Online Appendix

Loss in the Time of Cholera: Long-run Impact of a Disease Epidemic on the Urban Landscape

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Online Appendix A Additional Results

A.1 Additional Robustness Checks

A.1.1 Bandwidth Sensitivity

Figure SA4 in the Online Appendix assesses the sensitivity of the RD coefficients obtained from the polynomial specification of Equation (1) to the choice of bandwidth. Solid lines indicate the coefficient estimates using the bandwidth specified on the horizontal axis, while the dotted lines give the 90 and 95 percent confidence intervals of the estimate at each bandwidth. Note that, for all main outcomes in the analysis, the coefficient estimates are robust to bandwidth choice. In the pre-outbreak period (Panel (a)), the magnitude of the difference in rental values across the BSP boundary is consistently non-distinguishable from zero for any bandwidth. In the post-outbreak house value analysis (Panels (b)-(g)), the magnitudes of the coefficients are consistently higher than in the pre-outbreak period for any specified bandwidth. Furthermore, in most periods the coefficients are statistically significant for reasonable bandwidth choices.[2] For other outcomes, it is important to highlight that, in the case of the census results (Panels (j)-(l)), the estimated coefficients for all house occupancy characteristics are consistently higher in 1861 (post-outbreak) than in 1851 (pre-outbreak) for most reasonable bandwidths.

A.1.2 Trends prior to Outbreak

We assess whether there are differential trends in house values inside and outside the BSP area prior to the outbreak. If house values inside the BSP area were trending downward at a faster pace than outside the BSP area prior to the outbreak then house value differential later in time could be explained by this differential pre-outbreak trend. To assess this, we add rental data for the year 1846 to our analysis. This is the earliest year with data on both rental values and house numbers. We present the results in Table SA2 of the Online Appendix using data from both 1846 and 1853 (pre-outbreak) to create a household-level panel. Columns (1)-(3) present the results from the model below:

$$y_{it} = \beta_0 + \beta_1 BSP_i + \beta_2 Year1853_t + \beta_3 BSP_i \times Year1853_t + \mathbf{W}'_{it}\gamma + \epsilon_{it} \tag{SA1}$$

[2] One exception is rental values in 2015 (Panel (g)) which is statistically significant for a limited range of bandwidths. Recall, however, that the sample of rental values available in 2015 is relatively small. Also note that despite this, the estimated coefficients are consistently negative for any bandwidth.

where $Year1853_t$ is an indicator equal to 1 if the year is 1853 while other variables are defined in the main text. Column (1) estimates the model for the entire sample, column (2) restricts the analysis to houses within 100 meters of the boundary, and column (3) to the optimal bandwidth of 29 meters used in an RD design. We focus on the estimates of β_3 to assess the presence of differential pre-trends between houses inside BSP and outside. Note that the coefficients are close to zero and statistically insignificant suggesting that there were no differential pre-trends in rental prices. Columns (4) and (5) replicate the main RD design using only the 1846 data. Note that there is no evidence of different rental prices across the boundary. For graphical evidence, we plot the average rental values pre-outbreak (1846 and 1853) in Figure SA5 in the Online Appendix. Notice that rental prices inside and outside of the BSP catchment area were evolving very similarly prior to the outbreak in 1854.

A.1.3 Sewer Access Discontinuity

Note in the RD plots for the covariates that there is a noticeable discontinuity at the BSP boundary in whether houses have access to new sewers and whether houses have no sewer access. In an RD setting discontinuities in covariates are troublesome as they may confound the treatment effect.[3] In order to ensure that we have a sample where sewer access is smooth across the boundary, we drop all segments of the boundary where there is a visible change in no access to sewers and in access to new sewers near the boundary. Refer to Figure SA6 in the Online Appendix for a map of the boundary and the chosen segments. Once we omit these segments, notice that the RD plots look smooth across the boundary (Online Appendix Figure SA7).

We replicate the RD analysis using the restricted boundary. Results are presented in Table SA3 in the Online Appendix. The analysis shows that for the remaining segments there are no significant differences in house rental values prior to the outbreak (Panel A), while, as expected, the differences in 1864 are still statistically significant and larger than previously estimated. Therefore, although there are visible discontinuities in sewer access in the original analysis, they do not significantly affect the overall results.

[3] Notice, however, that there is actually a drop in no access and an increase in access to new sewers. This suggests that houses just inside the boundary may be richer prior to the outbreak. Therefore, our post-outbreak results may actually be a lower bound on the effect rather than confound it.

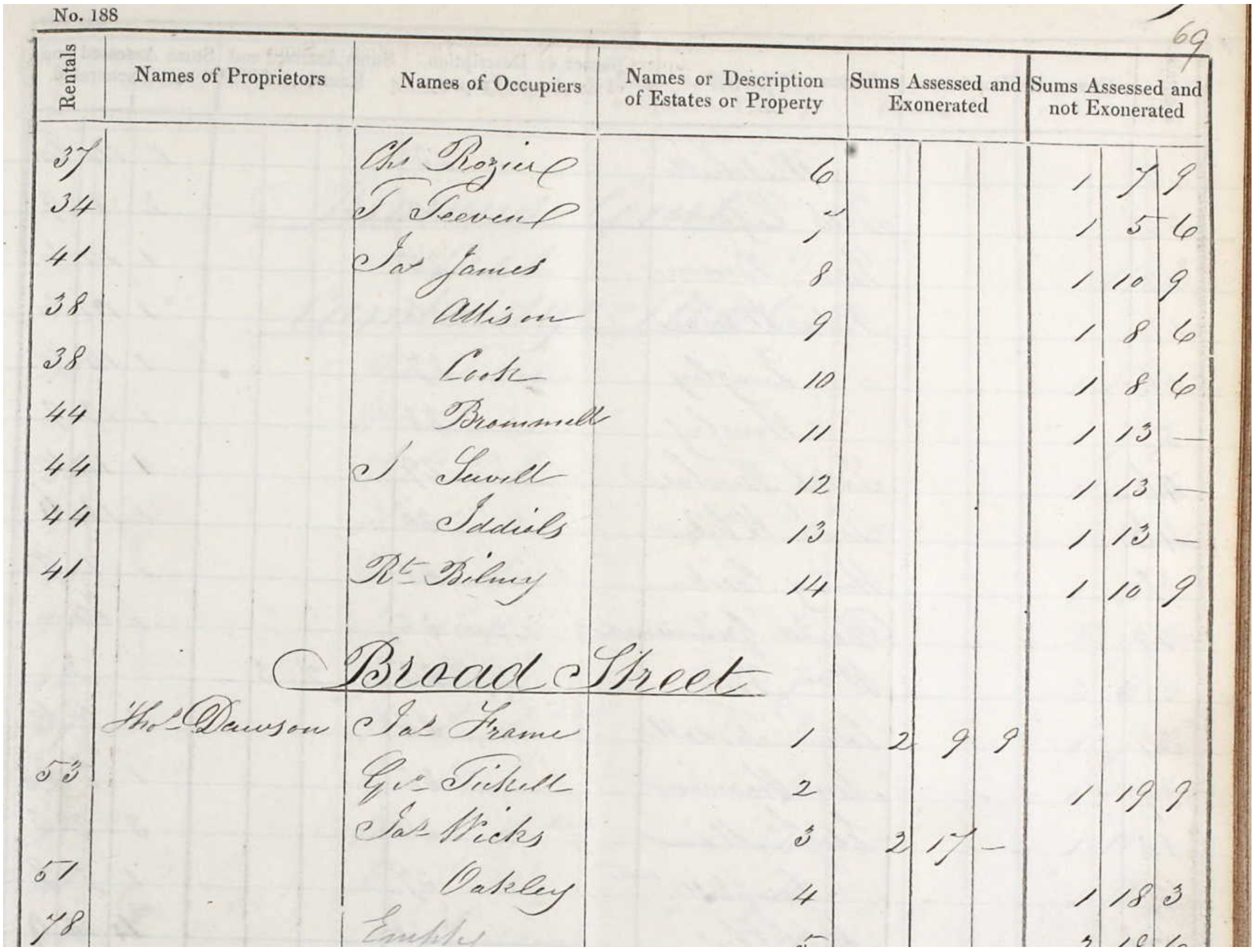
A.1.4 Irish Immigrants

Table 6 in the text uses immigrant status as a potential proxy for socioeconomic status. This section restricts the definition of immigrant family to Irish since this may be a better indicator of socioeconomic status. For the 1851 census, about 70 percent of all immigrants in our sample were Irish. In the 1861 census, the share was about 60 percent with the number of other immigrants increasing, particularly, German immigrants. Table SA4 in the Online Appendix presents the results restricting the sample to Irish immigrant households. Specifically, we look at (i) the number of households with Irish head in a given address, and (ii) the share of households with Irish head in a given address. Not surprisingly, given that most immigrant households were Irish, we find results that are very similar to the results presented in Table 6. In fact, the number and share of Irish headed households increases sharply at the boundary while no significant differences were present pre-outbreak. The results are robust to the method and bandwidth used.

A.1.5 Spatial Density of House Sales in Current Data (1995-2013)

The house sales data (1995-2013) used in the contemporary analysis shows a high density of sales at two locations within Soho. This can be concerning if unobserved characteristics of the area that explain the density nodes vary discontinuously across the BSP boundary. We note that while the 1853-1936 data consists of all houses in the parish (came from tax records of all houses in the area), the current data is, for the most part, houses that have been sold or rented in the area. Therefore, the more discontinuous patterns in the density for the current data is in part due to just not having a more complete dataset as in the historical dataset. With that being said, we investigate the two nodes (one west of the pump and inside the boundary and the other east of the pump and outside the boundary) and they are the result of large apartment buildings in those areas. The node inside the boundary is due to "Stirling Court", "City of Westminster Dwellings" buildings (both on Marshall Street) and "Sandringham Court" building (on Dufour's Street). The second node is due to the "10 Richmond" building (on Richmond Mews). We note, however, that in most cases these locations are beyond the optimal bandwidths used for the local linear regression and optimal band polynomial specifications. Thus the analysis is not significantly affected. Table SA5 in the Online Appendix shows results that replicate the analysis for the current data after dropping observations from these two areas of concern. Note that, given the loss of observations, statistical significance is lost in Columns 3 and 8, while the coefficients are slightly smaller for the wide bandwidth specifications (i.e., the ones that were mostly affected by the drop in observations). However, overall, the results are qualitatively similar to the previous results in Table 8.

A.2 Additional Figures and Tables



No. 188 69

Rentals	Names of Proprietors	Names of Occupiers	Names or Description of Estates or Property	Sums Assessed and Exonerated	Sums Assessed and not Exonerated
37		Chs Rozier	6		1 7 9
34		T. Seeven	7		1 5 6
41		Jas James	8		1 10 9
38		Allison	9		1 8 6
38		Cook	10		1 8 6
44		Brommell	11		1 13 —
44		J. Sewell	12		1 13 —
44		Iddiols	13		1 13 —
41		Rt. Bilney	14		1 10 9
		Broad Street			
	Thos Dawson	Jas Frame	1	2 9 9	
53		Geo Tickell	2		1 19 9
		Jas Wicks	3	2 17 —	
57		Oakley	4		1 18 3
78		Empty	5		[illegible]

Figure SA1: Land Tax records, Broad Street, 1853
Source: National Land Tax Assessment records

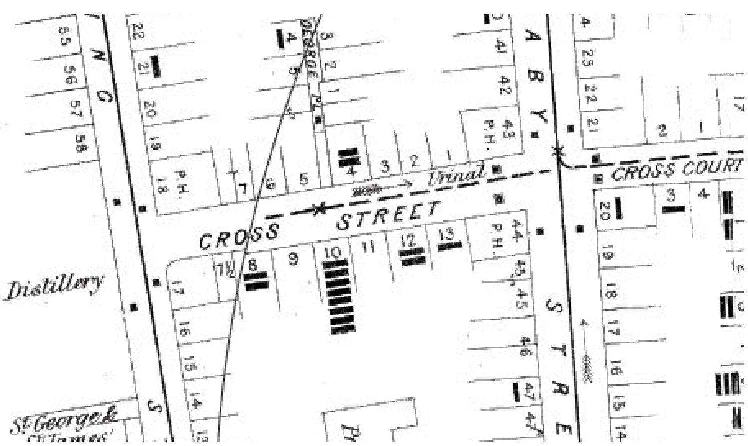


Figure SA2: Cholera Inquiry Committee (1855) Cholera Deaths Map
Notes: Black bars represent a cholera death

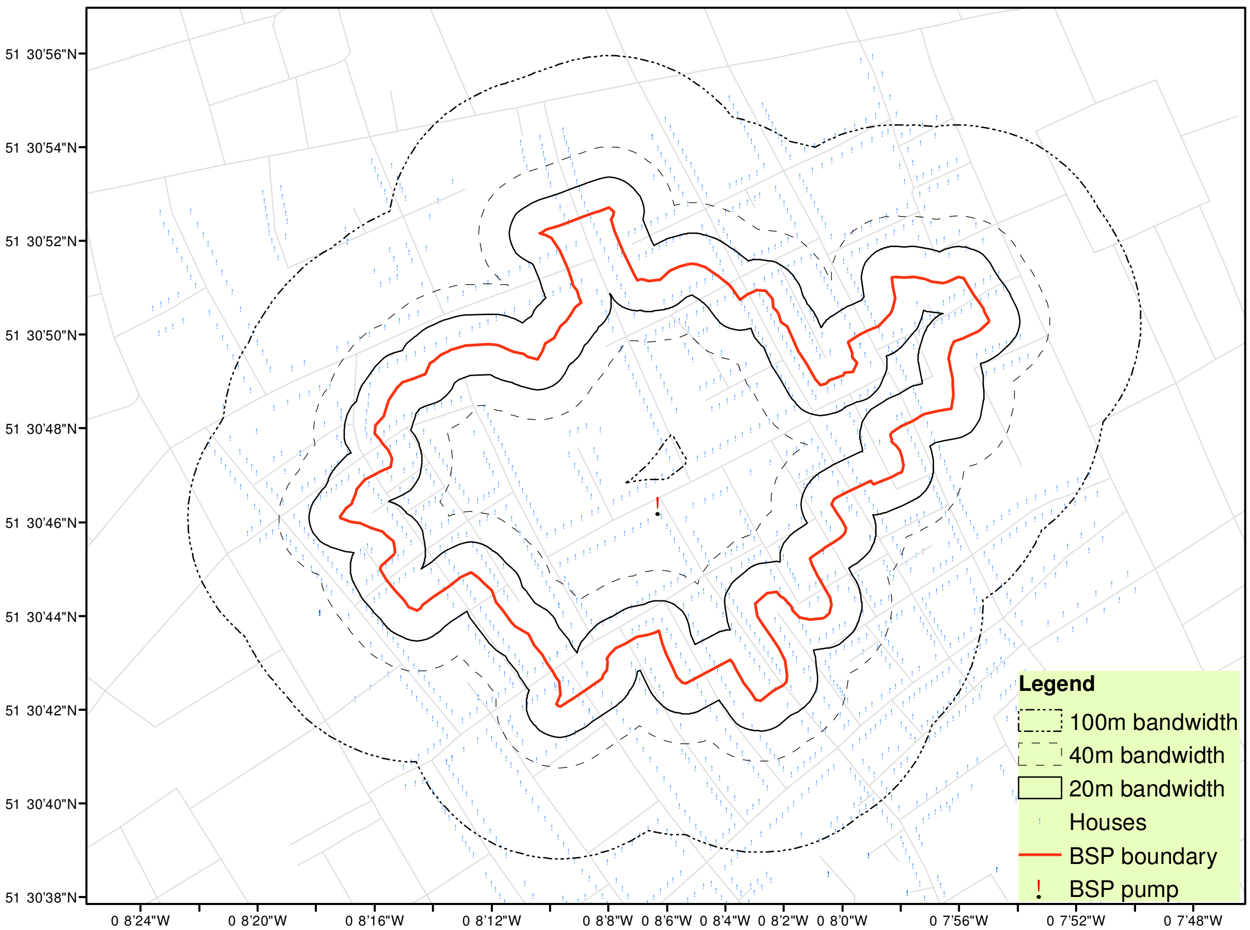
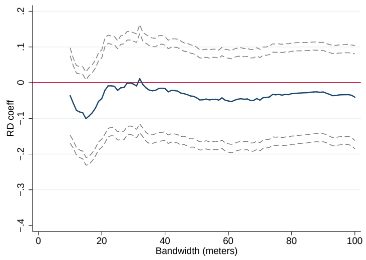
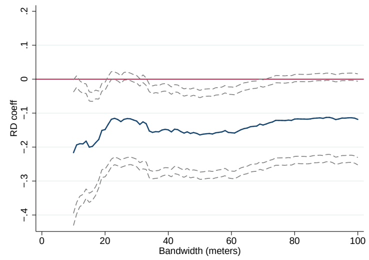


Figure SA3: BSP boundary with Distance Bandwidths

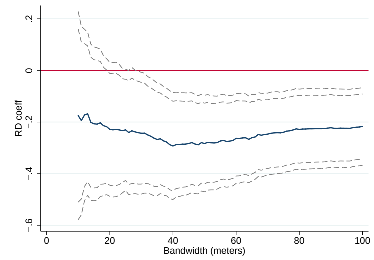
Notes: House locations are for the year 1853/1864 data.



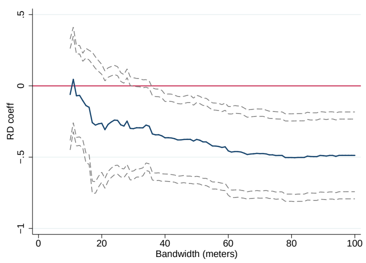
(a) Rental values (1853)



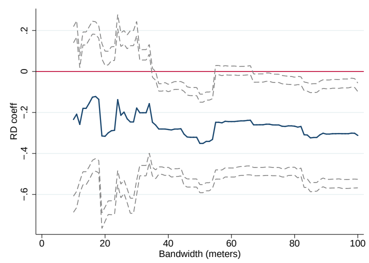
(b) Rental values (1864)



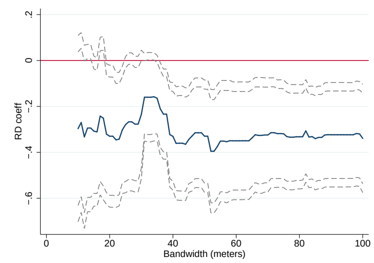
(c) Rental values (1894)



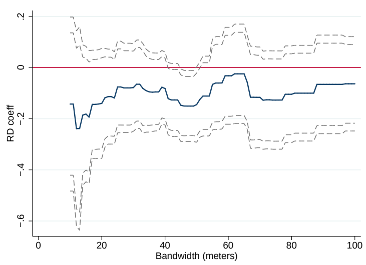
(d) Rental values (1936)



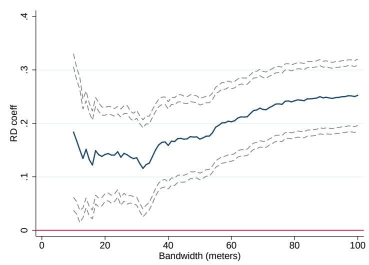
(e) House and Zoopla (1995-2013)



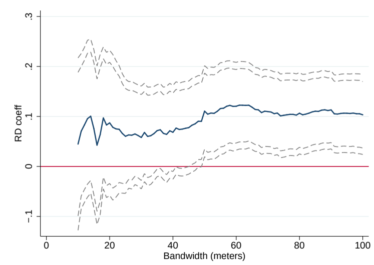
(f) Zoopla (1995-2013)



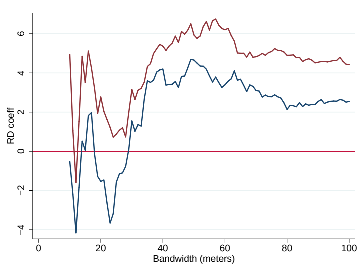
(g) Rental values (2015)



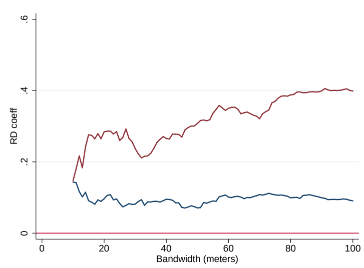
(h) At least one death



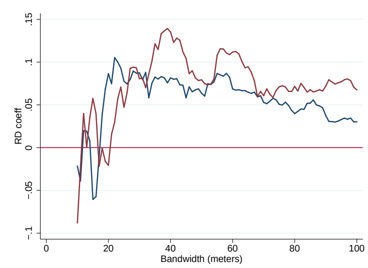
(i) Resident moved



(j) No. of occupants (1851, 1861)

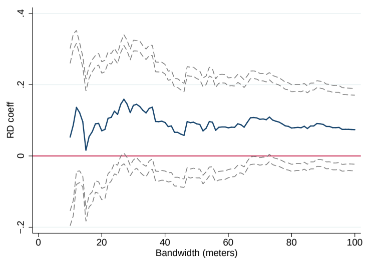


(k) No. of immigrants (1851, 1861)

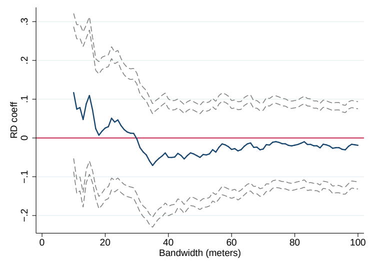


(l) Prop. of immigrants (1851, 1861)

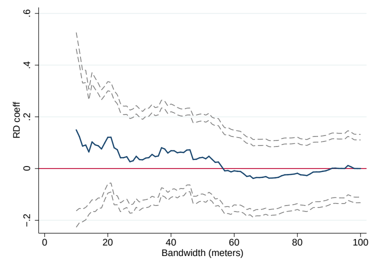
Figure SA4: Bandwidth Sensitivity-*Continues*



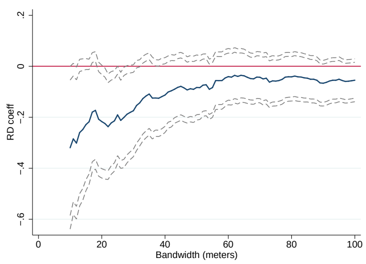
(m) Very poor (1899)



(n) Poor (1899)



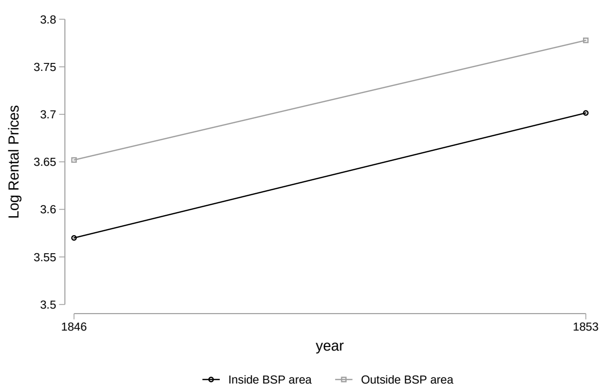
(o) Working poor (1899)



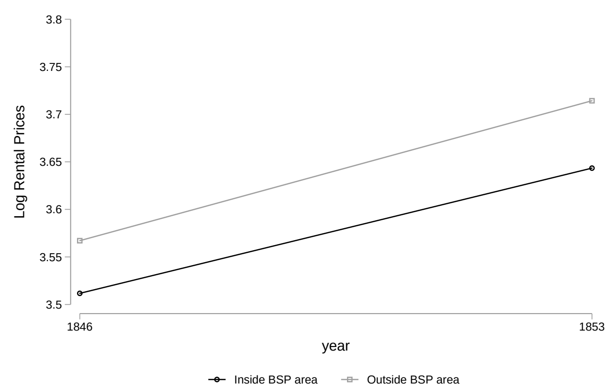
(p) Middle class (1899)

Figure SA4: Bandwidth Sensitivity-*Continued*

Notes: Dashed lines provide 90 and 95 percent confidence intervals. "RD coeff" refers to the coefficient estimate of BSP in Equation (1) using the polynomial specification. Outcome variables is specified in the figure caption. For Figures SA4j, SA4k, and SA4l solid blue line gives RD coefficient for 1851 census data while solid red line gives RD coefficient for 1861 census data.



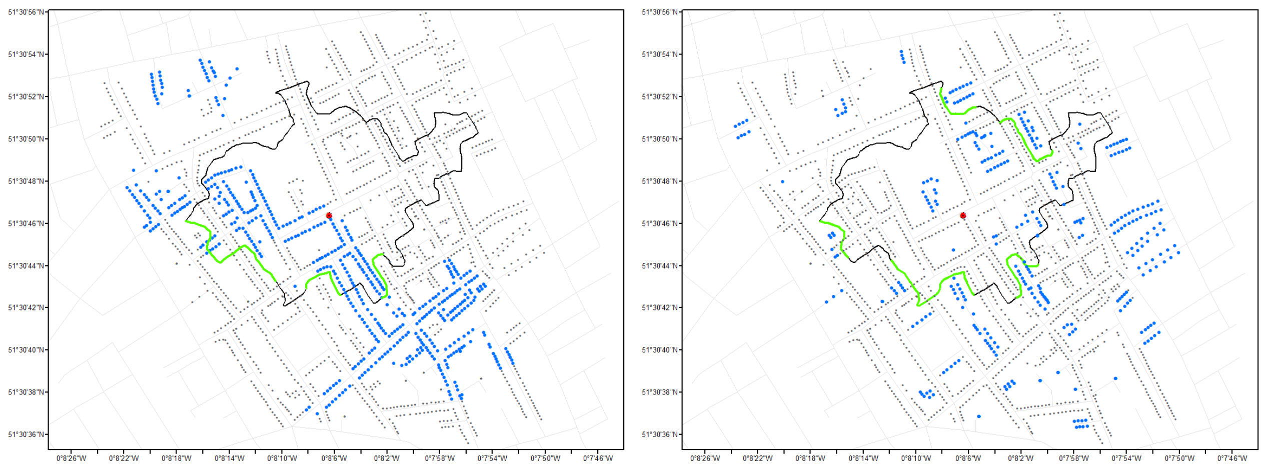
(a) All observations



(b) Within 30m of BSP boundary

Figure SA5: Mean Log Rental Prices in 1846 and 1853 by BSP status

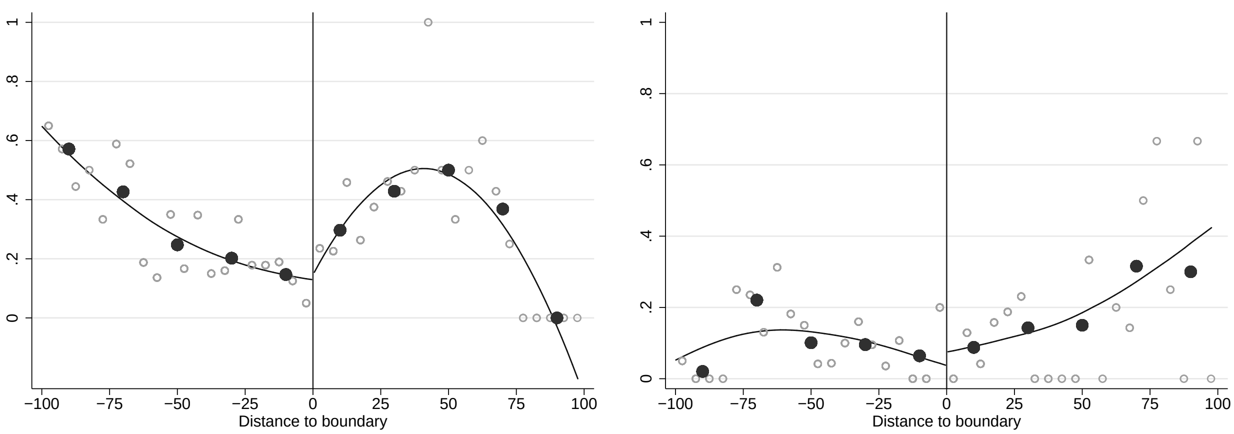
Notes: Solid black and gray lines give the mean log rental price inside and outside BSP area, respectively. Rental prices adjusted for inflation (1853 pounds).



(a) New Sewer (b) No Sewer access

Figure SA6: Omitted Segments

Notes: Blue dots indicate houses with access to new sewers (Panel a) and no access to sewers (Panel b). Highlighted segments of the boundary indicate segments where there is a visible change in the status of new sewers and no sewer access. Red dot indicates location of Broad Street pump. Black line delineated the BSP catchment area.



(a) New Sewer (b) No Sewer access

Figure SA7: RD Plots for Sewer Status: Restricted Sample

Notes: Solid dots give the average value of the specified variable for houses falling within 20 meter distance bins. Hollow dots give the average value of the specified variable for houses falling within 5 meter distance bins. "Distance to boundary" refers to the distance between a house and the closest point in the BSP boundary. The solid vertical line represents the BSP boundary. Negative/positive values of distance give the distance of houses inside/outside BSP area respectively. The solid line trends are the predicted values from a regression of the specified variable on a polynomial in distance to the boundary that uses a triangular kernel and a bandwidth of 200 meters.

Table SA1: Charles Booth class categorization

Class code	**Description of class**
A	The lowest class which consists of some occasional labourers, street sellers, loafers, criminals and semi-criminals. Their life is the life of savages, with vicissitudes of extreme hardship and their only luxury is drink.
B	Casual earnings, very poor. The labourers do not get as much as three days work a week, but it is doubtful if many could or would work full time for long together if they had the opportunity. Class B is not one in which men are born and live and die so much as a deposit of those who from mental, moral and physical reasons are incapable of better work.
C	Intermittent earning. 18s to 21s per week for a moderate family. The victims of competition and on them falls with particular severity the weight of recurrent depressions of trade. Labourers, poorer artisans and street sellers. This irregularity of employment may show itself in the week or in the year: stevedores and waterside porters may secure only one of two days' work in a week, whereas labourers in the building trades may get only eight or nine months in a year.
D	Small regular earnings. poor, regular earnings. Factory, dock, and warehouse labourers, carmen, messengers and porters. Of the whole section none can be said to rise above poverty, nor are many to be classed as very poor. As a general rule they have a hard struggle to make ends meet, but they are, as a body, decent steady men, paying their way and bringing up their children respectably
E	Regular standard earnings, 22s to 30s per week for regular work, fairly comfortable. As a rule the wives do not work, but the children do: the boys commonly following the father, the girls taking local trades or going out to service.
F	Higher class labour and the best paid of the artisans. Earnings exceed 30s per week. Foremen are included, city warehousemen of the better class and first hand lightermen; they are usually paid for responsibility and are men of good character and much intelligence.
G	Lower middle class. Shopkeepers and small employers, clerks and subordinate professional men. A hardworking sober, energetic class.
H	Upper middle class, servant keeping class.

Note: Source: **?**

Table SA2: Evaluating Evidence of Pre-Trends in Log Rental Prices (1846, 1853)

	Log Rentals (1846,1853)				
	Panel Estimates			RD Estimates	
				LLR	Polynomial
	(1)	(2)	(3)	(4)	(5)
Inside BSP area	-0.082	-0.023	-0.055	0.052	-0.011
	(0.067)	(0.077)	(0.091)	(0.122)	(0.034)
Year 1853	0.126	0.148	0.147		
	(0.018)	(0.016)	(0.024)		
Inside BSP area×Year 1853	0.006	-0.018	-0.016		
	(0.020)	(0.019)	(0.027)		
Observations	2605	2070	987	484	1000
Bandwidth (meters)	-	100	29.55	29.55	100

Note: Clustered standard errors by street block are shown in parenthesis. Year1853 equals 1 if year is 1853, 0 if year is 1846. Estimates in columns (4) and (5) use Log Rentals in 1846 as the outcome variable. LLR refers to Local Linear Regression. Sample in LLR specifications is restricted to be within an optimal bandwidth determined as in **?** and using a triangular weighting kernel. Polynomial RD specification uses a second degree polynomial in distance to the BSP boundary.

Table SA3: Analysis using Segments without Sharp Change in Sewage Status

	Local Linear Regression		Polynomial RD		
	Baseline	Controls	Optimal Band	Wide Band	Segment FE
	(1)	(2)	(3)	(4)	(5)
Panel A: Log Rental Prices (1853)					
Inside BSP area	0.048	0.055	-0.003	-0.069	-0.129
	(0.186)	(0.088)	(0.082)	(0.091)	(0.080)
Observations	203	222	203	494	494
Mean Outside BSP	47.01	53.68	47.01	48.63	48.63
Bandwidth (meters)	23.28	26.69	23.28	100	100
Panel B: Log Rental Prices (1864)					
Inside BSP area	-0.391	-0.276	-0.207	-0.186	-0.254
	(0.148)	(0.158)	(0.080)	(0.086)	(0.084)
Observations	214	164	214	483	483
Mean Outside BSP	48.48	59.97	48.48	50.24	50.24
Bandwidth (meters)	27.37	20.00	27.37	100	100

Note: Analysis omits houses located near segments of the boundary where access to new sewers or no access to sewer changes sharply. Sample in Local Linear Regression specifications is restricted to be within an optimal bandwidth determined as in **?** and using a triangular weighting kernel. Columns (2), (3), (4), (5) include a set of determinants of rental values as additional controls. Polynomial RD specifications use a second order polynomial in distance to the BSP boundary. Optimal band refers to a specification that uses the optimal bandwidth obtained from the Local Linear specification in Column (1). Wide Band refers to a specification that uses a bandwidth of 100m around the BSP boundary (this bandwidth encompasses almost all observations within the BSP area). Segment FE refers to a specification that adds a set of boundary specific fixed effects that denote the closest of five boundary segments to a given observation. Clustered standard errors by street block are shown in parenthesis.

Table SA4: Boundary Effects on House Occupancy Characteristics: Irish Immigrants

	Number of Irish Families at Address			Proportion of Irish Families at Address		
		Polynomial RD			Polynomial RD	
	LLR	Optimal Band	Wide Band	LLR	Optimal Band	Wide Band
	(1)	(2)	(3)	(4)	(5)	(6)
Panel A: Census Data (1851)						
Inside BSP area	0.010	0.034	0.066	0.034	0.036	0.035
	(0.145)	(0.145)	(0.063)	(0.037)	(0.040)	(0.035)
Observations	576	576	1115	552	552	1115
Mean Outside BSP	0.233	0.233	0.216	0.056	0.056	0.054
Bandwidth (meters)	35.51	35.51	100	33.79	33.79	100
Panel B: Census Data (1861)						
Inside BSP area	0.175	0.212	0.248	0.069	0.066	0.037
	(0.128)	(0.077)	(0.093)	(0.039)	(0.034	(0.016)
Observations	549	549	1004	375	375	810
Mean Outside BSP	0.206	0.206	0.194	0.057	0.057	0.068
Bandwidth (meters)	37.14	37.14	100	27.76	27.76	100

Note: Clustered standard errors shown in parenthesis. Regressions include controls for proximity to latrines and sewage. Irish families are defined by a household with a head born in Ireland. Optimal bandwidth determined as in **?** and using a triangular weighting kernel. Optimal band refers to a specification that uses the optimal bandwidth used in the Local Linear specification. Wide Band refers to a specification that uses a bandwidth of 100m around the BSP boundary (this bandwidth encompasses almost all observations within the BSP area). Census data acquired from The National Archives of the UK: Public Record Office. Number of observations in Panel B, Column (6) drop because of lack of data on total number of heads of households to calculate the ratio.

Table SA5: Boundary Effects on House Prices, Zoopla House Value Estimates, and Rental Prices 1995-2013, 2015, Restricted Sample

	House Prices and Zoopla Estimates			Zoopla Estimates Only			House Rental Prices		
		Polynomial RD			Polynomial RD			Polynomial RD	
	LLR	Optimal Band	Wide Band	LLR	Optimal Band	Wide Band	LLR	Optimal Band	Wide Band
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Inside BSP area	-0.290	-0.383	-0.249	-0.314	-0.293	-0.224	-0.290	-0.099	-0.276
	(0.097)	(0.215)	(0.192)	(0.125)	(0.149)	(0.098)	(0.227)	(0.077)	(0.146)
Observations	173	163	592	113	104	263	119	111	215
Mean Outside BSP	1005107.9	1005107.9	1151267.1	1049298.3	1049298.3	1308226.7	758.7	758.7	802.9
Bandwidth (meters)	26.60	24.15	100	34.86	30.75	100	49.10	41.15	100

Note: LLR refers to Local Linear Regression. Sample in LLR specifications is restricted to be within an optimal bandwidth determined as in **?** and using a triangular weighting kernel. Sample excludes areas with high density of houses near the BSP boundary. All specifications include a set of determinants of house values as additional controls. Polynomial RD specifications use a polynomial in distance to the BSP boundary. Order of polynomial determined using Akaike's criteria as suggested in Black et. al (2003). Optimal band refers to a specification that uses the optimal bandwidth used in a Local Linear specification without controls. Wide Band refers to a specification that uses a bandwidth of 100m around the BSP boundary (this bandwidth encompasses almost all observations within the BSP area). Clustered standard errors by postal code are shown in parenthesis.

Online Appendix B Proofs

Label apartments in the block $i = 1, ..., n$. Define a history at a negotiation opportunity at period $t \in \mathbb{Z}_+$, denoted by h_t, as a list comprising of the time, the apartment label and the negotiation outcome (previous tenant retained, new poor tenant hired, new rich tenant hired) for realized renegotiation opportunities preceding period t, plus the apartment label for the renegotiation opportunity at t. Let H_t be the set of all time t histories as above, and let $H = \underset{t \in \mathbb{Z}_+}{\cup} H_t$. Strategies of the landlord are defined as mappings from H to {poor,rich} (where it is implicitly assumed that action poor means retaining the previous tenant if her type is poor and hiring a new poor tenant otherwise; similarly action rich means retaining the old tenant if her type is rich and hiring a new rich tenant otherwise). For every $h \in H$, let $x(h)$ be the current number of poor tenants in other apartments at the time of the negotiation associated with h.

We assume that agreed upon rents are determinded by the landlord's strategy, through the maximum rent the chosen tenant is willing to pay, given the landlord's strategy. Thus, we assume that tenants correctly foresee the landlord's actions in the future, and that they have correct expectations on how the composition of the block changes over time. The landlord chooses a strategy maximizing his expected discounted rent revenue. An alternative, and simpler way of thinking about the landlord's strategies is the following. Let $\mathcal{T}$ be the set of all possible sequences of negotiation opportunities over time, with each member of the sequence indicating the time and apartment label of the negotiation. A typical $\sqcup \in \mathcal{T}$ is of the form $(t_0, i_0), (t_1, i_1), ...$ where $t_0 = 0$ and i_0 is the label of the initially vacant apartment. We refer to (t_k, i_k) as the kth negotiation in the sequence. Then we can define the landlord's strategy as a mapping that for every negotiation of every possible sequence in $\mathcal{T}$ allocates an action from {poor,rich}, in a way that if $\sqcup, \sqcup' \in \mathcal{T}$ are such that $(t_l, i_l) = (t'_l, i'_l)$ for $l = 1, ..., k$ then the action allocated to the kth negotiation has to be the same for the two sequences (actions can only be conditioned on past events, not on future ones). Defining strategies this way has the convenient feature that the set of strategies the same for different initial compositions of tenants. In particular, given two different histories h and h', and a continuation strategy s in the game starting at h, we can define a sequence-equivalent strategy s' in the game starting at h' as a strategy allocating the same action as s to every negotiation of every possible negotiation sequence.

Lemma 1: Let $h \in H$ and relabel apartments in the game starting at h' such that every apartment having a poor tenant at h also has a poor tenant at h'. Let s be any strategy in the game starting at h and let s' be a sequence equivalent strategy to s in the game starting with h'. Then the payoff that s yields to the landlord given h is weakly lower than the payoff s' yields given h'.

Proof: Since $x(h) \geq x(h')$ and s' is a sequence equivalent strategy to s, for any sequence of negotiations $\sqcup$ the number of poor tenants under s is weakly higher than under s'. Hence, at any future negotiation newly hired tenants expect in any future period weakly higher number of poor neighbors under s and are ready to pay weakly lower rent. As a result, the payoff that s yields to the landlord given h is weakly lower than the payoff s' yields given h'.

Theorem 1: The landlord always has an optimal strategy of the following form: there is $x^* \in \{0, ..., n-1\}$ such that at every history $h \in H$, if $x(h) \leq x^*$ then choose rich, and if $x(h) > x^*$ then choose poor.

Proof: To simplify notation below, denote the initial history, at $t = 0$, simply as h in this proof. First note that if $x(h) = 0$ then choosing rich at h and in all future negotiations is an optimal continuation strategy, as it results in the maximum possible negotiated wage (W^r) at every negotiation of the continuation game. Moreover, if $h' \in H$ is on the path of play given the landlord's continuation strategy at h, and $x(h') = 0$ then an optimal strategy has to choose rich at h' and at all successor histories on the path of play. This is because only those strategies can maximize the landlord's expected payoff given h', and at the same time maximize the rent for rich tenants retained/hired preceding h'.

Let x^* be largest number of initial poor tenants such that whenever $x(h) \leq x^*$, there exists an optimal strategy s given h such that rich is chosen at h. As shown above, the requirement holds for $x = 0$.

Assume $x^* \geq 1$ and consider $x(h) = 1$. Assume that the landlord is playing an optimal strategy which specifies acquiring a rich tenant at h. Note that for every immediate successor history h' of h, either $x(h') = 1$ or $x(h') = 0$. As shown above, in the latter case an optimal strategy of the landlord has to choose rich at h'. Next, for all h' such that $x(h') = 1$, change the continuation strategy that s specifies at h' to s itself (with the label of the negotiated apartment at h exchanged with the label of the negotiated apartment at h'). Since s is optimal at h, and the game starting at h' is equivalent (up to relabeling apartments) to the game starting at h, the new strategy s' is optimal conditional on h' and yields weakly higher continuation payoffs at every immediate successor h' of h. For now, fix the rich rent at h at the level it would be when s is played. Then s' with the old rent at h yields a weakly higher payoff for the landlord than s. Next, we can replace continuation strategies at all h'' that are immediate successors of h' that are immediate successors of h, with $x(h'') = 1$ to s. Analogous arguments as before establish that s'' is optimal conditional on h'' and yields weakly higher continuation payoffs at every immediate successor h'' of h' than s'. For now, keep rich rent levels agreed upon prior to h'' unchanged. Then s'' with the old rent levels prior to h'' yields a weakly higher payoff for the landlord than s'. Iterating the argument establishes that a continuation strategy that for any successor h' of h with $x(h) = 1$ chooses rich, fixing previous rich rents, yields a weakly higher payoff than s. Now revisit all the rents that were fixed at different steps of the iteration. Conditional on any history, the rich rent is maximized if landlord plays always rich strategy from that point on. Therefore all the rents fixed before can only increase. Hence, a continuation strategy that for any successor h' of h with $x(h) = 1$ chooses rich yields a weakly higher payoff than s, therefore it is optimal. Moreover, for any $h \in H$, there is an optimal strategy that for any h' that is a successor of h and satisfies $x(h') \in \{0, 1\}$, it specifies choosing rich at h', since the latter is the optimal continuation strategy at h' and among all continuation strategies at h', it maximizes the rent for rich tenants retained/hired preceding h'.

Iterating the previous argument establishes that there is an optimal strategy of the landlord, that for any h' that is a successor of h and statisfies $x(h') \in \{0, ..., \}$, specifies choosing rich at h'.

Assume next that in every optimal strategy s given h, poor is chosen at h (this in particular requires $x(h) > x^*$), but the always poor strategy is not optimal given h. Then there exists a successor $h' \in H$ such that for every history h'' preceding h' poor is chosen, but at h' rich is chosen. Note that s has to specify a continuation strategy at h' that is optimal given h', since at every history preceding h' a poor type is hired/retained, hence the rent obtained by the landlord is independent of the continuation strategy at h'. But below we show that it cannot be that s is optimal given both h and h', leading to a contradiction.

Let $W(x)$ be the expected discounted present value of all rents from rental agreements negotiated at or after time 0 when the initial number of poor tenants is x and the landlord chooses an optimal strategy.

$W(x)$ is the sum of the rent that is received from the tenant currently being hired plus the continuation utility received from future negotiated rents, given an optimal strategy. Assume that there is an optimal strategy for the owner to first hire a poor person, but $W(x)$ is greater than what he could get from an always poor strategy, which is equivalent to $W(x) > g^* = \frac{(1-\delta(1-q))W^p}{(1-\delta)(1-\delta(1-\frac{q}{n}))}$.

Then the continuation utility after current hire:

$$W(x) - \frac{W^p}{1-\delta(1-\frac{q}{n})} \leq \delta(1-q)\left(W(x) - \frac{W^p}{1-\delta(1-\frac{q}{n})}\right) + \\ + \delta q \frac{x+1}{n} W(x) + \delta q \left(1 - \frac{x+1}{n}\right) W(x+1)$$

From Lemma 1 we know that $W(x+1) > W(x)$ cannot be the case because if at the game starting with x poor the owner uses a sequence equivalent strategy to an optimal strategy of the game starting with $x+1$ poor, his payoffs (from noninitial rentors) are weakly higher. But if $W(x+1) \leq W(x)$, then from the inequality for the continuation utility we have $(1-\delta)W(x) \leq \frac{(1-\delta(1-q))W^p}{1-\delta(1-\frac{q}{n})}$ or, equivalently, $W(x) \leq g^*$, which contraddicts our assumption. This leads to a contradiction, establishing that $W(x) = g^*$ and if it is optimal to start with hiring a poor, then always poor must be an optimal strategy.

The above argument establishes that if in every optimal strategy s given h, poor is chosen at h then the always poor strategy is optimal given h. In particular, the always poor strategy is optimal if $x(h) = x^* + 1$ (provided $x^* < n-1$). Now assume that $x^* < n-2$, $x(h) = x^* + 2$, and there exists an optimal strategy s given h such that rich is chosen at h. But Lemma 1 establishes that for a history h' with $x(h') = x^* + 1$, the game starting at h' has a strategy that chooses rich at h', and yields a weakly higher expected payoff to the landlord than s does in the game starting at h. Moreover, note that the always poor strategy yields the same expected payoff to the landlord in both games. But then there exists an optimal strategy in the game starting at h' that chooses rich at h', contradicting the definition of x^*. Hence $x(h) = x^* + 2$ implies that there is an optimal strategy given h such that poor is chosen at every h' with $x(h') > x(h)$. Iterating the above argument establishes the same conclusion for any h such that $x(h) > x^*$.

Putting together the above-derived results yields that the strategy that specifies choosing rich at a history h' iff $x(h') \leq x^*$ is optimal given h, for any $x(h)$. ■

Note that the above optimal strategy of the landlord is optimal not only given h, but also given any successor history h'. Therefore the landlord does not need to be able to commit to follow the strategy – it is in his own interest to stick to it. Also note that the strategy implies either always retaining/hiring poor types or always retaining/hiring rich types, since if at the initial history a rich type is hired then the number of poor tenants is wekly lower at all subsequent negotiations, while if at the initial history a poor type is hired then the number of poor tenants is wekly lower at all subsequent negotiations.

Proof of Proposition 1

Consider a rich tenant, who pays r per period. If he realizes his outside option, he gets $V(out) = -\frac{W^r}{1-\delta}$. Let V_k denotes the expected continuation utility of a rich tenant renting an apartment for a general fixed r, given k current poor tenants, assuming that the landlord is following the always rich strategy. If the tenant has no poor neighbors, then next period three situations are possible: with probability $1-q$ no changes; with probability $q\frac{n-1}{n}$ one rich neighbour's contract expires; with probability $\frac{q}{n}$ the tenant's contract expires, in which case his continuation utility is equal to $V(out)$. Hence, we can write:

$$V_0 = -(r + c_0^r) + \delta[(1-q)V_0 + q\frac{n-1}{n}V_0 + \frac{q}{n}V(out)]$$
$$V_0 = \frac{-(r + c_0^r) + \delta\frac{q}{n}V(out)}{1-\delta(1-\frac{q}{n})} = \frac{-(r + c_0^r) - \frac{q}{n}\frac{\delta}{1-\delta}W^r}{1-\delta(1-\frac{q}{n})} \tag{SA2}$$

If the tenant has $k \geq 1$ poor neighbours, then next period four situations are possible: with probability $1 - q\frac{k+1}{n}$ no changes; with probability $q\frac{k}{n}$ one poor neighbour is replaced by a rich one; with probability $\frac{q}{n}x$ the tenant's contract expires and her continuation utility is equal to $V(out)$. Hence, we get:

$$V_k = -(r + c_k^r) + \delta[(1 - q\frac{k+1}{n})V_k + q\frac{k}{n}V_{k-1} + \frac{q}{n}V(out)] \tag{SA3}$$
$$V_k = \frac{\delta q\frac{k}{n}}{1-\delta(1-q\frac{k+1}{n})}V_{k-1} + \frac{\delta\frac{q}{n}V(out) - (r + c_k^r)}{1-\delta(1-q\frac{k+1}{n})}$$

Iterating, we obtain:

$$V_k = \frac{\delta\frac{q}{n}V(out) - r}{1-\delta(1-\frac{q}{n})} - \sum_{i=0}^{k}\frac{\frac{k!}{i!}(\delta\frac{q}{n})^{k-i}}{\prod_{j=i}^{k}(1-\delta(1-q\frac{j+1}{n}))}c_i^r$$

The apartment owner chooses rent r_x by making a rich tenant indifferent between renting and outside option: $V_x = V(out)$. Hence:

$$r_x = W_r - \left(1-\delta(1-\frac{q}{n})\right)\sum_{i=0}^{x}\frac{\frac{x!}{i!}(\delta\frac{q}{n})^{x-i}}{\prod_{j=i}^{x}(1-\delta(1-q\frac{j+1}{n}))}c_i = W_r - \sum_{i=0}^{x} a_{ix}c_i^r \tag{SA4}$$

$$a_{ix} = \left(1-\delta(1-\frac{q}{n})\right)\frac{\frac{x!}{i!}(\delta\frac{q}{n})^{x-i}}{\prod_{j=i}^{x}(1-\delta(1-q\frac{j+1}{n}))} \tag{SA5}$$

Consider the apartment owner, who has x poor tenants and follows the always rich strategy. His expected utility $U_{rich}(S_r, x)$ can be divided into the expected payoff from contacts agreed upon before time 0, $U_{curr}(S_r)$, and the expected payoff from contracts negotiated time 0 on, under the always rich strategy, f_x. The latter consists of the expected payoff from the time 0 contract and the expected payoff from all future contracts, denoted by h_x.

$$U_{rich}(S_r, x) = U_{curr}(S_r) + f_x = \frac{S_r}{1-\delta(1-\frac{q}{n})} + \frac{r_x}{1-\delta(1-\frac{q}{n})} + h_x$$

$$f_0 = \frac{W^r - c_0^r}{1-\delta(1-\frac{q}{n})} + \sum_{i=1}^{\infty}\delta^i q\frac{W^r - c_0^r}{1-\delta(1-\frac{q}{n})} = \frac{(1-\delta+\delta q)(W^r - c_0^r)}{(1-\delta)(1-\delta(1-\frac{q}{n}))}$$

As there are $k \geq 1$ poor tenants in the current period, then next period with probability $q(1-\frac{k}{n})$ a rich tenant's rent gets renegotiated to r_k, and with probability $q\frac{k}{n}$ a rich tenant replaces a poor one with a negotiated rent r_{k-1}. Therefore:

$$h_k = \delta[(1-q)h_k + q(1-\frac{k}{n})f_k + q\frac{k}{n}f_{k-1}]$$

$$(1-\delta(1-q))\left(f_k - \frac{r_k}{1-\delta(1-\frac{q}{n})}\right) = \delta q(1-\frac{k}{n})f_k + \delta q\frac{k}{n}f_{k-1}$$

$$f_k = \frac{\delta q\frac{k}{n}}{1-\delta(1-q\frac{k}{n})}f_{k-1} + \frac{1-\delta(1-q)}{(1-\delta(1-\frac{q}{n}))(1-\delta(1-q\frac{k}{n}))}r_k$$

Solving the difference equation, we get:

$$f_k = \frac{1-\delta+\delta q}{(1-\delta)(1-\delta(1-\frac{q}{n}))}W^r - \sum_{i=0}^{k} b_{ik}c_i \tag{SA6}$$

$$b_{ik} = (1-\delta(1-q))\frac{\frac{k!}{i!}(k+1-i)(\delta\frac{q}{n})^{k-i}}{\prod_{m=i}^{k+1}(1-\delta(1-q\frac{m}{n}))} \tag{SA7}$$

Now consider the always poor strategy. The owner's expected utility U_{poor} can be divided into expected payments from current contracts negotiated before time 0, $U_{curr}(S_r)$, and the expected payments from contracts negotiated at time 0 on when the landlord is playing the always poor strategy, denoted by g.

$$\begin{aligned} U_{poor}(S_r, x) &= U_{curr}(S_r) + g \\ g &= \frac{(1-\delta+\delta q)}{(1-\delta)(1-\delta(1-\frac{q}{n}))} W^p \end{aligned} \tag{SA8}$$

We can conclude that the apartment owner, having x poor tenants, prefers the always rich strategy to the always poor strategy if $f_x > g$ or, equivalently,

$$W^r - W^p > (1-\delta)\left(1-\delta(1-\frac{q}{n})\right)\left[\sum_{i=0}^{x} \frac{\frac{x!}{i!}(x+1-i)(\delta\frac{q}{n})^{x-i}}{\prod_{m=i}^{x+1}(1-\delta(1-q\frac{m}{n}))} c_i^r\right]$$

Online Appendix C Additional Model Extensions

Investments/maintenance

Consider the situation, when the owner has already got all n rich tenants and deviates to low investment for now (and then he is expected to invest low forever). Then he gains k in the current period, but next period with probability q he signs a contract with a rent $W^r - c_0^r - d$, which is by d lower than in the case of high investment strategy, therefore his expected loss from signing a worse contract is $\frac{d}{1-\delta(1-\frac{q}{n})}$. Hence a condition for high investment with two rich tenants is:

$$k - \delta q \frac{d}{1-\delta(1-\frac{q}{n})} \leq 0$$

Now assume this inequality holds and, moreover, the owner with $m-1$ poor tenants always invests high. Now consider the situation, when the owner still has m poor tenants and deviates to low investment until he has $m-1$ poor tenants. Then the landlord gains k in current period, next period with probability $\frac{m}{n}q$ he signs a contract with a new rich tenant with a rent $W^r - c_0^r - (1-\delta(1-\frac{q}{n}))d$ (a new rich tenant knows that her losses are just for one period, because after hiring her the landlord switches to high investment); with probability $(1-\frac{m}{n})q$ the owner renews a contract with an existing rich tenant. The tenant knows that she will face additional disutility d, while having m poor neighbours, that is equivalent to increasing c_m^r by d. Also if she moves into the state with $m-1$ poor she still faces d, but only for 1 period, which is equivalent to increasing c_{m-1}^r by $(1-\delta(1-q\frac{m}{n}))d$. Hence, from (6) we get that r_m decreases by $\frac{(1+\delta\frac{m}{n}q)(1-\delta+\delta\frac{q}{n})}{1-\delta+\delta\frac{m+1}{n}q}d$.

Hence a condition for high investment with m poor tenants is:

$$k - \delta\left[\frac{m}{n}qd + (1-\frac{m}{n})q\frac{1+\delta\frac{m}{n}q}{1-\delta+\delta\frac{m+1}{n}q}d\right] \leq 0$$

or, equivalently,

$$\frac{k}{d} \leq \frac{\delta\frac{q}{n}[n - m\delta + \frac{m(n+1)}{n}\delta q]}{1-\delta+\frac{m+1}{n}\delta q}$$

Poor types also willing to pay premium for rich neighbors

As long as we assume that $c_k^r - c_{k-1}^r > c_k^p - c_{k-1}^p$ for every $k \in \{1, ..., n-1\}$, that is the marginal willingness to pay to reduce the number of poor neighbors is always higher for rich types than for poor types, the result that either the always poor or the always rich strategy is optimal continues to hold. In the Online Appendix we derive the conditions in this extended model for the optimality of the always rich versus the always poor strategy. Fixing all other parameters, increasing any of the cost parameters c^p_k for $k \in \{x, ..., n-1\}$ decreases the payoffs from the always poor strategy, while not affecting the payoffs from the always rich strategy. In the case of $n = 2$, the condition for always rich being an optimal strategy is the following simple modification of the original condition:

$$W^r - W^p > \frac{\delta q}{1 - \delta + \delta q} c^r_0 + \frac{1 - \delta}{1 - \delta + \delta q} c^r_1 - c^p_1.$$

Here the payoff under the always rich strategy does not change. Consider the always poor strategy. Reinterpret costs c^p_k that poor tenants face from having k poor neighbours as costs d_{n-1-k} from having $n-1-k$ rich neighbours, so $d_i = c^p_{n-1-i}$. Then all formulas for the always rich strategy can be rewritten, if we redefine state as a number of rich tenants, use W^p instead of W^r and costs d_i instead of c_i.

$$\begin{aligned} U_{poor}(S_r, x) &= U_{current}(S_r) + g_{n-1-x} \\ g_k &= \frac{(1 - \delta + \delta q)}{(1-\delta)(1 - \delta(1 - \frac{q}{n}))} W^p - \sum_{i=0}^{k} b_{ik} d_i = \\ &= \frac{(1 - \delta + \delta q)}{(1-\delta)(1 - \delta(1 - \frac{q}{n}))} W^p - \sum_{j=n-1-k}^{n-1} b_{n-1-j,k} c^p_j, \end{aligned}$$

where b_{ik} are taken from (7).

We can conclude that a landlord, having x poor tenants prefers the always rich strategy to the always poor one if $f_x > g_x$ or, equivalently,

$$\begin{aligned} W^r - W^p > (1-\delta)\left(1 - \delta(1 - \frac{q}{n})\right) \sum_{i=0}^{x} \frac{\frac{x!}{i!}(x+1-i)(\delta \frac{q}{n})^{x-i}}{\prod\limits_{m=i}^{x+1} (1 - \delta(1 - q\frac{m}{n}))} c^r_i - \\ - (1-\delta)\left(1 - \delta(1 - \frac{q}{n})\right) \sum_{j=x}^{n-1} \frac{\frac{(n-1-x)!}{(n-1-j)!}(j+1-x)(\delta \frac{q}{n})^{j-x}}{\prod\limits_{m=n-1-j}^{n-x} (1 - \delta(1 - q\frac{m}{n}))} c^p_j \end{aligned}$$

Multiple owners

First note that since $W^r - c^r_1 < W^p$, it is always better for an owner to acquire a poor tenant if the other apartment has a poor tenant and the belief is that the other apartment owner plays an always poor strategy. Assume now that the owner of apartment 1 has a vacancy when the other apartment currently has a rich tenant, but he believes that the other apartment owner from now on will follow the always poor strategy. We call the owner's strategy initially rich if he acquires a rich tenant in such situations, and only.

If the owner chooses the always poor strategy, then he receives W^p in every period and $U_{poor} = \frac{W^p}{1-\delta}$.

Now we concentrate on the initially rich strategy. Consider a rich tenant, who pays rent r per period. If her contract gets renegotiated, she gets $V(out) = -\frac{W^r}{1-\delta}$. Her expected utility from renting depends on her current neighbour's type.

$$V(poor) = -(r + c_1^r) + \delta[(1-q)V(poor) + \frac{q}{2}V(poor) + \frac{q}{2}V(out)]$$

$$V(poor) = \frac{-(r + c_1^r) + \delta\frac{q}{2}V(out)}{1 - \delta(1 - \frac{q}{2})} = \frac{-(r + c_1^r) - \frac{q}{2}\frac{\delta}{1-\delta}W^r}{1 - \delta(1 - \frac{q}{2})}$$

$$V(rich) = -(r + c_0^r) + \delta[(1-q)V(rich) + \frac{q}{2}V(poor) + \frac{q}{2}V(out)]$$

$$V(rich) = \frac{-r + \delta\frac{q}{2}V(out)}{1 - \delta(1 - \frac{q}{2})} - \frac{1}{1 - \delta(1-q)}c_0^r - \frac{\delta\frac{q}{2}}{(1 - \delta(1-q))(1 - \delta(1 - \frac{q}{2}))}c_1^r$$

The apartment owner chooses rent r^* by making the tenant indifferent between renting and outside option: $V(rich) = V(out)$. Hence:

$$\frac{-r^* + \delta\frac{q}{2}V(out)}{1 - \delta(1 - \frac{q}{2})} - \frac{1}{1 - \delta(1-q)}c_0^r - \frac{\delta\frac{q}{2}}{(1 - \delta(1-q))(1 - \delta(1 - \frac{q}{2}))}c_1^r = V(out)$$

$$r^* = W^r - \frac{1 - \delta + \frac{1}{2}\delta q}{1 - \delta + \delta q}c_0^r - \frac{\frac{1}{2}\delta q}{1 - \delta + \delta q}c_1^r \tag{SA9}$$

The apartment owner prefers the initially rich strategy to the always poor strategy if $r^* > W^p$, which is equivalent to:

$$W^r - W^p > \frac{1 - \delta + \frac{1}{2}\delta q}{1 - \delta + \delta q}c_0^r + \frac{\frac{1}{2}\delta q}{1 - \delta + \delta q}c_1^r$$

This condition guarantees that (rich,rich) becomes an absorbing state.

No price discrimination

First consider the always poor strategy. If there is a vacancy and a poor tenant, the apartment owner sets a price W^p. As $r^* < W^p$, only poor type tenants apply. Then as we know from the proof of Proposition 1:

$$U_{poor} = \frac{r_0}{1-\delta+\delta\frac{q}{2}} + \frac{1-\delta+\delta q}{(1-\delta)(1-\delta+\delta\frac{q}{2})}W^p$$

Now consider the always rich strategy of the apartment owner. If there is a vacancy and a rich tenant, the owner sets a price $W^r - c_0^r$ and only rich types apply. If there is a vacancy and a poor tenant in the other apartment, then the owner sets a price r^*, which attracts both types of tenants. Under the always rich strategy the apartment owner's expected continuation payoff depends on the current state, which is determined by the current tenants' rents (we also use an index to distinguish between rich and poor types, who both can pay r^*).

$$U(W^r - c_0^r, W^r - c_0^r) = \frac{2(W^r - c_0^r)}{1-\delta}$$
$$U(W^r - c_0^r, r^*_{rich}) = \frac{r^*}{1-\delta+\delta\frac{q}{2}} + \frac{1-\delta+\delta q}{(1-\delta)(1-\delta+\delta\frac{q}{2})}(W^r - c_0^r)$$

First consider states with r^*_{rich}:

$$U(r^*_{poor}, r^*_{rich}) = 2r^* + \delta\left[\left(1-\frac{q}{2}\right)U(r^*_{poor}, r^*_{rich}) + \frac{q}{2}U(W^r - c_0^r, r^*_{rich})\right]$$
$$U(r^*_{poor}, r^*_{rich}) = \frac{2r^* + \delta\frac{q}{2}U(W^r - c_0^r, r^*_{rich})}{1-\delta+\delta\frac{q}{2}}$$
$$U(r_0, r^*_{rich}) = (r_0 + r^*) + \delta\left[\left(1-\frac{q}{2}\right)U(r_0, r^*_{rich}) + \frac{q}{2}U(W^r - c_0^r, r^*_{rich})\right]$$
$$U(r_0, r^*_{rich}) = \frac{r_0 + r^* + \delta\frac{q}{2}U(W^r - c_0^r, r^*_{rich})}{1-\delta+\delta\frac{q}{2}}$$

Further consider (r^*_{poor}, r^*_{poor}):

$$U(r^*_{poor}, r^*_{poor}) = 2r^* + \delta(1-q)U(r^*_{poor}, r^*_{poor}) +$$
$$+\delta q[\pi U(r^*_{poor}, r^*_{rich}) + (1-\pi)U(r^*_{poor}, r^*_{poor})]$$
$$U(r^*_{poor}, r^*_{poor}) = \frac{2(1-\delta)+\delta q(1+2\pi)}{(1-\delta+\delta\frac{q}{2})(1-\delta+\delta q\pi)}r^* +$$
$$+\frac{\delta^2\frac{q^2}{2}\pi}{(1-\delta+\delta\frac{q}{2})(1-\delta+\delta q\pi)}U(W^r - c_0^r, r^*_{rich})$$

Next, find the owner's utility at (r_0, r^*_{poor}):

$$U(r_0, r_p^*) = (r_0 + r^*) + \delta\,(1-q)\,U(r_0, r_p^*) +$$
$$+\delta\frac{q}{2}[\pi U(r_r^*, r_p^*) + (1-\pi)U(r_p^*, r_p^*)] + \delta\frac{q}{2}[\pi U(r_0, r_r^*) + (1-\pi)U(r_0, r_p^*)]$$
$$U(r_0, r_p^*) = \frac{r_0}{1-\delta+\delta\frac{q}{2}} + \frac{1-\delta+\delta q(1+\pi)}{(1-\delta+\delta\frac{q}{2})(1-\delta+\delta q\pi)}r^* +$$
$$+\frac{\delta^2\frac{q^2}{2}\pi}{(1-\delta+\delta\frac{q}{2})(1-\delta+\delta q\pi)}U(W^r - c_0^r, r^*_{rich})$$

The apartment owner's expected utility under the always rich strategy $U_{rich} = \pi U(r_0, r^*_{rich}) + (1-\pi)U(r_0, r^*_{poor})$, hence:

$$U_{rich} = \frac{r_0}{1-\delta+\delta\frac{q}{2}} + \frac{1-\delta+\delta q}{(1-\delta+\delta\frac{q}{2})(1-\delta+\delta q\pi)}r^* +$$
$$+\frac{\delta\frac{q}{2}\pi(1-\delta+\delta q)}{(1-\delta+\delta\frac{q}{2})(1-\delta+\delta q\pi)}U(W^r - c_0^r, r^*_{rich}) =$$
$$= \frac{r_0}{1-\delta+\delta\frac{q}{2}} + \frac{1-\delta+\delta q}{(1-\delta)(1-\delta+\delta\frac{q}{2})}W^r -$$
$$-\frac{\delta\frac{q}{2}[(1-\delta)(1+\pi)+2\delta q\pi]}{(1-\delta)(1-\delta+\delta\frac{q}{2})(1-\delta+\delta q\pi)}c_0^r - \frac{1-\delta+\delta\frac{q}{2}(1+\pi)}{(1-\delta+\delta\frac{q}{2})(1-\delta+\delta q\pi)}c_1^r$$

The always rich strategy gives higher expected payoff than the always poor ($U_{rich} > U_{poor}$), if:

$$W^r - W^p > \frac{\delta\frac{q}{2}[(1-\delta)(1+\pi)+2\delta q\pi]}{(1-\delta+\delta q)(1-\delta+\delta q\pi)}c_0^r + \frac{(1-\delta)(1-\delta+\delta\frac{q}{2}(1+\pi))}{(1-\delta+\delta q)(1-\delta+\delta q\pi)}c_1^r$$

Gentrification

Here we extend the baseline model to include four types of prospective renters: poor, middle-class, rich and very rich. Their outside options are correspondingly $-W^p$, $-W^m$, $-W^r$, $-W^v$ per period, where $W^p < W^m < W^r < W^v$. Let c_i^m be the cost to a middle-class tenant of having i poor neighbors, and assume it is increasing in i. Also assume that $W^r - c_{n-1}^m < W^p$, but $W^r - c_0^m > W^p$. Let $c_{i,j}^r$ be the cost for a rich tenant of having i poor neighbors and j middle-class neighbors, and let $c_{i,j}^r$ be increasing in both i and j. Furthermore, assume that if $i + j = i' + j'$ and $i > i'$ then $c_{i,j}^r > c_{i',j'}^r$. Let $c_{i,j,k}^v$ be the cost for a very rich tenant imposed by having i poor neighbors, j middle-class neighbors and k rich neighbors, and let $c_{i,j,k}^v$ be increasing in i , j and k. Assume also that if $i + j = i' + j''$ and $i > i'$, then $c_{i,j,k}^v > c_{i',j',k}^v$ and if $j + k = j' + k'$ and $j > j'$, then

$c^v_{i,j,k} > c^v_{i,j',k'}$. Lastly, assume that $W^p < W^m - c^m_0 < W^r - c^r_{0,0} < W^v - c^v_{0,0,0}$. Intuitively, these assumptions imply that all types are willing to pay a premium to avoid having neighbors of lower type, and higher types have a higher willingness to pay.

1) Consider the situation that when having all current poor tenants, the apartment owner prefers hiring poor tenants now and in the future when there is a vacancy to always hiring middle types, always hiring rich and always hiring very rich types.

From Proposition 1 we know that this happens if correspondingly:

$$W^m - W^p < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))} c^m_i$$

$$W^r - W^p < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))} c^r_{i,0}$$

$$W^v - W^p < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))} c^v_{i,0,0}$$

Assume also that, when having all current rich tenants, the apartment owner prefers hiring rich tenants now and in the future to always hiring very rich tenants. That happens if:

$$W^v - W^r < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))} c^v_{0,0,i} - c^r_{0,0}$$

2) Now suppose that the block becomes more attractive relative to other blocks, in that all types are willing to pay a higher rent than before. But differentially so: poor types by d_p, middle types by d_m, rich types by d_r and very rich by d_v, with $d_v > d_r > d_m > d_p$.

Then, having all current rich tenants, the apartment owner changes his behavior and starts hiring very rich tenants now and in the future if:

$$W^v - W^r < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))} c^v_{0,0,i} - c^r_{0,0} <$$
$$< W^v - W^r + d_v - d_r$$

Also after the change in block attractiveness, having all current poor tenants, the apartment owner prefers hiring middle tenants now and in the future when there is a vacancy, but doesn't prefer always hiring rich or very rich ones if:

$$W^m - W^p < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod\limits_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))} c_i^m <$$

$$< W^m - W^p + d_m - d_p$$

$$W^r - W^p + d_r - d_p < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod\limits_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))} c_{i,0}^r$$

$$W^v - W^p + d_v - d_p < \sum_{i=0}^{n-1} \frac{(1-\delta)\frac{(n-1)!}{i!}(n-i)(\delta\frac{q}{n})^{n-1-i}(1-\delta(1-\frac{q}{n}))}{\prod\limits_{m=i}^{n}(1-\delta(1-q\frac{m}{n}))} c_{i,0,0}^v$$

Therefore, there exists a parameter range for which originally both an all poor and an all rich block are stable, and after the increase in the attractiveness of the district, the poor block transitions to a middle-class one, while the rich block transforms to a very rich one.