Stock Market Wealth and the Real Economy:
A Local Labor Market Approach

Online Appendix

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A Data Details and Omitted Empirical Analyses

A.1 Details on the Capitalization Approach

A.1.1 Details on the IRS SOI

The IRS Statistics of Income (SOI) reports tax return variables aggregated to the zip code for 2004-2015 (and selected years before) and to the county for 1989-2015. Beginning in 2010 for the county files and in all available years for zip code files, the data aggregate all returns filed by the end of December of the filing year. Prior to 2010, the county files aggregate returns filed by the end of September of the filing year, corresponding to about 95% to 98% of all returns filed in that year. In particular, the county files before 2010 exclude some taxpayers who file form 4868, which allows a six month extension of the filing deadline to October 15 of the filing year.[1] To obtain a consistent panel, we first convert the zip code files to a county basis using the HUD USPS crosswalk file. We then implement the following algorithm: (i) for 2010 onward, use the county files; (ii) for 2004-2009, use the zip code files aggregated to the county level and adjusted by the ratio of 2010 dividends in the county file to 2010 dividends in the zip code aggregated file; (iii) for 1989-2003, use the county file adjusted by the ratio of 2004 dividends as just calculated to 2004 dividends

[1] See https://web.archive.org/web/20171019013107/https://www.irs.gov/statistics/soi-tax-stats-county-income-data-users-guide-and-record-layouts and https://web.archive.org/web/20190111012726/https://www.irs.gov/statistics/soi-tax-stats-individual-income-tax-statistics-zip-code-data-soi for data and documentation pertaining to the county and zip code files, respectively. For additional information on the timing of tax filings, see https://web.archive.org/web/20190211151353/https://www.irs.gov/newsroom/2019-and-prior-year-filing-season-statistics .

in the county files. We implement the same adjustment for labor income. We exclude from the baseline sample 74 counties in which the ratio of dividend income from the zip code files to dividend income in the county files exceeds 2 between 2004 and 2009, as the importance of late filers in these counties makes the extrapolation procedure less reliable for the period before 2004.[2]

Finally, since our benchmark analysis is at the quarterly frequency and the SOI income data is yearly data, we linearly interpolate the SOI data to obtain a quarterly series. Because the cross-sectional income distribution is persistent, measurement error arising from this procedure should be small.

A.1.2 Dividend Yield Adjustment

This section describes the county-specific dividend yield adjustment used in the capitalization of taxable county dividends. We start with the Barber and Odean (2000) data set, which contains a random sample of accounts at a discount brokerage, observed over the period 1991-96. The data contain monthly security-level information on financial assets held in the selected accounts. Graham and Kumar (2006) compare these data with the 1992 and 1995 waves of the SCF and show that the stock holdings of investors in the brokerage data are fairly representative of the overall population of retail investors.

We keep taxable individual and jointly owned accounts and remove margin accounts. We merge the monthly account positions data with the monthly CRSP stock price data and CRSP mutual funds data obtained from WRDS. Since our merge is based on CUSIP codes and mutual fund CUSIP codes are sometimes missing, we use a Fund Name-CUSIP crosswalk developed by Terry Odean and Lu Zheng. Additionally, we use an algorithm developed in Di Maggio, Kermani and Majlesi (forthcoming) based on minimizing the smallest aggregate price distance between mutual fund prices in household portfolios and in the CRSP fund-month data.[3] We drop household-month observations for which the

[2] Anecdotally, the filing extension option is primarily used by high-income taxpayers who may need to wait for additional information past the April 15 deadline (see e.g. Dale, Arden, "Late Tax Returns Common for the Wealthy," *Wall Street Journal*, March 29, 2013). Consistent with this, we find much less discrepancy in labor income than dividend income reported in the zip code and county files before 2010. Our results change little if we do not exclude the 74 counties from the analysis. For example, the coefficient for total payroll at the 7 quarter horizon changes from 2.18 to 2.27 (s.e.=0.67), and the coefficient for nontradable payroll changes from 3.23 to 2.67 (s.e.=0.83).

[3] We are grateful to Marco Di Maggio, Amir Kermani, and Kaveh Majlesi for sharing their codes.

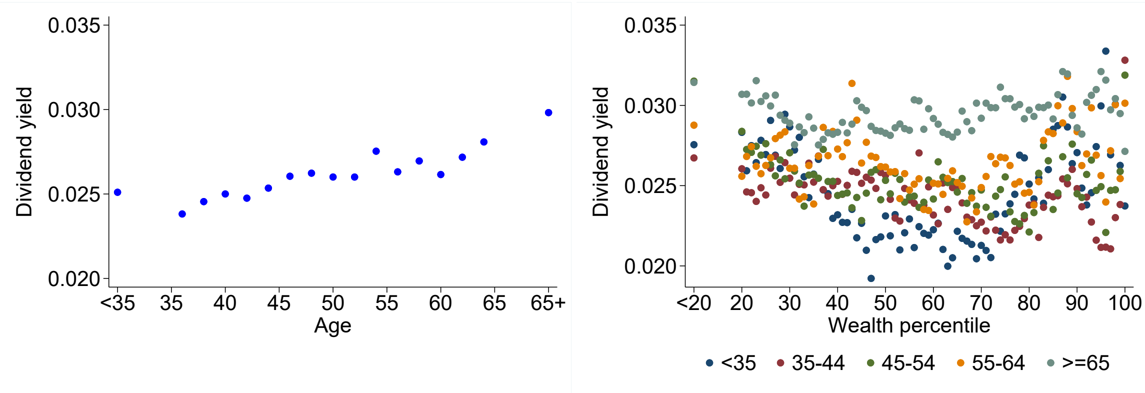

Figure A.1: Dividend Yield by Age and Wealth

Note: The figures plot dividend yields by age and wealth quantile based on the Barber and Odean (2000) data from a discount brokerage firm merged with data on CRSP stocks and mutual funds. Wealth denotes the total position equity among all taxable accounts that a household has in the discount brokerage firm.

value of total identified CRSP stocks and mutual funds is less than 95% of the value of the household's equity and mutual fund assets and also keep only identified CRSP stocks and mutual funds.[4] Finally, to be consistent with what we observe in the IRS-SOI data, we drop household-month observations with a zero dividend yield. Such households tend to be younger, hold few securities (around two on average), and hold only around 10% of total equity in the brokerage data.

We compute dividend yields by household and month using these data. Figure A.1 shows the average dividend yield by age of the household head (left panel) and by stock wealth percentile separately for different age bins (right panel), where household stock wealth is the total position equity in all accounts. As the figure shows, dividend yields increase with age. Moreover, within age bins, dividend yields have a weak relationship with wealth. These patterns motivate our focus on age.

Table A.1 reports average dividend yields by age bin (weighted by wealth), separately for each Census Region. A few features merit mention. First, dividend yield increases with age, consistent with the pattern shown in Figure A.1. Second, the age bin coefficients are precisely estimated and the R^2s are high. In column (5), which pools all geographic areas together, the five age bins explain 66% of the variation in dividend yield across households.

[4]We are able to match more than 95% of equity and mutual fund position-months. The main type of equity assets that we cannot match are foreign stocks.

Third, adding indicator variables for 10 wealth bins to the regression in column (6) has essentially no impact on the explanatory power of the regression or on the relative age bin coefficients.[5]

We combine the coefficients shown in columns (1)-(4) of Table A.1 with the county-year specific age structure from the Census Bureau and average wealth by age bin from the Survey of Consumer Finances (interpolated between SCF waves) to construct the wealth-weighted average of the age bin dividend yields in the county's Census region.[6] The resulting county-year yields account for time series variation in a county's age structure and in relative wealth of different age groups, but not for changes in market dividend yields over time. Therefore, we scale these dividend yields so that the average dividend yield in each year is equal to the dividend yield on the value-weighted CRSP portfolio.[7]

We end this section with a discussion of (implied) dividend yields in the SCF and how those compare to the dividend yield distribution in the Barber and Odean (2000) data. The SCF contains information on taxable dividend income reported on tax returns together with self-reported information on directly held stocks (and stock mutual funds). Therefore, it is tempting to use the SCF data directly to compute dividend yields by demographic groups and use those for the dividend yield adjustment or, even more directly, use the relationship between taxable dividend income and total stock wealth in the SCF to impute total stock wealth directly from taxable dividends rather than doing the two-step procedure that we perform here. Unfortunately, there is one key difficulty in implementing this procedure with SCF data; in the SCF, stock wealth is reported for the survey year (more specifically, at the time of the interview), while taxable dividend income is based on the *previous* year's tax return. This creates biases in any dividend yields computed as the ratio of (previous year) dividend income to (current year) stock wealth. The bias is larger (in magnitude) for participants that (dis-)save more (either actively or passively through capital gains that the household does not respond to). Moreover, as we show in Figure A.2, a very large share of respondent-wave observations (more than 45%) report zero dividend income and positive

[5]The age bin coefficients shift uniformly up by 0.37 to 0.38, reflecting the incorporation of average wealth.

[6]County population-by-age is available from the Census Bureau Interncensal population estimates (1990-2010) and Postcensal population estimates (2010-.). See https://www.census.gov/programs-surveys/popest.html.

[7]We also experimented with allowing the age-specific yields to vary with the CRSP yield, with almost no impact on our results.

Table A.1: Dividend Yields By Age

	Region 1	Region 2	Region 3	Region 4	Pooled	Pooled
	(1)	(2)	(3)	(4)	(5)	(6)
Right hand side variables:						
Age <35	2.81**	2.21**	2.28**	2.51**	2.45**	2.83**
	(0.16)	(0.19)	(0.25)	(0.18)	(0.11)	(0.15)
Age 35-44	2.48**	2.25**	2.43**	2.50**	2.43**	2.81**
	(0.11)	(0.16)	(0.18)	(0.14)	(0.08)	(0.12)
Age 45-54	2.65**	2.27**	2.51**	2.50**	2.49**	2.86**
	(0.16)	(0.09)	(0.30)	(0.08)	(0.08)	(0.13)
Age 55-64	3.00**	2.39**	2.40**	2.82**	2.69**	3.07**
	(0.11)	(0.14)	(0.20)	(0.10)	(0.08)	(0.13)
Age 65+	2.91**	2.73**	2.96**	3.27**	3.03**	3.40**
	(0.12)	(0.12)	(0.17)	(0.11)	(0.07)	(0.12)
Wealth bins	No	No	No	No	No	Yes
R^2	0.73	0.69	0.62	0.63	0.66	0.66
Individuals	1,965	1,586	2,192	3,556	9,299	9,299
Observations	73,486	60,987	83,112	133,149	350,734	350,734

Note: The table reports the coefficients from a regression of the account dividend yield on the variables indicated, at the account-month level. Standard errors in parentheses clustered by account. For readability, all coefficients multiplied by 100.

stock wealth.[8] A large share of those are respondents that establish direct holdings of stocks (or mutual funds) some time between the end of the tax return year and the survey date. An analogous extensive margin adjustment may be taking place for respondents that report zero stock wealth and positive dividend income for the previous year. In that case the implied dividend yield is infinite.

Even if one disregards these two groups and only considers respondents for which the implied dividend yield is between zero and one, there is still substantial dispersion (and a possible bias) in the implied dividend yields. Figure A.3 shows the median implied

[8]This is more than 2 times the account holders with zero dividend yield in the Barber and Odean (2000) data.

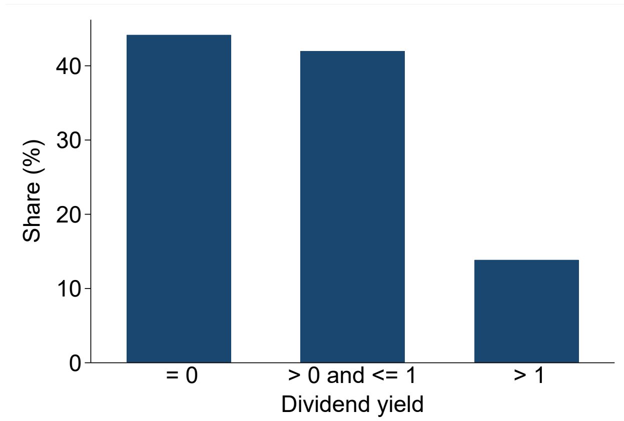

Figure A.2: SCF Implied Dividend Yield Categories

Note: The figure shows the distribution of implied dividend yields in the SCF based on a comparison of the reported dividend income from tax returns against reported directly held stock market wealth.

dividend yields and inter-quartile ranges for 5 age groups for the 1992 and 1995 waves of the SCF and compares them against the median dividend yields and inter-quartile ranges of (positive) dividend yields in the Barber and Odean (2000) data. Clearly the dividend yields in Barber and Odean (2000) are much more compressed around their median values compared to the SCF dividend yields. Moreover, the SCF dividend yields (conditional on being between zero and one) tend to be much higher than the Barber and Odean (2000) dividend yields.[9] Given these issues, we conclude that the SCF implied dividend yields cannot reliably be used for stock wealth imputation.

A.1.3 Non-taxable Stock Wealth Adjustment

The SOI data exclude dividends held in non-taxable accounts (e.g. defined contribution retirement accounts). In this section, we describe how we adjust for non-taxable stock wealth to arrive at the stock market wealth variable we use in our empirical analysis.

We begin by plotting in Figure A.4 the distribution of household holdings of corporate equity between taxable (directly held and non-IRA mutual fund) and non-taxable accounts using data from the Financial Accounts of the United States. Roughly 2/3 of corporate

[9]This is also reflected in the mean dividend yields (not shown) in the SCF, which are substantially higher than the medians, while in Barber and Odean (2000) the two are comparable.

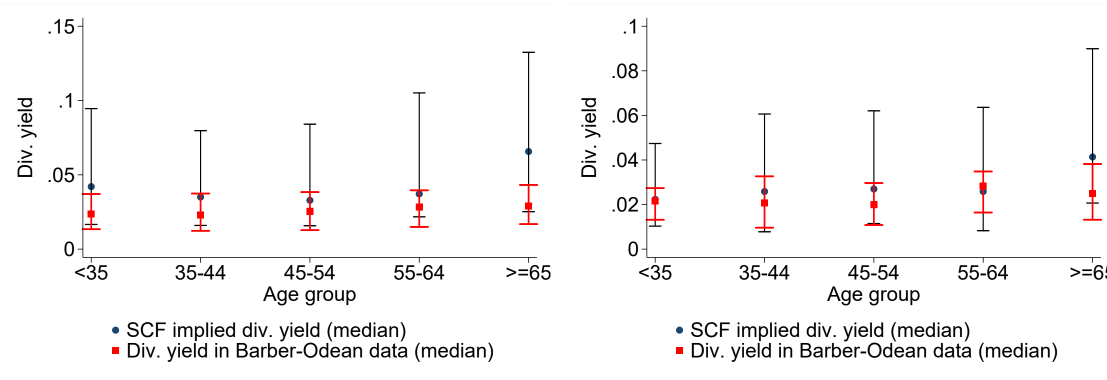

Figure A.3: Dividend yield distributions by age group in the SCF and Barber and Odean (2000) data for 1992 (left) and 1995 (right)

Note: Dots denote median values and bars show the inter-quartile range. The figures plot the distribution of implied dividend yields in the SCF (for dividend yields that are in $(0,1)$) and dividend yields in the Barber and Odean (2000) data from a discount brokerage firm (for positive dividend yields) by age group for 1992 and 1995.

equity owned by households is held in taxable accounts.[10]

We next use data from the SCF to examine the relationship between total stock market wealth and stock market wealth held in taxable accounts in the cross-section of U.S. households. We pool all waves from 1992 to 2016, consistent with the sample period for our benchmark analysis. We use the definition for stock-market wealth used in the Fed Bulletins.[11]. Stock-market wealth appears as "financial assets invested in stock". Following the Fed Bulletin definition of stock-market wealth, we define taxable stock wealth as the sum of direct holdings of stocks, stock mutual funds and other mutual funds, and 1/2 of the value of combination mutual funds. All variables are expressed in constant 2016 dollars. Table A.2 reports summary statistics for total stock wealth and taxable stock wealth.

Table A.3 reports the coefficients from regressions of total stock wealth on taxable stock wealth. There is a positive constant term, indicating that nontaxable stock market wealth is more evenly distributed than taxable wealth. The coefficient on taxable stock wealth

[10]Non-taxable retirement accounts here include only defined contribution accounts and exclude equity holdings of defined benefit plans. This definition accords with our empirical analysis since fluctuations in the market value of assets of defined benefit plans do not directly affect the future pension income of plan participants. The data plotted in Figure A.4 also include non-profit organizations, which hold about 10% of directly held equity and mutual fund shares.

[11]The precise definition is available here: https://www.federalreserve.gov/econres/files/bulletin.macro.txt

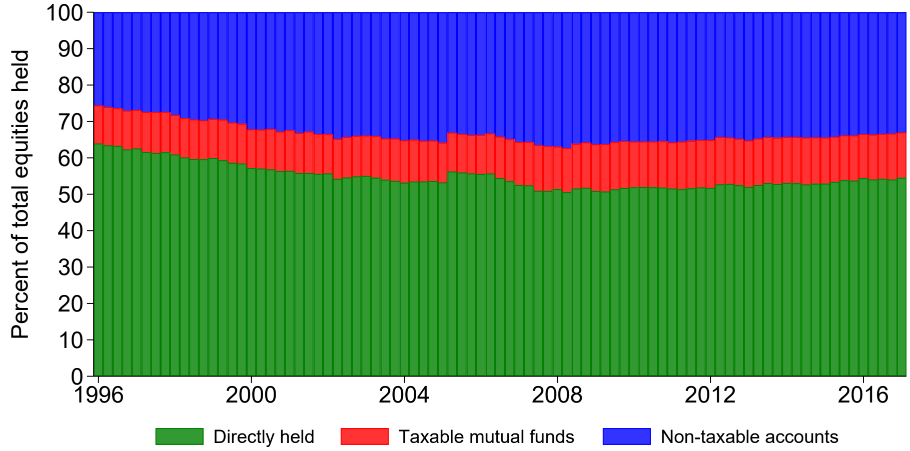

Figure A.4: Household Stock Market Wealth in the FAUS

Note: The figure reports household equity wealth as reported in the Financial Accounts of the United States. We define stock market wealth as total equity wealth (table B.101.e line 14, code LM153064475Q) less the market value of S-corporations (table L.223 line 31, code LM883164133Q) and similarly define directly held stock market wealth as directly held equity wealth (table B.101.e line 15, code LM153064105Q) less the market value of S-corporations. Taxable mutual funds are total mutual fund holdings of equity shares (table B.101.e line 21, code LM653064155Q) less equity held in IRAs, where we compute the latter by assuming the same equity share of IRAs as of all mutual funds, IRA mutual fund equity = IRA mutual funds at market value (table L.227 line 16, code LM653131573Q) × total equities held in mutual funds /total value of mutual funds (table B.101.e line 21, code LM653064155Q + table B.101.e line 12, code LM654022055Q). Non-taxable accounts include equities held through life insurance companies (table B.101.e line 17, code LM543064153Q), in defined contribution accounts of private pension funds (table B.101.e line 18, code LM573064175Q), federal government retirement funds (table B.101.e line 19, code LM343064125Q), and state and local government retirement funds (table B.101.e line 20, code LM223064213Q), and through mutual funds in IRAs.

is between 1.08 and 1.09 and the R^2 is around 0.91. Therefore, total stock wealth and taxable stock wealth vary almost one-for-one.

The high R^2 from these regressions suggests that we can use the relationship between total stock wealth, taxable stock wealth, and demographics in the SCF to account for non-taxable stock wealth at the county level. Specifically, we again use all waves of the SCF from 1992 to 2016. For each survey wave, we use a specification as in Column (2) of Table A.3. We then interpolate these coefficient estimates for years in which no survey took place. Finally, we use the estimate of (real) taxable stock wealth from capitalizing taxable dividend income and county-level demographic information on population shares in

Table A.2: Summary Statistics (values are in 2016 dollars).

Variable	Mean	Std. Dev.	Min	Max
total stock wealth	119,402	1,144,358	0	9.87×10^8
taxable stock wealth	65,428	1,001,526	0	9.84×10^8

different age bins and the college share (interpolated at yearly frequency from the decadal census and also extrapolated past 2010) to arrive at real total stock wealth for each county and year.

A.1.4 Non-public Companies

One remaining source of measurement error in our capitalization approach arises because dividend income reported on form 1040 includes dividends paid by private C-corporations. Such income accrues to owners of closely-held corporations and is highly concentrated at the top of the wealth distribution. Figure A.5 uses data from the Financial Accounts of the United States to plot the market value of equity issued by privately held C-corporations as a share of total equity issued by domestic C-corporations.[12] This share never exceeds 7% of total equity, indicating that as a practical matter dividend income from non-public C-corporations is small. Moreover, as described in Appendix A.1 our baseline sample excludes a small number of counties with a substantial share of dividend income reported by late filers who disproportionately own closely-held corporations. Therefore, non-public C-corporation wealth does not appear to meaningfully affect our results.

A.1.5 Return Heterogeneity

Similar to the dividend yield adjustment we also compute a county-specific stock market return. The systematic differences in dividend yields across households with different age

[12] Since 2015, table L.223 of the Financial Accounts of the United States has reported equity issued by domestic corporations separately by whether the corporation's equity is publicly traded, with the series extended back to 1996 using historical data. While obtaining market values of privately held corporations necessarily requires some imputations (Ogden, Thomas and Warusawitharana, 2016), we believe the results to be the best estimate of this split available and unlikely to be too far off.

Table A.3: Total stock wealth and taxable stock wealth

	(1)	(2)
Taxable stock wealth	1.09**	1.08**
	(0.01)	(0.01)
Age < 25		-12933.06**
		(1225.68)
Age 25-34		-22996.77**
		(1097.07)
Age 35-44		-2788.01*
		(1236.89)
Age 45-54		29412.54**
		(1790.46)
Age 55-64		64398.51**
		(2894.11)
Age 65+		34482.50**
		(2164.56)
College degree		102265.11**
		(2869.13)
Constant	48221.15**	
	(943.52)	
R^2	0.91	0.91
Observations	44,633	44,497

Note: The table reports coefficient estimates from regressing (real) total stock wealth on (real) taxable stock wealth, and household head demographics in the SCF using the pooled 1992-2016 waves. Robust standard errors in parenthesis. * denotes significance at the 5% level, and ** denotes significance at the 1% level.

that are the basis for our dividend yield adjustment in Appendix A.1.2 imply possible systematic differences in portfolio return characteristics across these same age groups. For example, it is well-known that stocks with higher dividend yields tend to be value stocks with a different return distribution than the stock market. Specifically, those stocks tend to have market betas below one. In that case the portfolio betas of households living in

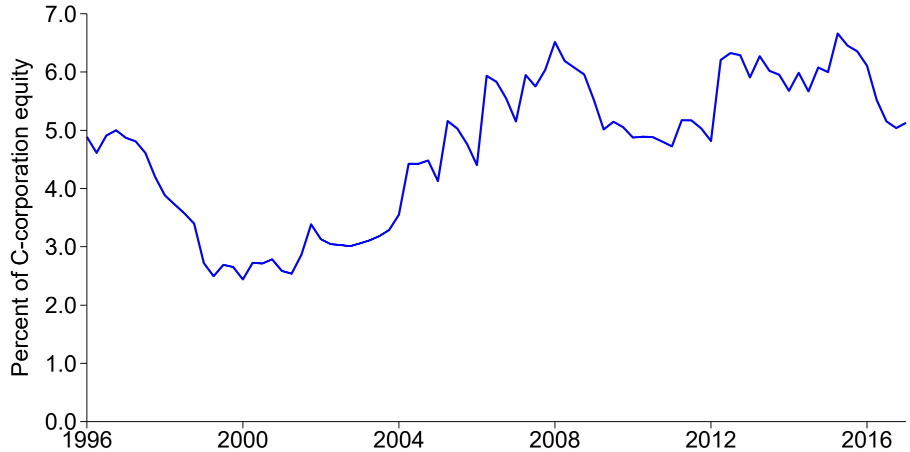

Figure A.5: Equity of Privately Held C-Corporations

Notes: The figure reports the market value of equity of privately held C-corporations as a share of total (privately held plus publicly-traded) equity of domestic C-corporations as reported in the Financial Accounts of the United States table L.223 lines 29 and 32.

counties with predominantly older households will be lower than those of households living in counties with predominantly younger households. In this section we first present evidence using the Barber and Odean (2000) data set that there is indeed a systematic (although quite small) relation between portfolio betas and age. Second, as with the dividend yield adjustment from Appendix A.1.2 we use this relationship and county demographic information to construct a county-specific beta and compute a county-specific stock market return.

We use the household portfolio data described in Appendix A.1.2 and construct value-weighted portfolios by age group (for the same 5 age groups as in Appendix A.1.2).[13] We then construct monthly returns for these portfolios by computing the weighted one-month return on the underlying CRSP assets.[14] Using these monthly returns we estimate

[13]One difference relative to the sample we use in Appendix A.1.2 is that we also include household-month observations that have zero dividends. The reason for keeping these households in this case is that we want to construct a county-level stock market return that will be applied to county-level stock market wealth, which also includes the stock wealth of households that hold only non-dividend paying stocks in their portfolios.

[14]Household positions are recorded at the beginning of a month, so similar to Barber and Odean (2000) we implicitly assume that each household holds the assets in their portfolio for the duration of the month.

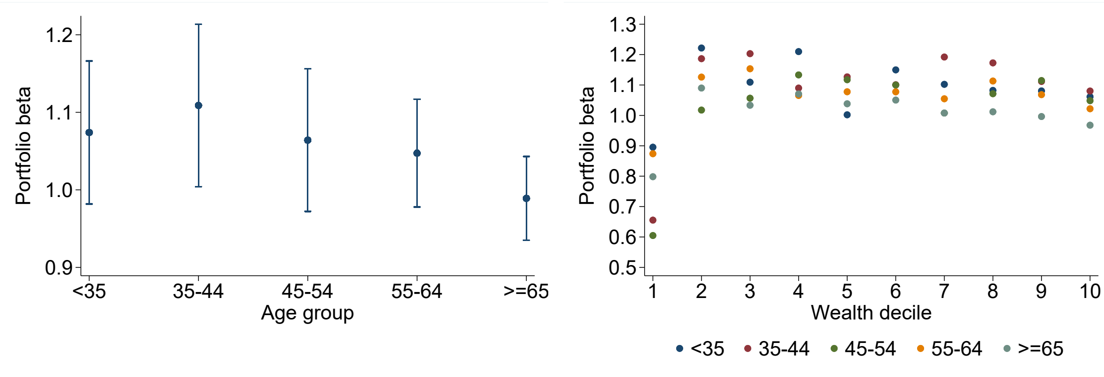

Figure A.6: Portfolio Beta by Age and Wealth

Note: The figures plot the portfolio betas by age and wealth quantile based on the Barber and Odean (2000) data from a discount brokerage firm merged with data on CRSP stocks and mutual funds. Wealth denotes the total position equity among all taxable accounts that a household has in the discount brokerage firm.

portfolio betas using the return on the CRSP value weighted index as the return on the market portfolio and the 3-month T-Bill yield as the risk free rate. Figure A.6 (left panel) plots the estimated portfolio betas together with a 95% confidence intervals. As the Figure shows there is a negative (albeit small in magnitude) relationship between beta and age with younger households having portfolios with higher beta (and beta above one) compared to older households.

We next use this relationship to construct a county-specific beta and from it a county-specific stock market return. Specifically, as with the dividend-yield adjustment, we combine the estimated betas shown in the left panel of Figure A.6 with the county-year specific age structure from the Census Bureau and average wealth by age bin from the Survey of Consumer Finances (interpolated between SCF waves) to construct the wealth-weighted average of the age bin portfolio betas for each county and year. Finally, we scale these betas so that the average beta in each year is equal to one (that is, we assume that on average counties hold the market portfolio). We then multiply CRSP total stock return by these county-year specific betas to arrive at a county-specific stock-market return.

Table A.4: Summary Statistics

Variable	Source	Mean	SD	Within county SD	Within county and state-quarter SD	Obs.
Quarterly total return on market	CRSP	0.019	0.067			94
Capitalized dividends/labor income	IRS SOI	2.316	1.177	0.628	0.309	269 057
Log empl., 8Q change	QCEW	0.025	0.053	0.047	0.032	272 942
Log payroll, 8Q change	QCEW	0.084	0.077	0.072	0.048	272 942
Log nontradable empl., 8Q change	QCEW	0.031	0.069	0.064	0.054	269 774
Log nontradable payroll, 8Q change	QCEW	0.081	0.089	0.084	0.064	269 774
Log tradable empl., 8Q change	QCEW	−0.018	0.130	0.123	0.106	258 856
Log tradable payroll, 8Q change	QCEW	0.045	0.158	0.151	0.128	258 856

Note: The table reports summary statistics. Within county standard deviation refers to the standard deviation after removing county-specific means. Within county and state-quarter standard deviation refers to the standard deviation after partialling out county and state-quarter fixed effects. All statistics weighted by 2010 population.

A.2 Summary Statistics

Table A.4 reports the mean and standard deviation of the 8 quarter change in the labor market variables. It also reports the standard deviation after removing county-specific means and state-quarter means, with the latter being the variation used in the main analysis.

A.3 County Demographic Characteristics and Stock Wealth

To more clearly illustrate that our empirical strategy does not depend on stock wealth to labor income being randomly assigned across counties, we correlate the (time-averaged) county level value of stock wealth to labor income with a number of county level demographics. Specifically, we use time-averaged data from the 1990, 2000 and 2010 US Census to compute the county level shares of individuals 25 years and older with bachelor degree or higher, median age of the resident population, share of retired workers receiving social security benefits, share of females, and share of the resident population identifying them-

Table A.5: County demographics regressions

	(1)	(2)	(3)	(4)	(5)	(6)
Bachelor degree or higher (%)	0.06**					0.09**
	(0.01)					(0.01)
Median age		0.10*				−0.04*
		(0.04)				(0.02)
Retired (%)			0.12**			0.31**
			(0.04)			(0.03)
Female (%)				0.19**		−0.06*
				(0.04)		(0.03)
White (%)					−0.00	−0.02**
					(0.00)	(0.00)
Population weighted	Yes	Yes	Yes	Yes	Yes	Yes
State FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.31	0.21	0.22	0.18	0.15	0.54
Observations	3,141	3,141	3,141	3,141	3,141	3,141

Note: The table reports coefficients and standard errors from regressing time-averaged total stock wealth by labor income on county demographics. Standard errors in parentheses are clustered by state. * denotes significance at the 5% level, and ** denotes significance at the 1% level.

selves as white.[15] Table A.5 reports the coefficient estimates from population weighted regressions of stock wealth to labor income on each demographic characteristics as well as a regression including all demographic characteristics (last column). All regressions include state fixed effects. Unsurprisingly, the share of retired workers and share with college degree are robustly positively related with the average stock wealth to labor income ratio in a county. The share of females and white is negatively related with stock wealth to labor income although the effects are smaller. Median age does not co-move with stock wealth to income after controlling for the other demographic characteristics.

[15]For the college share we use the American Community Survey rather than the 2010 US Census.

A.4 Coefficients on Control Variables

This appendix reproduces the baseline results in Table 1 including the coefficients on the main control variables.

A.5 Monte Carlo Simulation

In this section we perform Monte Carlo simulations to assess the possible impact of household-level MPC heterogeneity on our empirical estimates. We start by constructing a simulated data set containing the full distribution of household wealth by county. To do so, we first stratify the 2016 SCF into eight groups based on total 2015 income (less than \$75k, \$75k-\$100k, \$100k-\$200k, and \$200k+) and whether the household had any 2015 dividend income. For each group, we compute the share of households with positive stock wealth in 2016 and fit a log-normal distribution to the stock wealth of the households with positive stock wealth. We then obtain from the 2015 IRS SOI data the number of tax returns by county that have adjusted gross income in the same four income groups as in the SCF and within each income group the number of returns with dividend income. For each return in a county and income group-by-dividend indicator category, we first simulate whether the household holds stocks or not based on the estimated share in that category in the SCF. Next, for each simulated household with positive stock wealth, we draw their level of stock wealth from a log-normal distribution with mean and variance from the SCF distribution of stock wealth for the respective category. This process yields a simulated data set with 148,978,310 observations, of which 76,680,922 have positive stock wealth.

Table A.7 compares several moments in the simulated data and the actual data (2016 SCF for the first 5 moments and county-level capitalized dividend income from the 2015 IRS SOI for the remaining 2 moments). The simulated data capture very well key features of the actual data.

We perform two experiments using the simulated data. In both experiments, we assume a structure of household-level MPC heterogeneity out of stock wealth.[16] We then simulate the consumption change to a 1% increase in stock wealth, aggregate the wealth and consumption changes across households in a county and divide by the total number of

[16]We are agnostic in these experiments about the MPC of *non*-stock holders. In particular, as in our two agent model, there could be large differences in the MPCs of non-stock holders and stock holders even if there is little or no heterogeneity in MPCs among the group of stock-holders.

Table A.6: Baseline Results

	All		Non-traded		Traded	
	Emp.	W&S	Emp.	W&S	Emp.	W&S
	(1)	(2)	(3)	(4)	(5)	(6)
Right hand side variables:						
$S_{a,t-1}R_{a,t-1,t}$	0.77*	2.18**	2.02*	3.24**	−0.11	0.71
	(0.36)	(0.63)	(0.80)	(1.01)	(0.64)	(0.74)
Bartik predicted employment	0.86**	1.46**	0.59**	0.84**	1.66**	2.11**
	(0.08)	(0.14)	(0.10)	(0.10)	(0.19)	(0.25)
Labor income interaction	−1.11+	−2.65**	0.96	−0.92	1.70	1.92
	(0.62)	(0.87)	(0.99)	(1.19)	(1.92)	(2.12)
Business income interaction	1.08+	2.53**	−1.26	0.58	−1.63	−1.90
	(0.61)	(0.83)	(0.99)	(1.17)	(1.89)	(2.05)
Bond return interaction	−0.07	−0.14	3.58+	2.80	0.20	−0.51
	(0.82)	(1.39)	(1.87)	(2.32)	(1.20)	(1.81)
House price interaction	−1.55	5.45	−8.33*	2.29	−9.91	−4.88
	(3.28)	(4.40)	(4.14)	(5.25)	(6.32)	(6.87)
Horizon h	Q7	Q7	Q7	Q7	Q7	Q7
Pop. weighted	Yes	Yes	Yes	Yes	Yes	Yes
County FE	Yes	Yes	Yes	Yes	Yes	Yes
State × time FE	Yes	Yes	Yes	Yes	Yes	Yes
Shock lags	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.66	0.64	0.39	0.48	0.35	0.36
Counties	2,901	2,901	2,896	2,896	2,877	2,877
Periods	92	92	92	92	92	92
Observations	265,837	265,837	263,210	263,210	252,928	252,928

Note: The table reports coefficients and standard errors from estimating Eq. (1) for $h = 7$. Columns (1) and (2) include all covered employment and payroll; columns (3) and (4) include employment and payroll in NAICS 44-45 (retail trade) and 72 (accommodation and food services); columns (5) and (6) include employment and payroll in NAICS 11 (agriculture, forestry, fishing and hunting), NAICS 21 (mining, quarrying, and oil and gas extraction), and NAICS 31-33 (manufacturing). The shock occurs in period 0 and is an increase in stock market wealth equivalent to 1% of annual labor income. For readability, the table reports coefficients in basis points. Standard errors in parentheses and double-clustered by county and quarter. * denotes significance at the 5% level, and ** denotes significance at the 1% level.

returns to obtain the county-level average consumption and wealth change, and regress the change in county-average consumption on the change in county-average wealth. This yields

Table A.7: Comparison of simulated and actual data.

Moment	Simulated	Observed
Own stocks (percent)	51.5	53.6
Mean stock wealth	193,806	178,785
St. dev. stock wealth	1,682,979	1,680,982
Mean stock wealth (stocks> 0)	376,533	333,667
St. dev. stock wealth (stocks> 0)	2,331,120	2,285,270
Mean county stock wealth	140,077	121,557
St. dev. county stock wealth	63,871	84,879

Note: Simulated moments are based on simulated household-level data that uses information on stock ownership and stock wealth by 2015 dividend income (no dividend income vs. some dividend income) and total gross income group (4 groups: less than $75k, $75k-$100k, $100k-$200k, and $200k+) from the 2016 SCF and county-level information on number of returns in each (adjusted) gross income group and number of returns with dividend income by income group from the 2015 IRS SOI data. Observed moments are based on the 2016 SCF (for first 5 moments) as well as the 2015 county-level stock wealth (for the last 2 moments) based on capitalized dividend income, where the capitalization approach is described in Appendix A.1.

a cross-county coefficient that mirrors our actual empirical design.[17] We plot the regression coefficient and the true wealth-weighted average MPC as a function of the standard deviation of the MPC of stock holders.

The first experiment assumes the heterogeneity in MPCs is random across households. Specifically, MPCs are distributed uniformly over $[0.03 - k, 0.03 + k]$, where k is allowed to vary between 0 (no heterogeneity) and 0.03. The left panel of Figure A.7 plots the resulting regression coefficients and wealth-weighted MPCs as k varies. With random heterogeneity, the regression recovers an unbiased and precise estimate of the wealth-weighted average MPC out of stock wealth.

The second experiment assumes that the MPC declines in the amount of stock wealth according to the relationship $MPC = bW^{-a}$, where W denotes stock wealth and a parameterizes both the heterogeneity in MPCs and the strength of the relation between stock wealth and MPC. A value of $a = 0$ implies no heterogeneity, while positive values of a generate a negative relationship. For each value of a, we choose b such that the county-level regression coefficient roughly equals our empirical estimate of 0.03. The right

[17] Since we use change in county-level spending rather than growth in spending, we do not need to normalize the regressor by the level of spending as we do in Section 3.5.

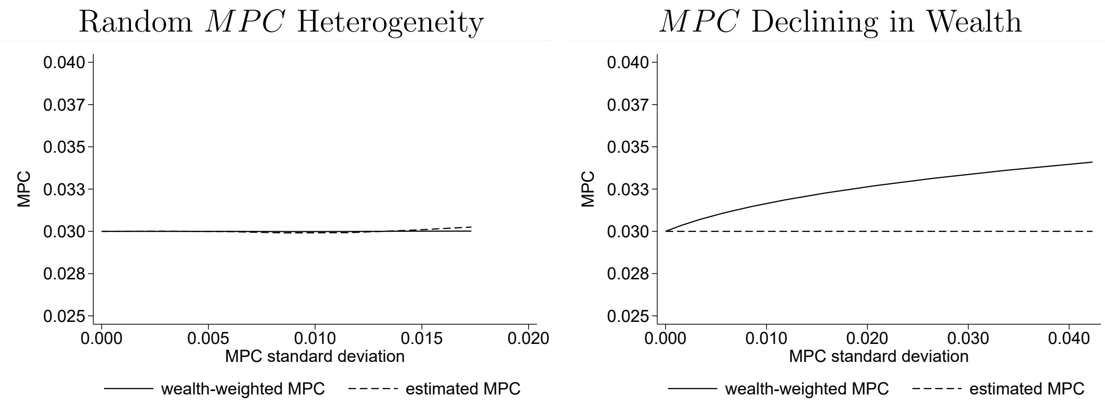

Figure A.7: Wealth-weighted MPC Versus County-level Regression Estimate

Note: The wealth-weighted MPC is computed based on simulated household-level data that uses information on stock ownership and stock wealth by 2015 dividend income (no dividend income vs. some dividend income) and total gross income group (4 groups: less than \$75k, \$75k-\$100k, \$100k-\$200k, and \$200k+) from the 2016 SCF and county-level information on number of returns in each (adjusted) gross income group and number of returns with dividend income by income group from the 2015 IRS SOI data. The estimated MPC is computed by aggregating the household-level changes in spending and wealth in response to a 1% stock return to the county level, dividing by the number of tax returns, and regressing the change in county-level spending per tax return on the change in county-level stock wealth per tax return and a constant term. In the left panel, household-level MPCs are drawn from a uniform distribution over $[0.03 - k, 0.03 + k]$, where k varies between 0 and 0.03. In the right panel, household-level MPCs are set to $MPC = bW^{-a}$, where W denotes stock wealth and a parameterizes the heterogeneity in MPCs and the strength of the relation between stock wealth and MPC, and is allowed to vary between 0 and 0.2, while b is chosen such that the county-level MPC estimate equals 0.03.

panel of Figure A.7 plots the regression coefficient and the wealth-weighted average MPC against the MPC of stock holders, for different levels of a. With no dispersion, the cross-county regression again exactly recovers the wealth-weighted MPC. More interesting, the wealth-weighted MPC remains very close to the county-level coefficient even for substantial dispersion in MPCs among stock-wealth holders. For example, an MPC standard deviation of 0.02, shown in the middle of the plot, corresponds to an MPC of stock owners at the 50th percentile that is double the MPC of stock owners at the 99th percentile, but the county-level estimate remains within 10% of the wealth-weighted average MPC. The assumed negative relationship between MPC and stock wealth implies that the regression coefficient always lies below the wealth-weighted MPC, making our estimates if anything a lower bound.

A.6 Evidence of Unit Income Elasticity of Nontradable Consumption in the Consumer Expenditure Survey

This appendix describes our analysis of the income elasticity of nontradable consumption using the interview module of the Consumer Expenditure Survey (CE). The CE interviews sampled households for up to four consecutive quarters about all expenditures over the prior three months on a detailed set of categories. We perform two sets of exercises. The first reports Engel curve estimation for selected expenditure categories, including our nontradable grouping of retail and restaurants. The second extends the Dynan and Maki (2001) and Dynan (2010) analysis of the conditional consumption expenditure response by stock holders to an increase in the stock market to consider different categories of consumption. Both exercises suggest a close to proportionate increase in consumption expenditure on nontradable and other goods.

Engel Curve Estimation. Table A.8 reports the elasticity of selected expenditure categories to total expenditure. We report two sets of specifications. The first uses the Almost Ideal Demand System of Deaton and Muellbauer (1980):

$$\frac{x_{i,j,t}}{X_{i,t}} = \alpha_{j,t} + \beta_j \ln X_{i,t} + \Gamma_j Z_i + u_{i,j,t}, \tag{A.1}$$

where $x_{i,j,t}$ is the expenditure by household i on good j in year t, $X_{i,t}$ is total expenditure by household i, $\alpha_{j,t}$ is a good-specific year fixed effect, and Z_i contains as included covariates categorical variables for age range, number of earners, and household size. To account for measurement error in $X_{i,t}$, we follow Aguiar and Bils (2015) and estimate Eq. (A.1) using instrumental variables with log after-tax income and income bins as excluded instruments. A value of β_j of 0 would indicate a unit income elasticity; more generally, the elasticity of good j at the sample mean expenditure share is equal to $\beta\times$expenditure share $+1$. The second Engel curve estimation procedure follows Aguiar and Bils (2015) and others and estimates:

$$\frac{x_{i,j,t} - \bar{x}_{j,t}}{\bar{x}_{j,t}} = \alpha_{j,t} + \beta_j \ln X_{i,t} + \Gamma_j Z_i + u_{i,j,t}, \tag{A.2}$$

where $\bar{x}_{j,t}$ is the cross-sectional average expenditure on good j in year t and estimation again proceeds via IV with the same set of excluded instruments. In this specification, β_j

Table A.8: Engel Curves in the Consumer Expenditure Survey

Category	Share	AIDS			Deviation	
		Coef.	SE	Elasticity	Elasticity	SE
Jewelry	0.21	0.003	0.000	2.269	1.913	0.079
Restaurants	3.80	0.015	0.000	1.401	1.198	0.013
Food at home	14.31	−0.081	0.001	0.437	0.418	0.005
Retail and restaurants	33.39	−0.007	0.002	0.978	0.895	0.008

Note: The table estimates Engel curves for selected categories using the Consumer Expenditure Survey. In the AIDS specification, the dependent variable is the expenditure share on the category indicated. In the deviation specification, the dependent variable is the percent difference in expenditure on the category indicated from the sample mean. In both specifications, the endogenous variable is log total household expenditure, the excluded instruments are log of after-tax income and categories of income and the included instruments are categorical variables for age range, number of earners, and household size as well as a year fixed effect.

directly gives the elasticity.

We report Engel curve estimates for jewelry, restaurant meals, food purchased for home consumption, and the total category of retail and restaurants, which includes the first three categories as well as all other retail purchases. We report results corresponding to our full sample of 1990-2016; we obtain similar results in sub-samples that address the possibility of estimate stability, for example due to changes in relative prices. Table A.8 shows that homotheticity does not hold across all sub-categories within retail and restaurants. Jewelry is a luxury good, with an elasticity around 2 across specifications. Meals at restaurants also have an elasticity above 1. Food at home is a necessity, with an elasticity around 0.4. However, the combined category of retail and restaurants has an elasticity of close to 1 — 0.98 using the AIDS specification and 0.9 using the Aguiar and Bils (2015) specification.

Response to Changes in the Stock Market. The CE does not ask directly about stock holdings. However, in the last interview the survey records information on security holdings. Dynan and Maki (2001) and Dynan (2010) use this information and the short panel structure of the survey to separately relate consumption growth of security holders and non-security holders to the change in the stock market. We follow the analysis in Dynan and Maki (2001) as closely as possible and extend it by measuring the response of retail and restaurant spending separately.[18]

The specification in Dynan and Maki (2001) is:

$$\Delta \ln C_{i,t} = \sum_{j=0}^{3} \beta_j \Delta \ln W_{t-j} + \Gamma' X_{i,t} + \epsilon_{i,t}, \tag{A.3}$$

where $\Delta \ln C_{i,t}$ is the log change in consumption expenditure by household i between the second and fifth CE interviews,[19] $\Delta \ln W_{t-j}$ is the log change in the Wilshire 5000 between the recall periods covered by the second and fifth interviews ($j = 0$) or over consecutive, non-overlapping 9 month periods preceding the second interview ($j = 1, 2, 3$), and $X_{i,t}$ contains monthly categorical variables to absorb seasonal patterns in consumption, taste shifters (age, age^2, family size), socioeconomic variables (race, high school completion, college completion), labor earnings growth between the second and fifth interviews, and year categorical variables. Thus, this specification attempts to address the causal identification challenge by controlling directly for contemporaneous labor income growth and including year categorical variables, the latter which isolate variation in recent stock performance for households interviewed during different months of the same calendar year. Following Mankiw and Zeldes (1991), the specification is estimated separately for households above and below a cutoff value for total securities holdings.

Table A.9 reports the results. The left panel contains our replication of table 2 in Dynan and Maki (2001) and Dynan (2010). We find very similar results to those papers. Notably, expenditure on nondurable goods and services rises on impact for households categorized as stock holders and continues to rise over the next 18 months following a positive stock return. This sluggish response accords with the sluggish adjustment of labor market variables in our main analysis. Summing over the contemporaneous and lag coefficients, the total elasticity of expenditure to increases in stock market wealth is about

[18] The Dynan and Maki (2001) sample covers the period 1983-1998. Dynan (2010) finds negligible consumption responses when extending the sample through 2008, possibly reflecting the deterioration in the quality of the CE sample in the more recent years and the difficulty in recruiting high income and high net worth individuals to participate. Since our purpose is to compare the responses of different categories of consumption, we restrict to periods when the data can capture an overall response.

[19] The first CE interview introduces the household to the survey but does not collect consumption information. Therefore, the span between the second and fifth interviews is the longest span available.

Table A.9: Consumption Responses in the Consumer Expenditure Survey

	Non-durable goods and services		Retail and restaurants	
	SH	Other	SH	Other
	(1)	(2)	(3)	(4)
Right hand side variables:				
Stock return	0.369	−0.015	0.198	−0.038
	(0.133)	(0.048)	(0.277)	(0.100)
Lag 1	0.385	0.074	0.519	0.121
	(0.151)	(0.053)	(0.312)	(0.109)
Lag 2	0.252	0.050	0.447	0.065
	(0.134)	(0.047)	(0.278)	(0.097)
Lag 3	0.039	0.038	0.104	0.135
	(0.103)	(0.037)	(0.220)	(0.077)
Sum of coefficients	1.044	0.146	1.268	0.283
R^2	0.02	0.01	0.02	0.01
Observations	4,086	28,329	4,026	28,376

Note: The estimating equation is: $\Delta \ln C_{i,t} = \sum_{j=0}^{3} \beta_j \Delta \ln W_{t-j} + \Gamma' X_{i,t} + \epsilon_{i,t}$, where $\Delta \ln C_{i,t}$ is the log change in consumption expenditure by household i between the second and fifth CE interviews in the consumption category indicated in the table header and $\Delta \ln W_{t-j}$ is the log change in the Wilshire 5000 between the recall periods covered by the second and fifth interviews ($j = 0$) or over consecutive, non-overlapping 9 month periods preceding the second interview ($j = 1, 2, 3$). All regressions include controls for calendar month and year of the final interview, age, age^2, family size, race, high school completion, college completion, and labor earnings growth between the second and fifth interviews. The sample is 1983-1998. Columns marked SH include households with more than $10,000 of securities.

1. In contrast, total expenditure by non-stock holders does not increase.

The right panel replaces the consumption measure with purchases of non-durable and durable goods from retail stores and purchases at restaurants. These categories provide the closest correspondence to all purchases made at stores in the retail or restaurant sectors.[20] The cumulative consumption responses of purchases of goods from retail stores and at

[20] Because we include durable goods, the categories in the right panel are not a strict subset of the categories in the left panel. We have experimented with excluding durable goods from the basket and obtain similar results.

restaurants are very similar to the responses of total non-durable goods and services, albeit estimated with less precision.

Overall, these results provide support for our assumption that expenditure on retail and restaurants moves proportionally with total expenditure, which we use to structurally interpret our empirical estimates in the paper. This conclusion holds both across households in the Engel curve analysis and within households in response to stock market changes. Even if one questions the causal identification of the Dynan and Maki (2001) framework for stock market changes, their specification still has the interpretation of the relative responses across categories to general demand shocks rather than to the stock market in particular.

B Model Details

In this appendix, we present the full model. In Section B.1, we describe the environment and define the equilibrium. For completeness, we repeat the key equations shown in the main text. In Section B.2, we provide a general characterization: specifically, we fully describe the long-run equilibrium, and we derive the equations for the short-run equilibrium that we solve subsequently. In Section B.3, we provide a closed-form solution for a benchmark case in which areas have the same stock wealth. In Section B.4, we log-linearize the equilibrium around the common-wealth benchmark and provide closed-form solutions for the log-linearized equilibrium with heterogeneous stock wealth. In Section B.5, we use our results to characterize the cross-sectional effects of shocks to stock prices. In Section B.6, we establish the robustness of the benchmark calibration of the model that we present in the main text. In Section B.7, we analyze the aggregate effects of shocks to stock prices (when monetary policy is passive) and compare the results with our earlier results on the cross-sectional effects. Finally, in Section B.8, we extend the model to incorporate uncertainty, and we show that our results are robust to obtaining the stock price fluctuations from alternative sources such as changes in households' risk aversion or perceived risk.

B.1 Environment and Definition of Equilibrium

Basic Setup and Interpretation. There are two factors of production: capital and labor. There is a continuum of measure one of areas (counties) denoted by subscript a.

Areas are identical except for their initial ownership of capital.

There is an infinite number of periods $t \in \{0, 1, 2...\}$. We view period 0 as the "the short run" with the key features that labor is specific to the area and nominal wages are (potentially) partially sticky. Therefore, local labor bill and the local labor in the short run are influenced by local aggregate demand. In contrast, periods $t \geq 1$ are "the long run" in which both factors are mobile cross areas. With appropriate monetary policy (that we describe subsequently), this mobility assumption implies outcomes in periods $t \geq 1$ are determined solely by productivity. (For simplicity, capital is mobile across areas in all periods including period 0).

Importantly, each area is populated by two types of agents denoted by superscript $i = s$ ("stockholders") and $i = h$ ("hand-to-mouth") with population mass $1 - \theta$ and θ, respectively (where $\theta \in (0, 1)$). Stockholders own (and trade) the capital, and also supply a fraction of the labor. They have a relatively low MPC that we estimate. Hand-to-mouth households hold no capital, and they supply the remaining fraction of labor. They have a much higher MPC equal to one. This heterogeneous MPC setup approximates the data better than a representative household model and enables us to calibrate the Keynesian multiplier. We also assume that the stockholders' labor supply is exogenous (or perfectly inelastic) but hand-to-mouth households' labor supply (in period 0) is endogenous (or somewhat elastic). This asymmetric labor supply assumption enables us to introduce some labor elasticity while abstracting away from the wealth effects on labor supply.

Our focus is to understand how fluctuations in the price of capital affects cross-sectional and aggregate outcomes in the short run. To this end, we will generate endogenous changes in the capital price in period 0 from exogenous permanent changes to the productivity of capital in period 1. We interpret these changes as capturing stock market fluctuations due to a "time-varying risk premium." We validate the risk premium interpretation in Section B.8, where we introduce uncertainty about capital productivity in period 1.

Goods and Production Technologies. For each period t, there is a composite tradable good that can be consumed everywhere. For each area a, there is also a corresponding nontradable good that can only be produced and consumed in area a. Labor and capital are perfectly mobile across the production technologies described below. We assume all production firms are competitive and not subject to nominal rigidities (we will assume nominal rigidities in the labor market).

The nontradable good in area a can be produced according to a standard Cobb-Douglas technology,

$$Y_{a,t}^N = \left(K_{a,t}^N/\alpha^N\right)^{\alpha^N} \left(L_{a,t}^N/\left(1-\alpha^N\right)\right)^{1-\alpha^N}. \tag{B.1}$$

Here, $L_{a,t}^N, K_{a,t}^N$ denote the quantity of labor and capital used by the nontradable sector in area a. The term $1-\alpha^N$ captures the share of labor in the nontradable sector.

In each period, the tradable good can be produced as a composite of tradable inputs across areas, where each input is produced according to a standard Cobb-Douglas technology:

$$Y_t^T = \left(\int_a \left(Y_{a,t}^T\right)^{\frac{\varepsilon-1}{\varepsilon}} da\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{B.2}$$

$$\text{where } Y_{a,t}^T = \left(K_{a,t}^T/\alpha^T\right)^{\alpha^T} \left(L_{a,t}^T/\left(1-\alpha^T\right)\right)^{1-\alpha^T}. \tag{B.3}$$

Here, $L_{a,t}^T, K_{a,t}^T$ denote the quantity of labor and capital used by the tradable sector in area a. The term $1-\alpha^T$ captures the share of labor in the tradable sector. The parameter, $\varepsilon > 0$, captures the elasticity of substitution across tradable inputs. When $\varepsilon > 1$ (resp. $\varepsilon < 1$), tradable inputs are gross substitutes (resp. gross complements).

Starting from period 1 onward, the tradable good can also be produced with another technology that uses only capital. This technology is linear,

$$\tilde{Y}_t^T = D^{1-\alpha^T} \tilde{K}_t^T \text{ for } t \geq 1. \tag{B.4}$$

Here, $\tilde{K}_t^T$ denotes the capital employed in the capital-only technology, and $\tilde{Y}_t^T$ denotes the tradable good produced via this technology (we use the tilde notation to distinguish them from K_t^T and Y_t^T). The term, $D^{1-\alpha^T}$, captures the capital productivity in period 1. This technology ensures that the rental rate of capital in the long run (periods $t \geq 1$) is a function of the exogenous parameter, D (with our normalization, it will be proportional to D). This in turn helps to generate fluctuations in the price of capital (in period 0) that are unrelated to current or future labor productivity.

Nominal Factor Returns and Prices. We let $P_{a,t}^N$ denote the nominal price of the nontradable good in period t and area a. We let P_t^T denote the price of the composite tradable good, and $P_{a,t}^T$ denote the price of the tradable input produced in area a

Likewise, we let $W_{a,t}$ denote the nominal wage for labor in period t and area a. We let R_t denote the nominal rental rate of capital in period t. There is a single rental rate for capital since capital is mobile across areas by assumption. Starting from period 1 onward, there is also a single wage (since labor is also mobile across areas), that is, $W_{a,t} = W_t$ for $t \geq 1$.

Capital Supply. In each period t, aggregate capital supply is exogenous and normalized to one,

$$\overline{K}_t \equiv 1. \tag{B.5}$$

Since capital is mobile across areas in all periods, we don't need to specify its location.

There are two financial assets. First, there is a claim on capital that pays R_t units in each period t. We let Q_t denote its nominal *cum-dividend* price. Thus, $Q_t - R_t$ denotes the nominal ex-dividend price. Second, there is also a risk-free asset in zero net supply. We denote the nominal gross risk-free interest rate between periods t and $t+1$ with R_t^f.

Heterogeneous Ownership of Capital. Stockholders in different areas start with zero units of the risk-free asset but they can differ in their endowments of aggregate capital. Specifically, we let $1 + x_{a,t}$ denote the share of aggregate capital held in area a in period t. For simplicity, capital wealth in an area is evenly distributed among stockholders: thus, each stockholder holds $(1 + x_{a,t}) / (1 - \theta)$ units of aggregate capital. The initial shares across areas $\{1 + x_{a,0}\}_a$, are exogenous and can be heterogeneous. The common-wealth benchmark corresponds to the special case with $x_{a,0} = 0$ for each a.

Households' Choice Between Nontradables and Tradables. Households of either type $i \in \{s, h\}$ consume the tradable good, $C_{a,t}^{i,T}$, and the nontradable good, $C_{a,t}^{i,N}$. We assume households' utility depends on these expenditures through a consumption aggregator given by:

$$C_{a,t}^i = \left(C_{a,t}^{i,N} / \eta\right)^{\eta} \left(C_{a,t}^{i,T} / (1 - \eta)\right)^{1-\eta}.$$

Here, η denotes the share of nontradables in spending.

In view of this assumption, we can formulate households' optimization problem in two steps. Consider the expenditure minimization problem in period t given a target consumption level $C^i_{a,t}$,

$$\min_{C^N_{a,t}, C^T_{a,t}} P^N_{a,t} C^N_{a,t} + P^T_{a,t} C^T_{a,t} \quad \text{(B.6)}$$
$$\left(C^N_{a,t}/\eta\right)^{\eta} \left(C^T_{a,t}/\left(1-\eta\right)\right)^{1-\eta} \geq C^i_{a,t}.$$

This problem is linearly homogeneous in $C^i_{a,t}$. Let $P_{a,t}$ (the unit cost or the ideal price index) denote the solution with $C^i_{a,t} = 1$. Then, given the price path, $\{P_{a,t}\}_{t=0}^{\infty}$, households first choose the path of their consumption (aggregator), $\left\{C^i_{a,t}\right\}_{t=0}^{\infty}$ (as we describe subsequently). Households then split their consumption $C^i_{a,t}$ between nontradables and tradables to solve problem $(B.6)$.

Throughout, we use $C^N_{a,t}, C^T_{a,t}$ to denote the total nontradable and tradable spending by the households in an area, that is,

$$\begin{aligned} C^N_{a,t} &= \left(1-\theta\right) C^{s,N}_{a,t} + \theta C^{h,N}_{a,t} \\ C^T_{a,t} &= \left(1-\theta\right) C^{s,T}_{a,t} + \theta C^{h,T}_{a,t}. \end{aligned} \quad \text{(B.7)}$$

Here, recall that $1-\theta$ and θ denote stockholders' and hand-to-mouth households' population share, respectively.

Stockholders' Labor Supply. In each period, stockholders' labor supply is still exogenous and the same across areas,

$$L^s_{a,t} = \overline{L} \text{ for each } a. \quad \text{(B.8)}$$

In contrast, hand-to-mouth households' labor is endogenous as we describe below.

Stockholders' Optimal Consumption-Saving and Portfolio Choice. Stockholders in area a have time separable log utility. They choose how much to consume and save and how to allocate savings across capital and the risk-free asset. We formulate their problem in period 0 as:

$$\max_{\left\{C^s_{a,t}, S_{a,t} \geq 0, \frac{1+x_{a,t+1}}{1-\theta}\right\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(1-\rho\right)^t \log C^s_{a,t} \quad \text{(B.9)}$$

$$P_{a,t}C^s_{a,t} + S_{a,t} = W_{a,t}\overline{L} + \frac{1+x_{a,t}}{1-\theta}Q_t + A^f_{a,t}$$
$$S_{a,t} = \frac{A^f_{a,t+1}}{R^f_t} + \frac{1+x_{a,t+1}}{1-\theta}\left(Q_t - R_t\right)$$
$$\text{with } A^f_{a,0} = 0 \text{ and } 1 + x_{a,0} \geq 0 \text{ given.}$$

Here, we use $1-\rho \in (0,1)$ to denote the one-period discount factor. The parameter $\rho \in (0,1)$ is inversely related to the discount factor and plays a central role in our analysis (as we will see, it will be equal to the marginal propensity to consume). We require savings (total asset holdings) $S_{a,t}$ to be nonnegative—this does not bind in equilibrium and helps to rule out Ponzi schemes.

The term, $\frac{1+x_{a,t+1}}{1-\theta}$ denotes the units of capital that the household purchases at the ex-dividend price, $\frac{1+x_{a,t+1}}{1-\theta}(Q_t - R_t)$. We normalize by $1-\theta$, so that $x_{a,t+1}$ denotes the total purchases in area a. Households invest the rest of their savings in the risk-free asset, $\frac{A^f_{a,t}}{R^f_t}$, which delivers $A^f_{a,t}$ units of cash in the next period. Areas start with the same cash positions for simplicity, $A^f_{a,0} = 0$ (which is zero to ensure market clearing), but heterogeneous capital positions, $\left\{1 + x_{a,0}\right\}_a$.

Hand-to-mouth Households' Labor Supply. Hand-to-mouth households are myopic (equivalently, they have time separable preferences with discount factor set equal to 0). Therefore, they spend their labor income in all periods

$$P_{a,t}C^h_{a,t} = W_{a,t}L^h_t. \tag{B.10}$$

Their labor supply is endogenous. For the purpose of endogenizing the labor supply, we work with a GHH functional form for the intra-period preferences between consumption and labor that eliminates the wealth effects on the labor supply. These effects seem counterfactual for business cycle analysis (Galí (2011)).

Specifically, recall that in each area there is a mass θ of hand-to-mouth households. Suppose each hand-to-mouth household corresponds to a "representative agent" that is subdivided into a continuum of worker types denoted by $\nu \in [0,1]$. These workers provide specialized labor services. A worker ν who specializes in providing a particular type of labor service has the utility function:

$$C^h_{a,t}(\nu) - \chi \frac{\left(L^h_{a,t}(\nu)\right)^{1+\varphi^h}}{1+\varphi^h}. \tag{B.11}$$

Since she is myopic, she is subject to the budget constraint:

$$P_{a,t}C^h_{a,t}(\nu) = W_{a,t}(\nu)\, L^h_{a,t}(\nu)\,. \tag{B.12}$$

Here, $L^h_{a,t}(\nu)$ denotes her labor and $C^h_{a,t}(\nu)$ denotes her consumption.

In each area a, there is also an intermediate firm that produces the (hand-to-mouth) labor services in the area by combining specific labor inputs from each worker type according to the aggregator:

$$L^h_{a,t} = \left(\int_0^1 L^h_{a,t}(\nu)^{\frac{\varepsilon_w - 1}{\varepsilon_w}}\, d\nu\right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}.$$

This leads to the labor demand equation:

$$L^h_{a,t}(\nu) = \left(\frac{W_{a,t}(\nu)}{W_{a,t}}\right)^{-\varepsilon_w} L^h_{a,t} \tag{B.13}$$

$$\text{where } W_{a,t} = \left(\int_0^1 W_{a,t}(\nu)^{1-\varepsilon_w}\, d\nu\right)^{1/(1-\varepsilon_w)}. \tag{B.14}$$

Here, $L^h_{a,t}$ denotes the equilibrium labor provided by the representative hand-to-mouth household. (The total labor by all hand-to-mouth households is $\theta L^h_{a,t}$).

In period 0, a fraction of the workers in an area, λ_w, reset their wages to maximize the intra-period utility function in $(B.11)$ subject to the budget constraints in $(B.12)$ and the labor demand equation in $(B.13)$. The remaining fraction, $1-\lambda_w$, have preset wages given by $\overline{W}$—the nominal level targeted by monetary policy (as we describe subsequently).

The wage level in an area is determined according to the ideal price index $(B.14)$. This index also ensures:

$$\int_0^1 W_{a,t}(\nu)\, L^h_{a,t}(\nu)\, d\nu = W_{a,t}L^h_{a,t}.$$

Substituting this into Eq. $(B.12)$, we obtain the budget constraint for the representative hand-to-mouth household that we stated earlier [cf. $(B.10)$]:

$$P_{a,t}C_{a,t}^{h} \equiv \int_{0}^{1} P_{a,t}C_{a,t}^{h}(\nu)\,d\nu = W_{a,t}L_{a,t}^{h}.$$

Here, we have defined $C_{a,t}^{h}$ as the consumption by the representative hand-to-mouth household.

Optimal Wage Setting and the Labor Supply. First consider the flexible workers that reset their wages in period 0. These workers optimally choose $\left(W_{a,t}^{flex}, L_{a,t}^{h,flex}\right)$ that satisfy:

$$\begin{aligned} W_{a,t}^{flex} &\equiv P_{a,t}\frac{\varepsilon_w}{\varepsilon_w - 1}MRS_{a,t} \qquad \text{(B.15)} \\ \text{where } MRS_{a,t} &= \chi\left(L_{a,t}^{h,flex}\right)^{\varphi^h} \text{ and } L_{a,t}^{h,flex} = \left(\frac{W_{a,t}^{flex}}{W_{a,t}}\right)^{-\varepsilon_w} L_{a,t}^{h}. \end{aligned}$$

In particular, workers set a real (inflation-adjusted) wage that is a constant markup over their marginal rate of substitution between labor and consumption (MRS). The functional form in $(B.11)$ ensures that the MRS depends on the level of labor supply but not on the level of consumption.

Note that $W_{a,t}^{flex}$ appears on both side of Eq. $(B.15)$. Solving for the fixed point, we further obtain:

$$\left(W_{a,t}^{flex}\right)^{1+\varphi^h\varepsilon_w} = \frac{\varepsilon_w}{\varepsilon_w - 1}\chi P_{a,t}W_{a,t}^{\varepsilon_w\varphi^h}\left(L_{a,t}^{h}\right)^{\varphi^h}. \qquad \text{(B.16)}$$

Next consider the sticky workers. These workers have a preset wage level, $\overline{W}$. They provide the labor services demanded at this wage level (as long as their markup remains positive, which is the case in our analysis since we focus on log-linearized outcomes).

Next we use $(B.14)$ to obtain an expression for the aggregate wage level and the (hand-to-mouth) labor supply:

$$W_{a,t} = \left(\lambda_w\left(W_{a,t}^{flex}\right)^{1-\varepsilon_w} + (1-\lambda_w)\,\overline{W}^{1-\varepsilon_w}\right)^{1/(1-\varepsilon_w)}$$

$$= \left(\lambda_w \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \chi W_{a,t}^{\varepsilon_w \varphi^h} P_{a,t} \left(L_{a,t}^h \right)^{\varphi^h} \right)^{(1-\varepsilon_w)/\left(1+\varphi^h \varepsilon_w\right)} + (1 - \lambda_w) \overline{W}^{1-\varepsilon_w} \right)^{1/(1-\varepsilon_w)} . \tag{B.17}$$

Here, the first line substitutes the wages of flexible and sticky workers. The second line substitutes the optimal wage for flexible workers from Eq. (B.16). This expression illustrates that greater hand-to-mouth labor in an area, $L_{a,t}^h$, creates wage pressure. The amount of pressure depends positively on the fraction of flexible workers, λ_w, and negatively on the labor supply elasticity, $1/\varphi^h$, as well as on the elasticity of substitution across labor types, ε_w. An increase in the local price index, $P_{a,t}$, also creates wage pressure.

It is also instructive to consider the (hand-to-mouth) labor supply in two special cases. First, consider the "frictionless" case without nominal rigidities: that is, suppose wages are fully flexible, $\lambda_w = 1$. All workers set the same wage, which implies $W_{a,t}^{flex} = W_{a,t}$. Using this observation Eq. (B.17) becomes:

$$\frac{W_{a,t}}{P_{a,t}} = \frac{\varepsilon_w}{\varepsilon_w - 1} \chi \left(L_{a,t}^h \right)^{\varphi^h} . \tag{B.18}$$

Hence, the frictionless hand-to-mouth labor supply in each area a is described by a neoclassical intra-temporal optimality condition. In particular, the real wage is a constant markup over the MRS between labor and consumption.

Next consider the case in which the nominal wage in the area is equal to the monetary policy target, $W_{a,t} = \overline{W}$. Substituting this expression into (B.17), we obtain,

$$\frac{\overline{W}}{P_{a,t}} = \frac{\varepsilon_w}{\varepsilon_w - 1} \chi \left(L_{a,t}^h \right)^{\varphi^h} . \tag{B.19}$$

This is equivalent to (B.18) (since $W_{a,t} = \overline{W}$). Hence, our model features a version of "the divine coincidence": stabilizing the nominal wage at the target ($\overline{W}$) is equivalent to stabilizing the labor supply at its frictionless level.

Monetary Policy. We assume monetary policy sets the nominal interest rate R_t^f to stabilize the average nominal wage at the target level $\overline{W}$:

$$\int_a W_{a,t} da = \overline{W} \text{ for each } t. \tag{B.20}$$

In periods $t \geq 1$, nominal wages are equated across regions (since labor is mobile). Therefore, Eq. $(B.20)$ implies $W_{a,t} = \overline{W}$ for each area, which in turn implies Eq. $(B.19)$. For these periods, monetary policy replicates the frictionless labor supply.

In period 0, wages are not necessarily equated across areas. Thus, monetary policy cannot stabilize labor supply in every area. For this period, the policy rule in $(B.20)$ can be thought of as stabilizing the labor supply "on average" at its frictionless level. When areas have common initial wealth (and therefore common initial wage, $W_{a,0} = \overline{W}$), monetary policy stabilizes the labor supply at its frictionless level also in period 0.

Market Clearing Conditions. First consider the nontradable good. Recall that we use $Y_{a,t}^N$ to denote nontradable production and $C_{a,t}^N$ to denote the total nontradable spending in an area [cf. $(B.1)$ and $(B.7)$]. Thus, we have the market clearing condition,

$$Y_{a,t}^N = C_{a,t}^N \text{ for each } a, t. \tag{B.21}$$

Next consider the composite tradable good. We use Y_t^T to denote the tradable production with the standard CES technology in either period, and $\tilde{Y}_t^T$ to denote the production with the capital-only technology in periods $t \geq 1$ [cf. $(B.2)$ and $(B.4)$]. We also use $C_{a,t}^T$ to denote the total tradable spending in an area [cf. $(B.22)$]. Thus, we have the market clearing conditions:

$$Y_0^T = \int C_{a,0}^T da. \tag{B.22}$$

$$Y_t^T + \tilde{Y}_t^T = \int_a C_{a,t}^T da \text{ for } t \geq 1. \tag{B.23}$$

There is a single market clearing condition for each period since the tradable good can be transported across areas costlessly.

Next consider the tradable good produced in area a. This market clearing condition is already embedded in our notation, since we use $Y_{a,t}^T$ to denote the tradable production in area a as well as the tradable input used in the CES production technology [cf. $(B.3)$ and $(B.2)$].

Next consider factor market clearing conditions. In period 0, for labor we have:

$$L_{a,0} = (1-\theta)\,\overline{L} + \theta L^h_{a,0} = L^N_{a,0} + L^T_{a,0} \text{ for each } a. \tag{B.24}$$

Labor supply comes from stockholders, who supply exogenous labor, $L^s_{a,0} = \overline{L}$, and hand-to-mouth households, who supply endogenous labor, $L^h_{a,0}$. Labor demand comes from nontradable and tradable production firms in the area. For capital, we have

$$1 = \int_a \left(K^N_{a,0} + K^T_{a,0}\right) da. \tag{B.25}$$

Capital supply is exogenous and normalized to one. Capital demand comes from nontradable and tradable production firms in all areas. There is a single market clearing condition since capital is mobile across areas.

For future periods $t \geq 1$, both factors are mobile across areas. Therefore, we have the following analogous market clearing conditions,

$$\int_a \left((1-\theta)\,\overline{L} + \theta L^h_{a,t}\right) da = \int_a \left(L^N_{a,t} + L^T_{a,t}\right) da \tag{B.26}$$

$$1 = \int_a \left(K^N_{a,t} + K^T_{a,t}\right) da + \tilde{K}^T_t \text{ for each } t \geq 1. \tag{B.27}$$

Capital demand reflects that capital can also be used with the alternative linear technology, $\tilde{K}^T_t$.

Finally, the asset market clearing conditions can be written as,

$$\int_a x_{a,t} da = 0 \text{ and } \int_a A^f_{a,t} = 0. \tag{B.28}$$

This condition ensures that the holdings of capital across areas sum to its supply (one). The second condition says the holdings of the risk-free asset sum to its supply (zero). We can then define the equilibrium as follows.

Definition 1 *Given an initial distribution of ownership of capital,* $\{x_{a,0}\}_a$ *(that sum to zero across areas), and otherwise symmetric regions, an equilibrium is a collection of cross-sectional and aggregate allocations together with paths of (nominal) factor prices,*

$\{\{W_{a,t}\}_a, R_t\}_t$, *goods prices,* $\left\{\{P^N_{a,t}\}_a, P^T_t\right\}_t$, *the asset price,* $\{Q_t\}_t$, *and the interest rate,* $\left\{R^f_t\right\}_t$, *such that:*

(i) Competitive firms maximize according to the production technologies described in $(B.1 - B.4)$.

(ii) Stockholders choose their consumption and portfolios optimally [cf. problem $(B.9)$*]. All households split their consumption between nontradable and tradable goods to solve the expenditure minimization problem* $(B.6)$.

(iii) Capital supply is exogenous and given by $(B.5)$. *Labor supply of stockholders is also exogenous and given by* $(B.8)$. *Labor supply of hand-to-mouth households is endogenous and satisfy Eq.* $(B.17)$.

(iv) Monetary policy stabilizes the average wage in each period at a particular level $\overline{W}$ *[cf.* $(B.20)$*].*

(v) Goods, factors, and asset markets clear [cf. Eqs. $(B.21 - B.28)$*].*

B.2 General Characterization of Equilibrium

We next provide a general characterization of equilibrium. In subsequent sections, we use this characterization to solve for the equilibrium under different specifications. Throughout, we assume the parameters satisfy:

$$D \geq \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L} \tag{B.29}$$

$$\chi = \frac{\varepsilon_w - 1}{\varepsilon_w}\left(\frac{1-\overline{\alpha}}{\overline{\alpha}}\right)^{\overline{\alpha}} \frac{1}{\overline{L}^{\overline{\alpha}+\varphi^h}} \tag{B.30}$$

The first condition ensures that the capital-only production technology is actually used when it is available, $\tilde{K}_t \geq 0$ for $t \geq 1$. The second condition ensures that in period 0 the frictionless hand-to-mouth labor supply (and thus, the frictionless aggregate labor supply) is the same as the stockholders' exogenous labor supply, $\overline{L}$. This is a symmetry assumption that simplifies the notation but otherwise does not play an important role.

We start by establishing general properties on the supply and the demand side that apply in all periods. We then fully characterize the equilibrium in periods $t \geq 1$ (long run). Finally, we derive the equations that characterize the equilibrium in period 0 (short run).

B.2.1 General Properties

Supply Side. First consider households' choice between nontradable and tradable goods. Households solve $(B.6)$, which implies:

$$P_{a,t} \equiv \left(P_{a,t}^{N}\right)^{\eta}\left(P_{t}^{T}\right)^{1-\eta} \tag{B.31}$$

$$P_{a,t}^{N}C_{a,t}^{i,N} = \eta P_{a,t}C_{a,t}^{i} \text{ and } P_{a,t}^{T}C_{a,t}^{i,T} = (1-\eta)\,P_{a,t}C_{a,t}^{i}. \tag{B.32}$$

Here, recall that $P_{a,t}$ (the unit cost or the ideal price index) denotes the solution to the problem with $C_{a,t}^{i}=1$. Aggregating across all households in an area, we further obtain

$$P_{a,t}^{N}C_{a,t}^{N} = \eta P_{a,t}C_{a,t} \text{ and } P_{t}^{T}C_{a,t}^{T} = (1-\eta)\,P_{a,t}C_{a,t}.$$

In view of the Cobb-Douglas aggregator, the shares of nontradables and tradables in household spending are constant.

Next consider optimization by firms that produce the nontradable good, which implies [cf. $(B.1)$]:

$$P_{a,t}^{N} = \left(W_{a,t}\right)^{1-\alpha^{N}} R_{t}^{\alpha^{N}} \tag{B.33}$$

$$w_{a,t}L_{a,t}^{N} = \left(1-\alpha^{N}\right)P_{a,t}^{N}Y_{a,t}^{N} \text{ and } R_{t}K_{a,t}^{N} = \alpha^{N}P_{a,t}^{N}Y_{a,t}^{N}. \tag{B.34}$$

Similarly, optimization by firms that produce the tradable input in an area implies [cf. $(B.3)$]:

$$P_{a,t}^{T} = \left(W_{a,t}\right)^{1-\alpha^{T}} R_{t}^{\alpha^{T}} \tag{B.35}$$

$$w_{a,t}L_{a,t}^{T} = \left(1-\alpha^{T}\right)P_{a,t}^{T}Y_{a,t}^{T} \text{ and } R_{t}K_{a,t}^{T} = \alpha^{T}P_{a,t}^{T}Y_{a,t}^{T}. \tag{B.36}$$

Here, we use $P_{a,t}^{T}$ to denote the price of the tradable input produced in an area. In view of Cobb-Douglas technologies, the shares of labor and capital in production of the nontradable good as well as the local tradable input are constant.

Next consider the firms that produce the composite tradable good with the CES production technology [cf. $(B.2)$]. These firms' optimization implies:

$$P_{t}^{T} = \left(\int_{a}\left(P_{a,t}^{T}\right)^{1-\varepsilon}da\right)^{1/(1-\varepsilon)} \tag{B.37}$$

$$P_{a,t}^T Y_{a,t}^T = \left(\frac{P_{a,t}^T}{P_t^T}\right)^{1-\varepsilon} P_t^T Y_t^T. \tag{B.38}$$

The unit cost of the composite tradable good is determined by the ideal price index. The share of tradable inputs from an area depends on the price in that area relative to the unit cost, $\frac{P_{a,t}^T}{P_t^T}$, as well as the elasticity of substitution across tradables, ε.

Finally, consider the firms that produce the composite tradable good in periods $t \geq 1$ with the linear technology [cf. $(B.4)$]. These firms' optimization implies,

$$P_t^T = R_t / D_t^{1-\alpha^T} \text{ as long as } \tilde{K}_t^T > 0 \text{ (for } t \geq 1\text{)}. \tag{B.39}$$

As we will verify below, the parametric restriction in $(B.29)$ ensures $\tilde{K}_t^T > 0$.

Recall also that we have the labor supply equation $(B.17)$ for each area a.

Demand Side. We next turn to the demand side. First consider the nontradable sector. Combining the market clearing condition $(B.21)$ with the factor shares in $(B.32)$ and $(B.34)$, we solve for the factor bills as:

$$\begin{aligned} W_{a,t} L_{a,t}^N &= \left(1-\alpha^N\right) \eta P_{a,t} C_{a,t} \\ R_t K_{a,t}^N &= \frac{\alpha^N}{1-\alpha^N} W_{a,t} L_{a,t}^N \end{aligned} \tag{B.40}$$

For the nontradable sector, the demand comes from the nontradable expenditure within the area. In view of the Cobb-Douglas technologies, this demand is split across factors in constant proportions.

Next consider the tradable sector. We combine the market clearing conditions $(B.22)$ and $(B.23)$ with the factor shares in $(B.32)$, $(B.36)$, and $(B.38)$ to solve:

$$\begin{aligned} W_{a,t} L_{a,t}^T &= \left(1-\alpha^T\right) \left(\frac{P_{a,t}^T}{P_t^T}\right)^{1-\varepsilon} \left((1-\eta) \int_a P_{a,t} C_{a,t} da - \tilde{Y}_t^T\right) \\ \text{and } R_t K_{a,t}^T &= \frac{\alpha^T}{1-\alpha^T} w_{a,t} L_{a,t}^T \\ \text{where } \tilde{Y}_0^T &= 0 \text{ and } \tilde{Y}_t^T = D_t^{1-\alpha^T} \tilde{K}_t^T \text{ for } t \geq 1. \end{aligned} \tag{B.41}$$

For the tradable sector (that use standard technologies), the demand comes from the tradable expenditure *from all areas.* The demand also depends on the relative price in that area, $\frac{P^T_{a,t}}{P^T_t}$, as well as the elasticity of substitution across tradable inputs, ε. The expression, $\tilde{Y}^T_t$ denotes the production of the composite tradable good via the alternative capital-only technology, which is zero in period 0 but not in periods $t \geq 1$ (as the technology is only available in periods $t \geq 1$).

Stockholders' Optimality Conditions. Finally, we characterize stockholders' optimality conditions at any period t [cf. problem $(B.9)$]. First consider their portfolio choice. Since there is no risk in capital (for simplicity), problem $(B.9)$ implies that stockholders take a non-zero position on capital if and only if its price satisfies, $\frac{Q_{t+1}}{Q_t - R_t} = R^f_t$. This implies,

$$\begin{aligned} Q_t &= R_t + \frac{Q_{t+1}}{R^f_t} \\ &= \sum_{n \geq 0}^{\infty} \frac{R_{t+n}}{R^f_t .. R^f_{t+n-1}}. \end{aligned} \tag{B.42}$$

Here, the second line rolls the equation forward to write the stock price as the present discounted value of the rental rate. We assume the transversality condition, $\lim_{n\to\infty} \frac{R_{t+n}}{R^f_t .. R^f_{t+n-1}} = 0$, which will hold in the equilibria we will characterize. Given the capital price in $(B.55)$, stockholders are indifferent between saving in the risk-free asset and in capital.

Next consider stockholders' consumption choice. Given the capital price in $(B.55)$, we can aggregate stockholders' budget constraints from time t onward to obtain a lifetime budget constraint at time t:

$$\sum_{n \geq 0}^{\infty} \frac{P_{a,t+n} C^s_{a,t+n}}{R^f_t .. R^f_{t+n-1}} = \sum_{n \geq 0}^{\infty} \frac{W_{a,t+n} \overline{L}}{R^f_t .. R^f_{t+n-1}} + \frac{1 + x_{a,t}}{1 - \theta} Q_t + A^f_{a,t}. \tag{B.43}$$

As before, we assume the transversality condition, $\lim_{n\to\infty} \frac{W_{a,t+n}\overline{L}}{R^f_t .. R^f_{t+n-1}} = 0$. In addition, the optimality condition for safe savings $A^f_{a,t+1}$ implies the Euler equation,

$$\frac{1}{P_{a,t+n-1} C^s_{a,t+n-1}} = \frac{(1-\rho) R^f_{t+n-1}}{P_{a,t+n} C^s_{a,t+n}} \text{ for each } t \geq 0, n \geq 1. \tag{B.44}$$

Solving this backward, we obtain $\frac{P_{a,t+n}C^s_{a,t+n}}{R^f_t..R^f_{t+n-1}} = (1-\rho)^n P_{a,t}C^s_{a,t}$. After substituting this into $(B.43)$ and calculating the sum, we obtain

$$P_{a,t}C^s_{a,t} = \rho\left(\sum_{n\geq 0}^{\infty}\frac{W_{a,t+n}\overline{L}}{R^f_t..R^f_{t+n-1}} + \frac{1+x_{a,t}}{1-\theta}Q_t + A^f_{a,t}\right). \tag{B.45}$$

Hence, in each period t, stockholders spend a fraction of their lifetime wealth. Their lifetime wealth consists of the present discounted value of their labor income as well as their stock wealth and cash at the beginning of the period. The marginal propensity to spend out of wealth is given by ρ.

B.2.2 Long Run Equilibrium

We next characterize the equilibrium further in periods $t \geq 1$. For these periods, labor (as well as capital) is mobile across areas. In addition, production technologies remain constant over time. In view of these features, we conjecture an equilibrium in which the economy immediately reaches a steady state in period $t = 1$. Specifically, we prove the following.

Proposition 1 *Suppose conditions $(B.29)$ and $(B.30)$ hold. Starting from period $t \geq 1$ onward, there is a steady-state equilibrium in which the capital-only technology is (weakly) used, $\tilde{K}^T_t \geq 0$. In this equilibrium, nominal wages, rental rates, price indices, hand-to-mouth labor, and aggregate labor are constant across areas and over time:*

$$W_{a,t} = \overline{W} \text{ and } R_t = \overline{W}D \tag{B.46}$$

$$P^T_{a,t} = \overline{W}D^{\alpha^T}, P^N_{a,t} = \overline{W}D^{\alpha^N}, P_{a,t} = \overline{W}D^{\overline{\alpha}} \text{ where } \overline{\alpha} = \eta\alpha^N + (1-\eta)\alpha^T \tag{B.47}$$

$$L^h_{a,t} = L^{h,long} \leq \overline{L} \text{ where } D^{-\overline{\alpha}} = \frac{\varepsilon_w}{\varepsilon_w - 1}\chi\left(L^{h,long}\right)^{\varphi^h}. \tag{B.48}$$

The interest rate and the price of capital are constant over time:

$$R^f_t = \frac{1}{1-\rho} \tag{B.49}$$

$$Q_t = \frac{\overline{W}D}{\rho}. \tag{B.50}$$

Stockholders' capital and cash holdings and consumption are constant over time and determined by their capital and cash holdings in period 1*:*

$$x_{a,t} = x_{a,1}, A^f_{a,t} = A^f_{a,1} \quad \text{(B.51)}$$

$$P_{a,t}C^s_{a,t} = \rho\left(\frac{\overline{W}\overline{L}}{\rho} + \frac{1+x_{a,1}}{1-\theta}\frac{\overline{W}D}{\rho} + A^f_{a,1}\right). \quad \text{(B.52)}$$

Proof. We first show factor and goods prices satisfy Eqs. (B.46) and (B.47). Since labor is mobile across areas, wages are equated across areas, $W_{a,t} \equiv W_t$. This proves $W_{a,t} = \overline{W}$ since monetary policy stabilizes the wage at the target level [cf. (B.20)]. Substituting this into the unit cost equations (B.35) and (B.37), we find $P^T_t = \overline{W}^{1-\alpha^T} R_t^{\alpha^T}$. Combining this with ($B$.39), we establish ($B$.46). Substituting Eq. ($B$.46) into the remaining unit cost equations (B.31) and (B.33), we also establish (B.47). Since the capital only technology is used (as we verify shortly), the rental rate is determined by the productivity of this technology, D. This provides a simple expression also for other prices.

Substituting the expression for the price index P_t into the frictionless labor supply equation (B.19), we also establish that hand-to-mouth labor is constant and given by (B.48). Consider how the solution changes with D. First consider the lowest level of D allowed by condition (B.29), $D = \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}$. In this case the solution is given by $L^{h,long} = \overline{L}$ in view of condition (B.30). Next note that increasing D decreases $L^{h,long}$. Intuitively, increasing the productivity of the capital-only technology draws capital from the standard technologies (as we verify shortly), which in turn lowers the labor supply. Therefore, the solution satisfies $L^{h,long} \leq \overline{L}$.

Next we verify that the capital-only technology is used in equilibrium, $\tilde{K}^T_t \geq 0$. To this end, we aggregate the factor demands used in the standard technologies across both sectors and across all areas to obtain [cf. Eqs. (B.40) and (B.41)]:

$$\overline{W}\left((1-\theta)\overline{L} + \theta L^{h,long}\right) = \begin{bmatrix} \left(1-\alpha^N\right)\eta\int_a P_{a,t}C_{a,t}da \\ +\left(1-\alpha^T\right)\left((1-\eta)\int_a P_{a,t}C_{a,t}da - \tilde{Y}^T_t\right) \end{bmatrix}$$

$$= (1-\overline{\alpha})\int_a P_{a,t}C_{a,t}da - \left(1-\alpha^T\right)\tilde{Y}^T_t$$

and

$$R_t\left(1-\tilde{K}^T_t\right) = \overline{\alpha}\int_a P_{a,t}C_{a,t}da - \alpha^T\tilde{Y}^T_t$$

Here, we have substituted the factor market clearing conditions $L_{a,t}^{T}+L_{a,t}^{N}=(1-\theta)\,\overline{L}+\theta L^{h,long}$ and $K_{a,t}^{T}+K_{a,t}^{N}+\tilde{K}_{t}^{T}=1$ [cf. $(B.26)$ and $(B.27)$].

Combining these expressions, we solve for the capital bill used in the standard technologies:

$$R_t\left(1-\tilde{K}_t^T\right)=\frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{W}\left((1-\theta)\,\overline{L}+\theta L^{h,long}\right)+\frac{\overline{\alpha}-\alpha^T}{1-\overline{\alpha}}\tilde{Y}_t.$$

After substituting $\tilde{Y}_t^T=R_t\tilde{K}_t^T$ and $R_t=\overline{W}D$, we find $\tilde{K}_t^T\equiv\tilde{K}^{T,long}(D)$ where:

$$D\left(1-\frac{1-\alpha^T}{1-\overline{\alpha}}\tilde{K}^{T,long}(D)\right)=\frac{\overline{\alpha}}{1-\overline{\alpha}}\left((1-\theta)\,\overline{L}+\theta L^{h,long}(D)\right). \tag{B.53}$$

Since $L^{h,long}(D)$ is a decreasing function, $\tilde{K}^{T,long}(D)$ that solves $(B.53)$ is an increasing function of D. Moreover, when $D=\frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}$, we have $L^{h,long}(D)=\overline{L}$, which implies $\tilde{K}^{T,long}(D)=0$. This proves $\tilde{K}^{T,long}(D)\geq 0$ for each $D\geq\frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}$ and establishes that the capital-only technology is used in equilibrium.

Finally, we verify that the constant interest rate path in $(B.49)$ corresponds to an equilibrium along with the asset price and allocations in $(B.50)$, $(B.51)$, and $(B.52)$.

Substituting $R_t^f=1/(1-\rho)$ into $(B.42)$, and using $(B.46)$, we establish that the stock price satisfies $(B.50)$. Substituting this expression along with Eq. $(B.49)$ and the solution for the wage and the rental rate into Eq. $(B.45)$, we establish that stockholders' consumption satisfies

$$P_{a,t}C_{a,t}^s=\rho\left(\frac{\overline{WL}}{\rho}+\frac{1+x_{a,t}}{1-\theta}\frac{\overline{W}D}{\rho}+A_{a,t}^f\right). \tag{B.54}$$

Note also that stockholders are indifferent between saving in capital and the risk-free asset. In particular, $x_{a,t+1}=x_{a,t}$ is a solution as long as the implied cash holding is non-negative, $A_{a,t+1}\geq 0$. To verify this, consider the stockholders' budget constraint with the equilibrium wage and the rental rate [cf. $(B.9)$]:

$$P_{a,t}C_{a,t}^s+\frac{A_{a,t+1}^f}{R_t^f}+\frac{1+x_{a,t+1}}{1-\theta}\left(Q_t-\overline{W}D\right)=\overline{WL}+\frac{1+x_{a,t}}{1-\theta}Q_t+A_{a,t}^f.$$

Substituting $x_{a,t+1}=x_{a,t}$ along with Eq. $(B.54)$, we obtain $A_{a,t+1}=A_{a,t}$. By induction, we further obtain $x_{a,t+1}=x_{a,1}, A_{a,t+1}=A_{a,1}$. Since $A_{a,1}\geq 0$, this verifies $A_{a,t+1}\geq 0$ and establishes $(B.51)$. Substituting this into $(B.45)$, we establish that stockholders' consumption is constant over time and given by $(B.52)$.

Note also that this allocation satisfies the asset market clearing conditions [cf. $(B.28)$], which implies that it also satisfies the aggregate goods market clearing conditions. In fact, aggregating Eq. $(B.52)$ across all areas, it is easy to verify that stockholders in the aggregate spend their labor income and capital income. Hand-to-mouth households spend their labor income. Since asset and goods markets clear, the conjectured interest rate path $(B.49)$ corresponds to an equilibrium, which completes the proof.■

Therefore, the economy reaches a steady state immediately in period $t = 1$. This simplifies the analysis as it enables us to focus on the allocations in period $t = 0$, which we turn to subsequently. Note also that using Proposition 1 together with Eqs. $(B.40)$ and $(B.41)$ we could characterize the labor employed in nontradable and tradable sectors separately for periods $t \geq 1$. We skip this step since it will not play an important role for our analysis of the equilibrium in period 0.

B.2.3 Short Run Equilibrium

We next characterize the conditions that determine the equilibrium in period 0. In subsequent sections, we use these conditions to solve the equilibrium for different specifications of initial wealth across areas.

Asset Price in Period 0. Using Eqs. $(B.42)$ and $(B.50)$, we obtain

$$Q_0 = R_0 + \frac{Q_1}{R_0^f} = R_0 + \frac{1}{R_0^f} \frac{\overline{W} D}{\rho}. \tag{B.55}$$

Hence, the stock price in the first period depends on the future productivity in the capital only technology, D, the current interest rate, R_0^f, and the current rental rate, R_0.

We next claim the rental rate satisfies

$$R_0 = \frac{\overline{\alpha}}{1 - \overline{\alpha}} \int_a W_{a,0} L_{a,0} da. \tag{B.56}$$

In view of the Cobb-Douglas technologies, the equilibrium rental rate of capital is proportional to the aggregate labor bill (and aggregate output). Combined with $(B.55)$, this describes the stock price in terms of the aggregate labor bill and the interest rate.

To prove the claim in $(B.56)$, we aggregate Eqs. $(B.40)$ and $(B.41)$ over the two sectors to obtain

$$
\begin{aligned}
W_{a,0}\left(L_{a,0}^N + L_{a,0}^T\right) &= \left(1-\alpha^N\right)\eta P_{a,0}C_{a,0} + \left(1-\alpha^T\right)\left(\frac{P_{a,0}^T}{P_0^T}\right)^{1-\varepsilon}(1-\eta)\int_a P_{a,0}C_{a,0}da \\
R_0\left(K_{a,0}^T + K_{a,0}^N\right) &= \alpha^N \eta P_{a,0}C_{a,0} + \alpha^T\left(\frac{P_{a,t}^T}{P_t^T}\right)^{1-\varepsilon}(1-\eta)\int_a P_{a,t}C_{a,t}da.
\end{aligned}
$$

Aggregating further across all areas and using the market clearing conditions $L_{a,0}^N + L_{a,0}^T = L_{a,0}$ and $K_{a,0}^N + K_{a,0}^T = 1$ [cf. $(B.24)$ and $(B.25)$] along with $(B.37)$, we obtain:

$$
\begin{aligned}
\int_a W_{a,0}L_{a,0}da &= (1-\overline{\alpha})\int_a P_{a,0}C_{a,0}da \\
R_0 &= \overline{\alpha}\int_a P_{a,0}C_{a,0}da.
\end{aligned}
$$

Here, recall that $\overline{\alpha} = \eta\alpha^N + (1-\eta)\alpha^T$ is the weighted-average capital share. Combining these expressions, we establish $(B.56)$.

Stockholders' Consumption in Period 0. It remains to characterize the households' consumption demand in period 0, which determines the labor demand and completes the characterization of equilibrium [cf. Eqs. $(B.40)$ and $(B.41)$]. Hand-to-mouth agents spend their income,

$$P_{a,t}C_{a,t}^h = W_{a,t}L_{a,t}^h. \tag{B.57}$$

Consider the stockholders. Note that their consumption is generally characterized by Eq. $(B.45)$. Using Proposition 1, and the assumption $A_{a,0}^f = 0$, we can write this as

$$P_{a,0}C_{a,0}^s = \rho\left(W_{a,0}\overline{L} + \frac{1}{R_0^f}\frac{\overline{WL}}{\rho} + \frac{1+x_{a,0}}{1-\theta}Q_0\right). \tag{B.58}$$

Hence, stockholders spend a fraction of their lifetime wealth, which is determined by their current and future labor income as well as their stock wealth.

Aggregating Eqs. $(B.57)$ and $(B.58)$ with households' population shares, we characterize the aggregate household demand in an area [cf. (4)]:

$$P_{a,0}C_{a,0} = \theta W_{a,0}L^h_{a,0} + \rho\left((1-\theta)\left(W_{a,0}\overline{L} + \frac{1}{R^f_0}\frac{\overline{WL}}{\rho}\right) + (1+x_{a,0})\,Q_0\right). \tag{B.59}$$

Hence aggregate demand in the area is determined by spending by the hand-to-mouth households (that depends on local wages) and the spending by stockholders (that depends on local wealth).

Labor Demand in Period 0. Combining Eq. $(B.59)$ with $(B.40)$, and substituting $\theta L^h_{a,0} = L_{a,0} - (1-\theta)\,\overline{L}$ (by definition), we calculate the labor demand in the nontradable sector as:

$$W_{a,0}L^N_{a,0} = \left(1-\alpha^N\right)\eta\left(\begin{array}{c} W_{a,0}\left(L_{a,0} - (1-\theta)\,\overline{L}\right) + \\ \rho\left(\begin{array}{c} (1-\theta)\left(W_{a,0}\overline{L} + \frac{1}{R^f_0}\frac{\overline{WL}}{\rho}\right) \\ + (1+x_{a,0})\,Q_0 \end{array}\right)\end{array}\right). \tag{B.60}$$

Likewise, we combine Eq. $(B.59)$ with $(B.41)$ to obtain the labor demand in the tradable sector as:

$$W_{a,0}L^T_{a,0} = \left(\frac{P^T_{a,0}}{P^T_0}\right)^{1-\varepsilon}\left(1-\alpha^T\right)(1-\eta)\left(\begin{array}{c} \int_a W_{a,0}\left(L_{a,0} - (1-\theta)\,\overline{L}\right)da + \\ \rho\left(\begin{array}{c} (1-\theta)\left(W_{a,0}\overline{L} + \frac{1}{R^f_0}\frac{\overline{WL}}{\rho}\right) \\ + (1+x_{a,0})\,Q_0 \end{array}\right)\end{array}\right). \tag{B.61}$$

After summing Eqs. $(B.60)$ and $(B.61)$, and using the labor market clearing condition $L_{a,0} = L^T_{a,0} + L^N_{a,0}$ [cf. $(B.24)$], we solve for the total labor demand in an area as follows,

$$\begin{aligned} W_{a,0}L_{a,0} = {} & \left(1-\alpha^N\right)\eta\left(\begin{array}{c} W_{a,0}\left(L_{a,0} - (1-\theta)\,\overline{L}\right) + \\ \rho\left(\begin{array}{c} (1-\theta)\left(W_{a,0}\overline{L} + \frac{1}{R^f_0}\frac{\overline{WL}}{\rho}\right) \\ + (1+x_{a,0})\,Q_0 \end{array}\right)\end{array}\right) \\ & + \left(\frac{P^T_{a,0}}{P^T_0}\right)^{1-\varepsilon}\left(1-\alpha^T\right)(1-\eta)\left(\begin{array}{c} \int_a W_{a,0}\left(L_{a,0} - (1-\theta)\,\overline{L}\right)da + \\ \rho\left(\begin{array}{c} (1-\theta)\left(W_{a,0}\overline{L} + \frac{1}{R^f_0}\frac{\overline{WL}}{\rho}\right) \\ + (1+x_{a,0})\,Q_0 \end{array}\right)\end{array}\right) \end{aligned} \tag{B.62}$$

The first line illustrates the local labor demand due to local spending on the nontradable good. The second line illustrates the local labor demand due to aggregate spending on the tradable good. While this expression looks complicated, it will be simplified once we log-linearize around the common wealth allocation.

Given the unit costs and the aggregate variables, Eq. $(B.62)$ is a collection of $|I|$ equations in $2|I|$ local variables, $\{L_{a,0}, W_{a,0}\}_{a \in I}$. Recall also that we have Eq. $(B.17)$ that determines the local labor supply of hand-to-mouth households in each area. After substituting $\theta L^h_{a,0} = L_{a,0} - (1-\theta)\overline{L}$, we write this expression as:

$$W_{a,0} = \left(\begin{array}{c} \lambda_w \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \chi W_{a,0}^{\varepsilon_w \varphi^h} P_{a,0} \left(\frac{L_{a,0} - (1-\theta)\overline{L}}{\theta} \right)^{\varphi^h} \right)^{(1-\varepsilon_w)/\left(1+\varphi^h \varepsilon_w\right)} \\ + (1-\lambda_w) \overline{W}^{1-\varepsilon_w} \end{array} \right)^{1/(1-\varepsilon_w)} . \quad \text{(B.63)}$$

This provides $|I|$ additional equations in $\{L_{a,0}, W_{a,0}\}_{a \in I}$. Thus, Eqs. $(B.62)$ and $(B.63)$ can be thought of as determining the equilibrium in labor markets in each area.

Recall also that we have characterized the aggregate variables earlier. In particular, the capital price is given $(B.55)$, which depends on the rental rate R_0 given by $(B.56)$ and the interest rate R_0^f. The interest rate is set by monetary policy to ensure the average nominal wage is equal to a target level, $\int_a W_{a,0} = \overline{W}$ [cf. $(B.20)$]. This completes the general characterization of equilibrium.

B.3 Benchmark Equilibrium with Common Stock Wealth

We next characterize the equilibrium in period 0 further in special cases of interest. In this section, we focus on a benchmark case in which areas have common wealth, $x_{a,0} = 0$ for each a, and provide a closed-form solution. In the next section, we log-linearize the equilibrium around this benchmark and provide a closed-form solution for the log-linearized equilibrium.

Labor Market Equilibrium. First consider the labor supply. By symmetry, wages, price indices, and labor are the same across areas. We denote these allocations by dropping the area subscript W_0, P_0, L_0^h, L_0. Then, the monetary policy rule $(B.20)$ implies $W_0 = \overline{W}$. Hence, in this case monetary policy ensures labor supply is at its frictionless level also in period 0 [cf. Eq. $(B.19)$]:

$$\frac{\overline{W}}{P_0} = \frac{\varepsilon_w}{\varepsilon_w - 1}\chi\left(L_0^h\right)^{\varphi^h}. \tag{B.64}$$

Next consider the labor demand. Using Eq. $(B.56)$ the rental rate of capital is given by:

$$R_0 = \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{W}L_0. \tag{B.65}$$

When wages are the same across all areas, the unit cost is given by $P_0 = \overline{W}^{1-\overline{\alpha}} R_0^{\overline{\alpha}}$ [cf. Eqs. $(B.31), (B.33)$, and $(B.37)$]. Combining this with Eq. $(B.65)$, we obtain,

$$P_0 = R_0^{\overline{\alpha}} \overline{W}^{1-\overline{\alpha}} = \left(\frac{\overline{\alpha}}{1-\overline{\alpha}}\right)^{\overline{\alpha}} L_0^{\overline{\alpha}} \overline{W} \text{ where } L_0 = (1-\theta)\overline{L} + \theta L_0^h. \tag{B.66}$$

After rearranging this expression, we obtain a labor demand equation

$$\frac{\overline{W}}{P_0} = \left(\frac{1-\overline{\alpha}}{\overline{\alpha}}\right)^{\overline{\alpha}} \left((1-\theta)\overline{L} + \theta L_0^h\right)^{-\overline{\alpha}}. \tag{B.67}$$

Eqs. $(B.64)$ and $(B.67)$ uniquely determines the hand-to-mouth labor. Condition $(B.30)$ ensures that the solution satisfies:

$$L_0^h = \overline{L}. \tag{B.68}$$

In sum, with common wealth, monetary policy ensures hand-to-mouth labor is at its frictionless level. In view of the normalizing condition $(B.30)$, this is the same as stockholders' labor supply. This ensures that the total labor is also at its frictionless level

$$L_0^T + L_0^N = L_0 = (1-\theta)\overline{L} + \theta L_0^h = \overline{L}. \tag{B.69}$$

Asset and Goods Market Equilibrium. Next consider the price of capital. Combining Eqs. $(B.65), (B.69)$ with Eq. $(B.55)$, we obtain:

$$Q_0 = \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{WL} + \frac{1}{R_0^f}\frac{\overline{W}D}{\rho}. \tag{B.70}$$

Next note that we can aggregate the labor demand Eq. $(B.62)$ to obtain:

$$\frac{\overline{WL}}{1-\overline{\alpha}} = \begin{array}{c} \theta\overline{WL}+ \\ \rho\left((1-\theta)\left(\overline{WL}+\frac{1}{R_0^f}\frac{\overline{WL}}{\rho}\right)+\frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{WL}+\frac{1}{R_0^f}\frac{\overline{W}D}{\rho}\right) \end{array}.$$

Rearranging terms, we obtain:

$$\begin{aligned} \overline{Y}_0 \equiv \frac{\overline{LW}}{1-\overline{\alpha}} &= M^A\rho\left[\frac{1}{R_0^f}\left((1-\theta)\frac{\overline{WL}}{\rho}+\frac{\overline{W}D}{\rho}\right)\right] \\ \text{where } M^A &= \frac{1}{1-(1-\overline{\alpha})(\theta+\rho(1-\theta))-\rho\overline{\alpha}} \\ &= \frac{1}{(1-\rho)(1-(1-\overline{\alpha})\theta)} \end{aligned} \tag{B.71}$$

Here, we have also defined the frictionless output $\overline{Y}_0$. The last line simplifies the multiplier. The expression says that the value of the stockholders' future claims (the bracketed term) should be at a particular level such that its direct spending effect, combined with the multiplier effects, are just enough to ensure output is equal to its frictionless level.

Using Eq. $(B.71)$, we characterize the equilibrium interest rate ("rstar"):

$$\begin{aligned} R_0^f &= (1-\overline{\alpha})M^A\frac{(1-\theta)\overline{L}+D}{\overline{L}} \\ &= \frac{1}{1-\rho}\frac{1-\overline{\alpha}}{1-(1-\overline{\alpha})\theta}\frac{(1-\theta)\overline{L}+D}{\overline{L}}. \end{aligned} \tag{B.72}$$

As expected, greater impatience (ρ) or greater future capital productivity (D) increases the equilibrium interest rate.

Using $(B.70)$ and $(B.72)$, we can also solve for the equilibrium price of capital as:

$$Q_0/\overline{W} = \frac{\overline{L}}{1-\overline{\alpha}}\left(\overline{\alpha}+\frac{1-\rho}{\rho}(1-(1-\overline{\alpha})\theta)\frac{D}{(1-\theta)\overline{L}+D}\right). \tag{B.73}$$

It is easy to check that (as long as $\theta < 1$) an increase in the future productivity of capital, D, also increases the equilibrium price of capital. The interest rate reacts to this change to ensure output is at its supply determined level. This mitigates the rise in the stock price somewhat but does not completely undo it, since some of the interest rate response is absorbed by stockholders' human capital wealth. (The last point is the difference from Caballero and Simsek (2020): here, "time-varying risk premium" translates into actual price movements because we have two different types of wealth and the "risk premium" varies only for one type of wealth.)

Next consider the determination of tradable and nontradable labor. Using $(B.60)$ and $(B.61)$, along with symmetry across areas, we obtain:

$$\frac{L_0^N}{L_0^T} = \frac{\left(1-\alpha^N\right)\eta}{\left(1-\alpha^T\right)\left(1-\eta\right)}.$$

Combining this with $L_0^N + L_0^T = \overline{L}$, we further solve:

$$\begin{aligned} L_0^N &= \frac{1-\alpha^N}{1-\overline{\alpha}}\eta\overline{L}, \\ L_0^T &= \frac{1-\alpha^T}{1-\overline{\alpha}}\left(1-\eta\right)\overline{L}. \end{aligned} \tag{B.74}$$

Hence, the labor employed in the nontradable and tradable sectors is determined by the share of the corresponding good in household spending, with an adjustment for the differences in the share of labor across the two sectors. The following result summarizes this discussion.

Proposition 2 *Suppose conditions* $(B.29)$ *and* $(B.30)$ *hold. Consider the equilibrium in period* 0 *when areas have common stock wealth,* $x_{a,0} = 0$ *for each* a*. All areas have identical allocations and prices. Nominal wages are given by* $W_0 = \overline{W}$*. Monetary policy ensures hand-to-mouth labor is at its frictionless level. This is equal to stockholders' labor,* $L_0^h = \overline{L}$*, which also implies* $L_0 = L_0^T + L_0^N = \overline{L}$ *[cf.* $(B.68 - B.69)$*]. The nominal interest rate is given by Eq.* $(B.72)$ *and the price of capital is given by Eq.* $(B.73)$*. The shares of labor employed in the nontradable and tradable sectors is given by Eq.* $(B.74)$*. An increase in the future productivity of capital* D *increases the interest rate and the price of capital but does not affect the labor market outcomes in period* 0*.*

B.4 Log-linearized Equilibrium with Heterogeneous Stock Wealth

We next consider the case with a more general distribution of stock wealth, $\{x_{a,0}\}_a$, that satisfies $\int_a x_{a,0} da = 0$. In this case, we log-linearize the equilibrium conditions around the common-wealth benchmark (for a fixed level of D), and we characterize the log-linearized equilibrium. To this end, we define the log-deviations of the local equilibrium variables around the common-wealth benchmark: $y = \log\left(Y/Y^b\right)$, where $Y \in \left\{L_{a,0}, L_{a,0}^N, L_{a,0}^T, W_{a,0}, P_{a,0}, P_{a,0}^T\right\}_a$. We also define the log-deviations of the endogenous aggregate variables: $y = \log\left(Y/Y^b\right)$, where $Y \in \left\{P_0^T, R_0, Q_0, R_0^f\right\}$. The following lemma simplifies the analysis (proof omitted).

Lemma 1 *Consider the log-linearized equilibrium conditions around the common-wealth benchmark. The solution to these equations satisfies* $\int_a l_{a,0} da = \int_a w_{a,0} da = 0$ *and* $p_0^T = r_0 = q_0 = r_0^f = 0$*. In particular, the log-linearized equilibrium outcomes for the aggregate variables are the same as their counterparts in the common-wealth benchmark.*

We next log-linearize the equilibrium conditions and characterize the log-linearized equilibrium outcomes for each area a. We start by Eqs. $(B.31)$, $(B.33)$, and $(B.37)$ that characterize the price indices in terms of nominal wages in an area. Log-linearizing Eqs. $(B.33)$ and $(B.37)$ we obtain,

$$\begin{aligned} p_{a,0}^N &= \left(1-\alpha^N\right) w_{a,0} \\ p_{a,0}^T &= \left(1-\alpha^T\right) w_{a,0}. \end{aligned} \tag{B.75}$$

Log-linearizing Eq. $(B.31)$, we further obtain,

$$p_{a,0} = \eta p_{a,0}^N = \eta\left(1-\alpha^N\right) w_{a,0}. \tag{B.76}$$

Next, we log-linearize the labor supply equation $(B.63)$ to obtain,

$$w_{a,0} = \frac{\lambda_w}{1+\varphi^h \varepsilon_w}\left(p_{a,0} + \varphi^h \varepsilon_w w_{a,0} + \varphi^h \frac{l_{a,0}}{\theta}\right).$$

After rearranging terms and simplifying, we obtain Eq. (7) from the main text:

$$w_{a,0} = \lambda \left(p_{a,0} + \varphi l_{a,0}\right) \tag{B.77}$$
$$\text{where} \qquad \lambda = \frac{\lambda_w}{1 + \left(1 - \lambda_w\right) \varphi^h \varepsilon_w} \text{ and } \varphi = \frac{\varphi^h}{\theta}$$

Note that we derive the wage flexibility and labor inelasticity parameters, λ and φ, in terms of the more structural parameters, $\lambda_w, \varphi, \varepsilon_w, \varphi^h, \theta$. As expected, wage flexibility is greater when a greater fraction of members adjust wages (greater λ_w), labor supply is more inelastic (greater φ^h), labor types are less substitutable (smaller ε_w). To understand the parameter φ, note that stockholders always supply the frictionless labor and thus their labor elasticity is effectively zero, $1/\varphi^s = 0$. Therefore, the aggregate "weighted-average" labor elasticity reflects the hand-to-mouth households' elasticity and their population share, $1/\varphi = \left(1 - \theta\right)/\varphi^s + \theta/\varphi^h = \theta/\varphi^h$.

Combining Eqs. $(B.76)$ and $(B.77)$, we obtain the reduced form labor supply equation:

$$w_{a,0} = \kappa l_{a,0}, \text{ where } \kappa = \frac{\lambda\varphi}{1 - \lambda\eta\left(1 - \alpha^N\right)}. \tag{B.78}$$

As expected, the wage adjustment parameter, κ, depends on the wage flexibility parameter, λ, and the inverse elasticity of the labor supply, φ. It also depends on the share of the nontradable sector and the share of labor in the nontradable sector, $\eta, 1 - \alpha^N$. These parameters capture the extent to which a change in local wages translate into local inflation, which creates further wage pressure.

Next, we log-linearize the labor demand equation $(B.62)$ to obtain,

$$\begin{aligned} \left(w_{a,0} + l_{a,0}\right)\overline{WL} \quad = \quad & \left(1 - \alpha^N\right)\eta\left[\theta\overline{WL}\left(w_{a,0} + \frac{l_{a,0}}{\theta}\right) + \rho\left(\begin{array}{c}\left(1-\theta\right)\overline{WL}w_{a,0} \\ +x_{a,0}Q_0\end{array}\right)\right] \\ & -p^T_{a,0}\left(\varepsilon - 1\right)\overline{W}L^T_0. \end{aligned} \tag{B.79}$$

Here, the first line captures the local expenditure on nontradable labor, which comes from both hand-to-mouth households and stockholders. Hand-to-mouth households' spending depends on the local wage, $w_{a,0}$, as well as the local aggregate labor $l_{a,0}$ (multiplied by $1/\theta$ to capture the implied local hand-to-mouth labor). Stockholders' spending depends on the local wage, $w_{a,0}$, as well as the local stock wealth, $x_{a,0}$. The second line captures the local expenditure on tradable labor that depends on the local price of nontradables, $p^T_{a,0}$, as well as the elasticity of substitution, $\varepsilon - 1$. The term, $\overline{W}L^T_0 = \left(1-\alpha^T\right)(1-\eta)\frac{\overline{WL}}{1-\overline{\alpha}}$, captures the expenditure on tradable labor in the common-wealth benchmark [cf. $(B.74)$].

After rearranging terms, and using Eq. $(B.78)$, we solve for the labor bill:

$$\begin{aligned}(w_{a,0}+l_{a,0})\,\overline{WL} &= \mathcal{M}\left(\left(1-\alpha^N\right)\eta\rho x_{a,0}Q_0 - p^T_{a,0}(\varepsilon-1)\,\overline{W}L^T_0\right), \qquad \text{(B.80)}\\ \text{where } \mathcal{M} &= \frac{1}{1-\left(1-\alpha^N\right)\eta\left\{\frac{\theta\kappa+1}{\kappa+1}+\rho\frac{\kappa(1-\theta)}{\kappa+1}\right\}}.\end{aligned}$$

Here, we have used $w_{a,0} = \kappa l_{a,0}$ to write the wage and the labor in terms of the labor bill. We have also defined, $\mathcal{M}$, which captures the local Keynesian multiplier effects. The term in set brackets can be thought of as a weighted-average MPCs out of labor income between hand-to-mouth households (MPC given by 1) and stockholders (MPC given by ρ). The relative weights, $\frac{\theta\kappa+1}{\kappa+1}$ and $\frac{\kappa(1-\theta)}{\kappa+1}$, capture the extent to which additional labor income is split between hand-to-mouth households and stockholders. This depends not only on the population shares (θ) but also on the wage adjustment parameter (κ), because agents have different labor supply elasticities (a simplifying assumption).

Finally, using Eq. $(B.75)$ to substitute for the price of tradables in terms of local wages, $p^T_{a,0} = \left(1-\alpha^T\right)w_{a,0}$, and using Eq. $(B.78)$ once more, we obtain the following closed-form solution:

$$\begin{aligned}w_{a,0}+l_{a,0} &= \frac{1+\kappa}{1+\kappa\zeta}\mathcal{M}\left(1-\alpha^N\right)\eta\rho\frac{x_{a,0}Q_0}{\overline{WL}} \qquad \text{(B.81)}\\ l_{a,0} &= \frac{1}{1+\kappa}(w_{a,0}+l_{a,0}) \qquad \text{(B.82)}\\ w_{a,0} &= \frac{\kappa}{1+\kappa}(w_{a,0}+l_{a,0}), \qquad \text{(B.83)}\\ \text{where } \zeta &= 1+(\varepsilon-1)\left(1-\alpha^T\right)\frac{L^T_0}{\overline{L}}\mathcal{M}\\ &= 1+(\varepsilon-1)\frac{\left(1-\alpha^T\right)^2}{1-\overline{\alpha}}(1-\eta)\,\mathcal{M}.\end{aligned}$$

Here, the last line defines the parameter, ζ, and the last line substitutes for L^T_0 from $(B.74)$. Eq. $(B.81)$ illustrates that the local spending on nontradables affects the local labor bill. Eqs. $(B.82)$ and $(B.83)$ illustrate that this also affects labor and wages according to the wage adjustment parameter, κ.

The term, $\frac{1+\kappa}{1+\kappa\zeta}$, in Eq. ($B$.81) captures the effect that works through exports. In particular, an increase in local spending increases local wages, which generates an adjustment of local exports. As expected, this adjustment is stronger when wages are more flexible (higher κ). The adjustment is also stronger when tradable inputs are more substitutable across regions (higher ε, which leads to higher ζ). In fact, when tradable inputs are gross substitutes ($\varepsilon > 1$, which leads to $\zeta > 1$), the export adjustment dampens the direct spending effect on the labor bill. When tradable inputs are gross complements ($\varepsilon < 1$, which leads to $\zeta < 1$), the export adjustment amplifies the direct spending effect.

Finally, consider the effect on local labor employed in nontradable and tradable sectors. First consider the tradable sector. Log-linearizing Eq. (B.61), we obtain

$$\begin{aligned} w_{a,0} + l^T_{a,0} &= -(\varepsilon - 1)\, p^T_{a,0} \\ &= -(\varepsilon - 1)\left(1 - \alpha^T\right) w_{a,0} \\ &= -(\varepsilon - 1)\left(1 - \alpha^T\right) \frac{\kappa}{1+\kappa\zeta} \mathcal{M}\left(1 - \alpha^N\right) \eta\rho \frac{x_{a,0} Q_0}{\overline{WL}}. \end{aligned} \tag{B.84}$$

Here, the third line uses Eqs. (B.83) and (B.81). These expressions illustrate that the export adjustment described above affects the tradable labor bill. While the effect of stock wealth on the tradable labor bill is ambiguous (as it depends on whether $\varepsilon > 1$ or $\varepsilon < 1$), we show that the effect on tradable labor is always (weakly) negative, $dl^T_{a,0}/dx_{a,0} \leq 0$. Intuitively, the increase in local wages always generate some substitution of labor away from the area. On the other hand, labor bill can increase or decrease depending on the strength of the income effect relative to this substitution effect.

Next consider the nontradable sector. Note that the total labor bill is the sum of nontradable and tradable labor bills:

$$\left(w_{a,0} + l_{a,0}\right) \overline{WL} = \left(w_{a,0} + l^N_{a,0}\right) \overline{W} L^N_0 + \left(w_{a,0} + l^T_{a,0}\right) \overline{W} L^T_0.$$

Substituting this into (B.80) we obtain

$$\begin{aligned} \left(w_{a,0} + l^N_{a,0}\right) \overline{W} L^N_0 &= \mathcal{M}\left[\left(1 - \alpha^N\right) \eta\rho x_{a,0} Q_0 - (\varepsilon - 1)\, p^T_{a,0} \overline{W} L^T_0\right] + (\varepsilon - 1)\, p^T_{a,0} \overline{W} L^T_0 \\ &= \mathcal{M}\left(1 - \alpha^N\right) \eta\rho x_{a,0} Q_0 - (\mathcal{M} - 1)(\varepsilon - 1)\, p^T_{a,0} \overline{W} L^T_0 \end{aligned}$$

After substituting $w_{a,0} + l^T_{a,0} = -(\varepsilon - 1)\, p^T_{a,0}$ from (B.84), normalizing by $\overline{WL}$, using Eq.

$(B.74)$, we further obtain:

$$w_{a,0} + l^N_{a,0} = \mathcal{M}\left(1-\overline{\alpha}\right)\rho \frac{x_{a,0}Q_0}{\overline{WL}} + \left(\mathcal{M}-1\right)\frac{1-\alpha^T}{1-\alpha^N}\frac{1-\eta}{\eta}\left(w_{a,0}+l^T_{a,0}\right). \tag{B.85}$$

This expression illustrates that greater stock wealth affects the nontradable labor bill due to a direct and an indirect effect. The direct effect is positive as it is driven by the impact of greater local wealth on local spending. There is also an indirect effect due to the impact of the stock wealth on the tradable labor bill—the multiplier effects of which accrue to the nontradable labor bill. The indirect effect has an ambiguous sign because stock wealth can decrease or increase the tradable labor bill depending on ε. Nonetheless, we show that the direct effect always dominates. Specifically, regardless of ε, we have $d\left(w_{a,0}+l^N_{a,0}\right)/dx_{a,0} > 0, dl^N_{a,0}/dx_{a,0} > 0$: that is, greater stock wealth increases the nontradable labor bill as well as nontradable labor. The following result summarizes this discussion.

Proposition 3 *Consider the model with Assumption D when areas have an arbitrary distribution of stock wealth, $\{x_{a,0}\}_a$, that satisfies $\int_a x_{a,0}da = 0$. In the log-linearized equilibrium, local labor and wages in a given area,$(l_{a,0}, w_{a,0})$, are characterized as the solution to Eqs. $(B.78)$ and $(B.80)$. The solution is given by Eqs. $(B.82)$ and $(B.83)$. Local labor bill in nontradables and tradable sectors are given by Eqs. $(B.84)$ and $(B.85)$. In particular, local labor and wages satisfy the following comparative statics with respect to stock wealth:*

$$dl_{a,0}/dx_{a,0} > 0, dw_{a,0}/dx_{a,0} \geq 0 \text{ and } d\left(l_{a,0}+w_{a,0}\right)/dx_{a,0} > 0.$$

Moreover, regardless of ε, the labor bill in the nontradable sector and the labor in each sector satisfy the following comparative statics:

$$d\left(l^N_{a,0}+w_{a,0}\right)/dx_{a,0} > 0, dl^N_{a,0}/dx_{a,0} > 0 \text{ and } dl^T_{a,0}/dx_{a,0} \leq 0.$$

Proof. Most of the proof is presented earlier. It remains to establish the comparative statics for the tradable labor, the nontradable labor and the nontradable labor bill.

First consider the tradable labor. Note that the first line of the expression in $(B.84)$ implies

$$l^T_{a,0} = -\left(1+\left(\varepsilon-1\right)\left(1-\alpha^T\right)\right)w_{a,0}. \tag{B.86}$$

Since $(\varepsilon - 1)\left(1 - \alpha^T\right) > -1$ (because $\varepsilon > 0$) and $dw_{a,0}/dx_{a,0} \geq 0$ (cf. Eq. $(B.83)$), this implies the comparative statics for the tradable labor, $dl^T_{a,0}/dx_{a,0} \leq 0$.

Next consider the nontradable labor. Note that $L_{a,0} = L^T_{a,0} + L^N_{a,0}$. Log-linearizing this expression, we obtain,

$$l^N_{a,0} L^N_{a,0} = l_{a,0}\overline{L} - l^T_{a,0} L^T_{a,0}.$$

Differentiating this expression with respect to $x_{a,0}$ and using $dl_{a,0}/dx_{a,0} > 0$ and $dl^T_{a,0}/dx_{a,0} \leq 0$, we obtain the comparative statics for the nontradable labor, $dl^N_{a,0}/dx_{a,0} > 0$. Combining this with $dw_{a,0}/dx_{a,0} \geq 0$, we further obtain the comparative statics for the nontradable labor bill, $d\left(l^N_{a,0} + w_{a,0}\right)/dx_{a,0} > 0$.■

B.5 Comparative Statics of Local Labor Market Outcomes

We next combine our results to investigate the impact of a change in aggregate stock wealth (over time) on local labor market outcomes. Specifically, consider the comparative statics of an increase in the future capital productivity from some D^{old} to $D^{new} > D^{old}$.

First consider the effect on the common-wealth benchmark. By Proposition 2, the equilibrium price of capital increases from Q^{old}_0 to $Q^{new}_0 > Q^{old}_0$. The labor market outcomes remain unchanged: in particular, $L_0 = \overline{L}, W_0 = \overline{W}, L^N_0/L_0 = \frac{1-\alpha^N}{1-\overline{\alpha}}\eta$ and $L^T_0/L_0 = \frac{1-\alpha^T}{1-\overline{\alpha}}(1-\eta)$.

Next consider the effect when areas have heterogeneous wealth. We use the notation $\Delta X = X^{new} - X^{old}$ for the comparative statics on variable X. Consider the effect on labor market outcomes, for instance, the (log of the) local labor bill $\log(W_{a,0}L_{a,0})$. Note that we have:

$$\log(W_{a,0}L_{a,0}) \simeq \log\left(\overline{WL}\right) + w_{a,0} + l_{a,0}.$$

Here, $w_{a,0}, l_{a,0}$ are characterized by Proposition 3 as linear functions of capital ownership, $x_{a,0}$; and the approximation holds up to first-order terms in capital ownership, $\{x_{a,0}\}_a$. Note also that the change of D does not affect $\log\left(\overline{WL}\right)$. Therefore, the comparative statics in this case can be written as,

$$\begin{aligned} \Delta \log(W_{a,0}L_{a,0}) &\simeq \Delta\left(w_{a,0} + l_{a,0}\right) \\ &= \left(w^{new}_{a,0} + l^{new}_{a,0}\right) - \left(w^{old}_{a,0} + l^{old}_{a,0}\right). \end{aligned}$$

Here, the approximation holds up to first-order terms in $\{x_{a,0}\}_a$. Put differently, up to a first order, the change of D affects the (log of the) local labor bill through its effect on the log-linearized equilibrium variables.

Recall that the log-linearized equilibrium is characterized by Proposition 3. In particular, considering Eq. $(B.81)$ for D^{old} and D^{new}, we obtain:

$$\begin{aligned} w_{a,0}^{old} + l_{a,0}^{old} &= \frac{1+\kappa}{1+\kappa\zeta}\mathcal{M}\left(1-\alpha^N\right)\eta\rho\frac{x_{a,0}Q_0^{old}}{\overline{W}L_0}, \\ w_{a,0}^{new} + l_{a,0}^{new} &= \frac{1+\kappa}{1+\kappa\zeta}\mathcal{M}\left(1-\alpha^N\right)\eta\rho\frac{x_{a,0}Q_0^{new}}{\overline{W}L_0}. \end{aligned}$$

These equations illustrate that the change of D affects the log-linearized equilibrium only through its effect on the price of capital, Q_0. Taking their difference, we obtain Eq. (10) in the main text that describes $\Delta\left(w_{a,0}+l_{a,0}\right)$.

Applying the same argument to Eqs. $(B.82), (B.85), (B.84)$, we also obtain Eqs. $(11), (12), (13)$ in the main text that describe, respectively, $\Delta l_{a,0}, \Delta\left(w_{a,0}+l_{a,0}^N\right), \Delta\left(w_{a,0}+l_{a,0}^T\right)$. These equations illustrate that an increase in local stock wealth due to a change in aggregate stock wealth has the same impact on local labor market outcomes as an increase of stock wealth in the cross section that we characterized earlier.

Comparative Statics of Local Consumption. We next derive the comparative statics of local consumption that we use in Section 5 (see Eq. (18)). For simplicity, we focus on the case $\varepsilon = 1$. Using $(B.62)$, we have

$$P_{a,0}C_{a,0} = \frac{W_{a,0}L_{a,0}^N}{\left(1-\alpha^N\right)\eta}.$$

Log-linearizing this expression around the common-wealth benchmark, we obtain

$$\begin{aligned} \left(p_{a,0}+c_{a,0}\right)P_0C_0 &= \left(w_{a,0}+l_{a,0}^N\right)\frac{\overline{W}L_0^N}{\left(1-\alpha^N\right)\eta} \\ &= \mathcal{M}\rho x_{a,0}Q_0 \end{aligned}$$

Here, the second line uses Eqs. $(B.85)$ and $(B.74)$, and observes that $w_{a,0}+l_{a,0}^T = 0$ when $\varepsilon = 1$. After rearranging terms, and considering the change from D^{old} to $D^{new} > D^{old}$, we obtain

$$\Delta\left(p_{a,0}+c_{a,0}\right)=\mathcal{M}\rho\frac{x_{a,0}\Delta Q_0}{P_0C_0}. \tag{B.87}$$

After an appropriate change of variables, this equation gives Eq. (18) in the main text.

B.6 Details of the Calibration Exercise

This appendix provides the details of the calibration exercise in Section 5. We start by summarizing the solution for the local labor market outcomes that we derived earlier. In particular, we write Eqs. $(B.81-B.85)$ as follows:

$$\begin{aligned}
\frac{\Delta\left(w_{a,0}+l_{a,0}\right)}{SR} &= \frac{1+\kappa}{1+\kappa\zeta}\mathcal{M}\left(1-\alpha^N\right)\eta\rho, \\
\frac{\Delta l_{a,0}}{SR} &= \frac{1}{1+\kappa}\frac{\Delta\left(w_{a,0}+l_{a,0}\right)}{SR} \qquad \text{(B.88)}\\
\frac{\Delta w_{a,0}}{SR} &= \frac{\kappa}{1+\kappa}\frac{\Delta\left(w_{a,0}+l_{a,0}\right)}{SR} \\
\frac{\Delta\left(w_{a,0}+l^T_{a,0}\right)}{SR} &= -\left(\varepsilon-1\right)\left(1-\alpha^T\right)\frac{\Delta w_{a,0}}{SR} \\
\frac{\Delta\left(w_{a,0}+l^N_{a,0}\right)}{SR} &= \mathcal{M}\rho\left(1-\overline{\alpha}\right)-\left(\mathcal{M}-1\right)\frac{\left(1-\alpha^T\right)^2}{1-\alpha^N}\frac{1-\eta}{\eta}\left(\varepsilon-1\right)\frac{\Delta w_{a,0}}{SR} \qquad \text{(B.89)}\\
\text{where } S &= \frac{x_{a,0}Q_{a,0}}{\overline{WL}_0}, R=\frac{\Delta Q_0}{Q_0} \\
\text{and } \mathcal{M} &= \frac{1}{1-\left(1-\alpha^N\right)\eta\left\{\frac{\theta\kappa+1}{\kappa+1}+\rho\frac{(1-\theta)\kappa}{\kappa+1}\right\}} \\
\text{and } \zeta &= 1+\left(\varepsilon-1\right)\frac{\left(1-\alpha^T\right)^2}{1-\overline{\alpha}}\left(1-\eta\right)\mathcal{M}.
\end{aligned}$$

Our calibration relies on two model equations that determine the key parameters κ and ρ. Specifically, we calibrate κ by using Eq. $(B.88)$, which replicates Eq. (19) from the main text. We calibrate ρ by using Eq. $(B.89)$ which generalizes Eq. (15) from the main text. For reasons we describe in the main text, we do not use the response of the tradable sector for calibration purposes (see Footnote 36).

Note that combining Eq. $(B.88)$ with the empirical coefficients for employment and the total labor bill from Table 1 (for quarter 7), we obtain:

$$0.77\% \leq \frac{1}{1+\kappa} 2.18\%$$

As we discuss in the main text, while the model makes predictions for total labor supply including changes in hours per worker, in the data we only observe employment. A long literature dating to Okun (1962) finds an elasticity of total hours to employment of 1.5. Applying this adjustment and using the coefficients for total employment and the total labor bill from Table 1 yields:

$$\begin{aligned} \frac{\Delta l_{a,0}}{S_{a,0} R_0} &= 1.5 \times 0.77\% \\ \frac{\Delta \left(w_{a,0} + l_{a,0}\right)}{S_{a,0} R_0} &= 2.18\%. \end{aligned}$$

Combining these with Eq. (19), we obtain:

$$\kappa = 0.9. \tag{B.90}$$

Thus, a one percent change in labor is associated with a 0.9% change in wages at a horizon of two years.

That leaves us with Eq. $(B.89)$ to determine the stock wealth effect parameter, ρ. In the main text, we focus on a baseline calibration that assumes unit elasticity for tradables, $\varepsilon = 1$, which leads to a particularly straightforward analysis. In this appendix, we first provide the details of the baseline calibration. We then show that this calibration is robust to considering a wider range for the tradable elasticity parameter, $\varepsilon \in [0.5, 1.5]$.

Throughout, we set the labor share parameters in the two sectors so that the weighted-average share of labor is equal to the standard empirical estimates [cf. (6)]:

$$1 - \overline{\alpha} = \frac{2}{3}.$$

To keep the calibration simple, we set the same labor share for the two sectors:

$$1 - \alpha^L = 1 - \alpha^N = \frac{2}{3}.$$

Eq. $(B.89)$ (when $\varepsilon = 1$) shows that our analysis is robust to allowing for heterogeneous labor share across the two sectors.

B.6.1 Details of the Baseline Calibration

Setting $\varepsilon = 1$, Eq. $(B.89)$ reduces to Eq. (15) in the main text,

$$\frac{\Delta\left(w_{a,0} + l_{a,0}^{N}\right)}{SR} = \mathcal{M}\left(1 - \overline{\alpha}\right)\rho.$$

Combining this expression with the empirical coefficient for the nontradable labor bill from Table 1 (for quarter 7), we obtain:

$$\mathcal{M}\left(1 - \overline{\alpha}\right)\rho = 3.23\% \text{ with } 1 - \overline{\alpha} = \frac{2}{3}. \tag{B.91}$$

We also require the local income multiplier to be consistent with empirical estimates from the literature, that is:

$$\mathcal{M} = \frac{1}{1 - \left(1 - \alpha^{N}\right)\eta\left\{\frac{\kappa\theta+1}{\kappa+1} + \rho\frac{\kappa(1-\theta)}{\kappa+1}\right\}} = 1.5 \tag{B.92}$$

After substituting $1 - \alpha^{N} = 2/3$, and rearranging terms, we obtain:

$$\eta\left\{\frac{\kappa\theta + 1}{1 + \kappa} + \rho\frac{(1-\theta)\kappa}{1 + \kappa}\right\} = 0.5. \tag{B.93}$$

Note also that we already have $\kappa = 0.9$. Hence, for a given ρ, the calibration of the multiplier provides a restriction in terms of the share of nontradables, η, and the fraction of hand-to-mouth households, θ. For instance, when $\eta = 0.5$, we require $\theta = 1$. In this case, we need the weighted-average MPC (the term inside the set brackets) to be one, which happens only if the hand-to-mouth population share is equal to one. More generally, increasing η decreases the implied θ.

Given Eq. $(B.92)$, Eq. $(B.91)$ determines the stock wealth effect parameter independently of the other parameters:

$$\rho = 3.23\%.$$

The parameter, η, is difficult to calibrate precisely because there is no good measure of the trade bill at the county level. We allow for a wide range of possibilities:

$$\eta \in \left[\underline{\eta}, \overline{\eta}\right], \text{ where } \underline{\eta} = 0.5 \text{ and } \underline{\eta} = 0.8. \tag{B.94}$$

For each η, we obtain the implied θ from Eq. $(B.93)$, which falls into the range:

$$\theta(\eta) \in \left[\underline{\theta}, \overline{\theta}\right], \text{ where } \underline{\theta} = \theta(\overline{\eta}) = 0.18 \text{ and } \overline{\theta} = \theta\left(\underline{\eta}\right) = 1. \tag{B.95}$$

B.6.2 Robustness of the Baseline Calibration

Next consider the case with a more general elasticity of substitution between tradable inputs, ε. In this case, Eq. $(B.89)$ is more complicated and given by:

$$\frac{\Delta\left(w_{a,0} + l_{a,0}^N\right)}{SR} = \mathcal{M}\rho\left(1 - \overline{\alpha}\right) - \left(\mathcal{M} - 1\right)\left(\varepsilon - 1\right)\frac{\left(1 - \alpha^T\right)^2}{1 - \alpha^N}\frac{1 - \eta}{\eta}\frac{\Delta w_{a,0}}{SR}.$$

In particular, the nontradable labor bill also depends on the effect on local wages. The intuition is that the change in local wages affects the tradable labor bill, which generates spillover effects on the local spending and the local nontradable labor bill. Consistent with this intuition, the magnitude of this effect depends on the elasticity ε and the multiplier $\mathcal{M}$ as well as the parameters, α^T, α^N, η.

Recall also that we have Eq. $(B.88)$ that describes the change in wages as a function of the change in the total labor bill:

$$\frac{\Delta w_{a,0}}{SR} = \frac{\kappa}{1 + \kappa}\frac{\Delta\left(w_{a,0} + l_{a,0}\right)}{SR}.$$

Substituting this expression into Eq. $(B.89)$, and using the empirical coefficients for the nontradable and the total labor bill from Table 1 (for quarter 7), we obtain the following generalization of $(B.91)$:

$$\mathcal{M}\rho\left(1 - \overline{\alpha}\right) = 3.23\% + \left(\mathcal{M} - 1\right)\left(\varepsilon - 1\right)\frac{\left(1 - \alpha^T\right)^2}{1 - \alpha^N}\frac{1 - \eta}{\eta}\frac{\kappa}{1 + \kappa}2.18\%. \tag{B.96}$$

Thus, the stock wealth effect parameter in this case is not determined independently of the remaining parameters. We have already calibrated $\kappa = 0.9$ and $\mathcal{M}$ =1.5 [cf. Eq.

(B.90) and (B.92)] as well as $1 - \overline{\alpha} = 1 - \alpha^T = 1 - \alpha^N = 2/3$. After substituting these, we obtain:

$$\rho = 3.23\% + \frac{1}{3}(\varepsilon - 1)\frac{1-\eta}{\eta}\frac{0.9}{1.9}2.18\%.$$

For any fixed ε, Eq. (B.96) describes ρ as a function of η, where η is required to lie in the range (B.94). Substituting this (as well as κ) into (B.93), we also obtain θ as a function of η.

Figure B.1 illustrates the possible values of ρ for $\varepsilon = 0.5$ (the left panel) and $\varepsilon = 1.5$ (the right panel). As the figure illustrates the implied values for ρ remain close to their corresponding levels from the baseline calibration with $\varepsilon = 1$. As expected, the largest deviations from the benchmark obtain when the share of nontradables is small—as trade has the largest impact on households' incomes in this case. However, ρ lies within 5% of its corresponding level from the baseline calibration even if we set $\eta = 0.5$.

The intuition for robustness can be understood as follows. As we described earlier, the additional effects emerge from the adjustment of the tradable labor bill due to a change in local wages. As long as wages do not change by much, the effect has a negligible effect on our baseline calibration. As it turns out, the value of κ that we find is such that the deviations from the benchmark are relatively small. Put differently, our analysis suggests that wages in an area do not change by much in response to stock wealth changes. Consequently, the tradable labor bill of the area also does not change by much either even if ε is somewhat different than 1.

B.7 Aggregation When Monetary Policy is Passive

So far, we assumed the monetary policy changes the interest rate to neutralize the impact of stock wealth changes on aggregate labor. In this appendix, we characterize the equilibrium under the alternative assumption that monetary policy leaves the interest rate unchanged in response to stock price fluctuations. In Section 6 of the main text, we use this characterization together with our calibration to describe how stock price fluctuations would affect aggregate labor market outcomes if they were not countered by monetary policy.

Specifically, consider some $\overline{D}$ and let $\overline{R}_0^f$ denote the "frictionless" interest rate that we

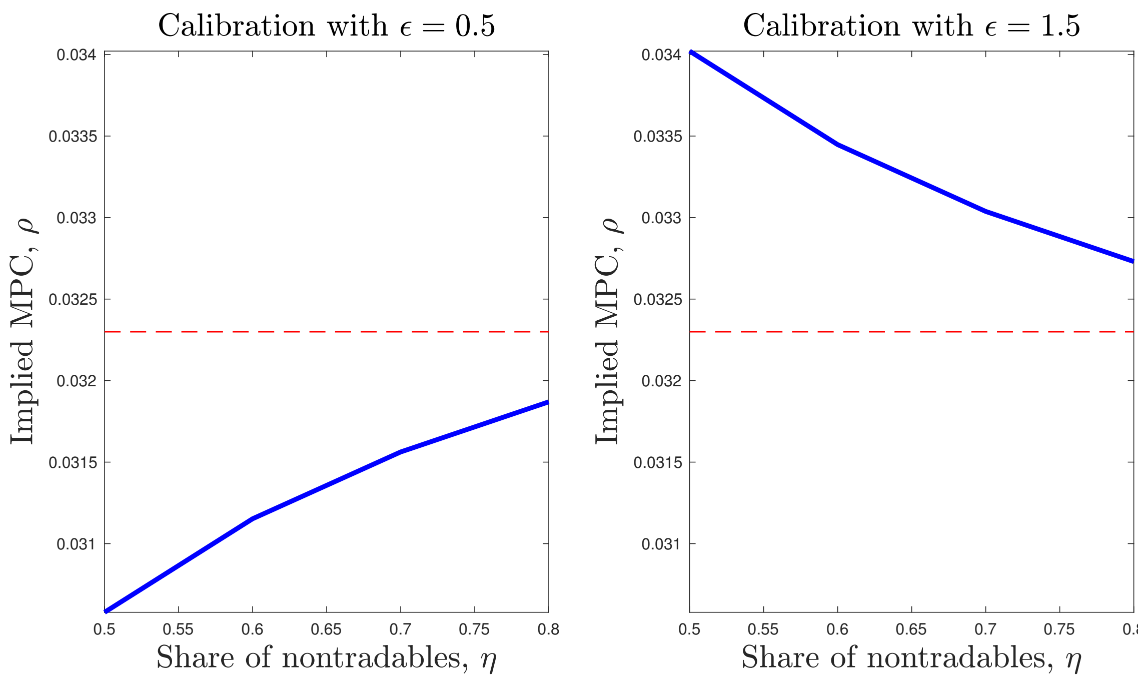

Figure B.1: Robustness to the elasticity of substitution between tradable inputs

Note: The left panel (resp. the right panel) illustrates the implied ρ as a function of η given $\varepsilon = 0.5$ (resp. $\varepsilon = 1.5$), as we vary η over the range in $(B.94)$. The red dashed lines illustrate the implied ρ for the baseline calibration with $\varepsilon = 1$.

characterized earlier corresponding to this level of productivity [$(B.72)$]:

$$\overline{R}_0^f = \frac{1}{1-\rho}\frac{1-\overline{\alpha}}{1-(1-\overline{\alpha})\,\theta}\frac{(1-\theta)\,\overline{L}+\overline{D}}{\overline{L}}. \tag{B.97}$$

Suppose the expected productivity D changes and is not necessarily equal to $\overline{D}$. In period 0, monetary policy leaves the interest rate unchanged at $\overline{R}_0^f$. Starting period $t \geq 1$ onward, monetary policy follows the same rule as before $(B.20)$. The model is otherwise the same as in Section B.1. Our goal is to understand how the change in expected D affects the aggregate equilibrium allocations in period 0 when the interest rate does not respond. For simplicity, we focus on the common-wealth benchmark, $x_{a,0} = 0$ (more generally, the results apply for the aggregate outcomes up to log linearization).

Most of our earlier analysis applies also in this case. In particular, Proposition 1 still applies and characterizes the equilibrium starting periods $t \geq 1$.

The differences concern the aggregate allocations in period 0. The analysis proceeds similar to Section B.3. Wages are the same across regions, W_a, but not necessarily equal to $\overline{W}$. Therefore, Eq. $(B.64)$ does not necessarily apply. Instead, we aggregate the labor supply Eq. $(B.63)$ to obtain

$$W_0^{1-\varepsilon_w} = \lambda_w \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \chi W_0^{\varepsilon_w \varphi^h} P_0 \left(\frac{L_0 - (1-\theta)\overline{L}_0}{\theta} \right)^{\varphi^h} \right)^{(1-\varepsilon_w)/\left(1+\varphi^h \varepsilon_w\right)} + (1-\lambda_w)\overline{W}^{1-\varepsilon_w}. \tag{B.98}$$

We also have the following analogues of Eqs. $(B.65)$ and $(B.66)$:

$$\begin{aligned} R_0 &= \frac{\overline{\alpha}}{1-\overline{\alpha}} W_0 L_0 \\ P_0 &= R_0^{\overline{\alpha}} W_0^{1-\overline{\alpha}} = \left(\frac{\overline{\alpha}}{1-\overline{\alpha}} \right)^{\overline{\alpha}} L_0^{\overline{\alpha}} W_0. \end{aligned} \tag{B.99}$$

This implies the price of capital is now given by:

$$Q_0 = \frac{\overline{\alpha}}{1-\overline{\alpha}} W_0 L_0 + \frac{1}{R_0^f} \frac{\overline{W} D}{\rho}. \tag{B.100}$$

Finally, we also aggregate Eq. $(B.62)$ to obtain the labor demand equation:

$$W_0 L_0 = (1-\overline{\alpha}) \left(\begin{array}{c} W_0 \left(L_0 - (1-\theta) \overline{L} \right) + \\ \rho \left((1-\theta) \left(W_0 \overline{L} + \frac{1}{R_0^f} \frac{\overline{WL}}{\rho} \right) + Q_0 \right) \end{array} \right). \tag{B.101}$$

The equilibrium is characterized by Eqs. $(B.98 - B.101)$ in four variables, (W_0, L_0, P_0, Q_0). When $D = \overline{D}$, these equations are satisfied with $L_0 = \overline{L}$ and $W_0 = \overline{W}$ and corresponding $\overline{Q}_0, \overline{P}_0$ [cf. $(B.97)$]. To characterize the equilibrium further, we next log-linearize the equations around the allocations corresponding to $D = \overline{D}$.

Log-linearized Aggregate Equilibrium. We start with the supply side. Log-linearizing Eq. $(B.99)$, we obtain:

$$p_0 = \overline{\alpha} l_0 + w_0. \tag{B.102}$$

Log-linearizing the labor supply equation ($B.98$), we obtain the aggregate analogue of (7) from the main text:

$$w_0 = \lambda \left(p_0 + \varphi l_0 \right) \tag{B.103}$$
$$\text{where} \qquad \lambda = \frac{\lambda_w}{1 + (1 - \lambda_w) \varphi^h \varepsilon_w} \text{ and } \varphi = \frac{\varphi^h}{\theta}.$$

Combining the last two equations, we further obtain:

$$w_0 = \kappa^A l_0, \text{ where } \kappa^A \equiv \frac{\lambda \left(\varphi + \overline{\alpha} \right)}{1 - \lambda} > \kappa = \frac{\lambda \varphi}{1 - \lambda \eta \left(1 - \alpha^N \right)}. \tag{B.104}$$

Here, κ^A denotes the *aggregate* wage adjustment parameter, and κ denotes the local wage adjustment as before [cf. ($B.78$)]. We discuss the comparison between κ^A and κ subsequently.

We next turn to the demand side. Log-linearizing Eq. ($B.100$), we obtain,

$$q_0 \overline{Q}_0 = \left(w_0 + l_0 \right) \frac{\overline{\alpha}}{1 - \overline{\alpha}} \overline{WL} + d \frac{1}{\overline{R}_0^f} \frac{\overline{WD}}{\rho}. \tag{B.105}$$

Log-linearizing the labor demand Eq. ($B.101$), we obtain,

$$\begin{aligned} \left(w_0 + l_0 \right) \overline{WL} &= \left(1 - \overline{\alpha} \right) \left(w_0 + \frac{l_0}{\theta} \right) \theta \overline{WL} + \rho \left(w_0 \left(1 - \theta \right) \overline{WL} + q_0 \overline{Q}_0 \right) \\ &= \left(w_0 + \frac{l_0}{\theta} \right) \theta \overline{WL} + \rho \left(\begin{array}{c} w_0 \left(1 - \theta \right) \overline{WL} \\ + \left(w_0 + l_0 \right) \frac{\overline{\alpha}}{1 - \overline{\alpha}} \overline{WL} + d \frac{1}{\overline{R}_0^f} \frac{\overline{WD}}{\rho} \end{array} \right). \end{aligned}$$

Here, the second line substitutes Eq. ($B.105$).

After rearranging terms to account for the multiplier effects, and using Eq. ($B.103$) to simplify the expression, we obtain the effect on the aggregate labor bill:

$$\left(w_0 + l_0 \right) \overline{WL} = \left(1 - \overline{\alpha} \right) \mathcal{M}^A \rho Q^A \tag{B.106}$$
$$\text{where } Q^A = d \frac{1}{\overline{R}_0^f} \frac{\overline{WD}}{\rho} \tag{B.107}$$
$$\text{and } \mathcal{M}^A = \frac{1}{1 - \left(1 - \overline{\alpha} \right) \left\{ \frac{\theta \kappa^A + 1}{\kappa^A + 1} + \rho \frac{(1 - \theta) \kappa^A}{\kappa^A + 1} \right\} - \overline{\alpha} \rho}$$

Here, Q^A denotes the exogenous part of the stock wealth—the valuation of future payoffs excluding current payoffs (that respond endogenously). This is multiplied by ρ to obtain total spending. This spending is then amplified by the *aggregate* multiplier, $\mathcal{M}^A$, which is different than the local multiplier, $\mathcal{M}$. We discuss the comparison of $\mathcal{M}^A$ and $\mathcal{M}$ subsequently. The amplified spending is then multiplied by the effective labor share, $1-\overline{\alpha}$, to obtain the aggregate labor bill.

Combining Eq. $(B.107)$ with Eq. $(B.104)$, we also obtain the separate effects on aggregate labor and wages:

$$l_0\overline{WL} = \frac{1}{\kappa^A+1}\left(1-\overline{\alpha}\right)\rho Q^A \quad \text{(B.108)}$$

$$w_0\overline{WL} = \frac{\kappa^A}{\kappa^A+1}\mathcal{M}^A\left(1-\overline{\alpha}\right)\rho Q^A \quad \text{(B.109)}$$

Substituting Eq. $(B.107)$ into Eq. $(B.105)$, we obtain the actual stock price (that incorporates the endogenous change in R_0):

$$q_0\overline{Q}_0 = \left(\overline{\alpha}\mathcal{M}^A\rho+1\right)Q^A. \quad \text{(B.110)}$$

Recall also that Eq. $(B.102)$ provides the solution for aggregate price index $p_0 = \alpha l_0 + w_0$.

Finally, considering Eqs. $(B.107)$ and $(B.108)$ for two different levels of future dividends, d^{old} and d^{new}, and taking the difference, we obtain Eqs. (21) and (22) in the main text.

Comparison with the Log-linearized Local Equilibrium. It is instructive to compare the log-linearized aggregate equilibrium with its counterpart we characterized earlier.

First consider the labor supply equations $(B.103)$ and $(B.104)$. Note that Eq. $(B.103)$ is the same as its local counterpart, Eq. $(B.77)$. Hence, controlling for prices as well as labor, the aggregate labor supply curve is the same as the local one. However, Eq. $(B.104)$ is different than its local counterpart, Eq. $(B.78)$. This is because the impact of aggregate nominal wages on the aggregate price index is greater than the impact of local wages on the local price index: specifically, we have $p_0 = \overline{\alpha}l_0 + w_0$ as opposed to $p_{0,a} = w_{0,a}\eta\left(1-\alpha^N\right)$ [cf. Eqs. $(B.102)$ and $(B.76)$]. The real wage $w-p$ increases locally whereas it decreases in the aggregate. Therefore, there is a positive neoclassical labor supply response locally whereas a negative one in the aggregate, with strength of both determined by the magnitude of the Frisch elasticity $1/\phi$.

To characterize these differences further, we rewrite the expressions for κ and κ^A to eliminate the wage stickiness parameter, λ, which gives:

$$\frac{1}{\kappa^A} = \frac{1}{1+\overline{\alpha}/\varphi}\left\{\frac{1}{\kappa} - \frac{1}{\varphi}\left(1-\eta\left(1-\alpha^N\right)\right)\right\}. \tag{B.111}$$

This expression calculates the aggregate labor response $1/\kappa^A$ in two steps. The term in set brackets starts with the local response but "cleanses" it from the local neoclassical effect to isolate the effect due to wage stickiness that extends to the aggregate. The term outside the set brackets adjusts the aggregate wage stickiness effect further for the aggregate neoclassical effect.

Next consider the aggregate labor bill equation $(B.107)$. Recall that its local counterpart is given by [cf. Eqs. $(B.82)$ and $(B.83)$]:

$$\frac{\left(l_{a,0}+w_{a,0}\right)\overline{WL}}{x_{a,0}Q_0} = \mathcal{M}\frac{1+\kappa}{1+\kappa\zeta}\left(1-\alpha^N\right)\eta\rho. \tag{B.112}$$

Hence, the aggregate effect differs from the local effect for three reasons. First, the direct spending effect is greater in the aggregate than at the local level, $\left(1-\overline{\alpha}\right)\rho > \eta\left(1-\alpha^N\right)\rho$. Here, the inequality follows since $1-\overline{\alpha} = \eta\left(1-\alpha^N\right)+\left(1-\eta\right)\left(1-\alpha^T\right)$. Intuitively, spending on tradables increases the labor bill in the aggregate but not locally. Second, the aggregate labor bill does not feature the export adjustment term, $\frac{1+\kappa}{1+\kappa\zeta}$, because this adjustment is across areas.

Third, the aggregate multiplier is different and typically greater than the local multiplier. To see this, note we can the local and the aggregate multipliers as:

$$\begin{aligned}
\mathcal{M}^A &= \frac{1}{1-m^A}, m^A = \left(1-\overline{\alpha}\right)\left\{\frac{\theta\kappa^A+1}{\kappa^A+1}+\rho\frac{\left(1-\theta\right)\kappa^A}{\kappa^A+1}\right\}+\overline{\alpha}\rho \\
\mathcal{M} &= \frac{1}{1-m}, m = \eta\left(1-\alpha^N\right)\left\{\frac{\theta\kappa+1}{\kappa+1}+\rho\frac{\left(1-\theta\right)\kappa}{\kappa+1}\right\}.
\end{aligned} \tag{B.113}$$

Here, m^A (resp. m) denote the additional spending induced by a dollar of income at the aggregate (resp. local) level. At the aggregate level, a dollar of income is split between labor and capital (according to their shares) and both components induce additional aggregate spending. At the local level, there are two differences. First, while the dollar is still split between labor and capital, the latter does not induce local spending—because capital is not held locally. Second, a fraction $1-\eta$ of the spending through labor income spills to other areas—because it is used to purchase tradables.

In view of these differences, if the additional (demand-induced) labor income were distributed symmetrically across households in the aggregate and in the local area, then the aggregate multiplier would always exceed the local multiplier. Formally, if the terms inside the set brackets were the same (which happens if $\kappa^A = \kappa$), then we would have $m^A > m$ since $1-\overline{\alpha} > \eta\left(1-\alpha^N\right)$ and $\overline{\alpha} > 0$. In our model, this comparison is slightly complicated by the fact that the aggregate and local wage flexibility terms are different, $\kappa^A \neq \kappa$, which changes the extent to which additional labor income accrues to wages compared to labor. This in turn affects the distribution of this income across stockholders and hand-to-mouth agents (that have heterogeneous MPCs), because these agents have heterogeneous labor supply elasticities (a simplifying assumption). As we will illustrate shortly, for our calibration these distributional effects are small and the slippage effects we described earlier dominate and imply that the aggregate multiplier is greater, $m^A > m$ and $\mathcal{M}^A > \mathcal{M}$.

Finally, going back to (B.112), note that as long as $\varepsilon \geq 1$ (and $\mathcal{M}^A > \mathcal{M}$), the aggregate effect is greater than the local effect. In this case, $\zeta \geq 1$ and thus the export adjustment also dampens the local effect relative to the aggregate effect. When $\varepsilon < 1$, the export adjustment tends to make the local effect greater than the aggregate effect. However, all other effects (the direct spending effect as well as the multiplier effect) tend to make the aggregate effect greater than the local effect.

Details and Robustness of the Aggregate Calibration. We next provide the details of the aggregate calibration exercise in Section 5. Most of the analysis is presented in the main text. Here, we show that our calibration of the aggregate wage adjustment coefficient, κ^A, is robust [cf. (B.111)]. We then verify that with our calibration the aggregate multiplier is greater than the local multiplier, $\mathcal{M}^A > \mathcal{M}$.

First consider the wage adjustment coefficient. Recall from Section B.6 that we take $1-\overline{\alpha} = 1-\alpha^N = \frac{2}{3}$. As we describe in Section 5, we also use $\varphi^{-1} = 0.5$ for the (effective) Frisch elasticity. Combining these observations with Eq. (B.111), and our estimate $\kappa = 0.9$, we obtain the aggregate wage adjustment coefficient as a function of the share of nontradables, $\kappa^A(\eta)$. Recall that we consider a wide range of parameters for the share of nontradables, $\eta \in [\underline{\eta}, \overline{\eta}]$, where $\underline{\eta} = 0.5$ and $\underline{\eta} = 0.8$ [cf. $(B.94)$]. Calculating the wage adjustment coefficient over this range, we obtain

$$\kappa^A(\eta) \in \left[\underline{\kappa}^A, \overline{\kappa}^A\right], \text{ where } \underline{\kappa}^A = \kappa^A(\overline{\eta}) = 1.32 \text{ and } \overline{\kappa}^A = \kappa^A(\underline{\eta}) = 1.5. \tag{B.114}$$

A higher κ^A implies a smaller labor response to a change in labor bill, $1/(1+\kappa^A)$ [cf. (21)]. Hence, the calibration we use in the main text, $\eta = \underline{\eta} = 0.5$ and $\kappa^A = \overline{\kappa}^A = 1.5$, implies the smallest aggregate labor response (a conservative calibration). Eq. $(B.114)$ illustrates further that this calibration is robust. With other choices for η, the implied κ^A (as well as the implied labor adjustment, $1/(1+\kappa^A)$) remains within 10% of our baseline calibration.

Next consider the aggregate multiplier. Recall from Section 5 that our baseline calibration implies $\rho = 3.23\%$. Recall also that, for each choice of η in $(B.94)$, we set the share of hand-to-mouth agents $\theta(\eta)$ that ensure the local multiplier is given by, $\mathcal{M}$ =1.5. Substituting these observations together with the implied $\kappa^A(\eta)$ from $(B.114)$ into $(B.113)$, we calculate the aggregate multiplier as a function of the share of nontradables, $\mathcal{M}^A(\eta)$.

Figure B.2 plots the possible values of the aggregate multiplier together with the local multiplier (which is 1.5 by assumption). As expected, the difference between the two multipliers is smallest when the share of nontradables is largest. Nonetheless, the implied aggregate multiplier exceeds the local multiplier for each level of η that we consider. This verifies that our calibration the aggregate multiplier is greater than the local multiplier, $\mathcal{M}^A > \mathcal{M}$.

B.8 Extending the Model to Incorporate Uncertainty

In this appendix, we generalize the baseline model to introduce uncertainty about capital productivity in period 1. We show that changes in households' risk aversion or perceived risk generate the same qualitative effects on the price of capital (as well as on "rstar") as in our baseline model. Moreover, conditional on a fixed amount of change in the price of capital, the model with uncertainty features the same *quantitative* effects on local labor market outcomes. Therefore, this exercise illustrates that our baseline analysis is robust to generating stock price fluctuations from alternative channels than the change in expected stock payoffs that we consider in our baseline analysis.

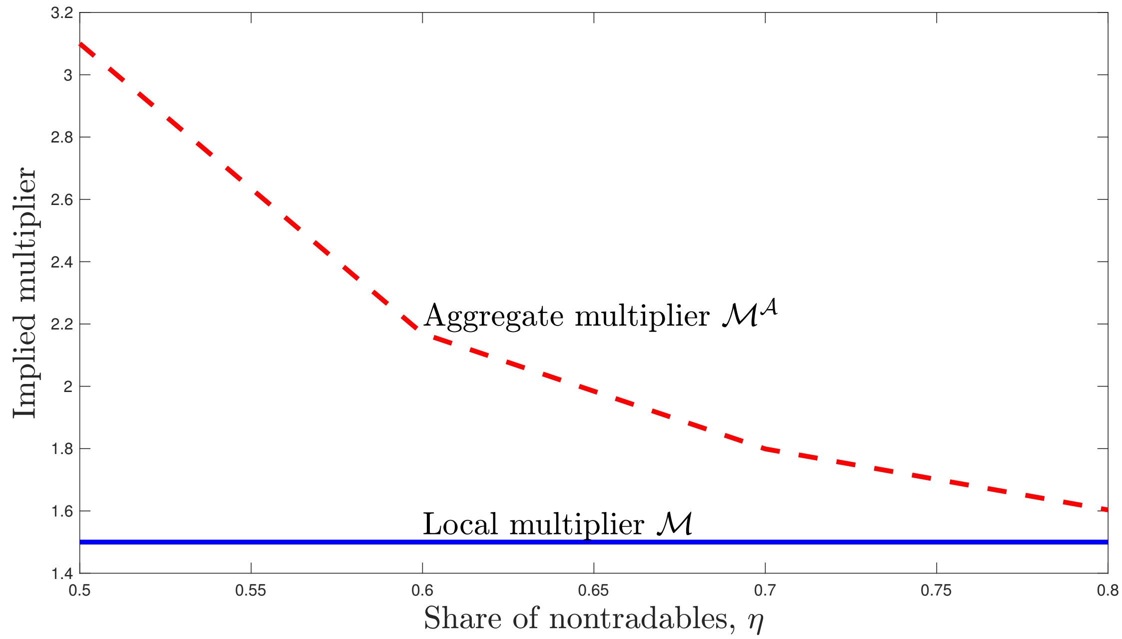

Figure B.2: Comparison between the aggregate and the local multipliers

Note: The solid line illustrates the implied aggregate multiplier $\mathcal{M}^{A}$ as a function of η, as we vary η over the range in $(B.94)$. The dashed line illustrates the local multiplier that we calibrate as, $\mathcal{M} = 1.5$.

The model is the same as in Section B.1 with two differences. First, there is uncertainty about the productivity of the future capital-only technology. Formally, we let $\mathcal{D} \subset [\frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}, \infty)$ denote a finite set of productivities. This domain ensures condition $(B.29)$ holds for each $D \subset \mathcal{D}$. Let $\pi(D)$ (with $\sum_{\mathcal{D}} \pi(D) = 1$) denote a probability distribution over $\mathcal{D}$. The productivity parameter D is uncertain in period 0 and it is realized in the beginning of period 1 with probability $\pi(D)$. Starting period 1 onward, there is no further uncertainty. The baseline model is the special case in which $\mathcal{D}$ has a single element. We denote the equilibrium allocations for periods $t \geq 1$ as a function of D, e.g., $C^{s}_{a,t}(D)$.

Second, to analyze the effect of risk aversion, we allow stockholders to have Epstein-Zin preferences that are more general than time-separable log utility. Specifically, we continue to assume the elasticity of intertemporal substitution is equal to one but allow for more general risk aversion.

Formally, we replace stockholders' preferences in (3) with the recursive utility defined by:

$$V_{a,t} = \left(C^s_{a,0}\right)^{\rho} U^{1-\rho}_{a,t+1} \text{ where } U_{a,t+1} = \left(E\left[V^{1-\gamma}_{a,t+1}\right]\right)^{1/(1-\gamma)}. \tag{B.115}$$

Here, $U^s_{a,t+1}$ captures a certainty-equivalent measure of the next period's continuation utility. The parameter, γ, captures relative risk aversion. The baseline model is the special case with $\gamma = 1$. The rest of the model is unchanged.

General Characterization of Equilibrium with Uncertainty. For periods $t \geq 1$, since there is no remaining uncertainty, our earlier analysis still applies. In particular, the utility function in $(B.115)$ becomes the same as in the baseline analysis. To see this, note that $U_{a,t+n} = V_{a,t+n}$ for each $t+n \geq t \geq 1$. Substituting this into $(B.115)$, taking logs, and iterating forward, we obtain:

$$\log V_{a,t} = \rho \sum_{n=0}^{\infty} (1-\rho)^n \log C^s_{a,t+n} \text{ for } t \geq 1.$$

This is equivalent to time separable log utility that we use in our baseline analysis [cf. (3)].

Therefore, Proposition 2 still applies and characterizes the equilibrium for periods $t \geq 1$. In particular, consumption is constant over time, $C^s_{a,t} = C^s_{a,1}(D)$ for each $t \geq 1$. Using this observation, we calculate,

$$V_{a,t} = C^s_{a,1}(D) \text{ for } t \geq 1. \tag{B.116}$$

Hence, for periods $t \geq 1$, the continuation utility is equal to consumption in period 1. Using Proposition 2, we also have an explicit characterization of this consumption:

$$P_{a,1}(D)\, C^s_{a,1}(D) = \rho\left(\frac{\overline{WL}}{\rho} + \frac{1+x_{a,1}}{1-\theta} Q_1(D) + A^f_{a,1}\right) \tag{B.117}$$

$$\text{where } P_1(D) = \overline{W} D^{\overline{\alpha}} \text{ and } Q_1(D) = \frac{\overline{W}D}{\rho} \tag{B.118}$$

For period 0, since there is uncertainty, stockholders' utility is different than before. Using Eqs. $(B.115), (B.116)$, and $(B.117)$, we write the stockholders' problem as [cf.

problem $(B.9)$]:

$$\max_{C^s_{a,0}, \frac{1+x_{a,1}}{1-\theta}} \rho \log C^s_{a,0} + (1-\rho) \log U_{a,1} \tag{B.119}$$

$$\text{where } U_{a,1} = \left(E\left[C^s_{a,1}(D)^{1-\gamma} \right] \right)^{1/(1-\gamma)}$$

$$\text{s.t. } P_{a,0} C^s_{a,0} + \frac{A^f_{a,1}}{R^f_0} + \frac{1+x_{a,1}}{1-\theta}(Q_0 - R_0) = W_{a,0}\overline{L} + \frac{1+x_{a,0}}{1-\theta} Q_0$$

$$\text{and } P_1(D) C^s_{a,1}(D) = \rho \left(\frac{\overline{WL}}{\rho} + \frac{1+x_{a,1}}{1-\theta} Q_1(D) + A^f_{a,1} \right)$$

The following lemma characterizes the solution to this problem.

Lemma 2 *Consider stockholders in area a. Their optimal consumption in period 0 satisfies:*

$$P_{a,0} C^s_{a,0} = \rho \left(W_{a,0}\overline{L} + \frac{1}{R^f_0} \frac{\overline{WL}}{\rho} + \frac{1+x_{a,0}}{1-\theta} Q_0 \right). \tag{B.120}$$

Their optimal portfolios are such that the risk-free interest rate satisfies,

$$\frac{1}{R^f_0} = E\left[M_{a,1}(D) \right] \tag{B.121}$$

and the price of capital satisfies,

$$Q_0 = R_0 + E\left[M_{a,1}(D) Q_1(D) \right] \text{ with } Q_1(D) = \frac{\overline{W}D}{\rho}, \tag{B.122}$$

where $M_{a,1}(D)$ denotes the nominal stochastic discount factor (SDF) *for area a (per unit time) and is given by*

$$M_{a,1}(D) = (1-\rho) \frac{P_{a,0} C^s_{a,0}}{P_1(D) C^s_{a,1}(D)} \frac{C^s_{a,1}(D)^{1-\gamma}}{E\left[C^s_{a,1}(D)^{1-\gamma} \right]}. \tag{B.123}$$

Eq. $(B.120)$ illustrates that the consumption wealth effect remains unchanged in this case [cf. Eq. $(B.59)$]. This is because we use Epstein-Zin preferences with an intertemporal elasticity of substitution equal to one. Eqs. $(B.121)$ and $(B.122)$ illustrate that standard asset pricing conditions apply in this setting. Specifically, the risk-free asset as well as capital are priced according to a stochastic discount factor (SDF) that might be specific to the area. Eq. $(B.123)$ characterizes the SDF. When $\gamma = 1$, the SDF has a familiar form corresponding to time-separable log utility. We relegate the proof of Lemma B.119 to the end of this section.

Since the optimal consumption Eq. $(B.120)$ remains unchanged (and the remaining features of the model are also unchanged), the rest of the general characterization in Section B.2 also applies in this case.

We next characterize the equilibrium further in the common-wealth benchmark.

Common-wealth Benchmark with Uncertainty. Consider the benchmark case with $x_{a,0} = 0$ for each a. Most of the analysis from Section B.3 also applies in this case. In particular, wages and labor are at their frictionless levels $W_0 = \overline{W}, L_0 = L_0^h = \overline{L}$. The rental rate, R_0, and the unit cost are given Eqs. $(B.65)$ and $(B.66)$.

The main difference concerns the pricing of stocks, which now reflects risk. To calculate the stochastic discount factor, note that $A_{a,1}^f = x_{a,1} = 0$ since areas are symmetric. Therefore, using Eqs. $(B.117)$ and $(B.118)$ stockholders' consumption in period 1 is given by,

$$\begin{aligned} P_1(D)\, C_1^s(D) &= \overline{WL} + \frac{\overline{W}D}{1-\theta} \\ \text{and } C_1^s(D) &= \frac{\overline{L} + \frac{D}{1-\theta}}{D^{\overline{\alpha}}}. \end{aligned} \tag{B.124}$$

Likewise, substituting $x_{a,0} = x_{a,1} = A_{a,1}^f = 0$ into the stockholders' budget constraint in $(B.119)$, we obtain stockholders' current expenditure:

$$P_0 C_0^s = W_0 \overline{L} + \frac{R_0}{1-\theta}.$$

Since stockholders' aggregate savings is zero, their aggregate spending is equal to the sum of their labor and capital income. Combining this with $W_0 = \overline{W}$ and $R_0 = \frac{\overline{\alpha}}{1-\overline{\alpha}} \overline{WL}$ [cf. $(B.65)$], we also calculate stockholders' spending in period 0 in terms of the parameters

$$P_0 C_0^s = \overline{WL} \left(1 + \frac{\overline{\alpha}}{1-\overline{\alpha}} \frac{1}{1-\theta}\right). \tag{B.125}$$

Combining Eqs. $(B.124)$ and $(B.125)$ with $(B.123)$, we also calculate the stochastic discount factor as

$$
\begin{aligned}
M_1(D) &= (1-\rho)\frac{P_0 C_0^s}{P_1(D)\,C_1^s(D)}\frac{C_1^s(D)^{1-\gamma}}{E\left[C_1^s(D)^{1-\gamma}\right]} \\
&= (1-\rho)\frac{\overline{L}\left(1+\frac{\overline{\alpha}}{1-\overline{\alpha}}\frac{1}{1-\theta}\right)}{\overline{L}+\frac{D}{1-\theta}}\frac{\left(\frac{\overline{L}+\frac{D}{1-\theta}}{D^{\overline{\alpha}}}\right)^{1-\gamma}}{E\left[\left(\frac{\overline{L}+\frac{D}{1-\theta}}{D^{\overline{\alpha}}}\right)^{1-\gamma}\right]} \qquad (B.126)
\end{aligned}
$$

Thus, in view of Lemma B.119, we obtain closed-form solutions for the interest rate and the price of capital:

$$
\frac{1}{R_0^f} = E\left[M_1(D)\right] \qquad (B.127)
$$

$$
Q_0/\overline{W} = \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}+E\left[M_1(D)\,\frac{D}{\rho}\right]. \qquad (B.128)
$$

When there is a single state, it is easy to check that Eqs. $(B.127)$ and $(B.128)$ give the same expression as in our baseline analysis [cf. $(B.72)$ and $(B.73)$]. Hence, these expressions generalize our baseline analysis to the case with uncertainty.

Here, we have arrived at these equations using a different method than in Section B.3. As before, we could also aggregate the labor demand and solve for the multiplier to obtain the following analogue of $(B.71)$:

$$
\begin{aligned}
\frac{\overline{LW}}{1-\overline{\alpha}} &= M^A\rho\left[\frac{1}{R_0^f}(1-\theta)\frac{\overline{WL}}{\rho}+E\left[M_1(D)\,\frac{\overline{W}D}{\rho}\right]\right] \\
\text{where } M^A &= \frac{1}{(1-\rho)\left(1-(1-\overline{\alpha})\,\theta\right)}
\end{aligned}
$$

As before, stockholders' future wealth should be at a particular level such that its direct spending effect, combined with the multiplier effects, are just enough to ensure output is equal to its frictionless level. Specifically, the term inside the set brackets is equal to a constant given by:

$$(1-\theta)\frac{1}{R_0^f}\frac{\overline{L}}{\rho}+E\left[M_1(D)\frac{D}{\rho}\right]=\frac{(1-\rho)(1-(1-\overline{\alpha})\theta)}{(1-\overline{\alpha})\rho}\overline{L}. \tag{B.129}$$

After substituting $\frac{1}{R_0^f}=E[M_1(D)]$ and the SDF from $(B.126)$, it can be checked that this equation indeed holds.

Recall that, in the baseline model without uncertainty, we generate fluctuations in Q_0 as well as R_0^f from changes in D. We next show that this aspect of the model also generalizes. In particular, after summarizing the above discussion, the following proposition establishes that changes in risk or risk aversion generate the same effects on asset prices as changes in future productivity in the baseline model.

Proposition 4 *Consider the model with uncertainty described earlier where D takes values in the finite set $\mathcal{D}\subset[\frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L},\infty)$ according to the probability distribution function $(\pi(D))_{\mathcal{D}}$. Suppose areas have common stock wealth, $x_{a,0}=0$ for each a. In equilibrium, all areas have identical allocations and prices. In period 0, nominal wages and labor are at their frictionless levels, $W_0=\overline{W}, L_0=\overline{L}$; the stochastic discount factor is given by Eq. $(B.126)$; the nominal interest rate is given by Eq. $(B.127)$; the price of capital is given by Eq. $(B.128)$; the shares of labor employed in the nontradable and tradable sectors are given by Eq. $(B.74)$.*

Consider any one of the following changes:

(i) Suppose $\gamma=1$ and the probability distribution, $\left(\pi^{old}(D)\right)_{\mathcal{D}}$, changes such that $(\pi^{new}(D))_{\mathcal{D}}$ first-order stochastically dominates $\left(\pi^{old}(D)\right)_{\mathcal{D}}$.

(ii) Suppose $\gamma=1$ and the probability distribution, $\left(\pi^{old}(D)\right)_{\mathcal{D}}$, changes such that $\left(\pi^{old}(D)\right)_{\mathcal{D}}$ is a mean-preserving spread of $(\pi^{new}(D))_{\mathcal{D}}$.

(iii) Suppose $\left(\pi^{old}(D)\right)_{\mathcal{D}}$ remains unchanged but risk-aversion decreases, $\gamma^{new}<\gamma^{old}$.

These changes increase Q_0 and reduce R_0^f in equilibrium but do not affect the labor market outcomes in period 0.

The first part is a generalization of the comparative statics exercise that we consider in the baseline model. It shows that the price of capital increases also if agents perceive greater capital productivity in the first-order stochastic dominance sense. The second part shows that a similar result obtains if agents' expected belief for capital productivity remains unchanged but they perceive less risk in capital productivity. For analytical tractability, these two parts focus on the case, $\gamma = 1$, which corresponds to time-separable log utility as in the baseline model. The last part considers the case with general γ, and shows that a similar result obtains also if agents' belief distribution remains unchanged but their risk aversion declines. We relegate the proof of Proposition 4 to the end of this section.

Comparative Statics of Local Labor Market Outcomes with Uncertainty. Recall that since the optimal consumption Eq. ($B.120$) remains unchanged, all equilibrium conditions for period 0 derived in Section B.2.3 continue to apply conditional on Q_0 and R_0^f. Therefore, the log-linearized equilibrium conditions derived in Section B.4 also continue to apply conditional on Q_0. Moreover, as we show in Section B.5, the comparative statics in Proposition 4 affect these conditions only through their effect on Q_0. It follows that, conditional on generating the same change in the price of capital, ΔQ_0, the model with uncertainty features the same *quantitative* effects on local labor market outcomes as in our our baseline model. Combining this result with the comparative static results in Proposition 4, we conclude that our baseline analysis is robust to generating stock price fluctuations from alternative sources such as changes in households' risk aversion or perceived risk about stock payoffs.

Proof of Lemma 2. To simplify the problem, consider the change of variables,

$$\tilde{S}_{a,0} = \frac{A_{a,1}^f + \overline{WL}/\rho}{R_0^f} + \frac{1 + x_{a,1}}{1 - \theta} \left(Q_0 - R_0 \right).$$

Here, $\tilde{S}_{a,0}$ can be thought of as the stockholder's "effective savings" that incorporates the present discounted value of her lifetime wealth in subsequent periods, $\frac{1}{R_0^f} \frac{\overline{WL}}{\rho}$. We also define

$$\omega_{a,1} \equiv \frac{1 + x_{a,1}}{1 - \theta} \frac{Q_0 - R_0}{\tilde{S}_{a,0}}.$$

Here, $\omega_{a,1}$ captures the fraction of the stockholder's effective savings that she invests in capital (recall that $Q_0 - R_0$ denotes the ex-dividend price of capital). The remaining fraction, $1 - \omega_{a,1}$, is invested in the risk-free asset. After substituting this notation into the budget constraints, the stockholder's problem can be equivalently written as,

$$\max_{\tilde{S}_{a,0},\omega_{a,1}} \rho \log C^s_{a,0} + (1-\rho)\log U_{a,1} \tag{B.130}$$

$$\text{where } U_{a,1} = \left(E\left[C^s_{a,1}(D)^{1-\gamma}\right]\right)^{1/(1-\gamma)}$$

$$\text{s.t. } P_{a,0}C^s_{a,0} + \tilde{S}_{a,0} = W_{a,0}\overline{L} + \frac{\overline{WL}}{\rho} + \frac{1+x_{a,0}}{1-\theta}Q_0$$

$$P_1(D)\,C^s_{a,1}(D) = \rho\tilde{S}_{a,0}\left(R^f_0 + \omega_{a,1}\left(\frac{Q_1(D)}{Q_0-R_0} - R^f_0\right)\right).$$

Here, $\frac{Q_1(D)}{Q_0-R_0}$ denotes the gross return on capital. When $\omega_{a,1}=0$, the stockholder does not invest in capital so her portfolio return is the gross risk-free rate, R^f_0. When $\omega_{a,1}=1$, the stockholder invests all of her savings in capital so her portfolio return is the gross return to capital, $\frac{Q_1(D)}{Q_0-R_0}$.

Next consider the optimality condition for $\tilde{S}_{a,0}$ in problem $(B.119)$. This gives:

$$\frac{\rho}{P_{a,0}C^s_{a,0}} = (1-\rho)\frac{U^{\gamma}_{a,1}}{U_{a,1}}E\left[C^s_{a,1}(D)^{-\gamma}\frac{1}{P_1(D)}\rho\left(R^f_0 + \omega_{a,1}\left(\frac{Q_1(D)}{Q_0-R_0} - R^f_0\right)\right)\right].$$

Using the budget constraint in period 1 to substitute for the return in terms of $C^s_{a,1}(D)$ and simplifying, we further obtain:

$$\begin{aligned}\frac{\rho}{P_{a,0}C^s_{a,0}} &= (1-\rho)\,U^{\gamma-1}_{a,1}E\left[C^s_{a,1}(D)^{-\gamma}\frac{C^s_{a,1}(D)}{\tilde{S}_{a,0}}\right]\\ &= (1-\rho)\,U^{\gamma-1}_{a,1}U^{1-\gamma}_{a,1}\frac{1}{\tilde{S}_{a,0}}\\ &= (1-\rho)\,\frac{1}{\tilde{S}_{a,0}}.\end{aligned}$$

Here, the second line uses $U^{1-\gamma}_{a,1} = E\left[C^s_{a,1}(D)^{1-\gamma}\right]$ (from the definition of the certainty-equivalent utility). The last line simplifies the expression. Combining the resulting expression with the budget constraint in period 0, we obtain,

$$P_{a,0}C^s_{a,0} = \rho\left(W_{a,0}\overline{L} + \frac{\overline{WL}}{\rho} + \frac{1+x_{a,0}}{1-\theta}Q_0\right).$$

This establishes $(B.120)$.

Next, to establish the asset pricing condition for the risk-free asset, consider the optimality condition for $A^f_{a,1}$ in the original problem $(B.119)$ (as this corresponds to saving in the risk-free asset). This gives:

$$\begin{aligned}\frac{\rho}{P_{a,0}C^s_{a,0}} &= (1-\rho)\,U^{\gamma-1}_{a,1}E\left[\frac{1}{P_1(D)\,C^s_{a,1}(D)^{\gamma}}\rho R^f_0\right] \\ &= (1-\rho)\frac{1}{E\left[C^s_{a,1}(D)^{1-\gamma}\right]}E\left[\frac{1}{P_1(D)\,C^s_{a,1}(D)^{\gamma}}\rho R^f_0\right] \qquad \text{(B.131)}\end{aligned}$$

Here, the second line substitutes $U^{1-\gamma}_{a,1} = E\left[C^s_{a,1}(D)^{1-\gamma}\right]$. Rearranging terms and substituting $M_{a,1}(D)$ from Eq. $(B.123)$, we obtain Eq. $(B.121)$.

Finally, to establish the asset pricing condition for capital, consider the optimality condition for $\omega_{a,1}$ in problem $(B.130)$. This gives:

$$E\left[\frac{C^s_{a,1}(D)^{-\gamma}}{P_{a,1}(D)}\rho\left(\frac{Q_1(D)}{Q_0-R_0} - R^f_0\right)\right] = 0.$$

Rearranging terms, we obtain,

$$\begin{aligned}Q_0 &= R_0 + \frac{1}{R^f_0}\frac{1}{E\left[\frac{1}{P_{a,1}(D)C^s_{a,1}(D)^{\gamma}}\right]}E\left[\frac{1}{P_{a,1}(D)\,C^s_{a,1}(D)^{\gamma}}Q_1(D)\right] \\ &= R_0 + (1-\rho)\frac{1}{E\left[C^s_{a,1}(D)^{1-\gamma}\right]}E\left[\frac{P_{a,0}C^s_{a,0}}{P_1(D)\,C^s_{a,1}(D)^{\gamma}}Q_1(D)\right] \\ &= R_0 + E\left[M_{a,1}(D)\,Q_1(D)\right].\end{aligned}$$

Here, the second line uses Eq. $(B.131)$ to substitute for $1/R^f_0$ and the last line substitutes for $M_{a,1}(D)$ from Eq. $(B.123)$. This establishes $(B.122)$. Note that we also have $Q_1(D) = \frac{\overline{W}D}{\rho}$ from $(B.118)$. This completes the proof of the lemma.■

Proof of Proposition 4. It remains to establish the comparative statics exercises. Recall that stockholders' future wealth satisfies $(B.129)$. Using $(B.128)$ and $R_0 = \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}$ we can rewrite this as:

$$\left(1-\theta\right)\frac{1}{R_0^f}\frac{\overline{L}}{\rho}+Q_0/\overline{W}-\frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}=\frac{\left(1-\rho\right)\left(1-\left(1-\overline{\alpha}\right)\theta\right)}{\left(1-\overline{\alpha}\right)\rho}\overline{L}.$$

Note that the probability distribution, $(\pi(D))_{\mathcal{D}}$, or the risk aversion, γ, affect this equation only through their effect on Q_0 and R_0^f. The equation then implies that if these changes increase Q_0 then they must also increase R_0^f. Therefore, it suffices to establish the comparative statics exercises for the price of capital, Q_0.

First consider the comparative statics exercises in parts (i) and (ii). After substituting $\gamma = 1$ into Eqs. $(B.128)$ and $(B.126)$, we obtain the following expression for the price of capital:

$$\begin{aligned} Q_0/\left(\overline{WL}\right) &= \frac{\overline{\alpha}}{1-\overline{\alpha}}+\frac{1-\rho}{\rho}\left(1-\theta+\frac{\overline{\alpha}}{1-\overline{\alpha}}\right)E\left[f\left(D\right)\right] \qquad \text{(B.132)} \\ \text{where } f\left(D\right) &= \frac{D}{\overline{L}\left(1-\theta\right)+D} \end{aligned}$$

Here, the second line defines the function $f : \mathbb{R}_+ \to \mathbb{R}_+$. Note that this function is strictly increasing and strictly concave: that is, $f'(D) > 0$ and $f''(D) < 0$ for $D > 0$. Combining this observation with Eq. $(B.132)$ proves the desired comparative statics. To establish (i), note that $E^{new}[f(D)] \geq E^{old}[f(D)]$ because $f(D)$ is increasing in D, and $\pi^{new}(D)$ first-order stochastically dominates $\pi^{old}(D)$. To establish (ii), note that $E^{new}[f(D)] \geq E^{old}[f(D)]$ because $f(D)$ is increasing and concave in D, and $\pi^{new}(D)$ second-order stochastically dominates $\pi^{old}(D)$ (which in turn follows because $\pi^{old}(D)$ is a mean-preserving spread of $\pi^{new}(D)$).

Finally, consider the comparative statics exercise in part (iii). In this case, Eqs. $(B.128)$ and $(B.126)$ imply,

$$\begin{aligned} Q_0/\left(\overline{WL}\right) &= \frac{\overline{\alpha}}{1-\overline{\alpha}}+\frac{1-\rho}{\rho}\left(1-\theta+\frac{\overline{\alpha}}{1-\overline{\alpha}}\right)\frac{E\left[f\left(D\right)g\left(D\right)^{1-\gamma}\right]}{E\left[g\left(D\right)^{1-\gamma}\right]}, \qquad \text{(B.133)} \\ \text{where } g\left(D\right) &= \frac{\overline{L}\left(1-\theta\right)+D}{D^{\overline{\alpha}}}. \end{aligned}$$

Here, the second line defines the function $g : \mathbb{R}_+ \to \mathbb{R}_+$. We first claim that this function is increasing in D over the relevant range. To see this, note that,

$$g'(D) = D^{-\overline{\alpha}-1}(1-\overline{\alpha})\left(D - (1-\theta)\frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}\right).$$

This is strictly positive since $D \geq \frac{\overline{\alpha}}{1-\overline{\alpha}}\overline{L}$ [cf. condition $(B.29)$]. Therefore, $g(D)$ is increasing in D over the relevant range.

Next note that Eq. $(B.133)$ can be rewritten as

$$Q_0/\left(\overline{WL}\right) = \frac{\overline{\alpha}}{1-\overline{\alpha}} + \frac{1-\rho}{\rho}\left(1-\theta+\frac{\overline{\alpha}}{1-\overline{\alpha}}\right)E^*\left[f(D)\right],$$

where $E^*[\cdot]$ denotes the expectations under the endogenous probability distribution $(\pi^*(D))_{\mathcal{D}}$, defined by,

$$\pi^*(D) = \frac{\pi(D)\, g(D)^{1-\gamma}}{\sum_{\tilde{D}\in\mathcal{D}} \pi\left(\tilde{D}\right) g\left(\tilde{D}\right)^{1-\gamma}} \text{ for each } D \in \mathcal{D}. \tag{B.134}$$

Hence, using our result from part (i), it suffices to show that $\pi^{*,new}(D)$ (which corresponds to $\gamma^{new} < \gamma^{old}$) first-order stochastically dominates $\pi^{*,old}(D)$.

To establish the last claim, define the cumulative distribution function corresponding to the endogenous probability distribution,

$$\Pi^*(D,\gamma) = \sum_{\tilde{D}\leq D} \pi^*\left(\tilde{D}\right) = \frac{\sum_{\tilde{D}\leq D, \tilde{D}\in\mathcal{D}} \pi\left(\tilde{D}\right) g\left(\tilde{D}\right)^{1-\gamma}}{\sum_{\tilde{D}\in\mathcal{D}} \pi\left(\tilde{D}\right) g\left(\tilde{D}\right)^{1-\gamma}} \text{ for each } D \in \mathcal{D}. \tag{B.135}$$

We made the dependence of the distribution function on γ explicit. To prove the claim, it suffices to show that $\frac{\partial \Pi^*(D,\gamma)}{\partial\gamma} \geq 0$ for each $D \in \mathcal{D}$ (so that a decrease in γ decreases $\Pi^*(D,\gamma)$ for each D and thus increases the distribution in the first-order stochastic dominance order). We have:

$$\begin{aligned}
&\frac{\partial \Pi^*(D,\gamma)}{\partial\gamma} \Big/ \left(\frac{\sum_{\tilde{D}\leq D} \pi\left(\tilde{D}\right) g\left(\tilde{D}\right)^{1-\gamma}}{\sum \pi\left(\tilde{D}\right) g\left(\tilde{D}\right)^{1-\gamma}}\right) \\
= \quad & -\frac{\sum_{\tilde{D}\leq D} \pi\left(\tilde{D}\right) g\left(\tilde{D}\right)^{1-\gamma} \log g\left(\tilde{D}\right)}{\sum_{\tilde{D}\leq D} \pi\left(\tilde{D}\right) g\left(\tilde{D}\right)^{1-\gamma}} + \frac{\sum \pi\left(\tilde{D}\right) g\left(\tilde{D}\right)^{1-\gamma} \log g\left(\tilde{D}\right)}{\sum \pi\left(\tilde{D}\right) g\left(\tilde{D}\right)^{1-\gamma}}
\end{aligned}$$

$$
\begin{aligned}
&= -\sum_{\tilde{D}\leq D}\frac{\pi^*\left(\tilde{D}\right)}{\Pi^*\left(D,\gamma\right)}\log g\left(\tilde{D}\right)+\sum\pi^*\left(\tilde{D}\right)\log g\left(\tilde{D}\right)\\
&= -E^*\left[\log g\left(\tilde{D}\right)\quad|\tilde{D}\leq D\right]+E^*\left[\log g\left(\tilde{D}\right)\right].
\end{aligned}
$$

Here, the second line substitutes the definition of the endogenous distribution and its cumulative distribution from Eqs. $(B.134)$ and $(B.135)$. The last line substitutes the unconditional and conditional expectations. It follows that $\frac{\partial\Pi^*(D,\gamma)}{\partial\gamma}\geq 0$ for some $D\in\mathcal{D}$ if and only if the unconditional expectation exceeds the conditional expectation, $E^*\left[\log g\left(\tilde{D}\right)\right]\geq E^*\left[\log g\left(\tilde{D}\right)|\quad\tilde{D}\leq D\right]$. This is true because $\log g\left(D\right)$ is increasing in D (since $g\left(D\right)$ is increasing). This proves the claim and completes the proof of part (iii).■