Appendices

Online Appendix for
Small and Large Firms over the Business Cycle

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A A simple model where size and financial constraints coincide

A.1 Overview of the model and the results

The baseline model The model is set in discrete time. Firms maximize the present discounted value of future payouts to equityholders, and use the constant discount rate $\frac{1}{1+r}$. The problem of a surviving firm, indexed by i, in period t, is:

$$\begin{aligned} \mathbf{V}_t(k_{i,t}) \quad &= \quad \max_{k_{i,t+1}} \quad \eta n_{i,t} + (1-\eta)\left(n_{i,t} - k_{i,t+1} + \frac{1}{1+r}\mathbf{V}_{t+1}(k_{i,t+1})\right) \\ \text{s.t.} \qquad n_{i,t} \quad &= \quad z_t k_{i,t}^{\zeta} + (1-\delta)k_{i,t} \\ [\lambda_{i,t}] \quad 0 \quad &\leq \quad n_{i,t} - k_{i,t+1} \end{aligned}$$

Here, $k_{i,t}$ are the firm's assets in place. The firm's operating profits are given by $\pi_{i,t} = z_t k_{i,t}^{\zeta}$, with $0 < \zeta < 1$ denoting the curvature of the profit function with respect to assets and z_t is an aggregate shock, which may capture aggregate changes in productivity, demand, or the cost of inputs.[68] Finally, $n_{i,t}$ is the firm's net worth, which is equal to the sum of its operating profits and the depreciated value of its capital stock.

There are two financial frictions in this environment. The first is that payouts to equityholders must be positive: $n_{i,t} \geq k_{i,t}$. The frictionless model is one where, by contrast, payouts to equityholders can take any sign without affecting their marginal benefit (or cost): $n_{i,t} \gtrless k_{i,t}$. The second is that firms are not allowed to borrow. Firms are therefore completely internally financed. Note that another way to express the financial constraint is that $\pi_{i,t} \geq i_{i,t} = k_{i,t+1} - (1-\delta)k_{i,t}$, so that operating profits must fully cover investment in each period. The shadow value of internal funds is $\nu_{i,t} = 1 + \lambda_{i,t}$; a firm is constrained, if and only if, $\nu_{i,t} > 1$. The stark assumption of pure internal financing is a useful benchmark that we later relax.

Finally, with probability η, a surviving firm exogenously exits at the beginning of the period. In this case, equityholders receive the firm's net worth as a payout. In order to focus the analysis on

[68] The curvature in the profit function may originate either in decreasing returns in production or in monopoly power. Depending on which specific microfoundation for the profit function is chosen, z_t will be given by a specific combination of aggregate productivity, the real wage rate, and aggregate demand for the industry's product.

intensive margin responses, we assume that replacement of each exiting firm occurs at a exogenously determined level of assets, k_e.

In stationary equilibrium ($z_t = z$ for all t), the frictionless model has the simple solution:

$$k_{i,t+1} = k^* \equiv \left(\frac{\zeta z}{r+\delta}\right)^{\frac{1}{1-\zeta}}, \quad \forall i,t. \tag{12}$$

At this value for $k_{i,t+1}$, the expected discounted marginal product of capital is equal to 1. In the frictionless model, all surviving firms have the same size. By contrast, in stationary equilibrium, the solution to the model with frictions is:

$$k_{i,t+1} = \begin{cases} n_{i,t} & \text{if } n_{i,t} < k^* \\ k^* & \text{if } n_{i,t} \geq k^* \end{cases}. \tag{13}$$

So long as $n_e = zk_e^{\zeta} + (1-\delta)k_e < k^*$, the stationary equilibrium also features a cross-section of firms of different sizes: firms are born small relative to their desired capital stock k^*, must save to reach it, and may fail to reach their optimal size due to the exogenous exit shock. (Details and proofs are reported in section A.2 below.)

The effects of an aggregate shock We consider the perfect foresight response of the model to a shock to z_t. Specifically, we assume that at time $t = -1$, $z_t = z$, and that the model is in its stationary equilibrium. Moreover, at time 0, firms learn that the future path of z_t, for $t \geq 0$, will be $z_t = z\exp\left(-\rho^t \epsilon\right)$, where $\epsilon > 0$ is a shock to productivity, and ρ is the persistence of the shock. This exercise is meant to approximate the response of the economy to a mean-reverting decline in productivity. The top panel of figure A1 shows the perfect foresight response of output to a temporary decline in z_t, starting from the steady-state described by (13).[69] In the model with frictions, the most responsive firms are the largest ones — there are differences in cylicality across firms of different sizes, but of the opposite sign as in the data.

Why are large firms more sensitive? The aggregate shock has two effects: it lowers all firms' net worth $n_{i,t} = z_t k_{i,t}^{\zeta} + (1-\delta)k_{i,t}$; but it also reduces the optimal unconstrained size of firms,

$$k^*_{t+1} = \left(\frac{\zeta z_{t+1}}{r+\delta}\right)^{\frac{1}{1-\zeta}}.$$

When the shock hits the economy, initially unconstrained firms (those with $n_{i,0} \geq k^*$) find themselves with financial slack: even though their net worth falls, it still remains above the new unconstrained threshold, $\underline{n}_1 = k^*_1$. As a result, these firms respond by paying out excess cash, and shrinking to $k_{i,1} = k^*_1$. By contrast, most constrained firms start from a point where $n_{i,0} < \underline{n}_1 = k^*_1$.

[69] The calibration of the model is described in section A.2 below; in particular, the choice of the exogenous exit rate and the entry size imply that in steady-state, 1% of firms are unconstrained. The path of the shock is $z_t = z\exp(-\rho^t\epsilon)z$; in all figures, we use $\rho = 0.8$ and $\epsilon = 0.01$.

That is, these firms are below their optimal size *even after* the aggregate shock. These firms' responses then only reflect changes in net worth. Because net worth is a linear function of the aggregate shock, whereas the optimal size is a convex function of the aggregate shock, the optimal size response tends to be larger than the net worth response.[70] Financial frictions, in this case, work like an adjustment cost, moderating the response of quantities.

Adding pro-cyclical external financing The previous example shows that restricted access to external finance alone is not sufficient to generate a size effect. We next add debt financing to the model and allow the borrowing constraint to be a function of both the firm's net worth and, crucially, of the aggregate shock. The firm's objective is now:

$$
\begin{aligned}
\mathbf{V}_t(k_{i,t}, b_{i,t}) \quad &= \max_{k_{i,t+1}, b_{i,t+1}} \quad \eta n_{i,t} + (1-\eta)\left(n_{i,t} - k_{i,t+1} + b_{i,t+1} + \frac{1}{1+r}\mathbf{V}_{t+1}(k_{i,t+1}, b_{i,t+1})\right) \\
& \qquad n_{i,t} \quad = \quad z_t k_{i,t}^{\zeta} + (1-\delta)k_{i,t} - (1+r)b_{i,t} \\
\text{s.t.} \qquad & \qquad b_{i,t+1} \quad \leq \quad \mathbf{b}(n_{i,t}; z_t) \\
& \qquad n_{i,t} + b_{i,t+1} \quad \geq \quad k_{i,t+1}
\end{aligned}
$$

where $\mathbf{b}(\cdot, \cdot)$ — the borrowing constraint — is a function of both the firm's net worth and the aggregate shock z_t. As before, firms cannot raise equity (i.e. issue negative dividends).[71]

The solution to the firm's problem is similar to the case with no borrowing; the details and proofs are reported in section A.3 below. Firms with high levels of net worth invest at the optimal level k^*_{t+1}, while firms with insufficient net worth are either partially or fully constrained. Partially or fully constrained firms do not issue any dividends. Fully constrained firms utilize all their borrowing capacity; that is, $k_{i,t+1} = n_{i,t} + \mathbf{b}(n_{i,t}, x_{t+1})$. Partially constrained firms invest at the currently optimal level, but pay zero dividends. There need not be partially constrained firms in equilibrium; the situation only occurs when fundamentals are such that firms may be constrained tomorrow, for example if z_t is rising sharply over time.[72]

As before, we construct the response to a one-time unanticipated and mean-reverting decline in z_t and compare the responses of small and large firms. The bottom panel of Figure A1 displays the sales, investment, dividend issuance, and debt financing response of small and large firms. These responses are constructed under the assumption that the borrowing constraint is sufficiently elastic with respect to the aggregate shock so as to generate greater sensitivity of investment among small firms.[73] Under this assumption, small firms will cut back on investment faster, and subsequently

[70] Below we show that a necessary condition for the response of net worth to be smaller than the response of the optimal investment target is that $\frac{\rho}{1-\zeta} \geq \frac{r+d}{\zeta}$. This condition is met in our calibration; it will be satisfied so long as the aggregate shock is not too transitory. It is clear that a purely transitory shock ($\rho = 0$) would only have a net worth effect and hence only cause constrained firms to respond.

[71] Additionally, we restrict attention to solutions which satisfy the following transversality condition: $\lim_{t\to\infty} (1+r)^{-t}\mathbf{V}_t(k_{i,t+1}, b_{i,t+1}) \leq 0$.

[72] The appendix provides detailed conditions under which the partially constrained regime exists. It is worth noting that it never exists in steady-state.

[73] The appendix derives a simple sufficient condition on the elasticity of the borrowing constraint with respect to

experience larger declines in sales than large firms. It is straightforward to understand why a highly procyclical borrowing constraint is necessary. Constrained firms' investment is given by their total financing capacity:

$$k_{i,t+1} = n_{i,t} + \mathbf{b}(n_{i,t}, z_t),$$

while unconstrained firms' investment is simply the optimal path $k^*_{t+1} = \left(\frac{\zeta z_{t+1}}{r+\delta}\right)^{\frac{1}{1-\zeta}}$. The latter is a convex function of the aggregate shock; intuitively, so long as the borrowing function is chosen so that the total borrowing capacity $n_{i,t} + \mathbf{b}(n_{i,t}, z_t)$ is a "more" convex function of the aggregate shock, the investment response of small/constrained firms will be larger.

However, a byproduct of the assumption of a procyclical borrowing constraint is that debt financing flows among small firms should also respond strongly to the aggregate shock. The bottom panel of Figure A1 reports the cumulative change in debt among small and large firms. The contraction in debt among small firms is deeper and more protracted than among large firms. This is the financial flipside of the greater sensitivity of investment which the model generates. The model thus suggests that if small firms display greater sensitivity in investment because of financial constraints, then, we should also expect to find greater sensitivity in debt flows.

A.2 Detailed results for the model with no external finance

Sufficient conditions for greater sensitivity First note that, in the stationary equilibrium of the model, the (gross) growth rate of the capital stock of a constrained firm is given by:

$$\begin{aligned} g_{i,cons} &= \frac{k_{i,t+1}}{k_{i,t}} \\ &= \frac{n_{i,t}}{k_{i,t}} \\ &= \frac{z k_{i,t}^{\zeta} + (1-\delta) k_{i,t}}{k_{i,t}} \\ &= 1 - \delta + z k_{i,t}^{1-\zeta} \\ &\geq 1 - \delta + \frac{1}{\zeta}(r+\delta) \equiv g_{cons} \end{aligned}$$

where the last line comes from the fact that $k_{i,t} \leq k^*$. Note that $g_{cons} > 1$. By contrast, in steady-state, the (gross) growth rate of unconstrained firms is $g_{uncons} = 1$.

Now consider a firm which is constrained at $t = -1$ and stays constrained at $t = 0$, when the

the aggregate shock that ensures the model generates greater sensitivity for investment.

shock occurs. Following similar steps, the gross growth rate of its capital stock will be given by:

$$\begin{aligned} g_{cons}^{(0)} &= \tfrac{k_{i,1}}{k_{i,0}} \\ &= \tfrac{n_{i,1}}{k_{i,0}} \\ &= \tfrac{z\exp(-\epsilon)k_{i,0}^{\zeta}+(1-\delta)k_{i,0}}{k_{i,0}} \\ &= 1-\delta+z\exp(-\epsilon)k_{i,0}^{1-\zeta} \\ &\geq 1-\delta+\tfrac{1}{\zeta}(r+\delta)\exp(-\epsilon) \\ &\approx g_{cons}-\tfrac{1}{\zeta}(r+\delta)\epsilon \end{aligned}$$

Thus, the drop in growth relative to g_{cons} is approximately:

$$\Delta g_{cons.} = -\frac{1}{\zeta}(r+\delta)\epsilon.$$

By contrast, for unconstrained firms, it is straightforward to see that the drop in growth relative to g_{unc} is:

$$\Delta g_{unc.} = -\frac{\rho}{1-\zeta}\epsilon.$$

Thus, for sales growth among large firms to fall more, relative to trend, that growth among small firms, it must be the case that:

$$\frac{\rho}{1-\zeta} \geq \frac{1}{\zeta}(r+\delta),$$

which holds in the calibration we study. Note here that in both the data and the model, growth among small and large firms is measured relative to its long-run average. The "de-trending" used in this derivation is approximate, in that it substitutes the long-run average growth rate of small firms for its lower bound, $g_{cons.}$, instead of the actual cross-sectional average growth rate of small firms in steady-state. However, the impulse responses reported are constructed using the actual long-run average growth rate of small firms in the stationary steady-state; this does not change the conclusion that small firms do not display greater sensitivity in this model.

Calibration of the model We construct a quarterly calibration of the model; in particular, we set $\zeta = 0.8$, $\delta = \frac{0.16}{4}$ and $r = \frac{0.02}{4}$. Additionally, we set:

$$z = \left(\frac{\zeta}{\delta+r}\right)^{-1},$$

This normalization implies that the steady-state size of unconstrained firms satisfies $\log(k^*) = 0$.

Given a value for the entry size k_e such that $k_e < \bar{k}$, there exists a unique integer $N \geq 2$ such that:

$$\mathbf{n}^{N-1}(k_e) < k^* \quad , \quad \mathbf{n}^{N}(k_e) \geq k^*,$$

where $\mathbf{n}(k) \equiv x^{1-\zeta}k^{\zeta} + (1-\delta)k$, and $\mathbf{n}^j(.)$ is the j-th iterate of $\mathbf{n}$. The stationary distribution is then a discrete distribution $\{\mu_j\}_{j=0}^N$, with $\sum_{j=0}^N \mu_j = 1$, supported on $N+1$ points $\{k_j\}_{j=0}^N$, where:

$$k_j = \begin{cases} \mathbf{n}^j(k_e) & \text{if } 0 \leq j \leq N-1 \\ k^* & \text{if } j = N \end{cases} \tag{14}$$

Given the exit rate η, and a mass of entering firms M, the distribution is given by:

$$\mu_j = \begin{cases} (1-\eta)^j M & \text{if } 0 \leq j \leq N-1 \\ \frac{(1-\eta)^N}{\eta} M & \text{if } j = N \end{cases} \tag{15}$$

We normalize $M = \frac{1}{\eta}$, so that the total mass of firms is 1 in steady-state. We then pick the entry size k_e to be $k_e = (0.001)k^*$, similar to the $p50/p99$ ratio of book assets in the QFR. Given that $\log(k^*) = 0$, this requires $\log(k_e) = \log(0.001)$. Given this choice of k_e, $N(k_e)$ is determined; given the calibration above, we have $N = 113$. We then pick η so that, in steady-state, 1% of firms are unconstrained: $\frac{(1-\eta)^N}{\eta} = 0.01$. This choice allows us to think of the size-conditional impulse response reported in the main text as also reflecting the behavior of constrained and unconstrained firms. Given all other parameters, matching this target requires $\eta = 0.040$. This exit rate is somewhat higher than what is observed among the firms of the balanced QFR panel. With a lower curvature of the profit function, it is straightforward to obtain lower implied exit rates; moreover, the qualitative implications of the model are independent of the value chosen for η.

A.3 Detailed results for the model with debt financing

Characterization of optimal policies The following lemma, and the figure that accompanies it, gives the solution to the problem of the firm with financial constraints. For brevity, the proofs of the lemma and the others that follow are omitted, but they are available from the authors upon request.

Lemma 1 (Constrained solution)**.** *Assume that the borrowing constraint is* $\mathbf{C}^1$ *and satisfies:*

$$\frac{\partial \mathbf{b}}{\partial n_{i,t}}(n_{i,t}, z_{t+1}) \geq 0, \quad \mathbf{b}(0, z_{t+1}) = 0;$$

$$\frac{\partial \mathbf{b}}{\partial z_{t+1}}(n_{i,t}, z_{t+1}) \geq 0.$$

Let $\{\underline{n}_t\}_{t\geq 0}$ *be the unique solution to:*

$$\begin{aligned} \underline{n}_t &= \max\left(\mathbf{c}^{-1}\left(k^*_{t+1}; z_{t+1}\right), -\left(\frac{1}{\zeta} - 1\right)(\delta + r_b)k^*_{t+1} + \frac{1}{1+r_b}\underline{n}_{t+1}\right), \\ \lim_{t\to+\infty}(1+r_b)^{-t}\underline{n}_t &\leq 0, \end{aligned} \tag{16}$$

where $\mathbf{c}(n,z) \equiv n + \mathbf{b}(n,z)$ *is the maximum investment capacity of a firm with net worth* n*, conditional on aggregate productivity being equal to* z*. The solution to the firm's problem takes one of three forms, corresponding to three regions for net worth:*

- ***If*** $n_{i,t} < \mathbf{c}^{-1}\left(k^*_{t+1}; z_{t+1}\right)$*, the firm is constrained:*

  $$k_{i,t+1} = \mathbf{c}\left(n_{i,t}, z_{t+1}\right), \quad d_{i,t} = 0, \quad \frac{1}{1+r_b} b_{i,t+1} = \mathbf{b}(n_{i,t}, z_{t+1}), \quad \mathbf{V}_t(k_{i,t}, b_{i,t}) < \mathbf{V}_t^{(unc)}\left(k_{i,t}, b_{i,t}\right).$$

  Investment is strictly smaller than the optimal unconstrained level: $k_{i,t+1} = \mathbf{c}\left(n_{i,t}, z_{t+1}\right) < k^*_{t+1}$*. The marginal value of net worth is strictly above* 1*.*

- ***If*** $n_{i,t} \in \left[\mathbf{c}^{-1}\left(k^*_{t+1}; z_{t+1}\right), -\left(\frac{1}{\zeta} - 1\right)(\delta + r_b)k^*_{t+1} + \frac{1}{1+r_b}\underline{n}_{t+1}\right]$*, the firm is partially constrained; it invests at the currently optimal scale, but issues no dividends:*

  $$k_{i,t+1} = k^*_{t+1}, \quad d_{i,t} = 0, \quad \frac{1}{1+r_b} b_{i,t+1} = n_{i,t} - k^*_{t+1}, \quad \mathbf{V}_t(k_{i,t}, b_{i,t}) < \mathbf{V}_t^{(unc)}\left(k_{i,t}, b_{i,t}\right).$$

  The marginal value of net worth is strictly above 1*.*

- ***If*** $n_{i,t} > \underline{n}_t$*, the firm is fully unconstrained, can invest at the optimal scale today and at all future dates:*

  $$k_{i,t+1} = k^*_{t+1}, \quad d_{i,t} \geq 0, \quad \frac{1}{1+r_b} b_{i,t+1} \leq \mathbf{b}(n_{i,t}, x_t), \quad \mathbf{V}_t(k_{i,t}, b_{i,t}) = \mathbf{V}_t^{(unc)}\left(k_{i,t}, b_{i,t}\right).$$

  The marginal value of net worth is equal to 1*.*

The lemma says that there are three possible regions for firms' policies: either firms are constrained, in that they issue no dividends, borrow as much as possible, and invest below the optimal level today; or, they are partially constrained, in that they issue no dividends, but invest at the optimal level today and borrow less (strictly) than the maximum possible; or, they are fully unconstrained. Firms move up across these three regions as their net worth increases.

In the constrained region, investment today is entirely constrained by the firms' investment capacity,

$$k_{i,t+1} = \mathbf{c}(n_{i,t}, z_{t+1}) = n_{i,t} + \mathbf{b}(n_{i,t}, z_{t+1}) < k^*_{t+1}.$$

So the responsiveness of these firms' investment to shocks depend on their effect on *current* net worth, and potentially future productivity. By contrast, in the partically constrained and unconstrained region, investment today depends only on fundamentals tomorrow $k_{i,t} = k^*_{i,t+1}$.

The partially constrained region need not exist. Namely, for it to exist, it needs to be the case that:

$$\mathbf{c}^{-1}\left(k^*_{t+1}; z_{t+1}\right) < -\left(\frac{1}{\zeta} - 1\right)(\delta + r_b)k^*_{t+1} + \frac{1}{1+r_b}\underline{n}_{t+1}.$$

The right-hand side of this equation is the level of net worth necessary today in order to be able to implement the unconstrained optimal plan *starting tomorrow*; the left-hand side is the level of net worth necessary to implement the unconstrained optimal level of investment *today*. So, the partially constrained region only exists if the fundamentals process is such that firms will need high(er) levels of net worth in the future in order to implement the unconstrained plan. Most likely, that will be when fundamentals are low today relative to what they will be in the future.

It is immediate to see that there are no partially constrained firms in the stationary steady-state of the model. Additionally, one can rule out the possibility by imposing some restrictions on the aggregate process $\{z_t\}_{t\geq 0}$ and on the borrowing constraint $\mathbf{c}$.

Lemma 2. *Let:*

$$\forall t \geq 0, \quad g_t \equiv -\left(\frac{1}{\zeta}-1\right)\frac{r_b+\delta}{1+r_b}\frac{k^*_{t+1}}{\mathbf{c}^{-1}\left(k^*_{t+1}, z_{t+1}\right)} + \frac{1}{1+r_b}\frac{\mathbf{c}^{-1}\left(k^*_{t+2}, z_{t+2}\right)}{\mathbf{c}^{-1}\left(k^*_{t+1}, z_{t+1}\right)}. \tag{17}$$

Assume that $\{z_t\}_{t\geq 0}$ is increasing and bounded from above, and that $\{g_t\}_{t\geq 0}$ is strictly decreasing. Let:

$$T \equiv \min\left\{ \quad t \geq 0 \quad s.t. \quad g_t \leq 1 \quad \right\}.$$

Then the net worth threshold $\{\underline{n}_t\}_{t\geq 0}$ is given by:

$$\underline{n}_t = \begin{cases} -\left(\frac{1}{\zeta}-1\right)\frac{r_b+\delta}{1+r_b}k^*_{t+1} + \frac{1}{1+r_b}\underline{n}_{t+1} & \text{if} \quad t \leq T-1, \\ \mathbf{c}^{-1}\left(k^*_{t+1}, z_{t+1}\right) & \text{if} \quad t \geq T. \end{cases} \tag{18}$$

In particular, if $g_0 \leq 1$, then the unconstrained threshold is always given by:

$$\underline{n}_t = \mathbf{c}^{-1}\left(k^*_{t+1}, z_{t+1}\right);$$

as a result, firms are never partially constrained.

This lemma essentially places a restriction on the fundamentals of the model that ensures that the unconstrained threshold $\underline{n}_t$ does not grow "too fast" in the wake of the shock. The calibration below (and the particular functional form for $\mathbf{c}$ chosen) satisfy the restriction provided by lemma 5. This ensures that firms are always completely constrained, or completely unconstrained, which simplifies the analysis of the model.

Borrowing constraint and sufficient conditions for greater sensitivity We assume that the borrowing constraint is given by:

$$\mathbf{b}(n_t, z_{t+1}) = \left(\frac{1}{\theta}\left(\frac{z_{t+1}}{z}\right)^{\alpha} - 1\right) n_t \quad , \quad \alpha \geq 0, \quad \theta \leq 1.$$

This parametrization captures some of the limit cases we are interested in. As $\theta \to 0$, the frictionless model obtains; when $\theta = 1$ and $\alpha = 0$, firms cannot borrow and the baseline model (with the addition of saving) obtains. Finally, the parameter α controls the sensitivity of the borrowing threshold to the aggregate shock, z_t; when $\alpha = 0$, the borrowing constraint only depends on net worth, and not on the shock; when $\alpha \in]0,1]$, the borrowing constraint is a concave function of the aggregate shock; and when $\alpha \in]1,+\infty]$, it is a convex function of the aggregate shock. Having a specific functional form will also allow us to plot impulse responses of the model.

Note that, given the functional form chosen for the borrowing constraint, the parameter α is irrelevant to the determination of the steady state. In what follows, we use:

$$\theta = 0.8,$$

implying a debt-to-asset ratio of about 0.2 in the version of the model with borrowing constraints that do not vary with productivity. This figure is consistent with the average net debt-to-asset ratio which we documented in the QFR data. We leave other parameters unchanged relative to the baseline model without borrowing.

The parameter α controls the ability for the model to generate greater sensitivity of sales and investment. To see this, first note that, following the same steps as in the model without borrowing, an approximation to the growth rate of constrained firms in the stationary steady-state of the model is:

$$g_{cons} = \frac{1}{\theta}\left(1 - \delta + \frac{1}{\zeta}(r+\delta)\right).$$

The impact growth rate on impact, on the other hand, can be bounded from below by:

$$g_{cons}^{(0)} \geq \tfrac{1}{\theta}\exp(-\alpha\epsilon)\left(1 - \delta + \tfrac{1}{\zeta}(r+\delta)\exp(-\epsilon)\right)$$

Thus, the impact response of growth among constrained firms, relative to the long-run steady-state, is:

$$\Delta g_{cons} = -\frac{1}{\zeta}(r+\delta)\epsilon - \frac{1}{\theta}\alpha\epsilon\left(\frac{1}{\zeta}(r+\delta) + (1-\delta)\right) + o(\epsilon).$$

The impact response of unconstrained firms is the same as in the previous model. Thus, greater sensitivity of small/constrained firms firms will obtain so long as:

$$\frac{\rho}{1-\zeta} \leq \frac{1}{\zeta}(r+\delta) + \frac{1}{\theta}\alpha\left(\frac{1}{\zeta}(r+\delta) + (1-\delta)\right),$$

and in particular, for sufficiently high values of α. In the reported impulse responses, we use $\alpha = 5$, which ensures that this condition holds.

B Economies of scope and the size effect

This appendix describes a simple equilibrium model which produces two simple empirical regularities: (a) firms operating across multiple industries are larger than single-industry firms, even within the industries were they compete; (b) the sales, prices, and output of multi-industry firms respond less than those of single-industry to aggregate shocks (in the case of this model, a shock to household income.) The model relies on two simple ingredients: non-homotheticity in the demand for each industry's products; and economies of scope across industries. Both elements must be present in order to deliver the predictions.

B.1 Model description

Representative household There is a representative household with linear preferences over an aggregate of goods produced by N different industries, described by:

$$C = \left(\sum_{n=1}^{N} C_n^{\rho} \right)^{\frac{1}{\rho}},$$

where $\rho \in [0,1[$ governs the elasticity of substitution across goods from different industries. In each industry, two firms operate; a firm of type S (for Single-line firm), and a firm of type M (for Multi-line firm.) The differences between these firms are described below. The total consumption of goods in each industry, C_n, is an aggregate of the goods produced by these two firms, taking the following form:

$$C_n = \left(\left(C_{n,S} - C_{n,S}^{*}\right)\right)^{\epsilon} + \left(C_{n,M} - C_{n,M}^{*}\right)^{\epsilon} \Big)^{\frac{1}{\epsilon}}. \tag{19}$$

Here, $\epsilon \in [0,1[$ governs the elasticity of substitution across goods within industries. Moreover, $C_{n,x}^{*}$, for $x \in \{S, M\}$., represents the inelastic portion of the demand for each firm's product, as explained below. Finally, the household has an endowment I of a numeraire good. The household's problem is therefore:

$$\max \left(\sum_{n=1}^{N} C_n^{\rho} \right)^{\frac{1}{\rho}} \quad \text{s.t.} \quad \sum_{n=1}^{N} P_{n,S} C_{n,S} + P_{n,M} C_{n,M} \leq I,$$

and subject to equation (1), for $n = 1, ..., N$. In each industry, the first-order conditions with respect to $\{C_{n,S}, C_{n,M}\}_{n=1}^{N}$ imply that:

$$\begin{aligned} C_{n,x} &= C_{n,x}^{*} + \left(\frac{P_{n,x}}{P_n} \right)^{-\frac{1}{1-\epsilon}} C_n, \quad x \in \{S, M\} \\ P_n &= \left(P_{n,S}^{-\frac{\epsilon}{1-\epsilon}} + P_{n,M}^{-\frac{\epsilon}{1-\epsilon}} \right)^{-\frac{1-\epsilon}{\epsilon}} \end{aligned} \tag{20}$$

In particular, a fraction $s_{n,x}$ of total demand for the good produced by firm x in sector n is price-inelastic, where:

$$s_{n,x}^{*} = \frac{C_{n,x}^{*}}{C_{n,x}} \in [0,1[, \qquad x \in \{S, M\}.$$

The household takes the level of inelastic demand associated with each industry and firm, $\left\{C^*_{n,x}\right\}_{n,x}$, as given. Finally, the other first-order conditions of household's problem imply that:

$$\begin{aligned} PC &= I - \sum_{n=1}^{N}\left(P_{n,S}C^*_{n,S} + P_{n,M}C^*_{n,M}\right) \\ P &= \left(\sum_{n=1}^{N} P_n^{-\frac{\rho}{1-\rho}}\right)^{-\frac{1-\rho}{\rho}} \end{aligned} \tag{21}$$

Firms There are two groups of firms in the economy: a group of N firms of type S, each of which operates in a specific industry; and a single firm of type M, which operates across all N industries. In the first stage, both types of firms invest in order to create a given level of price-inelastic demand for their product, $\left\{C^*_{n,x}\right\}_{n,x}$. In the second stage, both types of firms choose an output level $C_{n,x}$. At both stages, both types of firms take prices for their product as given. The main difference between S-firms and the M-firm is that, when investing in order to raise the level of inelastic demand, the M-firm enjoys economies of scope across industries.

Firms of type S In the second stage, a firm S in industry n chooses its variable input L in order to solve:

$$\begin{aligned} \max_{L_{n,S}} \quad & P_{n,S}Y_{n,S} - WL_{n,S} \\ \text{s.t.} \quad & Y_{n,S} \leq ZL^{\zeta}_{n,S}. \end{aligned}$$

Here, W is the price of the variable input and Z is the firm's productivity, and $\zeta < 1$ captures the degree of returns to scale in production. The solution to this problem can be written as:

$$Y_{n,S} = AP_{n,S}^{\frac{\zeta}{1-\zeta}}, \tag{22}$$

where $A \equiv \zeta^{\frac{\zeta}{1-\zeta}} Z^{\frac{1}{1-\zeta}} W^{-\frac{\zeta}{1-\zeta}}$. This leads to profits that are given by $\Pi_{n,S} = (1-\zeta)P_{n,S}Y_{n,S} = (1-\zeta)P_{n,S}\left(C^*_{n,S} + \left(\frac{P_{n,S}}{P_n}\right)^{-\frac{1}{1-\epsilon}} C_n\right)$. In the first stage, the S then firm solves:

$$\max_{C^*_{n,S}} \quad (1-\zeta)P_{n,S}\left(C^*_{n,S} + \left(\frac{P_{n,S}}{P_n}\right)^{-\frac{1}{1-\epsilon}} C_n\right) - \gamma\left(C^*_{n,S}\right)$$

The S firm faces a strictly increasing and convex cost of raising the inelastic portion of its demand, described by the function $\gamma(x)$. In what follows we assume this cost takes the form:

$$\gamma(x) = \frac{1-\zeta}{1+\delta} B x^{1+\delta}, \quad \delta > 0, \quad B > 0.$$

We require that $\frac{\zeta}{1-\zeta} \leq \frac{1}{\delta}$, which is a sufficient condition to guarantee the existence of a symmetric equilibrium. The first-order condition is:

$$P_{n,S} = \gamma' \left(C^*_{n,S} \right). \tag{23}$$

Firms of type M and economies of scope In the first stage, the problem of the firm of type $x = M$, which operates across industries, is:

$$\begin{aligned} \max_{\{L_{n,M}\}_{n=1}^{N}} \quad & \sum_{n=1}^{N} \left(P_{n,M} Y_{n,M} - W L_{n,M} \right) \\ \text{s.t.} \quad & Y_{n,M} \leq Z L_{n,M}^{\zeta}, \quad n = 1, ..., N. \end{aligned}$$

This problem is separable across industries; so it leads to the same solution as for the S-firms:

$$Y_{n,M} \;=\; A P_{n,M}^{\frac{\zeta}{1-\zeta}}. \tag{24}$$

In the first stage, we assume that the type-M firm potentially enjoys economies of scope in the production of the inelastic demand levels $\left\{ C^*_{n,M} \right\}_{n=1}^{N}$. Specifically, we assume that:

$$\Gamma \left(\left\{ C^*_{n,M} \right\}_{n=1}^{N} \right) = \left(\sum_{n=1}^{N} \gamma (C^*_{n,M})^{\alpha} \right)^{\frac{1}{\alpha}}, \quad \alpha \geq 1,$$

where $\gamma(.)$ is the cost function for the S-firms. When $\alpha = 1$, there are no economies of scope. When $\alpha > 1$, there are economies of scope. Indeed, when $\alpha > 1$, this cost function satisfies:

$$\Gamma \left(\left\{ C^*_{n,M} \right\}_{n=1}^{N} \right) = \left(\sum_{n=1}^{N} \gamma (C^*_{n,M})^{\alpha} \right)^{\frac{1}{\alpha}} < \sum_{n=1}^{N} \gamma (C^*_{n,M}) = \sum_{n=1}^{N} \Gamma \left(\left\{ 0, ..., 0, C^*_{n,M}, 0, ..., 0 \right\} \right).$$

In other words, when $\alpha > 1$, the cost function is sub-additive, and so exhibits economies of scope in the sense of Panzar and Willig (1981) or Tirole (1988).[74] In the first stage, the M-firm's profit maximization problem can then be written as:

$$\max_{\left\{ C^*_{n,M}, Y_{n,M} \right\}_{n=1}^{N}} \quad \sum_{n=1}^{N} P_{n,M} \left(C^*_{n,M} + \left(\frac{P_{n,M}}{P_n} \right)^{-\frac{1}{1-\epsilon}} C_n \right) - \left(\sum_{n=1}^{N} \gamma \left(C^*_{n,M} \right)^{\alpha} \right)^{\alpha}$$

The first-order conditions are:

$$P_{n,M} = \Gamma_M^{1-\alpha} \gamma (C_{n,M})^{\alpha-1} \gamma' \left(C^*_{n,M} \right), \quad n = 1, ..., N, \tag{25}$$

[74] The inequality follows from the fact that $\sum_i x_i^{\alpha} < \left(\sum_i x_i \right)^{\alpha}$ when $\alpha > 1$ and $x_i \geq 0$ for all i.

Finally, we close the model by assuming that there is an infinitely elastic supply of the variable input L at fixed price W.

B.2 Analysis

Equilibrium For given values of I, W, Z, an equilibrium of the model is a set of values for the endogenous variables:

$$P, C, MC, \Gamma_M, \{P_n, C_n, P_{n,S}, P_{n,M}, C_{n,S}, C_{n,M}, C^*_{n,S}, C^*_{n,M}\}_{n=1}^N$$

that solve the equations given above. Since the model is symmetric across sectors, in what follows, we only study symmetric equilibria, where:

$$\forall x \in \{S, M\}, \quad \forall n = 1, ..., N, \quad P_{n,x} = P_x, \quad C_{n,x} = C_x, \quad C^*_{n,x} = C^*_x,$$

Additionally, in a symmetric equilibrium, $\forall n = 1, ..., N, \quad P_n = N^{-\frac{1-\rho}{\rho}} P \equiv \tilde{P}, \quad C_n = N^{\frac{1}{\rho}} C \equiv \tilde{C}$. In this equilibrium, the total cost of raising inelastic demand for the M firm is given by $\Gamma_M = N^{\frac{1}{\alpha}} \gamma(C^*_M)$. Therefore, the first-order condition for investment in C^*_M in each industry are:

$$P_{n,M} = N^{\frac{1-\alpha}{\alpha}} \gamma'(C^*_M) = N^{\frac{1-\alpha}{\alpha}} B \left(C^*_M\right)^{\delta}.$$

Recall that for S-firms, this first-order condition is $P_{n,S} = B \left(C^*_S\right)^{\delta}$. In what follows, we denote $B_S = B$ the cost shifter for S-firms, and $B_M = N^{\frac{1-\alpha}{\alpha}} B < B = B_S$ the cost shifter for M-firms.

Analysis The equilibrium above has the following properties. When there are no economies of scope ($\alpha = 1$), within a particular industry, the M firm and the S firm invest to generate the same amount of inelastic demand. Moreover, they produce the same output, charge the same prices. Moreover, they respond identically to a shock to household income, I. This result is immediate; indeed, when there are no economies of scope, the two firms are identical.

However, when there are economies of scope ($\alpha > 1$), in each industry, the M-firm invests to generate more inelastic demand than the S firm. As a result, the inelastic portion of the firm's demand, s^*_x, is higher for the M-firm. Therefore, the M-firm charges higher prices, produces more, and has higher sales than the S-firm. Moreover, the M-firm is less responsive than the S-firm to a shock to household income. We discuss these results in the two lemmas that follow.

Lemma 3 (Economies of scope and firm size)**.** *Assume that there are economies of scope: $\alpha > 1$. Then:*

$$C^*_S < C^*_M, \quad C_S < C_M, \quad P_S < P_M, \quad S_S = P_S C_S < S_M = P_M C_M.$$

Proof. Consider the market clearing condition in each industry:

$$A P_x^{\frac{\zeta}{1-\zeta}} = C_x = Y_x = \left(\frac{P_x}{B_x}\right)^{\frac{1}{\delta}} + \left(\frac{P_x}{\tilde{P}}\right)^{-\frac{1}{1-\epsilon}} \tilde{C}, \quad x \in \{S, M\}.$$

The market clearing condition above implies:

$$\frac{B_x}{P_x}\frac{\partial P_x}{\partial B_x} = -\frac{\frac{1}{\delta}s_x}{\frac{\zeta}{1-\zeta}+\frac{1}{1-\epsilon}(1-s_x)-\frac{1}{\delta}s_{i,x}} \leq 0,$$

where the inequality holds because $\frac{\zeta}{1-\zeta} \geq \frac{1}{\delta}$, and where we defined $s_x = \frac{C_x^*}{C_x}$, the inelastic consumption share for each firm. Therefore, equilibrium prices are decreasing with B_x. But recall that, when there are economies of scope ($\alpha < 1$); $B_M = N^{\alpha-1}B < B = B_S$. Therefore, prices charged by the M-firm are higher. Additionally, the equilibrium share of inelastic demand for the M-firm is always higher:

$$s_M > s_S.$$

To see why, note that:

$$s_x = 1 - \frac{\tilde{P}^{\frac{1}{1-\epsilon}}\tilde{C}}{A}P_x^{-\frac{1}{1-\epsilon}-\frac{\zeta}{1-\zeta}}.$$

This expression is increasing with P_x. Since $P_M > P_S$, this implies that $s_M > s_S$. The rest of the results follow. □

Lemma 4 (Economies of scope and aggregate shocks)**.** *Assume that there are economies of scope: $\alpha > 1$. Then, in a response to a given shock to household income, I, sales of M-firms decline by strictly less than sales of S-firms.*

Proof. We denote by η_X the elasticity of variable X with to I. The market clearing condition can be log-linearized as:

$$\frac{\zeta}{1-\zeta}\eta_{P_x} = \frac{s_x}{\delta}\eta_{P_x}(1-s_x)\left(\eta_{\tilde{C}} - \frac{1}{1-\epsilon}\left(\eta_{P_x}-\eta_{\tilde{P}}\right)\right), \quad x \in \{S, M\}.$$

This implies that:

$$\eta_{P_x} = \frac{(1-s_x)(1-\zeta)(1-\epsilon)}{\zeta(1-\epsilon)+(1-s_x)(1-\zeta)-\frac{s_x}{\delta}(1-\zeta)(1-\epsilon)}\left(\eta_{\tilde{C}}+\frac{1}{1-\epsilon}\eta_{\tilde{P}}\right), \quad x \in \{S, M\}.$$

Note that sales elasticities are related to price elasticities through:

$$\eta_{S_x} = \frac{1}{1-\zeta}\eta_{P_x}, \quad x \in \{S, M\},$$

where $S_x = P_x C_x$ denotes the total sales of the firm of type x in one of the sectors. So,

$$\eta_{S_x} = \frac{(1-s_x)(1-\epsilon)}{\zeta(1-\epsilon)+(1-s_x)(1-\zeta)-\frac{s_x}{\delta}(1-\zeta)(1-\epsilon)}\left(\eta_{\tilde{C}}+\frac{1}{1-\epsilon}\eta_{\tilde{P}}\right).$$

Simple algebra, using the fact that $\zeta/(1-\zeta) > \frac{1}{\delta}$, then shows that:

$$\eta_{S_M} \leq \eta_{S_S} \iff s_M \geq s_S,$$

which, as discussed above, holds whenever $B_M < B_S$. Therefore, when there are economies of scope ($\alpha < 1$), the M-firm is less responsive to an aggregate shock than S-firms. □

C Non-size evidence of a financial accelerator

Finally, in this appendix, we document whether firms respond heterogeneously to recessions when conditioning directly on balance sheet characteristics, instead of size. Specifically, we provide event study plots comparing the evolution firm sales, inventories, and tangible investment around recessions, separating firms in groups of leverage, liquidity, bank-dependence, access to bond markets, and dividend issuance.

Figure A12 depicts the evolution of firms sales, inventories, and fixed capital comparing zero leverage firms (which account for roughly 20% of firm-quarter observations), and firms with positive leverage; we classify firms based on their four-quarter lagged debt to asset ratio. This plot is constructed using the same event study methodology as in section 5.2. As the plots show, the evolution of sales and investment at the two groups of firms is largely indistinguishable during recessions. The same holds true for liquidity: when sorting firms into low liquidity (firms with a cash to asset ratio of less than 0.2) and high liquidity (firms with a cash to asset ratio of greater than 0.2), we also find largely indistinguishable cumulative responses of sales, inventories, and investment.

The last row of Figure A12 sorts firms into bank-dependent and non-bank-dependent. The former are defined as firms with more than 90% of debt in the form of bank loans four quarters past. While bank dependent firms do qualitatively experience a sharper contraction in their sales and investment than non-bank dependent firms, the differences are, again, not statistically significant. Results based on leverage sorts would appear to be inconsistent with a financial accelerator mechanism. Under the financial accelerator mechanism, higher leverage firms should experience increases in the cost of external financing during recessions, leading to a faster decline in factor inputs and production relative to firms that do not rely on external financing. By contrast, the evidence provided above suggests that there is no sharp difference in the behavior of higher-leverage firms during recessions.

Figure A13 provides the event study plots for firms sorted on public debt market access (top row) and dividend issuance (bottom row). Firms with a history of accessing public debt markets contract their sales and inventories *faster* than firms with no history of market access. The financial accelerator mechanism would predict the opposite, as firms with access to bond markets should better be able to smooth sales and inventories over the business cycle. Moreover, the point estimates suggest that investment falls faster at firms without market access, but that the difference is not statistically significant. By contrast, firms sorted on dividend issuance do display statistically significant differences for inventory and investments in recessions: firms that issued dividends during the prior year also reduce inventories and investment more gradually than firms that did not.

D Measurement

D.1 The measurement framework used in this paper

The following paragraphs provide the details of the way in which we construct the size classification and growth measures used in section 3.

Sample selection Let i index firms and t index quarters. Let $x \in X$ index variables of interest; in the analysis, we use $X = \{\text{sales}, \text{inventory}, \text{NPPE stock}, \text{assets}\}$. Let:

$$\mathcal{I}_t(x) \quad \equiv \{ \quad i \quad \text{s.t.} \quad x_{i,t-4} > 0 \quad \text{and} \quad x_{i,t} > 0 \quad \} \tag{26}$$

We restrict attention to firms with strictly positive values of the variables of interest so as to compute log growth rates (see below). In order to be able to construct a consistent sample across variables of interest, we only consider firms $i \in \mathcal{I}_t$, where:

$$\mathcal{I}_t \equiv \bigcap_{x \in X} \mathcal{I}_t(x).$$

Size classification Let $a_{i,t}$ denote book assets. For every quarter t, we compute a set of percentiles,

$$\mathcal{P}_t = \left\{ \bar{a}_t^{(k)} \right\}_{k \in K},$$

where $K \subset [0, 100]$, $\bar{a}_t^{(0)} = 0$ and $\bar{a}_t^{(k)} = +\infty$. These percentiles are computed using the distribution of book assets of *all* firms, not only those firms $i \in \mathcal{I}_t$. Moreover, these percentiles are obtained using the Census-provided cross-sectional sampling weights $z_{i,t}$. We then define:

$$\mathcal{I}_t^{(k_1,k_2)} = \left\{ \quad i \in \mathcal{I}_t \quad \text{s.t.} \quad a_{i,t-4} \in \left[\bar{a}_{t-4}^{(k_1)}, \bar{a}_{t-4}^{(k_2)} \right[\quad \right\}. \tag{27}$$

In the case of the simple sample split between bottom 99% and top 1%, the small and large firms groups are defined as:

$$\begin{aligned} \mathcal{I}_t^{(\text{small})} &= \mathcal{I}_t^{(0,99)}, \\ \mathcal{I}_t^{(\text{large})} &= \mathcal{I}_t^{(99,100)} = \mathcal{I}_t \setminus \mathcal{I}_t^{(0,99)}. \end{aligned} \tag{28}$$

Growth rates For any $i \in \mathcal{I}_t$, we define growth rates as:

$$g_{i,t}(x) = \begin{cases} \log\left(\frac{x_{i,t}}{x_{i,t-4}}\right) & \text{if} \quad x \in \{\text{sales}, \text{inventory}, \text{NPPE stock}, \text{assets}\} \\ \frac{nppe_{i,t} - nppe_{i,t-4} + dep_{i,t-4,t}}{nppe_{i,t-4}} & \text{if} \quad x = \text{fixed investment}. \end{cases} \tag{29}$$

We focus on log growth-rates because they are easier to use in the decomposition of aggregate growth into firm-level growth rate discussed in section 4. Annual differences (instead of quarterly differences) are the main specification both because they are consistent with the size classification (which is based on one-year lags, so as to adequately capture initial size), and because they neutralize the issue of seasonal variation in the variables of interest. Cross-sectional averages of growth rates are then defined as:

$$\begin{aligned} \hat{g}_t^{(k_1,k_2)}(x) &\equiv \frac{1}{\mathcal{Z}_{t-4}^{(k_1,k_2)}} \sum_{i \in \mathcal{I}_t^{(k_1,k_2)}} z_{i,t-4} g_{i,t}(x) \\ Z_{t-4}^{(k_1,k_2)} &\equiv \sum_{i \in \mathcal{I}_t^{(k_1,k_2)}} z_{i,t-4}. \end{aligned} \tag{30}$$

and $z_{i,t-4}$ are the Census-provided cross-sectional sampling weights. Throughout, we analyze cross-sectional average time-series after de-meaning them (since the focus in not on long-term trends, but rather on the cyclicality of growth); we do not use any further detrending or filtering.

Robustness Our results for sales, inventory, the stock of net property, plant and equipment are robust to using half-growth rates of the form $2\frac{x_{i,t}-x_{i,t-4}}{x_{i,t}+x_{i,t-4}}$. Qualitatively and quantitatively, results do not change substantially whether one uses the one-year lagged or current weights in computing average growth rates of the form (30). Since the sample is titled toward larger firms, carrying the analysis using unweighted data ($z_{i,t} = 1, \forall (i,t)$) leads to qualitatively identical results, but somewhat smaller magnitudes.

D.2 The Gertler and Gilchrist (1994) measurement framework

The analysis of Gertler and Gilchrist (1994) centers around computing the cumulative change in revenue of an "aggregate" small and "aggregate" large firm. Revenues of the "aggregate" small firm are defined as the total sales of the group of firms which, starting from the smallest (by assets), account for a cumulative 30% of total sales at any point in time. Conversely, the revenues of the "aggregate" large firms are the total sales of firms which, starting from the largest (by assets), account for a cumulative 70% of revenue. This definition is driven by the fac that the publicly released QFR data only reports total sales of firms by bins of nominal asset size.

We next describe our implementation of the GG methodology. Let x denote nominal assets, let $\left\{x^{(1)}, ..., x^{(n)}\right\}$ denote the QFR's nominal asset bins' cutoffs, and let y denote nominal sales. For each quarter t, define $\underline{x}_t$ by:

$$\underline{x}_t = \max \left\{ x \in \left\{ x^{(1)}, ..., x^{(n)} \right\} \middle/ \frac{\sum_{x_{i,t} \leq x} y_{i,t}}{Y_t} \leq 0.3 \right\}$$

Furthermore, let $\underline{x}_t^+$ be the cutoff immediately above $\underline{x}_t$ in the list $\left\{x^{(1)}, ..., x^{(n)}\right\}$. Compute the weight w_t such that:

$$w_t \frac{\sum_{x_{i,t} \leq \underline{x}_t} y_{i,t}}{Y_t} + (1 - w_t) \frac{\sum_{x_{i,t} \leq \underline{x}_t^+} y_{i,t}}{Y_t} = 0.3$$

The growth rate of small firms' sales between time $t-1$ and t is then defined as:

$$G_t^{(small,GG)} = w_t \frac{\sum_{\{i/x_{i,t} \leq \underline{x}_t\}} y_{i,t}}{\sum_{\{i/x_{i,t-1} \leq \underline{x}_t\}} y_{i,t-1}} + (1-w_t) \frac{\sum_{\{i/x_{i,t} \leq \underline{x}_t^+\}} y_{i,t}}{\sum_{\{i/x_{i,t-1} \leq \underline{x}_t^+\}} y_{i,t-1}}.$$

The growth rate of large firms is defined analogously, using the cumulative sum of sales over the remaining bins of asset size. In our analysis, we use one-quarter lagged growth rates, consistent Gertler and Gilchrist (1994); moreover, we de-seasonalize the data by removing quarter fixed effects. Finally, consistent with GG, we de-mean the small and large growth series before computing cumulative growth rates in event study analyses.

Appendix Figure A4 reports changes in total sales of small and large firms constructed using this methodology, using publicly available QFR data from 1958q4 to 1991q4 — the period originally studied by Gertler and Gilchrist (1994). This graph most closely approximates figure II, p.321 from that paper. Appendix figure A4 reports the same event study responses, again for the GG growth rates, in the 1977q3 to 2014q1 period. These event study result highlight, in particular, the fact that using the GG methodology, sales large firms declined *more* than those of small firms during 2008q3. Finally, figure A6 reports the same event study response, constructed using the CM growth rates, that is, the growth rates for the top 1% and bottom 99% of firms by assets, the time series reported in Figure 1.

E Robustness results on the size effect

Robustness checks discussed in the main text Figure A2 reports estimates of the contribution of entering firms to employment in manufacturing. Table A1 reports estimates of the size effect after controlling for age. Table A2 contains estimates of the size effect under various levels of controls for industry effects, and using alternate size classifications.

Deflators In the main regression specification, Equation (1), sales are deflated using a common value-added price deflator for all firms in manufacturing. We use this deflator because, to our knowledge, at the quarterly frequency, there are no price deflators for output either at the manufacturing sector level or at more disaggregated levels within manufacturing. However, at the annual frequency, the BEA GDP by industry tables provide such indices. Table A3 compares estimates of the size effect in the Compustat quarterly sample when using different deflators. The top panel contains estimates for the same sample period as in the QFR, 1977q3-2014q1. Column 1 reports results from the specification of Equation (1), using the same quarterly value-added deflator as in that specification. The results of column 1 indicate a size effect in Compustat, though it is substantially smaller in magnitude than in the QFR, consistent with the fact there is less size variation in Compustat than in the QFR. Column 2 reports results from a specification that uses a manufacturing-wide deflator, but for gross output instead of value-added. Column 3 reports results from a specification that uses a separate annual gross output deflator for BEA subsector in manufacturing (the BEA subsectors in manufacturing correspond approximately to NAICS 3-digit industries). Estimates of the size effect in columns 1-3 are all very close in magnitude. Overall, these results indicate that estimates of the size effect in manufacturing are not sensitive to the choice of deflators.

Controlling for industry-quarter effects In most of our specifications, we absorb industry differences in cyclicality by controlling with set of dummies for industries interacted with GDP growth. An alternative approach is to instead control for a full set of industry-time fixed effects. The drawback of this approach is that it complicate the estimation of average marginal effects by size group, which are useful to our analysis of aggregate in Section 4. Column 4 of Table A3 reports results from estimating a specification with industry-quarter fixed effects in the Compustat manufacturing sample. The results from that specification are also very close in magnitude to those of specifications 1-3 of the same table, which are obtained using industries dummies interacted with GDP. This suggests that estimates of the relative size effect are robust to whether one uses our approach or whether one controls for industry-size effects.

Disaggregated estimates of the size effect Figure A3 contains disaggregated estimates of the size effect. Specifically, we disaggregate the smallest size category from our baseline analysis (the $[0, 90]$) group, into four smaller sub-groups, the $[0, 25]$, $[25, 50]$, $[50, 75]$ and $[75, 90]$ interquantile range for book assets. We then re-estimate the size effect using a specification identical to Equation (1), but with this richer size classification. (In particular, we use the durable/non-durable industry classification to control for differences in industry cyclicality.) The findings indicate that the size effects is in general homogeneous among the group of firms belonging to the $[0, 90]$ group. The notable exception is the $[50, 75]$ group for sales, which seems to display a higher cyclical sensitivity than average, though the difference is only marginally significant.

Sales and value added in manufacturing The income statements which firms reports to the QFR do not contain sufficient data to measure value added, and so our analysis focuses on sales as one of the main firm-level outcomes. A natural question, however, is whether the relative cyclicality of sales is also informative about the relative cyclicality of value added. Industry-level data indicate that the cyclical properties of value added may substantially differ from those of sales in certain sub-sectors of manufacturing. Table A5 reports the correlation between value added and output growth (at the annual frequency) in 18 BEA sub-industries of manufacturing between 1977 and 2014. (In the BEA's measures, output growth differs is the closest proxy for revenue or sales growth, and differs primarily because of inventory changes.) This correlation is lower than 0.3 for 2 out of 18 industries, Oil & Gas and Furniture and Related Products, which together accounted for 6.8 percent of nominal value added in manufacturing in 2001. In these industries, the differences between the behavior of output and sales can be large: for instance, in the Oil & Gas industry, the 2007-2009 decline in gross output was 4.7%, but value added in that industry increased, by 3.2%. Table A6 shows estimates of the size effect, when dropping firms which belong to BEA subsectors where the correlation between real output growth and real sales growth is low, as reported in table A5. The results are reported when dropping the 2, 5 or 8 sectors with the lowest correlation between real output growth and real value added growth. In the first case, the results are unchanged. In the latter two cases, the size effect is weaker in the 50%-75% size group, but remains comparable to its baseline estimate in the two other size groups. These results indicate that, in sectors where total value added growth and total output growth have similar cyclical properties, there is still a size effect for sales in the Compustat manufacturing sample. This is only suggestive evidence that the size effect for sales may translate to a size effect for value added; we recognize that direct measures of the cyclical behavior of value added across firm size groups would be needed to fully answer the question.

F The cyclicality of investment rates

In the QFR data, two cyclical properties of firm-level investment stand out. First, the contemporaneous correlation of firm-level investment with GDP growth, after controlling for industry effects, is slightly negative among the top 0.5% of firms, as reported in Table 4. Second, during recessions, the decline in investment among the top 1% of firms lags that of the bottom 99% of firms by 2-4 quarters, as indicated by the right panel of Figure 11. This appendix argues that the lag structure in investment among the largest firms can also be documented in two analogous data sources: the manufacturing segments of the annual and quarterly versions of Compustat.[75]

F.1 Data construction and summary statistics

Annual data Our source for the annual version of Compustat is the monthly update of the Fundamentals Annual file.[76] In order to obtain up-to-date industry identifiers, we merge this file with the Company file; whenever the 3-digit NAICS historical code is missing, we fill it with the next most recent available observation, using the Company file NAICS as the last (year 2017) NAICS observation.

In order to facilitate comparison with the QFR results, we focus on the following measure of investment:

$$ik_{i,t} = \frac{k_{i,t} - k_{i,t-1} + dep_{i,t}}{k_{i,t-1}}.$$

Here, $k_{i,t}$ is the stock of net property, plant and equipment reported on the balance sheet of firm i in year t, and $dep_{i,t}$ is depreciation reported in the firm's year t income statement.[77] Both $k_{i,t}$ and $dep_{i,t}$ are deflated using the BEA price index for manufacturing, as in the main text; the results also

[75] Replication code for this exercise is available from the authors upon request.

[76] We use the latest version of the `funda` file, available on WRDS at: `/wrds/comp/sasdata/nam/funda.sas7bdat`. We use only firm-year observations with strictly positive assets (variable `at`) and which satisfy the four standard screens INDL for industry format, STD for data format, D for population source and C for consolidation. The company file we use is the latest version available at: `/wrds/comp/sasdata/nam/company/company.sas7bdat`.

[77] We use fiscal year, variable `fyear`, to date our observations; replacing by the calendar year which most overlaps the firm's fiscal year does not change our results.

hold when using the BEA's 3-digit NAICS annual price indexes to deflate nominal values. We keep firm-year observations in sample if (a) t is between 1977 and 2014; (b) the firm-year observation is incorporated in the US (variable `fic` from the company file equal to "USA"); (c) the 3-digit NAICS code is between 311 and 339 in sample; (b) $k_{i,t}$ is non-missing and weakly larger than 1m$; (c) $dep_{i,t}$ is non-missing and weakly positive.

Each year, we create four size groups, corresponding to the four quartiles of the sample distribution of book assets. The average size of firms in each group over the 1977-2014 sample is reported in Table A8, after deflating book assets by the manufacturing price index. As in the main text, firms are then grouped according to their one-year lagged position in the firm size distribution. Relative to the overall sample, the regression sample is the subset of firm-year observations such that the firm is also present in sample one year prior; (b) total depreciation $dep_{i,t}$, in nominal terms, is weakly smaller than the one-year lagged stock of net property, plant and equipment. This latter criterion helps filter very large positive observation of $ik_{i,t}$. The resulting annual sample has 72363 firm-year observations.

Quarterly data We follow a similar procedure to construct the quarterly sample.[78] The fundamentals quarterly file does not contain NAICS 3-digit identifiers. Whenever possible, we use the 3D-NAICS identifier at the annual frequency, as described above; otherwise, we use the identifier from the company files. As in the QFR data, we construct year-on-year investment rates at the quarterly frequency for each firm: $ik^q_{i,t} = \frac{k^q_{i,t} - k^q_{i,t-4} + dep_{i,t-4,t}}{k^q_{i,t-4}}$. Here, t now denotes a quarter; $k^q_{i,t}$ denotes the net stock of property, plant and equipment (variable `ppentq`) deflated by the price index for manufacturing; we interpolate the annual time series in order to obtain quarterly data. The variable $dep_{i,t-4,t}$ denotes *total* depreciation over the preceding year, which we compute by taking the sum of reported depreciation in the four quarters up to and including quarter t. As in the annual data, we only keep observations for which $dep_{i,t-4,t} \geq k^q_{i,t-4}$ in nominal terms. Finally, we keep only observations with fiscal years between 1984 and 2014, since little data is available at the quarterly frequency prior to 1984. The resulting quarterly sample has 186784 firm-quarter observations.

Summary statistics Table A8 reports summary statistics for the average size and the average investment rate in the three different samples. QFR firms in the size-groups 1-2 (corresponding to the bottom 99% of the QFR distribution of book assets) are substantially smaller, on average, than firms in the bottom two size groups of the Compustat samples (the bottom 50% of the Compustat distribution of book assets). However, firms in group 4 (the top 0.5% of firms in QFR, and the top 25% of firms in Compustat) have comparable sizes (approximately 7bn$ on average). Measured investment rate among smaller firms (groups 1-3) are somewhat lower in the QFR than they are in Compustat; however, for the top size group, they have the same average magnitude. This suggests that the top quartile of Compustat firms represents relatively well the top 0.5% of firms in the

[78] We use the latest version of the `fundq` file, available on WRDS at: `/wrds/comp/sasdata/nam/fundq.sas7bdat`.

QFR, those with a differential investment behavior.

Additionally, Table A8 reports summary statistics for the entirety of the Compustat sample. These statistics are computed using the annual data. The balance sheet ratios reported are defined as follows in terms of Compustat variables: the debt to asset ratio is $\frac{\texttt{dlc+dltt}}{at}$; cash to asset ratio is $\frac{\texttt{che}}{at}$; net leverage is the difference between the debt and the cash to asset ratios; the short-term debt ratio is $\frac{\texttt{np}}{\texttt{dlc+dltt}}$; the trade credit ratio is $\frac{\texttt{ap}}{\texttt{lt}}$, where `lt` is the total of `lct`, `dltt`, `txditc` and `lo` (replacing individually missing variables, if at least one of the four is not missing); the intangible share is $\frac{\texttt{intan}}{\texttt{at}-\texttt{act}}$; zero leverage firms are those such that the debt to asset ratio is below 0.01; and negative equity firms are those such that the variable `teq` is not missing and negative.

F.2 The cyclical properties of investment

We first document unconditional estimates of the cyclicality of investment across size groups in Compustat data sources, and compare them to the QFR estimates. We use the same framework as in the main text, described in equation (1), in order to quantify this cyclicality; in particular, we use year-on-year GDP growth as our proxy for the state of the business cycle, and we control for durable/non-durable industry effects and their interaction with the year-on-year GDP growth. (The results are unchanged when controlling for 3D-NAICS effects in the same way). Table A9 reports the results, along with the estimates of the coefficients in the QFR data, which are identical to those reported in Table 4.

In both the quarterly and the annual Compustat, the baseline coefficient has the same magnitude and the opposite sign as the coefficient for the largest size group, group 4. In both cases, one cannot reject that the sum of the two coefficients is equal to 0.[79] The baseline industry group corresponding to the coefficient reported in the first line of Table A9 are firms in the durable sector; however, estimates of the average marginal effect of GDP growth on investment (not reported) convey the same message. In annual data, the point estimate for the average marginal effect is 0.066, with a 95% confidence interval of $[-0.118; 0.245]$; in quarterly data, those numbers are -0.057 and $[-0.297, 0.182]$. Thus, in Compustat data as well as in QFR data, investment at the largest firms does not display a significantly positive correlation with contemporaneous GDP growth.

We next turn to the question of whether investment declines among large firms also display a lag in Compustat data. We estimate the same simple event study response for investment as the one described in section 5.2 of the main text, using the Compustat quarterly sample. In order to focus on the lag among the largest firms in the data, we trace out the cumulative investment rates of the top size group — groups 3 and 4 from Table A9 — and the bottom size group — groups 1 and 2 from Table A9. Figure A7 reports the results. As in the QFR data, investment lags the start of the recession: the peak of the cumulative investment rate occurs three quarters after the start of the recession in both size groups. Moreover, there is a sharper slowdown in the investment rate among the bottom size group (the cumulative investment rate is between quarters 0, when the recession starts, and 3 is smaller in the bottom size groups than in the top size groups). The

[79]The t-statistic for the tests are -0.24 in annual data and 1.41 in annual data, respectively.

difference in lags between the top and the bottom size groups is less visible than in the QFR data. The fact that the typical size of firms in the bottom size groups is substantially larger in the QFR than in Compustat may explain this discrepancy.

Overall, these findings indicate that Compustat data shares the two salient features of the QFR investment rates — the fact that the very largest firms do not display a positive contemporaneous correlation with GDP growth, and the fact that investment declines seems to lag the beginning of recessions.

G Decompositions of aggregate growth

G.1 Baseline decomposition

Assume that all observations are equally weighted, that is:

$$z_{i,t} = 1 \quad \forall (i,t).$$

Let $\mathcal{I}_t^{(\text{small})} \subset \mathcal{I}_t$ denote the set of indexes of small firms, and $\mathcal{I}_t^{(\text{large})} = \mathcal{I}_t \setminus \mathcal{I}_t^{(\text{small})}$ be the set of large firms.[80] For some variable of interest $x \in \{\text{sales}, \text{inventory}, \text{NPPE stock}, \text{assets}\}$, and for some quarter t, define:

$$\begin{aligned}
X_t &= \sum_{i \in \mathcal{I}_t} x_{i,t}, & X_{t-4} &= \sum_{i \in \mathcal{I}_t} x_{i,t-4}, & G_t &= \frac{X_t}{X_{t-4}}, \\
X_t^{(\text{small})} &= \sum_{i \in \mathcal{I}_t^{(\text{small})}} x_{i,t}, & X_{t-4}^{(\text{small})} &= \sum_{i \in \mathcal{I}_t^{(\text{small})}} x_{i,t-4}, & G_{t-4}^{(\text{small})} &= \frac{X_t^{(\text{small})}}{X_{t-4}^{(\text{small})}}, \\
X_t^{(\text{large})} &= \sum_{i \in \mathcal{I}_t^{(\text{large})}} x_{i,t}, & X_{t-4}^{(\text{large})} &= \sum_{i \in \mathcal{I}_t^{(\text{large})}} x_{i,t-4}, & G_{t-4}^{(\text{large})} &= \frac{X_t^{(\text{large})}}{X_{t-4}^{(\text{large})}}.
\end{aligned} \tag{31}$$

These are simply totals for all firms and by group, along with their growth rates. Let $s_{t-4} = \frac{X_{t-4}^{(\text{small})}}{X_{t-4}}$ be the initial fraction of the aggregate value of x accounted for by small firms. Define the following firm-level growth rates and shares by:

$$\begin{aligned}
g_{i,t} &= \frac{x_{i,t}}{x_{i,t-4}} \\
w_{i,t-4} &= \begin{cases} \frac{x_{i,t-4}}{X_{t-4}^{(\text{small})}} & \text{if } i \in \mathcal{I}_t^{(small)} \\ \frac{x_{i,t-4}}{X_{t-4}^{(\text{large})}} & \text{if } i \in \mathcal{I}_t^{(large)} \end{cases}
\end{aligned} \tag{32}$$

First, note that the total growth of x for small firms (the growth rate $G_{t-4}^{(\text{small})}$ defined above) can be decomposed as:

$$G_t^{(small)} = \hat{g}_t^{(\text{small})} + \hat{cov}_t^{(\text{small})}, \tag{33}$$

[80] See appendix D for a formal definition of the size classification. Here, we refer to an arbitrary size classification, so long as it constitutes a partition of $\mathcal{I}_t$; in the counterfactuals that are reported next, we will focus on partition between the bottom 99% and top 1% by lagged book assets.

where:

$$\begin{aligned} \hat{g}_t^{(small)} &= \frac{1}{\#\mathcal{I}_t^{(\text{small})}} \sum_{i \in \mathcal{I}_t} g_{i,t} \\ \hat{cov}_t^{(\text{small})} &= \sum_{i \in \mathcal{I}_t^{(\text{small})}} \left(w_{i,t-4} - \frac{1}{\#\mathcal{I}_t} \right) \left(g_{i,t} - \hat{g}_t^{(small)} \right). \end{aligned} \tag{34}$$

The first term in this decomposition, $\hat{g}_t^{(\text{small})}$, is the cross-sectional average growth rate of the variable x. (Up to a constant and up to the approximation $log(x) \approx x - 1$ for x close to 1, this is the same variable as reported, for instance, in figure 1 for sales.) The second term can be interpreted as an (un-normalized) covariance, since $\frac{1}{\#\mathcal{I}_t^{(\text{small})}} = \frac{1}{\#\mathcal{I}_t^{(\text{small})}} \sum_{i \in \mathcal{I}_t^{(\text{small})}} w_{i,t-4}$. It captures the dependence between initial size (as proxied by the initial share of total size, $w_{i,t-4}$) and subsequent growth (as measured by $g_{i,t}$). Note that this decomposition is exact in any subset of $\mathcal{I}_t$; it holds for large firms as well, for example. Second, note that since $X_t = X_t^{(small)} + X_t^{(large)}$ and $X_{t-4} = X_{t-4}^{(small)} + X_{t-4}^{(large)}$, the following simple shift-share decomposition holds:

$$\begin{aligned} G_t &= s_{t-4} G_t^{(\text{small})} + (1 - s_{t-4}) G_t^{(\text{large})} \\ &= G_t^{(\text{large})} + s_{t-4} \left(G_t^{(\text{small})} - G_t^{(\text{large})} \right). \end{aligned} \tag{35}$$

Combining the two equations, we obtain the decomposition:

$$\begin{aligned} G_t &= \hat{g}_t^{(\text{large})} \\ &+ s_{t-4} \left(\hat{g}_t^{(\text{small})} - \hat{g}_t^{(\text{large})} \right) \\ &+ \hat{cov}_t, \end{aligned} \tag{36}$$

where the covariance term $\hat{cov}_t$ is given by:

$$\hat{cov}_t = \hat{cov}_t^{(\text{large})} + s_{t-4} \left(\hat{cov}_t^{(\text{small})} - \hat{cov}_t^{(\text{large})} \right).$$

G.2 The contribution of the covariance terms to the cyclicality of aggregate growth

In order to clarify the contribution of the term $\hat{cov}_t$ to business-cycle variation in G_t, it is useful to note that the analogous decomposition to (2) also holds within each firm group, namely:

$$\begin{aligned} G_t^{(\text{small})} &= g_t^{(\text{small})} + \hat{cov}_t^{(\text{small})}, \\ G_t^{(\text{large})} &= g_t^{(\text{large})} + \hat{cov}_t^{(\text{large})}. \end{aligned} \tag{37}$$

Let Y_t be a business-cycle indicator; for instance, $Y_t \equiv \Delta GDP_t$. We can then write the correlation between $G_t^{(\text{small})}$ and Y_t as:

$$corr(G_t^{(\text{small})}, Y_t) = \frac{\sigma_{\hat{g}_t^{(\text{small})}}}{\sigma_{G_t^{(\text{small})}}} corr\left(\hat{g}_t^{(\text{small})}, Y_t \right) + \frac{\sigma_{\hat{cov}_t^{(\text{small})}}}{\sigma_{G_t^{(\text{small})}}} corr\left(\hat{cov}_t^{(\text{small})}, Y_t \right). \tag{38}$$

Here, σ_Z denote the standard deviation of variable Z. Equation (38) breaks down the correlation between $G_t^{(\text{small})}$ and Y_t into a component originating from firm-level growth and a component originating from the covariance term. Of course, the same holds for large firms and for firms overall.

Table A10 reports the values of the different elements of the right-hand side of (38), when the variable of interest is sales. It shows that the covariance terms — whether it be for small firms, large firms or all firms — have a limited (although non-zero) contribution to business-cycle variation in aggregate growth. Of course, these terms are non-zero on average; in fact, their sample means are 0.13, 0.29 and 0.23 for small, large and all firms, respectively. The large *average* difference in the covariance term between small and large firms has a substantial effect on trends. Namely, within the small firm group, cumulative average firm-level growth tracks fairly closely the path of total sales. By contrast, for large firms, cumulative average firm-level growth falls far short of the trend in total sales, as documented in Figure A8.

But both the correlation to GDP growth of these covariance terms and their standard deviation relative to aggregate sales growth G_t are substantially smaller than for the cross-sectional average growth rates. For example, for large firms, the correlation between aggregate sales growth and GDP growth is 0.62 in the sample; this can be broken down into a contribution of $0.64 = 0.83 \times 0.77$, coming from the term $\frac{\sigma_{\hat{g}_t^{(\text{large})}}}{\sigma_{G_t^{(\text{large})}}} corr\left(\hat{g}_t^{(\text{large})}, Y_t\right)$, and $-0.02 = 0.45 \times (-0.05)$, coming from the term $\frac{\sigma_{\hat{cov}_t^{(\text{large})}}}{\sigma_{G_t^{(\text{large})}}} corr\left(\hat{cov}_t^{(\text{large})}, Y_t\right)$. This simple decomposition thus suggest that, up to first order, business-cycle variation in the covariance terms contribute little to aggregate growth; instead, average firm-level growth is the dominant factor.

G.3 Results using DHS growth rates

We finally replicate the decomposition results of Section 4 using an alternative set of measures of growth at the firm level: the bounded growth rates introduced by Davis, Haltiwanger and Schuh (1996) (henceforth DHS). For any variable x, these growth rates are given by:

$$\tilde{g}_{i,t} = \frac{x_{i,t} - x_{i,t-4}}{\frac{1}{2}\left(x_{i,t} + x_{i,t-4}\right)} \in [-2, 2] \,.$$

These growth rates are a second-order accurate approximation to the standard growth rate $\frac{x_{i,t}}{x_{i,t-4}} - 1$ in a neighborhood of 1; furthermore, they are bounded, and moments of the distribution of these growth rates are therefore not too sensitive to outliers.

Using the same steps as outlined in Appendix G, it is straightforward to verify that the following decomposition holds exactly:

$$\tilde{G}_t = \hat{\tilde{g}}_t^{(large)} + \tilde{s}_{t-4}(\hat{\tilde{g}}_t^{(large)} - \hat{\tilde{g}}_t^{(small)}) + c\hat{\tilde{o}}v_t^{(large)} + \tilde{s}_{t-4}(c\hat{\tilde{o}}v_t^{(large)} - c\hat{\tilde{o}}v_t^{(small)}),$$

where:

$$\tilde{G}_t = \frac{X_t - X_{t-4}}{\frac{1}{2}(X_t + X_{t-4})}$$

$$\tilde{s}_{t-4} = \frac{X_t^{(small)} + X_t^{(large)}}{X_t + X_{t-4}}$$

$$\hat{\tilde{g}}_t^{(small)} = \frac{1}{\#\mathcal{I}_t^{(\text{small})}} \sum_{i \in \mathcal{I}_t} \tilde{g}_{i,t}$$

$$\hat{c\tilde{o}v}_t^{(\text{small})} = \sum_{i \in \mathcal{I}_t^{(\text{small})}} \left(\tilde{w}_{i,t-4} - \frac{1}{\#\mathcal{I}_t} \right) \left(\tilde{g}_{i,t} - \hat{\tilde{g}}_t^{(small)} \right),$$

and $\hat{\tilde{g}}_t^{(large)}$, $\hat{c\tilde{o}v}_t^{(\text{large})}$ are similarly defined. In this decomposition, the weights appearing in the covariance terms are given by:

$$\tilde{w}_{i,t} = \frac{x_{i,t} + x_{i,t-4}}{\sum_{i \in \mathcal{I}_t} x_{i,t} + x_{i,t-4}}.$$

Thus, they capture not the initial size of the firm relative to other firms initially in the same size group, but its average size over the period between $t-4$ and t, relative to the average size of firms initially in the same size group.

When we apply this decomposition to the same sample as in Section 4, the two key results of the analysis using log growth rates still hold. First, the covariance terms in the decomposition account for a very small fraction of the overall correlation between aggregate growth and GDP growth; the lion's share of that correlation, instead, comes from the cross-sectional average components, $\hat{\tilde{g}}_t^{(small)}$ and $\hat{\tilde{g}}_t^{(large)}$. Table A11 makes this point; its contents are almost identical to those of Table 8 in the main text. Second, estimated elasticities of counterfactual time series for aggregate growth attempting to remove either the "greater sensitivity" or the cyclicality of small firms overall are very close to the actual elasticities of time series for aggregate growth. Table A12 reports these results; again, they are almost identical to the results from the same exercise conducted using log growth rates, and reported in Table 8 in the main text. The reason for the similarity between these results is simple: these two growth rates are very highly correlated at the firm level, in the sample of continuing firms used throughout in the main text.

G.4 An alternative decomposition

The following, complementary approach can be used to evaluate the relative contribution of small and large firms to aggregate fluctuations.[81] In general, aggregate growth G_t can be decomposed

[81] This decomposition is similar to the Shimer (2012) decomposition of fluctuations in the unemployment rate between changes in job finding rates and changes in employment exit rates. We thank an anonymous referee for suggesting to apply this decomposition to the question of this paper.

as:

$$
\begin{aligned}
G_t &= s_{t-4}G_t^{(\text{small})} + (1-s_{t-4})G_t^{(\text{large})} \\
&= \underbrace{s_{t-4}\bar{G}^{(\text{small})} + (1-s_{t-4})G_t^{(\text{large})}}_{\equiv \tilde{G}_t^{\text{large}}} + \underbrace{s_{t-4}G_t^{(\text{small})} + (1-s_{t-4})\bar{G}^{(\text{large})}}_{\equiv \tilde{G}_t^{\text{small}}} \\
&\quad - \underbrace{\left(s_{t-4}\bar{G}^{(\text{small})} + (1-s_{t-4})\bar{G}^{(\text{large})}\right)}_{\equiv -R_t} \\
&= \tilde{G}_t^{\text{large}} + \tilde{G}_t^{\text{small}} + R_t.
\end{aligned}
\tag{39}
$$

This decomposition separates aggregate (or total) growth into three terms. The first one, $\tilde{G}_t^{\text{large}}$, is equal to the total growth rate that would obtain, if total growth among small firms had no cyclical component (that is, were set equal to its sample mean, denoted here by $\bar{G}^{\text{small}}$). This term thus captures the contribution of large firms to business cycle fluctuations in aggregates. The second term represents the symmetric term, for small firms. The third term is a reallocation component: it represents the fluctuations in aggregates that would arise if only small firms' share, s_{t-4}, were to fluctuate over the cycle (while growth rates in each size group stayed equal to their sample mean).

Table A13 contains results from a variance decomposition based on equation (39). Each line reports the respective contribution of the terms $\tilde{G}_t^{\text{large}}$, $\tilde{G}_t^{\text{small}}$ and R_t to the variance of G_t (that is, the covariance of the term with G_t, divided by the variance of G_t). Consistent with the previous discussion, the last column shows that the contribution of the reallocation term R_t to fluctuations in sector-wide totals is negligible. Additionally, the first and second columns show that it is large firms that account for the bulk of the variance in the growth of total sales, inventory, fixed investment, and assets. Large firms' contribution to the variance in aggregate growth rates ranges from 70% to 90%, approximately in line with the averages shares reported in Figure 4.

H Additional results on the size effect and external financing

H.1 Triple Interaction Regressions

These regressions are meant to answer the following question: is the size effect weaker among groups of financially stronger firms? These regressions allow us to investigate if size effect meaningfully differs within group of financially strong or financially weak firms (see Sharpe (1994) for a similar analysis for Compustat firms investigating the relationship between financial frictions and employment). In order to measure financial strength, we use the same five ratios as in the horse-race regressions. We estimate a regression of the same form as (6), but where observations are effectively double sorted by their position in the firm size distribution and bins of a measure of financial strength. As in previous regressions, we also include industry fixed effects and interactions of industry effects and GDP growth.

Results are reported in Table A14. In this table, all estimates of the size effect are expressed relative to the bottom $[0, 90]$ group.[82] The first column is the baseline regression without triple interaction - the same regression as in Table 4. The coefficient -0.60, for instance, indicating that the sales elasticity to GDP of firms in the $[99.5, 100]$ group is 0.6 points lower than that of firms in the $[0, 90]$ group.

The second and third columns report similar elasticities when size and bank dependence categories are interacted. The estimates are organized by bank dependence groups; in order to keep the table readable, we have kept only two groups for bank dependence. Firm-year observations in the low bank-dependence group had a ratio of bank debt to total debt below 0.9 in the prior year, whereas firms in the high bank-dependence group had a ratio of bank debt to total debt over 0.9.[83] The reported coefficients denote relative elasticities within each bank dependence group. The estimates suggest that among firms with low to moderate bank dependence, the size estimate has the same sign, and a similar magnitude as in the unconditional regressions. Among highly bank-dependent, the size effect is slightly *smaller*, although the high minus low difference (reported in the right column) is not statistically significant. Had the size effect been a reflection of financial constraints, one might have expected it to be much weaker among firms with access to other sources of financing than bank debt; instead, it is somewhat stronger.

The following columns repeat this exercise for other proxies for financial constraints.[84] While results differ across measures of financial constraints, it is worth noting that, with the exception of the last indicator — firms' dividend issuance behavior — measures of the size effect are never statistically different across groups of financial strength proxies. Directionally, the estimates of the relative size effect for leverage and dividend issuance groups are consistent with the view that the size effect is weaker among financially stronger firms; on the other hand, estimates using liquidity and bond market access are not. Overall, the lack of significance in the cross-group differences in the size effect paired with its significance within group bolsters the view that the size effect may not be financially driven.

H.2 External financing and the size effect for the trade sector

Table A15 and Figures A9 and A10 speak to the financial origins of the size effect in the trade sector. They replicate, respectively, Table 9 and Figures A9 and A10 for the manufacturing sector. For Table 9, which documents how the size effect changes when other proxies for financial constraints are controlled for, we use three size groups: the $[0, 50]$, $[50, 90]$ and $[90, 100]$ interquantile range of book assets. Section 3.3 discusses this classification in more detail. For the event study regressions, the results of which are reported in figures A9 and A10, we classify firms in two groups, the bottom 90% and the top 10% by book assets.

[82] This is with the exception of regressions conditioning on bond market access where results are reported relative to the $[0, 99]$ group as there are too few observations with bond market access in the $[0, 90]$ group.

[83] In order to avoid creating non-overlapping groups, which would complicate disclosure of results, we are limited to using a grouping by financial strength indicators that is a coarser version of the grouping of Table 9.

[84] For leverage, we split the sample above and below 0.5. For liquidity, we use a 0.01 cash to asset ratio as the threshold between low and high liquidity. These choices correspond approximately to the top quartile of the distribution of leverage and the bottom quartile of the distribution of the cash to asset ratio.

H.3 The behavior of cash

Table A16 reports estimates of the cyclical sensitivity by firm size for three measures of cash holdings: cash growth, the cash to asset ratio, the growth rate of total financial assets, and the ratio of total financial assets to total assets. Estimates are obtained using our baseline framework, equation (1). Figure A11 reports the event study response of cash growth and the ratio of cash to total asset to the onset of a recession, estimated using the methodology described in section 5.2. Both speak to the possibility that financial constraints may induce small firms to hoard more cash during downturns than large firms. Empirically, the opposite seems to hold; the cash to asset ratio of small firms declines during recessions, while the cash to asset ratio of large firms increases.

I Additional results on the establishment composition of firms

Tables A17 to A19 report additional results regarding the matched QFR-DMI sample and the role of the establishment composition of firms in accounting for the size effect. For the manufacturing sector, table A17 reports results using continuous measures of the establishment composition of firms. For the trade sector, table A18 reports summary statistics, and table A18 reports results using continuous measures of the establishment composition of firms.

	Sales growth		Inventory growth		Fixed investment	
	(a)	(b)	(a)	(b)	(a)	(b)
$[90, 99] \times$ GDP growth $\times$ post-82	-0.242		-0.274		-0.290	
	(0.151)		(0.175)		(0.111)	
$[99, 99.5] \times$ GDP growth $\times$ post-82	-0.314		-0.518		-0.927	
	(0.059)		(0.013)		(0.001)	
$[99.5, 100] \times$ GDP growth $\times$ post-82	-0.622		-0.596		-1.158	
	(0.001)		(0.004)		(0.001)	
$[90, 99] \times$ GDP growth $\times$ post-82 $\times$ old		-0.109		-0.060		-0.076
		(0.154)		(0.173)		(0.178)
$[99, 99.5] \times$ GDP growth $\times$ post-82 $\times$ old		-0.530		-0.471		-1.495
		(0.024)		(0.023)		(0.001)
$[99.5, 100] \times$ GDP growth $\times$ post-82 $\times$ old		-0.462		-0.160		-0.482
		(0.021)		(0.120)		(0.015)
Observations	≈ 460000	≈ 460000	≈ 460000	≈ 460000	≈ 460000	≈ 460000
Firms	≈ 60000	≈ 60000	≈ 60000	≈ 60000	≈ 60000	≈ 60000
Adj. R^2	0.025	0.028	0.006	0.009	0.004	0.005
Clustering	Firm	Firm	Firm	Firm	Firm	Firm
Industry controls	D/ND	D/ND	D/ND	D/ND	D/ND	D/ND

Table A1: The size effect and age in the QFR manufacturing sample. The dependent variable in the first two columns is sales growth; in the third and fourth column, inventory growth; and in the fifth and sixth column, the investment rate. The first three lines report the estimated sensitivity of firm-level sales growth to GDP growth for different size groups relative to a baseline category, when size is defined in terms of quantiles of book assets, as in Section 3.1. The baseline category is the group of firm in the [0,90] interquantile range for size. In the first three lines, size and GDP growth are also interacted with an indicator for the post-1982 sample, in which a lower bound on firm age can be estimated. The baseline category for the post-1982 indicator is defined so that the coefficients reported in the first three lines should be interpreted as the *level* of the size effect in the post-1982 sample. We choose this approach, rather than a regression in the post-1982 subsample, so as to avoid creating and additional subsample, which complicates disclosure. Lines four through six report the estimated coefficients when the size effect is estimated conditional on being at least five years of age, as described in Section 3.3. This is done by further interacting size, GDP, and the post-1982 indicator with an indicator for whether the firm is at least 5 years of age; baseline categories are again defined so that the coefficients reported should be interpreted as the *level* of the size effect among the group of firms of at least five years of age. Robust p-values are reported in parentheses, with standard errors are clustered at the firm level.

	Sales growth				
	(1)	(2)	(3)	(4)	(5)
$[90, 99] \times$ GDP growth	-0.249	-0.160	-0.070	-0.177	-0.139
	(0.084)	(0.142)	(0.630)	(0.220)	(0.326)
$[99, 99.5] \times$ GDP growth	-0.382	-0.251	-0.256	-0.240	-0.052
	(0.009)	(0.081)	(0.117)	(0.103)	(0.745)
$[99.5, 100] \times$ GDP growth	-0.757	-0.600	-0.607	-0.535	-0.655
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	≈ 460000	≈ 460000	≈ 460000	≈ 460000	≈ 460000
Firms	≈ 60000	≈ 60000	≈ 60000	≈ 60000	≈ 60000
Adj. R^2	0.023	0.025	0.031	0.025	0.032
Clustering	Firm	Firm	Firm	Firm	Firm
Industry controls	None	D/ND	2D SIC/3D NAICS	D/ND	2D SIC/3D NAICS
Size classification	All manuf.	All manuf.	All manuf.	D/ND	2D SIC/3D NAICS

Table A2: The role of industry controls in the QFR sample. Each line reports the estimated semi-elasticity of the variable of interest with respect to GDP growth for a size group relative to firms in the smallest size group (the $0 - 90th$ interquantile range). In specifications (1)-(3), the size classification of firms is constructed by pooling all manufacturing firms. In specification (4), the size classification is constructed within the durable and non-durable industry. Finally, in specification (5), the size classification is constructed with each 3-digit NAICS industry (after 2000) or 2-digit SIC industry (before 2000). All values are deflated by the quarterly manufacturing price index. Robust p-values reported in parentheses.

	Sales growth			
	(1)	(2)	(3)	(4)
$[25, 50]$ × GDP growth	-0.18	-0.20	-0.20	-0.22
	(0.511)	(0.458)	(0.479)	(0.430)
$[50, 75]$ × GDP growth	-0.09	-0.10	-0.10	-0.12
	(0.746)	(0.691)	(0.707)	(0.657)
$[75, 100]$ × GDP growth	-0.25	-0.27	-0.26	-0.28
	(0.318)	(0.268)	(0.294)	(0.248)
Observations	238421	238421	238421	238421
Firms	6079	6079	6079	6079
Clustering	Firm	Firm	Firm	Firm
Industry controls	D/ND	D/ND	D/ND	None
Deflator type	Value-added	Output	Output	Output
Deflator level	Manufacturing	Manufacturing	BEA subsectors	BEA subsectors
Industry-quarter effects	No	No	No	Yes

Table A3: The effect of alternate deflators. The sample is the quarterly manufacturing sample from Compustat. The table shows results for the sample from 1977q3 to 2014q1 (the same dates for which the QFR is available). Reported are the semi-elasticities of sales growth to GDP growth for the top three quartiles of the size distribution, relative to the bottom quartile of the size distribution. Size is defined as the one-year lagged value of book assets. Industries are defined as the BEA sub-sectors for manufacturing, which approximately correspond to NAICS 3-digit groups. Specification (1) reports results from a specification identical to the main specification, equation 1. Specification (2) deflates sales by an output (instead of value-added) deflator, identical across manufacturing industries. Specification (3) deflates sales by output deflators specific to each BEA sub-sector. Finally, specification (4) adds for industry-quarter fixed effects instead of controlling for industry effects and their interaction with GDP growth. Standard errors are clustered at the firm level in all specifications.

	Sales growth				
	(1)	(2)	(3)	(4)	(5)
$[90, 99] \times$ GDP growth	-0.160	-0.160	-0.160	-0.160	-0.160
	(0.142)	(0.111)	(0.141)	(0.150)	(0.157)
$[99, 99.5] \times$ GDP growth	-0.251	-0.251	-0.251	-0.251	-0.251
	(0.143)	(0.122)	(0.157)	(0.156)	(0.165)
$[99.5, 100] \times$ GDP growth	-0.600	-0.600	-0.600	-0.600	-0.600
	(0.140)	(0.137)	(0.169)	(0.212)	(0.220)
Observations	≈ 460000	≈ 460000	≈ 460000	≈ 460000	≈ 460000
Firms	≈ 60000	≈ 60000	≈ 60000	≈ 60000	≈ 60000
Adj. R^2	0.033	0.032	0.034	0.035	0.034
Clustering	Firm	Quarter	Firm and quarter	Year	Firm and year
Industry controls	D/ND	D/ND	D/ND	D/ND	D/ND

Table A4: Clustering and the significance of the size effect in the QFR manufacturing sample. The dependent variable in all specifications is sales growth. The first column reports estimates from the baseline specification, where standard errors are clustered at the firm level. In the second column, standard errors are clustered at the quarter level; in the third column, standard errors are clustered at the firm and quarter levels; in the fourth column, standard errors are clustered at year level; and in the last column, standard errors are clustered at the firm and year levels. Standard errors are reported in parentheses.

BEA subsector	Name	Share of manufacturing nominal value added (2001)	Share of manufacturing nominal gross output (2001)	Correlation between value added growth and gross output growth (1977-2014)	
				Nominal	Real
336MO	Transportation Equipment	0.137	0.119	0.849	0.335
3250	Chemical Products	0.131	0.116	0.381	0.636
311A	Food and Beverage and Tobacco Products	0.119	0.150	-0.151	0.360
3340	Computer and Electronic Products	0.117	0.117	0.817	0.823
3320	Fabricated Metal Products	0.075	0.066	0.875	0.896
3330	Machinery	0.072	0.069	0.893	0.892
3240	Petroleum and Coal Products	0.047	0.056	0.474	0.059
3260	Plastics and Rubber Products	0.043	0.044	0.531	0.730
338A	Miscellaneous Manufacturing	0.039	0.030	0.496	0.367
3220	Paper Products	0.036	0.041	0.623	0.389
3350	Electrical Equipment, Appliances, and Components	0.030	0.029	0.540	0.527
3230	Printing and Related Support Activities	0.029	0.027	0.864	0.699
3270	Nonmetallic Mineral Products	0.028	0.024	0.743	0.723
3310	Primary Metal Products	0.027	0.036	0.905	0.430
3370	Furniture and Related Products	0.021	0.019	0.864	0.002
3210	Wood Products	0.019	0.023	0.843	0.597
313T	Textile Mills and Textile Product Mills	0.017	0.020	0.665	0.691
315A	Apparel and Leather and Applied Products	0.013	0.015	0.551	0.446

Table A5: Correlation between gross output and value added at the BEA sub-sector level. The data are from the BEA's GDP by industry tables, specifically: https://www.bea.gov/sites/default/files/2018-04/GDPbyInd_GO_1947-2017.xlsx for gross output and https://www.bea.gov/sites/default/files/2018-04/GDPbyInd_VA_1947-2017.xlsx for value added. The data are annual from 1977 to 2014.

	Outcome variable: sales growth (log)			
	Compustat quarterly, manufacturing, 1977q4-2014q1			
	(1)	(2)	(3)	(4)
25-50% $\times$ GDP growth	-0.22	-0.21	-0.13	-0.33
	(0.430)	(0.449)	(0.670)	(0.340)
50-75% $\times$ GDP growth	-0.12	-0.12	0.02	-0.02
	(0.657)	(0.671)	(0.951)	(0.958)
75-100% $\times$ GDP growth	-0.28	-0.31	-0.25	-0.36
	(0.248)	(0.219)	(0.367)	(0.255)
Observations	238421	231184	196667	169069
Firms	6079	5917	4993	4351
Deflator type	Output	Output	Output	Output
Deflator level	BEA subsectors	BEA subsectors	BEA subsectors	BEA subsectors
Industry $\times$ GDP growth control	No	No	No	No
Industry-quarter effects	Yes	Yes	Yes	Yes

Table A6: The size effect in quarterly Compustat, including and excluding sub-sectors in which total sales and total gross output have a low correlation. All specifications include industry-quarter fixed effects. Specification (1) is identical to specification (4) in column table 1, and includes all subsectors. Specifications (2), (3) and (4) exclude, respectively, the 2, 5 and 8 BEA subsectors with the lowest correlation between real output growth and real value added growth; see table A5 for details on those sectors. Standard errors are clustered at the firm level in all specifications.

Romer date dropped	CM growth rates (firm-level)			GG growth rates		
	Δ_S	Δ_L	$\frac{\Delta_S - \Delta_L}{\Delta_L}$	Δ_S	Δ_L	$\frac{\Delta_S - \Delta_L}{\Delta_L}$
None	-9.2%	-5.8%	58%	-9.5%	-8.4%	14%
1978q3	-10.3%	-8.0%	30%	-8.7%	-8.7%	0%
1979q4	-5.3%	-2.4%	121%	-6.6%	-4.7%	39%
1988q4	-10.0%	-4.7%	115%	-8.0%	-7.4%	8%
1994q2	-14.5%	-10.4%	40%	-14.0%	-13.2%	6%
2008q3	-5.7%	-3.7%	55%	-8.4%	-5.9%	42%

Table A7: Event study results using the data from 1977q3 to 2014q1. The table reports the average cumulative change in sales for small (columns 2 and 5) and large (columns 3 and 6) firms in the three years following Romer dates, and their relative magnitude (columns 4 and 7.) Columns 5 to 7 report results using growth rates constructed following the Gertler and Gilchrist (1994) methodology and using publicly available data (GG growth rates), while columns 2 to 4 report results using growth rates constructed using micro data and reported in Figure 1 (CM growth rates). Each line reports the results dropping one Romer date from the set of six Romer dates identified by Romer and Romer (1989, 1994) and Kudlyak and Sanchez (2017) for the 1977q3 to 2014q1 sample: three original Romer dates, 1978q3, 1979q4, 1988q3, and two additional dates, 1994q2 and 2008q3.

	Size group			
	(1)	**(2)**	**(3)**	**(4)**
Assets (2009 m$)				
QFR	2.0	48.8	626.0	6766.3
Compustat (annual)	22.6	94.4	375.7	7348.9
Compustat (quarterly)	26.1	113.2	452.9	8835.6
Investment rate				
QFR	26.50%	24.91%	21.89%	20.36%
Compustat (annual)	30.93%	32.00%	27.94%	21.87%
Compustat (quarterly)	28.83%	31.89%	28.90%	22.69%

Table A8: Summary statistics for the QFR sample and the two Compustat samples. Each column corresponds to a different size group. For QFR data, size groups are defined as in the main text. For Compustat (annual and quarterly), size groups are quartiles of the distribution of book assets (variable `at` in the annual files and `atq` in the quarterly files). Assets are nominal book values deflated by the BEA price deflator for manufacturing value added, as in the main text. See appendix E for details on the construction of the annual and quarterly Compsutat samples and the computation of investment rates.

	QFR	Compustat (annual)	Compustat (quarterly)
GDP growth	0.912	1.082	0.537
	(0.258)	(0.306)	(0.174)
Size group 2 × GDP growth	−0.299	−0.235	−0.103
	(0.157)	(0.197)	(0.223)
Size group 3 × GDP growth	−0.687	−0.329	−0.250
	(0.194)	(0.189)	(0.216)
Size group 4 × GDP growth	−1.257	−0.921	−0.572
	(0.355)	(0.260)	(0.199)
N	≈ 460000	72363	186784
nr. firms	≈ 60000	6550	5944
adj. R^2	0.003	0.022	0.017
industry controls	yes	yes	yes
s.e. clustering	Firm	Firm	Firm

Table A9: Investment cyclicality by size in the QFR data (first column) and for the annual and quarterly Compustat samples (second and third columns. The baseline coefficient (first line) refers to firms in the durable sector. All values are deflated by the quarterly manufacturing price index. Standard errors reported in parentheses.

	Small firms	Large firms	All firms
$corr(G_t, Y_t)$	0.68	0.62	0.65
$\frac{\sigma_{\hat{g}_t}}{\sigma_{G_t}}$	1.02	0.83	0.89
$corr(\hat{g}_t, Y_t)$	0.84	0.77	0.80
$\frac{\sigma_{c\hat{o}v_t}}{\sigma_{G_t}}$	0.54	0.45	0.41
$corr(c\hat{o}v_t, Y_t)$	−0.32	−0.05	−0.15

Table A10: Decomposition of the correlation of aggregate sales growth with GDP growth. The decomposition used is $corr(G_t, Y_t) = \frac{\sigma_{\hat{g}_t}}{\sigma_{G_t}} corr\,(\hat{g}_t, Y_t) + \frac{\sigma_{c\hat{o}v_t}}{\sigma_{G_t}} corr\,(c\hat{o}v_t, Y_t)$, where Y_t is year-on-year GDP growth, G_t is year-on-year growth in total sales, $\hat{g}_t$ is year-on-year average firm-level growth, and $c\hat{o}v_t$ is a term capturing the covariance between initial size and subsequent growth. The results are reported for all firms (first column), small firms (second column) and large firms (third column). See Appendix G for more details on the decomposition.

	Small firms	Large firms	All firms
$corr(\tilde{G}_t, Y_t)$	0.68	0.61	0.65
$\frac{\sigma_{\hat{g}_t}}{\sigma_{\tilde{G}_t}}$	0.97	0.81	0.86
$corr(\hat{\tilde{g}}_t, Y_t)$	0.84	0.77	0.80
$\frac{\sigma_{c\hat{o}v_t}}{\sigma_{\tilde{G}_t}}$	0.51	0.45	0.41
$corr(c\hat{\tilde{o}}v_t, Y_t)$	−0.26	−0.04	−0.10

Table A11: Decomposition of the correlations of aggregate sales growth among all firms, small firms, and large firms, to GDP growth, constructed using DHS growth rates. See Appendix G for details on the decomposition.

	Actual β	Counterfactual 1 $\beta^{(1)}$	Counterfactual 2 $\beta^{(2)}$
Sales	2.285	2.174	2.263
	(0.339)	(0.339)	(0.362)
Inventory	0.918	0.758	0.768
	(0.225)	(0.250)	(0.226)
Fixed investment	0.583	0.576	0.567
	(0.145)	(0.151)	(0.148)
Total assets	0.876	0.791	0.857
	(0.121)	(0.129)	(0.745)
Observations	143	143	143

Table A12: Cyclical sensitivities of aggregate sales, inventory, fixed investment, and total assets using DHS growth rates. Each line reports the estimated slope in regressions of the form $Z_t = \alpha + \beta \log\left(\frac{GDP_t}{GDP_{t-4}}\right) + \epsilon_t$. The first column reports results for $Z_t = G_t$, where G_t is the actual aggregate growth rate. The second column uses $Z_t = G_t^{(1)}$, where $G_t^{(1)}$ is a counterfactual aggregate growth rate series in which we have assumed that the average firm-level growth rate of small and large firms is equal (so that small firms do not have greater average sensitivity to business cycles than large firms). The third column uses $Z_t = G_t^{(2)}$, where $G_t^{(2)}$ is another counterfactual time series in which we have also assumed that the covariance between initial size and subsequent growth is also the same between small and large firms. Heteroskedasticity robust standard errors in parentheses.

	$\tilde{G}_t^{\text{large}}$	$\tilde{G}_t^{\text{small}}$	R_t
Sales	0.792	0.207	0.001
Inventory	0.723	0.276	0.001
Fixed investment	0.963	0.035	0.002
Total assets	0.924	0.071	0.005
Observations	143	143	143

Table A13: Variance decomposition for total sales, total inventory investment, total fixed investment, and total assets. Each column shows the contribution of a different term ($\tilde{G}_t^{\text{large}}$, $\tilde{G}_t^{\text{small}}$ or R_t) to the variance of G_t, i.e. the covariance between G_t and the term, divided by the variance of G_t (or alternatively, the coefficient in a single-variable OLS regression of the term on G_t).

Panel A	Baseline	Bank dependence			Leverage			Liquidity		
		Low	High	Diff	Low	High	Diff	High	Low	Diff
$[90, 99] \times$ GDP growth	−0.160	−0.094	−0.286	−0.192	−0.195	0.010	0.205	−0.257	0.179	0.436
	(0.260)	(0.597)	(0.219)	(0.507)	(0.198)	(0.978)	(0.585)	(0.113)	(0.508)	(0.155)
$[99, 99.5] \times$ GDP growth	−0.251	−0.270	−0.184	0.085	−0.182	−0.570	−0.387	−0.384	0.042	0.376
	(0.080)	(0.114)	(0.471)	(0.779)	(0.230)	(0.115)	(0.316)	(0.043)	(0.876)	(0.223)
$[99.5, 100] \times$ GDP growth	−0.600	−0.616	−0.429	0.187	−0.530	−0.974	−0.444	−0.604	−0.482	0.122
	(0.000)	(0.001)	(0.165)	(0.584)	(0.000)	(0.007)	(0.257)	(0.000)	(0.073)	(0.687)
N	≈ 460000		≈ 460000			≈ 460000			≈ 460000	
nr. firms	≈ 60000		≈ 60000			≈ 60000			≈ 60000	
adj. R^2	0.025		0.025			0.025			0.025	
industry controls	yes		yes			yes			yes	
s.e. clustering	Firm		firm-level			firm-level			firm-level	

Panel B	Baseline	Bond market access			Dividend issuance		
		Yes	No	Diff	High	low	Diff
$[90, 99] \times$ GDP growth	−0.160				−0.178	−0.146	0.032
	(0.260)				(0.268)	(0.675)	(0.931)
$[99, 99.5] \times$ GDP growth	−0.251	−2.907	−0.421	2.486	−0.150	−0.385	−0.235
	(0.080)	(0.211)	(0.001)	(0.196)	(0.426)	(0.262)	(0.544)
$[99.5, 100] \times$ GDP growth	−0.600	−3.568	−0.832	2.736	−0.395	−0.758	−0.363
	(0.000)	(0.095)	(0.001)	(0.124)	(0.059)	(0.026)	(0.357)
N	≈ 460000		≈ 460000			≈ 460000	
nr. firms	≈ 60000		≈ 60000			≈ 60000	
adj. R^2	0.025		0.025			0.025	
industry controls	yes		yes			yes	
s.e. clustering	Firm		firm-level			firm-level	

Table A14: Triple-interaction regressions. The dependent variable is sales growth. The columns marked baseline, bank dependence, leverage, liquidity, bond market access, and dividend issuance each correspond to one regression. For each financial indicator, coefficients are reported by sub-groups corresponding to firms which are less likely to be financially constrained (left column) and firms which are more likely to be financially constrained (middle column). The coefficients shown are differences in elasticities to GDP growth relative to the $[0, 90]$ size group *within each sub-group*. That is, the coefficient −0.616 in the right column of the bank dependence regression indicates that *within* firms with low bank dependence, the top 0.5% of firms has an elasticity of sales growth −0.616 points smaller than the $[0, 90]$ group. The last column, denoted Diff, reports the difference across groups of the size effect, as well as its significance level. The Bond market access regressions only compare the $[0, 99]$ group to others in order to avoid violating disclosure limits as there are too few observations with a bond issuance history in the $[0, 90]$ size group. Table 9 describes the groups of financial constraints in more detail. Standard errors clustered at the firm level. Robust p-values in parentheses.

	Baseline	(1)	(2)	(3)	(4)
$[50, 90] \times$ GDP growth	−0.335	−0.302	−0.347	−0.319	−0.351
	(0.125)	(0.165)	(0.113)	(0.146)	(0.112)
$[90, 100] \times$ GDP growth	−0.910	−0.883	−0.915	−0.890	−0.961
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Bank share [0.10,0.90] $\times$ GDP growth		0.075			
		(0.740)			
Bank share $< 0.10 \times$ GDP growth		−0.101			
		(0.642)			
Leverage [0.15,0.50] $\times$ GDP growth			−0.168		
			(0.535)		
Leverage (0,0.15] $\times$ GDP growth			−0.104		
			(0.736)		
Leverage $= 0 \times$ GDP growth			−0.307		
			(0.354)		
Liquidity [0.01,0.20] $\times$ GDP growth				−0.169	
				(0.396)	
Liquidity $> 0.20 \times$ GDP growth				−0.499	
				(0.163)	
Market access $\times$ GDP growth					0.098
					(0.667)
Observations	$\approx$ 120000	$\approx$ 120000	$\approx$ 120000	$\approx$ 120000	$\approx$ 120000
Firms	$\approx$ 10000	$\approx$ 10000	$\approx$ 10000	$\approx$ 10000	$\approx$ 10000
Adj. R^2	0.017	0.019	0.018	0.018	0.017
Industry controls	yes	yes	yes	yes	yes
Clustering	Firm	Firm	Firm	Firm	Firm

Table A15: Regression of sales growth on firm size and proxies for financial constraints (model 6), for firms in the trade sector. Each column is a separate regression. All coefficients are the semi-elasticity with respect to GDP growth, relative to a baseline group. For size, the baseline group is the $[0, 90]$ group. For the bank share, the reference group is the group of firms with more than 90% of bank debt, as a fraction of total debt. For leverage, the reference group is the group of firms with a ratio of debt to assets above 50%. For liquidity, the reference group is the group of firms with a cash to asset ratio below 1%. For market access, the reference group is the group of firms that have never issued a bond or commercial paper in the past. Robust p-values in parentheses.

	Financial assets growth	Financial to total assets	Cash growth	Cash to total assets
$[90, 99] \times$ GDP growth	-1.368	-0.080	-1.426	-0.084
	(0.001)	(0.016)	(0.001)	(0.010)
$[99, 99.5] \times$ GDP growth	-1.169	-0.009	-1.724	-0.045
	(0.006)	(0.792)	(0.001)	(0.142)
$[99.5, 100] \times$ GDP growth	-1.142	-0.032	-1.681	-0.001
	(0.006)	(0.290)	(0.001)	(0.961)
Observations	$\approx$ 460000	$\approx$ 460000	$\approx$ 460000	$\approx$ 460000
Firms	$\approx$ 60000	$\approx$ 60000	$\approx$ 60000	$\approx$ 60000
Adj. R^2	0.001	0.001	0.001	0.001
Clustering	Firm	Firm	Firm	Firm
Industry controls	D/ND	D/ND	D/ND	D/ND

Table A16: The size effect for financial assets and cash in the QFR manufacturing sample. Each column reports estimates of a specification similar to the baseline framework, Equation (1), but where the outcome variable is a measure of changes in the financial assets or cash held by the firm. In the first column, the dependent variable is the log growth rate of total financial assets. In the second column, the dependent variable is the one-year change in the level of total financial assets, divided by one-year lagged total assets. The third and fourth columns are similarly defined, but using cash specifically, as opposed to total financial assets. See Tables 1 and 2 for a comparison of the ratios of financial assets and cash to total assets across different size groups. Robust p-values in parentheses.

	Sales growth			
	(1)	(2)	(3)	(4)
[90, 99] × GDP growth	−0.222		−0.267	−0.261
	(0.395)		(0.333)	(0.344)
[99, 99.5] × GDP growth	−0.221		−0.331	−0.317
	(0.423)		(0.204)	(0.223)
[99.5, 100] × GDP growth	−0.802		−0.297	−0.309
	(0.001)		(0.296)	(0.279)
Nr. of lines of business × GDP growth		−0.018	−0.011	−0.020
		(0.019)	(0.117)	(0.037)
Nr. of establishments × GDP growth				0.004
				(0.130)
Observations	≈ 200000	≈ 200000	≈ 200000	≈ 200000
Firms	≈ 30000	≈ 30000	≈ 30000	≈ 30000
Adj. R^2	0.033	0.032	0.034	0.035
Industry controls	D/ND	D/ND	D/ND	D/ND
Clustering	Firm	Firm	Firm	Firm

Table A17: The size effect and the establishment composition of firms in the QFR manufacturing sample, using continuous measures of the establishment composition of firms. The dependent variable in all specifications is sales growth. The first three lines report the estimated sensitivity of firm-level sales growth to GDP growth for different size groups relative to a baseline category, when size is defined in terms of quantiles of book assets, as in Section 3.1; the baseline category is the group of firm in the [0,90] inter-quantile range for size. The fourth line reports the estimated coefficient on the interaction between the number of lines of business and GDP growth. A firm's number of lines of business is the total number of distinct SIC-4 digit codes of the collection of establishments that make up the firm in a given quarter. The fifth line reports estimated coefficient on the interaction between the number of establishments and GDP growth. Establishment counts and the industry composition of establishments for a given firm are obtained by merging the QFR with DnB, as discussed in section 6. Robust p-values in parentheses.

	Employment (000's)	Establishments (00's)	Lines of business (00's)	Manufacturing index	Trade index
$[0, 50]$	0.522	0.184	0.045	0.104	0.879
	(0.025)	(0.009)	(0.001)	(0.005)	(0.005)
$[50, 90]$	1.994	0.642	0.071	0.123	0.836
	(0.112)	(0.046)	(0.002)	(0.006)	(0.006)
$[90, 100]$	16.95	2.326	0.154	0.133	0.797
	(4.63)	(0.382)	(0.010)	(0.011)	(0.013)
Observations	≈ 80000	≈ 80000	≈ 80000	≈ 80000	≈ 80000
Firms	≈ 10000	≈ 1000	≈ 1000	≈ 1000	≈ 1000
Adj. R^2	0.060	0.138	0.445	0.174	0.911

Table A18: Summary statistics for the QFR trade sample merged to the DnB database. Each line corresponds to a different size group; size groups are defined based on the cross-sectional distribution of book assets, as described in Section 3.1. Each column reports the coefficients in a regression of a particular outcome variable on a full set of dummies, excluding the constant; that is, they are the conditional mean of each outcome variable by size group in the matched sample. The numbers in parentheses are standard errors, clustered at the firm level. The first column reports the conditional mean for employment (in thousands). The second column reports the conditional mean of the number of establishments (in hundreds). The third column reports the conditional mean of the number of lines of business (that is, the distinct number of SIC-4 digit codes in the collection of all establishments belonging to a particular firm; in hundreds). The fourth column reports the conditional mean of the manufacturing index, defined as the fraction of establishments with an SIC-4 digit code in manufacturing. The fifth column reports the conditional mean of the trade index, defined as the fraction of establishments with an SIC-4 digit not in retail or wholesale trade.

	Sales growth			
	(1)	(2)	(3)	(4)
$[50, 90] \times$ GDP growth	-0.202		-0.135	-0.103
	(0.453)		(0.616)	(0.701)
$[90, 100] \times$ GDP growth	-0.669		-0.433	-0.359
	(0.024)		(0.156)	(0.240)
Nr. of lines of business $\times$ GDP growth		-0.034	-0.027	-0.016
		(0.000)	(0.001)	(0.065)
Nr. of establishments $\times$ GDP growth				-0.069
				(0.348)
Observations	≈ 80000	≈ 80000	≈ 80000	≈ 80000
Firms	≈ 10000	≈ 10000	≈ 10000	≈ 10000
Adj. R^2	0.021	0.021	0.021	0.022
Industry controls	D/ND	D/ND	D/ND	D/ND
Clustering	Firm	Firm	Firm	Firm

Table A19: The size effect and the establishment composition of firms in the QFR trade sample, using continuous measures of the establishment composition of firms. The dependent variable in all specifications is sales growth. The first three lines report the estimated sensitivity of firm-level sales growth to GDP growth for different size groups relative to a baseline category, when size is defined in terms of quantiles of book assets, as in Section 3.1; the baseline category is the group of firm in the [0,50] inter-quantile range for size. The fourth line reports the estimated coefficient on the interaction between the number of lines of business and GDP growth. A firm's number of lines of business is the total number of distinct SIC-4 digit codes of the collection of establishments that make up the firm in a given quarter. The fifth line reports estimated coefficient on the interaction between the number of establishments and GDP growth. Establishment counts and the industry composition of establishments for a given firm are obtained by merging the QFR with DnB. Robust p-values shown in parentheses.

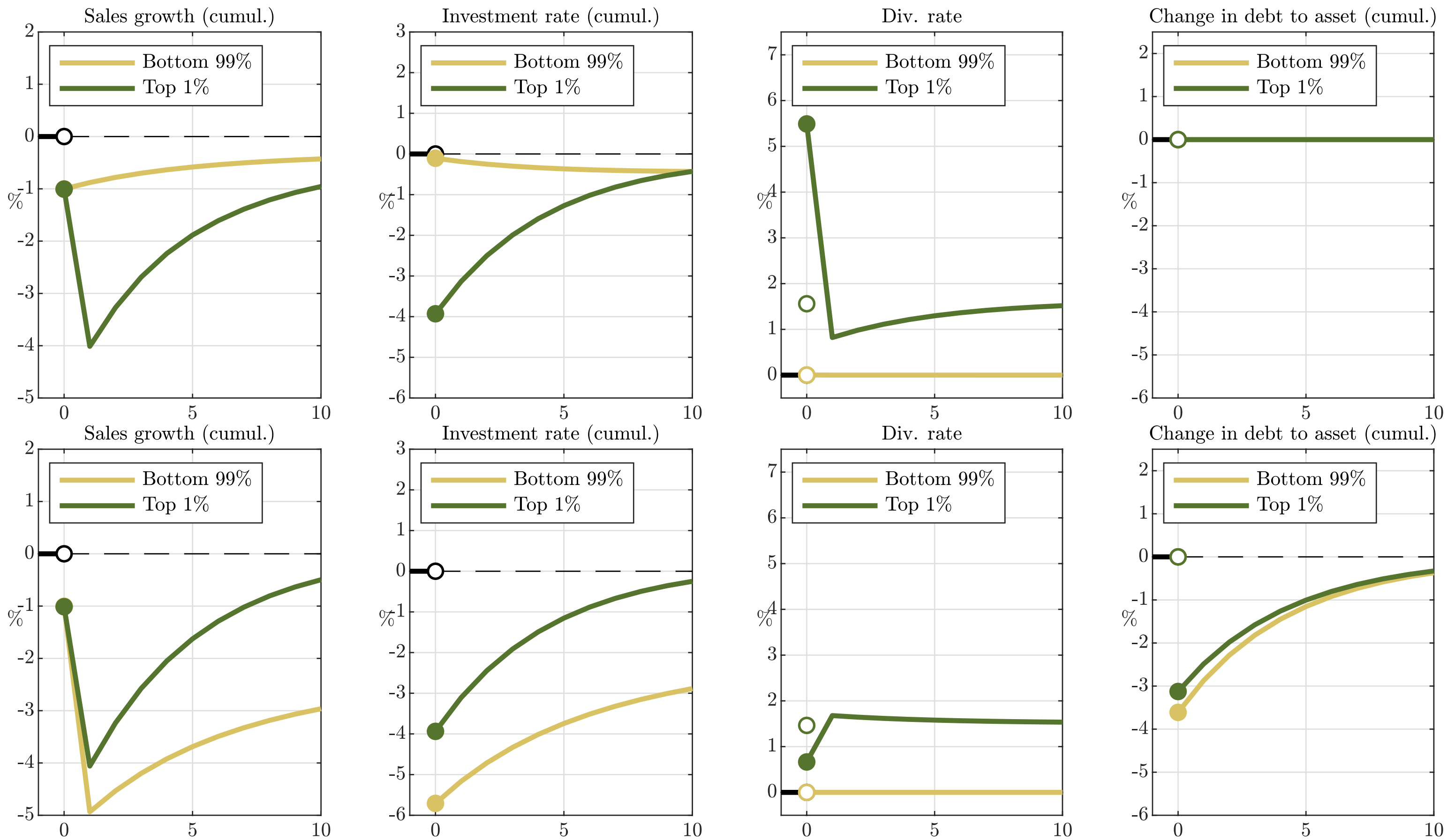


Figure A1: Impulse responses to an aggregate shock in the models of section 5.2. The green lines correspond to firms in the top 1% of the one-quarter lagged distribution of book assets, and the yellow lines correspond to firms in the bottom 99%; book assets in the model are defined as $k_{i,t}$. The top row reports impulse responses in the model with no external financing. The bottom row show the impulse responses in the model with borrowing.

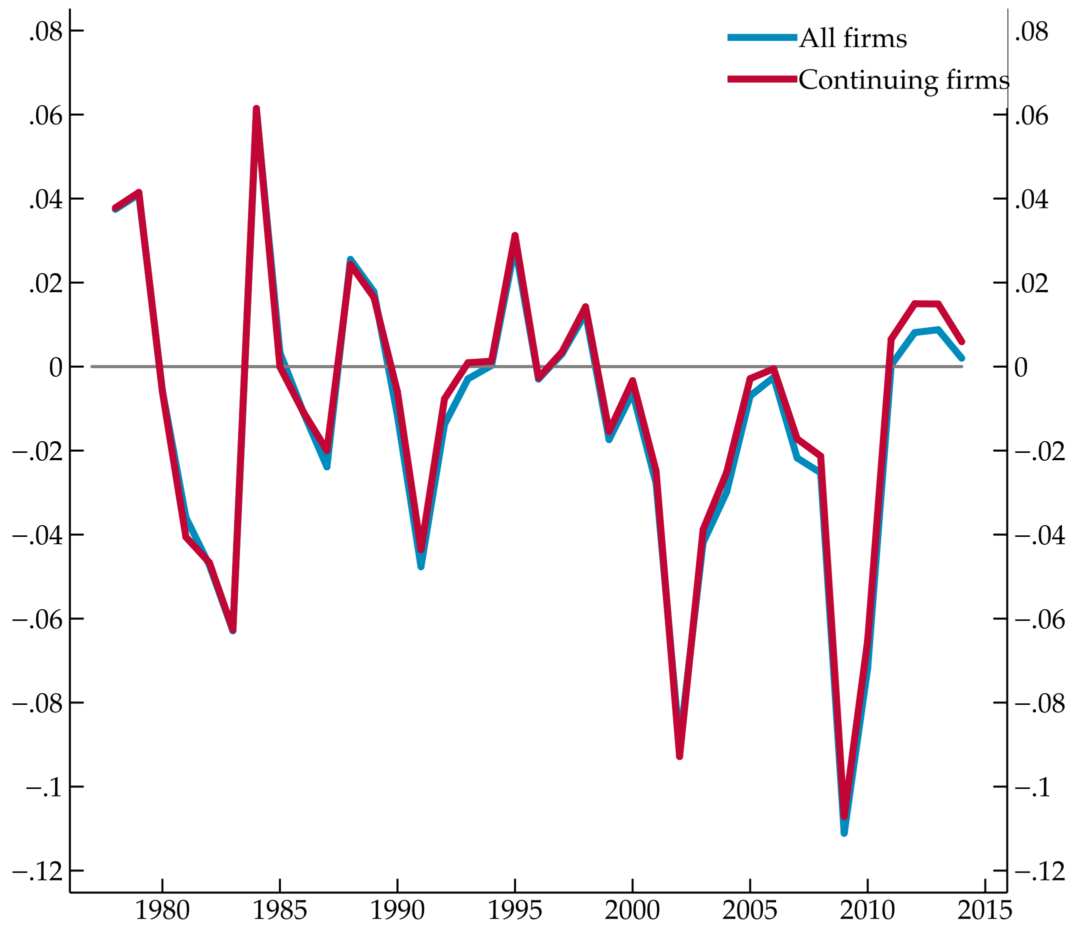


Figure A2: The blue line is employment growth at firms with initial firm size of 10 employees or more in manufacturing from the Business Dynamics Statistics (BDS) from 1978-2014. The red line is employment growth at continuing firms with initial firms size of 10 employees or more in manufacturing. Employment growth at continuing firms is defined as the change in employment less net entry (entry - exits). Entry and exit are restricted to firms over 10 employees. BDS data by industry and initial firm size available from `https://www.census.gov/ces/dataproducts/bds/data_firm.html`.

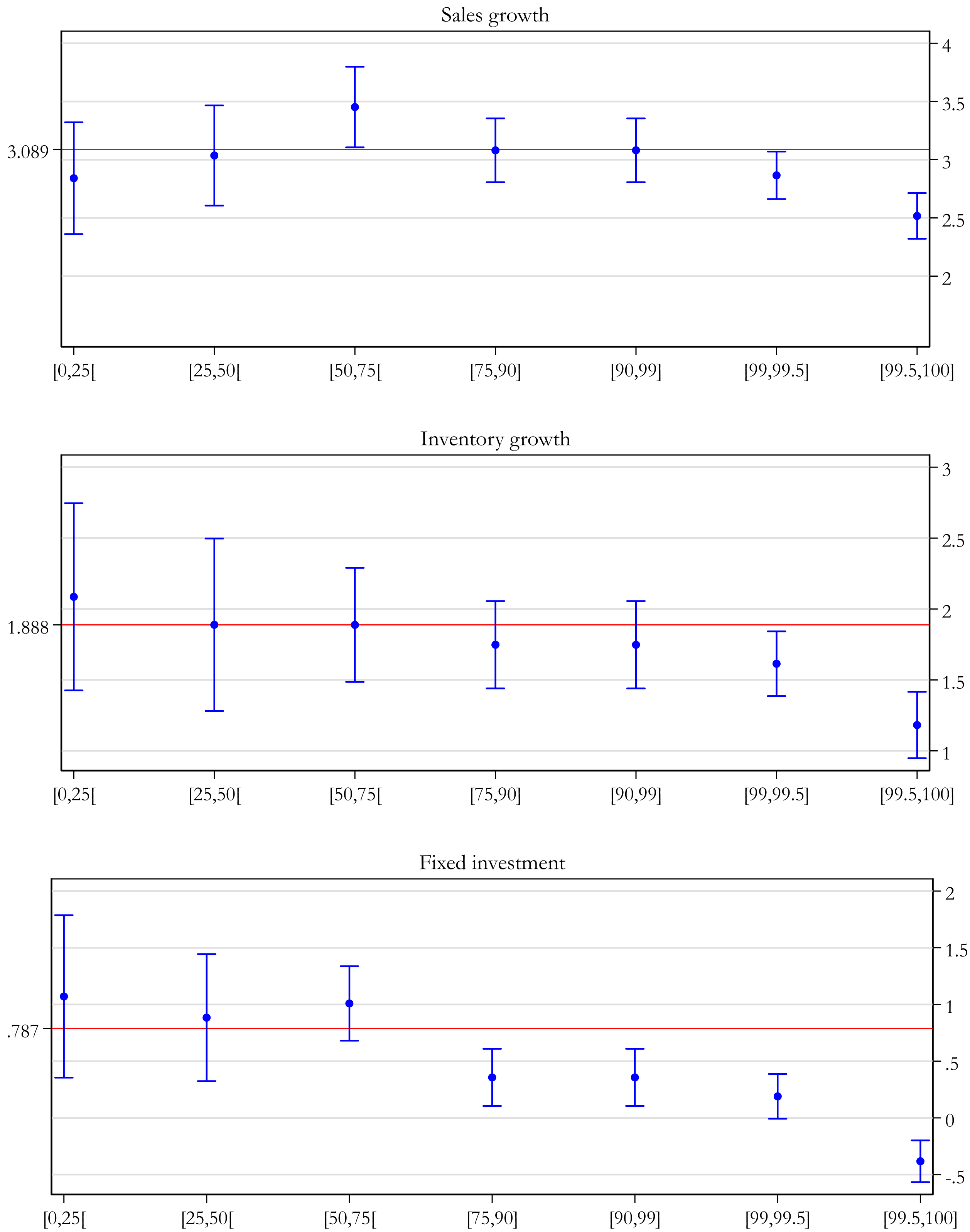
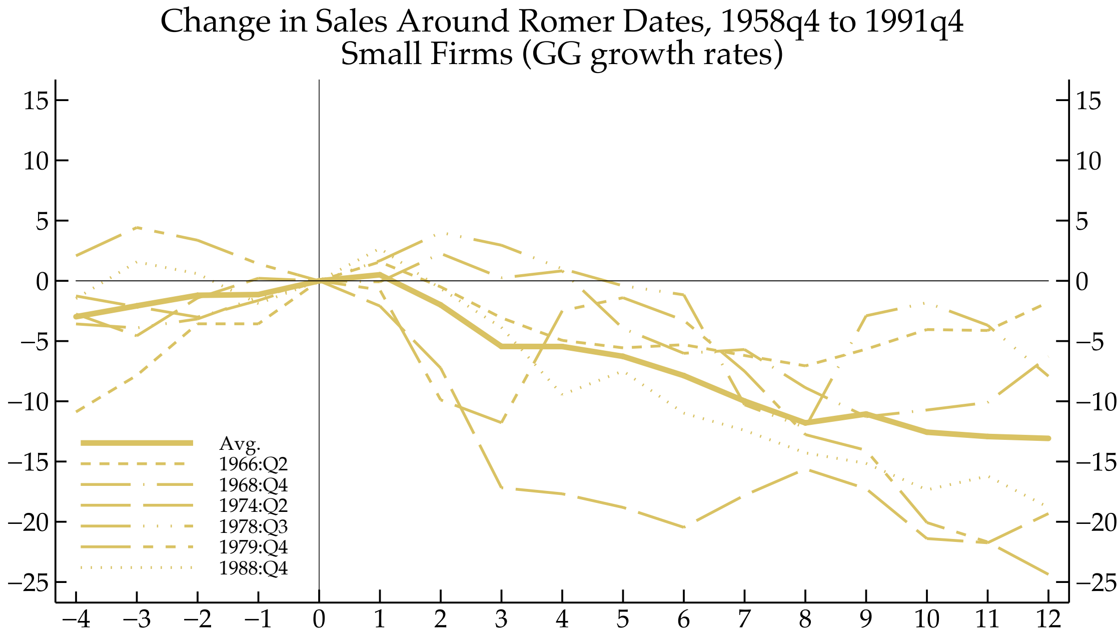


Figure A3: Average marginal effect of GDP growth on sales growth, inventory growth, and fixed investment, by disaggregated size groups and unconditionally. The marginal effects are computed a model similar to (1), but a larger number of size groups. Size is defined as described in Section 3.1 — that is, in terms of the one-year lagged cross-sectional distribution of book assets — but in this figure, we use seven size groups: the $[0, 25]$, $[0, 50]$, $[50, 75]$, $[75, 90]$, $[90, 99]$, $[99, 99.5]$ and $[99.5, 100]$ groups, thus disaggregating the smallest size group from our baseline specification into four sub-groups. Blue: conditional average marginal effect by size group, with $+/-2s.e.$ confidence interval. Red: unconditional average marginal effect.



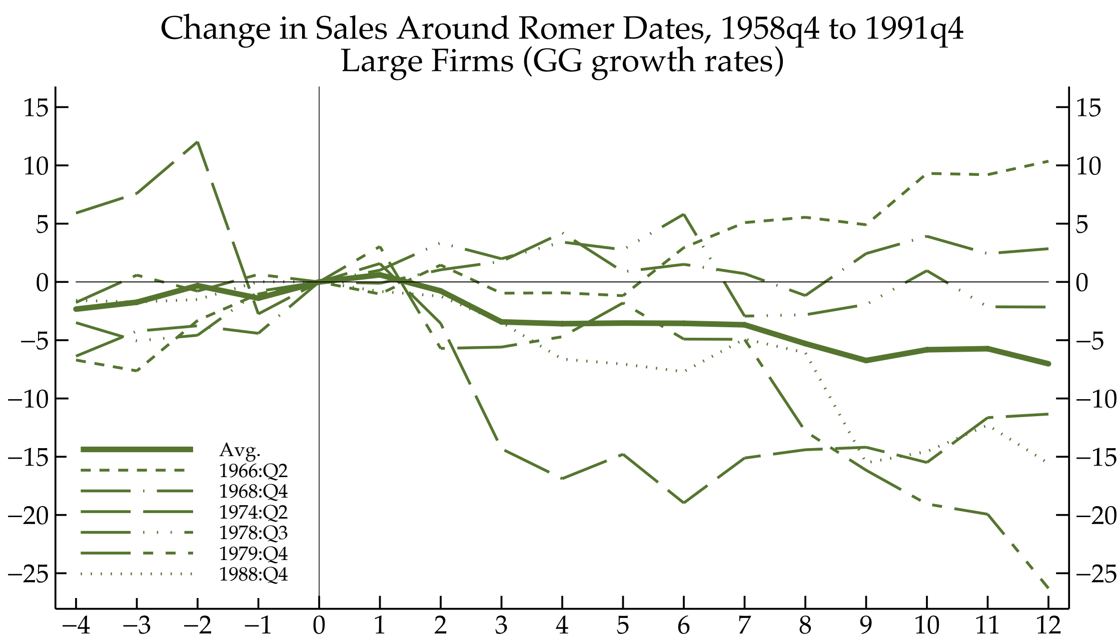
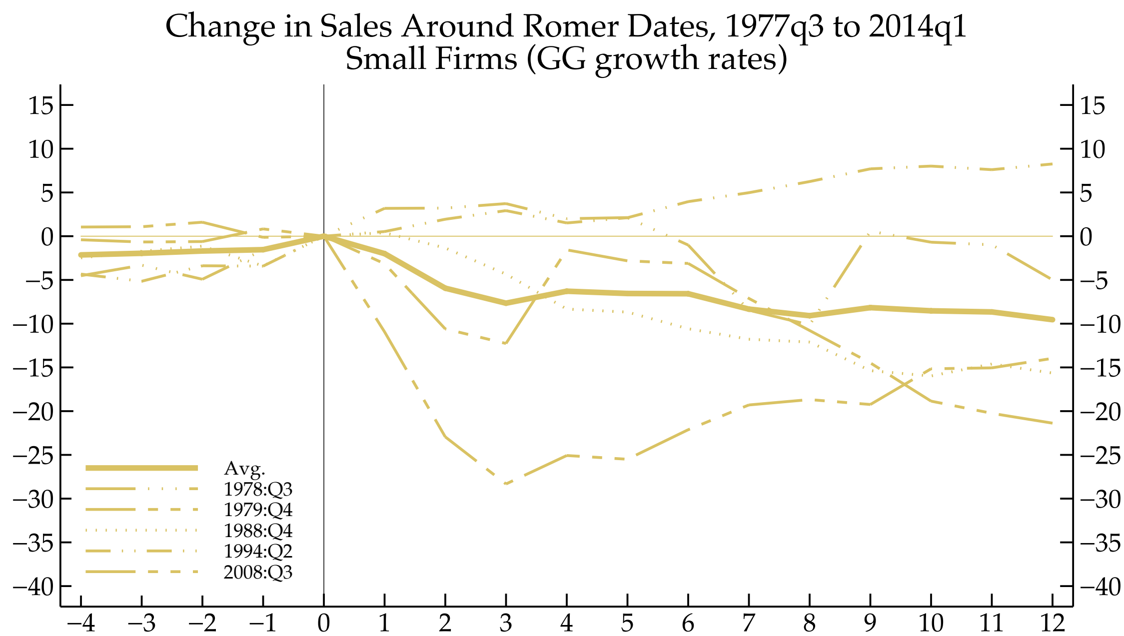


Figure A4: Event study of sales growth around Romer dates, using the publicly available QFR data and the Gertler and Gilchrist (1994) methodology for constructing small and large firm growth rates. The event study uses only data from 1958q4 to 1991q4, consistent with the original Gertler and Gilchrist (1994) analysis. The methodology used for constructing small and large firm growth rates is described in Appendix D. The growth rates are de-seasonalized by eliminating quarter fixed effects, and de-meaned in the 1954q1 to 1994q1 sample before constructing the event study responses.



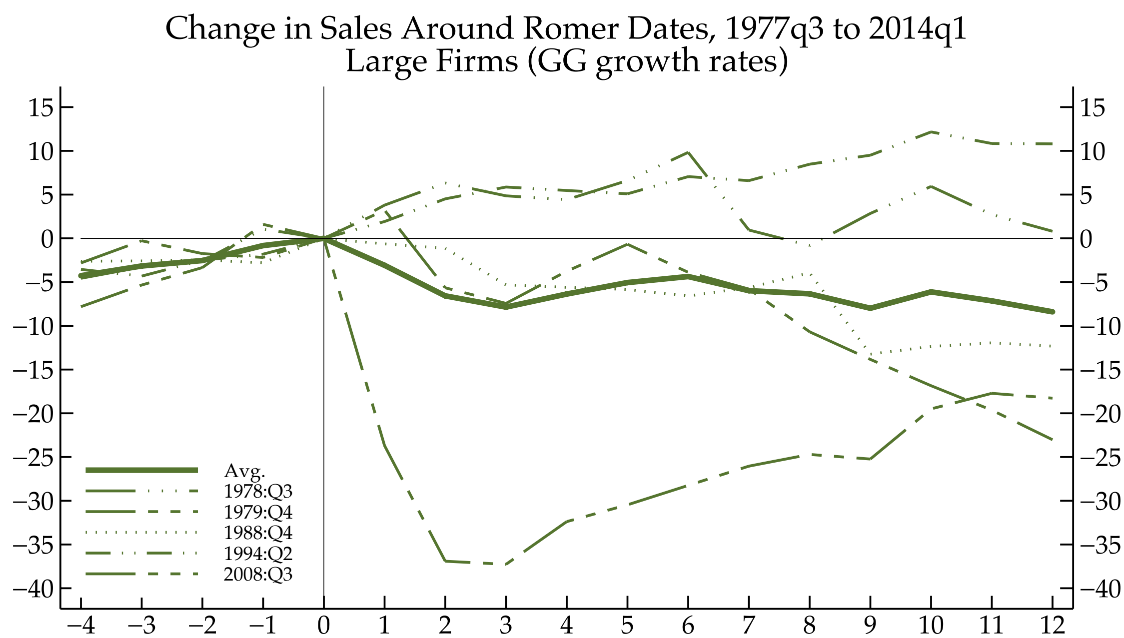
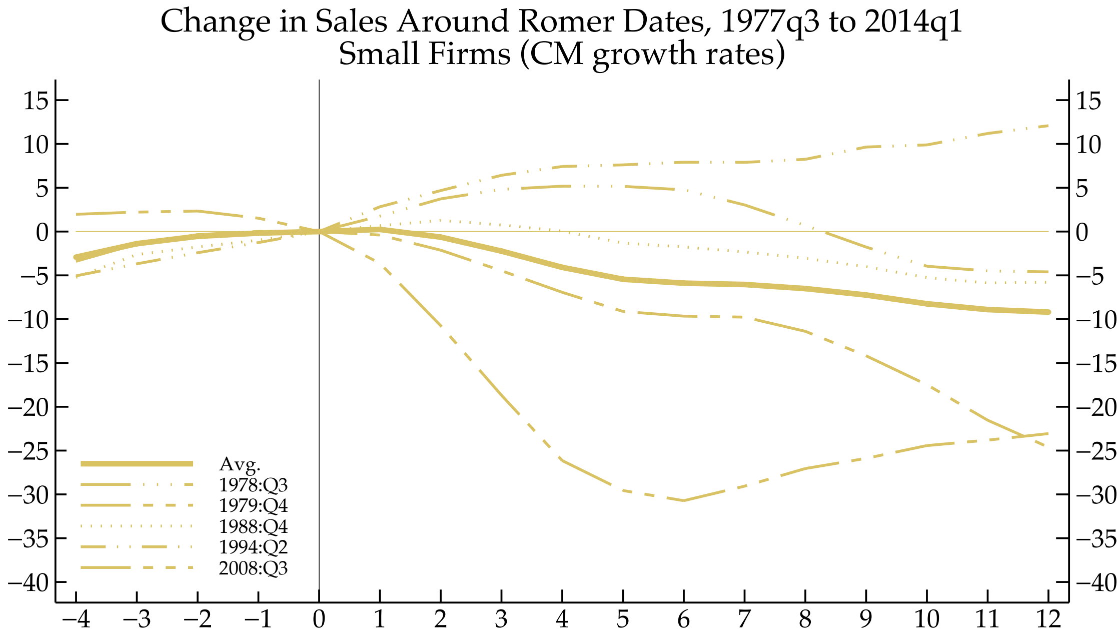


Figure A5: Event study of sales growth around Romer dates, using the publicly available QFR data and the Gertler and Gilchrist (1994) methodology for constructing small and large firm growth rates. The event study uses the data from 1977q3 to 2014q1, over which the micro data is available and the CM growth rates can be constructed. The methodology used for constructing small and large firm growth rates is described in Appendix D. The growth rates are de-seasonalized by eliminating quarter fixed effects, and de-meaned in the 1977q3 to 2014q1 sample before constructing the event study responses.



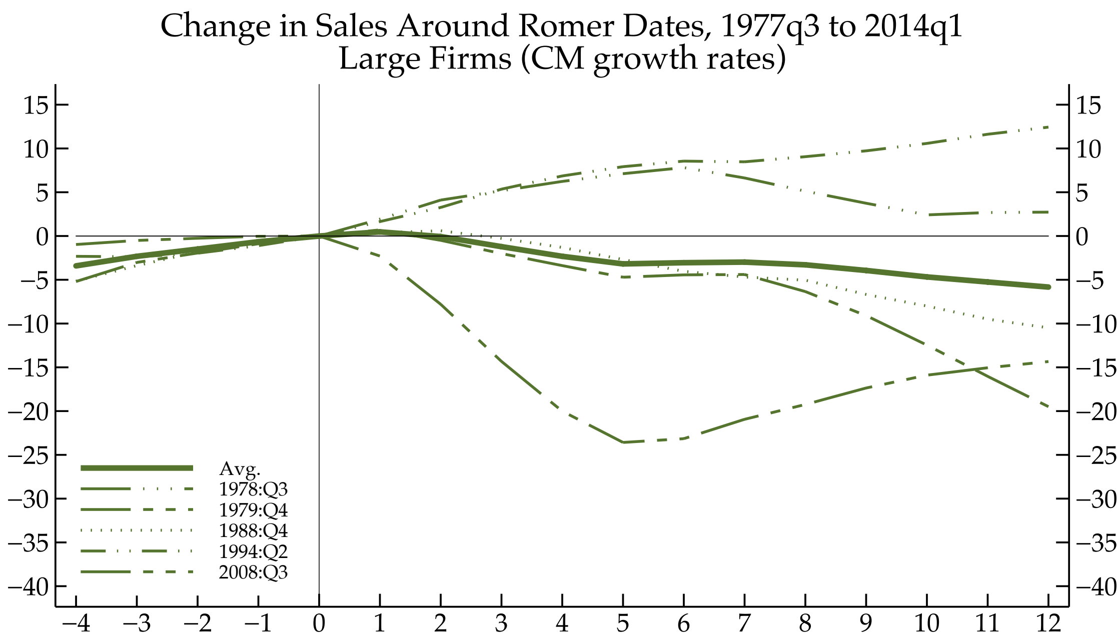


Figure A6: Event study of sales growth around Romer dates, using the average equal-weighted growth rates constructed from the micro data underlying the QFR. The time series used to construct these event study graphs are reported in Figure 1. The event study uses the data from 1977q3 to 2014q1. The growth rates are de-meaned before constructing the event study responses.

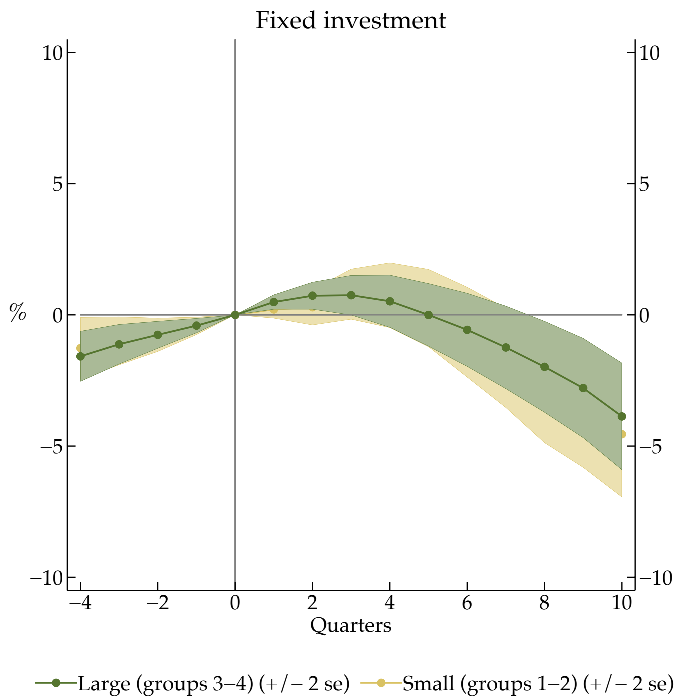


Figure A7: The behavior of fixed investment after the start of a recession in the quarterly Compustat sample. The graph reports the cumulative investment rate relative to the beginning of the recession; see section 5.2 for details on the estimation. Shaded areas are +/- 2 standard error bands. See appendix D for details on the definition of size groups. Recession start dates are 1981q3, 1990q3, 2001q1, and 2007q4.

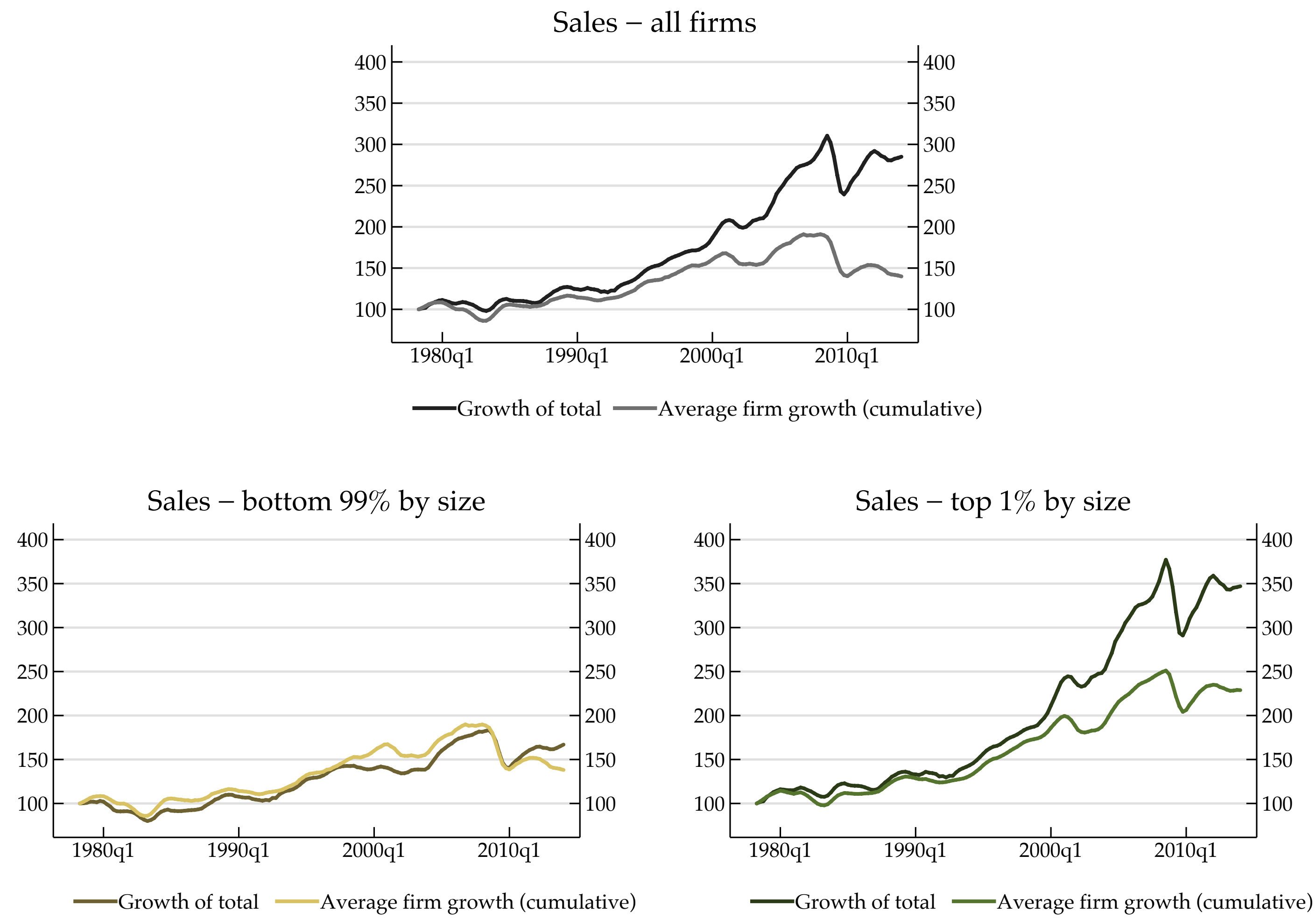


Figure A8: Aggregate sales and average within-firm cumulative growth rate of sales. Each panel reports total annual sales normalized to 100 at the beginning of the sample, and the cumulative firm-level average growth rate of sales, for a different group of firms, also normalized to 100 at the beginning of the sample.

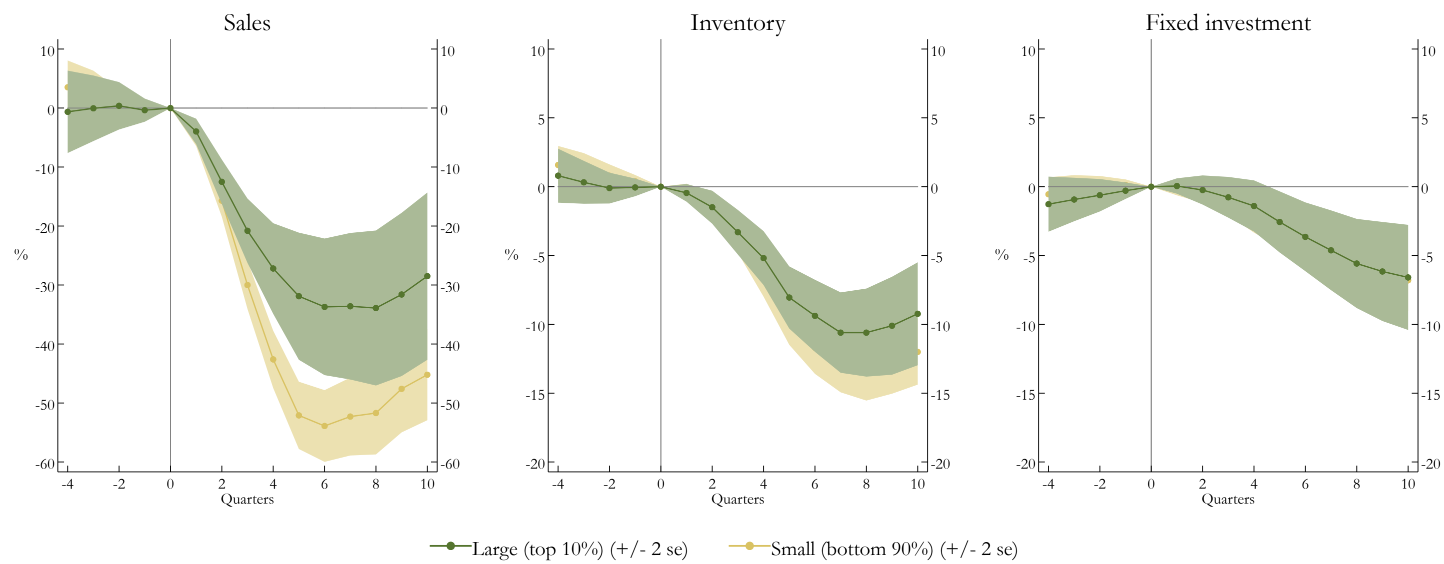


Figure A9: The behavior of sales, inventory and fixed capital after the start of a recession in the trade sector. Each graph reports the cumulative change in a variable of interest after the beginning of a recession. Shaded areas are +/- 2 standard error bands. All growth rates are computed year-on-year and expressed at the quarterly frequency. Recession start dates are 1981q3, 1990q3, 2001q1, and 2007q4. See section 5.2 for more details on the construction of these cumulative responses.

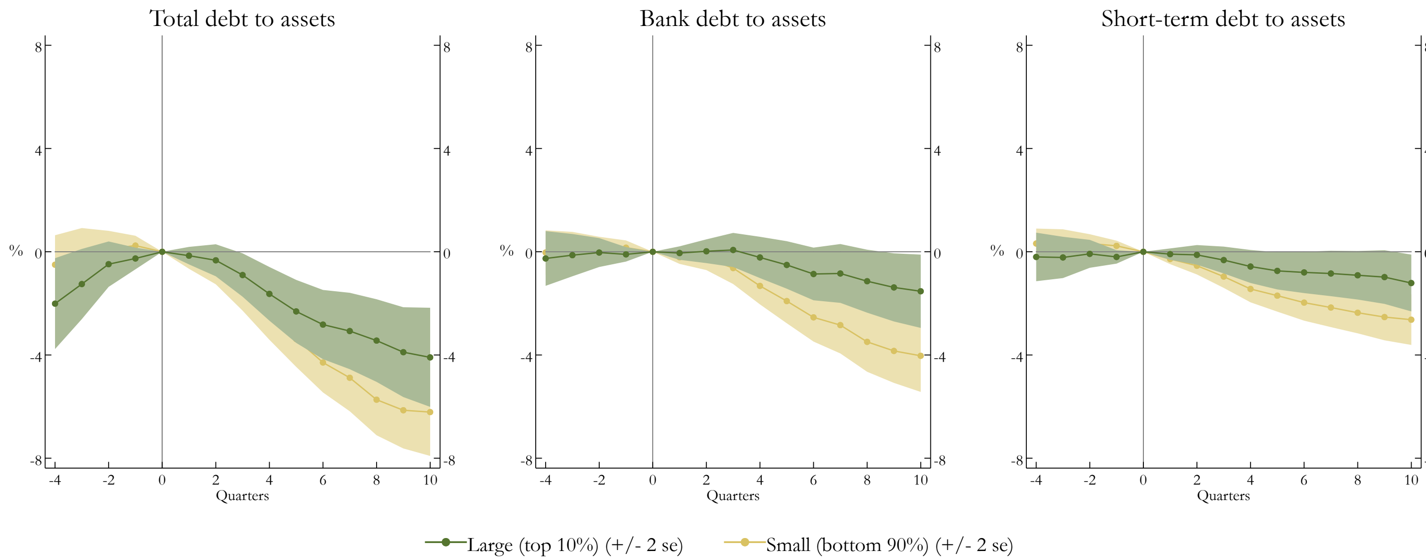


Figure A10: The behavior of debt overall, bank debt, and short-term debt after the start of a recession in the trade sector. Each panel reports changes relative to quarter 0 (the recession start date), computed using the cumulative sum of average growth rate of each size group. Growth rates at the firm-level are computed as $\frac{x_{i,t}-x_{i,t-4}}{assets_{i,t-4}}$, where $x \in \{\text{all debt}, \text{bank debt}, \text{short-term debt}\}$. Size groups are defined with a four-quarter lag. See section 5.2 for more details on the construction of these cumulative responses.

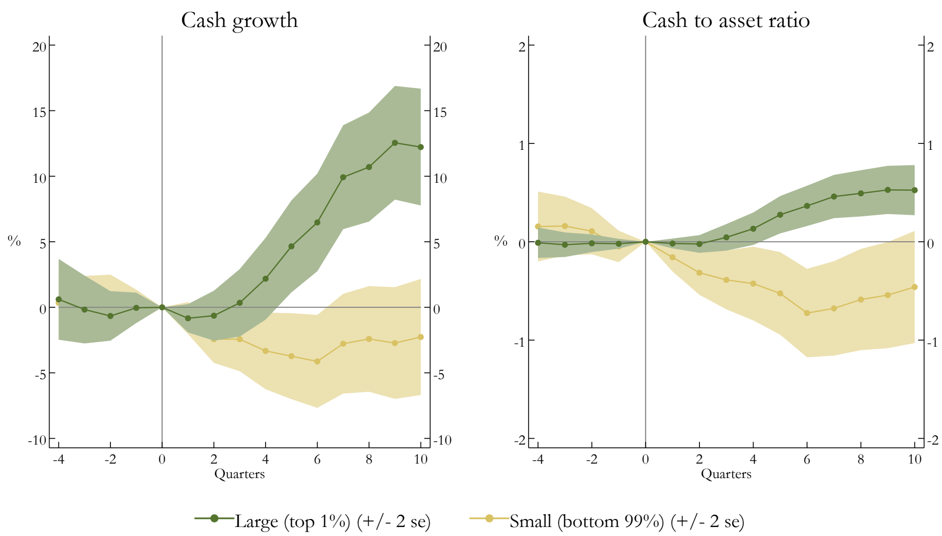


Figure A11: The behavior of cash growth and of the ratio of cash to total assets during recessions. The left panel reports the cumulative change in log cash around a recession start. The second panel reports the cumulative change in the ratio of cash to assets. Each graph reports the cumulative change in one of the two variables variable of interest after the beginning of a recession. Shaded areas are +/- 2 standard error bands. All changes are computed year-on-year and expressed at the quarterly frequency. Recession start dates are 1981q3, 1990q3, 2001q1, and 2007q4. See section 5.2 for more details.

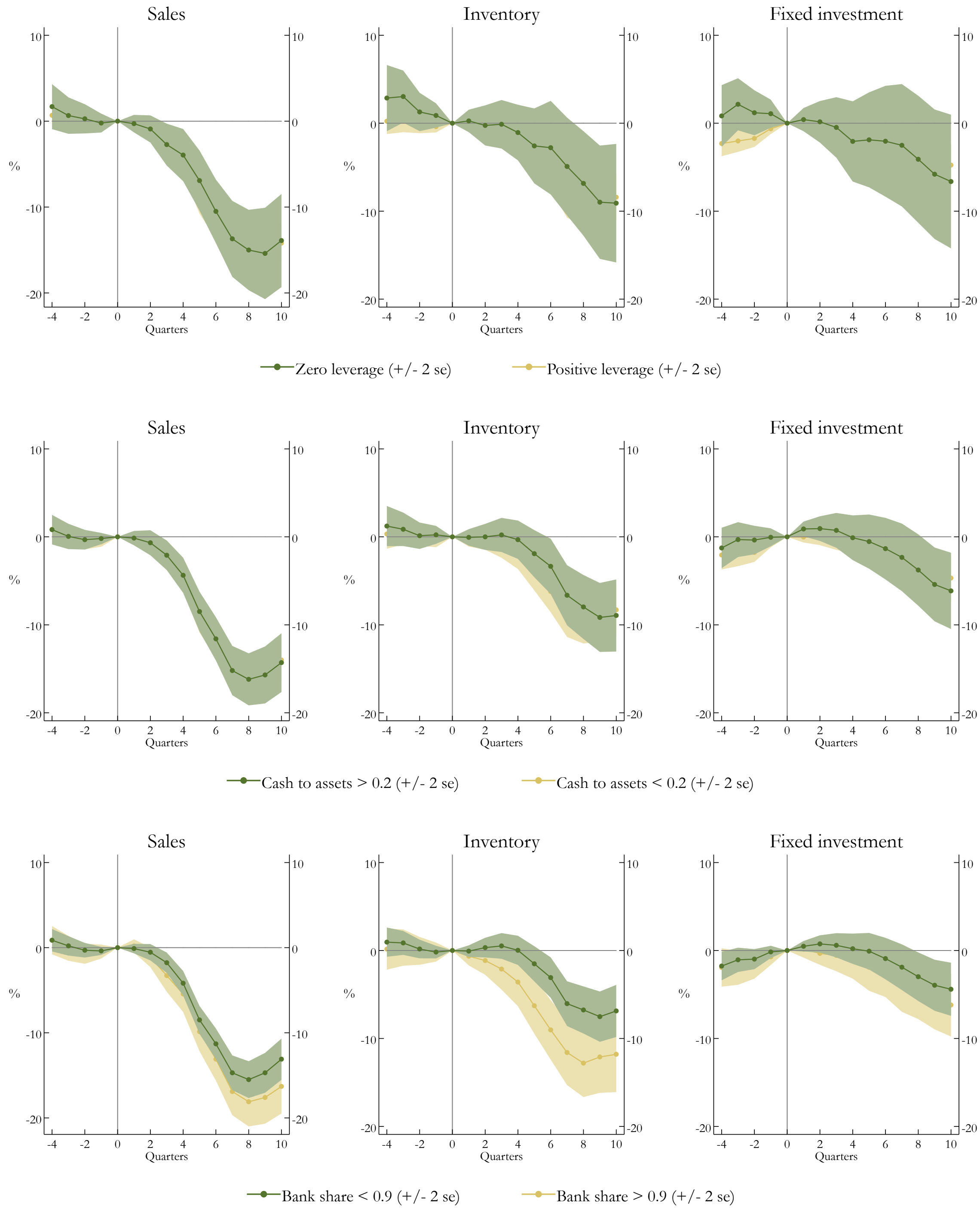


Figure A12: Sales, inventory and fixed capital after the start of a recession, across firms sorted by leverage, liquidity and bank dependence. Each graph reports the cumulative change in a variable of interest after the beginning of a recession; see section 5.2 for details on the estimation. Shaded areas are +/- 2 standard error bands. Variable definitions are given in appendix (D). Top row: firms sorted based on lagged leveraged; middle row: firms sort based on lagged cash-to-asset ratio; bottom row: firms sorted on bank dependence.

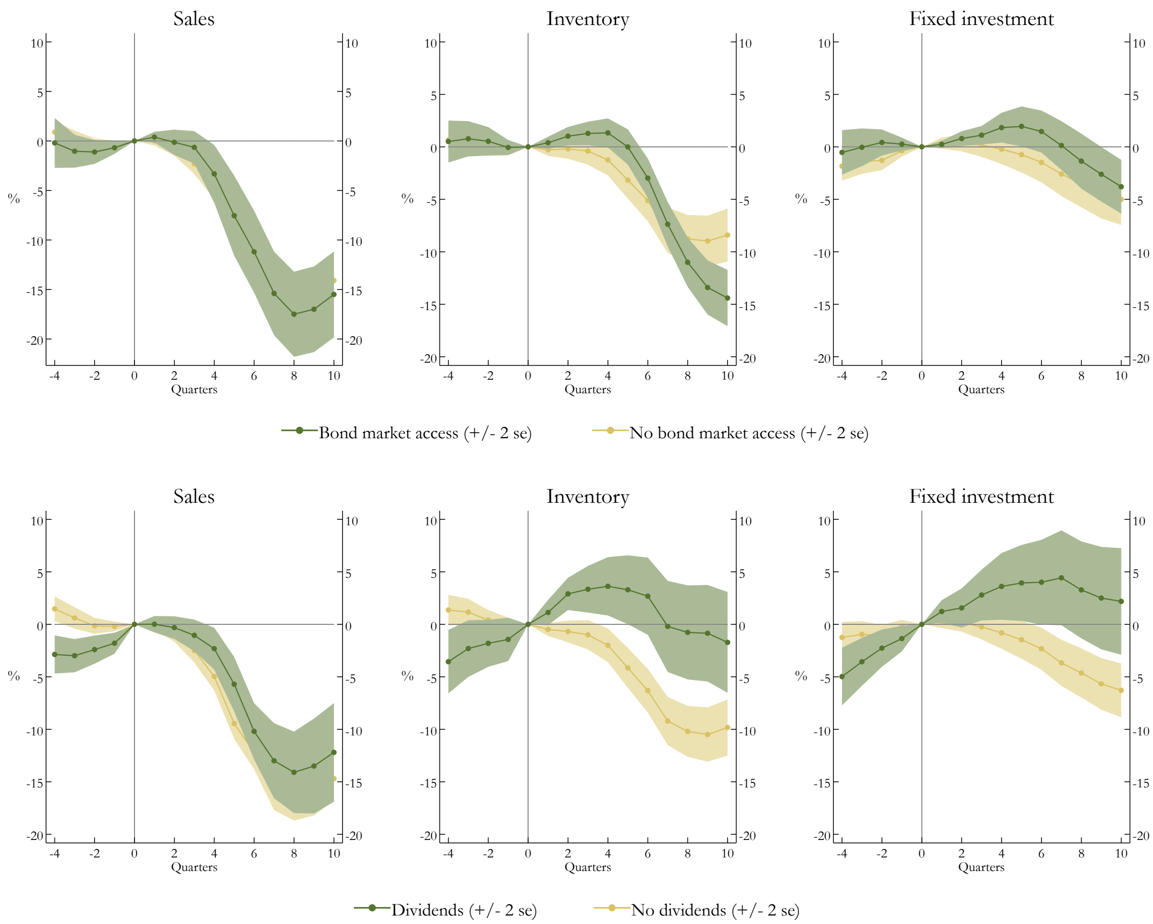


Figure A13: Sales, inventory and fixed capital after the start of a recession, across firms sorted by market access and dividend issuance. Each graph reports the cumulative change in a variable of interest after the beginning of a recession; see section 5.2 for details on the estimation. Shaded areas are +/- 2 standard error bands. Variable definitions are given in appendix D. Top row: firms sorted based on lagged access to bond market; bottom row: firms sort based on lagged dividend issuance.