Online Appendix for "Going Negative at the Zero Lower Bound"

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This set of online appendices complement the paper "Going Negative at the Zero Lower Bound: The Effects of Negative Nominal Interest Rates". Apenddix A provide additional results and proofs related to banks. Appendix B provides additional empirical results. Appendix C provides the equilibrium equations and steady state of the full DSGE model. Finally, Appendix D provides a couple of additional figures.

Appendix A Bank Related Derivations

A.1 Loan Market

Here I solve the problem of an agent (in the model described in Section III it would represent a firm) that has to decide how much to borrow from each bank subject to a CES constraint. As discussed by GNSS, and in Appendix A.3, there are several ways to justify the CES constraint, some of them are: switching costs, asymmetric information, menu costs and regulatory restrictions. Agent s seeks a total amount of real loans equal to $l_t(s)$, he borrows an amount $l_t(s,j)$ from each bank j and faces the following constraint:

$$\left[\int_0^1 l_t(s,j)^{(\varepsilon_t^l-1)/\varepsilon_t^l} dj\right]^{\varepsilon_t^l/(\varepsilon_t^l-1)} \geq l_t(s),$$

which indicates that the loans he gets from individual banks are aggregated via a CES aggregator into the total loans he obtains. ε_t^l is the elasticity of substitution between banks, which for now I allow to vary with time. This elasticity will be assumed to be greater than one, as traditional in the monopolistic competition framework. Each bank charges the agent a net interest rate $i_t^l(j)$. Demand for this agent can be derived from minimizing over $l_t(s,j)$ the total repayment (including principal) due to the continuum of banks j:

$$\int_0^1 (1+i_t^l(j)) l_t(s,j) dj,$$

subject to the constraint given above. I assume that loan customers minimize total repayment to banks $(1+i^l)$ instead of net interest payments (i^l). Total payments are more suitable that interest payments, since in the full dynamic model there is a time difference between when loans are taken out and when they are repaid. The gross formulation also leads to expressions that are a lot more suitable in a ZLB context.

The Lagrangian for this problem is:

$$\mathcal{L} = \int_0^1 (1+i_t^l(j)) l_t(s,j) dj - \lambda\left(\left[\int_0^1 l_t(s,j)^{(\varepsilon_t^l-1)/\varepsilon_t^l} dj\right]^{\varepsilon_t^l/(\varepsilon_t^l-1)} - l_t(s)\right).$$

The F.O.C. w.r.t. $l_t(s,j)$ yields:

$$l_t(s,j) = \left(\frac{1+i_t^l(j)}{\lambda}\right)^{-\varepsilon_t^l} l_t(s),$$

where:

$$\lambda \quad = \quad \left(\int_0^1 (1+i_t^l(j))^{1-\varepsilon_t^l} dj\right)^{\frac{1}{1-\varepsilon_t^l}}.$$

So, denoting $1+i_t^l \equiv \lambda$, one can write demand for a particular bank j coming from client s as:

$$l_t(s,j) = \left(\frac{1+i_t^l(j)}{1+i_t^l}\right)^{-\varepsilon_t^l} l_t(s).$$

A.2 Deposit Market

Now I analyze the problem of an agent that instead of needing to borrow from banks wants to lend to banks via deposits (this will represent households in the full model). Assume that households want to maximize total repayment from deposits subject to total deposits (as aggregated through a CES aggregator) being smaller or equal to the amount available to deposit. In this case demand by agent s seeking an amount of real deposits equal to $d_t(s)$ can be derived from maximizing over $d_t(s,j)$ the total amount obtained from the continuum of banks j:

$$\int_0^1 (1+i_t^d(j))d_t(s,j)dj,$$

subject to:

$$\left[\int_0^1 d_t(s,j)^{(\varepsilon_t^d-1)/\varepsilon_t^d} dj\right]^{\varepsilon_t^d/(\varepsilon_t^d-1)} \leq d_t(s),$$

where the elasticity ε_t^d is required to be smaller than -1, which means that the exponent of the terms inside the integral is greater than one, implying this function is convex. The Lagrangian for this problem is:

$$\mathcal{L} = \int_0^1 (1+i_t^d(j))d_t(s,j)dj - \lambda\left(\left[\int_0^1 d_t(s,j)^{(\varepsilon_t^d-1)/\varepsilon_t^d} dj\right]^{\varepsilon_t^d/(\varepsilon_t^d-1)} - d_t(s)\right).$$

This is exactly the same problem as in the loan case, which means that the solution will be the same. In this case the solution can be written as:

$$d_t(s,j) = \left(\frac{1+i_t^d(j)}{1+i_t^d}\right)^{-\varepsilon_t^d} d_t(s).$$

But remember that now ε_t^d is negative (in particular smaller than -1), which means that customers put more deposits in a particular bank the higher that bank's deposit rate is.

A.3 Microfoundation of Bank CES

Here I provide simple microfoundations for the use of a CES aggregator across individual banks. One possible objection to such a setup could be that it is implausible that all consumers borrow from all banks, since a more accurate representation of reality probably is that each consumer borrows from one (or at most two) banks. Here I show how a model where each consumer chooses to borrow from a single bank and is subject to an stochastic utility of borrowing from each bank can deliver the same demand for loans as the one that emerges from the CES approach. The different stochastic utilities across individuals of borrowing from specific banks can represent proximity, switching costs, tastes, or asymmetric information. The presentation is inspired by Anderson et al. (1988) and Anderson et al. (1989).

Assume that there is an individual consumer that lives for two periods, denoted 0 and 1. This consumer has a total income of $\overline{Y}$ in the second period and he can consume in both periods. To consume in period 1 is easy for this consumer, he can do it directly, but to consume in period 0 he most borrow against his future income $\overline{Y}$ through one of a continuum of banks between zero and one (indexed with j). The decision process of this consumer happens in two stages. In the first stage, the consumer decides which bank he wants to borrow from, and in the second stage he chooses the amount he wants to borrow. Suppose that the outcome of the first stage is that the consumer decides to borrow from bank j. Assume that the direct utility function of the consumer conditional on his choice of bank j is:

$$U(C_{0j}, C_1) = \ln(C_{0j}) + \beta \ln(C_1),$$

Where β is the discount factor between periods. The first period, second period, and aggregate budget constraints of the consumer (again conditional on the choice of bank j) are:

$$\begin{aligned} C_{0j} &= B_j \\ C_1 &= \overline{Y} - (1 + i^l_j) B_j \\ (1 + i^l_j) C_{0j} + C_1 &= \overline{Y}, \end{aligned}$$

where $1 + i^l_j$ is the interest rate charged between periods 0 and 1 by bank j (which is known by the consumer with certainty). The solution to this problem is:

$$C_{0j} = \frac{\overline{Y}}{1+\beta} \frac{1}{1+i^l_j}, \qquad C_1 = \frac{\beta}{1+\beta} \overline{Y},$$

and the indirect utility function conditional on borrowing from bank j is:

$$v_j = (1+\beta)(\ln(\overline{Y}) - \ln(1+\beta)) + \beta \ln(\beta) - \ln(1 + i^l_j).$$

Then, as in Anderson et al. (1988), assume that the first stage (the bank choice stage), is described by a stochastic utility approach:

$$V_j = v_j + \mu \varepsilon_j,$$

where μ is a positive constant and ε_j is a random variable with zero mean and unit variance. Assuming that the ε_j random variables are independently and identically distributed with type-1 extreme value distribution (Gumbel), then the probability for a consumer of choosing bank j is given by:

$$Pr(j) = Pr(V_j = \max_r V_r) = \frac{e^{v_j/\mu}}{\int_0^1 e^{v_r/\mu} dr} = \frac{(1+i_j^l)^{-\frac{1}{\mu}}}{\int_0^1 (1+i_r^l)^{-\frac{1}{\mu}} dr},$$

as in McFadden (1973). Substituting $1/\mu$ for $\varepsilon^l - 1$ (which is positive since $\varepsilon^l > 1$) the previous expression can be rewritten as:

$$Pr(j) = \frac{(1+i_j^l)^{1-\varepsilon^l}}{\int_0^1 (1+i_r^l)^{1-\varepsilon^l} dr} = \left(\frac{1+i_j^l}{1+i^l}\right)^{1-\varepsilon^l},$$

where i^l is the aggregate loan rate defined in Appendix A.1. Multiplying B_j by this probability one obtains:

$$B_j Pr(j) = \frac{\overline{Y}}{1+\beta}\frac{1}{1+i^l}\left(\frac{1+i_j^l}{1+i^l}\right)^{-\varepsilon^l}.$$

Interpret $\frac{\overline{Y}}{1+\beta}\frac{1}{1+i^l}$ as aggregate borrowing and denote it L. Additionally, interpret $B_j Pr(j)$ as the amount borrowed from bank j once the whole population of consumers are taken into account, and denote this by L_j. Then:

$$L_j = \left(\frac{1+i_j^l}{1+i^l}\right)^{-\varepsilon^l} L,$$

which is the same expression that one obtains directly from the CES aggregator. This shows that a heterogeneous borrower approach with stochastic utility and extreme value shocks works as a microfoundation for the CES aggregator in the case of a homogeneous borrower. A similar process can be followed to microfound deposit supply.

A.4 Solution to the Simple Bank Problem

Recall the following maximization problem for an individual bank j (which is one out of a continuum of identical banks between zero and one) described in Section I:

$$\begin{aligned}
\max_{i_j^l, L_j, i_j^d, D_j, H_j} \quad & (1+i_j^l)L_j + (1+i)H_j - (1+i_j^d)D_j \\
L_j &= \left(\frac{1+i_j^l}{1+i^l}\right)^{-\varepsilon^l} L \\
D_j &= \begin{cases} \left(\frac{1+i_j^d}{1+i^d}\right)^{-\varepsilon^d} D & \text{if } i_j^d \geq 0 \\ 0 & \text{if } i_j^d < 0 \end{cases} \qquad (1) \\
L_j + H_j &= F_j + D_j
\end{aligned}$$

$$H_j \geq 0.$$

Proposition 1: *Consider the bank problem described in equation (1), additionally assume that* $\varepsilon^l > 1$, $\varepsilon^d < -1$, *and* $D > L > F$. *The solution is described by several regimes that apply depending on the level of the policy rate* i. *Regime 1 applies when* $i \geq \tilde{\imath}$, *Regime 2 applies when* $\underline{i} \leq i < \tilde{\imath}$, *Regime 3A applies when* $\underline{\underline{i}} \leq i < \underline{i}$, *and Regime 3B applies when* $i < \underline{\underline{i}}$. *The thresholds are given by:*

$$\begin{aligned}
\tilde{\imath} &= -\frac{1}{\varepsilon^d} > 0 \\
\underline{i} &= \frac{\left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1} - \frac{1}{\varepsilon^l - 1}\frac{L}{F} - 1}{1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{D}{F} - \left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}} < 0 \\
\underline{\underline{i}} &= \frac{\left(1 + \frac{D}{F}\right)^{\frac{1}{\varepsilon^l}} - 1 - \frac{D}{\varepsilon^l F}}{1 + \frac{D}{F} - \left(1 + \frac{D}{F}\right)^{\frac{1}{\varepsilon^l}}} < \underline{i},
\end{aligned}$$

Regime 1: *In this regime all banks obtain an amount of deposits* $D_j = D$, *lend out an amount* $L_j = L$, *and hold an amount of reserves* $H_j = F + D - L > 0$. *All banks set the same loan rate and deposit rate, whose expressions are given by:*

$$1 + i_j^l = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i), \qquad 1 + i_j^d = \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i).$$

Bank return on equity is given by:

$$ROE_j \equiv \frac{F_j'}{F_j} - 1 = (1 + i)\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{1}{1 - \varepsilon^d}\frac{D}{F}\right) - 1.$$

Regime 2: *In this regime all banks obtain an amount of deposits* $D_j = D$, *lend out an amount* $L_j = L$, *and hold an amount of reserves* $H_j = F + D - L > 0$. *All banks set the same loan rate and deposit rate, whose expressions are given by:*

$$1 + i_j^l = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i), \qquad i_j^d = 0.$$

Bank return on equity is given by:

$$ROE_j = \frac{1}{\varepsilon^l - 1}\frac{L}{F} + i\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{D}{F}\right).$$

Regime 3A: *In this regime a fraction of banks* $\mu(i)$ *sets a negative deposit rate, doesn't obtain any deposits and lends out its equity at the loan rate:*

$$1 + i_{ND}^l = \left(\frac{F}{L}\right)^{-\frac{1}{\varepsilon}}(1 + i^l),$$

where ND *stands for "No Deposits" and* i^l *is the aggregate loan rate, given by:*

$$1 + i^l = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i)\left(\frac{1 - \mu(i)}{1 - \mu(i)\,(F/L)^{\frac{\varepsilon^l - 1}{\varepsilon^l}}}\right)^{\frac{1}{1 - \varepsilon^l}}.$$

These banks don't keep any reserves at the central bank. The remaining fraction of banks $(1-\mu(i))$ sets a zero deposit rate and obtains an amount of deposits D, lends out an amount:

$$L_D = \left(\frac{L^{\frac{\varepsilon-1}{\varepsilon}} - \mu(i) F^{\frac{\varepsilon-1}{\varepsilon}}}{1-\mu(i)} \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

at a rate: $1+i_D^l = \frac{\varepsilon^l}{\varepsilon^l-1}(1+i)$, and keeps an amount of reserves at the central bank given by: $H_D = F + D - L_D$. The fraction of banks not taking deposits $\mu(i)$ is defined implicitly by the following expression:

$$\frac{\varepsilon^l}{\varepsilon^l-1}(1+i)\frac{L_D}{L}\left(\left(\frac{L_D}{L}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} - 1 \right) = \mu \cdot (1+i) \cdot \left(\frac{F}{L} + \frac{D}{L} - \frac{L_D}{L} \right) - \mu \cdot \frac{D}{L}.$$

Regime 3B: *In this regime a fraction of banks μ^* sets a negative deposit rate, doesn't obtain any deposits and just lends out its equity at the loan rate i_{ND}^l. These variables are given by:*

$$\mu^* = \frac{(1+D/F)^{\frac{\varepsilon^l-1}{\varepsilon^l}} - (L/F)^{\frac{\varepsilon^l-1}{\varepsilon^l}}}{(1+D/F)^{\frac{\varepsilon^l-1}{\varepsilon^l}} - 1}$$

$$1+i_{ND}^l = \frac{\frac{D}{F}}{\left(1+\frac{D}{F}\right)^{\frac{\varepsilon^l-1}{\varepsilon^l}} - 1},$$

These banks don't keep any reserves at the central bank. The remaining fraction of banks $(1-\mu^*)$ sets a zero deposit rate and obtains an amount of deposits D, lends out an amount $L_D = F + D$ at a rate i_D^l, where:

$$1+i_D^l = \frac{\frac{1}{F/D+1}}{1-\left(1+\frac{D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}}$$

These banks also don't keep any reserves at the central bank. In this case the aggregate loan rate is given by:

$$1+i^l = \frac{\frac{D}{F}\left(\frac{F}{L}\right)^{\frac{1}{\varepsilon^l}}}{\left(1+\frac{D}{F}\right)^{\frac{\varepsilon^l-1}{\varepsilon^l}} - 1}$$

Proof: First verify the claims about the thresholds. Since $\varepsilon^d < 0$ it is obvious that $\tilde{\imath} > 0$. To prove that $\underline{i}$ is negative I will prove that the denominator is positive and the numerator is negative. First the denominator:

$$1 + \frac{1}{\varepsilon^l-1}\frac{L}{F} + \frac{D}{F} - \left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l-1} > \frac{1}{\varepsilon^l-1}\frac{L}{F}\left(1 - \left(\frac{L}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\right) + \frac{D}{F} - \left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}}.$$

The last expression is positive because $D/F > L/F > (L/F)^{1/\varepsilon^l}$ and $L/F > 1$ which implies $\left(\frac{L}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} < 1$. The numerator of $\underline{i}$ is negative, to see this write it as a function of L/F, $\text{numerator}_{\underline{i}} = f(L/F)$, where:

$$f(x) = x^{1/\varepsilon^l}\frac{\varepsilon^l}{\varepsilon^l - 1} - \frac{x}{\varepsilon^l - 1} - 1$$

Since the second derivative is always negative for $x > 1$, the function is maximized where $f'(x) = 0$, which is at $x = 1$. Additionally $f(1) = 0$, which implies that the function is always negative for values of x greater than 1. Since $L/F > 1$ this means that the numerator of $\underline{i}$ is always negative. In the explanation for Regime 3A I will prove that $\underline{i} > \underline{\underline{i}}$. Now I proceed to prove the claims about the regimes.

Regime 1: In this regime both rates are set according to their unconstrained F.O.C.s, which due to the concavity of the objective function guarantees that banks achieve a maximum. The loan and deposit subproblems are independent, due to the presence of positive reserves. The unconstrained loan subproblem is to maximize $(i^l_j - i)L_j$ subject to loan demand. The solution is given by:

$$1 + i^l_j = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i).$$

Similarly, the unconstrained deposit subproblem is to maximize $(i - i^d_j)D_j$ subject to deposit supply. The solution is given by:

$$1 + i^d_j = \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i).$$

No bank has incentives to deviate since they are all acting optimally. If a bank decided to stop keeping reserves at the central bank it would have to lend more and this requires lowering its loan rate, which is not optimal. If a bank decided to stop taking deposits it would stop earning its deposit spread, which is also suboptimal. The constrain $H > 0$ is satisfied in this regime, since $H = F + D - L > 0$. The deposit ZLB is satisfied as long as the deposit rate paid by banks is greater or equal than zero, which occurs as long as:

$$i \geq -\frac{1}{\varepsilon^d} \equiv \tilde{\iota}.$$

In this regime:

$$ROE_j \equiv \frac{F'_j}{F_j} - 1 = (1 + i)\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{1}{1 - \varepsilon^d}\frac{D}{F}\right) - 1.$$

Regime 2: In this regime the constraint that reserves are positive holds for the same reason as in the previous regime: $H_j = F + D - L > 0$. All banks set a zero deposit rate, so the deposit ZLB also holds. Since banks still hold reserves at the central bank they solve the unconstrained loan subproblem, which yields the same solution as it did in Regime 1. In this regime ROE is given by:

$$ROE_j = \frac{1}{\varepsilon^l - 1}\frac{L}{F} + i\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{D}{F}\right).$$

Banks don't want to start taking deposits by setting a positive deposit rate, since their unconstrained problem would deliver a negative rate. Banks also don't want to deviate to stop having reserves at the central bank, since this would require deviating from their maximizing loan rate. An individual bank might want to deviate to set a negative deposit rate, obtain no deposits and just lend out its full equity at the rate that allows to do so. This deviating bank would need to lend out F, so it could charge a loan rate of:

$$i^l_j \;=\; \left(\frac{F}{L}\right)^{-\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i) - 1.$$

Banks are better by not deviating if:

$$\begin{aligned}\left(\frac{F}{L}\right)^{-\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i) - 1 \;&\leq\; \frac{1}{\varepsilon^l - 1}\frac{L}{F} + i\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{D}{F}\right) \\ i \;&\leq\; \frac{\left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l-1} - 1 - \frac{1}{\varepsilon^l-1}\frac{L}{F}}{1 + \frac{1}{\varepsilon^l-1}\frac{L}{F} + \frac{D}{F} - \left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l-1}} \equiv \underline{i}.\end{aligned}$$

It is also easy to check than in this case banks that deviate to not taking deposits wouldn't want to keep reserves at the central bank, since the rate that they earn on loans is bigger than the policy rate.

Regime 3A: Since banks not taking deposits lend an amount equal to F, then the amount lent by banks taking deposits (denoted L_D) has to equal:

$$L_D \;=\; \left(\frac{L^{\frac{\varepsilon^l-1}{\varepsilon^l}} - \mu F^{\frac{\varepsilon^l-1}{\varepsilon^l}}}{1-\mu}\right)^{\frac{\varepsilon^l}{\varepsilon^l-1}}.$$

Since banks taking deposits are still keeping reserves at the central bank they must still satisfy their unconstrained loan subproblem F.O.C. which delivers:

$$(1 + i^l_D) = \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i).$$

Banks taking deposits must lend out L_D, which requires:

$$1 + i^l \;=\; \left(\frac{L_D}{L}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i).$$

Additionally, banks that do not take deposits must lend out F, which requires:

$$1 + i^l_{ND} \;=\; \left(\frac{L_D}{F}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i).$$

It is now possible to calculate the ROE for both types of banks:

$$\frac{F'_{ND}}{F_{ND}} - 1 \;=\; i^l_{ND},$$

and:

$$\frac{F'_D}{F_D} - 1 \quad = \quad \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i)\frac{L_D}{F} + (1+i)\frac{F + D - L_D}{F} - \frac{D}{F} - 1.$$

So, the profits of the two types of banks are equal when:

$$\begin{aligned} i^l_{ND} &= \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i)\frac{L_D}{F} + (1+i)\frac{F + D - L_D}{F} - \frac{D}{F} - 1 \\ 1 + \frac{i}{1+i}\frac{D}{F} &= \frac{\varepsilon^l}{\varepsilon^l - 1}\frac{L_D}{F}\left[\left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} - \frac{1}{\varepsilon^l}\right]. \end{aligned}$$

This expression of i and μ can be written as $F(i,\mu) = 0$, where:

$$F(i,\mu) \quad = \quad 1 + \frac{i}{1+i}\frac{D}{F} - \frac{\varepsilon^l}{\varepsilon^l - 1} g(\mu)^{\frac{1}{\varepsilon^l}} + \frac{g(\mu)}{\varepsilon^l - 1},$$

and:

$$g(\mu) = \frac{L_D}{F} = \left(\frac{(L/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \mu}{1 - \mu}\right)^{\frac{\varepsilon^l}{\varepsilon^l - 1}}.$$

Since $g(\mu)$ is an increasing function of $\frac{a-\mu}{1-\mu}$, where $a = (L/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} > 1$, it is evident that $g'(\mu) > 0$. Also notice that:

$$\frac{\partial F}{\partial \mu} \quad = \quad \frac{g'(\mu)}{\varepsilon^l - 1}\frac{(L/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - 1}{(L/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \mu} > 0.$$

Using the implicit function theorem this means that the expression $F(i,\mu) = 0$ implicitly defines μ as a function of i. The derivative of $\mu(i)$ is given by:

$$\frac{\partial \mu}{\partial i} = -\frac{\frac{\partial F}{\partial i}}{\frac{\partial F}{\partial \mu}},$$

which is negative because:

$$\frac{\partial F}{\partial i} = \frac{1 + i - i}{(1+i)^2}\frac{D}{F} = \frac{1}{(1+i)^2}\frac{D}{F} > 0.$$

The level $\mu(i) = 0$ is a solution when:

$$1 + i + i\frac{D}{F} \quad = \quad (1+i)\frac{\varepsilon^l}{\varepsilon^l - 1}\frac{L}{F}\left[\left(\frac{L}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} - \frac{1}{\varepsilon^l}\right]$$

Which occurs when $i = \underline{i}$. The requirement that the amount of reserves held at the central bank by commercial banks taking deposits must be positive can be expressed as:

$$
\begin{aligned}
0 &\leq F + D - \left(\frac{L^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \mu F^{\frac{\varepsilon^l - 1}{\varepsilon^l}}}{1 - \mu} \right)^{\frac{\varepsilon^l}{\varepsilon^l - 1}} \\
\mu &\leq \frac{(1 + D/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - (L/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}}}{(1 + D/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - 1} \equiv \overline{\mu}.
\end{aligned}
$$

It is easy to see that this limit level $\overline{\mu}$ is greater than zero and smaller than one. When $\mu = \overline{\mu}$ the amount lent by banks taking deposits is $1 + D/F$, this is obvious from the fact that this limit was derived from $H_D = 0$, and that occurs when $1 + \frac{D}{F} = \frac{L_D}{F}$. This can be introduced into the expression for $\mu(i)$ to obtain the level of the policy rate that delivers $\overline{\mu}$:

$$
\begin{aligned}
1 + \frac{i}{1+i}\frac{D}{F} &= \frac{\varepsilon^l}{\varepsilon^l - 1}\frac{L_D}{F}\left[\left(\frac{L_D}{F} \right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} - \frac{1}{\varepsilon^l} \right] \\
i &= \frac{\left(1 + \frac{D}{F}\right)^{\frac{1}{\varepsilon^l}} - 1 - \frac{D}{\varepsilon^l F}}{1 + \frac{D}{F} - \left(1 + \frac{D}{F}\right)^{\frac{1}{\varepsilon^l}}} \equiv \underline{\underline{i}}.
\end{aligned}
$$

This interest rate is smaller than $\underline{i}$, since at $\underline{i}$ it is the case that $\mu = 0$, at $\underline{\underline{i}}$ it is the case that $0 < \mu < 1$, and $\mu(i)$ is a decreasing function of i. Now I want to show that i^l is a decreasing function of i. Recall that:

$$
1 + i^l = \left(\frac{L_D}{L} \right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i),
$$

which means that as i increases, there are two effects on i^l, the increase in i directly increases i^l, but the fall in μ brought about by the increase in i lowers L_D/F and this tends to lower i^l. To see which effect dominates take derivatives:

$$
\frac{\partial i^l}{\partial i} = \frac{1}{\varepsilon^l}\left(\frac{L_D}{L} \right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} \frac{\partial (L_D/L)}{\partial i}\frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i) + \left(\frac{L_D}{L} \right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1}.
$$

To obtain the derivative $\frac{\partial (L_D/F)}{\partial i}$ apply the implicit function theorem directly to:

$$
F(i, L_D/F) = 1 + \frac{i}{1+i}\frac{D}{F} - \frac{\varepsilon^l}{\varepsilon^l - 1}\frac{L_D}{F}\left[\left(\frac{L_D}{F} \right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} - \frac{1}{\varepsilon^l} \right],
$$

to obtain:

$$
\frac{\partial (L_D/F)}{\partial i} = -\frac{\frac{\partial F}{\partial i}}{\frac{\partial F}{\partial L_D/F}} = -\frac{\frac{1}{(1+i)^2}\frac{D}{F}}{\frac{1}{\varepsilon^l - 1}\left(1 - \left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\right)}.
$$

Introducing this in the expression for $\frac{\partial i^l}{\partial i}$ one gets:

$$\frac{\partial i^l}{\partial i} = \left(\frac{L_D}{L}\right)^{\frac{1}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1} - \left(\frac{L_D}{L}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} \frac{\frac{1}{1+i}\frac{D}{L}}{1 - \left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}}.$$

This is equal to zero if:

$$\frac{\varepsilon^l}{\varepsilon^l - 1}\left(1 - \left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\right) = \frac{1}{1+i}\frac{D}{L_D}.$$

At $\underline{\underline{i}}$, where $L_D/F = 1 + D/F$ the previous expression becomes:

$$(1+\underline{\underline{i}})\frac{\varepsilon^l}{\varepsilon^l - 1}\left(1 - \left(1 + \frac{D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\right) = \frac{D}{L_D}$$

$$\frac{1}{1 + \frac{D}{F} - \left(1 + \frac{D}{F}\right)^{\frac{1}{\varepsilon^l}}}\left(1 + \frac{D}{F} - \left(1 + \frac{D}{F}\right)^{\frac{1}{\varepsilon^l}}\right) = 1$$

Which is always true. This proves that at $i = \underline{\underline{i}}$, the derivative $\frac{\partial i^l}{\partial i}$ is equal to zero. It is also possible to show that the second derivative of i^l w.r.t. i is negative:

$$\frac{\partial^2 i^l}{\partial i^2} = -\frac{1}{\varepsilon^l}\left(\frac{L_D}{L}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}} \frac{\varepsilon^l}{\varepsilon^l - 1} \frac{\frac{1}{(1+i)^2}\frac{D}{L}}{\frac{1}{\varepsilon^l - 1}\left(1 - \left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\right)} + \frac{1}{(1+i)^2} \frac{\frac{D}{L}}{\left(\frac{L_D}{L}\right)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \left(\frac{F}{L}\right)^{\frac{\varepsilon^l - 1}{\varepsilon^l}}}$$
$$- \frac{\frac{1}{1+i}\frac{D}{L}}{\left(\left(\frac{L_D}{L}\right)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \left(\frac{F}{L}\right)^{\frac{\varepsilon^l - 1}{\varepsilon^l}}\right)^2} \frac{\varepsilon^l - 1}{\varepsilon^l}\left(\frac{L_D}{L}\right)^{-\frac{1}{\varepsilon^l}} \frac{\frac{1}{(1+i)^2}\frac{D}{L}}{\frac{1}{\varepsilon^l - 1}\left(1 - \left(\frac{L_D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}\right)}$$

The first two terms cancel out and then it is clear than the second derivative of i^l w.r.t. i is negative, which means that throughout the whole Regime 3A the highest i^l is achieved at $i = \underline{\underline{i}}$. In this regime ROE is given by:

$$ROE_j = \frac{F_j'}{F_j} - 1 = i^l_{ND} = \left(\frac{(L/F)^{\frac{\varepsilon^l - 1}{\varepsilon^l}} - \mu(i)}{1 - \mu(i)}\right)^{\frac{1}{\varepsilon^l - 1}} \frac{\varepsilon^l}{\varepsilon^l - 1}(1+i) - 1$$

Regime 3B: Banks not taking deposits lend out F:

$$1 + i^l_{ND} = \left(\frac{F}{L}\right)^{-\frac{1}{\varepsilon}}(1 + i^l)$$

Banks taking deposits lend out $F + D$:

$$1 + i^l_D = \left(\frac{F + D}{L}\right)^{-\frac{1}{\varepsilon}}(1 + i^l)$$

ROE for both types of banks must be equalized.

$$\begin{aligned}(1+i^l_{ND}) &= (1+i^l_D)\left(1+\frac{D}{F}\right)-\frac{D}{F}\\ 1+i^l &= \frac{\frac{D}{F}\left(\frac{F}{L}\right)^{\frac{1}{\varepsilon^l}}}{\left(1+\frac{D}{F}\right)^{\frac{\varepsilon^l-1}{\varepsilon^l}}-1}\end{aligned}$$

Recall that in Regime 3A, when $\mu=\overline{\mu}$, it is the case that:

$$1+i^l = \left(\frac{L_D}{F}\right)^{\frac{1}{\varepsilon^l}}\left(\frac{F}{L}\right)^{\frac{1}{\varepsilon^l}}\frac{\varepsilon^l}{\varepsilon^l-1}(1+\underline{i}) = \frac{\frac{D}{F}\left(\frac{F}{L}\right)^{\frac{1}{\varepsilon^l}}}{\left(1+\frac{D}{F}\right)^{\frac{\varepsilon^l-1}{\varepsilon^l}}-1}$$

This means that the highest loan rate charged in Regime 3A is precisely the same rate charged in Regime 3B. It is easy to see that both types of banks satisfy all constraints and have no profitable deviations. ROE for all banks in this regime is given by:

$$ROE_j = \frac{F'_j}{F_j}-1 = i^l_{ND} = \left(\frac{F}{L}\right)^{-\frac{1}{\varepsilon}}(1+i^l)-1 = \frac{\left(1+\frac{D}{F}\right)^{\frac{1}{\varepsilon^l}}-1}{1-\left(1+\frac{D}{F}\right)^{\frac{1-\varepsilon^l}{\varepsilon^l}}}$$

A.5 Static Problem of the Bank with a Target Leverage Ratio

In this section I describe a more general version of the bank model described in Section I. Here, as in Section III, banks are going to be subject to a cost of deviating from a target level of loan-to-equity ratio. The bank pays a quadratic cost (parameterized by a coefficient κ and proportional to outstanding bank net worth) whenever the loan-to-equity ratio $L(j)/F(j)$ deviates from the target value ν.

Banks also face costs of issuing loans given by μ^l and benefits of issuing deposits given by μ^d, these are per dollar of loan or deposit issued, and they could be positive or negative. The cost of issuing loans is positive (the bank has to monitor the borrowers, pay loan originators, etc), while the cost of issuing deposits could plausibly be negative, as it could be seen as a benefit that the bank receives for having a large deposit base, for example attracting more customers or obtaining more publicity (that is why I will depict them as a benefit in my notation).

I will make the assumption that $D+F>L>F$, which indicates that the total amount of loans being demanded is possible to cover with the total amount of deposits being supplied plus total bank equity, but it is not possible to cover simply with bank equity. The problem of the bank is:

$$\begin{aligned}\max_{i^l(j),L(j),i^d(j),D(j),H(j)} \quad & (1+i^l(j)-\mu^l)L(j)+(1+i)H(j)-(1+i^d(j)-\mu^d)D(j)\\ & -\left[\kappa\nu\frac{L(j)}{F(j)}\left(\ln\left(\frac{L(j)}{F(j)}\right)-\ln\nu-1\right)+\kappa\nu^2\right]F(j)\\ \text{s.t.} \quad & \\ L(j) &\leq \left(\frac{1+i^l(j)}{1+i^l}\right)^{-\varepsilon^l}L\end{aligned}$$

$$
\begin{aligned}
D(j) &= \begin{cases} \left(\frac{1+i^d(j)}{1+i^d}\right)^{-\varepsilon^d} D & \text{if } i^d(j) \geq 0 \\ 0 & \text{if } i^d(j) < 0 \end{cases} \\
L(j) + H(j) &= F(j) + D(j) \\
H(j) &\geq 0
\end{aligned}
$$

In Regime 1 all banks charge the same loan rate, which is set according to the unconstrained loan subproblem F.O.C.:

$$
1 + i^l(j) = \varepsilon^l(i^l(j) - i - \mu^l) - \kappa\nu\varepsilon^l \left(\ln\left(\frac{L(j)}{F(j)}\right) - \ln(\nu)\right).
$$

In equilibrium, since all the banks are symmetric, this becomes:

$$
1 + i^l = \frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i + \mu^l) + \kappa\nu\frac{\varepsilon^l}{\varepsilon^l - 1}\left(\ln\left(\frac{L}{F}\right) - \ln(\nu)\right).
$$

So, it is clear that in this regime the loan interest rate increases not only with the policy rate and the cost of issuing loans, but also with the amount of loans being made (L). So this is a type of loan supply curve. In this regime the F.O.C. for the deposit rate is:

$$
0 = -\left(\frac{1 + i^d(j)}{1 + i^d}\right)^{-\varepsilon^d} D - \varepsilon^d(i + \mu^d - i^d(j))\left(\frac{1 + i^d(j)}{1 + i^d}\right)^{-\varepsilon^d - 1}\frac{D}{1 + i^d},
$$

hence the solution for the deposit rate is:

$$
1 + i^d(j) = \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i + \mu^d).
$$

Notice that this net deposit rate is only greater than zero if:

$$
-\frac{1}{\varepsilon^d} - \mu^d \leq i.
$$

Defining $\tilde{\imath} \equiv -\frac{1}{\varepsilon^d} - \mu^d$, I can say that as long as $i \geq \tilde{\imath}$ commercial banks want to set nonnegative deposit rates. In this regime bank resources at the end of the period are given by:

$$
\begin{aligned}
F' &= (1+i)F + (i^l - i - \mu^l)L + (i - i^d + \mu^d)D - \left[\kappa\nu\frac{L}{F}\left(\ln\left(\frac{L}{F}\right) - \ln\nu - 1\right) + \kappa\nu^2\right]F \\
&= (1+i)F + \left(\frac{1}{\varepsilon^l - 1}(1 + i + \mu^l) + \kappa\nu\frac{\varepsilon^l}{\varepsilon^l - 1}\left(\ln\left(\frac{L}{F}\right) - \ln(\nu)\right)\right)L \\
&+ \frac{1}{1 - \varepsilon^d}(1 + i + \mu^d)D - \left[\kappa\nu\frac{L}{F}\left(\ln\left(\frac{L}{F}\right) - \ln\nu - 1\right) + \kappa\nu^2\right]F \\
&= (1+i)\left(F + \frac{1}{\varepsilon^l - 1}L + \frac{1}{1 - \varepsilon^d}D\right) + \frac{\mu^l}{\varepsilon^l - 1}L + \frac{\mu^d}{1 - \varepsilon^d}D \\
&+ \kappa\nu L\left(\ln\left(\frac{L}{F}\right) - \ln(\nu)\right)\frac{1}{\varepsilon^l - 1} + \kappa\nu F\left(\frac{L}{F} - \nu\right).
\end{aligned}
$$

Approximating $\nu(\ln(L/F) - \ln(\nu))$ with its first order Taylor approximation around the steady state $(L/F - \nu)$, and approximating νF as L then the previous expression can be written as:

$$\frac{F'}{F} \;=\; \left(1 + \frac{1+\mu^l}{\varepsilon^l - 1}\frac{L}{F} + \frac{1+\mu^d}{1-\varepsilon^d}\frac{D}{F}\right) + i\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{1}{1-\varepsilon^d}\frac{D}{F}\right) + \frac{\varepsilon^l}{\varepsilon^l - 1}\kappa\frac{L}{F}\left(\frac{L}{F} - \nu\right).$$

Once $i < \tilde{\imath}$ commercial banks start setting a deposit rate of exactly zero and continue to obtain deposits. In this case bank resources at the end of the period are given by:

$$\frac{F'}{F} \;=\; \left(1 + \frac{1+\mu^l}{\varepsilon^l - 1}\frac{L}{F} + \mu^d\frac{D}{F}\right) + i\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{D}{F}\right) + \frac{\varepsilon^l}{\varepsilon^l - 1}\kappa\frac{L}{F}\left(\frac{L}{F} - \nu\right).$$

All banks setting a zero deposit rate and obtaining deposits continues to be an equilibrium until any single bank would have an incentive to deviate. This deviating bank would set its loan rate to lend out its full equity:

$$1 + i^l_{DEV} \;=\; \left(\frac{F}{L}\right)^{-\frac{1}{\varepsilon^l}}(1 + i^l)$$

Equity at the end of the period for the deviating bank would be given by:

$$F'_{DEV} = (1 + i^l_{DEV} - \mu^l)F_{DEV} - \delta_i\left[\kappa\nu^2 - \kappa\nu - \kappa\nu\ln\nu\right]F_{DEV},$$

where δ_i is an indicator that can be set equal to 1 if one wishes to still include the cost of deviating from target leverage when banks are not taking deposits and equal to zero if one wishes not to include it. Since the "bank" is no longer taking household deposits it could be argued that it is no longer a "bank" and should not be subject to regulatory oversight. In order to discover the level of the policy rate at which it would start paying off to deviate I compare the ROE from the two scenarios:

$$\begin{aligned} &\left(1 + \frac{1+\mu^l}{\varepsilon^l - 1}\frac{L}{F} + \mu^d\frac{D}{F}\right) + i\left(1 + \frac{1}{\varepsilon^l - 1}\frac{L}{F} + \frac{D}{F}\right) + \frac{\varepsilon^l}{\varepsilon^l - 1}\kappa\frac{L}{F}\left(\frac{L}{F} - \nu\right) \\ = \;& \left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}}\left(\frac{\varepsilon^l}{\varepsilon^l - 1}(1 + i + \mu^l) + \kappa\nu\frac{\varepsilon^l}{\varepsilon^l - 1}\left(\ln\left(\frac{L}{F}\right) - \ln(\nu)\right)\right) - \mu^l - \delta_i(\kappa\nu^2 - \kappa\nu - \kappa\nu\ln\nu), \end{aligned}$$

Defining:

$$\underline{i} = -\frac{1 + \frac{1+\mu^l}{\varepsilon^l-1}\frac{L}{F} + \mu^d\frac{D}{F} + \frac{\varepsilon^l\kappa}{\varepsilon^l-1}\frac{L}{F}\left(\frac{L}{F} - \nu\right) - \left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}}\frac{\varepsilon^l}{\varepsilon^l-1}\left(1 + \mu^l + \kappa\nu\left(\ln\left(\frac{L}{F}\right) - \ln(\nu)\right)\right) + \mu^l + \delta_i(\kappa\nu^2 - \kappa\nu - \kappa\nu\ln\nu)}{1 + \frac{1}{\varepsilon^l-1}\frac{L}{F} + \frac{D}{F} - \left(\frac{L}{F}\right)^{\frac{1}{\varepsilon^l}}\frac{\varepsilon^l}{\varepsilon^l-1}},$$

then it is possible to say that Regime 2 is active when $\tilde{\imath} > i \geq \underline{i}$. For my baseline calibration I obtain $\tilde{\imath} = 0.5\%$ and $\underline{i} \approx -2\%$ at the annual level with $\delta_i = 0$ (with $\delta_i = 1$ $\underline{i}$ is much more negative). Notice that when $\mu^d = \mu^l = \kappa = 0$ the expressions just given simplify to the ones given in Section I.

A.6 Dynamic Bank Problem

First, I will modify the problem of the borrowing firm from the one described in Appendix A.1 so that it is consistent with the stochastic nature of loan returns described in the banking framework of Section III. The reasons for this change are sketched in Section III.E.

There will be a firm seeking to obtain a known amount of funding from a continuum of banks. Each bank j will charge a multiple $m(j)$ of the return of the project (so they would earn $m(j)$ times the return of the project if they were allocated the whole project) and they will be allocated a fraction $\gamma(j)$ of the total project. Hence bank j will have to be paid an amount $m(j)\gamma(j)$ of the total return of the project. The firm will want to minimize this amount subject to the CES aggregation of the $\gamma(j)$ being equal to one. That is, they want to minimize:

$$\int_0^1 m(j)\gamma(j)dj,$$

subject to:

$$\left[\int_0^1 \gamma(j)^{(\varepsilon_t^b-1)/\varepsilon_t^b} dj\right]^{\varepsilon_t^b/(\varepsilon_t^b-1)} \geq 1.$$

The F.O.C. implies that:

$$\gamma(j) = \left(\frac{m(j)}{\lambda}\right)^{-\varepsilon_t^b},$$

where:

$$\lambda = \left[\int_0^1 m(j)^{1-\varepsilon_t^b} dj\right]^{\frac{1}{1-\varepsilon_t^b}},$$

so, denoting $m \equiv \lambda$, I can write demand for a particular bank j as:

$$\gamma(j) = \left(\frac{m(j)}{m}\right)^{-\varepsilon_t^b}.$$

Since all the banks will be symmetric, they will all charge $m(j) = m$ and they will all be allocated a fraction of 1 of the total project (remember there is a measure 1 of banks so this is consistent with the total fraction allocated being 1).

With this loan demand schedule it is now possible to solve the dynamic problem of the bank. As mentioned in the main text, the bank problem is:

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{s+1} \Lambda_{t,t+s+1} DIV_{j,t+s+1}$$

Notice that the dividend given in period $t+1$ comes from the activities performed in period t and is given by $(1-\omega)X_{t+1}$. If the bank were choosing its dividend distribution endogenously then this would be a more complicated problem, since the bank would have to decide when to retain more equity depending on expected spreads and when to give more dividends. As mentioned above, here banks don't independently optimize their dividend distribution but instead take as given that a fraction $(1-\omega)$ of "profits" $X_{j,t+1}$ are distributed as dividends.

The problem of the bank can then be expressed using the Bellman equation:

$$V(F_j, \Xi) \;=\; \max_{\gamma_j,\mu_j,i_j^d,D_j} \mathbb{E}\left\{\beta\Lambda DIV_j' + \beta\Lambda V(F_j', \Xi')\right\},$$

where Ξ denotes the aggregate state variables that influence the value of being a bank. I have also removed time indexes so that, for example, F_j represents $F_{j,t}$ and F_j' represents $F_{j,t+1}$. Λ stands for $\Lambda_{t,t+1}$. With this notation, the maximization problem is subject to the following constraints (ignoring the non-negativity of the deposit rate for notational convenience):

$$\begin{aligned}
\gamma_j &= \left(\frac{m_j}{m}\right)^{-\varepsilon^l} \\
D_j &= \left(\frac{1+i_j^d}{1+i^d}\right)^{-\varepsilon^d} \mathbf{D} \\
DIV_j' &= (1-\omega)X_j' \\
F_j' &= F_j(1-\varsigma)(1+\pi') + \omega X_j' \\
X_j' &= iF_j + [(1+(i_j^l)')m_j - 1 - \mu^l - i]\gamma_j \mathbf{L} + (i + \mu^d - i_j^d)D_j - F_j(1-\varsigma)\pi' \\
&- \left[\kappa\nu\frac{\gamma_j \mathbf{L}}{F_j}\left(\ln\left(\frac{\gamma_j \mathbf{L}}{F_j}\right) - \ln\nu - 1\right) + \kappa\nu^2\right] F_j
\end{aligned}$$

The first equation describes loan demand. The second equation describes deposit supply. The third equation describes dividends paid as a function of X_j. The fourth equation describes the evolution of bank equity depending on X_j. Finally, the fifth equation is the definition of "profits" X_j. The F.O.C. w.r.t. i_j^d yields the following:

$$0 \;=\; \mathbb{E}\left\{\beta\Lambda(1-\omega)\frac{\partial X_j'}{\partial i_j^d} + \beta\Lambda\frac{\partial V(F_j', \Xi')}{\partial F_j'}\omega\frac{\partial X_j'}{\partial i_j^d}\right\}$$

Since $\frac{\partial X_j'}{\partial i_j^d}$ is deterministic (known in period t), it can exit the expectation operator and the optimality condition becomes:

$$1 + i_j^d = \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i + \mu^d)$$

Returning to time notation, this can be expressed as:

$$1 + i_{j,t}^d \;=\; \frac{\varepsilon^d}{\varepsilon^d - 1}(1 + i_t + \mu_t^d)$$

Which is equation (17) in the main body of the paper.

If the loan rate problem was also deterministic (where the return to the time t decision is known in period t) then it would work the same way as the deposit problem and the solution would be similar. But the loan rate problem is not deterministic, since the return to a loan is not know at time t. The F.O.C. w.r.t. m_j is:

$$0 \;=\; \mathbb{E}\left\{\Lambda\left[1 - \omega + \omega\frac{\partial V(F_j', \Xi')}{\partial F_j'}\right]\frac{\partial X_j'}{\partial m_j}\right\}$$

Recall that the derivative of γ_j w.r.t. m_j is:

$$\frac{\partial \gamma_j}{\partial m_j} = -\varepsilon^l \frac{\gamma_j}{m_j}$$

With this, the F.O.C. for m_j can be expressed as:

$$\frac{\partial X_j'}{\partial m_j'} = \varepsilon^l \frac{\gamma_j \mathbf{L}}{m_j}\left[(1+(i_j^l)')m_j\frac{1-\varepsilon^l}{\varepsilon^l} + (1+\mu^l+i) + \kappa\nu\left(\ln\left(\frac{\gamma_j \mathbf{L}}{F_j}\right) - \ln(\nu)\right)\right]$$

Introduce this in the previous equation for the F.O.C. w.r.t. to m_j, while using the fact that in equilibrium all banks are identical and hence $\gamma_j = 1$ and $m_j = 1$, to obtain:

$$0 = \mathbb{E}\left\{\Lambda\left[1-\omega+\omega\frac{\partial V(F',\Xi')}{\partial F'}\right]\left[(1+(i^l)')\frac{1-\varepsilon^l}{\varepsilon^l} + (1+\mu^l+i) + \kappa\nu\left(\ln\left(\frac{\mathbf{L}}{F}\right) - \ln(\nu)\right)\right]\right\}$$

Denote $T' = \Lambda\left[1-\omega+\omega\frac{\partial V(F',\Xi')}{\partial F'}\right]$, then the previous equation can be simplified to:

$$\begin{aligned}
\mathbb{E}\left\{T'(1+(i^l)')\right\}\frac{\varepsilon^l-1}{\varepsilon^l} &= \mathbb{E}\left\{T'\right\}\left[(1+\mu^l+i) + \kappa\nu\left(\ln\left(\frac{\mathbf{L}}{F}\right) - \ln(\nu)\right)\right] \\
\frac{\mathbb{E}\left\{T'(1+(i^l)')\right\}}{\mathbb{E}\left\{T'\right\}} &= \frac{\varepsilon^l}{\varepsilon^l-1}\left[(1+i+\mu^l) + \kappa\nu\left(\ln\left(\frac{\mathbf{L}}{F}\right) - \ln(\nu)\right)\right]
\end{aligned}$$

Notice that T' is unknown, because it is a complicated object that contains the derivative of the value function with respect to equity, which depends on the state variables in Ξ. However, since I will log-linearize the system, then the previous condition is equivalent to:

$$\mathbb{E}_t(1+i_{t+1}^l) = \frac{\varepsilon^l}{\varepsilon^l-1}\left[(1+i_t+\mu_t^l) + \kappa\nu\left(\ln\left(\frac{\mathbf{L}_t}{F_t}\right) - \ln(\nu)\right)\right]$$

Which is equation (16) in the paper.

A.7 Heterogeneous Bank Model

Recall that the functional form for the costs of deviating from the target level of loan-to-equity ratio is:

$$-\kappa\nu\frac{L_j}{F_j}\left(\ln\left(\frac{L_j}{F_j}\right) - \ln\nu - 1\right) - \kappa\nu^2.$$

Denote this function by $f(x)$, where $x = L/F$, then:

$$\begin{aligned}
f(x) &= -\kappa\nu x\left(\ln(x) - \ln\nu - 1\right) - \kappa\nu^2 \\
f'(x) &= -\kappa\nu\left(\ln(x) - \ln\nu\right) \\
f''(x) &= -\frac{\kappa\nu}{x}.
\end{aligned}$$

Hence the second order approximation to the new functional form (around $x = \nu$ which is the steady state value of L/F) is the following:

$$f(x) \approx^2 f(\nu) + f'(\nu)(x-\nu) + \frac{f''(\nu)}{2}(x-\nu)^2$$

$$= \quad -\frac{\kappa}{2}(x-\nu)^2.$$

This means that the logarithmic specification that I have been using can be approximated around the steady state to the second order as a simple quadratic function. The reason that the logarithmic specification is convenient is because it allows to solve the heterogeneous bank model in a simple way (after using an approximation). When using the specification in logs, the bank problem (abstracting from the managerial cost, the exogenous costs and benefits of issuing loans and deposits, the stochastic nature of the loan return, and the constraints that reserves are nonnegative and deposits rates are nonnegative) is:

$$\max_{H_j,L_j,D_j,i_j^l,i_j^d} \quad (1+i)H_j + (1+i_j^l)L_j - (1+i_j^d)D_j - \left[\kappa\nu\frac{L_j}{F_j}\left(\ln\left(\frac{L_j}{F_j}\right) - \ln(\nu) - 1\right) + \kappa\nu^2\right]F_j$$

$$\text{s.t.} \qquad L_j = \left(\frac{1+i_j^l}{1+i^l}\right)^{-\varepsilon^l} L, \qquad D_j = \left(\frac{1+i_j^d}{1+i^d}\right)^{-\varepsilon^d} D, \qquad L_j + H_j = F_j + D_j.$$

The F.O.C. with respect to the deposit rate is unchanged, and it implies that all banks set the same deposit rate. The F.O.C. w.r.t. the loan rate is:

$$1+i_j^l \quad = \quad \frac{\varepsilon^l}{\varepsilon^l - 1}\left(1 + i + \kappa\nu\left(\ln\left(\frac{L_j}{F_j}\right) - \ln(\nu)\right)\right).$$

Taking natural logs one gets:

$$\ln(1+i_j^l) \quad = \quad \ln\left(\frac{\varepsilon^l}{\varepsilon^l - 1}\right) + \ln\left(1 + i + \kappa\nu\left(\ln\left(\frac{L_j}{F_j}\right) - \ln(\nu)\right)\right).$$

I can approximate this as:[1]

$$i_j^l \quad = \quad \ln\left(\frac{\varepsilon^l}{\varepsilon^l - 1}\right) + i + \kappa\nu\ln(L_j) - \kappa\nu\ln(F_j) - \kappa\nu\ln(\nu).$$

This is linear in the net rates and the logs of quantities, which is convenient because demand is also linear in those things (after a similar approximation):

$$\ln(L_j) \quad \approx \quad -\varepsilon^l(i_j^l - i^l) + \ln(L).$$

Introduce this in the previous equation for the loan rate and simplify to obtain:

$$i_j^l \quad = \quad \frac{1}{1+\kappa\nu\varepsilon^l}\left(\ln\left(\frac{\varepsilon^l}{\varepsilon^l - 1}\right) + i + \kappa\nu\varepsilon^l i^l + \kappa\nu\ln(L) - \kappa\nu\ln(F_j) - \kappa\nu\ln(\nu)\right).$$

Now introduce i_j^l in the expression for $\ln(L_j)$ to obtain:

$$\ln(L_j) \quad = \quad -\frac{\varepsilon^l}{1+\kappa\nu\varepsilon^l}\left(\ln\left(\frac{\varepsilon^l}{\varepsilon^l - 1}\right) + i - \kappa\nu\ln(F_j) - \kappa\nu\ln(\nu) - i^l - \frac{\ln(L)}{\varepsilon^l}\right).$$

[1]This assumes that interest rates are small and that $\kappa\nu\left(\ln\left(\frac{L_j}{F_j}\right) - \ln(\nu)\right)$ is also small, these are very plausible things to assume for all the parameter values and shock sizes that I will use. In numerical simulations I confirmed that the approximation works very well.

It can also be shown that the aggregate loan rate is given by

$$i^l = \ln\left(\frac{\varepsilon^l}{\varepsilon^l - 1}\right) + i + \kappa\nu\left(\ln\left(\frac{L}{\tilde{F}} - \nu\right)\right),$$

where:

$$\tilde{F} = \left(\int_0^1 F_j^{\frac{\varepsilon^l - 1}{\varepsilon^l + 1/(\kappa\nu)}} dj\right)^{\frac{\varepsilon^l + 1/(\kappa\nu)}{\varepsilon^l - 1}}.$$

Hence, the equations for bank-level loan rate and loan amount can be expressed as:

$$\begin{aligned} i_j^l &= \ln\left(\frac{\varepsilon^l}{\varepsilon^l - 1}\right) + i + \kappa\nu\left(\ln\left(\frac{L}{\tilde{F}} - \nu\right)\right) - \frac{\kappa\nu}{1 + \kappa\nu\varepsilon^l}\ln\left(\frac{F_j}{\tilde{F}}\right) \\ \ln(L_j) &= \ln(L) + \frac{\kappa\nu\varepsilon^l}{1 + \kappa\nu\varepsilon^l}\ln\left(\frac{F_j}{\tilde{F}}\right). \end{aligned}$$

These two equations can be rewritten as

$$\begin{aligned} i_j^l &= \alpha + \beta i - \frac{\kappa\nu}{1 + \kappa\nu\varepsilon^l}\ln(F_j) \\ \ln(L_j) &= \alpha' + \beta' i + \frac{\kappa\nu\varepsilon^l}{1 + \kappa\nu\varepsilon^l}\ln(F_j), \end{aligned}$$

where:

$$\begin{aligned} \alpha &= \ln\left(\frac{\varepsilon^l}{\varepsilon^l - 1}\right) + \kappa\nu\left(\ln\left(\frac{L}{\tilde{F}} - \nu\right)\right) + \frac{\kappa\nu}{1 + \kappa\nu\varepsilon^l}\ln(\tilde{F}) \\ \alpha' &= \ln(L) - \frac{\kappa\nu\varepsilon^l}{1 + \kappa\nu\varepsilon^l}\ln(\tilde{F}) \\ \beta &= 1 \\ \beta' &= 0. \end{aligned}$$

Appendix B Additional Empirical Results

B.1 More Summary Statistics

Table 7: Summary Statistics for Banking Variables 1990-2017

	All Countries		NR Countries		Other Countries	
	Mean	N	Mean	N	Mean	N
Rate on Av. Earning Assets	4.57	80086	4.17	56385	5.53	23701
Deposit Rate	1.02	31615	0.89	19884	1.24	11731
Net Interest Margin	2.46	80441	2.20	56408	3.07	24033
ROAA	0.48	80545	0.32	56481	0.84	24064
ROAE	5.78	80202	4.41	56455	9.03	23747
Log of Net Loans	6.60	84721	6.47	60239	6.91	24482
Log of Total Customer Deposits	6.71	83532	6.58	59388	7.04	24144
Log of Equity	4.48	85240	4.27	60568	5.00	24672
Log of Total Assets	7.13	85311	7.03	60605	7.39	24706
Customer Deposits to Assets ratio	0.72	83599	0.71	59446	0.75	24153
Net Loans to Assets ratio	0.62	84823	0.61	60291	0.66	24532

Notes: This table contains more summary statistics for banking variables in the period 1990-2017 split between negative rate and non-negative rates regions.

B.2 Linear Results

As a starting point to study the relationships between the policy rate and the variables of interest one could run linear regressions of the following type:

$$y_{b,t} = \alpha_b + \delta_t + \beta i_{c(b),t} + \varepsilon_{b,t}, \tag{2}$$

where $y_{b,t}$ is some outcome variable for bank b, in country $c(b)$ and year t, and $i_{c(b),t}$ is the policy rate in that country and year. The regressions include a bank fixed effect (α_b) and a year fixed effect (δ_t). The results of these regressions without a lag of the dependent variable are given in Table 8, while the ones that include a lag are given in Table 9. In all these regressions the coefficient on the policy rate is positive and significant, which means that the loan rate, deposit rate and ROAE all move together with the policy rate and in the same direction. These results are well known in the literature. The first lag of the dependent variable is also positive and significant, while if a second lag is included (results not shown) it is generally nonsignificant. This indicates that roughly two-thirds of the effects of the policy rate on the loan rate and the deposit rate happens in the first year and the remaining one-third happens during the second year. With return on equity the first year effect is roughly 85% of the total.

The fact that movements in the policy rate do not fully translate to the deposit rate is qualitatively and quantitatively consistent with the idea of the "deposit channel of monetary policy" developed in Drechsler et al. (2017). In my results the spread between the policy rate and the deposit rate increases by 40 bps with a 100 bps increase in the policy rate, while in their results the increase in the policy rate is 54 bps. The numbers are very similar, and can easily be accounted for by the difference in time periods and countries being analyzed.

Table 8: Linear regressions for main variables of interest, no lag

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.428***	0.425***	0.928***
	(0.027)	(0.039)	(0.236)
N	80078	31554	80199
R squared	0.931	0.846	0.401
Mean dep. var.	4.575	1.015	5.779

Notes: This table contains the results of the linear regressions described in equation (2) for the main variables of interest. SE in parenthesis, clustering is done at the Country-Year Level. Stars: * for p<.10, ** for p<.05, and *** for p<.01.

Table 9: Linear regressions for main variables of interest, including a lag

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.322***	0.324***	0.730***
	(0.017)	(0.032)	(0.227)
L.Rate on Av. Earning Assets	0.480***		
	(0.023)		
L.Deposit Rate		0.499***	
		(0.046)	
L.ROAE			0.333***
			(0.033)
N	74096	28209	74209
R squared	0.954	0.916	0.473
Mean dep. var.	4.491	0.927	5.655

Notes: Linear regressions for main variables of interest, including a lag. Clustering is done at the Country-Year Level. Stars: * for p<.10, ** for p<.05, and *** for p<.01.

B.3 Locally-Weighted Regressions

The results of the locally weighted regressions described in section II.B, with and without a lag of the dependent variable respectively, are shown in Figures 9 and 10. The results are consistent with the predictions of the model. The loan rate decreases with the policy rate both for high and low levels of the policy rate, and at a similar rate in both cases. The deposit rate does not react much to the policy rate at low levels of the policy rate but does react for high levels. Return on equity reacts strongly to the policy rate at low rates but reacts less at high rates.

While somewhat informative, this approach does not allow for the identification of the break point, because the residualized measure has no direct connection with the underlying policy rate. In the figures I draw a vertical line at certain level of the residualized policy, but this level varies depending on whether or not the regressions include a lag, and (even though it is informed by the average level at which the residualized measure would hit the 50 basis point threshold) it is chosen somewhat arbitrarily. That is why in the text I focus on the regression threshold framework which allows for a very clear analysis of the threshold.

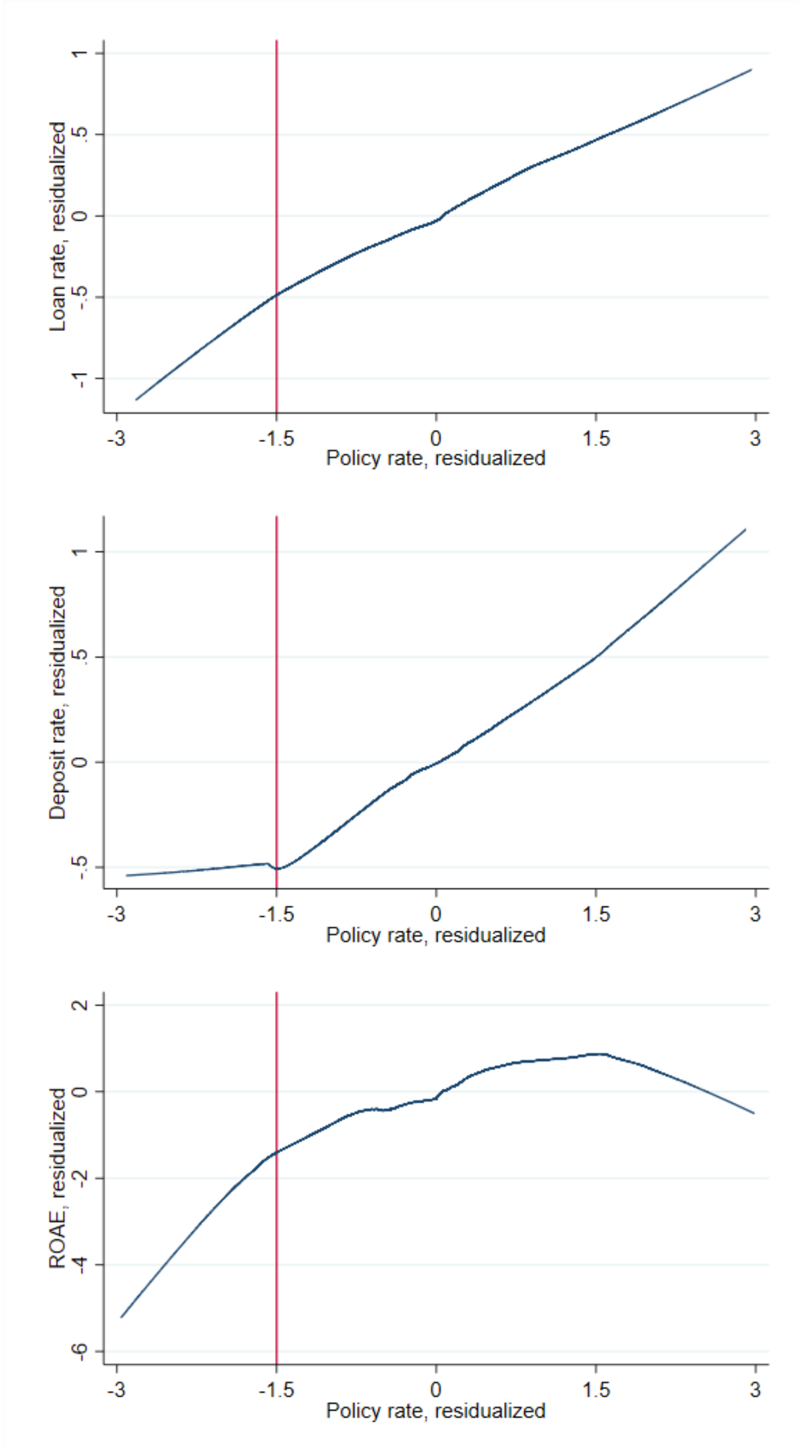

Figure 9: Locally-weighted regressions, including a lag

Notes: This figure contains the behavior of the loan rate, deposit rate and return on average equity (ROAE) with respect to the policy rate in the selected sample of banks. All quantities have been residualized using bank fixed effects, year fixed effects and one lag of the dependent variable, and clustered at the country level. The graphs show the line from a locally weighted regression using tricube weighting with 0.5 bandwidth.

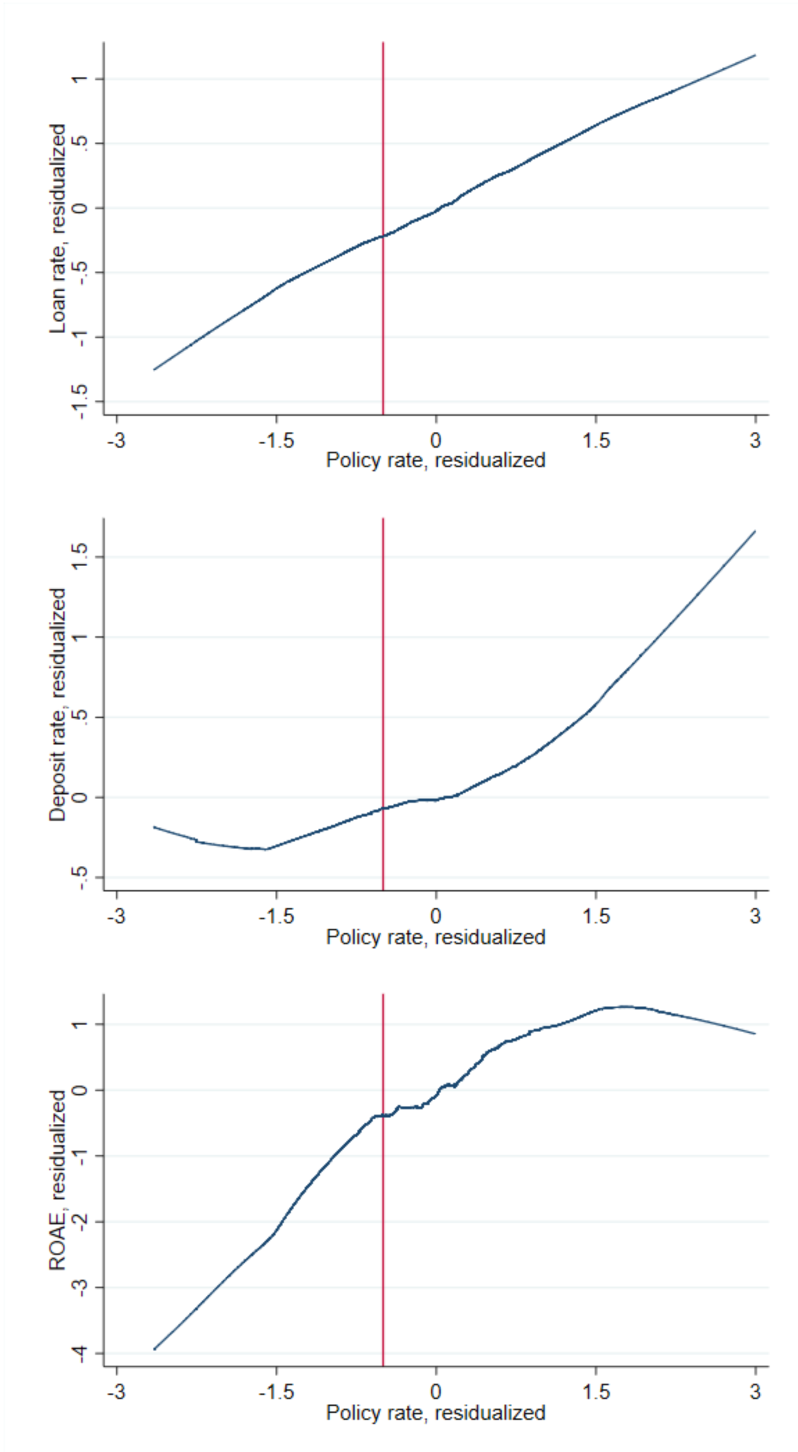

Figure 10: Locally-weighted regressions, no lag

Notes: This figure contains the behavior of the loan rate, deposit rate and return on average equity (ROAE) with respect to the policy rate in the selected sample of banks. All quantities have been residualized using bank fixed effects and year fixed effects, and clustered at the country level. The graphs show the line from a locally weighted regression using tricube weighting with 0.5 bandwidth.

B.4 Robustness of Threshold Effects

In this appendix I document the robustness of the results presented in section II.B to different modifications of the baseline specification. Table 11 contains the results of the regressions that include a lag of the dependent variable. Tables 10 and 12 present the results of regressions that allow for a break in level at the threshold $\tilde{\imath}$. Table 13 contains the results of regressions that include bank-level time-varying characteristics, like the amount of bank equity or bank assets (either contemporaneous or lagged one period). Finally, Tables 14 and 15 present the results of regressions that control for indicators of banking or financial crises. The tables present the results for the Reinhart and Rogoff (2014b) indicator for all crises, but the results are similar if one uses their systemic indicator, the indicators in Laeven and Valencia (2013b) or the Romer and Romer (2017b) indicator (data: Laeven and Valencia, 2013a; Romer and Romer, 2017a; Reinhart and Rogoff, 2014a). In all of these cases the results are qualitatively similar to the ones from the baseline specification.

Table 10: Main regressions, change in level at $\tilde{\imath}$ and no lag

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.398***	−0.195	4.804***
	(0.132)	(0.156)	(1.191)
$(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	−0.001	0.591***	−4.020***
	(0.127)	(0.150)	(1.113)
$\mathbb{1}(i \geq \tilde{\imath})$	0.337***	0.280**	0.364
	(0.098)	(0.120)	(0.867)
$\beta_1 + \beta_2$	0.398***	0.396***	0.784***
s.e.$(\beta_1 + \beta_2)$	0.026	0.043	0.224
N	80078	31554	80199
R squared	0.931	0.851	0.407
Mean dep. var.	4.575	1.015	5.779

Notes: This table is equivalent to table 2 but when one allows for the existence of a break at the threshold $\tilde{\imath}$ (taken to be 50 basis points in this table). S.E. are in parentheses. Clustering is done at the country-year level. Stars: * for p<.10, ** for p<.05, *** for p<.01.

Table 11: Main regressions, lag of dependent variable

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.391***	0.134*	3.495***
	(0.088)	(0.077)	(0.855)
$(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	−0.070	0.200**	−2.847***
	(0.086)	(0.091)	(0.812)
L.Rate on Av. Earning Assets	0.479***		
	(0.023)		
L.Deposit Rate		0.492***	
		(0.046)	
L.ROAE			0.326***
			(0.032)
$\beta_1 + \beta_2$	0.320***	0.334***	0.648***
s.e.$(\beta_1 + \beta_2)$	0.017	0.032	0.208
N	74096	28209	74209
R squared	0.954	0.917	0.475
Mean dep. var.	4.491	0.927	5.655

Notes: This table is equivalent to table 2 but when one allows for a lag of the dependent variable. S.E. are in parentheses. Clustering is done at the country-year level. Stars: * for p<.10, ** for p<.05, *** for p<.01.

Table 12: Main regressions, lag and change in level at $\tilde{\imath}$

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.367***	0.145	3.488***
	(0.106)	(0.098)	(1.002)
$(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	−0.050	0.193*	−2.841***
	(0.100)	(0.101)	(0.924)
$\mathbb{1}(i \geq \tilde{\imath})$	0.048	−0.019	0.013
	(0.076)	(0.082)	(0.843)
L.Rate on Av. Earning Assets	0.477***		
	(0.023)		
L.Deposit Rate		0.493***	
		(0.047)	
L.ROAE			0.326***
			(0.032)
$\beta_1 + \beta_2$	0.317***	0.338***	0.647***
s.e.$(\beta_1 + \beta_2)$	0.019	0.035	0.223
N	74096	28209	74209
R squared	0.954	0.917	0.475
Mean dep. var.	4.491	0.927	5.655

Notes: This table is equivalent to table 2 but when one allows for a lag of the dependent variable and for the existence of a break at the threshold $\tilde{\imath}$ (taken to be 50 basis points in this table). S.E. are in parentheses. Clustering is done at the country-year level. Stars: * for p<.10, ** for p<.05, *** for p<.01.

Table 13: Main regressions, lag and controls for bank equity and assets

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.382***	0.107	3.950***
	(0.087)	(0.072)	(0.928)
$(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	−0.061	0.232***	−3.302***
	(0.085)	(0.086)	(0.879)
Log of Equity	−0.054	−0.040	3.467***
	(0.038)	(0.037)	(0.787)
Log of Total Assets	0.072*	0.229***	−2.791***
	(0.038)	(0.048)	(0.661)
L.Rate on Av. Earning Assets	0.479***		
	(0.023)		
L.Deposit Rate		0.485***	
		(0.046)	
L.ROAE			0.310***
			(0.032)
$\beta_1 + \beta_2$	0.321***	0.339***	0.648***
s.e.$(\beta_1 + \beta_2)$	0.017	0.031	0.200
N	74037	28185	74182
R squared	0.954	0.918	0.482
Mean dep. var.	4.491	0.927	5.663

Notes: This table is equivalent to table 2 but when one allows for a lag of the dependent variable and controls for the level of bank equity and assets for each individual bank. S.E. are in parentheses. Clustering is done at the country-year level. Stars: * for p<.10, ** for p<.05, *** for p<.01.

Table 14: Main regressions, controlling for Reinhart and Rogoff's banking crisis indicators, no lag

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.704***	−0.243	9.082***
	(0.204)	(0.217)	(1.763)
$(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	−0.301	0.670***	−8.460***
	(0.204)	(0.210)	(1.742)
Reinhart and Rogoff BCI	0.047	0.302***	−1.689***
	(0.075)	(0.087)	(0.633)
$\beta_1 + \beta_2$	0.403***	0.427***	0.622***
s.e.$(\beta_1 + \beta_2)$	0.028	0.038	0.210
N	68960	24716	69049
R squared	0.928	0.863	0.422
Mean dep. var.	4.854	1.149	5.872

Notes: This table is equivalent to table 2 but when one controls for the Banking Crisis Indicator of Reinhart and Rogoff. S.E. are in parentheses. Clustering is done at the country-year level. Stars: * for p<.10, ** for p<.05, *** for p<.01.

Table 15: Main regressions, controlling for Reinhart and Rogoff's banking crisis indicators, with a lag

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.370^{**}	-0.153	6.922^{***}
	(0.153)	(0.158)	(1.536)
$(i-\tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	-0.045	0.484^{***}	-6.357^{***}
	(0.151)	(0.164)	(1.516)
Reinhart and Rogoff BCI	0.039	0.146^{**}	-1.436^{**}
	(0.070)	(0.066)	(0.676)
L.Rate on Av. Earning Assets	0.446^{***}		
	(0.025)		
L.Deposit Rate		0.442^{***}	
		(0.050)	
L.ROAE			0.301^{***}
			(0.034)
$\beta_1 + \beta_2$	0.325^{***}	0.332^{***}	0.566^{***}
s.e.$(\beta_1 + \beta_2)$	0.018	0.034	0.217
N	63001	21418	63074
R squared	0.949	0.918	0.479
Mean dep. var.	4.782	1.056	5.737

Notes: This table is equivalent to table 2 but when one controls for the Banking Crisis Indicator of Reinhart and Rogoff as well as a lag of the dependent variable. S.E. are in parentheses. Clustering is done at the country-year level. Stars: * for p<.10, ** for p<.05, *** for p<.01.

Table 16: Main regressions when $\tilde{\imath} = 0$

	(1) Loan Rate	(2) Deposit Rate	(3) ROAE
Policy Rate	0.467	0.036	3.597***
	(0.310)	(0.324)	(1.168)
$(i - \tilde{\imath}) * \mathbb{1}(i \geq \tilde{\imath})$	-0.039	0.399	-2.685**
	(0.305)	(0.331)	(1.113)
$\beta_1 + \beta_2$	0.428***	0.435***	0.912***
s.e.$(\beta_1 + \beta_2)$	0.027	0.039	0.232
N	80078	31554	80199
R squared	0.931	0.846	0.402
Mean dep. var.	4.575	1.015	5.779

Notes: This table is equivalent to table 2 but when the threshold level $\tilde{\imath}$ is taken to be 0 instead of 50 basis points. S.E. are in parentheses. Clustering is done at the country-year level. Stars: * for p<.10, ** for p<.05, *** for p<.01.

B.5 Additional Threshold Tests

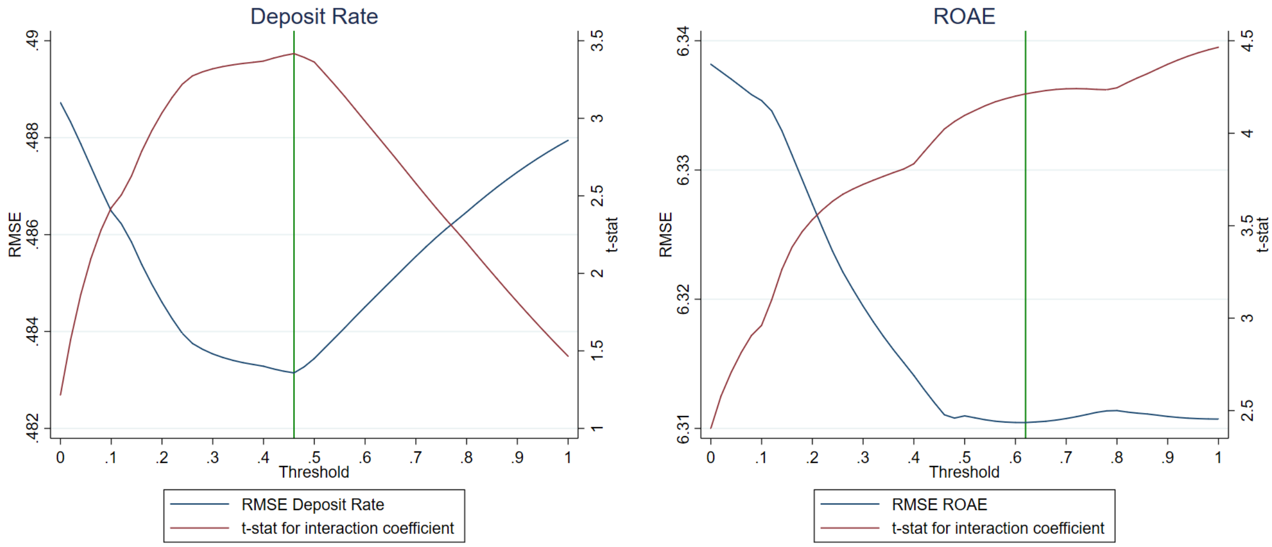

Figure 11: RMSE for threshold tests

Notes: This figure plots the root mean squared error (RMSE) and the t-stat on the interaction coefficient (β_2) for the regression in (6) across different values of $\tilde{\iota}$. The dependent variable is the deposit rate in the left panel, and return on average equity (ROAE) in the right panel.

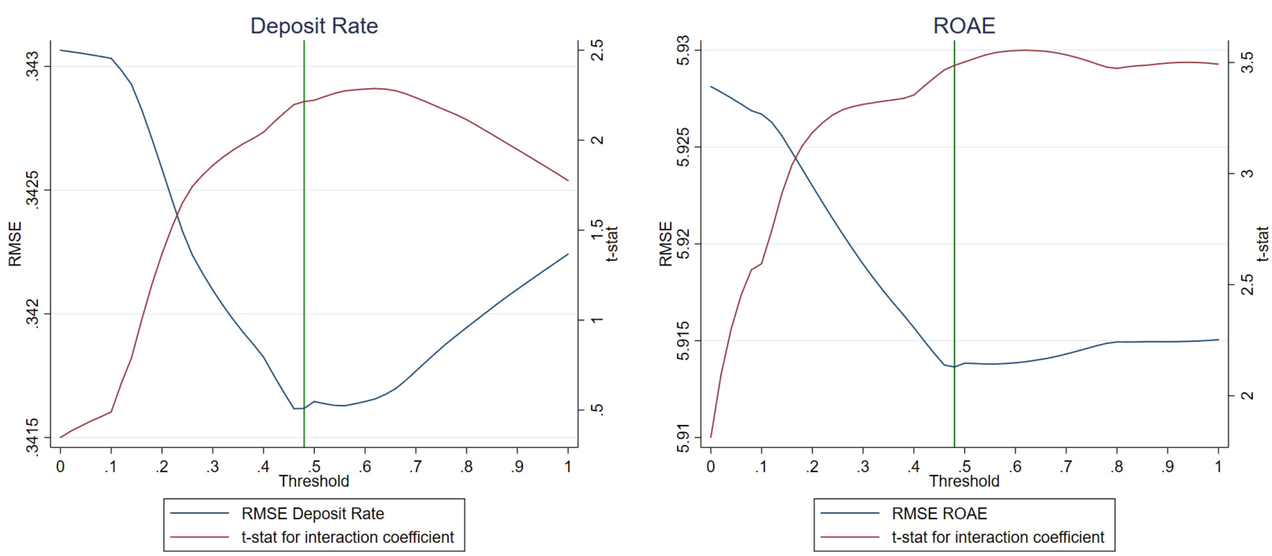

Figure 12: RMSE for threshold tests, lag

Notes: These figures contain the RMSE and t-stat on the interaction coefficient β_2 for Regression (6) with a lag of the dependent variable for the deposit rate and ROAE for different values of the threshold level $\tilde{\iota}$.

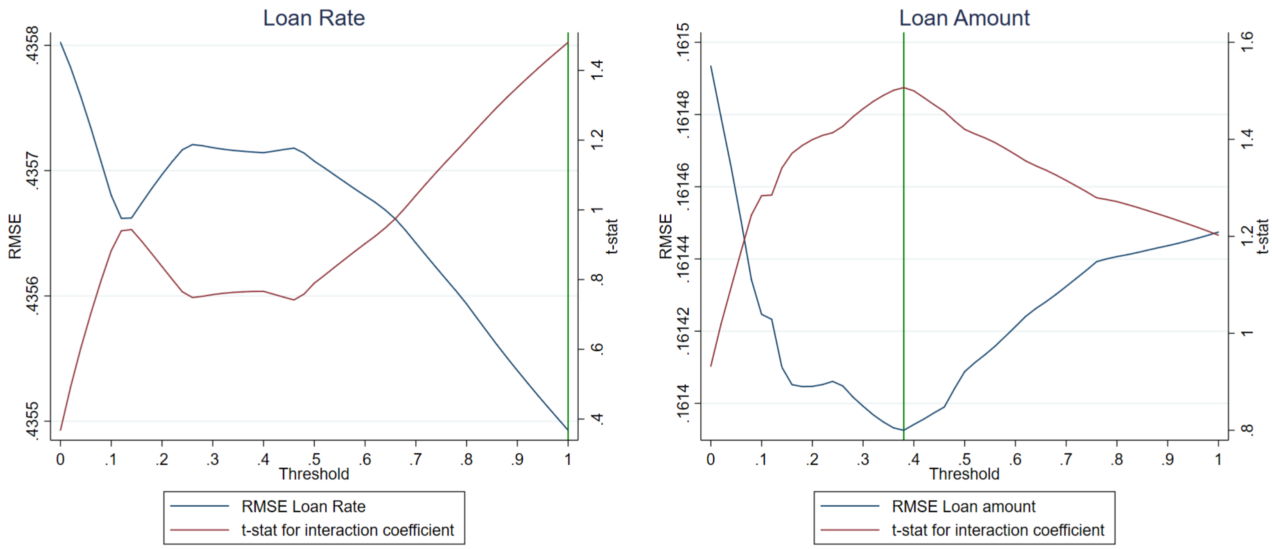

Figure 13: RMSE for threshold tests, other variables

Notes: These figures contain the RMSE and t-stat on the interaction coefficient β_2 for Regression (6) with a lag of the dependent variable for the loan rate and the loan amount for different values of the threshold level $\tilde{\iota}$.

As pointed out by Andrews (1993) and Hansen (1999), inference in the presence of an unknown threshold is complicated by the presence of a nuisance parameter, this comes from the fact that the break point is not present under the null-hypothesis. Andrews (1993) proposes the use of supremum statistics to solve this issue and Hansen (1999) proposes a bootstrap-based method. The Hansen methodology is only theoretically applicable for nondynamic panels and it requires a balanced panel, so to apply it in my context I turn my dataset into a balanced panel and do not include the lag of the dependent variable. Running the Hansen (1999) procedure on data for the deposit rate (a balanced panel between 2009 and 2016 with 1986 banks per year) identifies a break point at 62 basis points and rejects the null-hypothesis of no break point at the 1% level. Running it on data for ROAE (a balanced panel between 2006 and 2016 with 3401 banks per year) identifies a break point at 47 basis points and also rejects the null-hypothesis of no break point at the 1% level.[2] The full results of the Hansen (1999) procedure are shown in table 17.

Another possible test for the threshold level using equation (6) for the deposit rate, that utilizes information on both β_1 and β_2, is based on Chay and Munshi (2015). In my setup, as in theirs, if the true threshold is picked, β_1 should be close to zero and β_2 should be positive. If a threshold candidate below the true threshold is picked, β_1 should still be estimated as zero but β_2 should be estimated as a smaller quantity. If a threshold candidate above the true threshold is picked, β_1 should now be estimated as positive but β_2 should be estimated as a higher quantity. The authors develop a test for the joint hypothesis that

[2] I choose the starting year of the panel to maximize the total number of observations remaining in the balanced panel. This year turns out to be 2009 for the case of the deposit rate and 2006 for ROAE. Running the test for the deposit rate starting the balanced panel in 2006 produces similar results.

Table 17: **Main regressions with the Hansen (1999) procedure**

	(1) Deposit Rate	(2) ROAE
Slope below threshold	-0.188	5.575^{***}
	(0.169)	(1.192)
Slope above threshold	0.471^{***}	1.022^{***}
	(0.076)	(0.161)
Threshold lower bound	0.503	0.464
Threshold estimate	0.625^{***}	0.465^{***}
Threshold upper bound	0.990	0.500
N	15888	37411
R squared	0.358	0.078
Mean dep. var.	0.784	4.846

Notes: This table contains the results of regressing the deposit rate and ROAE on bank fixed effects, time fixed effects, and the policy rate, but allowing for a different slope with respect to the policy rate above and below the threshold level $\tilde{\imath}$. These regressions incorporate the fact that the threshold level $\tilde{\imath}$ is unknown and uncertain. The thresholds (together with their 95% confidence bands) are estimated using the Hansen (1999) procedure. The threshold is estimated at 62 basis points for the deposit rate and at 46 basis points for ROAE. The stars in the threshold estimate represent that the null hypothesis of no threshold brek is rejected at the 1% level (they do not represent that the threshold is significantly different from zero, although this is also true). The results for the loan rate are not shown since the Hansen procedure doesn't identify a break for that variable. S.E. are in parentheses. Clustering is done at the country-year level. Stars: * for p<.10, ** for p<.05, *** for p<.01.

$\beta_1 = 0$ and $\beta_2 > 0$ based on the test statistic:

$$\Upsilon^{CM} = \frac{\left[\phi\left(\frac{\hat{\beta}_1}{h}\right)\right]^2}{\left[\phi\left(\epsilon\right)\right]^2} \frac{\hat{\beta}_2^2}{\hat{V}_{\beta_2}}, \tag{3}$$

where ϕ is a symmetric and continuous function that reaches its maximum value at zero (the authors use the normal p.d.f.), h is a scale parameter, ϵ is the value below which the normalized baseline slope coefficient, $\frac{\hat{\beta}_1}{h}$, is treated as "zero", and $\hat{V}_{\beta_2}$ is the estimated variance of β_2. The authors use the normalization by h because $\hat{\beta}_1$ will be further away from zero when the outcome variable has a larger mean or variance, the authors set h to be the standard deviation of the outcome under consideration, in my case the standard deviation of the deposit rate. I will set $\epsilon = 0$, as the authors do, to be conservative. The statistic is distributed as a chi-square with one degree of freedom. The statistic is presented in Figure 14 for the case that does not include a lag of the dependent variable, the statistic is maximized at $\tilde{\imath} = 0.46$ (46 basis points). The p-value of the statistic is given in the secondary axis, and the green horizontal line indicates the 5% threshold for rejection of the null. This means that for thresholds between 10 and 80 basis points there is evidence that $\beta_1 = 0$ and $\beta_2 > 0$ which is what the model predicts. Like the Hansen (1999) procedure, the Chay and Munshi procedure was in theory developed for a nondynamic panel, that is why I do not include the lag of the dependent variable, but including it yields similar results.

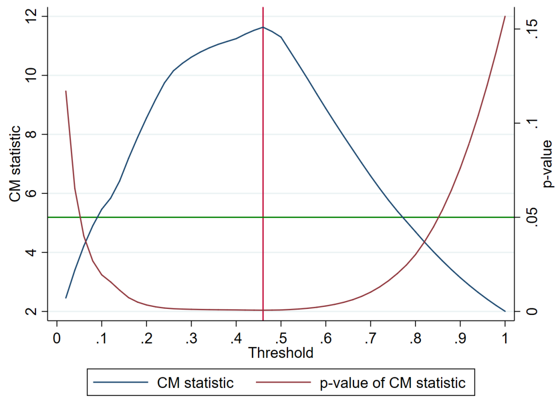

Figure 14: CM statistic for the deposit rate

Notes: This figure contains the Chay and Munshi (CM) statistic for the deposit rate based on equation (6) and its p-value, for different values of the threshold level $\tilde{\iota}$.

B.6 Quintile Description

Table 18: CDA for different quintiles

Quintile	Mean	Std. Dev.	Min.	Max.	N
First	0.43	0.18	0.01	0.63	16726
Second	0.70	0.03	0.63	0.75	16717
Third	0.78	0.02	0.75	0.82	16721
Fourth	0.85	0.02	0.82	0.88	16738
Fifth	0.91	0.02	0.88	0.96	16697

Notes: Description of the CDA ratio variable for different quintiles

B.7 Additional Structural Estimation Results

The following table summarizes the results of the regressions in equation (21), when assuming that $\nu = 9$ (the mean loan-to-equity ratio in my dataset). Column 1 presents the results when the regression includes two lags of the dependent variable and $F_{b,t-1}$ is instrumented with $F_{b,t-3}$, to avoid potential endogeneity with the two lags of the dependent variable. Column 2 presents the results when the regression includes just one lag of the dependent variable and $F_{b,t-1}$ is instrumented with $F_{b,t-2}$. Column 3 presents the results when the regression does not include any lags of the dependent variable and $F_{b,t-1}$ is not instrumented.

The results are similar for the three models and deliver an estimate of κ between 15 and 35 basis points, together with an estimate of ε^l between 40 and 60 (at the annual level).

Table 19: Aggregate structural estimation of κ

	(1)	(2)	(3)
γ_{la}	0.4549	0.3584	0.6624
γ_{lr}	-0.0100	-0.0086	-0.0104
κ	0.0020	0.0015	0.0034
ε^l	45.5052	41.5496	63.8244
$\mathrm{i}^l - i$	0.0222	0.0244	0.0158

Notes: This table contains the results of the aggregate structural estimation of κ and ε^l described in equation (21).

These estimates of ε^l would deliver a steady state wedge between the loan rate and the policy rate of between 1.5% and 2.5% which is reasonable. The baseline specification will be the first one, since it is necessary to include the lags of the dependent variable to control for the sluggishness in rate setting and lending behavior. The following table contains the region-level results of the regressions for Denmark, Sweden, Canada, Australia and Norway.

Table 20: Structural estimation of κ and ε^l, part 2

	(1)	(2)	(3)	(4)	(5)
	DKK	SEK	CAD	AUD	NOK
γ_{la}	0.7844	0.6058	1.2854	0.1653	-0.5330
γ_{lr}	-0.0047	-0.0030	0.0588	-0.0081	0.0099
κ	0.0024	0.0008	0.0229	0.0011	-0.0007
ε^l	168.6147	202.4669	-21.8763	20.3058	54.0099
$\mathrm{i}^l - i$	0.0059	0.0050	-0.0447	0.0505	0.0187

Notes: This table contains the results of the country level structural estimation of κ and ε^l described in equation (21) for the 5 smallest regions.

Appendix C Model Solution

C.1 Equilibrium Equations

The equilibrium is characterized by the relevant equations for each of the types of agents in the model. Households have an intratemporal condition for labor supply, an Euler equation, the definition of the marginal utility of consumption and the definition of the stochastic discount factor:

$$
\begin{aligned}
\chi N_t^{\frac{1}{\eta}} &= \phi_t \frac{W_t}{P_t} \\
1 &= \mathbb{E}_t\left(\beta \Lambda_{t,t+1}(1+i_t^d)\frac{P_t}{P_{t+1}}\right) \\
\phi_t &= (C_t - hC_{t-1})^{-\sigma} - \beta h \frac{\varphi_{t+1}}{\varphi_t}\mathbb{E}_t(C_{t+1}-hC_t)^{-\sigma} \\
\Lambda_{t,t+1} &= \frac{\phi_{t+1}}{\phi_t}\frac{\varphi_{t+1}}{\varphi_t}.
\end{aligned}
$$

Intermediate goods firms have their production function, a labor demand equation and the definition of the return on capital:

$$
\begin{aligned}
Y_t^m &= A_t(\xi_t K_t)^{\alpha} N_t^{1-\alpha} \\
(1-\alpha)\frac{P_t^m}{P_t}\frac{Y_t^m}{N_t} &= \frac{W_t}{P_t} \\
1+i_t^l &= \frac{\frac{Q_t}{P_t}\xi_t(1-\delta)+\frac{P_t^m}{P_t}\alpha\frac{Y_t^m}{K_t}}{\frac{Q_{t-1}}{P_{t-1}}}\frac{P_t}{P_{t-1}}.
\end{aligned}
$$

Capital producing firms have the evolution of capital and the F.O.C. for the price of capital:

$$
\begin{aligned}
K_{t+1} &= (1-\delta)\xi_t K_t + I_t \\
\frac{Q_t}{P_t} &= 1 + f\left(\frac{I_t}{I_{t-1}}\right) + f'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} - \mathbb{E}_t\beta\Lambda_{t,t+1} f'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2.
\end{aligned}
$$

Retail firms have equations for price setting, the evolution of prices, the dispersion of prices and the relationship between final output and intermediate output:

$$
\begin{aligned}
1 &= (1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{1-\theta} + \gamma\left(\frac{P_{t-1}}{P_t}\right)^{1-\theta} \\
\theta\Gamma_t^1 &= (\theta-1)\Gamma_t^2 \\
\Gamma_t^1 &= \phi_t\varphi_t\frac{P_t^m}{P_t}Y_t + \gamma\beta\mathbb{E}_t\left(\frac{P_t}{P_{t+1}}\right)^{-\theta}\Gamma_{t+1}^1 \\
\Gamma_t^2 &= \phi_t\varphi_t\frac{P_t^*}{P_t}Y_t + \gamma\beta\mathbb{E}_t\frac{P_t^*}{P_{t+1}^*}\left(\frac{P_t}{P_{t+1}}\right)^{-\theta}\Gamma_{t+1}^2 \\
Y_t^m &= Y_t v_t^p
\end{aligned}
$$

$$v_t^p = \gamma \left(\frac{P_{t-1}}{P_t}\right)^{-\theta} v_{t-1}^p + (1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{-\theta}.$$

Banks have equations for the deposit rate, the loan rate, bank profits, bank equity evolution and the bank balance sheet constraint:

$$\begin{aligned}
1+i_t^d &= \frac{\varepsilon^d}{\varepsilon^d-1}(1+i_t+\mu_t^d) \\
\mathbb{E}_t(1+i_{t+1}^l) &= \frac{\varepsilon^l}{\varepsilon^l-1}(1+i_t+\mu_t^l) + \kappa\nu\frac{\varepsilon^l}{\varepsilon^l-1}\left(\ln\left(\frac{L_t}{F_t}\right)-\ln(\nu)\right) \\
\frac{P_t}{P_{t-1}}\frac{X_t}{P_t} &= i_{t-1}\frac{F_{t-1}}{P_{t-1}} + (i_t^l-\mu_{t-1}^l-i_{t-1})\frac{L_{t-1}}{P_{t-1}} + (i_{t-1}+\mu_{t-1}^d-i_{t-1}^d)\frac{D_{t-1}}{P_{t-1}} \\
&- \Psi\left(\frac{L_{t-1}}{F_{t-1}};\kappa,\nu\right)\frac{F_{t-1}}{P_{t-1}} - \frac{F_{t-1}}{P_{t-1}}(1-\varsigma)\pi_t \\
\frac{F_t}{P_t} &= (1-\varsigma)\frac{F_{t-1}}{P_{t-1}} + \omega\frac{X_t}{P_t} \\
\frac{L_t}{P_t}+\frac{H_t}{P_t} &= \frac{F_t}{P_t}+\frac{D_t}{P_t}.
\end{aligned}$$

Finally one has the resource constraint, Taylor rule and the condition saying that total loans must equal the value of capital:

$$\begin{aligned}
Y_t &= C_t + I_t + G_t + f\left(\frac{I_t}{I_{t-1}}\right)I_t + \mu_t^l\frac{P_{t-1}}{P_t}\frac{L_{t-1}}{P_{t-1}} - \mu_t^d\frac{P_{t-1}}{P_t}\frac{D_{t-1}}{P_{t-1}} \\
&+ \varsigma\frac{P_{t-1}}{P_t}\frac{F_{t-1}}{P_{t-1}} + \Psi\left(\frac{L_{t-1}}{F_{t-1}};\kappa,\nu\right)\frac{P_{t-1}}{P_t}\frac{F_{t-1}}{P_{t-1}} \\
i_t &= (1-\rho_i)\left(\bar{\imath} + \psi_\pi(\pi_t-\overline{\pi}) + \psi_y\frac{Y_t-Y_t^*}{\overline{Y}}\right) + \rho_i i_{t-1} + \epsilon_t^i \\
\frac{L_t}{P_t} &= \frac{Q_t}{P_t}K_{t+1}.
\end{aligned}$$

This is a system of 23 equations in 23 unknowns $(N, \phi, W/P, \Lambda, i^d, \pi, C,\ Y^m, K, P^m/P, i^l, Q/P,\ I, P^*/P,\ \Gamma^1, \Gamma^2, Y,\ v^p, i, L/P, F/P, X/P, D/P)$ which can be used to solve for the equilibrium. The processes for the shocks are given by:

$$\begin{aligned}
A_t &= \overline{A}^{1-\rho_a}A_{t-1}^{\rho_a}e^{\epsilon_t^a} \\
G_t &= \overline{G}^{1-\rho_g}G_{t-1}^{\rho_g}e^{\epsilon_t^g} \\
H_t &= \overline{H}^{1-\rho_h}H_{t-1}^{\rho_h}e^{\epsilon_t^h} \\
\xi_t &= \xi_{t-1}^{\rho_\xi}e^{\epsilon_t^\xi} \\
\varphi_t &= \varphi_{t-1}^{\rho_\varphi}e^{\epsilon_t^\varphi}.
\end{aligned}$$

A_t is the technology of intermediate good producers, G_t is government expenditure in goods, H_t is the total amount of central bank reserves, ξ_t is capital efficiency and φ_t is the shock to the discount factor. Additionally $\mu_t^l = \mu^l$ and $\mu_t^d = \mu^d$. I choose $\overline{A} = 1$ as a normalization,

$\overline{G}$ is pinned down by the equation $\frac{\overline{G}}{\overline{Y}} = 0.2$ and $\overline{H}$ is pinned down by the parameter $\overline{H}/\overline{F}$ which as I explain in the calibration section is set to 2. The steady state value of capital efficiency ξ is 1, as is the steady state value of φ, which is just a normalization.

C.2 Steady State

In steady state with zero inflation the equations simplify a lot. For the retailers, for example, $P^* = P$, $v^p = 1$, $Y^m = Y$ and $\frac{P^m}{P} = \frac{\theta-1}{\theta}$. I can also get rid of the equations for Λ and Q/P. This way the 6 equations for retailers plus 2 additional equations are eliminated. Then I get rid of superfluous equations, like $i = \bar{\imath}$, the balance sheet of the banks (which just defines D/P), the equation for X/P, the equation for F/P and the equation for L/P. I obtain the following system:[3]

$$
\begin{aligned}
\chi N^{\frac{1}{\eta}} &= \phi \frac{W}{P} \\
1 &= \beta(1+i^d) \\
\phi &= C^{-\sigma}(1-h)^{-\sigma}(1-\beta h) \\
Y &= AK^\alpha N^{1-\alpha} \\
(1-\alpha)\frac{\theta-1}{\theta}\frac{Y}{N} &= \frac{W}{P} \\
\alpha\frac{\theta-1}{\theta}\frac{Y}{K} + (1-\delta) &= (1+i^l) \\
1+i^d &= \frac{\varepsilon^d}{\varepsilon^d-1}(1+i+\mu^d) \\
1+i^l &= \frac{\varepsilon^l}{\varepsilon^l-1}(1+i+\mu^l) \\
Y &= C+I+G+\mu^l\frac{L}{P} - \mu^d\frac{D}{P} + \varsigma\frac{F}{P} \\
I &= \delta K.
\end{aligned}
$$

Then I can get rid of the Euler equation and the bank equations defining interest rates, since $1+i^d = \frac{1}{\beta}$, $1+i = \frac{\varepsilon^d-1}{\varepsilon^d}\frac{1}{\beta} - \mu^d$ and $1+i^l = \frac{\varepsilon^l}{\varepsilon^l-1}\left(\frac{\varepsilon^d-1}{\varepsilon^d}\frac{1}{\beta} - \mu^d + \mu^l\right)$. I can also get rid of investment (it is just δK), ρ, the real wage and the definition of output (I assume $G = gY$). I obtain the 3 equation system:

$$
\begin{aligned}
\chi N^{\frac{1}{\eta}} &= C^{-\sigma}(1-h)^{-\sigma}(1-\beta h)(1-\alpha)\frac{\theta-1}{\theta}AK^\alpha N^{-\alpha} \\
\alpha\frac{\theta-1}{\theta}AK^{\alpha-1}N^{1-\alpha} + (1-\delta) &= \frac{\varepsilon^l}{\varepsilon^l-1}\left(\frac{\varepsilon^d-1}{\varepsilon^d}\frac{1}{\beta} - \mu^d + \mu^l\right) \\
(1-g)AK^\alpha N^{1-\alpha} &= C + \left(\delta + \mu^l - \mu^d\left(1+\frac{H/F-1}{\nu}\right) + \frac{\varsigma}{\nu}\right)K.
\end{aligned}
$$

[3]Recall that $\frac{L/P}{F/P} = \nu$ and $L/P = K$, hence I know that $F/P = K/\nu$, also from the balance sheet of the banks I know $\frac{L}{F} + \frac{H}{F} = \frac{D}{F} + 1$ so $\frac{D/P}{F/P} = \nu + \frac{H}{F} - 1$, which implies $D/P = \left(\nu + \frac{H}{F} - 1\right)\frac{K}{\nu}$

The capital labor ratio can be obtained from the second equation:

$$\begin{aligned}
\alpha\frac{\theta-1}{\theta}AK^{\alpha-1}N^{1-\alpha}+(1-\delta) &= \frac{\varepsilon^l}{\varepsilon^l-1}\left(\frac{\varepsilon^d-1}{\varepsilon^d}\frac{1}{\beta}-\mu^d+\mu^l\right) \\
Z\equiv\frac{K}{N} &= \left(\frac{\alpha A(\theta-1)}{\theta\left(\frac{\varepsilon^l}{\varepsilon^l-1}\left(\frac{\varepsilon^d-1}{\varepsilon^d}\frac{1}{\beta}-\mu^d+\mu^l\right)+\delta-1\right)}\right)^{\frac{1}{1-\alpha}}.
\end{aligned}$$

I can introduce this into the other two equations and further introduce the first in the third to obtain:

$$N^{\sigma+\frac{1}{\eta}} = \frac{(\theta-1)(1-\alpha)AZ^{\alpha}(1-\beta h)}{\theta\chi(1-h)^{\sigma}\left((1-g)AZ^{\alpha}-\left(\delta+\mu^l-\mu^d\left(1+\frac{H/F-1}{\nu}\right)+\frac{\varsigma}{\nu}\right)Z\right)^{\sigma}}.$$

And from this variable all the other ones can be backed out. In particular consumption is given by:

$$C = \left(\frac{\chi\theta N^{\frac{1}{\eta}}}{AZ^{\alpha}(1-\alpha)(\theta-1)(1-h)^{-\sigma}(1-\beta h)}\right)^{-\frac{1}{\sigma}}.$$

Appendix D Additional Figures

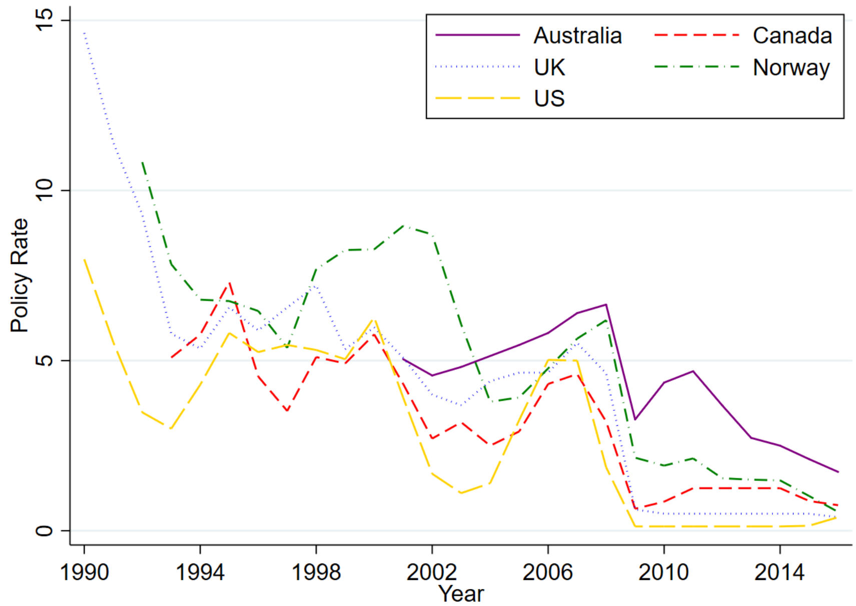

Figure 15: Policy rate across years, positive rates

Notes: This figure contains the policy rate across years for the regions in the sample not setting negative rates.

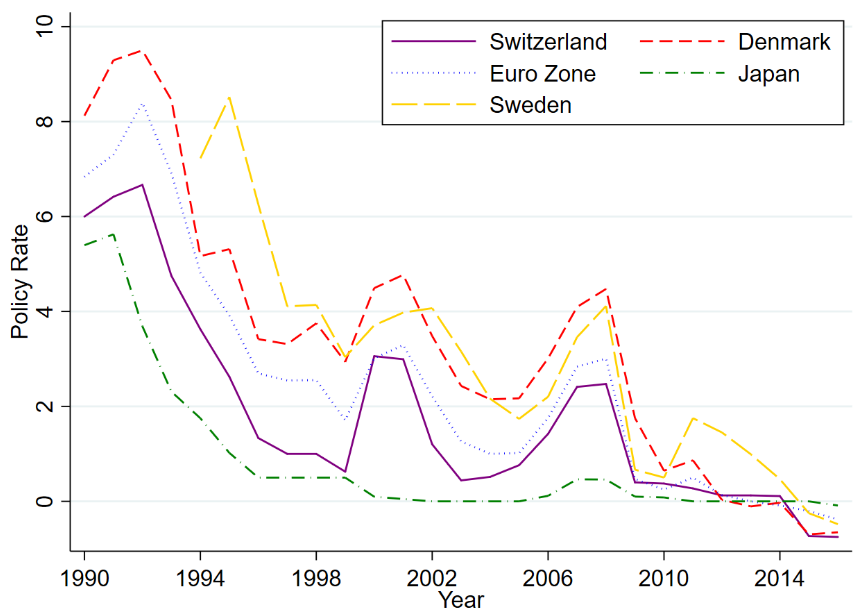

Figure 16: Policy rate across years, negative rates

Notes: This figure contains the policy rate across years for the regions in the sample setting negative rates.

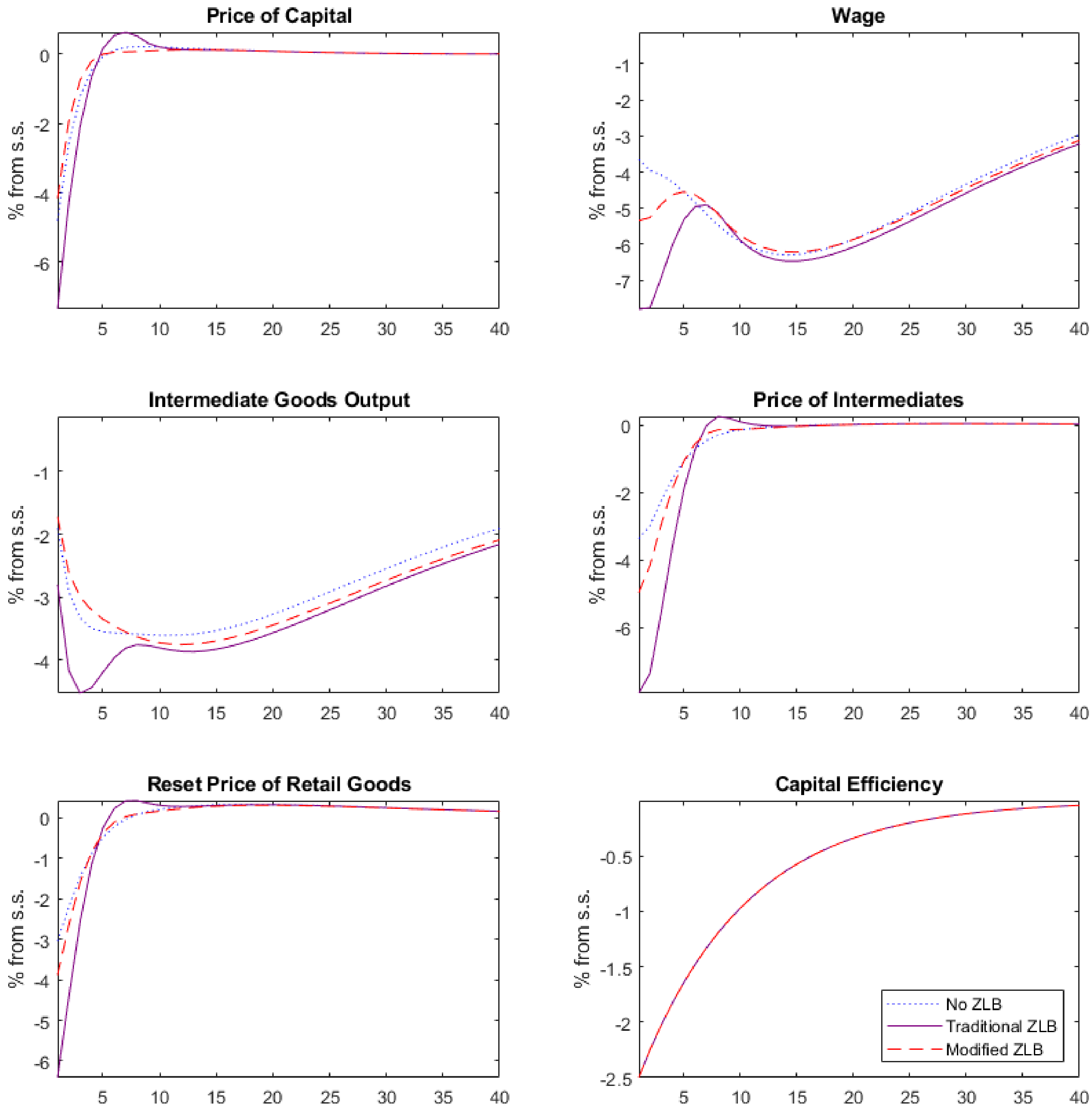

Figure 17: IRF's to capital productivity shock

Notes: IRF's to capital productivity shock under "no ZLB" (blue dotted line), "traditional ZLB" (purple solid line) and "modified ZLB" (red dashed line) when $\kappa = 12.5$ basis points. The x axis is given in quarters and the y axis is given in percent deviations from steady state.

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