Online Appendix

Optimal Policy under Dollar Pricing

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A Related Papers

Table A1: Comparison to the literature

<table>
<tr><th></th><th>DSX</th><th>CP</th><th>GT</th><th>CDGG</th><th>BBGRU</th><th>CDL</th><th>EM</th></tr>
<tr><td colspan="8">Environment:</td></tr>
<tr><td># of countries</td><td colspan="2">two</td><td>three</td><td colspan="2">SOE</td><td>two</td><td>continuum</td></tr>
<tr><td>preferences</td><td colspan="6">log-linear</td><td>general</td></tr>
<tr><td>intermediates</td><td colspan="6">no</td><td>yes</td></tr>
<tr><td>asset markets</td><td colspan="4">complete</td><td colspan="2">one bond</td><td>arbitrary</td></tr>
<tr><td>nominal frictions</td><td colspan="3">fully rigid</td><td>Calvo</td><td>fully rigid</td><td>Calvo</td><td>arbitrary</td></tr>
<tr><td>currency choice</td><td>rationalized</td><td colspan="5">exogenous</td><td>endogenous</td></tr>
<tr><td colspan="8">Non-U.S. policy:</td></tr>
<tr><td>terms of trade</td><td colspan="6">exogenous to MP</td><td>endogenous</td></tr>
<tr><td>optimal target</td><td colspan="7">price stabilization</td></tr>
<tr><td>allocation</td><td colspan="7">inefficient</td></tr>
<tr><td>implementation</td><td colspan="6">inward-looking</td><td>outward-looking</td></tr>
<tr><td>capital controls</td><td colspan="5">–</td><td>small</td><td>inefficient</td></tr>
<tr><td>trade policy</td><td colspan="6">–</td><td>efficient</td></tr>
<tr><td colspan="8">U.S. policy:</td></tr>
<tr><td>rents motive</td><td colspan="3">yes</td><td colspan="2">–</td><td>yes</td><td>yes</td></tr>
<tr><td>dynamic ToT motive</td><td colspan="3">no</td><td colspan="2">–</td><td>no</td><td>yes</td></tr>
<tr><td>gains from DCP</td><td>negative</td><td>–</td><td>negative</td><td colspan="3">–</td><td>ambiguous</td></tr>
<tr><td>cooperative policy</td><td colspan="3">monetary</td><td colspan="2">–</td><td>monetary</td><td>monetary+fiscal</td></tr>
</table>

Note: DSX stands for Devereux, Shi and Xu (2007), CP for Corsetti and Pesenti (2007), GT for Goldberg and Tille (2009), CDGG for Casas et al. (2017), BBGRU for Basu et al. (2020), CDL for Corsetti, Dedola and Leduc (2020), and EM for this paper.

B Proofs for Section 1

B.1 Proof of Lemma 1

We have already shown that the conditions (4) – (10) are necessary for an allocation and prices to form part of an equilibrium. Now we show that these conditions are also sufficient. The proof is constructive. Start with an allocation and prices that satisfy these conditions. We choose wages W_{it} to satisfy the labor supply condition (1) for all countries i and time periods t. We then choose the nominal exchange rate $\mathcal{E}_{it}$ for all non-U.S. countries $i \neq 0$ to satisfy the relative demand for foreign goods (2). Domestic nominal interest rate R_{it} can be chosen to satisfy the Euler equation (3). Finally, set the government transfers T_{it} to satisfy the households' flow budget constraint. By using the country's budget constraint (8), the flow profits from local firms Π_{it}^{f} from the problems of domestic sellers and exporters, the market clearing condition (7), and the zero net supply of domestic bonds (9), one can verify that this choice of transfers would also satisfy the government's flow budget constraint

$$T_{it} = (\tau_i - 1) \frac{W_{it}}{A_{it}} C_{iit} + (\tau_i^* - 1) \frac{W_{it}}{A_{it}} h(S_{it}) C_t^*. \tag{A1}$$

B.2 Proof of Lemma 2

To prove the first part, note that all non-U.S. countries are small open economies, and thus they take all foreign variables as given. So for them it does not matter in which variables foreign strategies are formulated.

The U.S. moves first and takes as given the best response of non-U.S. variables to the U.S. actions. At the second stage, non-U.S. countries take all global variables, including the U.S. actions, as given. Proposition 1 states that in this case the optimal non-U.S. policy is to set $\pi_{iit} = 1$. This condition (11) along with conditions (4) – (8) is enough to pin down all local non-U.S. variables $\left\{C_{iit}, C_{it}^*, L_{it}, B_{it}^h, S_{it}, \pi_{iit}\right\}$ as functions of global variables. Thus, the best response functions are uniquely determined by conditions (4) – (8) and (11) regardless of which variable is used by non-U.S. countries to formulate their strategies.

To prove the second part, note that the non-U.S. inflation π_{iit} does not depend on the U.S. actions, $\pi_{iit} = 1$, as is stated in Proposition 1. Therefore, the optimal policy condition (11) in a sequential game can be viewed instead as a fixed non-U.S. strategy in a simultaneous game, while the rest of the non-U.S. variables are still determined by conditions (4) – (8).

The third part of the lemma follows from Proposition 1, which states that the optimal policy is time consistent, and thus condition (11) stays the same under discretion.

B.3 Proof of Lemma 3

Efficient allocation To solve for the efficient allocation in one country, we allow the planner to choose all quantities in this country directly. However, the planner has to take international prices as given and respect the country's budget constraint as well as foreign demand for her own goods. Thus, the social planner is subject only to the market clearing condition (7) and the country's budget constraint (8):

$$\max_{\left\{C_{iit}, C_{it}^*, L_{it}, \left\{B_{it}^h\right\}_h, S_{it}\right\}_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right)$$

$$\text{s.t.} \quad A_{it} L_{it} = C_{iit} + h(S_{it}) C_t^*,$$

$$\sum_{h \in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h \in H_{t-1}} \left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) B_{it}^h = S_{it} h(S_{it}) C_t^* - C_{it}^* + \psi_{it}.$$

Here the planner can choose any export price in dollars or, equivalently, the terms of trade S_{it}. By construction, the planner does not have to pay any price-adjustment costs.

Let's denote the Lagrangian multiplier for the market clearing condition as λ_{it} and for the budget constraint as μ_{it}. Then the FOCs are

$$\begin{aligned} U_{C_{iit}} - \lambda_{it} &= 0 \\ U_{C^*_{it}} + \mu_{it} &= 0 \\ U_{L_{it}} + \lambda_{it} A_{it} &= 0 \\ \mu_{it} \mathcal{Q}^h_t - \beta \mathbb{E}_t \mu_{it+1} \left(\mathcal{Q}^h_{t+1} + \mathcal{D}^h_{t+1} \right) &= 0 \\ -\lambda_{it} h'(S_{it}) C^*_t - \mu_{it} \left[h(S_{it}) + S_{it} h'(S_{it}) \right] C^*_t &= 0 \end{aligned}$$

Use the first FOC to find λ_{it}, the second to find μ_{it}, and substitute for these Lagrange multipliers in all other conditions. Then we arrive at the labor supply condition

$$-\frac{U_{L_{it}}}{U_{C_{iit}}} = A_{it}, \tag{A2}$$

the no-arbitrage condition (4), and the export price-setting condition

$$S_{it} = \frac{\varepsilon}{\varepsilon - 1} \frac{U_{C_{iit}}}{U_{C^*_{it}}}. \tag{A3}$$

Flexible-price allocation Now let's solve for the flexible-price allocation. Under flexible prices, the domestic price setting condition (5) collapses to

$$U_{C_{iit}} = \frac{-U_{L_{it}}}{A_{it}},$$

and the export price setting condition (6) collapses to

$$U_{C^*_{it}} S_{it} = \frac{\varepsilon}{\varepsilon - 1} \frac{-U_{L_{it}}}{A_{it}} = \frac{\varepsilon}{\varepsilon - 1} U_{C_{iit}},$$

where the second equality uses the previous condition. Note that the first of these conditions is the same as the efficient labor supply condition (A2) and the second one is equivalent to the efficient export price-setting condition (A3). Finally, the no-aribtrage condition (4) is part of the private sector equilibrium conditions, and therefore it holds under flexible prices as well. Thus, we have shown that the flexible-price equilibrium conditions coincide with the planner's optimality conditions.

Equilibrium under producer currency pricing First, let's set up equilibrium conditions under PCP. Note that conditions (1) – (4) and (7) – (8) are independent of the pricing assumptions and thus stay the same. As under DCP, the domestic producers set their prices in local currency, and thus their price-setting condition remains equivalent to (5). But the problem of exporters changes to

$$\{P^*_{it}/\mathcal{E}_{it}\} = \underset{\{P_t\}}{\operatorname{argmax}}\, \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(P_t - \frac{W_{it}}{A_{it}} \right) h\left(\frac{P_t}{\mathcal{E}_{it} P^*_t} \right) C^*_t - \Omega^* \left(\frac{P_t}{P_{t-1}} \right) W_{it} \right],$$

since export prices are now sticky in local currency, not in dollars.

Second, let's show that the monetary policy targeting $\pi_{iit} = 1$ leads to the efficient allocation. This monetary policy rule implies that the prices of domestic producers are always equal to their marginal costs,

$$U_{C_{iit}} = \frac{-U_{L_{it}}}{A_{it}}.$$

As before, this condition is equivalent to the efficient labor supply condition (A2). Next, plug in the SDF $\Theta_{i0,t}$ from (3) and wages W_{it} from households' optimality condition (1) into the export price-setting condition:

$$\{P_{it}^*/\mathcal{E}_{it}\} = \underset{\{P_t\}}{\operatorname{argmax}}\, \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{P_t}{P_{iit}} - 1 \right) U_{C_{iit}} h \left(\frac{P_t}{\mathcal{E}_{it} P_t^*} \right) C_t^* - \Omega^* \left(\frac{P_t}{P_{t-1}} \right) (-U_{L_{it}}) \right],$$

where we have used the fact that $U_{C_{iit}} = -U_{L_{it}}/A_{it}$. Recall that $\pi_{iit} = 1$ implies constant domestic prices P_{iit}. Thus, by choosing

$$P_{it}^*/\mathcal{E}_{it} = \frac{\varepsilon}{\varepsilon - 1} P_{iit}$$

domestic exporters can maintain their optimal markup without paying any price-adjustment costs.

We have shown that under PCP the nominal exchange rate $\mathcal{E}_{it}$ replicates the path of the flexible dollar prices, and thus the resulting allocation is efficient.

C Proofs for Section 2

C.1 Kimball demand and intermediate inputs

We allow for pricing-to-market and heterogeneity between domestic and exporting firms and show robustness of the optimal policy.

Cost minimization We assume that production functions of domestic producers and exporters are given by

$$Y_{it}^d = A_{it}^d F \left(L_{it}^d, X_{iit}^d, X_{it}^{d*} \right),$$

$$Y_{it}^e = A_{it}^e G \left(L_{it}^e, X_{iit}^e, X_{it}^{e*} \right),$$

where $F(\cdot)$ and $G(\cdot)$ are both constant returns to scale. Thus, the labor intensity and productivity shocks might differ across two types of firms. X_{iit}^d and X_{it}^{d*} are domestic and foreign intermediates used by domestic firms, while X_{iit}^e and X_{it}^{e*} are intermediates used by exporters. Different production functions allow the model to capture, among other things, the fact that exporting firms are also the largest importers and that consumers might not have direct access to foreign goods, but rather have to buy them from local retailers.

The cost minimization problem for domestic producers is

$$\min_{L_{it}^d, X_{iit}^d, X_{it}^{d*}} W_{it} L_{it}^d + P_{iit} X_{iit}^d + \mathcal{E}_{it} P_t^* X_{it}^{d*}$$

$$\text{s.t. } A_{it}^d F \left(L_{it}^d, X_{iit}^d, X_{it}^{d*} \right) = Y_{it}^d.$$

The first-order conditions are

$$W_{it} = \lambda_{it} A_{it}^d F_{L_{it}^d}$$

$$P_{iit} = \lambda_{it} A_{it}^d F_{X_{iit}^d}$$

$$\mathcal{E}_{it} P_t^* = \lambda_{it} A_{it}^d F_{X_{it}^{d*}}$$

The solution to this problem can be described by the system of optimality conditions

$$\frac{W_{it}}{P_{iit}} = \frac{F_{L_{it}^d}}{F_{X_{iit}^d}}, \quad \frac{\mathcal{E}_{it} P_t^*}{P_{iit}} = \frac{F_{X_{it}^{d*}}}{F_{X_{iit}^d}},$$

and by the total cost function

$$C^d\left(W_{it}, P_{iit}, \mathcal{E}_{it}P_t^*\right)Y_{it}^d/A_{it}^d = W_{it}L_{it}^d + P_{iit}X_{iit}^d + \mathcal{E}_{it}P_t^*X_{it}^{d*},$$

which is linear in Y_{it}^d due to constant returns to scale. Moreover, the cost function is homogenous of degree 1 in input prices, and thus we can rewrite the marginal cost as

$$C^d\left(W_{it}, P_{iit}, \mathcal{E}_{it}P_t^*\right)/A_{it}^d \equiv \frac{W_{it}}{A_{it}^d}c^d\left(\frac{P_{iit}}{W_{it}}, \frac{\mathcal{E}_{it}P_t^*}{W_{it}}\right).$$

Also, we can replace relative prices P_{iit}/W_{it} and $\mathcal{E}_{it}P_t^*/W_{it}$ with $-U_{C_{iit}}/U_{L_{it}}$ and $-U_{C_{it}^*}/U_{L_{it}}$ by using households' optimality conditions (1) and (2). Finally, note that the Lagrange multiplier λ_{it} has to be equal to the marginal cost, and thus

$$\frac{W_{it}}{A_{it}^d F_{L_{it}^d}} = \frac{W_{it}}{A_{it}^d}c^d\left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C_{it}^*}}{-U_{L_{it}}}\right). \tag{A4}$$

Similarly, the optimality conditions for exporters are

$$\frac{-U_{L_{it}}}{U_{C_{iit}}} = \frac{G_{L_{it}^e}}{G_{X_{iit}^e}}, \quad \frac{U_{C_{it}^*}}{U_{C_{iit}}} = \frac{G_{X_{it}^{e*}}}{G_{X_{iit}^e}},$$

and their marginal cost is $\frac{W_{it}}{A_{it}^e}c^e\left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C_{it}^*}}{-U_{L_{it}}}\right)$.

Kimball demand Instead of the CES bundle, we assume that both local and foreign varieties are combined via the Kimball (1995) aggregator, e.g. demand for an individual domestic variety solves the following expenditure minimization problem:

$$\min_{\{C_{iit}(\omega)\}} \int P_{iit}(\omega)\, C_{iit}(\omega)\, \mathrm{d}\omega$$

$$\text{s.t.} \int \Upsilon\left(\frac{C_{iit}(\omega)}{C_{iit}}\right)\mathrm{d}\omega = 1,$$

where $\Upsilon(1) = \Upsilon'(1) = 1$, $\Upsilon'(\cdot) > 0$ and $\Upsilon''(\cdot) < 0$.[1] The first-order conditions lead to the demand function

$$C_{iit}(\omega) = h\left(\frac{P_{iit}(\omega)}{\mathcal{P}_{iit}}\right)C_{iit},$$

where $h(z) \equiv \Upsilon'^{-1}(z)$ and the price index $\mathcal{P}_{iit}$ is implicitly defined by

$$\int \Upsilon\left(h\left(\frac{P_{iit}(\omega)}{\mathcal{P}_{iit}}\right)\right)\mathrm{d}\omega = 1.$$

We also define another price index P_{iit} to express expenditures as $P_{iit}C_{iit} \equiv \int P_{iit}(\omega)\, C_{iit}(\omega)\, \mathrm{d}\omega$. This price index is then given by

$$P_{iit} \equiv \int P_{iit}(\omega)\, h\left(\frac{P_{iit}(\omega)}{\mathcal{P}_{iit}}\right)\mathrm{d}\omega.$$

Note, however, that in equilibrium all domestic producers are going to be symmetric and hence, for any ω, $P_{iit}(\omega) = P_{iit} = \mathcal{P}_{iit}$.[2]

[1] The bundles of intermediates X_{iit} and X_{it}^* are similarly defined.

[2] The CES demand is a special case with $\Upsilon(x) = 1 + \frac{\varepsilon}{\varepsilon-1}\left(x^{\frac{\varepsilon-1}{\varepsilon}} - 1\right)$ and $P_{iit} = \mathcal{P}_{iit}$.

The expenditure minimization problem for imported varieties is similar to the one for domestic varieties considered above, and leads to demand function

$$C_{jit}(\omega) = h\left(\frac{P^*_{jt}(\omega)}{\mathcal{P}^*_t}\right) C^*_{it},$$

where the two price indices $\mathcal{P}^*_t$ and P^*_t are defined by

$$1 = \int\int h\left(\frac{P^*_{jt}(\omega)}{\mathcal{P}^*_t}\right) \mathrm{d}\omega \mathrm{d}j, \quad P^*_t \equiv \int\int P^*_{jt}(\omega)\, h\left(\frac{P^*_{jt}(\omega)}{\mathcal{P}^*_t}\right) \mathrm{d}\omega \mathrm{d}j.$$

In contrast to the case of domestic prices, these price indices do not coincide because of the cross-country differences. However, both of them are taken as given by a small open economy.

Price setting The problem of a domestic firm can then be written as

$$\{P_{iit}\} = \arg\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty} \Theta_{i0,t}\left[\left(P_t - \tau_i \frac{W_{it}}{A^d_{it}} c^d\left(\frac{P_{iit}}{W_{it}}, \frac{\mathcal{E}_{it}P^*_t}{W_{it}}\right)\right) h\left(\frac{P_t}{\mathcal{P}_{iit}}\right) Y_{iit} - \Omega\left(\frac{P_t}{P_{t-1}}\right) W_{it}\right],$$

where the demand shifter Y_{iit} combines the demand from consumers, domestic producers, and exporters, $Y_{iit} \equiv C_{iit} + X^d_{iit} + X^e_{iit}$. Also, the production subsidy corrects for the time-invariant markup, $\frac{h'(1)}{1+h'(1)}\tau_i = 1$. Together with equilibrium relationships, this price-setting condition can be rewritten as

$$\{1\} = \arg\max_{\{p_t\}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t\left[\left(U_{C_{iit}} p_t - \tau_i \frac{-U_{L_{it}}}{A^d_{it}} c^d\left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C^*_{it}}}{-U_{L_{it}}}\right)\right) h(p_t)\, Y_{iit} - \Omega\left(\frac{p_t}{p_{t-1}}\pi_{iit}\right)(-U_{L_{it}})\right]. \tag{A5}$$

The problem of an exporter is

$$\{P^*_{it}\} = \arg\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty} \Theta_{i0,t}\left[\left(\mathcal{E}_{it}P_t - \frac{W_{it}}{A^e_{it}} c^e\left(\frac{P_{iit}}{W_{it}}, \frac{\mathcal{E}_{it}P^*_t}{W_{it}}\right)\right) h\left(\frac{P_t}{\mathcal{P}^*_t}\right) Y^*_t - \Omega^*\left(\frac{P_t}{P_{t-1}}\right) W_{it}\right],$$

where the foreign demand shifter is given by $Y^*_t \equiv \int\left(C^*_{jt} + X^{d*}_{jt} + X^{e*}_{jt}\right)\mathrm{d}j$, and we assume that there is no production subsidy, $\tau^*_i = 1$. Similarly, this condition can be rewritten as

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t\left[\left(U_{C^*_{it}} S_t - \frac{-U_{L_{it}}}{A^e_{it}} c^e\left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C^*_{it}}}{-U_{L_{it}}}\right)\right) h\left(S_t \frac{P^*_t}{\mathcal{P}^*_t}\right) Y^*_t - \Omega^*\left(\frac{S_t}{S_{t-1}}\pi^*_t\right)(-U_{L_{it}})\right], \tag{A6}$$

where as before $S_{it} \equiv P^*_{it}/P^*_t$ and $\pi^*_t \equiv P^*_t/P^*_{t-1}$.

The key difference of pricing in the export market compared to the domestic market is that the optimal markup is time-varying. The reason is that the optimal markup depends on the prices of competitors. In the domestic market, all firms are symmetric and thus the relevant relative price, $P_{iit}(\omega)/\mathcal{P}_{iit}$, is always 1. In the export market, only exporters from one country are symmetric, $P^*_{it}(\omega) = P^*_{it}$, but they compete with exporters from all over the world, and thus the relevant relative price, $P^*_{it}/\mathcal{P}^*_t$, is time-varying.

Market clearing Finally, the goods market clearing condition (7) splits into one condition for domestic goods

$$A^d_{it} F\left(L^d_{it}, X^d_{iit}, X^{d*}_{it}\right) = C_{iit} + X^d_{iit} + X^e_{iit},$$

one condition for exported goods

$$A_{it}^{e} G\left(L_{it}^{e}, X_{iit}^{e}, X_{it}^{e*}\right) = h\left(S_{it}\frac{P_t^*}{\mathcal{P}_t^*}\right) Y_t^*,$$

and one condition for labor

$$L_{it} = L_{it}^{d} + L_{it}^{e} + \Omega\left(\pi_{iit}\right) + \Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi_t^*\right).$$

Proof of Proposition 1 The full policy problem is

$$\max_{\left\{C_{iit}, C_{it}^*, L_{it}, X_{iit}^{d}, X_{it}^{d*}, X_{iit}^{e}, X_{it}^{e*}, L_{it}^{d}, L_{it}^{e}, \left\{B_{it+1}^{h}\right\}_h, \pi_{iit}, S_{it}\right\}_t} \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right)$$

$$\text{s.t. } A_{it}^{d} F\left(L_{it}^{d}, X_{iit}^{d}, X_{it}^{d*}\right) = C_{iit} + X_{iit}^{d} + X_{iit}^{e},$$

$$A_{it}^{e} G\left(L_{it}^{e}, X_{iit}^{e}, X_{it}^{e*}\right) = h\left(S_{it}\frac{P_t^*}{\mathcal{P}_t^*}\right) Y_t^*,$$

$$L_{it} = L_{it}^{d} + L_{it}^{e} + \Omega\left(\pi_{iit}\right) + \Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi_t^*\right),$$

$$\frac{-U_{L_{it}}}{U_{C_{iit}}} = \frac{F_{L_{it}^{d}}}{F_{X_{iit}^{d}}}, \quad \frac{U_{C_{it}^*}}{U_{C_{iit}}} = \frac{F_{X_{it}^{d*}}}{F_{X_{iit}^{d}}},$$

$$\frac{-U_{L_{it}}}{U_{C_{iit}}} = \frac{G_{L_{it}^{e}}}{G_{X_{iit}^{e}}}, \quad \frac{U_{C_{it}^*}}{U_{C_{iit}}} = \frac{G_{X_{it}^{e*}}}{G_{X_{iit}^{e}}},$$

$$\sum_{h\in H_t}\mathcal{Q}_t^h B_{it+1}^h - \sum_{h\in H_{t-1}}\left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) B_{it}^h = S_{it} h\left(S_{it}\frac{P_t^*}{\mathcal{P}_t^*}\right) Y_t^* - C_{it}^* - X_{it}^{d*} - X_{it}^{e*} + \psi_{it},$$

$$\beta\mathbb{E}_t \frac{U_{C_{it+1}^*}}{U_{C_{it}^*}}\frac{\mathcal{Q}_{t+1}^h + \mathcal{D}_{t+1}^h}{\mathcal{Q}_t^h} = 1,$$

$$\{1\} = \arg\max_{\{p_t\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(U_{C_{iit}} p_t - \tau_i \frac{-U_{L_{it}}}{A_{it}^{d}} c^{d}\left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C_{it}^*}}{-U_{L_{it}}}\right)\right) h\left(p_t\right)\left(C_{iit} + X_{iit}^{d} + X_{iit}^{e}\right)\right.$$
$$\left. -\Omega\left(\frac{p_t}{p_{t-1}}\pi_{iit}\right)\left(-U_{L_{it}}\right)\right],$$

$$\{S_{it}\} = \arg\max_{\{S_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(U_{C_{it}^*} S_t - \frac{-U_{L_{it}}}{A_{it}^{e}} c^{e}\left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C_{it}^*}}{-U_{L_{it}}}\right)\right) h\left(S_t\frac{P_t^*}{\mathcal{P}_t^*}\right) Y_t^* - \Omega^*\left(\frac{S_t}{S_{t-1}}\pi_t^*\right)\left(-U_{L_{it}}\right)\right].$$

We guess (and verify later) that some of the constraints are not binding. Then the Lagrangian based on the constraints that do bind is

$$\mathcal{L} = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \Big\{ U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right) + \lambda_{it}^d \left[A_{it}^d F\left(L_{it}^d, X_{iit}^d, X_{it}^{d*}\right) - C_{iit} - X_{iit}^d - X_{iit}^e \right]$$
$$+ \lambda_{it}^e \left[A_{it}^e G\left(L_{it}^e, X_{iit}^e, X_{it}^{e*}\right) - h\left(S_{it} \frac{P_t^*}{\mathcal{P}_t^*}\right) Y_t^* \right] + \lambda_{it}^l \left[L_{it} - L_{it}^d - L_{it}^e - \Omega\left(\pi_{iit}\right) - \Omega^*\left(\frac{S_{it}}{S_{it-1}} \pi_t^*\right) \right]$$
$$+ \mu_{it} \left[\sum_{h \in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h \in H_{t-1}} \left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) B_{it}^h - S_{it} h\left(S_{it} \frac{P_t^*}{\mathcal{P}_t^*}\right) Y_t^* + C_{it}^* + X_{it}^{d*} + X_{it}^{e*} \right] \Big\}.$$

The corresponding optimality conditions are:

- wrt C_{iit}:
$$0 = U_{C_{iit}} - \lambda_{it}^d,$$
- wrt C_{it}^*:
$$0 = U_{C_{it}^*} + \mu_{it},$$
- wrt L_{it}:
$$0 = U_{L_{it}} + \lambda_{it}^l,$$
- wrt X_{iit}^d:
$$0 = \lambda_{it}^d \left(A_{it}^d F_{X_{iit}^d} - 1 \right),$$
- wrt X_{it}^{d*}:
$$0 = \lambda_{it}^d A_{it}^d F_{X_{it}^{d*}} + \mu_{it},$$
- wrt X_{iit}^e:
$$0 = -\lambda_{it}^d + \lambda_{it}^e A_{it}^e G_{X_{iit}^e},$$
- wrt X_{it}^{e*}:
$$0 = \lambda_{it}^e A_{it}^e G_{X_{it}^{e*}} + \mu_{it},$$
- wrt L_{it}^d:
$$0 = \lambda_{it}^d A_{it}^d F_{L_{it}^d} - \lambda_{it}^l,$$
- wrt L_{it}^e:
$$0 = \lambda_{it}^e A_{it}^e G_{L_{it}^e} - \lambda_{it}^l,$$
- wrt B_{it+1}^h:
$$0 = \mu_{it} \mathcal{Q}_t^h - \beta \mathbb{E}_t \mu_{it+1} \left(\mathcal{Q}_{t+1}^h + \mathcal{D}_{t+1}^h \right),$$
- wrt π_{iit}:
$$\{\pi_{iit}\} = \arg\max_{\{\pi\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ -\lambda_{it}^l \Omega(\pi) \right\},$$
- wrt S_{it}:
$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ -\lambda_{it}^e h\left(S_t \frac{P_t^*}{\mathcal{P}_t^*}\right) Y_t^* - \lambda_{it}^l \Omega^*\left(\frac{S_t}{S_{t-1}} \pi_t^*\right) - \mu_{it} S_t h\left(S_t \frac{P_t^*}{\mathcal{P}_t^*}\right) Y_t^* \right\}.$$

Note that the last two conditions are formulated in terms of argmax because we do not impose differentiability on functions Ω and Ω^*.

Use the first three FOCs to subsitute for λ_{it}^d, μ_{it}, and λ_{it}^l. Then the FOC wrt L_{it}^d implies

$$\frac{U_{C_{iit}}}{-U_{L_{it}}} = \frac{1}{A_{it}^d F_{L_{it}^d}}.$$

This condition ensures that prices of domestic sellers are constant, $\pi_{iit} = 1$. Indeed, the price-setting condition (A5) implies that domestic firms without any price-adjustment costs would choose their prices according to

$$0 = U_{C_{iit}} h\left(p_t\right) + U_{C_{iit}} p_t h'\left(p_t\right) - h'\left(p_t\right) \tau_i \frac{-U_{L_{it}}}{A_{it}^d} c^d,$$

and in equilibrium with $p_t = 1$ and $\tau_i = h(1)/h'(1) + 1$, it collapses to

$$\frac{U_{C_{iit}}}{-U_{L_{it}}} = \frac{c^d}{A_{it}^d}.$$

Use the expression for the marginal cost (A4) to see the equivalence between the two conditions. Thus, the optimal policy implies $\pi_{iit} = 1$, which is consistent with the optimality condition wrt π_{iit} and with the private agents' price-setting condition (A5).

Now we verify our guesses. The FOC wrt B_{it+1}^h is equivalent to the no-arbitrage condition (4), once we plug in the value of μ_{it}. Thus, we have verified that this constraint indeed does not bind. The FOC wrt X_{iit}^d implies $F_{X_{iit}^d} = 1/A_{it}^d$. Then the FOC wrt X_{it}^{d*} can be rewritten as

$$\frac{U_{C_{iit}}}{U_{C_{it}^*}} = \frac{1}{A_{it}^d F_{X_{it}^{d*}}} = \frac{F_{X_{iit}^d}}{F_{X_{it}^{d*}}},$$

which verifies that one of the firms' optimality conditions does not bind. Similarly, the FOC wrt L_{it}^d leads to the other optimality condition for domestic producers.

The FOCs wrt X_{iit}^e and X_{it}^{e*} can be combined to show

$$\frac{U_{C_{iit}}}{U_{C_{it}^*}} = \frac{G_{X_{iit}^e}}{G_{X_{it}^{e*}}},$$

and FOCs wrt X_{iit}^e and L_{it}^e lead to

$$\frac{U_{C_{iit}}}{-U_{L_{it}}} = \frac{G_{X_{iit}^e}}{G_{L_{it}^e}}.$$

Thus, all firms' optimality conditions do not bind in the optimal policy problem.

Next, we can rewrite the optimality condition wrt S_{it} as

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \left(U_{C_{it}^*} S_t - \frac{-U_{L_{it}}}{A_{it}^e G_{L_{it}^e}} \right) h\left(S_t \frac{P_t^*}{\mathcal{P}_t^*} \right) Y_t^* - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi_t^* \right) \left(-U_{L_{it}}\right) \right\},$$

and recall the marginal cost condition (A4) to arrive at

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \left(U_{C_{it}^*} S_t - \frac{-U_{L_{it}}}{A_{it}^e} c^e \left(\frac{U_{C_{iit}}}{-U_{L_{it}}}, \frac{U_{C_{it}^*}}{-U_{L_{it}}} \right) \right) h\left(S_t \frac{P_t^*}{\mathcal{P}_t^*} \right) Y_t^* - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi_t^* \right) \left(-U_{L_{it}}\right) \right\}.$$

This condition is identical to the private sector price-setting condition (A6), and thus this condition also does not bind in the optimal policy problem.

We have shown that there exists a set of values of Lagrange multipliers such that all optimality conditions are satisfied under our policy, $\pi_{iit} = 1$. Since this policy is feasible, that is all constraints of the policy problem are satisfied, this policy is optimal.

Finally, we show that this policy is time consistent. To see this, note that the private agents' expectations enter the policy problem only through the no-arbitrage condition (4) and and the two price-setting conditions (A5) and (A6). We have shown that all of these constraints do not bind under the optimal policy. Thus, the policymaker under commitment does not use policy to influence private agents' expectations. Moreover, the optimal policy stays the same regardless of how (and whether) the policy can affect these expectations. Therefore, the optimal policy under commitment coincides with the optimal policy under discretion, and thus it is time consistent.

Proof of Proposition 2 Augment the policy problem in the previous section with a set of state-contingent taxes $\left\{\tau_{it}^{h}\right\}$ that enter the no-arbitrage condition (4). The solution to the problem stays the same since the no-arbitrage condition (4) was not binding even in the absence of these instruments. Moreover, after substituting out the equilibrium value of the Lagrange multiplier μ_{it}, the FOC wrt B_{it+1}^{h} coincides with the no-arbitrage condition (4). This implies that the optimal allocation can be decentralized with zero taxes, $\tau_{it}^{h} = 0$.

C.2 Calvo pricing

Equilibrium conditions Under Calvo friction, there is a price dispersion, which affects all aggregate quantities. In particular, the market clearing condition (7) becomes

$$A_{it}L_{it} = \Delta_{iit}C_{iit} + \Delta_{it}^{*}h\left(S_{it}\right)C_{t}^{*},$$

$$\text{where } \Delta_{iit} \equiv \int h\left(\frac{P_{iit}\left(\omega\right)}{P_{iit}}\right)\mathrm{d}\omega, \text{ and } \Delta_{it}^{*} \equiv \int h\left(\frac{P_{it}^{*}\left(\omega\right)}{P_{it}^{*}}\right)\mathrm{d}\omega.$$

Then each price index has a non-trivial dynamics, that is

$$P_{iit}^{1-\varepsilon} = \lambda P_{iit-1}^{1-\varepsilon} + (1-\lambda)\,\tilde{P}_{iit}^{1-\varepsilon}, \quad P_{it}^{*1-\varepsilon} = \lambda P_{it-1}^{*1-\varepsilon} + (1-\lambda)\,\tilde{P}_{it}^{*1-\varepsilon}, \tag{A7}$$

where a share $1-\lambda$ of firms can adjust their prices, $\tilde{P}_{iit}$ and $\tilde{P}_{it}^{*}$ the prices chosen by the firms that do adjust. And solving for the dynamics of price dispersion yields

$$\Delta_{iit} = \lambda\Delta_{iit-1}\pi_{iit}^{\varepsilon} + (1-\lambda)\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\pi_{iit}^{\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}},$$

$$\Delta_{it}^{*} = \lambda\Delta_{it-1}^{*}\pi_{it}^{*\varepsilon} + (1-\lambda)\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\pi_{it}^{*\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}}. \tag{A8}$$

The problem of an exporter can be written as

$$\tilde{P}_{it}^{*} = \arg\max_{P_t} \mathbb{E}\sum_{k=0}^{\infty}\Theta_{it,t+k}\lambda^{k}\left(\mathcal{E}_{it+k}P_{t} - \frac{W_{it+k}}{A_{it+k}}\right)h\left(\frac{P_{t}}{P_{t+k}^{*}}\right)C_{t+k}^{*}.$$

Then the price-setting condition is just the first-order condition of this problem,

$$\mathbb{E}\sum_{k=0}^{\infty}\Theta_{it,t+k}\lambda^{k}\left(\mathcal{E}_{it+k}\tilde{P}_{it}^{*} - \frac{\varepsilon}{\varepsilon-1}\frac{W_{it+k}}{A_{it+k}}\right)h\left(\frac{\tilde{P}_{it}^{*}}{P_{t+k}^{*}}\right)C_{t+k}^{*} = 0. \tag{A9}$$

The domestic price-setting condition is similar, but the domestic subsidy eliminates the monopolistic competition distortion, $\frac{\varepsilon\tau_i}{\varepsilon-1}=1$,

$$\mathbb{E}\sum_{k=0}^{\infty}\Theta_{it,t+k}\lambda^k\left(\tilde{P}_{iit}-\frac{W_{it+k}}{A_{it+k}}\right)h\left(\frac{\tilde{P}_{iit}}{P_{iit+k}}\right)C_{iit+k}=0. \tag{A10}$$

Next, rewrite condition (A9) recursively. First, rewrite it as

$$\tilde{P}^*_{it}F_t=\mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\lambda^k\frac{\varepsilon}{\varepsilon-1}\frac{W_{it+k}}{A_{it+k}}\frac{U_{C_{iit+k}}}{P_{iit+k}}P^{*\varepsilon}_{t+k}C^*_{t+k},$$

$$\text{where } F_t\equiv\mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\lambda^k\frac{U_{C_{iit+k}}}{P_{iit+k}}\mathcal{E}_{it+k}P^{*\varepsilon}_{t+k}C^*_{t+k}.$$

Then, separate the first term from the rest of the sum on the right hand side,

$$\tilde{P}^*_{it}F_t=\frac{\varepsilon}{\varepsilon-1}\frac{W_{it}}{A_{it}}\frac{U_{C_{iit}}}{P_{iit}}P^{*\varepsilon}_tC^*_t+\beta\lambda\mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\lambda^k\frac{\varepsilon}{\varepsilon-1}\frac{W_{it+1+k}}{A_{it+1+k}}\frac{U_{C_{iit+1+k}}}{P_{iit+1+k}}P^{*\varepsilon}_{t+1+k}C^*_{t+1+k}.$$

Use the law of iterated expectations and an iterated version of the same equation to rewrite this as

$$\tilde{P}^*_{it}F_t=\frac{\varepsilon}{\varepsilon-1}\frac{W_{it}}{A_{it}}\frac{U_{C_{iit}}}{P_{iit}}P^{*\varepsilon}_tC^*_t+\beta\lambda\mathbb{E}_t\tilde{P}^*_{it+1}F_{t+1}.$$

And note that the definition of F_t could also be written recursively as

$$F_t=\frac{U_{C_{iit}}}{P_{iit}}\mathcal{E}_{it}P^{*\varepsilon}_tC^*_t+\beta\lambda\mathbb{E}_tF_{t+1}.$$

Finally, use the households' optimality conditions (1) – (2) to further rewrite it as

$$\tilde{P}^*_{it}F_t=\frac{\varepsilon}{\varepsilon-1}\frac{-U_{L_{it}}}{A_{it}}P^{*\varepsilon}_tC^*_t+\beta\lambda\mathbb{E}_t\tilde{P}^*_{it+1}F_{t+1}, \tag{A11}$$

$$F_t=\frac{U_{C^*_{it}}}{P^*_t}P^{*\varepsilon}_tC^*_t+\beta\lambda\mathbb{E}_tF_{t+1}. \tag{A12}$$

Recursive equations (A11) and (A12) are equivalent to the single price-setting condition (A9). Similar expressions for the domestic price-setting condition (A10) are

$$\tilde{P}_{iit}G_t=\frac{-U_{L_{it}}}{A_{it}}P^{\varepsilon}_{iit}C_{iit}+\beta\lambda\mathbb{E}_t\tilde{P}_{iit+1}G_{t+1}, \tag{A13}$$

$$G_t=\frac{U_{C_{iit}}}{P_{iit}}P^{\varepsilon}_{iit}C_{iit}+\beta\lambda\mathbb{E}_tG_{t+1}. \tag{A14}$$

Policy problem and optimality conditions The full policy problem can be written as

$$\max_{\left\{C_{iit},C^*_{it},L_{it},\left\{B^h_{it+1}\right\}_h,\Delta_{iit},\Delta^*_{it},P_{iit},P^*_{it},\tilde{P}_{iit},\tilde{P}^*_{it}\right\}_t}\mathbb{E}\sum_{t=0}^{\infty}\beta^tU\left(C_{iit},C^*_{it},L_{it},\xi_{it}\right)$$

$$\text{s.t. } A_{it}L_{it}=\Delta_{iit}C_{iit}+\Delta^*_{it}h\left(\frac{P^*_{it}}{P^*_t}\right)C^*_t,$$

$$\sum_{h\in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h\in H_{t-1}} \left(\mathcal{Q}_t^h + D_t^h\right) B_{it}^h = \frac{P_{it}^*}{P_t^*} h\left(\frac{P_{it}^*}{P_t^*}\right) C_t^* - C_{it}^* + \psi_{it},$$

$$\beta \mathbb{E}_t \frac{U_{C_{it+1}^*}}{U_{C_{it}^*}} \frac{\mathcal{Q}_{t+1}^h + \mathcal{D}_{t+1}^h}{\mathcal{Q}_t^h} = 1,$$

$$P_{iit}^{1-\varepsilon} = \lambda P_{iit-1}^{1-\varepsilon} + (1-\lambda)\tilde{P}_{iit}^{1-\varepsilon}, \quad P_{it}^{*1-\varepsilon} = \lambda P_{it-1}^{*1-\varepsilon} + (1-\lambda)\tilde{P}_{it}^{*1-\varepsilon},$$

$$\Delta_{iit} = \lambda \Delta_{iit-1}\left(\frac{P_{iit}}{P_{iit-1}}\right)^{\varepsilon} + (1-\lambda)\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P_{iit}}{P_{iit-1}}\right)^{\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}},$$

$$\Delta_{it}^* = \lambda \Delta_{it-1}^*\left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{\varepsilon} + (1-\lambda)\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{\varepsilon-1}\right)^{\frac{-\varepsilon}{1-\varepsilon}}.$$

$$\mathbb{E}\sum_{k=0}^{\infty} \beta^k \lambda^k \left(U_{C_{iit+k}} \frac{\tilde{P}_{iit}}{P_{iit+k}} - \frac{-U_{L_{it+k}}}{A_{it+k}}\right)\left(\frac{\tilde{P}_{iit}}{P_{iit+k}}\right)^{-\varepsilon} C_{iit+k} = 0, \tag{A15}$$

$$\mathbb{E}\sum_{k=0}^{\infty} \beta^k \lambda^k \left(U_{C_{it+k}^*} \frac{\tilde{P}_{it}^*}{P_{t+k}^*} - \frac{\varepsilon}{\varepsilon-1}\frac{-U_{L_{it+k}}}{A_{it+k}}\right)\left(\frac{\tilde{P}_{it}^*}{P_{t+k}^*}\right)^{-\varepsilon} C_{t+k}^* = 0. \tag{A16}$$

where we used households' optimality conditions (1) – (2) to rewrite the price-setting conditions (A10) and (A9).

As before, we guess (and verify later) that the no-arbitrage condition (4), as well as both price-setting conditions do not bind. Then the price index constraints (A7) are the only ones that contain $\tilde{P}_{iit}$ and $\tilde{P}_{it}^*$, and therefore they can be dropped from this problem. Then the optimality conditions for the relaxed problem are:

- wrt C_{iit}:
$$0 = U_{C_{iit}} - \lambda_{it}\Delta_{iit},$$
- wrt C_{it}^*:
$$0 = U_{C_{it}^*} + \mu_{it},$$
- wrt L_{it}:
$$0 = U_{L_{it}} + \lambda_{it} A_{it},$$
- wrt B_{it+1}^h:
$$0 = \mu_{it}\mathcal{Q}_t^h - \beta\mathbb{E}_t \mu_{it+1}\left(\mathcal{Q}_{t+1}^h + D_{t+1}^h\right),$$
- wrt Δ_{iit}:
$$0 = -\lambda_{it} C_{iit} + \chi_{it} - \beta\mathbb{E}_t \chi_{it+1}\lambda\left(\frac{P_{iit+1}}{P_{iit}}\right)^{\varepsilon},$$
- wrt Δ_{it}^*:
$$0 = -\lambda_{it} h\left(\frac{P_{it}^*}{P_t^*}\right) C_t^* + \chi_{it}^* - \beta\mathbb{E}_t \chi_{it+1}^*\lambda\left(\frac{P_{it+1}^*}{P_{it}^*}\right)^{\varepsilon},$$
- wrt P_{iit}:

$$
\begin{aligned}
0 = & -\chi_{it}\lambda\Delta_{iit-1}\varepsilon P_{iit}^{-1}\left(\frac{P_{iit}}{P_{iit-1}}\right)^{\varepsilon} + \beta\mathbb{E}_t\chi_{it+1}\lambda\Delta_{iit}\varepsilon P_{iit}^{-1}\left(\frac{P_{iit+1}}{P_{iit}}\right)^{\varepsilon} \\
& + \chi_{it}\lambda\varepsilon\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P_{iit}}{P_{iit-1}}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}}\left(\frac{P_{iit}}{P_{iit-1}}\right)^{\varepsilon-2}\frac{1}{P_{iit-1}} \\
& - \beta\mathbb{E}_t\chi_{it+1}\varepsilon\lambda\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P_{iit+1}}{P_{iit}}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}}\left(\frac{P_{iit+1}}{P_{iit}}\right)^{\varepsilon-1}\frac{1}{P_{iit}},
\end{aligned}
$$

- wrt P_{it}^*:

$$
\begin{aligned}
0 = & \lambda_{it}\Delta_{it}^*\varepsilon\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon-1}\frac{1}{P_t^*}C_t^* - \mu_{it}\left(1-\varepsilon\right)\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\frac{1}{P_t^*}C_t^* \\
& - \chi_{it}^*\lambda\Delta_{it-1}^*\varepsilon\left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{\varepsilon-1}\frac{1}{P_{it-1}^*} + \beta\mathbb{E}_t\chi_{it+1}^*\lambda\Delta_{it}^*\varepsilon\left(\frac{P_{it+1}^*}{P_{it}^*}\right)^{\varepsilon}\frac{1}{P_{it}^*} \\
& + \chi_{it}^*\varepsilon\lambda\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}}\left(\frac{P_{it}^*}{P_{it-1}^*}\right)^{\varepsilon-2}\frac{1}{P_{it-1}^*} \\
& - \beta\mathbb{E}_t\chi_{it+1}^*\lambda\varepsilon\left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P_{it+1}^*}{P_{it}^*}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}}\left(\frac{P_{it+1}^*}{P_{it}^*}\right)^{\varepsilon-1}\frac{1}{P_{it}^*}.
\end{aligned}
$$

The first four FOCs imply

$$U_{C_{iit}} = \Delta_{iit}\frac{-U_{L_{it}}}{A_{it}},$$

and verify that the no-arbitrage condition (4) does not bind.

Next we guess that $\Delta_{iit} = 1$. Then the domestic-price setting condition (A15) can be satisfied with $\tilde{P}_{iit} = P_{iit+k}$ for any k. Together with the price index constraint (A7), they imply constant domestic prices $P_{iit} = P_{iit-1}$. Then the recursive equations of the domestic price-setting condition (A13) and (A14) become equivalent to each other and collapse to the single equation

$$G_t P_{iit}^{1-\varepsilon} = \frac{-U_{L_{it}}}{A_{it}}C_{iit} + \beta\lambda\mathbb{E}_t G_{t+1}P_{iit}^{1-\varepsilon}.$$

The FOC wrt P_{iit} is trivially satisfied, while the FOC wrt Δ_{iit} reduces to

$$\chi_{it} = \frac{-U_{L_{it}}}{A_{it}}C_{iit} + \beta\lambda\mathbb{E}_t\chi_{it+1}.$$

This is equivalent to the recursive form of the domestic price-setting condition once we set $\chi_{it} = G_t P_{iit}^{1-\varepsilon}$. Thus, this FOC is also satisfied under our guess.

Finally, we rewrite the FOC wrt Δ_{it}^* as

$$\chi_{it}^* P_{it}^{*\varepsilon} = \frac{-U_{L_{it}}}{A_{it}}P_t^{*\varepsilon}C_t^* + \beta\lambda\mathbb{E}_t\chi_{it+1}^*P_{it+1}^{*\varepsilon},$$

and the FOC wrt P^*_{it} as

$$
\begin{aligned}
0 = {} & \varepsilon \frac{-U_{L_{it}}}{A_{it}} \frac{\Delta^*_{it}}{P^*_t} \left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon-1} C^*_t + (1-\varepsilon) \frac{U_{C^*_{it}}}{P^*_t} \left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon} C^*_t \\
& - \chi^*_{it} \lambda\varepsilon \left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-1} \frac{1}{P^*_{it-1}} \left[\Delta^*_{it-1} - \left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}} \left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{-1}\right] \\
& + \beta \mathbb{E}_t \chi^*_{it+1} \lambda\varepsilon \left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{\varepsilon} \frac{1}{P^*_{it}} \left[\Delta^*_{it} - \left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}} \left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{-1}\right].
\end{aligned}
$$

Let's plug in the expression for $-U_{L_{it}}/A_{it}$ from the first FOC to the second,

$$
\begin{aligned}
0 = {} & \frac{1-\varepsilon}{\varepsilon} \frac{U_{C^*_{it}}}{P^*_t} \left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon} C^*_t \\
& - \chi^*_{it} \lambda \left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-1} \frac{1}{P^*_{it-1}} \left[-\frac{1}{\lambda}\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{-\varepsilon} \Delta^*_{it} + \Delta^*_{it-1} - \left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}} \left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{-1}\right] \\
& - \beta \mathbb{E}_t \chi^*_{it+1} \lambda \left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{\varepsilon} \frac{1}{P^*_{it}} \left(\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{\varepsilon-1}\right)^{\frac{-1}{1-\varepsilon}} \left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{-1}.
\end{aligned}
$$

Note that the price index constraint (A7) implies

$$
\frac{1}{1-\lambda} - \frac{\lambda}{1-\lambda}\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-1} = \left(\frac{\tilde{P}^*_{it}}{P^*_{it}}\right)^{1-\varepsilon},
$$

so that this FOC becomes

$$
\begin{aligned}
0 = {} & \frac{1-\varepsilon}{\varepsilon} \frac{U_{C^*_{it}}}{P^*_t} \left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon} C^*_t - \beta \mathbb{E}_t \chi^*_{it+1} \lambda \left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{\varepsilon} \frac{1}{\tilde{P}^*_{it+1}}. \\
& - \chi^*_{it} \lambda \left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-1} \frac{1}{P^*_{it-1}} \left[-\frac{1}{\lambda}\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{-\varepsilon} \Delta^*_{it} + \Delta^*_{it-1} - \frac{P^*_{it-1}}{\tilde{P}^*_{it}}\right]
\end{aligned}
$$

Use the same price index constraint to rewrite the constraint (A8) as

$$
-\frac{1}{\lambda}\left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{-\varepsilon} \Delta^*_{it} + \Delta^*_{it-1} = -\frac{1-\lambda}{\lambda}\left(\frac{\tilde{P}^*_{it}}{P^*_{it-1}}\right)^{-\varepsilon}
$$

and plug it in to get

$$
\begin{aligned}
0 = {} & \frac{1-\varepsilon}{\varepsilon} \frac{U_{C^*_{it}}}{P^*_t} \left(\frac{P^*_{it}}{P^*_t}\right)^{-\varepsilon} C^*_t - \beta \mathbb{E}_t \chi^*_{it+1} \lambda \left(\frac{P^*_{it+1}}{P^*_{it}}\right)^{\varepsilon} \frac{1}{\tilde{P}^*_{it+1}}. \\
& + \chi^*_{it} \lambda \left(\frac{P^*_{it}}{P^*_{it-1}}\right)^{\varepsilon-1} \frac{1}{P^*_{it-1}} \left[\frac{1-\lambda}{\lambda}\left(\frac{\tilde{P}^*_{it}}{P^*_{it-1}}\right)^{1-\varepsilon} + 1\right] \frac{P^*_{it-1}}{\tilde{P}^*_{it}}
\end{aligned}
$$

Again, use the price index constraint (A7) and rearrange to get

$$\frac{\varepsilon}{\varepsilon-1}\frac{\chi_{it}^{*}P_{it}^{*\varepsilon}}{\tilde{P}_{it}^{*}}=\frac{U_{C_{it}^{*}}}{P_{t}^{*}}P_{t}^{*\varepsilon}C_{t}^{*}+\beta\lambda\mathbb{E}_{t}\frac{\varepsilon}{\varepsilon-1}\frac{\chi_{it+1}^{*}P_{it+1}^{*\varepsilon}}{\tilde{P}_{it+1}^{*}}.$$

If $\chi_{it}^{*}=F_{t}\tilde{P}_{it}^{*}P_{it}^{*-\varepsilon}\left(\varepsilon-1\right)/\varepsilon$, this FOC becomes equivalent to the private sector condition (A12), while the FOC wrt Δ_{it}^{*} becomes equivalent to (A11). As we have shown earlier, these two conditions together are equivalent to the export price-setting condition (A16), and thus we have verified that this condition does not bind.

To conclude, we have shown that there exists a set of Lagrange multipliers such that the system of first-order conditions is satisfied under the optimal policy of $P_{iit}=P_{iit-1}$.

C.3 Sticky wages

Let's assume that each household ω choose their own wage $W_{it}\left(\omega\right)$ to provide a unique variety of labor $L_{it}\left(\omega\right)$. Firms combine different varieties according to the CES technology

$$L_{it}=\left(\int L_{it}\left(\omega\right)^{\frac{\epsilon-1}{\epsilon}}\mathrm{d}\omega\right)^{\frac{\epsilon}{\epsilon-1}},$$

so that their demand for labor is

$$L_{it}\left(\omega\right)=\left(\frac{W_{it}\left(\omega\right)}{W_{it}}\right)^{-\epsilon}L_{it},\quad W_{it}=\left(\int W_{it}\left(\omega\right)^{1-\epsilon}\mathrm{d}\omega\right)^{\frac{1}{1-\epsilon}}.$$

Taking this demand function as given, households set their wages subject to the price-adjustment costs

$$\max_{\left\{C_{iit},C_{it}^{*},L_{t},W_{t},\mathcal{B}_{it+1}^{i},\left\{B_{it+1}^{h}\right\}_{h}\right\}_{t}}\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}U\left(C_{iit},C_{it}^{*},L_{t},\xi_{it}\right)$$

$$\begin{aligned}\text{s.t. }P_{iit}C_{iit}+\mathcal{E}_{it}P_{t}^{*}C_{it}^{*}&=\mathcal{E}_{it}P_{t}^{*}\left[\sum_{h}\left(\mathcal{Q}_{t}^{h}+\mathcal{D}_{t}^{h}\right)B_{it}^{h}-\sum_{h}\mathcal{Q}_{t}^{h}B_{it+1}^{h}\right]+\mathcal{B}_{it}^{i}-\frac{\mathcal{B}_{it+1}^{i}}{R_{it}}\\&\quad+\tau_{i}^{w}W_{t}L_{t}+\Pi_{it}^{f}+T_{it}-\Omega\left(\frac{W_{t}}{W_{t-1}}\right)W_{it},\\L_{t}&=\left(\frac{W_{t}}{W_{it}}\right)^{-\epsilon}L_{it}.\end{aligned}$$

Similarly to the production subsidy, we impose a constant labor tax τ_{i}^{w} to correct for the monopolistic competition distortion. This problem leads to the following wage-setting condition

$$\left\{W_{it}\right\}=\underset{\left\{W_{t}\right\}}{\arg\max}\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left\{U\left(C_{iit},C_{it}^{*},\left(\frac{W_{t}}{W_{it}}\right)^{-\epsilon}L_{it},\xi_{it}\right)+\tau_{i}^{w}\frac{U_{C_{iit}}}{P_{iit}}W_{t}\left(\frac{W_{t}}{W_{it}}\right)^{-\epsilon}L_{it}-\frac{U_{C_{iit}}}{P_{iit}}\Omega\left(\frac{W_{t}}{W_{t-1}}\right)W_{it}\right\}.$$

Domestic sellers can set their prices flexibly, and thus $P_{iit}=W_{it}/A_{it}$. We can use this expression to substitute for P_{iit} and arrive at

$$\left\{1\right\}=\underset{\left\{w_{t}\right\}}{\arg\max}\,\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left\{U\left(C_{iit},C_{it}^{*},w_{t}^{-\epsilon}L_{it},\xi_{it}\right)+U_{C_{iit}}A_{it}\left[\tau_{i}^{w}w_{t}^{1-\epsilon}L_{it}-\Omega\left(\frac{w_{t}}{w_{t-1}}\pi_{wit}\right)\right]\right\},\qquad\text{(A17)}$$

where $\pi_{wit} \equiv W_{it}/W_{it-1}$.

Then, we can set up the optimal policy problem as

$$\max_{\left\{C_{iit}, C_{it}^*, L_{it}, \left\{B_{it+1}^h\right\}_h, \pi_{wit}, S_{it}\right\}_t} \mathbb{E}\sum_{t=0}^{\infty} \beta^t U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right)$$

$$\text{s.t. } A_{it}L_{it} = C_{iit} + h\left(S_{it}\right)C_t^* + A_{it}\left[\Omega\left(\pi_{wit}\right) + \Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi_t^*\right)\right],$$

$$\sum_{h\in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h\in H_{t-1}} \left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) B_{it}^h = S_{it}h\left(S_{it}\right)C_t^* - C_{it}^* + \psi_{it},$$

$$\beta\mathbb{E}_t \frac{U_{C_{it+1}^*}}{U_{C_{it}^*}} \frac{\mathcal{Q}_{t+1}^h + \mathcal{D}_{t+1}^h}{\mathcal{Q}_t^h} = 1,$$

$$\{1\} = \arg\max_{\{w_t\}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t \left\{U\left(C_{iit}, C_{it}^*, w_t^{-\epsilon}L_{it}, \xi_{it}\right) + U_{C_{iit}}A_{it}\left[\tau_i^w w_t^{1-\epsilon}L_{it} - \Omega\left(\frac{w_t}{w_{t-1}}\pi_{wit}\right)\right]\right\},$$

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{it}^*}S_t - U_{C_{iit}}\right)h\left(S_t\right)C_t^* - U_{C_{iit}}A_{it}\Omega^*\left(\frac{S_t}{S_{t-1}}\pi_t^*\right)\right].$$

As before, we guess (and verify) that some of the constraints do not bind, and formulate the Lagrangian as

$$\mathcal{L} = \mathbb{E}\sum_{t=0}^{\infty} \beta^t \left\{U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right) + \lambda_{it}\left[A_{it}L_{it} - C_{iit} - h\left(S_{it}\right)C_t^* - A_{it}\left[\Omega\left(\pi_{wit}\right) + \Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi_t^*\right)\right]\right]\right.$$
$$\left.+\mu_{it}\left[\sum_{h\in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h\in H_{t-1}} \left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) B_{it}^h - S_{it}h\left(S_{it}\right)C_t^* + C_{it}^* - \psi_{it}\right]\right\}.$$

Then the optimality conditions are:

- wrt C_{iit}:
$$0 = U_{C_{iit}} - \lambda_{it},$$
- wrt C_{it}^*:
$$0 = U_{C_{it}^*} + \mu_{it},$$
- wrt L_{it}:
$$0 = U_{L_{it}} + \lambda_{it}A_{it},$$
- wrt B_{it+1}^h:
$$0 = \mu_{it}\mathcal{Q}_t^h - \beta\mathbb{E}_t\mu_{it+1}\left(\mathcal{Q}_{t+1}^h + \mathcal{D}_{t+1}^h\right),$$
- wrt π_{wit}:
$$\{\pi_{wit}\} = \arg\max_{\{\pi\}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t \left\{-\lambda_{it}A_{it}\Omega\left(\pi\right)\right\},$$
- wrt S_{it}:
$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t \left\{-\lambda_{it}h\left(S_t\right)C_t^* - \lambda_{it}A_{it}\Omega^*\left(\frac{S_t}{S_{t-1}}\pi_t^*\right) - \mu_{it}S_th\left(S_t\right)C_t^*\right\}.$$

We use the first two FOCs to substitute for λ_{it} and μ_{it}. Then the FOC wrt L_{it} becomes $U_{C_{iit}}A_{it} = -U_{L_{it}}$. This condition together with the wage-setting condition (A17) implies constant wages, $\pi_{wit} = 1$. To see this, note that without wage-adjustment costs, the wage-setting condition (A17) becomes

$$w_t = \frac{\epsilon}{\epsilon-1}\frac{1}{\tau_i^w}\frac{-U_{L_{it}}}{U_{C_{iit}}A_{it}},$$

so that under $\tau_i^w = \frac{\epsilon}{\epsilon-1}$ the wage-setting condition does not bind. The FOC wrt B^h_{it+1} leads to the no-arbitrage condition (4). Finally, the optimality condition wrt S_{it} can be rewritten as

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left\{\left(U_{C^*_{it}}S_t - U_{C_{iit}}\right)h\left(S_t\right)C^*_t - U_{C_{iit}}A_{it}\Omega^*\left(\frac{S_t}{S_{t-1}}\pi^*_t\right)\right\},$$

which is exactly the same as the exporters' price-setting condition, and thus this constraint also does not bind.

C.4 Fraction of exporters with flexible prices

Let's assume that share $0 < \alpha < 1$ of exporters set their prices in producer currency (PCP), while the rest $1-\alpha$ set their prices in dollars as before. The problem of PCP-exporters can be written as

$$\{P^*_{iit}\} = \arg\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\frac{U_{C_{iit}}}{P_{iit}}\left[\left(P_t - \frac{W_{it}}{A_{it}}\right)h\left(\frac{P_t}{\mathcal{E}_{it}P^*_t}\right)C^*_t - \Omega^*\left(\frac{P_t}{P_{t-1}}\right)W_{it}\right],$$

which together with other equilibrium conditions leads to

$$\{S_{iit}\} = \arg\max_{\{S_t\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(U_{C^*_{it}}S_t - \frac{-U_{L_{it}}}{A_{it}}\right)h\left(S_t\right)C^*_t - \Omega^*\left(\frac{S_t}{S_{t-1}}\frac{U_{C^*_{it}}}{U_{C^*_{it-1}}}\frac{U_{C_{iit-1}}}{U_{C_{iit}}}\pi_{iit}\right)\left(-U_{L_{it}}\right)\right], \quad \text{(A18)}$$

where $S_{iit} \equiv P^*_{iit}/\left(\mathcal{E}_{it}P^*_t\right)$. And the country's budget constraint changes to

$$\sum_{h\in H_t}\mathcal{Q}^h_t B^h_{it+1} - \sum_{h\in H_{t-1}}\left(\mathcal{Q}^h_t + \mathcal{D}^h_t\right)B^h_{it} = (1-\alpha)S_{it}h\left(S_{it}\right)C^*_t + \alpha S_{iit}h\left(S_{iit}\right)C^*_t - C^*_{it} + \psi_{it}.$$

Now we can set up the optimal policy problem,

$$\max_{\left\{C_{iit},C^*_{it},L_{it},\left\{B^h_{it+1}\right\}_h,\pi_{iit},S_{it},S_{iit}\right\}_t} \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(C_{iit},C^*_{it},L_{it},\xi_{it}\right)$$

$$\begin{aligned}\text{s.t. } A_{it}L_{it} = {} & C_{iit} + (1-\alpha)h\left(S_{it}\right)C^*_t + \alpha h\left(S_{iit}\right)C^*_t \\ & + A_{it}\left[\Omega\left(\pi_{iit}\right) + (1-\alpha)\Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi^*_t\right) + \alpha\Omega^*\left(\frac{S_{iit}}{S_{iit-1}}\frac{U_{C^*_{it}}}{U_{C^*_{it-1}}}\frac{U_{C_{iit-1}}}{U_{C_{iit}}}\pi_{iit}\right)\right],\end{aligned}$$

$$\sum_{h\in H_t}\mathcal{Q}^h_t B^h_{it+1} - \sum_{h\in H_{t-1}}\left(\mathcal{Q}^h_t + \mathcal{D}^h_t\right)B^h_{it} = (1-\alpha)S_{it}h\left(S_{it}\right)C^*_t + \alpha S_{iit}h\left(S_{iit}\right)C^*_t - C^*_{it} + \psi_{it},$$

$$\beta\mathbb{E}_t\frac{U_{C^*_{it+1}}}{U_{C^*_{it}}}\frac{\mathcal{Q}^h_{t+1} + \mathcal{D}^h_{t+1}}{\mathcal{Q}^h_t} = 1,$$

$$\{1\} = \arg\max_{\{p_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{iit}} p_t - \tau_i \frac{-U_{L_{it}}}{A_{it}} \right) h(p_t) C_{iit} - \Omega \left(\frac{p_t}{p_{t-1}} \pi_{iit} \right) (-U_{L_{it}}) \right],$$

$$\{S_{iit}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C^*_{it}} S_t - \frac{-U_{L_{it}}}{A_{it}} \right) h(S_t) C^*_t - \Omega^* \left(\frac{S_t}{S_{t-1}} \frac{U_{C^*_{it}}}{U_{C^*_{it-1}}} \frac{U_{C_{iit-1}}}{U_{C_{iit}}} \pi_{iit} \right) (-U_{L_{it}}) \right],$$

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C^*_{it}} S_t - U_{C_{iit}} \right) h(S_t) C^*_t - U_{C_{iit}} A_{it} \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t \right) \right].$$

We guess (and verify later) that some of the constraints do not bind. Similarly, we also guess that in equilibrium, PCP-exporters need not pay the price-adjustment costs, that is

$$\Omega^* \left(\frac{S_{iit}}{S_{iit-1}} \frac{U_{C^*_{it}}}{U_{C^*_{it-1}}} \frac{U_{C_{iit-1}}}{U_{C_{iit}}} \pi_{iit} \right) = 0.$$

Now formulate the Lagrangian as

$$\begin{aligned}
\mathcal{L} = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \Bigg\{ & U(C_{iit}, C^*_{it}, L_{it}, \xi_{it}) + \lambda_{it} \left[A_{it} L_{it} - C_{iit} - (1-\alpha) h(S_{it}) C^*_t - \alpha h(S_{iit}) C^*_t \right] \\
& - \lambda_{it} A_{it} \left[\Omega(\pi_{iit}) + (1-\alpha) \Omega^* \left(\frac{S_{it}}{S_{it-1}} \pi^*_t \right) \right] \\
& + \mu_{it} \left[\sum_{h \in H_t} \mathcal{Q}^h_t B^h_{it+1} - \sum_{h \in H_{t-1}} \left(\mathcal{Q}^h_t + \mathcal{D}^h_t \right) B^h_{it} - (1-\alpha) S_{it} h(S_{it}) C^*_t - \alpha S_{iit} h(S_{iit}) C^*_t + C^*_{it} \right] \Bigg\}
\end{aligned}$$

with the corresponding optimality conditions

- wrt C_{iit}:
$$0 = U_{C_{iit}} - \lambda_{it},$$
- wrt C^*_{it}:
$$0 = U_{C^*_{it}} + \mu_{it},$$
- wrt L_{it}:
$$0 = U_{L_{it}} + \lambda_{it} A_{it},$$
- wrt B^h_{it+1}:
$$0 = \mu_{it} \mathcal{Q}^h_t - \beta \mathbb{E}_t \mu_{it+1} \left(\mathcal{Q}^h_{t+1} + \mathcal{D}^h_{t+1} \right),$$
- wrt π_{iit}:
$$\{\pi_{iit}\} = \arg\max_{\{\pi\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ -\lambda_{it} A_{it} \Omega(\pi) \right\},$$
- wrt S_{it}:
$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t (1-\alpha) \left\{ -\lambda_{it} h(S_t) C^*_t - \lambda_{it} A_{it} \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t \right) - \mu_{it} S_t h(S_t) C^*_t \right\},$$

- wrt S_{iit}:

$$\{S_{iit}\} = \arg\max_{\{S_t\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\alpha\left\{-\lambda_{it}h\left(S_t\right)C_t^* - \mu_{it}S_t h\left(S_t\right)C_t^*\right\}.$$

As before, the first 5 conditions imply that the monetary policy stabilizes domestic prices, $\pi_{iit} = 1$ and $U_{C_{iit}}A_{it} = -U_{L_{it}}$, while the price-setting condition of domestic sellers (5) and the no-arbitrage condition (4) do not bind. The optimality condition wrt S_{it} can be rewritten exactly as the price-setting condition (6).

The optimality condition wrt S_{iit} can be rewritten as

$$\{S_{iit}\} = \arg\max_{\{S_t\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(U_{C_{it}^*}S_t - \frac{-U_{L_{it}}}{A_{it}}\right)h\left(S_t\right)C_t^*\right],$$

which is exactly the same as the private price-setting condition (A18) without price-adjustment costs Ω^*. This condition can be simplified further to

$$S_{iit} = \frac{\varepsilon}{\varepsilon-1}\frac{-U_{L_{it}}}{U_{C_{it}^*}A_{it}} = \frac{\varepsilon}{\varepsilon-1}\frac{U_{C_{iit}}}{U_{C_{it}^*}}\frac{W_{it}}{P_{iit}A_{it}} = \frac{\varepsilon}{\varepsilon-1}\frac{U_{C_{iit}}}{U_{C_{it}^*}},$$

where the second equality follows from the households' optimality condition (1), and the last equality holds becase stable domestic prices $\pi_{iit} = 1$ imply $P_{iit} = W_{it}/A_{it}$. Moreover, this condition implies

$$\frac{S_{iit}}{S_{iit-1}}\frac{U_{C_{it}^*}}{U_{C_{it-1}^*}}\frac{U_{C_{iit-1}}}{U_{C_{iit}}}\pi_{iit} = 1,$$

and thus it is feasible even when price-adjustment costs Ω^* are present since setting optimal export prices for PCP-exporters does not require ever adjusting their prices.

Thus, we have first solved a relaxed planner's problem with fewer constraints, and then we showed that the same allocation can be achieved in the full problem with all of the constraints.

C.5 Sector-specific labor

Let's change preferences to

$$U\left(C_{iit}, C_{it}^*, L_{iit}, L_{it}^*, \xi_{it}\right),$$

where L_{iit} is used by domestic sellers both for production and for the price-adjustment costs, while L_{it}^* is used by exporters. Then the households' optimality conditions (1) – (2) are replaced with

$$U_{C_{iit}} = -U_{L_{iit}}\frac{P_{iit}}{W_{iit}}, \quad U_{C_{it}^*} = -U_{L_{iit}}\frac{\mathcal{E}_{it}P_t^*}{W_{iit}}, \quad \frac{U_{L_{iit}}}{W_{iit}} = \frac{U_{L_{it}^*}}{W_{it}^*}.$$

After some derivations, the optimal policy problem can be formulated as

$$\max_{\left\{C_{iit},C_{it}^*,L_{iit},L_{it}^*,\left\{B_{it+1}^h\right\}_h,\pi_{iit},S_{it}\right\}_t} \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(C_{iit}, C_{it}^*, L_{iit}, L_{it}^*, \xi_{it}\right)$$

$$\text{s.t. } A_{it}L_{iit} = C_{iit} + A_{it}\Omega\left(\pi_{iit}\right),$$

$$A_{it}L_{it}^* = h\left(S_{it}\right)C_t^* + A_{it}\Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi_t^*\right),$$

$$\sum_{h\in H_t}\mathcal{Q}_t^h B_{it+1}^h - \sum_{h\in H_{t-1}}\left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right)B_{it}^h = S_{it}h\left(S_{it}\right)C_t^* - C_{it}^* + \psi_{it},$$

$$\beta\mathbb{E}_t \frac{U_{C^*_{it+1}}}{U_{C^*_{it}}} \frac{\mathcal{Q}^h_{t+1} + \mathcal{D}^h_{t+1}}{\mathcal{Q}^h_t} = 1,$$

$$\{1\} = \arg\max_{\{p_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{iit}} p_t - \tau_i \frac{-U_{L_{it}}}{A_{it}} \right) h(p_t) C_{iit} - \Omega \left(\frac{p_t}{p_{t-1}} \pi_{iit} \right) (-U_{L_{it}}) \right],$$

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C^*_{it}} S_t - \frac{-U_{L^*_{it}}}{A_{it}} \right) h(S_t) C^*_t - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t \right) (-U_{L^*_{it}}) \right].$$

We guess (and verify) that some of the constraints do not bind, and formulate the Lagrangian

$$\begin{aligned}
\mathcal{L} = \mathbb{E} \sum_{t=0}^{\infty} \beta^t \{ & U(C_{iit}, C^*_{it}, L_{iit}, L^*_{it}, \xi_{it}) + \lambda_{iit} [A_{it} L_{it} - C_{iit} - A_{it} \Omega(\pi_{iit})] \\
& + \lambda^*_{it} \left[A_{it} L^*_{it} - h(S_{it}) C^*_t - A_{it} \Omega^* \left(\frac{S_{it}}{S_{it-1}} \pi^*_t \right) \right] \\
& + \mu_{it} \left[\sum_{h \in H_t} \mathcal{Q}^h_t B^h_{it+1} - \sum_{h \in H_{t-1}} \left(\mathcal{Q}^h_t + \mathcal{D}^h_t \right) B^h_{it} - S_{it} h(S_{it}) C^*_t + C^*_{it} - \psi_{it} \right] \}
\end{aligned}$$

with the corresponding optimality conditions

- wrt C_{iit}:
$$0 = U_{C_{iit}} - \lambda_{iit},$$
- wrt C^*_{it}:
$$0 = U_{C^*_{it}} + \mu_{it},$$
- wrt L_{iit}:
$$0 = U_{L_{iit}} + \lambda_{iit} A_{it},$$
- wrt L^*_{it}:
$$0 = U_{L^*_{it}} + \lambda^*_{it} A_{it},$$
- wrt B^h_{it+1}:
$$0 = \mu_{it} \mathcal{Q}^h_t - \beta \mathbb{E}_t \mu_{it+1} \left(\mathcal{Q}^h_{t+1} + \mathcal{D}^h_{t+1} \right),$$
- wrt π_{iit}:
$$\{\pi_{iit}\} = \arg\max_{\{\pi\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \{ -\lambda_{iit} A_{it} \Omega(\pi) \},$$
- wrt S_{it}:
$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ -\lambda^*_{it} h(S_t) C^*_t - \lambda^*_{it} A_{it} \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t \right) - \mu_{it} S_t h(S_t) C^*_t \right\}.$$

As before, the first 6 conditions imply that the monetary policy stabilizes domestic prices, $\pi_{iit} = 1$ and $U_{C_{iit}} A_{it} = -U_{L_{it}}$, while the price-setting condition of domestic sellers (5) and the no-arbitrage condition (4) do not bind. The optimality condition wrt S_{it} can be rewritten as

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \left(U_{C^*_{it}} S_t - \frac{-U_{L^*_{it}}}{A_{it}} \right) h(S_t) C^*_t - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t \right) (-U_{L^*_{it}}) \right\},$$

which is the same as the price-setting condition of exporters. Thus, this constraint does not bind.

C.6 Endogenous currency choice

The problem of a representative exporter is to choose not only the path of export prices, but also the currency, in which the prices are set:

$$\max_{\{P_t^k\},k} \mathbb{E}\sum_{t=0}^{\infty} \Theta_{i0,t}\left[\left(\mathcal{E}_{it}P_t^k/\mathcal{E}_{kt} - \tau_i^* MC_{it}\right) h\left(\frac{P_t^k/\mathcal{E}_{kt}}{P_t^*}\right) C_t^* - \Omega^*\left(\frac{P_t^k}{P_{t-1}^k}\right) W_{it}\right],$$

where P_t^k is a price sticky in currency k. To make the argument in a most transparent way, we focus on a special case of a static model $\beta \to 0$ with fully sticky prices $\Omega^* \to \infty$ and drop time subscript t — see Gopinath, Itskhoki and Rigobon (2010) for a dynamic setting. Following Engel (2006) and Mukhin (2022), one can show that to the second-order approximation, the currency choice problem is equivalent to

$$\min_k \mathbb{E}\big(\tilde{p}_i^* + e_k\big)^2,$$

where $\tilde{p}_i^*$ is the desired dollar price that maximizes static profits of a firm in a given state of the world, and both $\tilde{p}_i^*$ and e_k are measured in log deviations from the steady-state values. Intuitively, given the nominal rigidities that do not allow the firm to adjust prices after the realization of shocks, the exporter chooses currency k such that the optimal price expressed in that currency $\tilde{p}_i^* + e_k$ is most stable. To the first-order approximation, the optimal price can be expressed as a weighted average of firm's marginal costs and the prices of competitors:

$$\tilde{p}_i^* = (1-\delta)(mc_i - e_i) + \delta p^*,$$

where parameter $\delta = \frac{\epsilon(1)}{\epsilon(1)+\vartheta(1)-1} \in [0,1)$ reflects complementarities in price setting and depends on elasticity of demand $\vartheta(x) \equiv -\frac{\partial \log h(x)}{\partial \log x}$ and superelasticity of demand $\epsilon(x) \equiv \frac{\partial \log \vartheta(x)}{\partial \log x}$. Assume for simplicity that the marginal costs of exporters coincide with the costs of domestic firms and are stabilized by the monetary policy, $mc_i = 0$. It follows that as long as exporters from other countries set their prices in dollars $p^* = 0$, the problem of exporters in country i is to minimize $\mathbb{E}\big(e_k - (1-\delta)e_i\big)^2$. When complementarities in price setting are sufficiently strong $\delta \to 1$, it is optimal to choose U.S. currency with $e_k = 0$ (exchange rate of the dollar against itself). The symmetry across economies ensures that the DCP equilibrium can be sustained at the global level. Mukhin (2022) shows that the incentives of exporters to set prices in dollars are further strengthened if — in line with the empirical evidence from Amiti, Itskhoki and Konings (2014) — the share of foreign intermediates is higher for exporters and are robust to partially adjusting prices.

The same analysis applies to local firms with the currency choice problem

$$\min_k \mathbb{E}\big(\tilde{p}_{ii} - e_i + e_k\big)^2 = \min_k \mathbb{E}\big((1-\delta)mc_i + \delta p_{ii} - e_i + e_k\big)^2,$$

where $\tilde{p}_{ii}$ is the desired price of a representative domestic firm expressed in local currency. The monetary policy ensures that $mc_i = 0$ and as long as other local firms choose PCP $p_{ii} = 0$, it is optimal to set prices in local currency, i.e. $k = i$. Thus, it is possible to sustain an equilibrium with PCP in local markets and DCP in international trade under the optimal policy described in Proposition 1.

Finally, consider the policy that takes into account its effects on firms' currency choice. While PCP in local market allows the monetary policy to stabilize local demand, DCP in exports is the main source of inefficiency in the economy. To make exporters switch to a local currency, the monetary policy needs to ensure that the desired price is more stable in currency i than in dollars

$$\mathbb{E}\big(\tilde{p}_i^* + e_i\big)^2 < \mathbb{E}\tilde{p}_i^{*2} \qquad \Leftrightarrow \qquad \frac{\mathbb{E}\tilde{p}_i^* e_i}{\tilde{p}_i^{*2}} < -\frac{1}{2}.$$

Assuming that exporters from other economies are pricing in dollars, the optimal price is

$$\tilde{p}_i^* = (1-\delta)(mc_i - e_i).$$

and the planner needs to deviate from stabilizing domestic prices and closing the local wedge $\bar{\tau}_{ii}$ to ensure that exporters choose PCP. Such policy is clearly suboptimal if the economy is relative closed and its welfare depends primarily on the local margin.

C.7 Domestic dollarization

Proposition A1 *Assume that preferences $U\left(C_{it}, L_{it}, \xi_{it}\right)$ are separable, $U_{C_{it}L_{it}} = 0$, and that they are CES with respect to domestic goods C_{iit}, dollarized domestic goods C_{iit}^*, and imported goods C_{it}^*:*

$$C_{it} = \left[(1-\gamma^*-\gamma)^{\frac{1}{\theta}} C_{iit}^{\frac{\theta-1}{\theta}} + \gamma^* C_{iit}^{*\frac{\theta-1}{\theta}} + \gamma C_{it}^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}},$$

where $0 < \gamma^* < 1-\gamma$ reflects the share of domestic producers, whose prices are sticky in dollars. Also augment the policy problem with the full set of state-contingent capital controls $\left\{\tau_{it}^h\right\}_h$, time-varying production subsidy to exporters τ_{it}^*, and a constant subsidy on price-adjustment costs for domestic dollarized producers τ_i^Ω. Then the optimal monetary policy stabilizes prices in domestic currency, $\pi_{iit} = 1$.

First, note that the CES preferences imply the following demand structure:

$$C_{iit}\left(\omega\right) = \left(1-\gamma^*-\gamma\right)\left(\frac{P_{iit}\left(\omega\right)}{P_{iit}}\right)^{-\varepsilon}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} C_{it},$$

$$C_{iit}^*\left(\omega\right) = \gamma^*\left(\frac{P_{iit}^*\left(\omega\right)}{P_{iit}^*}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}P_{iit}^*}{P_{it}}\right)^{-\theta} C_{it}, \quad C_{it}^* = \gamma\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta} C_{it},$$

$$P_{iit} = \left(\int P_{iit}\left(\omega\right)^{1-\varepsilon}\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}}, \quad P_{iit}^* = \left(\int P_{iit}^*\left(\omega\right)^{1-\varepsilon}\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}},$$

$$P_{it} = \left(\left(1-\gamma^*-\gamma\right)P_{iit}^{1-\theta} + \gamma^*\left(\mathcal{E}_{it}P_{iit}^*\right)^{1-\theta} + \gamma\left(\mathcal{E}_{it}P_t^*\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}.$$

Now we can write the problem of a dollarized domestic producer as

$$\left\{P_{iit}^*\right\} = \arg\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t \frac{U_{C_{iit}}}{P_{iit}}\left[\left(\mathcal{E}_{it}P_t - \tau_i\frac{W_{it}}{A_{it}}\right)\gamma^*\left(\frac{P_t}{P_{iit}^*}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}P_{iit}^*}{P_{it}}\right)^{-\theta} C_{it} - \tau_i^\Omega\Omega\left(\frac{P_t}{P_{t-1}}\right)W_{it}\right],$$

where as before $\tau_i = \frac{\varepsilon-1}{\varepsilon}$. Together with other equilibrium conditions, this leads to

$$\left\{P_{iit}^*\right\} = \arg\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(\frac{U_{C_{it}^*}}{P_t^*}P_t - \tau_i\frac{-U_{L_{it}}}{A_{it}}\right)\frac{\gamma^*}{\tau_i^\Omega}\left(\frac{P_t}{P_{iit}^*}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}P_{iit}^*}{P_{it}}\right)^{-\theta} C_{it} - \Omega\left(\frac{P_t}{P_{t-1}}\right)\left(-U_{L_{it}}\right)\right]. \tag{A19}$$

Then the full policy problem becomes

$$\max_{\left\{C_{it},L_{it},\left\{B_{it+1}^h\right\}_h,S_{it},P_{iit},P_{iit}^*,P_{it},\mathcal{E}_{it},\tau_{it}^h,\tau_{it}^*\right\}_t} \mathbb{E}\sum_{t=0}^{\infty}\beta^t U\left(C_{it},L_{it},\xi_{it}\right)$$

$$\text{s.t. } A_{it}L_{it} = (1-\gamma^*-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} C_{it} + \gamma^*\left(\frac{\mathcal{E}_{it}P_{iit}^*}{P_{it}}\right)^{-\theta} C_{it} + S_{it}^{-\varepsilon}C_t^*$$
$$+ A_{it}\left[\Omega\left(\frac{P_{iit}}{P_{iit-1}}\right) + \Omega\left(\frac{P_{iit}^*}{P_{iit-1}^*}\right) + \Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi_t^*\right)\right],$$

$$\sum_{h\in H_t}\mathcal{Q}_t^h B_{it+1}^h - \sum_{h\in H_{t-1}}\left(\mathcal{Q}_t^h+\mathcal{D}_t^h\right)B_{it}^h = S_{it}^{1-\varepsilon}C_t^* - \gamma\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta}C_{it} + \psi_{it},$$

$$\beta\mathbb{E}_t\frac{U_{C_{it+1}^*}}{U_{C_{it}^*}}\tau_{it}^h\frac{\mathcal{Q}_{t+1}^h+\mathcal{D}_{t+1}^h}{\mathcal{Q}_t^h} = 1, \tag{A20}$$

$$(1-\gamma^*-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} + \gamma^*\left(\frac{\mathcal{E}_{it}P_{iit}^*}{P_{it}}\right)^{1-\theta} + \gamma\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta} = 1. \tag{A21}$$

$$\{P_{iit}\} = \arg\max_{\{P_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(\frac{U_{C_{iit}}}{P_{iit}}P_t - \tau_i\frac{-U_{L_{it}}}{A_{it}}\right)(1-\gamma^*-\gamma)\left(\frac{P_t}{P_{iit}}\right)^{-\varepsilon}\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}C_{it}\right.$$
$$\left. -\Omega\left(\frac{P_t}{P_{t-1}}\right)(-U_{L_{it}})\right], \tag{A22}$$

$$\{P_{iit}^*\} = \arg\max_{\{P_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(\frac{U_{C_{it}^*}}{P_t^*}P_t - \tau_i\frac{-U_{L_{it}}}{A_{it}}\right)\frac{\gamma^*}{\tau_i^{\Omega}}\left(\frac{P_t}{P_{iit}^*}\right)^{-\varepsilon}\left(\frac{\mathcal{E}_{it}P_{iit}^*}{P_{it}}\right)^{-\theta}C_{it} - \Omega\left(\frac{P_t}{P_{t-1}}\right)(-U_{L_{it}})\right].$$

$$\{S_{it}\} = \arg\max_{\{S_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(U_{C_{it}^*}S_t - \tau_{it}^*\frac{-U_{L_{it}}}{A_{it}}\right)S_t^{-\varepsilon}C_t^* - \Omega^*\left(\frac{S_t}{S_{t-1}}\pi_t^*\right)(-U_{L_{it}})\right]. \tag{A23}$$

Note that the labor supply condition (1) is redundant since it's the only constraint that contains nominal wages W_{it}. And the relative demand constraint (2) is already plugged in through the CES demand functions.

Now we can drop the no-arbitrage condition (A20) from this policy problem since it's the only constraint that contains the capital controls tax τ_{it}^h. Similarly, we drop the export price-setting condition (A23) because of the time-varying production subsidy τ_{it}^*. Next, we guess (and verify later) that the remaining two price-setting conditions (A22) and (A19) do not bind. Finally, we define relative prices as

$$p_{iit} \equiv \frac{P_{iit}}{P_{it}}, \quad p_{iit}^* \equiv \frac{P_{iit}^*}{P_t^*}, \quad e_{it} \equiv \frac{\mathcal{E}_{it}P_t^*}{P_{it}}. \tag{A24}$$

Then the Lagrangian of the relaxed policy problem can be written as

$$\mathcal{L} = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\Big\{U\left(C_{it},L_{it},\xi_{it}\right)+\lambda_{it}\left[A_{it}L_{it}-\left(1-\gamma^*-\gamma\right)p_{iit}^{-\theta}C_{it}-\gamma^*\left(e_{it}p_{iit}^*\right)^{-\theta}C_{it}-S_{it}^{-\varepsilon}C_t^*\right]$$
$$-\lambda_{it}A_{it}\left[\Omega\left(\frac{p_{iit}}{p_{iit-1}}\pi_{it}\right)+\Omega\left(\frac{p_{iit}^*}{p_{iit-1}^*}\pi_t^*\right)+\Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi_t^*\right)\right]$$
$$+\eta_{it}\left[\left(1-\gamma^*-\gamma\right)p_{iit}^{1-\theta}+\gamma^*\left(e_{it}p_{iit}^*\right)^{1-\theta}+\gamma e_{it}^{1-\theta}-1\right]$$
$$+\mu_{it}\left[\sum_{h\in H_t}\mathcal{Q}_t^hB_{it+1}^h-\sum_{h\in H_{t-1}}\left(\mathcal{Q}_t^h+\mathcal{D}_t^h\right)B_{it}^h-S_{it}^{1-\varepsilon}C_t^*+\gamma e_{it}^{-\theta}C_{it}-\psi_{it}\right]\Big\},$$

where $\pi_{it}\equiv P_{it}/P_{it-1}$. The corresponding optimality conditions are

- wrt C_{it}:
$$0=U_{C_{it}}-\lambda_{it}\left[\left(1-\gamma^*-\gamma\right)p_{iit}^{-\theta}+\gamma^*\left(e_{it}p_{iit}^*\right)^{-\theta}\right]+\mu_{it}\gamma e_{it}^{-\theta},$$
- wrt L_{it}:
$$0=U_{L_{it}}+\lambda_{it}A_{it},$$
- wrt B_{it+1}^h:
$$0=\mu_{it}\mathcal{Q}_t^h-\beta\mathbb{E}_t\mu_{it+1}\left(\mathcal{Q}_{t+1}^h+\mathcal{D}_{t+1}^h\right),$$
- wrt e_{it}:
$$0=\lambda_{it}\gamma^*\theta\left(e_{it}p_{iit}^*\right)^{-\theta}C_{it}+\eta_{it}\gamma^*\left(1-\theta\right)\left(e_{it}p_{iit}^*\right)^{1-\theta}+\eta_{it}\gamma\left(1-\theta\right)e_{it}^{1-\theta}-\theta\mu_{it}\gamma e_{it}^{-\theta}C_{it},$$
- wrt π_{it}:
$$\{\pi_{it}\}=\arg\max_{\{\pi_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left\{-\lambda_{it}A_{it}\Omega\left(\frac{p_{iit}}{p_{iit-1}}\pi_t\right)\right\},$$
- wrt S_{it}:
$$\{S_{it}\}=\arg\max_{\{S_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left\{-\lambda_{it}S_t^{-\varepsilon}C_t^*-\lambda_{it}A_{it}\Omega^*\left(\frac{S_t}{S_{t-1}}\pi_t^*\right)-\mu_{it}S_t^{1-\varepsilon}C_t^*\right\},$$
- wrt p_{iit}:
$$\{p_{iit}\}=\arg\max_{\{p_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left\{\eta_{it}\left(1-\gamma^*-\gamma\right)p_t^{1-\theta}-\lambda_{it}\left(1-\gamma^*-\gamma\right)p_t^{-\theta}C_{it}-\lambda_{it}A_{it}\Omega\left(\frac{p_t}{p_{t-1}}\pi_{it}\right)\right\},$$
- wrt p_{iit}^*:
$$\{p_{iit}^*\}=\arg\max_{\{p_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left\{\eta_{it}\gamma^*\left(e_{it}p_t\right)^{1-\theta}-\lambda_{it}\gamma^*\left(e_{it}p_t\right)^{-\theta}C_{it}-\lambda_{it}A_{it}\Omega\left(\frac{p_t}{p_{t-1}}\pi_t^*\right)\right\}.$$

Use the FOC wrt L_{it} to substitute λ_{it} for $-U_{L_{it}}/A_{it}$. The FOC wrt π_{it} implies that the optimal policy stabilizes prices of domestic producers in domestic currency, $\Omega\left(\frac{p_{iit}}{p_{iit-1}}\pi_{it}\right)=0$. Then the optimality condition wrt p_{iit}

collapses to

$$\eta_{it} p_{iit} = \frac{\theta}{\theta - 1} \lambda_{it} C_{it} = \frac{\theta}{\theta - 1} \frac{-U_{L_{it}}}{A_{it}} C_{it}.$$

We can use it to substitute for η_{it}. Then the FOC wrt e_{it} becomes

$$\mu_{it} = \frac{-U_{L_{it}}}{A_{it}} \frac{e_{it}^{\theta}}{\gamma} \left[\gamma^* \left(e_{it} p_{iit}^* \right)^{-\theta} - \frac{\gamma^* \left(e_{it} p_{iit}^* \right)^{1-\theta} + \gamma e_{it}^{1-\theta}}{p_{iit}} \right], \tag{A25}$$

Use this expression for μ_{it} to rewrite the FOC wrt C_{it} as

$$0 = U_{C_{it}} - \frac{-U_{L_{it}}}{A_{it}} \frac{1}{p_{iit}} \left[\gamma^* \left(e_{it} p_{iit}^* \right)^{1-\theta} + \gamma e_{it}^{1-\theta} + \left(1 - \gamma^* - \gamma \right) p_{iit}^{1-\theta} \right].$$

Use the price index constraint (A21) and arrive at

$$U_{C_{it}} p_{iit} = \frac{-U_{L_{it}}}{A_{it}}.$$

Note that this condition confirms that the price-setting constraint of domestic producers (A22) is satisfied under the optimal policy with no adjustment to their prices.[3] Thus, we have verified that this constraint does not bind.

Now rewrite the optimality condition wrt p_{iit}^* as

$$\{p_{iit}^*\} = \arg\max_{\{p_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{\theta}{\theta - 1} U_{C_{it}} e_{it} p_t - \frac{-U_{L_{it}}}{A_{it}} \right) \gamma^* \left(e_{it} p_t \right)^{-\theta} C_{it} - \Omega \left(\frac{p_t}{p_{t-1}} \pi_t^* \right) \left(-U_{L_{it}} \right) \right\}.$$

Go back from relative prices to absolute ones using (A24), and use the fact that $U_{C_{it}}/P_{it} = U_{C_{iit}}/P_{iit}$ together with the households' optimality condition (2)

$$\{P_{iit}^*\} = \arg\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left\{ \left(\frac{\theta}{\theta - 1} \frac{U_{C_{it}^*}}{P_t^*} P_t - \frac{-U_{L_{it}}}{A_{it}} \right) \gamma^* \left(\frac{P_t}{P_{iit}^*} \right)^{-\theta} \left(\frac{\mathcal{E}_{it} P_{iit}^*}{P_{it}} \right)^{-\theta} C_{it} - \Omega \left(\frac{P_t}{P_{t-1}} \right) \left(-U_{L_{it}} \right) \right\}.$$

This condition is equivalent to the private price-setting condition (A19) once we set $\tau_i^{\Omega} = (\varepsilon - 1)/\theta$. To see this, note that the two conditions differ by the demand elasticity, θ vs ε. Then the social instantaneous gains from marginally adjusting price P_{iit}^* are

$$\theta \left(-\frac{U_{C_{it}^*}}{P_t^*} + \frac{-U_{L_{it}}}{A_{it} P_{iit}^*} \right) \gamma^* \left(\frac{\mathcal{E}_{it} P_{iit}^*}{P_{it}} \right)^{-\theta} C_{it}.$$

And the similar private gains from (A19) are

$$\frac{\varepsilon - 1}{\tau_i^{\Omega}} \left(-\frac{U_{C_{it}^*}}{P_t^*} + \frac{\varepsilon}{\varepsilon - 1} \tau_i \frac{-U_{L_{it}}}{A_{it} P_{iit}^*} \right) \gamma^* \left(\frac{\mathcal{E}_{it} P_{iit}^*}{P_{it}} \right)^{-\theta} C_{it}.$$

Under $\frac{\varepsilon}{\varepsilon - 1} \tau_i = 1$ and $\frac{\varepsilon - 1}{\tau_i^{\Omega}} = \theta$, the two coincide, while the social and private price-adjustment costs coincide by construction. Thus, the price-setting constraint (A19) does not bind. This conculdes our proof.

In addition, we can also back out capital controls τ_{it}^h and production subsidy τ_{it}^* that are required to support

[3] Also use the fact that under the CES demand, $U_{C_{it}}/P_{it} = U_{C_{iit}}/P_{iit}$.

this equilibrium. To do this, go back to the expression for μ_{it}, (A25), and rewrite it as

$$\mu_{it} = U_{C^*_{it}} \left[\left(\frac{P_{iit}}{\mathcal{E}_{it} P^*_{iit}} - 1 \right) \frac{\gamma^*}{\gamma} \left(\frac{P^*_{iit}}{P^*_t} \right)^{1-\theta} - 1 \right].$$

Plug it to the optimality condition wrt B^h_{it+1}

$$1 = \beta \mathbb{E}_t \frac{U_{C^*_{it+1}}}{U_{C^*_{it}}} \frac{\left(\frac{P_{iit+1}}{\mathcal{E}_{it+1} P^*_{iit+1}} - 1 \right) \frac{\gamma^*}{\gamma} \left(\frac{P^*_{iit+1}}{P^*_{t+1}} \right)^{1-\theta} - 1}{\left(\frac{P_{iit}}{\mathcal{E}_{it} P^*_{iit}} - 1 \right) \frac{\gamma^*}{\gamma} \left(\frac{P^*_{iit}}{P^*_t} \right)^{1-\theta} - 1} \frac{\mathcal{Q}^h_{t+1} + \mathcal{D}^h_{t+1}}{\mathcal{Q}^h_t},$$

and compare it with the private no-arbitrage condition (A20) to back out τ^h_{it}. Note that capital controls are not used whenever the prices of domestic goods are equalized, $\mathcal{E}_{it} P^*_{iit} = P_{iit}$.

Similarly, manipulate the optimality condition wrt S_{it}

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C^*_{it}} \left[1 + \frac{\gamma^*}{\gamma} \left(\frac{P^*_{iit}}{P^*_t} \right)^{1-\theta} \left(1 - \frac{P_{iit}}{\mathcal{E}_{it} P^*_{iit}} \right) \right] S_t - \frac{-U_{L_{it}}}{A_{it}} \right) S_t^{-\varepsilon} C^*_t \right.$$
$$\left. - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi^*_t \right) (-U_{L_{it}}) \right],$$

and compare it with the private price-setting condition (A23) to back out τ^*_{it}. Similarly, note that production subsidy is not used whenever the prices of domestic goods are equalized, $\mathcal{E}_{it} P^*_{iit} = P_{iit}$.

C.8 Policy rates

Iterate forward the Euler equation (3) for local bonds to back out the nominal interest rates:

$$\frac{U_{Ciit}}{P_{iit}} = \beta R_{it} \mathbb{E}_t \frac{U_{Ciit+1}}{P_{iit+1}} = \lim_{T \to \infty} \beta^T \mathbb{E}_t \left(\prod_{\tau=0}^{T-1} R_{it+\tau} \right) \frac{U_{Ciit+T}}{P_{iit+T}}.$$

Assume stationarity, so that the long-run values of all real variables are constant,[4] while the monetary policy stabilizes P_{iit}. It follows that $\lim_{T \to \infty} \frac{U_{Ciit+T}}{P_{iit+T}} = \text{const}$ and $\frac{U_{Ciit}}{P_{iit}}$ is equal to the expected present value of future interest rates — the characteristic of the monetary policy we focus on henceforth.

Recall the version of the model with intermediate goods from Section C.1. Under the optimal monetary policy, the nominal marginal costs of local firms are constant, i.e.

$$MC^d_{it} = \frac{C^d \left(W_{it}, P_{iit}, \mathcal{E}_{it} P^*_t \right)}{A^d_{it}} = \frac{C^d \left(-U_{L_{it}} / \left(U_{C_{iit}} / P_{iit} \right), P_{iit}, \mathcal{E}_{it} P^*_t \right)}{A^d_{it}} = \text{const}.$$

It follows that the monetary policy has to react to foreign shocks: $U_{L_{it}}$ fluctuates with foreign demand for domestic products and import prices $\mathcal{E}_{it} P^*_t$ directly affect the marginal costs. Moreover, because both import and export prices are sticky in dollars, the dollar exchange rate $\mathcal{E}_{it}$ has a disproportionately large effect on local monetary policy through both channels.

If only import prices are sticky in dollars, then any U.S. shock that leads to an appreciation of the dollar results in higher prices of imported goods in other economies. To keep MC^d_{it} constant, non-U.S. monetary authorities have to increase $U_{C_{iit}} / P_{iit}$, which corresponds to higher interest rates. On the other hand, if only export prices

[4]While the stationarity is in general not guaranteed under incomplete markets, one can ensure it by adding infinitely small portfolio adjustment costs (see Schmitt-Grohé and Uribe 2003).

are sticky in dollars, then an appreciation of the dollar lowers foreign demand for exported goods. The export sector demand for labor goes down lowering $U_{L_{it}}$ and making non-U.S. monetary policy to decrease $U_{C_{iit}}/P_{iit}$, which corresponds to higher interest rates.

C.9 Proof of Proposition 3

For all remaining results in this Section, we return to our baseline setup described in Section 1.

Let's add an export tax τ_{it}^{E} and a revenue subsidy for exporters τ_{it}^{R} to the environment described in Section 1. The export tax is applied on top of prices set by firms, and all proceeds go to the government. The revenue subsidy applies to the revenue of exporters and it's funded by the government. Then, the exporter's problem becomes

$$\{P_{it}^{*}\} = \arg\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[\left(\tau_{it}^{R} \mathcal{E}_{it} P_t - \frac{W_{it}}{A_{it}} \right) h\left(\tau_{it}^{E} \frac{P_t}{P_t^{*}} \right) C_t^{*} - \Omega^{*}\left(\frac{P_t}{P_{t-1}} \right) W_{it} \right], \tag{A26}$$

where the export tax τ_{it}^{E} affects the demand function $h\,()$, but otherwise has no effect on firm's profits, while the revenue subsidy τ_{it}^{R} affects firm's revenue, but does not directly affects the demand. The government's budget constraint (A1) changes to

$$T_{it} = (\tau_i - 1) \frac{W_{it}}{A_{it}} C_{iit} + \mathcal{E}_{it} P_t^{*} \psi_{it} + \left[\left(1 - \tau_{it}^{R}\right) + \left(\tau_{it}^{E} - 1\right) \right] \mathcal{E}_{it} P_{it}^{*} h\left(\tau_{it}^{E} S_{it}\right) C_t^{*},$$

and the last two terms become non-zero relative to our baseline case without fiscal instruments. Thus, the two new fiscal instruments are revenue neutral whenever $\tau_{it}^{R} = \tau_{it}^{E}$.

Now we set the monetary policy to stabilize domestic prices, $\pi_{iit} = 1$, which together with the price-setting condition (5) implies $A_{it} = -U_{L_{it}}/U_{C_{iit}}$. Recall that this is identical to (A2), one of the conditions for the efficient allocation.

The export tax is set to achieve an optimal markup for exported goods,

$$\tau_{it}^{E} \mathcal{E}_{it} P_{it}^{*} = \frac{\varepsilon}{\varepsilon - 1} \frac{W_{it}}{A_{it}}.$$

Since monetary policy achieves $P_{iit} = W_{it}/A_{it}$, we can rewrite it as

$$\tau_{it}^{E} S_{it} = \frac{\varepsilon}{\varepsilon - 1} \frac{P_{iit}}{\mathcal{E}_{it} P_t^{*}} = \frac{\varepsilon}{\varepsilon - 1} \frac{U_{C_{iit}}}{U_{C_{it}^{*}}},$$

where the second equality uses the household's optimality condition (2). This condition is equivalent to the efficiency condition (A3), since $\tau_{it}^{E} P_{it}^{*}$ is the price faced by foreign consumers, not P_{it}^{*}. Moreover, since monetary policy achieves $P_{iit} = const$, and the revenue subsidy ensures that $P_{it}^{*} = const$, we get that this condition is equivalent to $\tau_{it}^{E} \mathcal{E}_{it} = 1$.

Lastly, the revenue subsidy τ_{it}^{R} should stabilize dollar prices of exporters. Note that without any price-adjustment costs, the solution to the exporters' problem (A26) would be characterised by

$$\tau_{it}^{R} \mathcal{E}_{it} P_{it}^{*} = \frac{\varepsilon}{\varepsilon - 1} \frac{W_{it}}{A_{it}}.$$

To be consistent with the previous condition, we just need to set $\tau_{it}^{R} = \tau_{it}^{E}$.

Thus, we have shown that under $\tau_{it}^{R} = \tau_{it}^{E} = 1/\mathcal{E}_{it}$, both efficiency conditions (A2) – (A3) are satisfied, and thus the efficient allocation is implemented. Moreover, this combination of fiscal instruments is revenue neutral. Intuitively, the export tax τ_{it}^{E} allows to set optimal dollar prices bypassing the nominal rigidity. The revenue subsidy τ_{it}^{R} then transfers all the revenue from the export tax back to exporters. Because in the absence of sticky prices, exporters would choose the same prices as the planner, the sizes of two transfers are exactly the same, and this fiscal intervention is revenue neutral.

D Proofs for Section 3

D.1 Proof of Proposition 4

The U.S. maximizes its welfare

$$\max \mathbb{E} \sum_{t=0}^{\infty} \beta^t U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right)$$

subject to three blocks of constraints. The first block characterizes the U.S. economy, that is it determines the U.S. variables $\left\{C_{iit}, C_{it}^*, L_{it}, \left\{B_{it+1}^h\right\}_h, S_{it}, \pi_{iit}\right\}$ for given U.S. policy and global variables $\left\{C_t^*, \pi_t^*, \mathcal{Q}_t^h\right\}$:

$$A_{it} L_{it} = C_{iit} + h\left(S_{it}\right) C_t^* + A_{it} \Omega^* \left(\frac{S_{it}}{S_{it-1}} \pi_t^*\right),$$

$$\sum_{h \in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h \in H_{t-1}} \left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) B_{it}^h = S_{it} h\left(S_{it}\right) C_t^* - C_{it}^* + \psi_{it},$$

$$\beta \mathbb{E}_t \frac{U_{C_{it+1}^*}}{U_{C_{it}^*}} \frac{\mathcal{Q}_{t+1}^h + \mathcal{D}_{t+1}^h}{\mathcal{Q}_t^h} = 1,$$

$$\frac{U_{C_{it}^*}/U_{C_{it-1}^*}}{U_{C_{iit}}/U_{C_{iit-1}}} = \frac{\pi_t^*}{\pi_{iit}},$$

$$\{1\} = \arg\max_{\{p_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{iit}} p_t - \tau_i \frac{-U_{L_{it}}}{A_{it}}\right) h\left(p_t\right) C_{iit}\right],$$

$$\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{it}^*} S_t - \frac{-U_{L_{it}}}{A_{it}}\right) h\left(S_t\right) C_t^* - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi_t^*\right) \left(-U_{L_{it}}\right)\right].$$

Note that due to flexible domestic prices, the price-setting condition (5) does not have price-adjustment costs Ω and is independent of π_{iit}. Because of that, we can drop constraint (10) along with variable π_{iit}. Also, we can further simplify the price-setting constraint (5) to $U_{C_{iit}} = -U_{L_{it}}/A_{it}$.

The second block of constraints characterizes the non-U.S. economy in country j, that is it determines non-U.S. variables $\left\{C_{jjt}, C_{jt}^*, L_{jt}, \left\{B_{jt+1}^h\right\}_h, S_{jt}\right\}$ in each country j for given global variables $\left\{C_t^*, \pi_t^*, \mathcal{Q}_t^h\right\}$:

$$A_{jt} L_{jt} = C_{jjt} + h\left(S_{jt}\right) C_t^* + A_{jt} \Omega^* \left(\frac{S_{jt}}{S_{jt-1}} \pi_t^*\right),$$

$$\sum_{h \in H_t} \mathcal{Q}_t^h B_{jt+1}^h - \sum_{h \in H_{t-1}} \left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) B_{jt}^h = S_{jt} h\left(S_{jt}\right) C_t^* - C_{jt}^* + \psi_{jt},$$

$$\beta \mathbb{E}_t \frac{U_{C_{jt+1}^*}}{U_{C_{jt}^*}} \frac{\mathcal{Q}_{t+1}^h + \mathcal{D}_{t+1}^h}{\mathcal{Q}_t^h} = 1,$$

$$U_{C_{jjt}} = \frac{-U_{L_{jt}}}{A_{jt}},$$

$$\{S_{jt}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{jt}^*} S_t - \frac{-U_{L_{jt}}}{A_{jt}}\right) h\left(S_t\right) C_t^* - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi_t^*\right) \left(-U_{L_{jt}}\right)\right],$$

where we already plugged in the optimal poicy $\pi_{jjt} = 1$.

Finally, the third block of constraints consists of global balances and it determines the global variables $\left\{C_t^*, \pi_t^*, \mathcal{Q}_t^h\right\}$ for given non-U.S. variables:

$$\int C_{jt}^* \mathrm{d}j = C_t^*, \quad \int S_{jt} h\left(S_{jt}\right) \mathrm{d}j = 1, \quad \int B_{jt+1}^h \mathrm{d}j = 0.$$

Note, however, that due to Walras' law, one of these global constraints follows from the others. In fact, let's integrate the budget constraint (8) over all non-U.S. countries j

$$\sum_{h \in H_t} \mathcal{Q}_t^h \int B_{jt+1}^h \mathrm{d}j - \sum_{h \in H_{t-1}} \left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) \int B_{jt}^h \mathrm{d}j = \int S_{jt} h\left(S_{jt}\right) \mathrm{d}j C_t^* - \int C_{jt}^* \mathrm{d}j + \int \psi_{jt} \mathrm{d}j.$$

Use the two of the global balances, $\int B_{jt+1}^h \mathrm{d}j = 0$ and $\int C_{jt}^* \mathrm{d}j = C_t^*$, as well as the restriction on the system of exogenous shocks, $\int \psi_{jt} \mathrm{d}j = 0$, to arrive at the last global balance $\int S_{jt} h\left(S_{jt}\right) \mathrm{d}j = 1$.

Overall, the U.S. policymaker maximizes its welfare subject to three blocks of constraints over the U.S. variables, non-U.S. variables, and the global variables. Instead of explicitly characterizing solution to this problem as a function of fundamentals only, we describe it implicitly. Specifically, suppose that the U.S. policy is formulated in terms of global demand C_t^*. Then, for a given U.S. policy, that is a state-contingent path $\{C_t^*\}_t$, one can solve the second and the third blocks of constraints for all the variables in them. Then the solution to this system can be written as a function of U.S. policy only. That is we implicitly describe this solution in terms of functions of $\{C_t^*\}_t$, in particular we denote

$$\pi_t^* = \pi_t^*\left(\{C_t^*\}_t\right), \quad \mathcal{Q}_t^h = \mathcal{Q}_t^h\left(\{C_t^*\}_t\right).$$

Then the full U.S. policy problem can be rewritten as

$$\begin{aligned}
&\max_{\left\{C_{iit}, C_{it}^*, L_{it}, \left\{B_{it+1}^h\right\}_h, S_{it}, C_t^*\right\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right) \\
&\text{s.t.} \quad A_{it} L_{it} = C_{iit} + h\left(S_{it}\right) C_t^* + A_{it} \Omega^* \left(\frac{S_{it}}{S_{it-1}} \pi_t^*\left(\{C_t^*\}_t\right)\right), \\
&\sum_{h \in H_t} \mathcal{Q}_t^h\left(\{C_t^*\}_t\right) \left(B_{it+1}^h - B_{it}^h\right) - \sum_{h \in H_{t-1}} \mathcal{D}_t^h B_{it}^h = S_{it} h\left(S_{it}\right) C_t^* - C_{it}^* + \psi_{it}, \\
&\beta \mathbb{E}_t \frac{U_{C_{it+1}^*}}{U_{C_{it}^*}} \frac{\mathcal{Q}_{t+1}^h\left(\{C_t^*\}_t\right) + \mathcal{D}_{t+1}^h}{\mathcal{Q}_t^h\left(\{C_t^*\}_t\right)} = 1, \\
&U_{C_{iit}} = \frac{-U_{L_{it}}}{A_{it}}, \\
&\{S_{it}\} = \arg\max_{\{S_t\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[\left(U_{C_{it}^*} S_t - \frac{-U_{L_{it}}}{A_{it}}\right) h\left(S_t\right) C_t^* - \Omega^* \left(\frac{S_t}{S_{t-1}} \pi_t^*\left(\{C_t^*\}_t\right)\right) \left(-U_{L_{it}}\right)\right].
\end{aligned} \tag{A27}$$

Now we guess (and verify later) that the last three constraints are not binding. Then the Lagrangian to the relaxed policy problem is

$$\mathcal{L} = \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left\{U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right) + \lambda_{it}\left[A_{it}L_{it} - C_{iit} - h\left(S_{it}\right)C_t^* - A_{it}\Omega^*\left(\frac{S_{it}}{S_{it-1}}\pi_t^*\left(\{C_t^*\}_t\right)\right)\right]\right.$$
$$\left. + \mu_{it}\left[\sum_{h\in H_t}\mathcal{Q}_t^h\left(\{C_t^*\}_t\right)\left(B_{it+1}^h - B_{it}^h\right) - \sum_{h\in H_{t-1}}\mathcal{D}_t^h B_{it}^h - S_{it}h\left(S_{it}\right)C_t^* + C_{it}^*\right]\right\}.$$

The corresponding optimality conditions are:

- wrt C_{iit}:
$$0 = U_{C_{iit}} - \lambda_{it},$$
- wrt C_{it}^*:
$$0 = U_{C_{it}^*} + \mu_{it},$$
- wrt L_{it}:
$$0 = U_{L_{it}} + \lambda_{it}A_{it},$$
- wrt B_{it+1}^h:
$$0 = \mu_{it}\mathcal{Q}_t^h\left(\{C_t^*\}_t\right) - \beta\mathbb{E}_t\mu_{it+1}\left(\mathcal{Q}_{t+1}^h\left(\{C_t^*\}_t\right) + \mathcal{D}_{t+1}^h\right),$$
- wrt C_t^*:
$$0 = -\lambda_{it}h\left(S_{it}\right) - \mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\lambda_{it+k}A_{it+k}\Omega^{*\prime}\left(\frac{S_{it+k}}{S_{it+k-1}}\pi_{t+k}^*\left(\{C_t^*\}_t\right)\right)\frac{S_{it+k}}{S_{it+k-1}}\frac{\partial\pi_{t+k}^*}{\partial C_t^*}$$
$$-\mu_{it}S_{it}h\left(S_{it}\right) + \mathbb{E}_t\sum_{k=0}^{\infty}\beta^k\mu_{it+k}\sum_{h\in H_{t+k}}\left(B_{it+k+1}^h - B_{it+k}^h\right)\frac{\partial\mathcal{Q}_{t+k}^h\left(\{C_t^*\}_t\right)}{\partial C_t^*},$$
- wrt S_{it}:
$$\{S_{it}\} = \arg\max_{\{S_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[-\lambda_{it}h\left(S_t\right)C_t^* - \lambda_{it}A_{it}\Omega^*\left(\frac{S_t}{S_{t-1}}\pi_t^*\left(\{C_t^*\}_t\right)\right) - \mu_{it}S_th\left(S_t\right)C_t^*\right].$$

We use the first two FOCs to substitute for values of λ_{it} and μ_{it}. Then the FOC wrt L_{it} verifies our guess that the constraint (A27) is not binding. The FOC wrt B_{it+1}^h verifies that the no-arbitrage condition (4) is not binding. The optimality condition wrt S_{it} becomes

$$\{S_{it}\} = \arg\max_{\{S_t\}}\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\left(U_{C_{it}^*}S_t - U_{C_{iit}}\right)h\left(S_t\right)C_t^* - U_{C_{iit}}A_{it}\Omega^*\left(\frac{S_t}{S_{t-1}}\pi_t^*\left(\{C_t^*\}_t\right)\right)\right],$$

which becomes identical to the private price-setting condition (6) once we use (A27). Thus, we have verified all our guesses.

Finally, to characterize the optimal U.S. monetary policy, we rewrite the FOC wrt C_t^* as

$$0 = \left(U_{C_{it}^*} - \frac{U_{C_{iit}}}{S_{it}}\right) S_{it} h\left(S_{it}\right) - \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k U_{C_{it+k}^*} \sum_{h \in H_{t+k}} \left(B_{it+k+1}^h - B_{it+k}^h\right) \frac{\partial \mathcal{Q}_{t+k}^h \left(\{C_t^*\}_t\right)}{\partial C_t^*}$$
$$- \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k U_{C_{iit+k}} A_{it+k} \Omega^{*\prime} \left(\frac{S_{it+k}}{S_{it+k-1}} \pi_{t+k}^* \left(\{C_t^*\}_t\right)\right) \frac{S_{it+k}}{S_{it+k-1}} \frac{\partial \pi_{t+k}^*}{\partial C_t^*}.$$

Use (A27) and the household's optimality conditions (1) – (2) to further rewrite it as

$$0 = \left(1 + \frac{U_{L_{it}}}{A_{it} S_{it} U_{C_{it}^*}}\right) S_{it} h\left(S_{it}\right) C_t^* - \mathbb{E}_t \sum_{k=0}^{\infty} \left(\beta^k \frac{U_{C_{it+k}^*}}{U_{C_{it}^*}}\right) \left(\frac{W_{it+k}}{P_{t+k}^*} \Omega_{t+k}^{*\prime}\right) \left(\frac{S_{it+k}}{S_{it+k-1}} \pi_{t+k}^* \frac{\partial \log \pi_{t+k}^*}{\partial \log C_t^*}\right)$$
$$- \mathbb{E}_t \sum_{k=0}^{\infty} \left(\beta^k \frac{U_{C_{it+k}^*}}{U_{C_{it}^*}}\right) \left(\sum_{h \in H_{t+k}} \mathcal{Q}_{t+k}^h \left(B_{it+k+1}^h - B_{it+k}^h\right) \frac{\partial \log \mathcal{Q}_{t+k}^h}{\partial \log C_t^*}\right).$$

This is equivalent to (12), once the appropriate notation is used, including $\pi_{it+k}^* \equiv \frac{S_{it+k}}{S_{it+k-1}} \pi_{t+k}^*$.

D.2 Proof of Corollary 4.1

To prove the result, it is sufficient to focus on productivity shocks alone. We also assume that prices are fully sticky and that the preferences are defined by

$$U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right) = \frac{C_{it}^{1-\sigma}}{1-\sigma} - L_{it}, \quad C_{it} = \left[(1-\gamma)^{\frac{1}{\theta}} C_{iit}^{\frac{\theta-1}{\theta}} + \gamma C_{it}^{*\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}.$$

Next, we consider a version of the model with complete asset markets. Then the no-arbitrage condition (4) and the budget constraint (8) change to

$$\beta \mathbb{E}_t \frac{U_{C_{it+1}^*}}{U_{C_{it}^*}} \frac{Z_t P_t^*}{Z_{t+1} P_{t+1}^*} = 1, \tag{A28}$$

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t Z_t P_t^* \gamma \left[\left(\frac{P_{it}^*}{P_t^*}\right)^{1-\varepsilon} Y_t^* - \left(\frac{\mathcal{E}_{it} P_t^*}{P_{it}}\right)^{-\theta} C_{it}\right] = 0,$$

where Z_t is the price of an Arrow-Debreu security that pays one dollar in a specific state of the world, and the global demand Y_t^* is defined as $Y_t^* \equiv \int \left(\frac{\mathcal{E}_{jt} P_t^*}{P_{jt}}\right)^{-\theta} C_{jt} \mathrm{d}j$. Finally, we also impose $\sigma\theta = 1$.

Note that in equilibrium, the welfare is equal to the value of the Lagrangian as all constraints hold with equality. Thus, instead of comparing welfare across countries we can compare the values of the Lagrangians. Next, to eliminate the first-order differences in optimal policy across countries we consider the autarky limit $\gamma \to 0$. However, at the point of $\gamma = 0$, all countries are ex-ante symmetric and achieve the same welfare, or have the same Lagrangians, $\left(\mathcal{L}^{US} - \mathcal{L}^{nUS}\right)|_{\gamma=0} = 0$. Instead, we focus on the limit $\gamma \to 0$, as the welfare across countries starts to differ as soon as we deviate from the autarky point:

$$\lim_{\gamma \to 0} \frac{\mathcal{L}^{US} - \mathcal{L}^{nUS}}{\gamma} = \lim_{\gamma \to 0} \left(\frac{\mathrm{d}\mathcal{L}^{US}}{\mathrm{d}\gamma} - \frac{\mathrm{d}\mathcal{L}^{nUS}}{\mathrm{d}\gamma}\right) = \frac{\mathrm{d}\mathcal{L}^{US}}{\mathrm{d}\gamma}\Big|_{\gamma=0} - \frac{\mathrm{d}\mathcal{L}^{nUS}}{\mathrm{d}\gamma}\Big|_{\gamma=0}.$$

Non-U.S. Recall the policy problem of a non-U.S. economy from Section C.1. Write down the Lagrangian for this problem, keeping only the binding constraints:

$$
\mathcal{L}^{nUS} \equiv \mathbb{E}\left[\frac{C_{it}^{1-\sigma}}{1-\sigma} - L_{it} + \lambda_{it}\left(A_{it}L_{it} - (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} C_{it} - \gamma\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^*\right)\right.
$$

$$
\left. -\mu_i\gamma Z_t\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta} C_{it}\right) + \eta_{it}\left(1-(1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} - \gamma\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta}\right)\right].
$$

We fix all primitives of the model as we change only the openness parameter γ and investigate how it affects the value of $\mathcal{L}^{nUS}$. Parameter γ enters $\mathcal{L}^{nUS}$ both directly and indirectly through the equilibrium values of the global variables (Y_t^*, Z_t, P_t^*) and of the local non-U.S. variables (C_{it}, L_{it}, etc.). From the envelope theorem, the effects of the latter variables are all zero: the optimality conditions for the non-U.S. economy ensure that the derivatives of the Lagrangian with respect to all local variables (including the Lagrange multipliers) are zero. Then we need to consider only the partial derivative wrt γ and the derivatives wrt all global variables:

$$
\frac{\mathrm{d}\mathcal{L}^{nUS}}{\mathrm{d}\gamma} = \mathbb{E}\left[\lambda_{it}\left(\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} C_{it} - \left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^*\right) + \eta_{it}\left(\left(\frac{P_{iit}}{P_{it}}\right)^{1-\theta} - \left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta}\right)\right.
$$

$$
- \mu_i Z_t\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta} C_{it}\right) - \gamma\left(\lambda_{it}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} + \mu_i Z_t P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\right)\frac{\mathrm{d}Y_t^*}{\mathrm{d}\gamma}
$$

$$
- \gamma\left(\lambda_{it}\varepsilon\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* + \mu_i Z_t\left(P_{it}^*\varepsilon\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - (1-\theta)\,P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta} C_{it}\right)\right) P_t^{*-1}\frac{\mathrm{d}P_t^*}{\mathrm{d}\gamma}
$$

$$
\left. -\gamma\eta_{it}\frac{1-\theta}{P_t^*}\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta}\frac{\mathrm{d}P_t^*}{\mathrm{d}\gamma} - \mu_i\gamma\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta} C_{it}\right)\frac{\mathrm{d}Z_t}{\mathrm{d}\gamma}\right].
$$

We evaluate this derivative in the autarky limit $\gamma = 0$. Note that all terms with the derivatives of the global variables drop out. Moreover, the price index constraint implies $P_{it} = P_{iit}$, and the optimal policy (the marginal cost stabilization) (A27) collapses to $C_{it}^{\sigma} = A_{it}$. Also, solving for the optimal policy yields $\lambda_{it} = C_{it}^{-\sigma}$ and $\eta_{it} = \frac{\theta}{1-\theta}C_{it}^{1-\sigma}$. Finally, the budget constraint implies

$$
\mathbb{E}\mu_i Z_t\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - P_t^*\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{-\theta} C_{it}\right) = 0,
$$

since μ_i is just a constant. After using all of these conditions, we arrive at

$$
\frac{\mathrm{d}\mathcal{L}^{nUS}}{\mathrm{d}\gamma}\,|_{\gamma=0} = \mathbb{E}\left[\frac{1}{1-\theta}A_{it}^{\frac{1}{\sigma}-1} - A_{it}^{-1}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon} Y_t^* - \frac{\theta}{1-\theta}A_{it}^{\frac{1}{\sigma}-1}\left(\frac{\mathcal{E}_{it}P_t^*}{P_{it}}\right)^{1-\theta}\right].
$$

U.S. Recall from Section D.1 that the U.S. chooses global variables Y_t^*, Z_t, and P_t^*. Therefore, all global terms drop out from $\mathrm{d}\mathcal{L}^{US}/\mathrm{d}\gamma$ due to the envelope theorem. Also, one can show that the global constraints do not bind at the autarky point $\gamma = 0$. Crucially, the autarky limit also implies that the optimal U.S. policy is exactly the same as the non-U.S. policy and stabilizes domestic marginal costs. Therefore, repeating the same steps as above results in the same expression up to the $\mathcal{E}_{it} = 1$.

The difference Denote all U.S. variables with a subscript i and all variables of a non-U.S. country with j. Use the ex-ante symmetry of all non-U.S. countries so that $P_t^* = P_{jt}^*$, but keep $P_t^* \neq P_{it}^*$. Assume that shocks in all countries are identically distributed and hence, $\mathbb{E}A_{it}^{\frac{1}{\sigma}-1}=\mathbb{E}A_{jt}^{\frac{1}{\sigma}-1}$. Then the difference in welfare becomes

$$\frac{\mathrm{d}\left(\mathcal{L}^{US}-\mathcal{L}^{nUS}\right)}{\mathrm{d}\gamma}\mid_{\gamma=0}=\mathbb{E}\left[\left(\left(A_{jt}^{-1}-A_{it}^{-1}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\right)Y_t^*+\frac{\theta}{1-\theta}\left(A_{jt}^{\frac{1}{\sigma}-1}\left(\frac{\mathcal{E}_{jt}P_t^*}{P_{jt}}\right)^{1-\theta}-A_{it}^{\frac{1}{\sigma}-1}\left(\frac{P_t^*}{P_{it}}\right)^{1-\theta}\right)\right]\right].$$

To get rid of the nominal exchange rate $\mathcal{E}_{jt}$, use the risk-sharing condition (A28), which in a static model with ex-ante symmetric non-U.S. countries reduces to $\mathcal{E}_{jt}C_{jt}^{-\sigma}/P_{jt}=Z_t$. For the U.S., the same condition becomes $C_{it}^{-\sigma}/P_{it}=\Lambda_i Z_t$, where Λ_i is a constant that describes the wealth of the U.S. relative to the rest of the world. Combined with the marginal cost stabilization, this condition implies $P_{it}A_{it}\Lambda_i Z_t=1$. Substitute these risk-sharing conditions along with $\frac{1}{\sigma}=\theta$ into the definition of the global demand:

$$Y_t^*\equiv\int\left(\frac{\mathcal{E}_{jt}P_t^*}{P_{jt}}\right)^{-\theta}C_{jt}\mathrm{d}j=P_t^{*-\theta}P_{it}^{\theta}\Lambda_i^{\theta}A_{it}^{\theta}.$$

After using these conditions, the welfare difference reduces to

$$\frac{\mathrm{d}\left(\mathcal{L}^{US}-\mathcal{L}^{nUS}\right)}{\mathrm{d}\gamma}\mid_{\gamma=0}=\mathbb{E}\left[\left(A_{it}^{\theta}A_{jt}^{-1}-A_{it}^{\theta-1}\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}\right)P_t^{*-\theta}P_{it}^{\theta}\Lambda_i^{\theta}+\frac{\theta}{\theta-1}\left(\frac{P_t^*}{P_{it}}\right)^{1-\theta}A_{it}^{\theta-1}\left(1-\Lambda_i^{\theta-1}\right)\right].$$

To get rid of prices P_t^* and P_{it}^*, we use the U.S. export price setting, which under domestic marginal cost stabilization is just $P_{it}^*=\frac{\varepsilon}{\varepsilon-1}P_{iit}$, and the non-U.S. export price setting (see Section D.1), which under the optimal policy collapses to

$$\mathbb{E}\left(\mathcal{E}_{jt}P_t^*-\frac{\varepsilon}{\varepsilon-1}P_{jjt}\right)\frac{C_{jt}^{-\sigma}}{P_{jt}}Y_t^*=0.$$

Once again, substitute in the risk-sharing, other conditions from above, and $\frac{1}{\sigma}=\theta$ to simplify this expression to

$$P_t^*=P_{it}^*\Lambda_i\frac{\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}}{\mathbb{E}A_{it}^{\theta-1}}.$$

To get rid of the wealth constant Λ_i, we use the U.S. budget constraint

$$\mathbb{E}Z_t\left(P_{it}^*\left(\frac{P_{it}^*}{P_t^*}\right)^{-\varepsilon}Y_t^*-P_t^*\left(\frac{P_t^*}{P_{it}}\right)^{-\theta}C_{it}\right)=0,$$

which after the same manipulations reduces to

$$\Lambda_i=\left(\frac{\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}}{\mathbb{E}A_{it}^{\theta-1}}\right)^{\frac{1-\varepsilon}{\theta+\varepsilon-1}}.$$

Using all these conditions results in

$$\frac{\mathrm{d}\left(\mathcal{L}^{US}-\mathcal{L}^{nUS}\right)}{\mathrm{d}\gamma}\mid_{\gamma=0}=\left(\frac{\theta}{\theta-1}\frac{\varepsilon}{\varepsilon-1}-1\right)\left(1-\left(\frac{\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}}{\mathbb{E}A_{it}^{\theta-1}}\right)^{\frac{(1-\varepsilon)(\theta-1)}{\theta+\varepsilon-1}}\right)\frac{\left(\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}\right)^{\frac{\theta}{\theta+\varepsilon-1}}}{\left(\mathbb{E}A_{it}^{\theta-1}\right)^{\frac{1-\varepsilon}{\theta+\varepsilon-1}}}\left(\frac{P_t^*}{P_{it}}\right)^{-\theta}.$$

As long as $\theta>0$ and $\varepsilon>1$, this difference is non-negative whenever $\mathbb{E}A_{it}^{\theta-1}\leq\mathbb{E}A_{it}^{\theta}A_{jt}^{-1}$. Take a second-order approximation to express this condition as $-2\theta\left(\mathbb{E}a_{it}^2-\mathbb{E}a_{it}a_{jt}\right)\leq 0$, which is true since $\mathbb{E}a_{it}a_{jt}\leq\mathbb{E}a_{it}^2$.

D.3 Proof of Proposition 5

We now allow for asymmetric trade flows, so that import price indices can vary country by country. Specifically, the price indices are determined by

$$\int \frac{P_{jt}^*}{\mathcal{P}_{it}^*} h_{ji}\left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*}\right) \mathrm{d}j = 1,$$

where $\varpi_{jit} \equiv \frac{P_{jt}^*}{\mathcal{P}_{it}^*} h_{ji}\left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*}\right)$ is the market share of country j in country's i imports, $\int \varpi_{jit} \mathrm{d}j = 1$. $\mathcal{P}_{it}^*$ is the dollar import price index in country i. Also, because each country has its own import bundle, in this section we assume that international securities pay in dollars, not in units of import bundle as before.

Note that the price-setting condition (6) drops out from the policy problem due to availability of state-dependent production subsidies in each country. Similarly, we drop the no-arbitrage condition (4) due to the presence of state-contingent taxes (capital controls) $\{\tau_{it}^h\}$ that can implement any feasible portfolio choice. We guess (and verify later) that the price-setting condition (5) does not bind. Moreover, we ignore constraint (10) as well as all other equilibrium conditions for the U.S. because we drop the U.S. welfare with all of its variables from the objective function. The reason is that the U.S. has zero size in the global economy, and thus we can neglect the effect of their welfare on the total welfare. But the U.S. policy has significant effects on global outcomes, and thus we maximize our global objective function with respect to policies in all countries, including the U.S.

Then the policy problem of a global planner could be written as

$$\max_{\left\{C_{iit}, C_{it}^*, L_{it}, \left\{B_{it+1}^h\right\}_h, \pi_{iit}, P_{it}^*, \mathcal{P}_{it}^*, \mathcal{Q}_t^h\right\}_{i,t}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t \int U\left(C_{iit}, C_{it}^*, L_{it}, \xi_{it}\right) \mathrm{d}i$$

$$\text{s.t. } A_{it} L_{it} = C_{iit} + \int h_{ij}\left(\frac{P_{it}^*}{\mathcal{P}_{jt}^*}\right) C_{jt}^* \mathrm{d}j + A_{it}\left[\Omega\left(\pi_{iit}\right) + \Omega^*\left(\frac{P_{it}^*}{P_{it-1}^*}\right)\right],$$

$$\sum_{h \in H_t} \mathcal{Q}_t^h B_{it+1}^h - \sum_{h \in H_{t-1}} \left(\mathcal{Q}_t^h + \mathcal{D}_t^h\right) B_{it}^h = P_{it}^* \int h_{ij}\left(\frac{P_{it}^*}{\mathcal{P}_{jt}^*}\right) C_{jt}^* \mathrm{d}j - \mathcal{P}_{it}^* C_{it}^* + \psi_{it},$$

$$\int \frac{P_{jt}^*}{\mathcal{P}_{it}^*} h_{ji}\left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*}\right) \mathrm{d}j = 1, \quad \int B_{jt+1}^h \mathrm{d}j = 0.$$

To prove the result, it's enough to consider just four of the optimality conditions:

- wrt C_{iit}:
$$0 = U_{C_{iit}} - \lambda_{it},$$
- wrt C_{it}^*:
$$0 = U_{C_{it}^*} - \int \lambda_{jt} h_{ji}\left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*}\right) \mathrm{d}j + \int \mu_{jt} P_{jt}^* h_{ji}\left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*}\right) \mathrm{d}j - \mu_{it} \mathcal{P}_{it}^*,$$
- wrt L_{it}:
$$0 = U_{L_{it}} + \lambda_{it} A_{it},$$
- wrt π_{iit}:
$$\{\pi_{iit}\} = \arg\max_{\{\pi_t\}} \mathbb{E}\sum_{t=0}^{\infty} \beta^t \left[-\lambda_{it} A_{it} \Omega\left(\pi_t\right)\right].$$

Use the FOC wrt C_{iit} to substitute for λ_{it}. Then the FOC wrt L_{it} implies marginal cost stabilization in each country, $U_{C_{iit}} = -U_{L_{it}}/A_{it}$. Thus, the optimality condition wrt π_{iit} is also satisfied as $\pi_{iit} = 1$. Thus, we have shown the monetary policy in each non-U.S. country stabilizes domestic prices, and the price-setting condition (5) does not bind.

Rewrite the FOC wrt C^*_{it} as

$$0 = \frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}} + \int \frac{U_{L_{jt}}}{A_{jt} P^*_{jt}} \varpi_{jit} \mathrm{d}j + \int \mu_{jt} \varpi_{jit} \mathrm{d}j - \mu_{it}.$$

We can further regroup terms to arrive at

$$\left(\mu_{it} - \frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}} \right) = \int \left(\mu_{jt} - \frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}} \right) \varpi_{jit} \mathrm{d}j + \int \left[1 + \frac{U_{L_{jt}} \mathcal{P}^*_{jt}}{A_{jt} U_{C^*_{jt}} P^*_{jt}} \right] \frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}} \varpi_{jit} \mathrm{d}j,$$

which is equivalent to equation (14).

Next, ϖ_{jit} can be interpreted as a Markov kernel with a corresponding invariant measure $\upsilon_{it} \geq 0$. Multiply all terms in equation (14) by υ_{it} and integrate:

$$\int \upsilon_{it} \left(\mu_{it} - \frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}} \right) \mathrm{d}i = \int \int \upsilon_{it} \varpi_{jit} \left(\mu_{jt} - \frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}} \right) \mathrm{d}j \mathrm{d}i + \int \int \upsilon_{it} \varpi_{jit} \frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}} \tilde{\tau}^*_{jt} \mathrm{d}j \mathrm{d}i,$$

Use the fact that $\int \varpi_{jit} \upsilon_{it} \mathrm{d}i = \upsilon_{jt}$ to obtain

$$\int \upsilon_{it} \left(\mu_{it} - \frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}} \right) \mathrm{d}i = \int \upsilon_{jt} \left(\mu_{jt} - \frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}} \right) \mathrm{d}j + \int \upsilon_{jt} \frac{U_{C^*_{jt}}}{\mathcal{P}^*_{jt}} \tilde{\tau}^*_{jt} \mathrm{d}j.$$

It follows that the optimal U.S. policy rule is given by

$$\int \upsilon_{it} \frac{U_{C^*_{it}}}{\mathcal{P}^*_{it}} \tilde{\tau}^*_{it} \mathrm{d}i = 0.$$

E Numerical Analysis

Equilibrium conditions The preferences and production technology are described at the beginning of Section 4.1. The demand for individual domestic products within a region i can be expressed as

$$C_{iit}(\omega) = \left(\frac{P_{iit}(\omega)}{P_{iit}} \right)^{-\varepsilon} C_{iit}, \quad P_{iit} = \left(\int_0^1 P_{iit}(\omega)^{1-\varepsilon} \mathrm{d}\omega \right)^{\frac{1}{1-\varepsilon}}.$$

The demand for individual products that are imported from j to i is

$$C_{jit}(\omega) = \left(\frac{P^*_{jt}(\omega)}{P^*_{jt}} \right)^{-\varepsilon} \left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}} \right)^{-\eta} C^*_{it}, \quad P^*_{jt} = \left(\int_0^1 P^*_{jt}(\omega)^{1-\varepsilon} \mathrm{d}\omega \right)^{\frac{1}{1-\varepsilon}},$$

where P^*_{jt} is the export price index of region j, and $\mathcal{P}^*_{it}$ is the import price index of region i.

Since the U.S. exporters may charge different prices for their customers from other U.S. regions and the rest of the world, the import price index $\mathcal{P}^*_{it}$ can be different for the U.S. and the non-U.S. regions. We define it as[5]

$$\mathcal{P}^*_{it} = \begin{cases} \left(nP^{*1-\eta}_{0t} + P^{*1-\eta}_{t}\right)^{\frac{1}{1-\eta}}, & \text{if } i > n, \\ \left(nP^{*1-\eta}_{00t} + P^{*1-\eta}_{t}\right)^{\frac{1}{1-\eta}}, & \text{if } i \leq n, \end{cases} \tag{A29}$$

where P^*_{00t} is the bundle of prices that U.S. exporters charge when they ship their products to other regions within the U.S., and P^*_t is the bundle of export prices from the rest of the world,

$$P^*_{00t} = \left(\int_0^1 P^*_{00t}(\omega)^{1-\varepsilon}\,\mathrm{d}\omega\right)^{\frac{1}{1-\varepsilon}}, \quad P^*_t = \left(\int_n^1 P^{*1-\eta}_{jt}\mathrm{d}j\right)^{\frac{1}{1-\eta}}. \tag{A30}$$

Finally, the demand for all imported and domestic products can be expressed as

$$C_{iit} = (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta} C_{it}, \quad C^*_{it} = \gamma\left(\frac{\mathcal{E}_{it}\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta} C_{it},$$

$$P_{it} = \left((1-\gamma)\,P^{1-\theta}_{iit} + \gamma\left(\mathcal{E}_{it}\mathcal{P}^*_{it}\right)^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{A31}$$

The firms from a U.S. region $i \leq n$ have three sources of demand for its products: from the same region, from other U.S. regions, from the rest of the world. The product market clearing condition for a U.S. region i is

$$A_{it}X^{\alpha}_{it}N^{1-\alpha}_{it} = (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \gamma\int_0^n \left(\frac{P^*_{iit}}{\mathcal{P}^*_{jt}}\right)^{-\eta}\left(\frac{\mathcal{P}^*_{jt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j$$
$$+\,\gamma\int_n^1 \left(\frac{P^*_{it}}{\mathcal{P}^*_{jt}}\right)^{-\eta}\left(\frac{\mathcal{E}_{jt}\mathcal{P}^*_{jt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j,$$

where we have used the symmetry of all firms within a region. We further use the fact that in equilibrium all U.S. regions should have symmetric outcomes (as there are no region-specific shocks within the U.S.), and obtain

$$A_{it}X^{\alpha}_{it}N^{1-\alpha}_{it} = (1-\gamma)\left(\frac{P_{iit}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \gamma n\left(\frac{P^*_{iit}}{\mathcal{P}^*_{it}}\right)^{-\eta}\left(\frac{\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \gamma\left(\frac{P^*_{it}}{\mathcal{P}^*_{t}}\right)^{-\eta} Y^*_t, \tag{A32}$$

where $Y^*_t \equiv \int_n^1 \left(\frac{\mathcal{E}_{jt}\mathcal{P}^*_{jt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt})\,\mathrm{d}j$ is the rest of the world demand, and $\mathcal{P}^*_t$ without a region subscript denotes the non-U.S. import price index, $\mathcal{P}^*_t \equiv \mathcal{P}^*_{jt}$ for $j > n$. The product market clearing condition for a non-U.S. region $j > n$ is similar, but the export prices are the same for both U.S. and non-U.S. regions,

$$A_{jt}X^{\alpha}_{jt}N^{1-\alpha}_{jt} = (1-\gamma)\left(\frac{P_{jjt}}{P_{jt}}\right)^{-\theta}(C_{jt}+X_{jt}) + \gamma n\left(\frac{P^*_{jt}}{\mathcal{P}^*_{it}}\right)^{-\eta}\left(\frac{\mathcal{P}^*_{it}}{P_{it}}\right)^{-\theta}(C_{it}+X_{it}) + \gamma\left(\frac{P^*_{jt}}{\mathcal{P}^*_{t}}\right)^{-\eta} Y^*_t. \tag{A33}$$

We assume that international bonds are denominated in units of rest-of-the-world import bundle with the price P^*_t. Moreover, there are quadratic portfolio adjustment costs. Similar to the Rotemberg price-adjustment costs, these cost are set in labor units. Then, the labor market clearing condition can be written as

$$L_{it} = N_{it} + \frac{\varphi}{2}(1-\gamma)(\pi_{iit}-1)^2 + \frac{\varphi}{2}\gamma(\pi^*_{it}-1)^2 + \frac{\upsilon}{2}B^2_{it+1}, \tag{A34}$$

[5]Note that we have used symmetry across all U.S. regions to write this formula, as $P^*_{it} = P^*_{0t}$ and $P^*_{iit} = P^*_{00t}$ for $i \leq n$.

where $\varphi > 0$ is the Rotemberg price-adjustment parameter, $\upsilon > 0$ is the portfolio-adjustment parameter, and B_{it+1} is the amount of bonds. We assume that the steady state bond position is zero for all countries. Then the no-arbitrage condition for international bonds becomes

$$\beta \mathbb{E}_t \frac{C_{it}^{\sigma}}{C_{it+1}^{\sigma}} \frac{P_{it}}{P_{it+1}} \frac{\mathcal{E}_{it+1}}{\mathcal{E}_{it}} \frac{P_{t+1}^*}{P_t^*} = \frac{1}{R_t} + \upsilon \frac{P_{it} C_{it}^{\sigma} L_{it}^{\phi}}{\mathcal{E}_{it} P_t^*} B_{it+1}, \tag{A35}$$

where R_t is the bonds' gross interest rate. Since portfolio-adjustment costs are set in labor units, they are proportional to wages W_{it}, and we have already used the household's labor supply condition $W_{it} = P_{it} C_{it}^{\sigma} L_{it}^{\phi}$.

For a non-U.S. region $j > n$, the region's budget constraint can be written as

$$\begin{aligned} \frac{B_{jt+1}}{R_t} - B_{jt} = \gamma n \frac{\mathcal{P}_{it}^*}{P_t^*} \left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*} \right)^{1-\eta} \left(\frac{\mathcal{P}_{it}^*}{P_{it}} \right)^{-\theta} (C_{it} + X_{it}) + \gamma \frac{\mathcal{P}_t^*}{P_t^*} \left(\frac{P_{jt}^*}{\mathcal{P}_t^*} \right)^{1-\eta} Y_t^* \\ - \gamma \frac{\mathcal{P}_t^*}{P_t^*} \left(\frac{\mathcal{E}_{jt} \mathcal{P}_t^*}{P_{jt}} \right)^{-\theta} (C_{jt} + X_{jt}) + \psi_{jt}, \end{aligned} \tag{A36}$$

where the right-hand side reflects the exports to the U.S., the exports to the rest of the world, the imports, and the financial shock. Similarly, the budget constraint of a single U.S. region $i \le n$ is

$$\frac{B_{it+1}}{R_t} - B_{it} = \gamma n \frac{\mathcal{P}_{0t}^*}{P_t^*} \left(\frac{P_{iit}^*}{\mathcal{P}_{0t}^*} \right)^{1-\eta} \left(\frac{\mathcal{P}_{0t}^*}{P_{0t}} \right)^{-\theta} (C_{0t} + X_{0t}) + \gamma \frac{\mathcal{P}_t^*}{P_t^*} \left(\frac{P_{it}^*}{\mathcal{P}_t^*} \right)^{1-\eta} Y_t^* - \gamma \frac{\mathcal{P}_{it}^*}{P_t^*} \left(\frac{\mathcal{P}_{it}^*}{P_{it}} \right)^{-\theta} (C_{it} + X_{it}) + \psi_{it}.$$

Next, we use the symmetry of all U.S. regions and integrate this budget constraint over i. Then we use the fact that exports to other U.S. regions are equal to the imports from other U.S. regions, and as a result we arrive at

$$\frac{B_{it+1}}{R_t} - B_{it} = \gamma \frac{\mathcal{P}_t^*}{P_t^*} \left(\frac{P_{it}^*}{\mathcal{P}_t^*} \right)^{1-\eta} Y_t^* - \gamma \left(\frac{P_t^*}{\mathcal{P}_{it}^*} \right)^{-\eta} \left(\frac{\mathcal{P}_{it}^*}{P_{it}} \right)^{-\theta} (C_{it} + X_{it}) + \psi_{it}, \tag{A37}$$

where the right-hand side shows the net exports of the U.S. (per region) plus the financial shock.

A single domestic firm in any region i solves the following price-setting problem

$$\{P_{iit}\} = \arg\max_{\{P_t\}} \mathbb{E} \sum_{t=0}^{\infty} \Theta_{i0,t} \left[(P_t - \tau_i MC_{it}) \left(\frac{P_{iit}(\omega)}{P_{iit}} \right)^{-\varepsilon} (C_{iit} + X_{iit}) - (1-\gamma) \tau_{Rii} \frac{\varphi}{2} \left(\frac{P_t}{P_{t-1}} - 1 \right)^2 W_{it} \right],$$

where $\Theta_{i0,t}$ is the stochastic discount factor $\Theta_{i0,t} \equiv \beta^t P_{i0} C_{i0}^{\sigma} / (P_{it} C_{it}^{\sigma})$, and marginal costs depend both on wages and the price of intermediates,

$$MC_{it} \equiv \frac{P_{it}^{\alpha} W_{it}^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} A_{it}} = P_{it} \frac{C_{it}^{\sigma(1-\alpha)} L_{it}^{\phi(1-\alpha)}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} A_{it}}.$$

Simplified, this price-setting condition can be rewritten as

$$\begin{aligned} \pi_{iit} (\pi_{iit} - 1) L_{it}^{\phi} = \beta \mathbb{E}_t \pi_{iit+1} (\pi_{iit+1} - 1) L_{it+1}^{\phi} \\ - \frac{\varepsilon - 1}{\varphi \tau_{Ri}} \left(\frac{P_{iit}}{P_{it}} - \frac{C_{it}^{\sigma(1-\alpha)} L_{it}^{\phi(1-\alpha)}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} A_{it}} \right) C_{it}^{-\sigma} \left(\frac{P_{iit}}{P_{it}} \right)^{-\theta} (C_{it} + X_{it}), \end{aligned} \tag{A38}$$

where $\pi_{iit} \equiv P_{iit}/P_{iit-1}$. Here we use the time-invariant subsidy τ_i to get rid of the markup, $\tau_i \varepsilon / (\varepsilon - 1) = 1$, and also add the subsidy τ_{Ri} on the price-adjustment costs.

A single exporter firm in a non-U.S. region $j > n$ solves the following price-setting problem

$$\{P_{jt}^*\} = \arg\max_{\{P_t\}} \mathbb{E}\sum_{t=0}^{\infty}\Theta_{j0,t}\left[\left(\mathcal{E}_{jt}P_t - \tau_j^* MC_{jt}\right)\gamma\left(\frac{P_{jt}^*(\omega)}{P_{jt}^*}\right)^{-\varepsilon} Y_{jt}^* - \gamma\tau_{Rj}^*\frac{\varphi}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 W_{jt}\right],$$

$$\text{where } Y_{jt}^* \equiv \left(\frac{P_{jt}^*}{\mathcal{P}_t^*}\right)^{-\eta} Y_t^* + n\left(\frac{P_{jt}^*}{\mathcal{P}_{it}^*}\right)^{-\eta}\left(\frac{\mathcal{P}_{it}^*}{P_{it}}\right)^{-\theta}\left(C_{it} + X_{it}\right),$$

so the demand for the firm's products includes the demand from the U.S. and from the rest of the world. This problem leads to the following price-setting condition

$$\pi_{jt}^*\left(\pi_{jt}^* - 1\right)L_{jt}^{\phi} = \beta\mathbb{E}_t\pi_{jt+1}^*\left(\pi_{jt+1}^* - 1\right)L_{jt+1}^{\phi} - \frac{\varepsilon - 1}{\varphi\tau_{Rj}^*}\left(\frac{P_{jt}^*}{P_{jt}} - \frac{\varepsilon\tau_j^*}{\varepsilon - 1}\frac{C_{jt}^{\sigma(1-\alpha)}L_{jt}^{\phi(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_{jt}}\right)C_{jt}^{-\sigma}Y_{jt}^*, \quad \text{(A39)}$$

where $\pi_{jt}^* \equiv P_{jt}^*/P_{jt-1}^*$.

A U.S. exporter from region $i \leq n$ solves a similar problem, but is subject to (potentially) different subsidies when exporting to other U.S. regions and to the rest of the world. Then, the corresponding two price-setting conditions can by ultimately expressed as

$$\begin{aligned}\pi_{iit}^*\left(\pi_{iit}^* - 1\right)L_{it}^{\phi} &= \beta\mathbb{E}_t\pi_{iit+1}^*\left(\pi_{iit+1}^* - 1\right)L_{it+1}^{\phi} \\ &\quad - \frac{\varepsilon - 1}{\varphi\tau_{Rii}^*}\left(\frac{P_{iit}^*}{P_{it}} - \frac{\varepsilon\tau_{ii}^*}{\varepsilon - 1}\frac{C_{it}^{\sigma(1-\alpha)}L_{it}^{\phi(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_{it}}\right)C_{it}^{-\sigma}\left(\frac{P_{iit}^*}{\mathcal{P}_{it}^*}\right)^{-\eta}\left(\frac{\mathcal{P}_{it}^*}{P_{it}}\right)^{-\theta}\left(C_{it} + X_{it}\right),\end{aligned} \quad \text{(A40)}$$

$$\begin{aligned}\pi_{it}^*\left(\pi_{it}^* - 1\right)L_{it}^{\phi} &= \beta\mathbb{E}_t\pi_{it+1}^*\left(\pi_{it+1}^* - 1\right)L_{it+1}^{\phi} \\ &\quad - \frac{\varepsilon - 1}{\varphi\tau_{Ri}^*}\left(\frac{P_{it}^*}{P_{it}} - \frac{\varepsilon\tau_i^*}{\varepsilon - 1}\frac{C_{it}^{\sigma(1-\alpha)}L_{it}^{\phi(1-\alpha)}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}A_{it}}\right)C_{it}^{-\sigma}\left(\frac{P_{it}^*}{\mathcal{P}_t^*}\right)^{-\eta}\frac{Y_t^*}{1-n},\end{aligned} \quad \text{(A41)}$$

where $\pi_{iit}^* \equiv P_{iit}^*/P_{iit-1}^*$.

Also, as a part of the cost minimization problem with Cobb-Douglas production function, each firm always chooses to spend share α on intermediates, and thus to set

$$\frac{X_{it}}{L_{it}} = \frac{\alpha}{1-\alpha}C_{it}^{\sigma}N_{it}^{\phi}, \quad \text{(A42)}$$

where once again we have used the labor supply condition $W_{it}/P_{it} = C_{it}^{\sigma}L_{it}^{\phi}$.

Finally, to close the global equilibrium, we need to add the balance on global international trade or, the same, the balance on international bond,

$$(1-n)B_{jt+1} + nB_{it+1} = 0,$$

where $j > n$ denotes a representative non-U.S. region and $i \leq n$ denotes a representative U.S. region. However, due to Walras' law, this condition follows from the budget constraints (A36) and (A37) and the structure of financial shocks, i.e. $(1-n)\psi_{jt} + n\psi_{it} = 0$.

The non-U.S. policy problem The planner chooses $\left\{C_{jt}, X_{jt}, L_{jt}, N_{jt}, P_{jjt}, P_{jt}, P^*_{jt}, \mathcal{E}_{jt}, \pi_{jjt}, \pi^*_{jt}, B_{jt+1}\right\}_t$ in a representative non-U.S. economy $j > n$ to maximize

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_{jt}^{1-\sigma}}{1-\sigma}-\frac{L_{jt}^{1+\phi}}{1+\phi}\right]$$

subject to the product market clearing (A33), firm's optimality condition (A42), the budget constraint (A36), the labor market clearing (A34), the no-arbitration condition (A35), the price index (A31), the price-setting conditions (A38) and (A39), and the definitions of inflation rates $\pi_{jjt} = P_{jjt}/P_{jjt-1}$ and $\pi^*_{jt} = P^*_{jt}/P^*_{jt-1}$. Note that a single non-U.S. economy takes all foreign variables as given.

When we explore the robustness of our results to the presence of terms of trade externality, we assume that there are no subsidies on exporters, that is $\tau^*_j = \tau^*_{Rj} = 1$. Following the literature, we also assume that production subsidy on domestic producers eliminates domestic markups, $\tau_j = (\varepsilon - 1)/\varepsilon$, and there is no subsidy on their price-adjustment costs, $\tau_{Rj} = 1$.

The U.S. policy problem To solve for the global equilibrium, we assume that the world economy consists of 3 types of countries. There are large U.S. that consist of n regions and we denote its representative region by i. The rest $1-n$ economies make their decisions independently of each other, but all of them have perfectly correlated shocks. So in equilibrium all of them have the same outcomes, and we denote a representative rest-of-the-world region by j. Finally, to evaluate the welfare and the response of a non-U.S. economy to idiosyncratic shocks, we add a zero-size country that we denote by k and that has its own shocks. We ignore this country while solving for the optimal U.S. policy since its zero size implies that it can not affect any of the global variables, but we compute its equilibrium allocation when make appropriate comparisons.

Overall, there are five uncorrelated shocks in the global economy: productivity A_{it} and financial ψ_{it} shocks in the U.S., productivity A_{kt} and financial ψ_{kt} shocks in a small open economy k, productivity A_{jt} shocks in the rest of the world. By construction, the financial shock in the rest of the world is the opposite of the U.S. financial shock, $\psi_{jt} = -n\psi_{it}/(1-n)$.

Solving the U.S. problem, we assume that non-U.S. economies set the optimal time-invariant subsidies

$$\tau_j = \frac{\varepsilon-1}{\varepsilon},\ \tau_{Rj} = 1,\ \tau^*_j = \frac{\eta}{\eta-1}\frac{\varepsilon-1}{\varepsilon},\ \tau^*_{Rj} = \frac{\varepsilon-1}{\eta-1}.$$

Production subsidies set the optimal markups for both destinations, while the price-adjustment subsidy corrects the firm-specific elasticity for the region-specific elastiticity. Under these values of subsidies, the optimal policy in non-U.S. economies reduces to domestic price stabilization, $\pi_{jjt} = 1$. Similarly, we assume that the U.S. subsidies set the optimal markups and correct the demand elasticity for exporters to the rest of the world,

$$\tau_i = \tau^*_{ii} = \frac{\varepsilon-1}{\varepsilon},\ \tau_{Ri} = \tau^*_{Rii} = 1,\ \tau^*_i = \frac{\eta}{\eta-1}\frac{\varepsilon-1}{\varepsilon},\ \tau^*_{Ri} = \frac{\varepsilon-1}{\eta-1}.$$

The planner in the U.S. economy chooses the U.S. quantities $\{C_{it}, X_{it}, L_{it}, N_{it}, B_{it+1}\}_t$, the U.S. prices $\{P_{iit}, P^*_{iit}, P_{it}, P^*_{it}, \mathcal{P}^*_{it}, \pi_{iit}, \pi^*_{iit}, \pi^*_{it}\}_t$, and $\{C_{jt}, X_{jt}, L_{jt}, N_{jt}, P_{jjt}, P_{jt}, P^*_{jt}, \mathcal{E}_{jt}, P^*_t, \mathcal{P}^*_t, R_t, \pi_{jjt}, \pi^*_{jt}, B_{jt+1}\}_t$ in the rest of the world to maximize the U.S. welfare

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_{it}^{1-\sigma}}{1-\sigma}-\frac{L_{it}^{1+\phi}}{1+\phi}\right]$$

subject to the product market clearing in the U.S. (A32) and in the non-U.S. (A33), firm's optimality conditions (A42) in both countries, the budget constraints (A37) and (A36), the labor market clearing conditions (A34) in both countries, the no-arbitration conditions (A35) in both countries, the price-setting conditions of domestic sellers

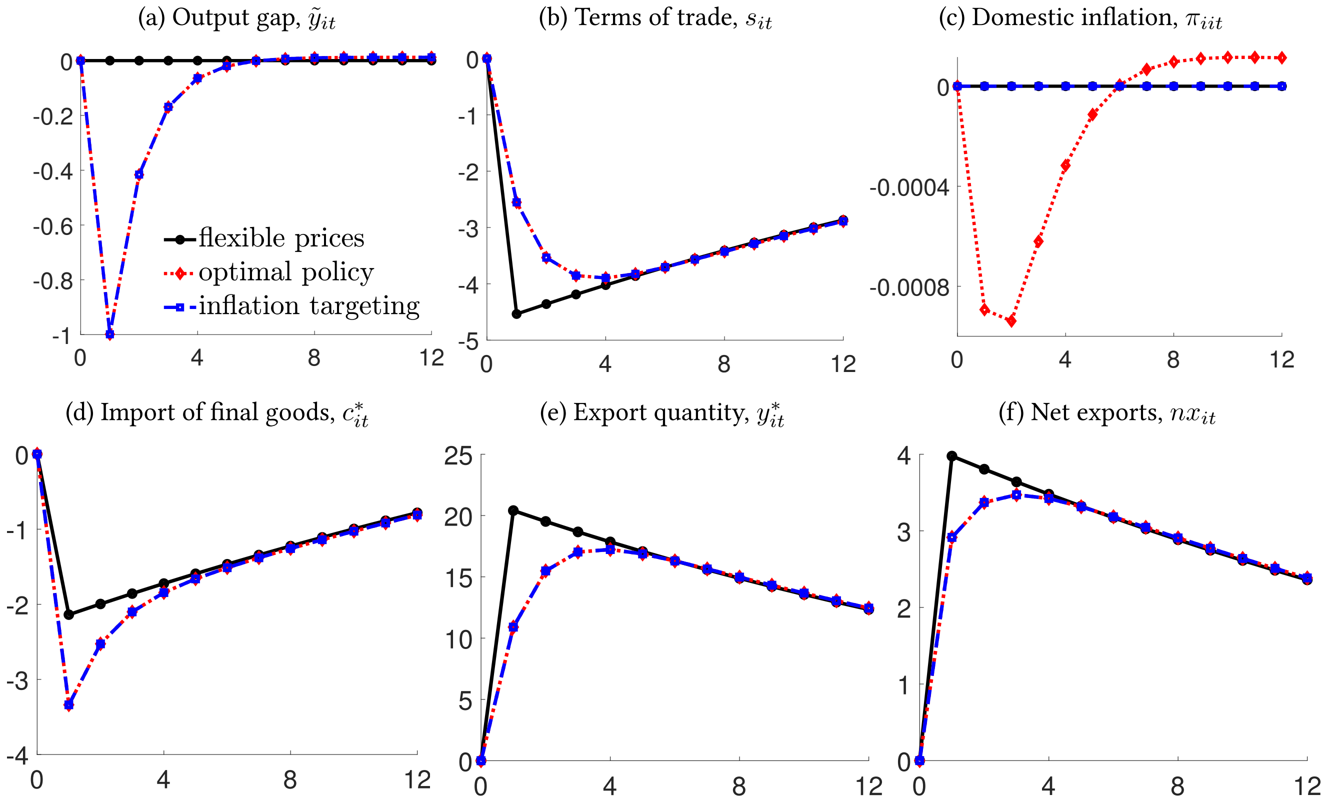
(A38) in both countries, the price-setting conditions of the non-U.S. exporters (A39) and of the U.S. exporters (A41) and (A40), all price index constraints (A29) and (A31) in both countries, the global price index constraint (A30), definitions of five inflation rates, and the optimal policy rule in the non-U.S. economies, $\pi_{jjt} = 1$.[6]

Calibration of price-adjustment costs Following Faia and Monacelli (2008), we linearize the domestic price-setting condition (A38) around the non-stochastic steady state. The elasticity of inflation to the real marginal cost is $(\varepsilon - 1)/\varphi$. This statistic is directly comparable with the Phillips curve derived in a Calvo model, where the same elsticity is $(1-\delta)(1-\beta\delta)/\delta$. Here $1-\delta$ is the probability of resetting the price in any given period. Thus, the average frequency of the price adjustment in the Calvo model is $1/(1-\delta)$, which we equalize to 3 quarters. Then, using our calibrated value of $\beta = 0.99$ and $\varepsilon = 11$, we set $\varphi = 60$ to match elasticities in two models.

Solution methods We use first-order approximations around the non-stochastic steady state for impulse response functions. For welfare comparisons, we follow Fernández-Villaverde, Rubio-Ramírez and Schorfheide (2016) and compute the second-order approximation to the value function. We then calculate the difference between this value function under uncertainty and the value function in a deterministic model with perfect foresight. This difference reflects the welfare costs of uncertainty, which we then convert to consumption units. Finally, we confirm the accuracy of our approximate solution by calculating the Euler equation errors. Following Den Haan (2010), we compute a dynamic version to check whether the errors accumulate over time.

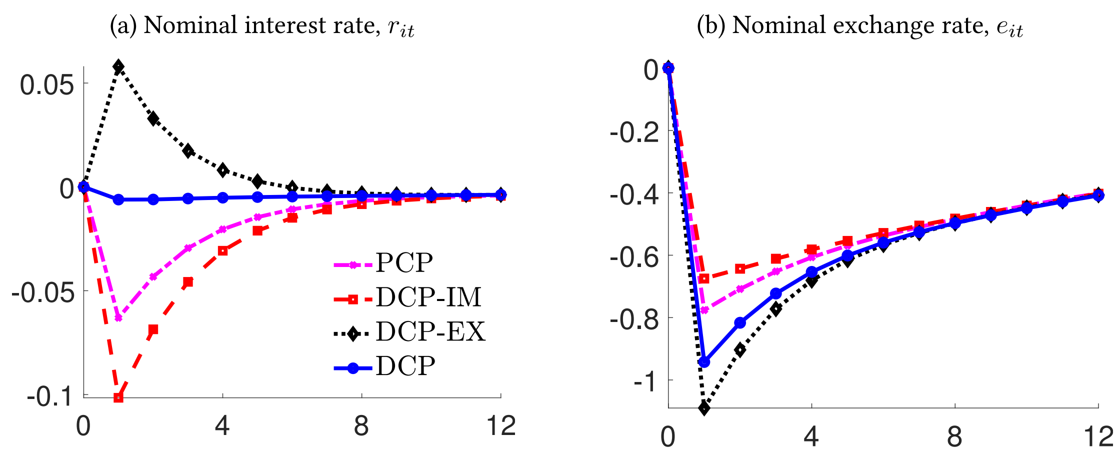
[6]Implicitly, we also include definitions of demand shifters Y_t^* and Y_{jt}^* to the set of constraints. Also, the labor market clearing condition (A34) for the U.S. should include the inflation costs from π_{iit}^* as well.

Figure A1: Impulse responses to local productivity shock a_{it} in a non-U.S. economy



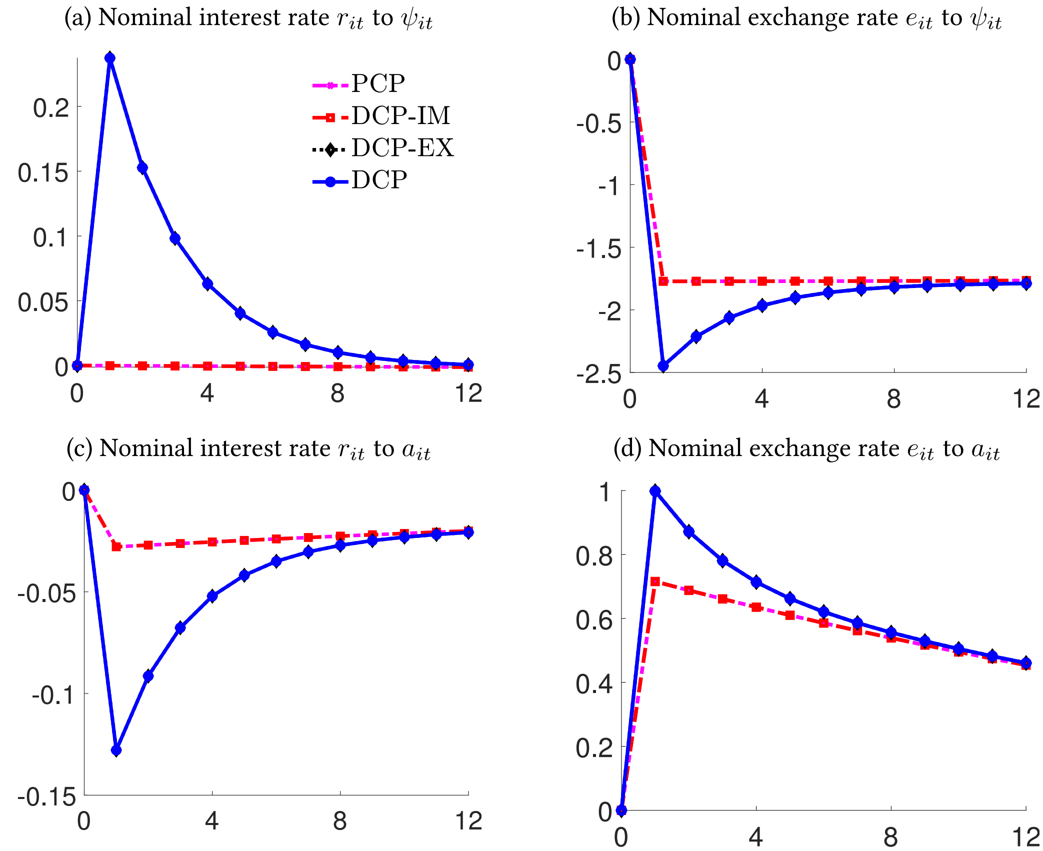
Note: see notes to Figure 1.

Figure A2: Policy response to U.S. productivity shock a_{it} in a non-U.S. economy



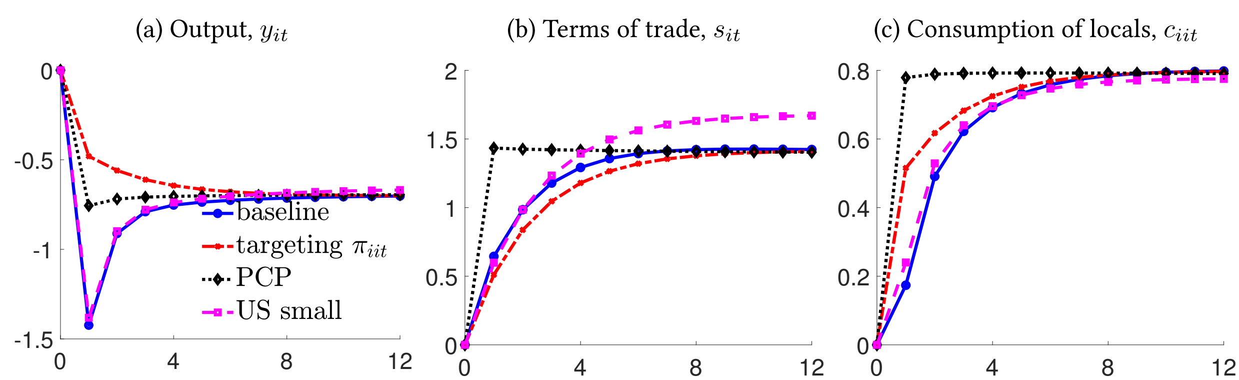
Note: see notes to Figure 3.

Figure A3: Policy response to local shocks in a non-U.S. economy



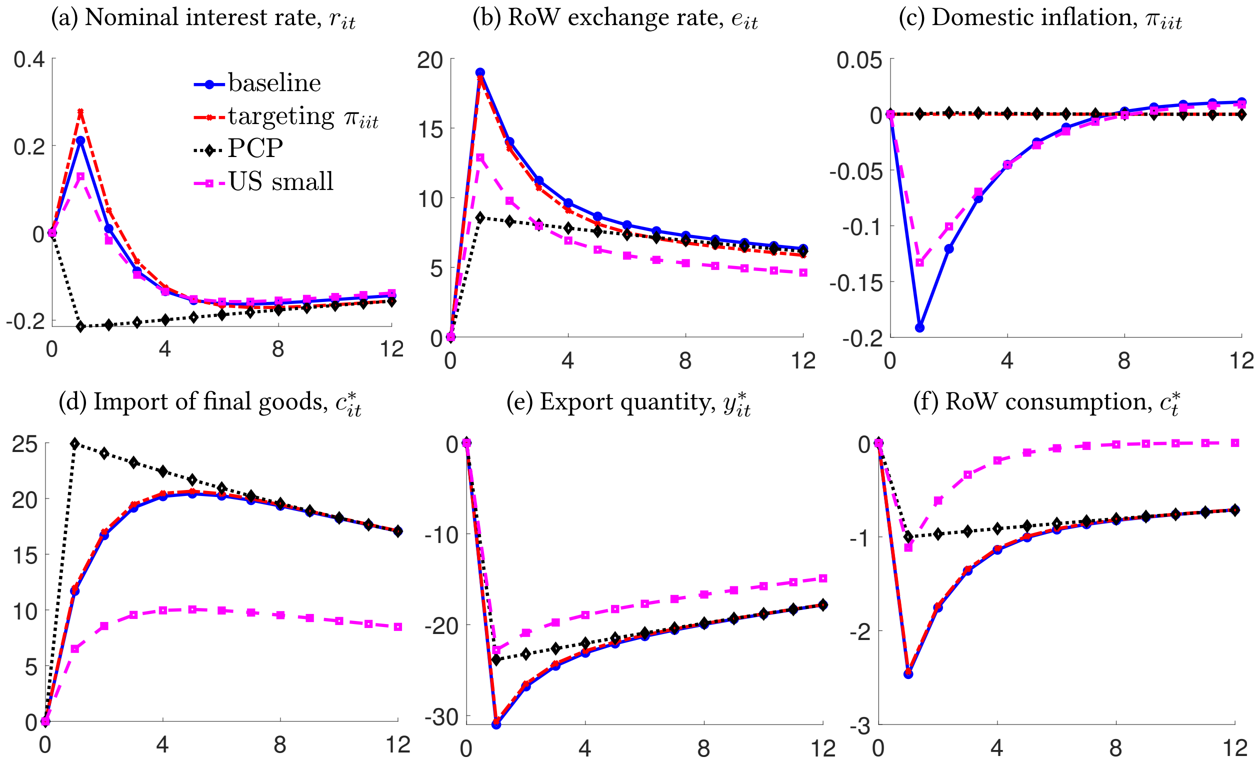
Note: see notes to Figure 3.

Figure A4: Impulse responses to local financial shock ψ_{it} in the U.S.: supplement



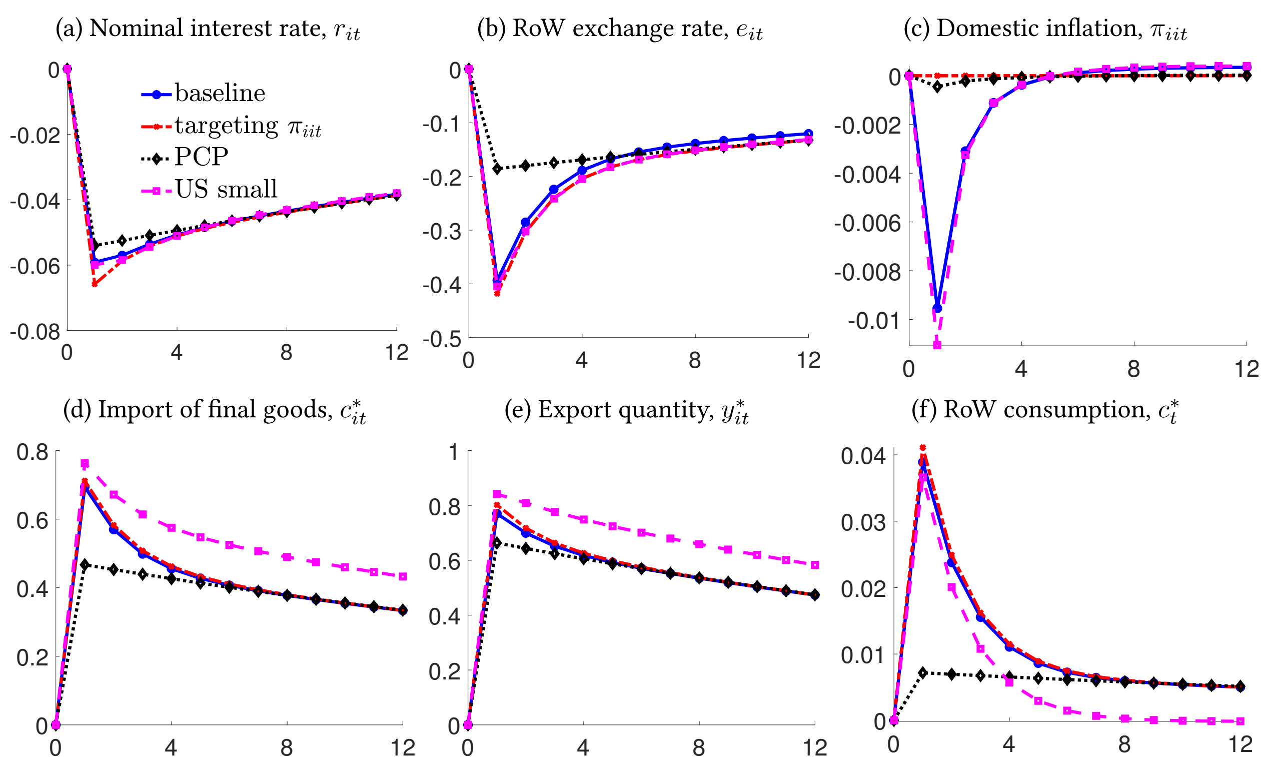
Note: see notes to Figure 4.

Figure A5: Impulse responses to local financial shock ψ_{it} in the U.S. under financial autarky



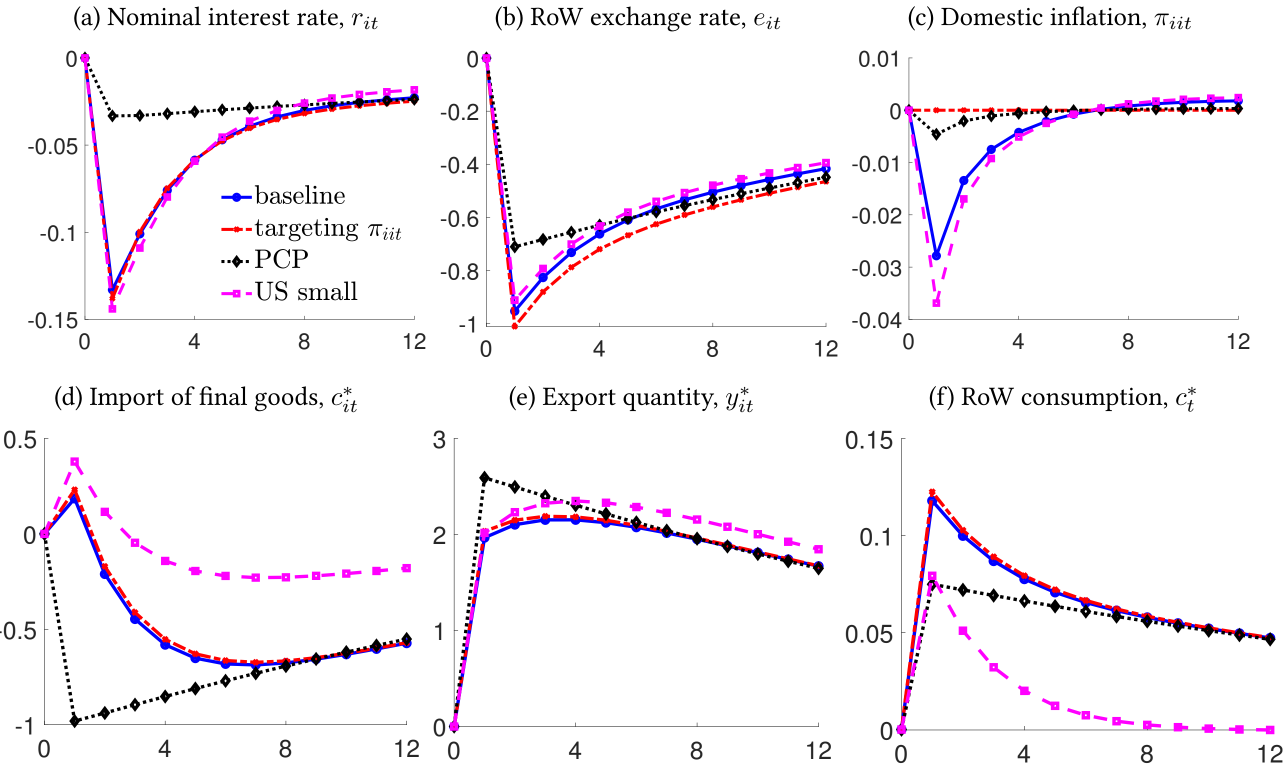
Note: see notes to Figure 4.

Figure A6: Impulse responses to local productivity shock a_{it} in the U.S. under financial autarky



Note: see notes to Figure 4.

Figure A7: Impulse responses to local productivity shock a_{it} in the U.S.



Note: see notes to Figure 4.

Table A2: Empirical and simulated moments

Moments	Data	Model	Moments	Data	Model
A. Exchange rate disconnect:			D. International business cycle moments:		
$\rho(\Delta e)$	≈ 0	-0.1	$\sigma(\Delta c)/\sigma(\Delta gdp)$	0.82	0.61
$\sigma(\Delta e)/\sigma(\Delta gdp)$	5.2	3.1	$\sigma(\Delta l)/\sigma(\Delta gdp)$	0.62	0.67
$\sigma(\Delta e)/\sigma(\Delta c)$	6.3	5.1	$\text{corr}(\Delta c, \Delta gdp)$	0.64	0.71
B. Real exchange rate and the PPP:			$\text{corr}(\Delta l, \Delta gdp)$	0.72	0.61
$\rho(q)$	0.96	0.99	$\text{corr}(\Delta gdp, \Delta gdp^*)$	0.35	0.34
$\sigma(\Delta q)/\sigma(\Delta e)$	0.99	0.81	$\text{corr}(\Delta c, \Delta c^*)$	0.30	0.31
$\text{corr}(\Delta q, \Delta e)$	0.99	1.00	E. Trade moments:		
C. Backus-Smith correlation:			$\sigma(\Delta nx)/\sigma(\Delta q)$	0.10	0.28
$\text{corr}(\Delta q, \Delta c - \Delta c^*)$	-0.20	-0.20	$\text{corr}(\Delta nx, \Delta q)$	≈ 0	0.73

Note: empirical moments are from Chari, Kehoe and McGrattan (2002) and Itskhoki and Mukhin (2021) and are estimated for the U.S. against selected countries for the period from 1973–2017. The simulated moments are obtained from the baseline model with a large U.S. and calibrated shocks.

Table A3: Welfare losses from shocks when the U.S. is a large economy

Shock	non-U.S.			U.S.		
	optimal (1)	$\tilde{y}_{it} = 0$ (2)	PCP (3)	optimal (4)	$\pi_{iit} = 0$ (5)	PCP (6)
Productivity a_{it}:						
local	0.03	0.12	0.02	0.04	0.04	0.04
foreign	0.00	0.08	0.00	–	–	–
global	0.02	0.02	0.02	0.02	0.02	0.02
Financial ψ_{it}:						
local	3.33	3.90	3.30	3.13	3.23	3.56
foreign	-0.06	1.86	-0.15	–	–	–
Total	3.32	5.81	3.19	3.19	3.29	3.61

Note: welfare losses from shocks in equivalent changes of the steady-state consumption (%). Columns 1, 3, 4, 6 assume the optimal monetary policy, column 2 shows the welfare of a non-U.S. economy that targets output gap, and column 5 shows the U.S. welfare when it targets domestic prices. "Foreign" corresponds to a shock in a non-U.S. economy for the U.S. and in the U.S. for a non-U.S. economy. Both "local" and "foreign" include only idiosyncratic shocks, while "global" represents shocks common to all economies. "Total" can differ from the sum of other rows. The values of parameters are the same as in Table 1, except for $n = 0.2$.