# Online Appendix to: Concentration Thresholds for Horizontal Mergers

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## 1 Merger Enforcement Statistics

**Table 7:** FTC horizontal merger enforcement statistics, 1996-2011: Percent of mergers enforced among those receiving a second request, as a function of the post-merger level of the Herfindahl index and the merger-induced change in the Herfindahl index. The total number of mergers receiving a second request are shown in brackets. [Source: U.S. Federal Trade Commission (Table 3.1, 2013).]

| | ***Change in HHI:*** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ***Post-merger HHI:*** | **0-99** | **100-199** | **200-299** | **300-499** | **500-799** | **800-1199** | **1200-2499** | **2500+** | **Total** |
| **0-1799** | 0% [14] | 35.4% [48] | 48.7% [39] | 60.7% [28] | 30.0% [10] | 0.0% [1] | — [0] | — [0] | 40.0% [140] |
| **1800-1999** | 0.0% [4] | 55.6% [9] | 45.5% [11] | 75.0% [16] | 70.6% [17] | — [0] | — [0] | — [0] | 59.6% [57] |
| **2000-2399** | 33.3% [3] | 14.3% [7] | 46.7% [15] | 56.8% [44] | 72.7% [44] | 50.0% [4] | — [0] | — [0] | 58.1% [117] |
| **2400-2999** | 33.3% [3] | 66.7% [6] | 54.5% [11] | 75.0% [24] | 75.9% [58] | 72.2% [36] | — [0] | — [0] | 71.7% [138] |
| **3000-3999** | 25.0% [4] | 60.0% [5] | 71.4% [7] | 64.3% [14] | 64.1% [39] | 77.2% [92] | 73.6% [53] | — [0] | 71.5% [214] |
| **4000-4999** | — [0] | 50.0% [4] | 50.0% [2] | 83.3% [6] | 71.4% [14] | 81.8% [22] | 95.8% [71] | — [0] | 87.4% [119] |
| **5000-6999** | 100% [1] | 100% [6] | 80.0% [10] | 88.9% [9] | 100% [19] | 91.3% [23] | 87.9% [165] | 90.4% [52] | 89.5% [285] |
| **7000+** | — [0] | — [0] | 100% [1] | 100% [1] | 100% [3] | 100% [9] | 96.3% [27] | 99.2% [248] | 99.0% [289] |
| **Total** | 13.8% [29] | 44.7% [85] | 54.2% [96] | 66.9% [142] | 72.5% [204] | 78.6% [187] | 88.0% [316] | 97.7% [300] | 77.6% [1359] |

# 2 Proofs

## 2.1 Proof of Corollary 3

To see part (i), note that the pre-merger marginal cost of product $k \in m$ can be written as

$$c^k = p^k \left[ 1 - \frac{1}{\sigma(1 - s_m) + s_m} \right],$$

where we have used equations (7) and (10). For the merged firm to charge the same prices for all of its products (implying that its post-merger market share is $s_M = s_m + s_n$), the post-merger marginal cost of product $k$ has to satisfy

$$\overline{c}^k = p^k \left[ 1 - \frac{1}{\sigma(1 - s_M) + s_M} \right].$$

Combining, we obtain:

$$\phi^k = \frac{\overline{c}^k - c^k}{c^k} = -\frac{s_M - s_m}{(1 - s_m)[\sigma(1 - s_M) + s_M]}.$$

To see part (ii), note that:

$$\begin{aligned}
\frac{\overline{T}_M}{T_m + T_n} &= \frac{\sum_{k \in M} b^k \left( (1 - \phi) c^k \right)^{1-\sigma}}{\sum_{k \in m} b^k \left( c^k \right)^{1-\sigma} + \sum_{k \in n} b^k \left( c^k \right)^{1-\sigma}} \\
&= \frac{(1 - \phi)^{1-\sigma} \sum_{k \in M} b^k \left( c^k \right)^{1-\sigma}}{\sum_{k \in m} b^k \left( c^k \right)^{1-\sigma} + \sum_{k \in n} b^k \left( c^k \right)^{1-\sigma}} \\
&= (1 - \phi)^{1-\sigma}.
\end{aligned}$$

The assertion then follows from applying Proposition 2.

## 2.2 Proof of Corollary 5

To see part (i), note that the pre-merger marginal cost of product $k \in m$ can be written as

$$c^k = p^k - \frac{\lambda}{1 - s_m},$$

where we have used equations (15) and (16) For the merger to leave all prices unchanged (implying that the post-merger market share is $s_M = s_m + s_n$), the post-merger marginal cost of product $k$ has to satisfy

$$\overline{c}^k = p^k - \frac{\lambda}{1 - s_M}.$$

Combining, we obtain:

$$\Delta c^k = \overline{c}^k - c^k = \frac{\lambda}{1-s_m} - \frac{\lambda}{1-s_M} = \frac{\lambda(s_M - s_m)}{(1-s_M)(1-s_m)}.$$

To see part (ii), note that:

$$\begin{aligned}\frac{\overline{T}_M}{T_m + T_n} &= \frac{\sum_{k\in M} \exp\left(\frac{b^k - c^k - \Delta c}{\lambda}\right)}{\sum_{k\in m} \exp\left(\frac{b^k-c^k}{\lambda}\right) + \sum_{k\in n} \exp\left(\frac{b^k-c^k}{\lambda}\right)} \\ &= \frac{\exp\left(\frac{-\Delta c}{\lambda}\right) \sum_{k\in M} \exp\left(\frac{b^k-c^k}{\lambda}\right)}{\sum_{k\in m} \exp\left(\frac{b^k-c^k}{\lambda}\right) + \sum_{k\in n} \exp\left(\frac{b^k-c^k}{\lambda}\right)} \\ &= \exp\left(\frac{-\Delta c}{\lambda}\right).\end{aligned}$$

The assertion then follows from applying Proposition 3.

## 2.3 Proof of Proposition 4

If the post-merger type $\overline{T}_M$ falls short by a small fraction of the level that would restore consumer surplus after the merger, the shortfall in consumer surplus is given by

$$\begin{aligned}-\frac{dCS(A^*)}{dA}\frac{dA}{d\overline{T}_M}\overline{T}_M &= -\frac{\frac{\overline{T}_M}{A^*} S'\left(\frac{\overline{T}_M}{A^*}\right)}{\frac{\overline{T}_M}{A^*} S'\left(\frac{\overline{T}_M}{A^*}\right) + \sum_{f\notin M} \frac{T_f}{A^*} S'\left(\frac{T_f}{A^*}\right)} \\ &= -\frac{S^{-1}(s_M) S'\left(S^{-1}(s_M)\right)}{S^{-1}(s_M) S'\left(S^{-1}(s_M)\right) + \sum_{f\notin M} S^{-1}(s_f) S'\left(S^{-1}(s_f)\right)} \qquad (1)\end{aligned}$$

where the first equality follows from applying the implicit function theorem to the adding-up condition

$$S\left(\frac{\overline{T}_M}{A^*}\right) + \sum_{f\notin M} S\left(\frac{\overline{T}_f}{A^*}\right) + \frac{A^0}{A^*} = 1.$$

As the number of outsiders is finite, it is straightforward to see that a sum-preserving spread of the outsiders' market shares can be decomposed into a finite number of steps where at each step there is a sum-preserving spread of market shares involving only two outsiders. We now prove that at any such step the denominator on the r.h.s. of equation (1) decreases, from which the result follows.

Let $t_f \equiv T_f/A^*$ and suppose that $t_f > t_g$. We need to show that an increase in $t_f$ and a decrease in $t_g$ such that $S(t_f) + S(t_g)$ remains unchanged induces a reduction in

$t_f S'(t_f) + t_g S'(t_g)$. We have:

$$\left.\frac{d\,[t_f S'(t_f) + t_g S'(t_g)]}{dt_f}\right|_{S(t_f)+S(t_g)=\text{const.}} = S'(t_f)\left[\frac{t_f S''(t_f)}{S'(t_f)} - \frac{t_g S''(t_g)}{S'(t_g)}\right].$$

As $S'(\cdot) > 0$, we thus only need to show that the elasticity of $S'$ is decreasing, i.e.,

$$\frac{d}{dt}\frac{tS''(t)}{S'(t)} < 0.$$

From the proof of Proposition 9 in Nocke and Schutz (2019), we have:

$$S'(t) = \frac{1}{t}\frac{S(t)(1 - S(t))(1 - \alpha S(t))}{1 - S(t) + \alpha S(t)^2},$$
$$S''(t) = -\frac{\alpha}{t^2}\frac{(2 - S(t))S(t)^2(1 - S(t))(1 - \alpha S(t))}{[1 - S(t) + \alpha S(t)^2]^3},$$

where $\alpha = 1$ if demand is of the MNL form and $\alpha = (\sigma - 1/\sigma) < 1$ if it is of the CES form. It follows that

$$\frac{tS''(t)}{S'(t)} = -\frac{\alpha(2 - S(t))S(t)}{[1 - S(t) + \alpha S(t)^2]^2}.$$

We thus have

$$\frac{d}{dt}\frac{tS''(t)}{S'(t)} < 0$$

if and only if

$$[(2 - S(t))S'(t) - S(t)S'(t)][1 - S(t) + \alpha S(t)^2] > 2(2 - S(t))S(t)[-S'(t) + 2\alpha S(t)S'(t)],$$

i.e.,

$$1 + \alpha S(t)^3 > 3\alpha S(t)^2.$$

It can easily be verified that this inequality holds, for any $\alpha \in (0, 1]$ if $S(t) \leq 0.65$.

# 3 Empirical Results for Brewing Mergers using Revenue Shares

Here we present the tables and figures for the empirical analysis of Section 4 when markets shares and the Herfindahl index are revenue-based rather than volume-based.

**Table 8:** Regression of the Required Synergy on Functions of the Herfindahl and the Change in the Herfindahl (Revenue-based)

Dependent Variable: Synergy Required to Prevent Consumer Harm

| | RCNL-1 | | | RCNL-3 | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| hhi | .019 | −.826 | −.636 | .080 | −1.15 | −.883 |
| | (.032) | (.182) | (.170) | (.046) | (.264) | (.266) |
| | [.61] | [−4.54] | [−3.75] | [1.74] | [−4.35] | [−3.32] |
| delta | 2.30 | 2.73 | 2.45 | 3.02 | 3.50 | 2.79 |
| | (.060) | (.270) | (.324) | (.086) | (.390) | (.508) |
| | [38.55] | [10.12] | [7.58] | [35.06] | [8.97] | [5.49] |
| hhi × delta | | −2.84 | −3.66 | | −2.42 | −3.13 |
| | | (1.17) | (1.08) | | (1.70) | (1.70) |
| | | [−2.42] | [−3.38] | | [−1.42] | [−1.84] |
| $\text{hhi}^2$ | | 1.69 | 1.38 | | 2.37 | 1.89 |
| | | (.374) | (.360) | | (.541) | (.566) |
| | | [4.53] | [3.82] | | [4.38] | [3.35] |
| $\text{delta}^2$ | | 2.66 | 7.57 | | .791 | 10.08 |
| | | (1.47) | (2.30) | | (2.12) | (3.61) |
| | | [1.82] | [3.29] | | [.37] | [2.79] |
| constant | −.015 | .088 | .068 | −.033 | .117 | .096 |
| | (.008) | (.022) | (.020) | (.011) | (.031) | (.031) |
| | [−1.88] | [4.03] | [3.43] | [−2.91] | [3.71] | [3.06] |
| Sample | Full | Full | Restricted | Full | Full | Restricted |
| # Observations | 390 | 390 | 343 | 390 | 390 | 343 |
| $R^2$ | .85 | .86 | .80 | .83 | .84 | .74 |

Notes: Dependent variable measured as 0.01 for 1% synergy, hhi is the revenue-based post-merger Herfindahl index scaled between 0 and 1, and delta is the merger-induced change in the revenue-based Herfindahl index scaled between 0 and 1. Standard errors are in parentheses; *t*-statistics are in square brackets.

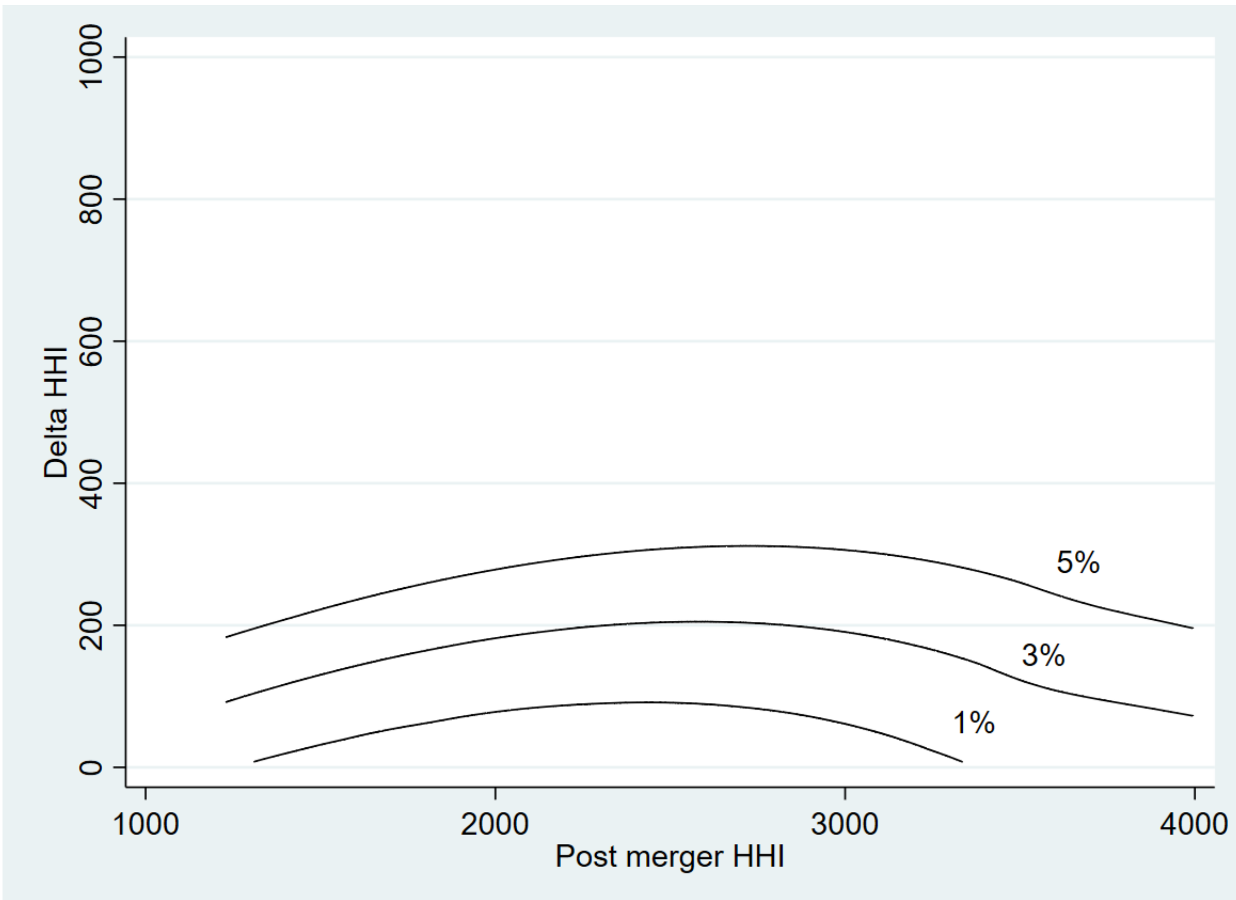


**Figure 11:** Contour plot showing the combinations of the post-merger Herfindahl (labelled here as "post_hhi_val") and the merger-induced change in the Herfindahl ("delta_hhi_val") that have no effect on consumer surplus if there is a 1%, 3%, and 5% synergy due to the merger. Points above (respectively, below) a contour line correspond to mergers that are expected to harm (respectively, benefit) consumers. Based on estimates in Table 8, column (5).

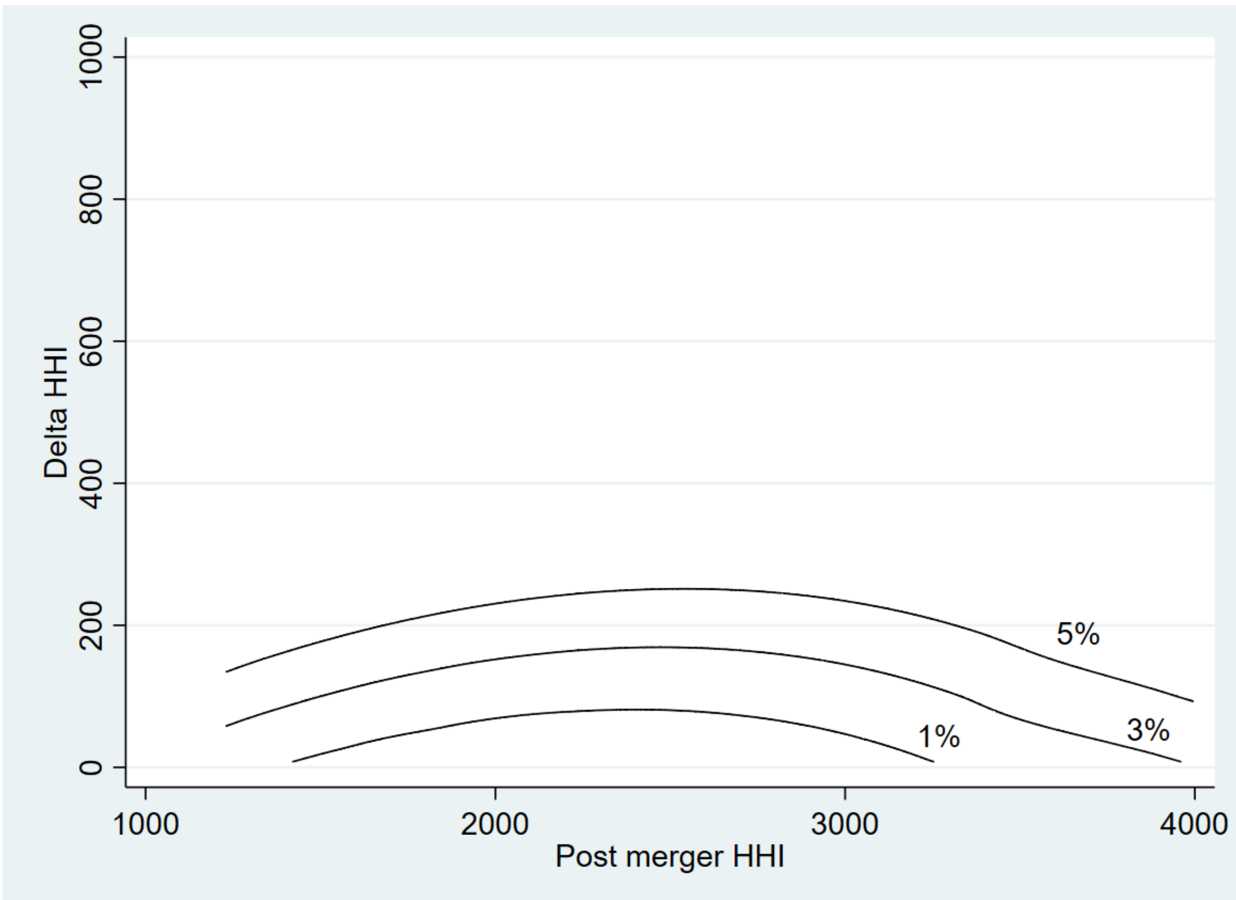


**Figure 12:** Contour plot showing the combinations of the post-merger Herfindahl (labelled here as "post_hhi_val") and the merger-induced change in the Herfindahl ("delta_hhi_val") that have no effect on consumer surplus if there is a 1%, 3%, and 5% synergy due to the merger. Points above (respectively, below) a contour line correspond to mergers that are expected to harm (respectively, benefit) consumers. Based on estimates in Table 8, column (6).

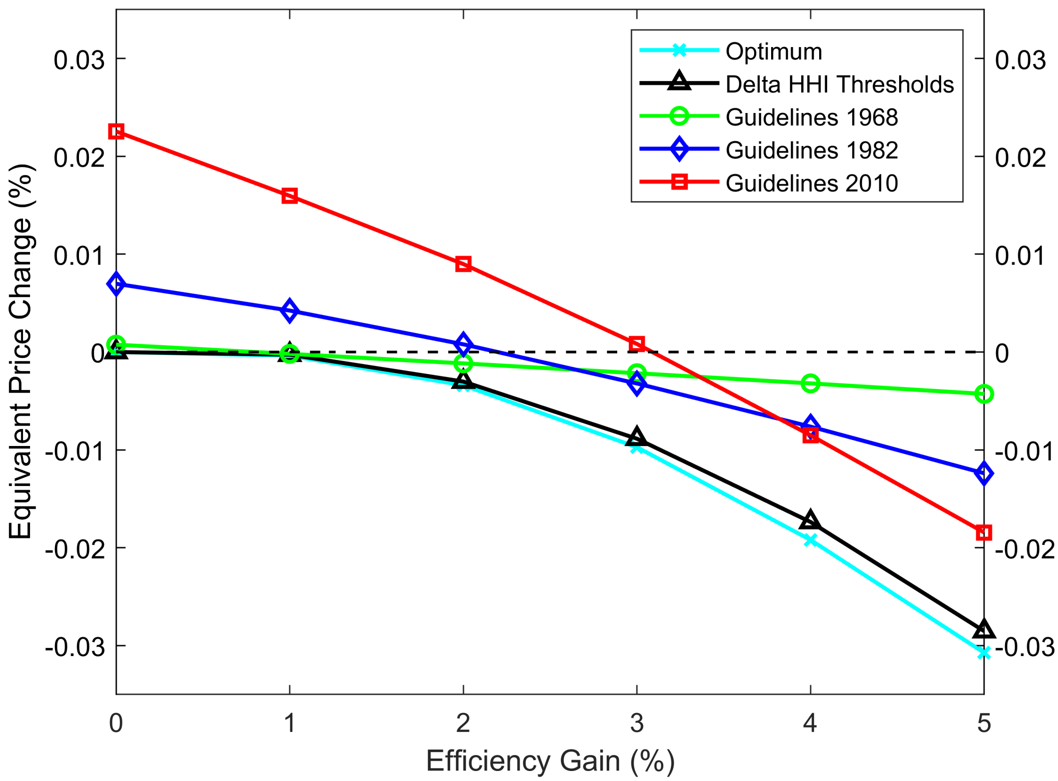


**Figure 13:** Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 *Guidelines*' thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on $\Delta$ HHI (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-1 model and value shares.

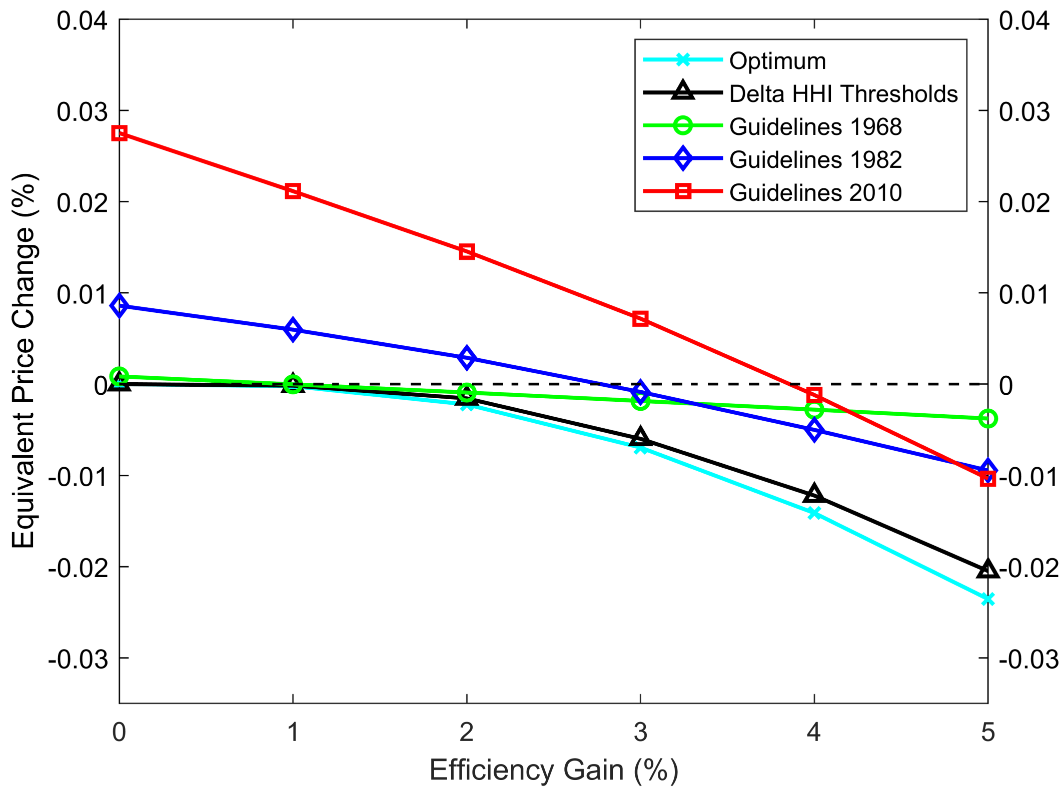


**Figure 14:** Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 *Guidelines*' thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on $\Delta$ HHI (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-3 model and value shares.

**Table 9:** Share of Mergers with 3% Efficiency Gain That Harm Consumers Under 2010 *Guidelines'* Screening Thresholds (Revenue-based)

| **Merger Guidelines' Screening Zone** | | **RCNL-1** | **RCNL-3** |
|---|---|---|---|
| *Green Zone (Safe Harbor)* | | *0.11* | *0.13* |
| | HHI $< 1500$ | 0.50 | 0.61 |
| | HHI $\in (1500, 2500)$ and $\Delta H < 100$ | 0.00 | 0.00 |
| | HHI $> 2500$ and $\Delta H < 100$ | 0.00 | 0.00 |
| *Yellow Zone* | | *0.62* | *0.75* |
| | HHI $\in (1500, 2500)$ and $\Delta H > 100$ | 0.74 | 0.81 |
| | HHI $> 2500$ and $\Delta H \in (100, 200)$ | 0.18 | 0.48 |
| *Red Zone (Anticompetitive Presumption)* | | *0.95* | *0.99* |

**Table 10:** Share of Mergers with 3% Efficiency Gain That Harm Consumers Under 2010 *Guidelines'* Screening Thresholds (Revenue-based)

| **Merger Guidelines' Screening Zone** | | **RCNL-1** | **RCNL-3** |
|---|---|---|---|
| *Green Zone (Safe Harbor)* | | *0.04* | *0.06* |
| | HHI $< 1500$ | 0.17 | 0.28 |
| | HHI $\in (1500, 2500)$ and $\Delta H < 100$ | 0.00 | 0.00 |
| | HHI $> 2500$ and $\Delta H < 100$ | 0.00 | 0.00 |
| *Yellow Zone* | | *0.38* | *0.46* |
| | HHI $\in (1500, 2500)$ and $\Delta H > 100$ | 0.48 | 0.58 |
| | HHI $> 2500$ and $\Delta H \in (100, 200)$ | 0.00 | 0.00 |
| *Red Zone (Anticompetitive Presumption)* | | *0.76* | *0.89* |

# 4 Implications of the Hypothetical Monopolist Test

In this appendix, we study the implications of the hypothetical monopolist test (HMT) for market definition—and the relationship to the conditions under which a merger does not hurt consumers. We do so in two theoretical models: in the homogeneous-goods Cournot model and in the CES model of differentiated-goods price competition.

## 4.1 Implications of the HMT in the Cournot Model

Suppose we have a symmetric Cournot model with $|\mathcal{F}|$ firms and a constant elasticity demand function $P(\sum_i q_i)$ with elasticity $\varepsilon$ and cost per unit $c$. The individual firm first-order condition is

$$(P(Q) - c) + P'(Q)q_i = 0 \tag{2}$$

Imposing symmetry ($q_i = q$ for all $i$) and rearranging we get the equilibrium condition:

$$\frac{p-c}{p} = \frac{1}{|\mathcal{F}|\varepsilon} \tag{3}$$

so

$$p = \left(\frac{|\mathcal{F}|\varepsilon}{|\mathcal{F}|\varepsilon - 1}\right) c \tag{4}$$

or

$$p - c = \left(\frac{1}{|\mathcal{F}|\varepsilon - 1}\right) c. \tag{5}$$

**Upward pricing pressure.** The upward pricing pressure (really downward output pressure) created by a merger of two symmetric firms is $P'(Q)q$. So a merger will not reduce output if the merger-induced improvement in cost per unit ($-\Delta c$) is less than $P'(Q)q$, which can be rewritten, using (2), as[1]

$$|\Delta c| > (p - c)$$

or, substituting from (5) and rearranging,

$$\frac{|\Delta c|}{c} > \left(\frac{1}{|\mathcal{F}|\varepsilon - 1}\right)$$

For an efficiency improvement that reduces marginal cost to $(1-e)c$, this would be satisfied for $(|\mathcal{F}|, \varepsilon)$ such that

$$|\mathcal{F}|\varepsilon > \frac{1+e}{e} \tag{6}$$

**The HMT.** The HMT restricts what can qualify as a relevant market, and hence implicitly the $(|\mathcal{F}|, \varepsilon)$ elasticities we might be dealing with. Roughly, a market qualifies as relevant/valid

[1] A merger of two symmetric firms results in the derivative of the firm's profit taking value

$$(P(Q) - (c + \Delta c)) + P'(Q)2q_i$$

if quantities don't change. Using (2), this derivative is negative (resulting in the merged firm having an incentive to reduce output) if $-\Delta c + P'(Q)q_i < 0$.

if a merger of all firms would result in a price increase of at least 5%.[2] This can be stated as:

$$(1.05)\left(\frac{|\mathcal{F}|\varepsilon}{|\mathcal{F}|\varepsilon-1}\right)c<\left(\frac{\varepsilon}{\varepsilon-1}\right)c$$

or equivalently,

$$1.05|\mathcal{F}|(\varepsilon-1)<(|\mathcal{F}|\varepsilon-1)$$

Alternatively, given $\varepsilon$ the HMT is passed if

$$|\mathcal{F}|>\frac{1}{1.05-0.05\varepsilon} \tag{7}$$

**Putting these together.** Figure 15 shows these two relationships together for the case of $e = 0.03$. CS-decreasing mergers are those that lie below the hatched (red) curve $|\mathcal{F}|\varepsilon = 34.33$, while the homogeneous-goods market passes the HMT to the left of the solid (blue) curve (where equation (7) is satisfied). As is evident in the figure, for elasticities below 11, the HMT is always satisfied and so for the elasticities in typical "natural" markets asserted in merger cases, the HMT would always be passed.

Figure 15 also allows us to assess, when demand takes a constant elasticity form, what insisting on the narrowest HMT-supported market would imply. To do this we ask what the smallest subset of firms is that would pass the HMT (this exercise can be thought of as considering a case in which the firms' products are very close to homogeneous, and selecting a subset of firms is choosing a smallest set of firms that are close to the merging firms).

Note that when demand takes a constant elasticity form, when we conduct the HMT holding fixed the outputs of the "out of market" firms, the residual demand elasticity facing the "in market" firms is still $\varepsilon$. Hence, Figure 15 can be used to find, given $\varepsilon$, the smallest number of firms satisfying the HMT. As Figure 15 makes clear, if $\varepsilon$ is less than 11, this would always consist of a two-firm market (i.e., the narrowest relevant market would consist of the merging firms).

Similar conclusions hold if one considers instead efficiency gains of 5%.

## 4.2 Implications of the HMT in the CES Model of Price Competition

Consider an industry consisting of $|\mathcal{F}|$ symmetric firms, each of type $T$. Denoting by $\mu \equiv (p^k - c^k)/p^k \in (0,1)$ the (common) percentage markup on any product $k$, and $\sigma > 1$ the

[2]More accurately, it says that a hypothetical monopolist would find a 5% price increase profitable. So the condition we use above is somewhat more restrictive when monopoly profit is a concave function (the monopolist could optimally increase price less than 5% but still find a 5% increase profitable).

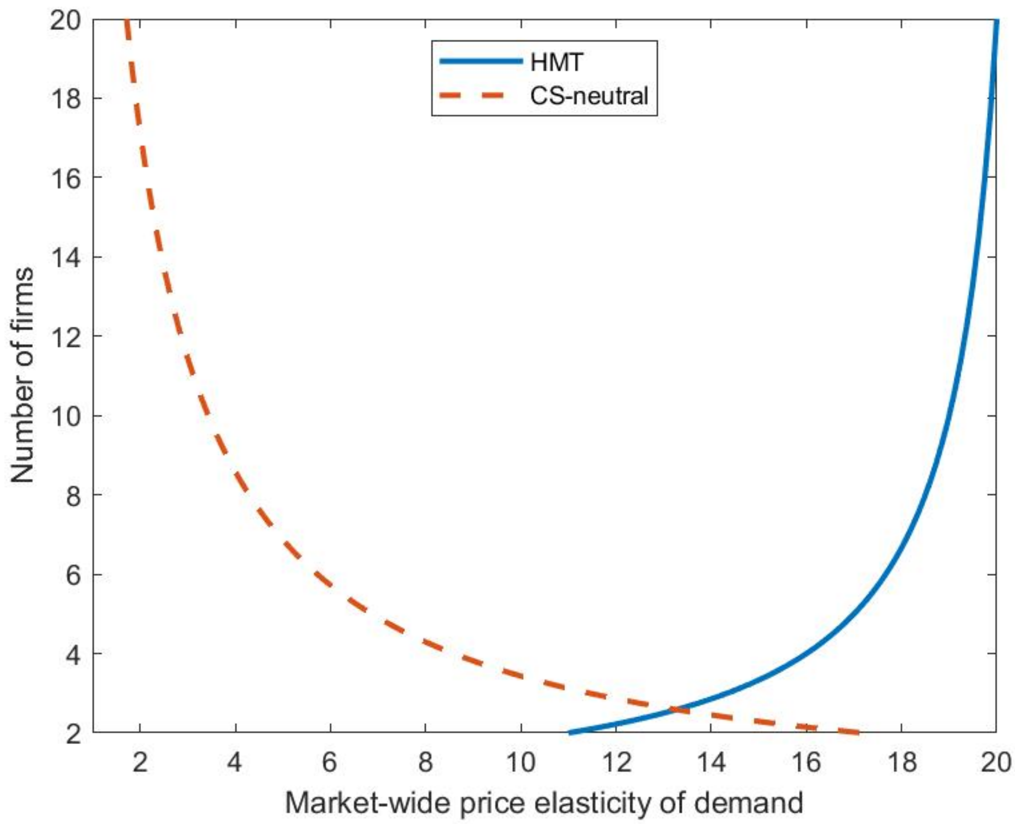


**Figure 15:** The symmetric-firm Cournot model with constant price elasticity of demand, with the elasticity depicted on the horizontal axis and the number of firms on the vertical axis. The HMT is satisfied is to the left of the solid (blue) curve; assuming an efficiency of 3 percent ($e = 0.03$), a two-firm merger is CS-decreasing below the hatched (red) curve.

elasticity of substitution, we have for any firm $f \in \mathcal{F}$:

$$\sum_{k \in f} b^k (p^k)^{1-\sigma} = \sum_{k \in f} b^k (c^k)^{1-\sigma} \left(\frac{c^k}{p^k}\right)^{\sigma-1} = T_f (1-\mu_f)^{\sigma-1} = T(1-\mu)^{\sigma-1},$$

where the last equality follows by symmetry. The equilibrium aggregator $A$ can therefore be written as

$$A = A^0 + |\mathcal{F}| T (1-\mu)^{\sigma-1},$$

where $A^0 > 0$ is the value of the outside good.

In the following, we normalize $A^0 \equiv 1$, and let $\varepsilon = \sigma - (\sigma-1)(1-s^0) \in (1, \sigma)$ denote the (pre-merger) market-wide price elasticity, and $s^0 = A^0/A$ the (pre-merger) market share (in value) of the outside option. Hence, the (pre-merger) equilibrium value of the aggregator is given by

$$A^* = \frac{\sigma-1}{\varepsilon-1}.$$

As $A^* = 1 + |\mathcal{F}| T (1-\mu^*)^{\sigma-1}$, we obtain

$$T = \frac{\sigma-\varepsilon}{(\varepsilon-1)|\mathcal{F}|(1-\mu^*)^{\sigma-1}}.$$

The equilibrium markup $\mu^*$ is uniquely determined by

$$\sigma \mu^* \left(1 - \frac{\sigma-1}{\sigma} \frac{\frac{\sigma-\varepsilon}{(\varepsilon-1)|\mathcal{F}|}}{1 + \frac{\sigma-\varepsilon}{\varepsilon-1}}\right) = 1.$$

Rewriting,

$$\mu^* = \frac{|\mathcal{F}|}{(|\mathcal{F}|-1)\sigma + \varepsilon},$$

implying

$$T = \frac{\sigma-\varepsilon}{(\varepsilon-1)|\mathcal{F}|} \left(\frac{(|\mathcal{F}|-1)\sigma+\varepsilon}{(|\mathcal{F}|-1)\sigma+\varepsilon-|\mathcal{F}|}\right)^{\sigma-1}.$$

The (pre-merger) joint equilibrium profit is given by

$$\Pi^* = \mu^*(1-s^0) = \frac{(\sigma-\varepsilon)|\mathcal{F}|}{(\sigma-1)[(|\mathcal{F}|-1)\sigma+\varepsilon]}. \tag{8}$$

**The HMT.** Consider now an increase in all prices by 5 percent. The resulting markup is

$$\hat{\mu} = \frac{0.05 + \mu^*}{1.05}.$$

The joint profit is then given by

$$\hat{\Pi} = \hat{\mu} \cdot \frac{|\mathcal{F}|T(1-\hat{\mu})^{\sigma-1}}{1+|\mathcal{F}|T(1-\hat{\mu})^{\sigma-1}} = \frac{0.05+\mu^*}{1.05} \cdot \frac{|\mathcal{F}|T(1-\mu^*)^{\sigma-1}}{1.05^{\sigma-1}+|\mathcal{F}|T(1-\mu^*)^{\sigma-1}}.$$

Rewriting,

$$\begin{aligned}
\hat{\Pi} =& \frac{0.05+\frac{|\mathcal{F}|}{(|\mathcal{F}|-1)\sigma+\varepsilon}}{1.05} \cdot \frac{\frac{\sigma-\varepsilon}{(\varepsilon-1)}}{1.05^{\sigma-1}+\frac{\sigma-\varepsilon}{(\varepsilon-1)}} \\
=& \frac{0.05+\frac{|\mathcal{F}|}{(|\mathcal{F}|-1)\sigma+\varepsilon}}{1.05} \cdot \frac{\sigma-\varepsilon}{(\varepsilon-1)1.05^{\sigma-1}+\sigma-\varepsilon} \\
=& \frac{\{|\mathcal{F}|+0.05[(|\mathcal{F}|-1)\sigma+\varepsilon]\}(\sigma-\varepsilon)}{\{(\varepsilon-1)1.05^{\sigma}+1.05(\sigma-\varepsilon)\}[(|\mathcal{F}|-1)\sigma+\varepsilon]}.
\end{aligned}$$

The price increase is (weakly) profitable if and only if $\Pi^* \leq \hat{\Pi}$, i.e.,

$$\frac{|\mathcal{F}|}{(\sigma-1)} \leq \frac{|\mathcal{F}|+0.05[(|\mathcal{F}|-1)\sigma+\varepsilon]}{(\varepsilon-1)1.05^{\sigma}+1.05(\sigma-\varepsilon)},$$

i.e.,

$$|\mathcal{F}|(\varepsilon-1)1.05^{\sigma}+1.05N(\sigma-\varepsilon) \leq (\sigma-1)\{|\mathcal{F}|+0.05[(|\mathcal{F}|-1)\sigma+\varepsilon]\} \tag{9}$$

**Upward pricing pressure.** Now, let us consider the conditions under which a merger between two firms does not hurt consumers. Let $\overline{T}$ and $\overline{\mu}$ denote the post-merger type and markup (of the merged firm), and let $t \equiv \overline{T}/(2T)$. For the merger to be CS-neutral requires that

$$2T(1-\mu^*)^{\sigma-1} = \overline{T}(1-\overline{\mu})^{\sigma-1},$$

i.e.,

$$\overline{\mu} = 1 - t^{-\frac{1}{\sigma-1}}(1-\mu^*).$$

It also requires that

$$\sigma\overline{\mu}\left(1-\frac{\sigma-1}{\sigma}\frac{\overline{T}(1-\overline{\mu})^{\sigma-1}}{A^*}\right) = 1.$$

Combining, the merger is CS-neutral if and only if

$$\sigma\left[1-t^{-\frac{1}{\sigma-1}}\frac{(|\mathcal{F}|-1)\sigma+\varepsilon-|\mathcal{F}|}{(|\mathcal{F}|-1)\sigma+\varepsilon}\right]\left[1-\left(\frac{\sigma-\varepsilon}{\sigma}\right)\frac{2}{|\mathcal{F}|}\right] = 1,$$

or

$$t^{\frac{1}{\sigma-1}} = \left(\frac{(|\mathcal{F}|-1)\sigma+\varepsilon-|\mathcal{F}|}{(|\mathcal{F}|-1)\sigma+\varepsilon}\right)\left(\frac{(|\mathcal{F}|-2)\sigma+2\varepsilon}{(|\mathcal{F}|-2)\sigma+2\varepsilon-|\mathcal{F}|}\right).$$

Let us now replace type efficiencies by cost efficiencies: suppose that the merged firm produces the same set of products as the merger partners did jointly before the merger but

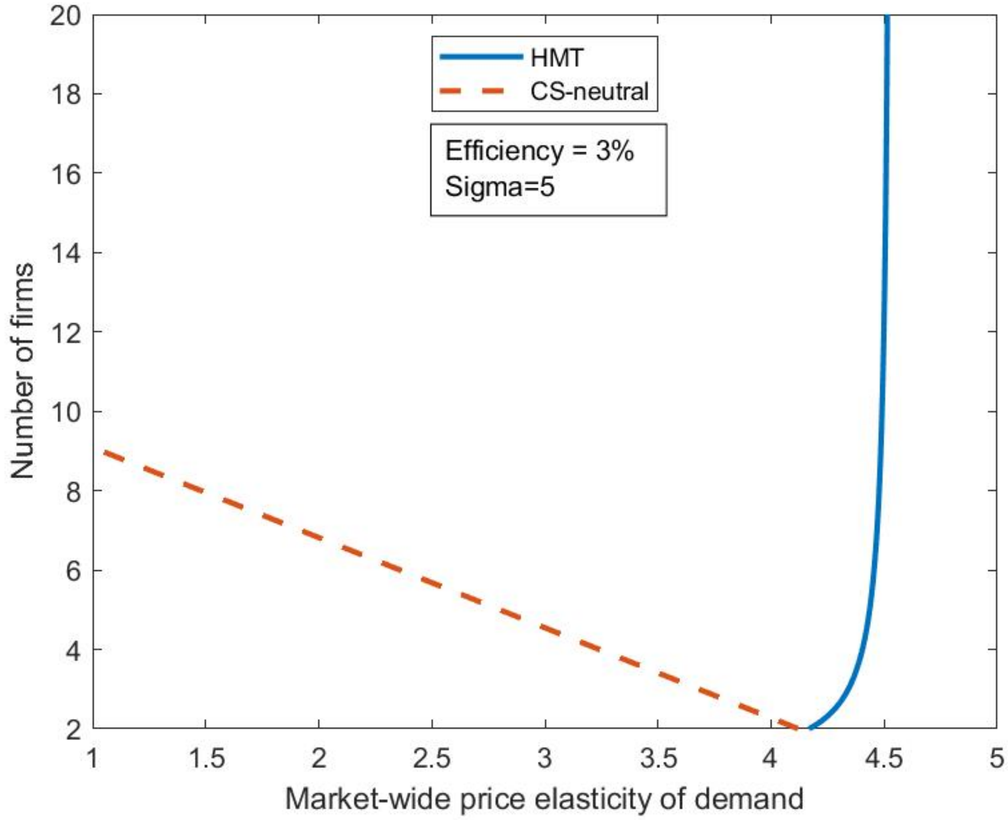


**Figure 16:** The symmetric-firm CES model with $\sigma = 5$. The market-wide price elasticity of demand is depicted on the horizontal axis and the number of firms on the vertical axis. The HMT is satisfied is to the left of the solid (blue) curve; assuming an efficiency of 3 percent ($e = 0.03$), a two-firm merger is CS-decreasing below the hatched (red) curve.

that the post-merger marginal cost of any product $k$ satisfies $\bar{c}^k = (1-e)c^k$, where $c^k$ is the pre-merger marginal cost. The post-merger type $\overline{T}$ then satisfies

$$\overline{T} = 2(1-e)^{1-\sigma}T,$$

so that $t = (1-e)^{1-\sigma}$. Hence, the merger is CS-neutral if and only if

$$(1-e) = \left(\frac{(|\mathcal{F}|-1)\sigma + \varepsilon}{(|\mathcal{F}|-1)\sigma + \varepsilon - |\mathcal{F}|}\right)\left(\frac{(|\mathcal{F}|-2)\sigma + 2\varepsilon - |\mathcal{F}|}{(|\mathcal{F}|-2)\sigma + 2\varepsilon}\right); \tag{10}$$

it is CS-decreasing if the l.h.s. is larger than the r.h.s., and CS-increasing if the l.h.s. is smaller than the r.h.s.

**Putting these together.** For a given level of $\sigma$ and $e$, we can now plot (9) and (10) in $(|\mathcal{F}|, \varepsilon)$-space. Figure 16 shows the case of $\sigma = 5$ and $e = 0.03$. Here the entire market passes the HMT provided that the market-wide elasticity is less than 4 (and in any case in which the merger would reduce consumer surplus), and the narrowest HMT-supported market consists of just the merging firms in all of these cases. As in the Cournot case, similar conclusions hold if one considers instead efficiency gains of 5%.

# 5 Performance of Alternative Merger Approval Policies

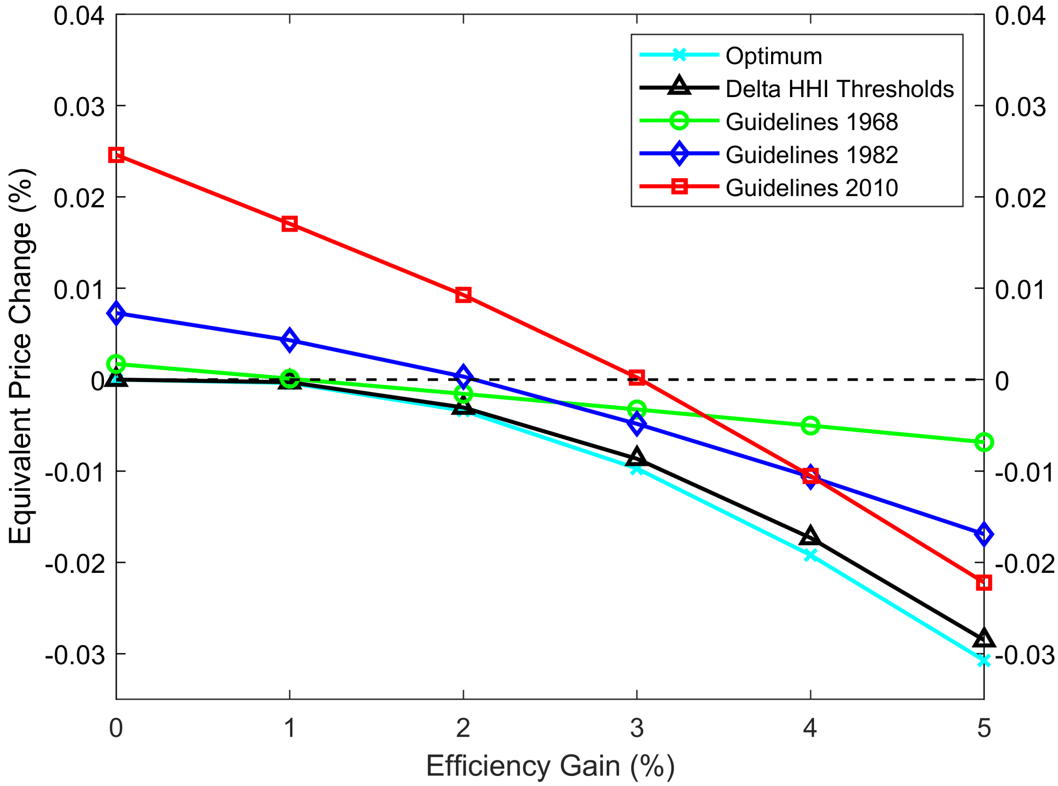


**Figure 17:** Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 *Guidelines*' thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on $\Delta$ HHI (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-1 model and volume shares, and assuming that an approval decision in the yellow zone (of the 1982 and 2010 *Guidelines*) is correct with 85% probability.

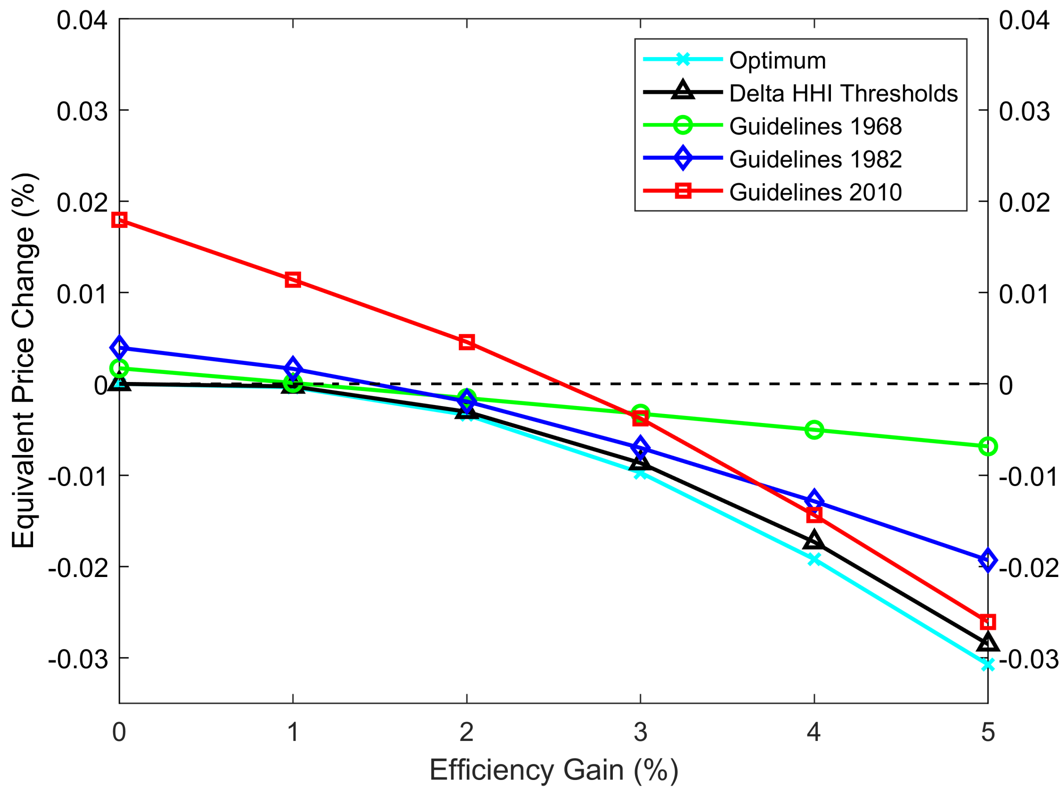


**Figure 18:** Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 *Guidelines*' thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on $\Delta$ HHI (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-1 model and volume shares, and assuming that an approval decision in the yellow zone (of the 1982 and 2010 *Guidelines*) is correct with 95% probability.

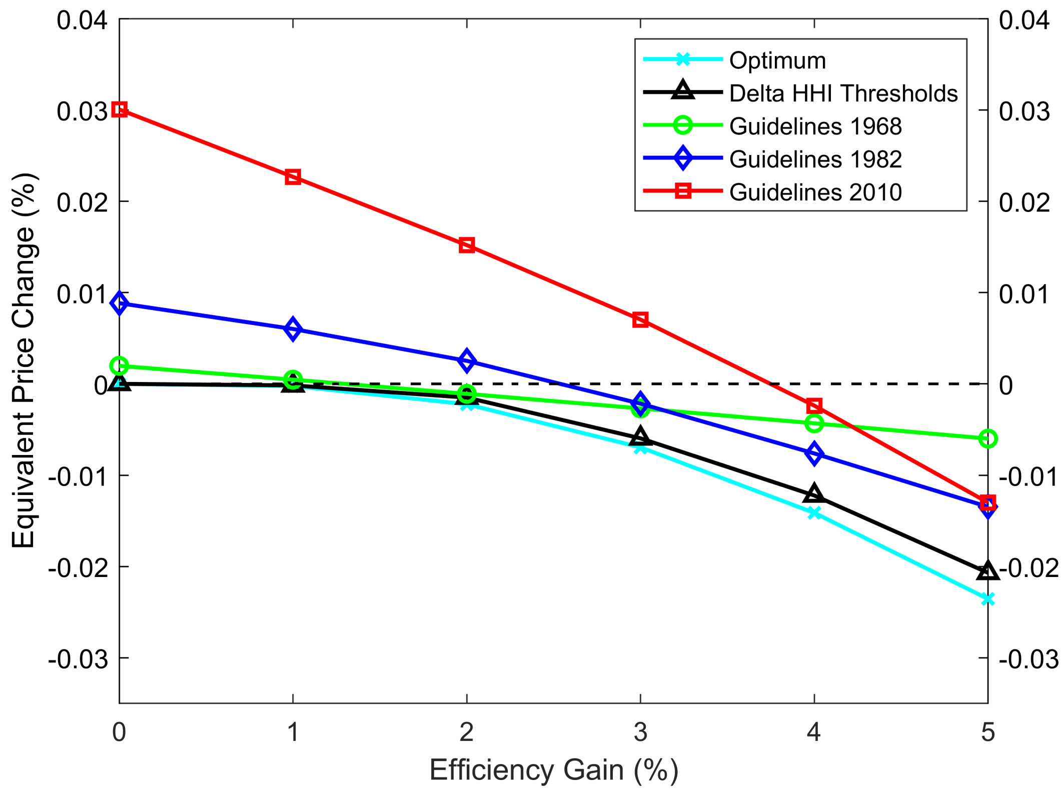


**Figure 19:** Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 *Guidelines*' thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on $\Delta$ HHI (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-3 model and volume shares, and assuming that an approval decision in the yellow zone (of the 1982 and 2010 *Guidelines*) is correct with 85% probability.

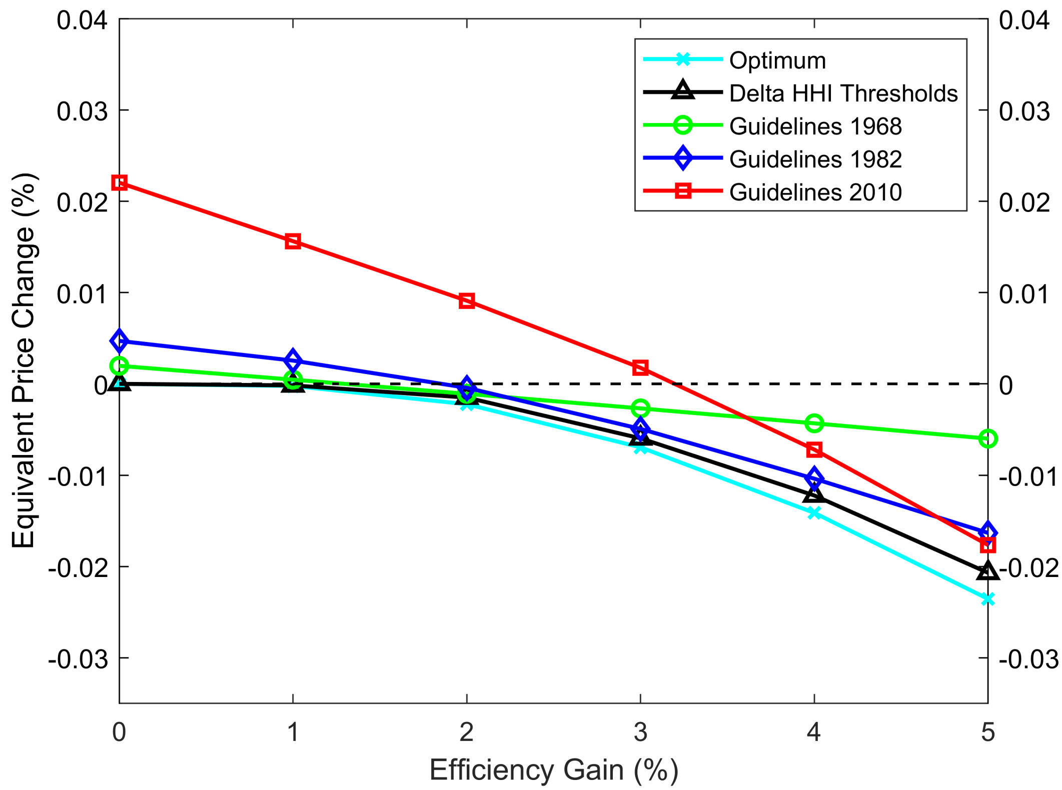


**Figure 20:** Graph showing the performance (measured by the induced percentage change in all prices) of alternative approval policies as a function of the merger-induced efficiency gains. The depicted policies are: the 1968, 1982 and 2010 *Guidelines*' thresholds (green circles, blue diamonds and orange squares, respectively), a simple threshold policy based only on $\Delta$ HHI (grey triangles) and the optimal policy (light blue crosses). Based on the RCNL-3 model and volume shares, and assuming that an approval decision in the yellow zone (of the 1982 and 2010 *Guidelines*) is correct with 95% probability.