details.

Turning from the equilibria to the first-best planner solution, as $\beta \to 1$, the perpetual social benefit of product variety looms larger relative to the one-time setup cost of operating both firms. At the baseline parameterization in Table 1 in the main paper, the first-best long-run industry structure switches from monopoly to duopoly between $\beta = 0.965$ and $\beta = 0.97$.

Social vs. private discount factor. In the main paper, we use the same discount factor for firms and the first-best planner. We now explore what happens if the social discount factor $\beta^{FB} \in [\beta, 1)$ diverges from the private discount factor β, thus rendering the planner more patient than firms. In doing so, we hold fixed the aggressive and accommodative equilibria that arise at the baseline parameterization in Table 1 in the main paper, including the private discount factor β, and re-compute the first-best planner solution using the social discount factor β^{FB}. Deadweight loss is given by

$$DWL_{\beta^{FB},\beta} = TS^{FB}_{\beta^{FB}} - TS_{\beta^{FB},\beta}$$

β^{FB}	aggr. eqbm.	accom. eqbm.
0.9524	13.06%	4.54%
0.96	12.53%	2.62%
0.965	12.20%	1.31%
0.97	12.20%	0.30%
0.98	14.02%	0.19%
0.985	15.05%	0.19%
0.99	16.13%	0.18%
0.995	17.24%	0.14%

Table OA6: Relative deadweight loss $\frac{DWL_{\beta^{FB},\beta}}{VA_{\beta^{FB}}}$ for aggressive and accommodative equilibrium and social discount factor β^{FB}. Baseline parameterization.

$$= \sum_{t=0}^{\infty} \left(\beta^{FB}\right)^t \sum_{\mathbf{e}} \mu_t^{FB}\left(\mathbf{e};\beta^{FB}\right) TS^{FB}(\mathbf{e};\beta^{FB}) - \sum_{t=0}^{\infty} \left(\beta^{FB}\right)^t \sum_{\mathbf{e}} \mu_t\left(\mathbf{e};\beta\right) TS(\mathbf{e};\beta)$$

and the maximum value added of the industry by $VA_{\beta^{FB}} = TS^{FB}_{\beta^{FB}} - TS^{\varnothing}_{\beta^{FB}}$ with $TS^{\varnothing}_{\beta^{FB}} = \frac{v-p_0}{1-\beta^{FB}}$. Our notation makes explicit that $\mu_t(\mathbf{e};\beta)$ is the probability that the industry is in state $\mathbf{e}$ in period t and that $TS(\mathbf{e};\beta)$ is the total surplus in state $\mathbf{e}$ as derived from the equilibrium for the private discount factor β, whereas $\mu_t^{FB}(\mathbf{e};\beta^{FB})$ and $TS^{FB}(\mathbf{e};\beta^{FB})$ are derived from the first-best planer solution for the social discount factor β^{FB}. We compute $TS_{\beta^{FB},\beta}$ using the social discount factor β^{FB} as if the more patient planner evaluates the behavior of less patient firms given by $\mu_t(\mathbf{e};\beta)$ and $TS(\mathbf{e};\beta)$.

Table OA6 shows relative deadweight loss $\frac{DWL_{\beta^{FB},\beta}}{VA_{\beta^{FB}}}$ for various values of the social discount factor β^{FB}.[14] For the aggressive equilibrium, $\frac{DWL_{\beta^{FB},\beta}}{VA_{\beta^{FB}}}$ increases as β^{FB} increases, albeit modestly. This is consistent with the bound on the pricing distortion in Proposition 2 in the main paper. For the accommodative equilibrium, $\frac{DWL_{\beta^{FB},\beta}}{VA_{\beta^{FB}}}$ decreases as β^{FB} increases, dropping well below 1% between $\beta^{FB} = 0.965$ and $\beta^{FB} = 0.97$. This reflects not only the bounded pricing distortion in Proposition 1 in the main paper but also the above noted fact that first-best long-run industry structure switches from monopoly to duopoly. The non-pricing distortion thus becomes very small.

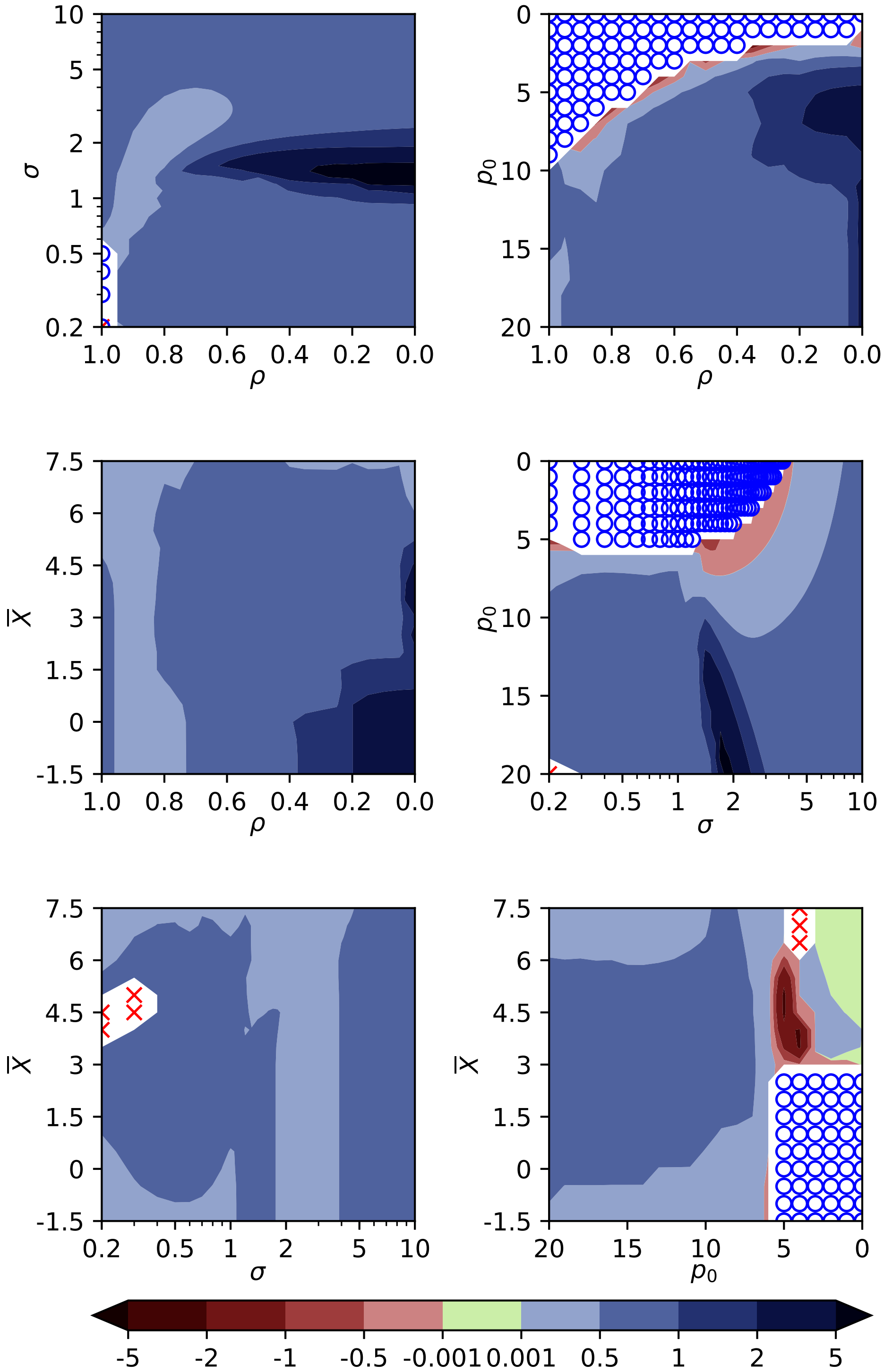

Figure OA17: Scaled pricing distortion $\frac{DWL_\beta^{PR}}{DWL_\beta}$. Equilibria within parameterizations weighted equally. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

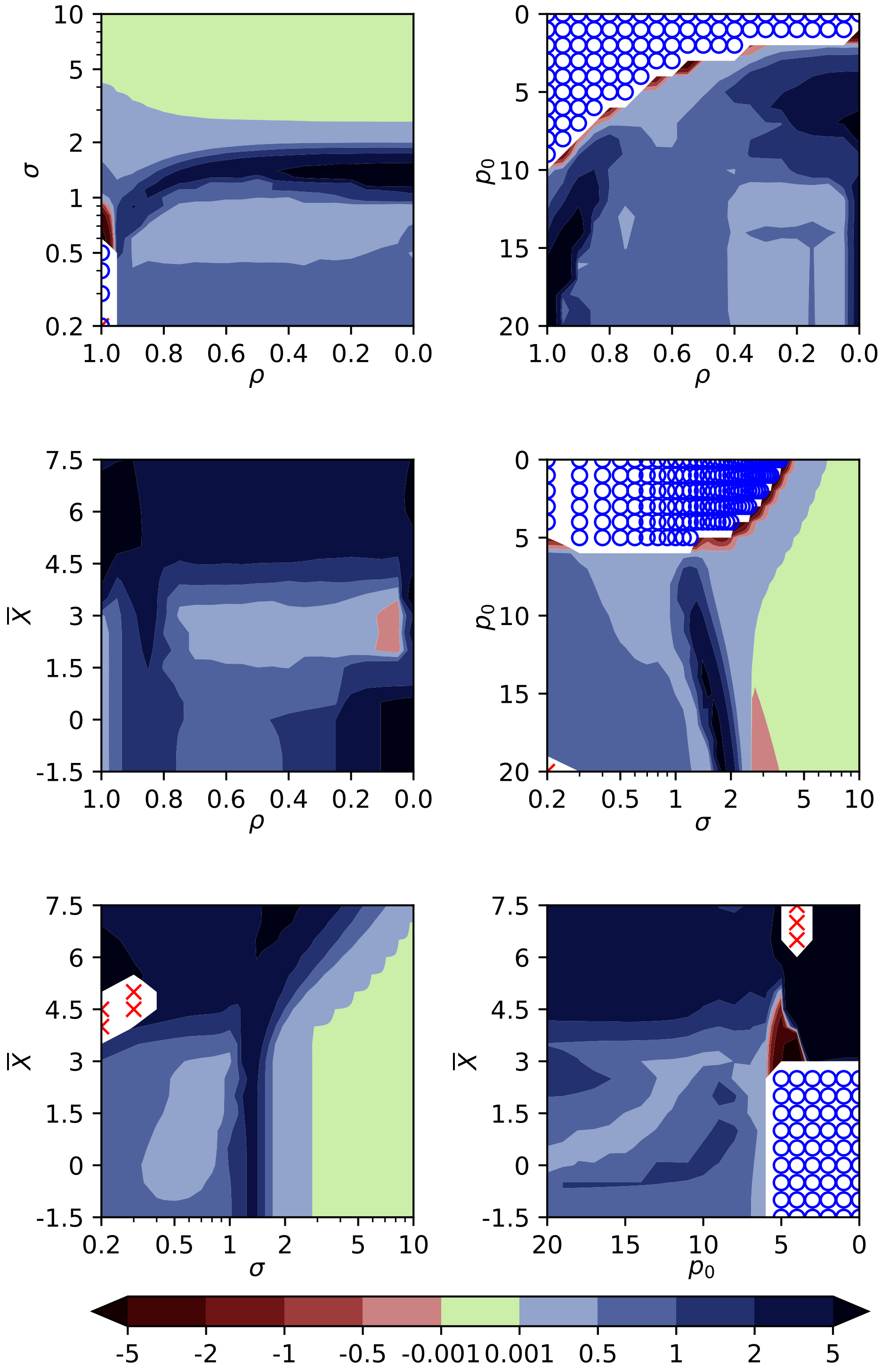

Figure OA18: Scaled exit and entry distortion $\frac{DWL_\beta^{EE}}{DWL_\beta}$. Equilibria within parameterizations weighted equally. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

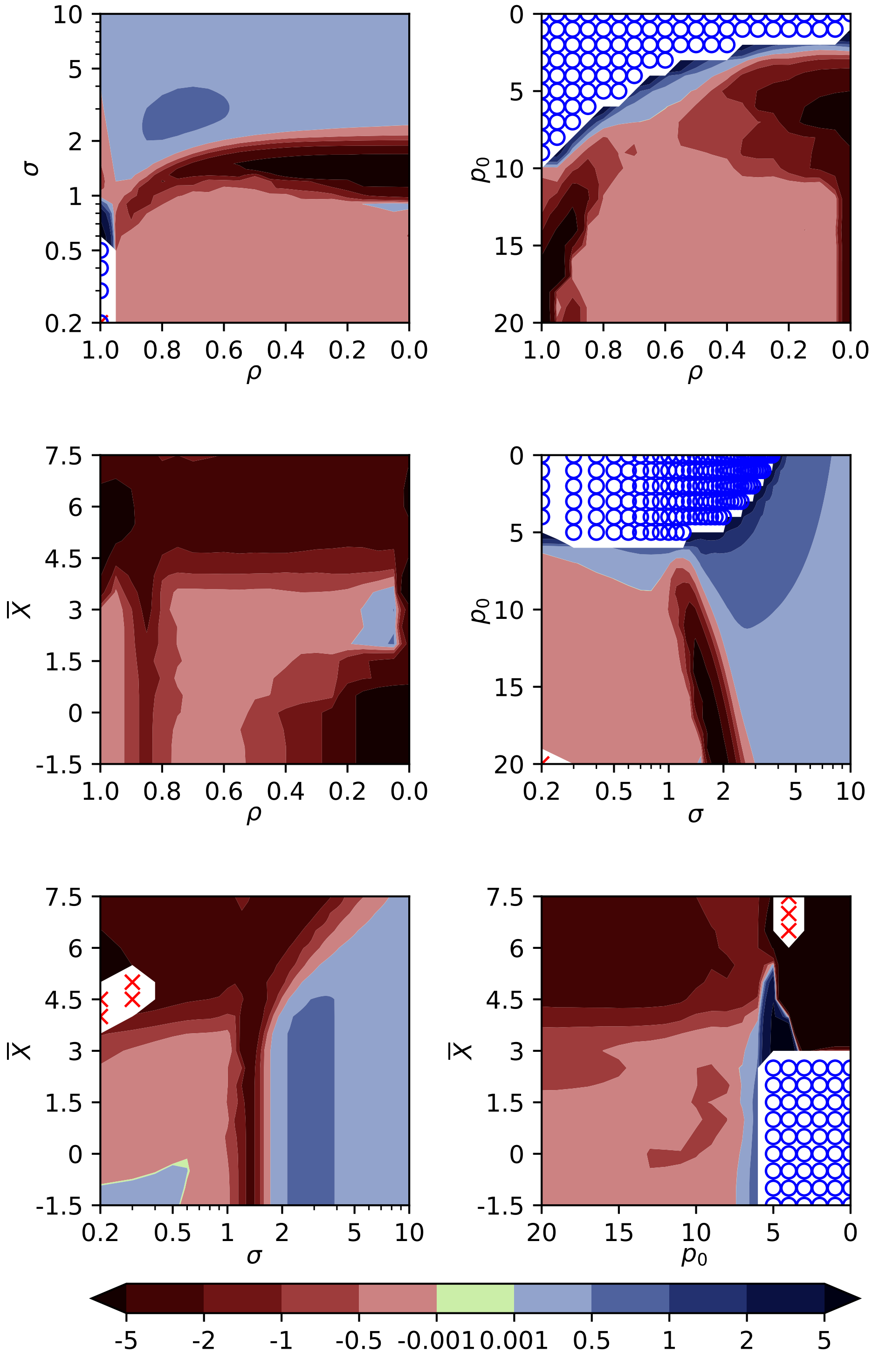

Figure OA19: Scaled market structure distortion $\frac{DWL_{\beta}^{MS}}{DWL_{\beta}}$. Equilibria within parameterizations weighted equally. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

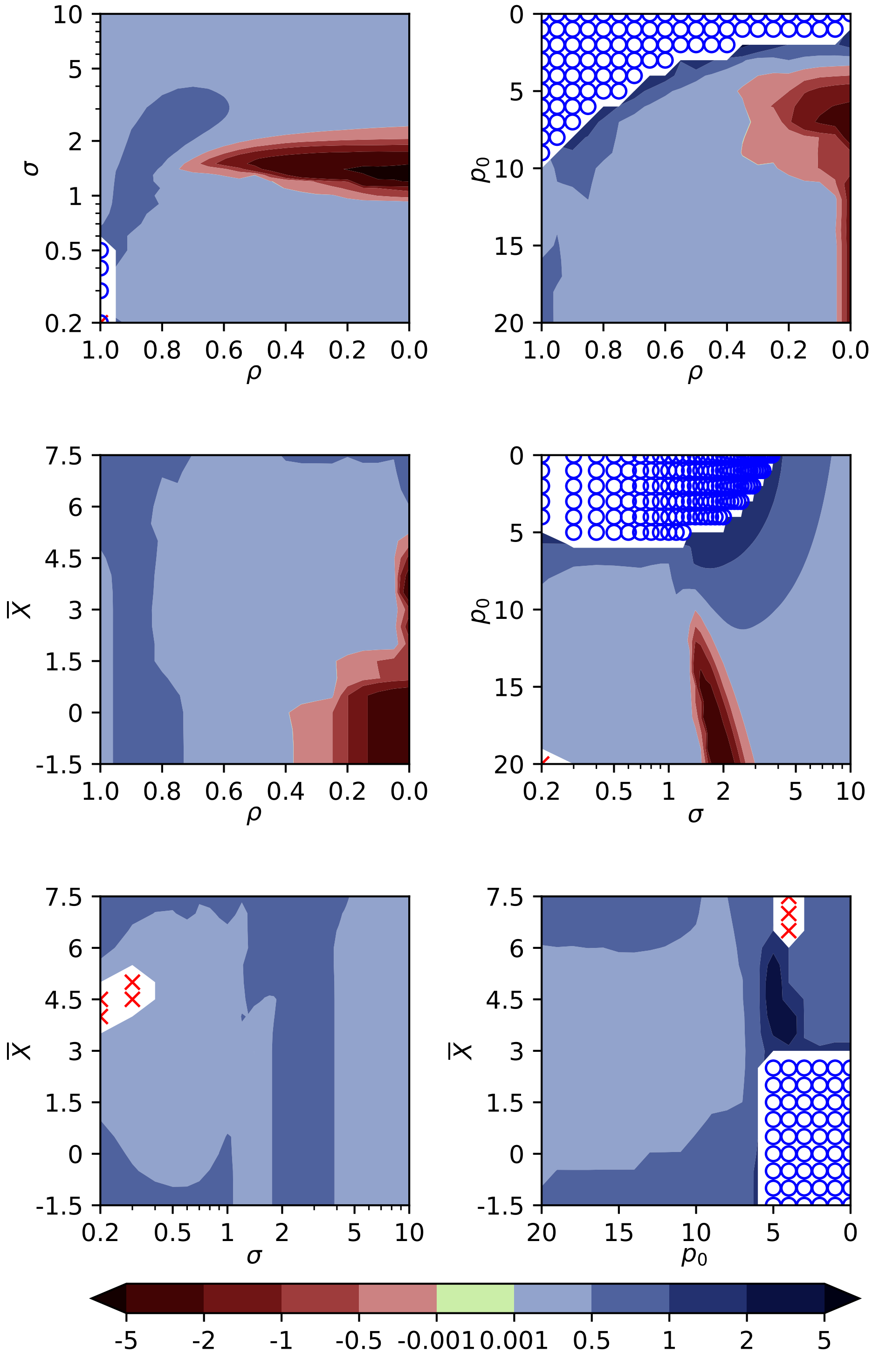

Figure OA20: Scaled non-pricing distortion $\frac{DWL_{\beta}^{NPR}}{DWL_{\beta}}$. Equilibria within parameterizations weighted equally. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

OA6 Why does dynamic competition lead to low deadweight loss?

OA6.1 Decomposition

Complementing Results 5–7 in the main paper, Figures OA17–OA20 show the scaled pricing distortion $\frac{DWL_\beta^{PR}}{DWL_\beta}$, scaled exit and entry distortion $\frac{DWL_\beta^{EE}}{DWL_\beta}$, scaled market structure distortion $\frac{DWL_\beta^{MS}}{DWL_\beta}$, and scaled non-pricing distortion $\frac{DWL_\beta^{NPR}}{DWL_\beta}$, averaged across equilibria within parameterizations, for the six slices through the equilibrium correspondence. Red, green, and blue indicates negative, zero, and positive values, respectively, and darker shades indicate larger values (in absolute value).

OA6.2 Why is the best equilibrium so good?

Linear demand. Consider a representative consumer who allocates her income I among the inside goods that are offered by the incumbent firms at prices $\mathbf{p} = (p_1, p_2)$, an outside good at an exogenously given price p_0, and a numeraire good. Substituting the budget constraint into the utility function, the maximization problem of the representative consumer is

$$\max_{Q_0,Q_1,Q_2} \sum_{n=0}^{2} a_n Q_n - \frac{b}{2}\sum_{n=0}^{2} Q_n^2 - \theta b\left(Q_0Q_1 + Q_0Q_2 + Q_1Q_2\right) + I - \sum_{n=0}^{2} p_n Q_n,$$

where $a_0 > 0$, $a_1 > 0$, $a_2 > 0$, $b > 0$, and $\theta \in [0, 1)$ are parameters. The parameter θ governs the degree of product differentiation, with higher values of θ corresponding to weaker product differentiation.

The first-order conditions in matrix form are:

$$\begin{bmatrix} 1 & \theta & \theta \\ \theta & 1 & \theta \\ \theta & \theta & 1 \end{bmatrix} \begin{bmatrix} Q_0 \\ Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} \frac{a_0 - p_0}{b} \\ \frac{a_1 - p_1}{b} \\ \frac{a_2 - p_2}{b} \end{bmatrix}.$$

Solving yields the demand functions

$$\begin{aligned}
Q_0 &= D_0(\mathbf{p}) = \frac{1}{b(2\theta+1)(1-\theta)}\left(\left(1+\theta\right)a_0 - \theta a_1 - \theta a_2 - (1+\theta)p_0 + \theta p_1 + \theta p_2\right), \\
Q_1 &= D_1(\mathbf{p}) = \frac{1}{b(2\theta+1)(1-\theta)}\left(-\theta a_0 + \left(1+\theta\right)a_1 - \theta a_2 + \theta p_0 - (1+\theta)p_1 + \theta p_2\right), \\
Q_2 &= D_2(\mathbf{p}) = \frac{1}{b(2\theta+1)(1-\theta)}\left(-\theta a_0 - \theta a_1 + \left(1+\theta\right)a_2 + \theta p_0 + \theta p_1 - (1+\theta)p_2\right).
\end{aligned}$$

The aggregate demand for the inside goods is

$$D_T(\mathbf{p}) = \sum_{n=1}^{2} D_n(\mathbf{p}) = \frac{1}{b(2\theta+1)(1-\theta)}\left[-2\theta a_0 + a_1 + a_2 + 2\theta p_0 - (p_1 + p_2)\right].$$

[14] In contrast to $\frac{DWL_{\beta^{FB},\beta}}{VA_{\beta^{FB}}}$, $DWL_{\beta^{FB},\beta}$ increases mechanically as β^{FB} increases and is therefore difficult to compare across values of β^{FB}.

To prevent $D_T(\mathbf{p}) < 0$, we maintain $-2\theta a_0 + a_1 + a_2 + 2\theta p_0 > 0$. We compute the price elasticity of aggregate demand as the percentage change in aggregate demand $D_T(\mathbf{p})$ that from results a one-percent change in prices $\mathbf{p}$:

$$\eta_T(\mathbf{p}) = \left.\frac{\partial D_T(\lambda\mathbf{p})}{\partial\lambda}\frac{\lambda}{D_T(\lambda\mathbf{p})}\right|_{\lambda=1} = \frac{-(p_1+p_2)}{-2\theta a_0 + a_1 + a_2 + 2\theta p_0 - (p_1+p_2)}.$$

Note that the absolute value $|\eta_T(\mathbf{p})|$ of this price elasticity increases in $p_1 + p_2$. Moreover, the quantity of the outside good demanded $D_0(\mathbf{p})$ increases in $p_1 + p_2$. Thus, as the prices of the inside goods decrease, the aggregate demand for the inside goods becomes less price elastic, and at the same time, the quantity of the outside good demanded decreases.

OA6.3 Aggressive and accommodative equilibria

Complementing the classification of equilibria in Appendix B in the main paper, Figures OA21–OA23 show the share of aggressive, accommodative, and unclassified equilibria, respectively, for the six slices through the equilibrium correspondence. Darker shades of blue indicate larger shares.

References

Besanko, David, Ulrich Doraszelski, Yaroslav Kryukov, and Mark Satterthwaite. 2010. "Learning-by-doing, organizational forgetting, and industry dynamics." *Econometrica*, 78(2): 453–508.

Bolton, Patrick, and Joseph Farrell. 1990. "Decentralization, duplication, and delay." *Journal of Political Economy*, 98(4): 803–826.

Bulow, Jeremy, and Paul Klemperer. 1999. "The generalized war of attrition." *Journal of Political Economy*, 89(1): 175–189.

Cabral, Luis M.B. 1993. "Experience advantages and entry dynamics." *Journal of Economic Theory*, 59: 403–416.

Doraszelski, Ulrich, and Juan F. Escobar. 2010. "A theory of regular Markov perfect equilibria in dynamic stochastic games: Genericity, stability, and purification." *Theoretical Economics*, 5: 369–402.

Doraszelski, Ulrich, and Mark Satterthwaite. 2010. "Computable Markov-perfect industry dynamics." *Rand Journal of Economics*, 41(2): 215–243.

Kulkarni, Vidyadhar G. 1995. *Modeling and analysis of stochastic systems.* London:Chapman and Hall.

Smith, J. Maynard. 1974. "Theory of games and the evolution of animal conflicts." *Journal of Theoretical Biology*, 47: 209–221.

Tirole, Jean. 1988. *The theory of industrial organization.* Cambridge:MIT Press.

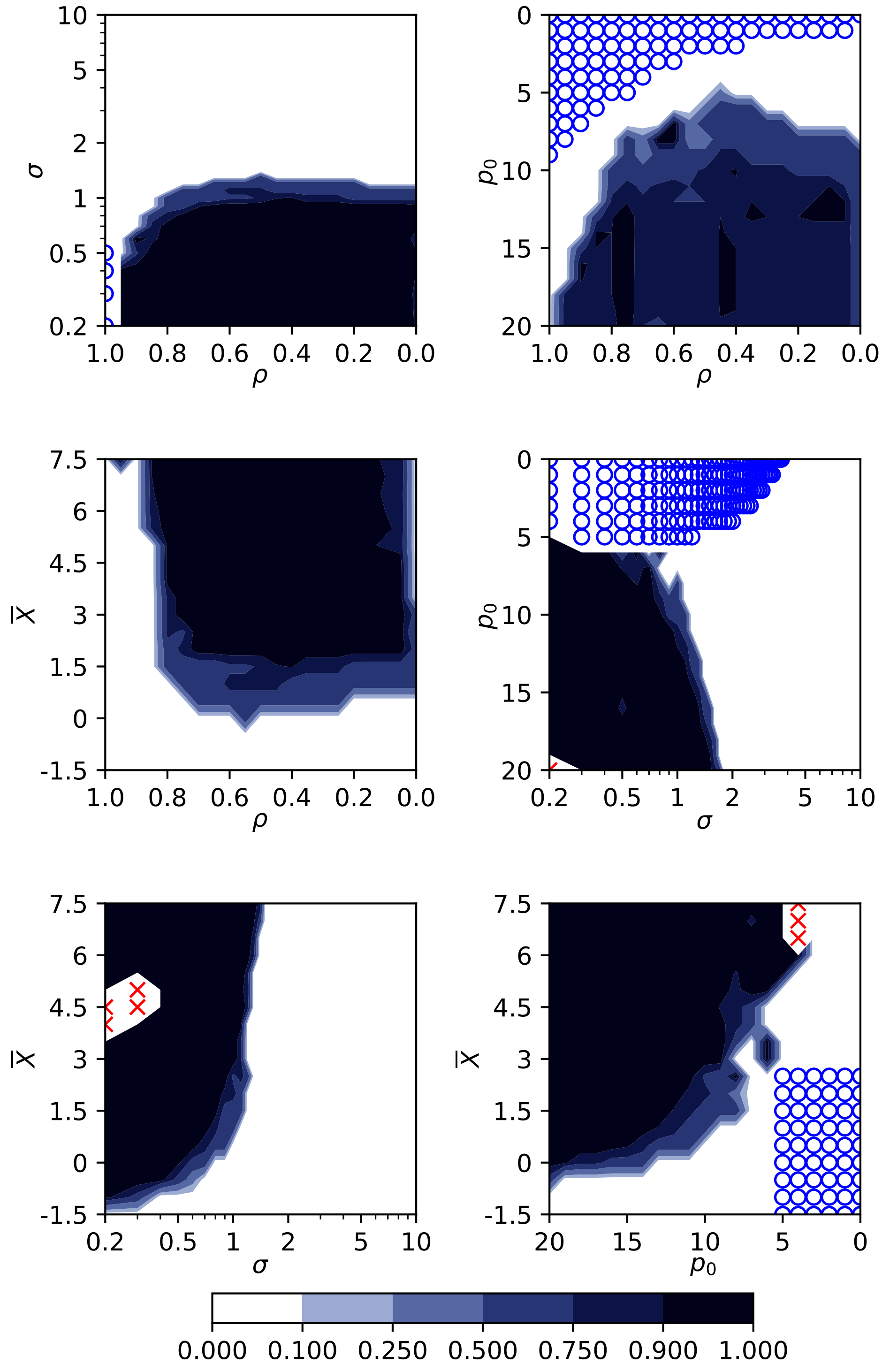

Figure OA21: Share of aggressive equilibria. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

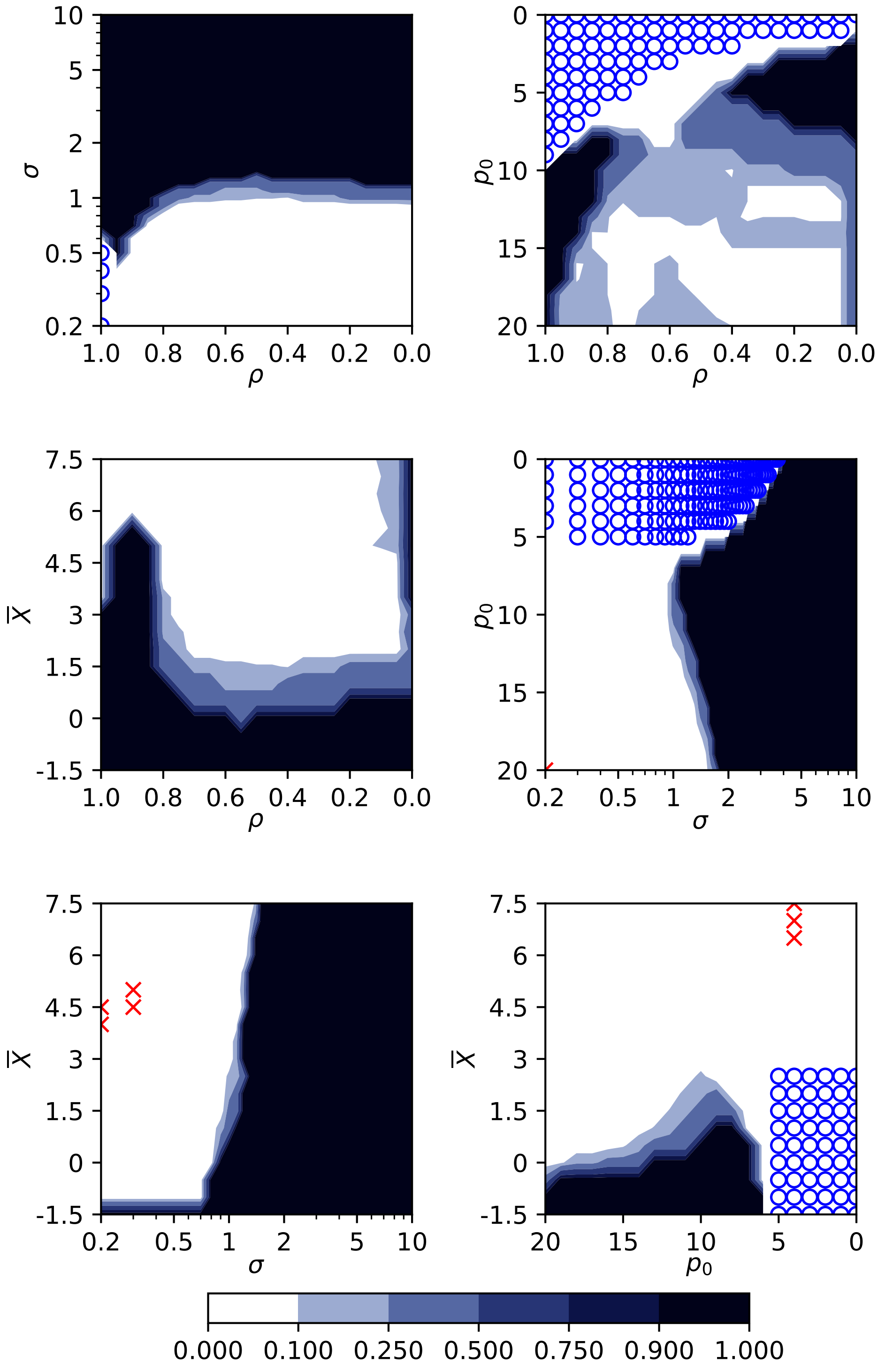

Figure OA22: Share of accommodative equilibria. Slices through equilibrium correspondence. Blue circle indicates non-viable industry. Red cross indicates computation failure.

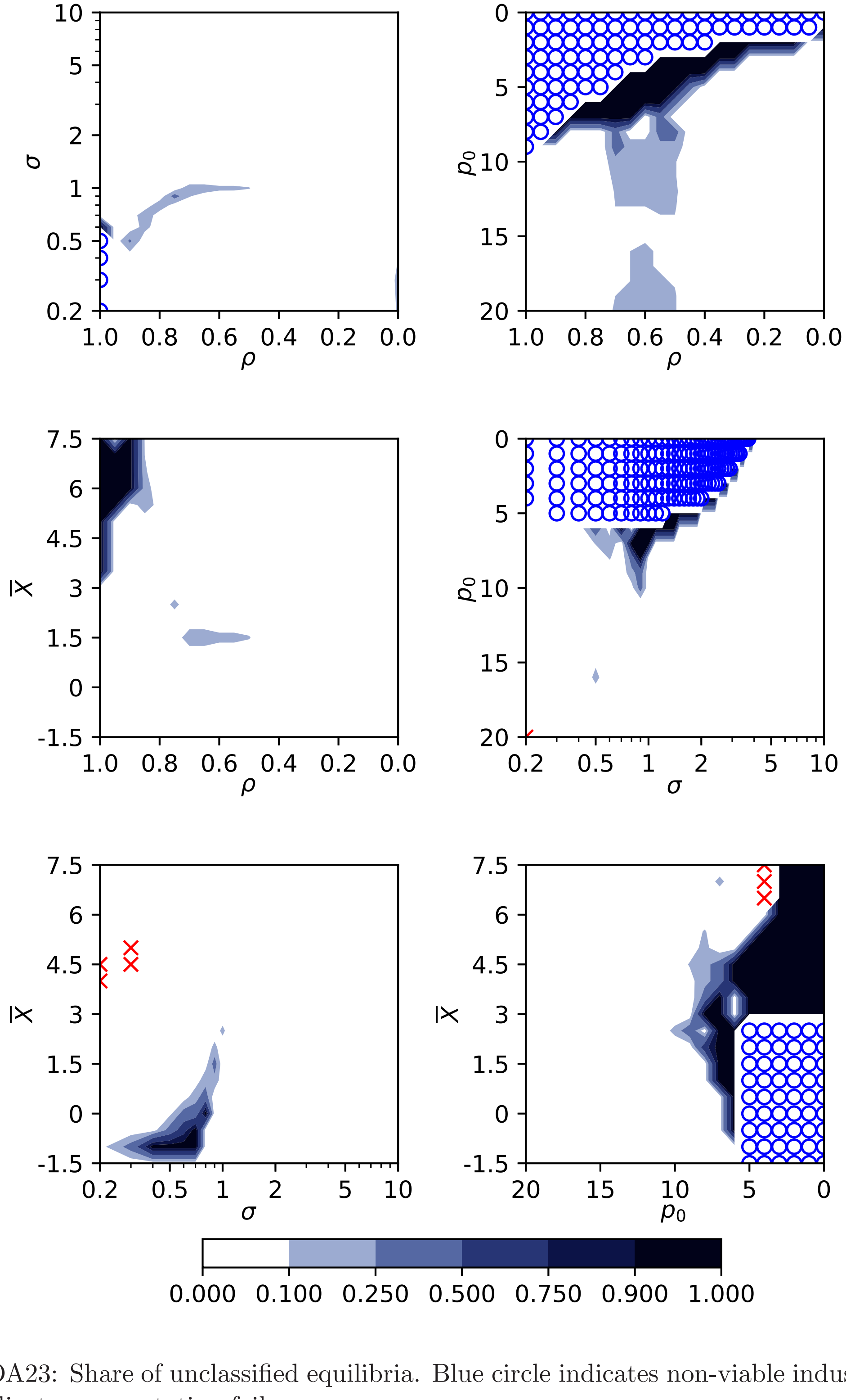

Figure OA23: Share of unclassified equilibria. Blue circle indicates non-viable industry. Red cross indicates computation failure.

Vettas, Nikolaos. 1998. "On entry, exit, and coordination with mixed strategies." *European Economic Review*, 44(2): 1557–1576.