Online Appendices
for
Delegation in Veto Bargaining

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The in-print appendix of the paper is Appendix A; hence, this document begins with Appendix B. For convenience, we recall:

Condition LQ. *For some* $\gamma \in [0,1]$, $u(a) = -(1-\gamma)|1-a| - \gamma(1-a)^2$.

B. Proofs of Corollaries 1, 2, and 3

B.1. Proof of Corollary 1

Since u is concave, u' is decreasing on $[0,1]$. Recall $\kappa \geq 0$. Hence, if the type density f is decreasing on $[0,1]$, then $\kappa F - u'f$ is increasing on $[0,1]$. The result follows from Proposition 1.

B.2. Proof of Corollary 2

As $\kappa F(v) - u'(v)f(v)$ is continuous on $[0,1]$, it is increasing on $[0,1]$ if its derivative is positive for all $v \in [0,1)$. The derivative is $(\kappa - u''(v))f(v) - u'(v)f'(v)$, which is larger than $-u''(v)f(v) - u'(v)f'(v)$. The latter function is positive for all $v \in [0,1)$ if

$$\inf_{v\in[0,1)} \frac{-u''(v)}{u'(v)} \geq \sup_{v\in[0,1)} \frac{f'(v)}{f(v)}.$$

The RHS above is finite since f is continuously differentiable and strictly positive on $[0,1]$. Therefore, $\kappa F(v) - u'(v)f(v)$ is increasing on $[0,1]$ when the LHS above is sufficiently large. The result follows from Proposition 1.

B.3. Proof of Corollary 3

Assume Condition LQ. We prove the result by establishing that (i) logconcavity of f on $[0,1]$ ensures that the conditions of either Proposition 2 or Proposition 3 are satisfied, and (ii) if $\gamma > 0$ (equivalently, given Condition LQ, u is strictly concave) or f is strictly logconcave on $[0,1]$, then among interval delegation sets there is a unique optimum.

As introduced in Section 4, Proposer's expected utility from delegating the interval $[c,1]$ with $c \in [0,1]$ is:

$$W(c) \equiv u(0)F(c/2) + u(c)(F(c) - F(c/2)) + \int_c^1 u(v)f(v)\mathrm{d}v. \tag{B.1}$$

As shorthand for the function in condition (i) of Proposition 3, define

$$G(v) := \kappa F(v) - u'(v)f(v). \tag{B.2}$$

We establish some properties of the W and G functions.

Lemma B.1. *Assume Condition LQ and f is logconcave on $[0,1]$. The functions W and G defined by* (B.1) *and* (B.2) *are respectively quasiconcave and quasiconvex on $[0,1]$, both strictly so if either $\gamma > 0$ or f is strictly logconcave on $[0,1]$. Furthermore, for any $c^* \in \arg\max_{c\in[0,1]} W(c)$, $G'(c^*/2) \leq 0$ if $c^* > 0$ and $G'(c^*) \geq 0$ if $c^* < 1$.*

Proof. The proof proceeds in four steps. Throughout, we restrict attention to the domain $[0,1]$ for the type density. Step 1 shows that G is (strictly) quasiconvex and that $\{v : G'(v) = 0\}$ is connected. Step 2 shows that W can be expressed in terms of G'. Step 3 establishes that given any maximizer c^* of W, G is decreasing on $[0, c^*/2]$ and increasing on $[c^*, 1]$. Step 4 establishes the (strict) quasiconcavity of W. Note that under Condition LQ, $\kappa \equiv \inf_{v\in[0,1)} -u''(v) = 2\gamma$, $u'(v) = 1-\gamma+2\gamma(1-v)$, and hence $G(v) = 2\gamma F(v) - (1-\gamma+2\gamma(1-v))f(v)$.

Step 1: We first establish that G is (strictly) quasiconvex and that $\{v : G'(v) = 0\}$ is connected. Logconcavity of f implies that its modes (i.e., maximizers) are connected, and moreover $f'(v) = 0 \implies v$ is a mode. Denote by Mo the smallest mode. Since

$$G'(v) = 4\gamma f(v) - (1-\gamma+2\gamma(1-v))f'(v), \tag{B.3}$$

it holds that $\operatorname{sign} G'(v) = \operatorname{sign} \beta(v)$, where

$$\beta(v) := 4\gamma - \frac{f'(v)}{f(v)}(1-\gamma+2\gamma(1-v)).$$

On the domain $[0,\mathrm{Mo})$, f'/f is positive and decreasing by logconcavity. Furthermore, $1-\gamma+2\gamma(1-v)$ is positive and decreasing. As the product of positive decreasing functions is decreasing, β is increasing on the domain $[0,\mathrm{Mo})$. Since $\beta(v) \geq 0$ when $v \geq \mathrm{Mo}$, it follows that β is upcrossing (once strictly positive, it stays positive), and hence G is quasiconvex.

We claim $\{v : \beta(v) = 0\}$ is connected, which implies the same about $\{v : G'(v) = 0\}$. If $\gamma = 0$ then $\beta(v) = 0 \iff f'(v) = 0$, which is a connected set, as noted earlier. If $\gamma > 0$, then the conclusion follows because β is increasing on $[0,\mathrm{Mo})$, $\beta(v) > 0$ for $v > \mathrm{Mo}$ (as $f'(v) \leq 0$), and β is continuous. Furthermore, analogous observations imply that if either f is strictly logconcave or $\gamma > 0$, then $|\{v : G'(v) = 0\}| \leq 1$ and so G is strictly quasiconvex.

Step 2: We now show that

$$W'(c) = \int_{c/2}^{c} (v-c)G'(v)\mathrm{d}v. \tag{B.4}$$

The derivation is as follows:

$$
\begin{aligned}
W'(c) =&(F(c) - F(c/2))(1+\gamma-2\gamma c) - \frac{c}{2}f(c/2)(1+\gamma-\gamma c)\\
=&(1+\gamma-2\gamma c)\left[\int_{c/2}^{c} f(v)\mathrm{d}v - \frac{c}{2}f(c/2)\right] - \gamma\frac{c^2}{2}f(c/2)\\
=&-(1+\gamma-2\gamma c)\int_{c/2}^{c}(v-c)f'(v)\mathrm{d}v - \gamma\frac{c^2}{2}f(c/2)\\
=&-\int_{c/2}^{c}(v-c)(1+\gamma-2\gamma v)f'(v)\mathrm{d}v + 2\gamma\left[-\int_{c/2}^{c}(v-c)^2 f'(v)\mathrm{d}v - \left(\frac{c}{2}\right)^2 f(c/2)\right]\\
=&-\int_{c/2}^{c}(v-c)(1+\gamma-2\gamma v)f'(v)\mathrm{d}v + 2\gamma\int_{c/2}^{c}2(v-c)f(v)\mathrm{d}v\\
=&\int_{c/2}^{c}(v-c)G'(v)\mathrm{d}v.
\end{aligned}
$$

The first equality above is obtained by differentiating (B.1) and using $u'(c) = 1+\gamma-2\gamma c$ and $u(c) - u(0) = c(1+\gamma-\gamma c)$; the third and fifth equalities use integration by parts; the last equality involves substitution from (B.3); and the remaining equalities follow from algebraic manipulations.

Step 3: We now establish that for any $c^* \in \arg\max_{c\in[0,1]} W(c)$, $c^* > 0 \implies G'(c^*/2) \leq 0$ and $c^* < 1 \implies G'(c^*) \geq 0$.

By Step 1, there exist v_* and v^* with $0 \leq v_* \leq v^* \leq 1$ such that $G'(v) < 0$ on $[0, v_*)$, $G'(v) = 0$ on (v_*, v^*), and $G'(v) > 0$ on $(v^*, 1]$. By (B.4), $c \in (0, v_*) \implies W'(c) > 0$, and $c/2 \in (v^*, 1) \implies W'(c) < 0$. Since c^* is optimal, $c^* > 0 \implies W'(c^*) \geq 0 \implies c^*/2 \leq v^* \implies G'(c^*/2) \leq 0$. Similarly, $c^* < 1 \implies W'(c^*) \leq 0 \implies c^* \geq v_* \implies G'(c^*) \geq 0$.

Step 4: Finally we establish that W is quasiconcave, strictly if $\gamma > 0$ or f is strictly logconcave. For this it is sufficient to establish that if $c > 0$ and $W'(c) = 0$, then $W''(c) \leq 0$, with a strict inequality if $\gamma > 0$ or f is strictly logconcave.

Differentiating (B.4),

$$W''(c) = \frac{c}{4}G'(c/2) - (G(c) - G(c/2)). \tag{B.5}$$

Integrating by parts,

$$\int_{c/2}^{c}[(v-c)G'(v) + G(v)]\mathrm{d}v = [(v-c)G(v)]_{c/2}^{c} = \frac{c}{2}G(c/2).$$

Now fix any $c > 0$ such that $W'(c) = 0$ (if no such c exists, W is monotonic and hence quasiconcave). By (B.4) and the above integration by parts, $G(c/2) = (2/c)\int_{c/2}^{c} G(v)\mathrm{d}v$, which, because G is quasiconvex by Step 1, implies $G(c/2) \le G(c)$, with a strict inequality if $\gamma > 0$ or f strictly logconcave. Similarly $G'(c/2) \le 0$, and hence from (B.5) we conclude that $W''(c) \le 0$, with a strict inequality if $\gamma > 0$ or f is strictly logconcave. □

We build on Lemma B.1 to establish Corollary 3 by verifying the conditions of Proposition 2 and Proposition 3.

Proof of Corollary 3. If the interval delegation set $[c^*, 1]$ is optimal then c^* must maximize $W(c)$ defined in (B.1). Hence if W is strictly quasiconcave—as is the case if $\gamma > 0$ or f is strictly logconcave on $[0, 1]$, by Lemma B.1—there can be at most one interval that is optimal. So it suffices to establish that if $c^* \in \arg\max_{c\in[0,1]} W(c)$ then $[c^*, 1]$ is optimal.

To that end, we verify that if $c^* = 1$ the conditions of Proposition 2 are satisfied and, if $c^* < 1$, then conditions (i)–(iii) of Proposition 3 are satisfied. Note that condition (i) is immediate from Lemma B.1. As conditions (ii) and (iii) are vacuous for $c^* = 0$ we need only consider $c^* \in (0, 1]$. For any $c^* \in (0, 1)$ conditions (ii) and (iii) are jointly equivalent to

$$(u'(0) - \kappa s)\,\frac{F(c^*/2) - F(s)}{c^*/2 - s} \le u'(c^*)\frac{F(c^*) - F(c^*/2)}{c^*/2} \le (u'(c^*) + \kappa(c^* - t))\,\frac{F(t) - F(c^*/2)}{t - c^*/2}$$

for all $s \in [0, c^*/2)$ and $t \in (c^*/2, c^*]$. Substituting into the middle expression from the first-order condition $W'(c^*) = 0$ (i.e., setting expression (2) equal to zero and rearranging) yields

$$(u'(0) - \kappa s)\,\frac{F(c^*/2) - F(s)}{c^*/2 - s} \le (u(c^*) - u(0))\frac{f(c^*/2)}{c^*} \le (u'(c^*) + \kappa(c^* - t))\,\frac{F(t) - F(c^*/2)}{t - c^*/2} \quad \text{(B.6)}$$

for all $s \in [0, c^*/2)$ and $t \in (c^*/2, c^*]$. So if (B.6) holds for $c^* \in (0, 1)$ then the conditions in Proposition 3 are verified. On the other hand, since the condition in Proposition 2 is equivalent to the right-most term in (B.6) being larger than the left-most term for all $s \in [0, c^*/2)$ and $t \in (c^*/2, c^*]$ when $c^* = 1$, (B.6) holding for $c^* = 1$ implies the condition in Proposition 2. Accordingly, we fix a $c^* > 0$ and verify the two inequalities of (B.6) in turn.

First inequality of (B.6): Using $u'(a) = 1 + \gamma - 2\gamma a$, $\kappa = 2\gamma$, and $\frac{u(a)-u(0)}{a} = 1 + \gamma - \gamma a$, the first inequality of (B.6) reduces to

$$(1 + \gamma - 2\gamma s)\frac{F(c^*/2) - F(s)}{c^*/2 - s} \le (1 + \gamma - \gamma c^*)f(c^*/2) \quad \forall s \in [0, c^*/2).$$

It follows from L'Hopital's rule that the above inequality holds with equality in the limit as $s \to c^*/2$. Hence it is sufficient to demonstrate that the LHS of the inequality is increasing for all $s \in [0, c^*/2)$. For any $s \in [0,1]$ let

$$D(s) := (1+\gamma-\gamma c^*)(F(c^*/2)-F(s)) - (c^*/2-s)(1+\gamma-2\gamma s)f(s), \tag{B.7}$$

and observe that

$$\frac{\partial}{\partial s}\left[(1+\gamma-2\gamma s)\frac{F(c^*/2)-F(s)}{c^*/2-s}\right] = \frac{1}{(c^*/2-s)^2}D(s).$$

So it is sufficient to show that, for all $s \in [0, c^*/2)$, $D(s) \geq 0$. This holds because $D(c^*/2) = 0$ and, for all $s < c^*/2$,

$$\begin{aligned} D'(s) &= (c^*/2-s)[4\gamma f(s) - (1+\gamma-2\gamma s)f'(s)] \quad \text{differentiating (B.7) and simplifying} \\ &= (c^*/2-s)G'(s) \quad \text{substituting from (B.3)} \\ &\leq 0 \quad \text{by Lemma B.1.} \end{aligned} \tag{B.8}$$

Second inequality of (B.6): Using $u'(a) = 1+\gamma-2\gamma a$, $\kappa = 2\gamma$, and $\frac{u(a)-u(0)}{a} = 1+\gamma-\gamma a$, the second inequality of (B.6) reduces to

$$(1+\gamma-\gamma c^*)f(c^*/2) \leq (1+\gamma-2\gamma t)\frac{F(t)-F(c^*/2)}{t-c^*/2} \quad \forall t \in (c^*/2, c^*].$$

Using L'Hopital's rule for the limit as $t \to c^*/2$ and the fact that $W'(c^*) \geq 0$ by optimality of $c^* > 0$, it follows that

$$\lim_{t\to c^*/2}(1+\gamma-2\gamma t)\frac{F(t)-F(c^*/2)}{t-c^*/2} = (1+\gamma-\gamma c^*)f(c^*/2) \leq (1+\gamma-2\gamma c^*)\frac{F(c^*)-F(c^*/2)}{c^*/2}.$$

Hence it is sufficient to show that $(1+\gamma-2\gamma t)\frac{F(t)-F(c^*/2)}{t-c^*/2}$ is quasiconcave for $t \in (c^*/2, c^*]$. Note that

$$\frac{\partial}{\partial t}\left[(1+\gamma-2\gamma t)\frac{F(t)-F(c^*/2)}{t-c^*/2}\right] = \frac{1}{(t-c^*/2)^2}D(t),$$

where D is defined in (B.7), and so

$$\operatorname{sign}\frac{\partial}{\partial t}\left[(1+\gamma-2\gamma t)\frac{F(t)-F(c^*/2)}{t-c^*/2}\right] = \operatorname{sign} D(t).$$

Since $D(c^*/2) = 0$, it follows that $(1+\gamma-2\gamma t)\frac{F(t)-F(c^*/2)}{t-c^*/2}$ is quasiconcave for $t \in (c^*/2, c^*]$ if

D is quasiconcave. D is quasiconcave because, as was shown in (B.8), $D'(t) = (c^*/2 - t)G'(t)$, which is positive then negative on $(c^*/2, c^*]$ by the quasiconvexity of G (Lemma B.1). □

C. Proof of Proposition 4

Proof of Proposition 4(i). Let $H(a,c)$ denote the cumulative distribution function of the action implemented under the interval delegation set $[c,1]$. That is,

$$H(a,c) = \begin{cases} 0 & \text{if } a < 0 \\ F(c/2) & \text{if } 0 \leq a < c \\ F(a) & \text{if } c \leq a < 1 \\ 1 & \text{if } 1 \leq a. \end{cases}$$

Consider any $0 \leq c_L < c_H \leq 1$. The difference $H(\cdot, c_L) - H(\cdot, c_H)$ is upcrossing: once strictly positive, it stays positive.

Given any pair of Proposer utilities, u_1 and u_2, where u_1 is strictly more risk averse than u_2, define $K : [0,1] \times \{1,2\} \to \mathbb{R}$ by $K(a,i) := u_i'(a)$. It holds that $\frac{\partial \log K(a,i)}{\partial a}$ is strictly increasing in i, and hence K is strictly totally positive of order 2. It follows from the variation diminishing property (Karlin, 1968, Theorem 3.1 on p. 21) that

$$S(i) := \int_0^1 K(a,i)\,[H(a,c_L) - H(a,c_H)]\,\mathrm{d}a$$

satisfies

$$S(1) \geq (>) 0 \implies S(2) \geq (>) 0.$$

Equivalently,

$$\int_0^1 u_1'(a)\,[H(a,c_L) - H(a,c_H)]\,\mathrm{d}a \geq (>) 0 \implies \int_0^1 u_2'(a)\,[H(a,c_L) - H(a,c_H)]\,\mathrm{d}a \geq (>) 0.$$

Integrating by parts, we obtain

$$\int_0^1 u_1'(a)\,[H(\mathrm{d}a,c_L) - H(\mathrm{d}a,c_H)] \leq (<) 0 \implies \int_0^1 u_2'(a)\,[H(\mathrm{d}a,c_L) - H(\mathrm{d}a,c_H)] \leq (<) 0.$$

A standard monotone comparative statics argument (Milgrom and Shannon, 1994) then implies that $C^*(u_2) \geq_{SSO} C^*(u_1)$. □

Proof of Proposition 4(ii). Let density $f(v)$ strictly dominate density $g(v)$ in likelihood ratio on the unit interval: i.e., for all $0 \leq v_L < v_H \leq 1$, $f(v_L)g(v_H) < f(v_H)g(v_L)$. Let $w(c,v)$ denote Proposer's payoff under the interval delegation set $[c,1]$ when Vetoer's type is v. We have

$$w(c,v) = \begin{cases} u(0) & \text{if } v < c/2 \\ u(c) & \text{if } v \in (c/2, c) \\ u(v) & \text{if } v \in (c, 1). \\ u(1) & \text{if } v > 1. \end{cases}$$

Consider any $0 \leq c_L < c_H \leq 1$. The difference $w(c_H, \cdot) - w(c_L, \cdot)$ is upcrossing: once strictly positive, it stays positive. As in the proof of Proposition 4(i), it follows from the variation diminishing property (Karlin, 1968, Theorem 3.1 on p. 21) that

$$\int_0^1 \left[w(c_H, v) - w(c_L, v)\right] g(v)\mathrm{d}v \geq (>)0 \implies \int_0^1 \left[w(c_H, v) - w(c_L, v)\right] f(v)\mathrm{d}v \geq (>)0.$$

A standard monotone comparative statics argument (Milgrom and Shannon, 1994) then implies that $C^*(f) \geq_{SSO} C^*(g)$. □

D. Proof of Proposition 5

Let a_U and a_I denote proposals in some noninfluential and influential cheap-talk equilibria, respectively (the latter may not exist). It is straightforward that $a_U > 0$ and, if it exists, $a_I \in (0,1)$. Since Proposition 5's conclusion is trivial for full delegation ($c^* = 0$), it suffices to establish that any optimal interval delegation set $[c^*, 1]$ with $c^* \in (0,1)$ has $c^* < \min\{a_I, a_U\}$. (By convention, $\min\{a_I, a_U\} = a_U$ if a_I does not exist.)

Plainly, a_U is a noninfluential equilibrium proposal if and only if

$$a_U \in \arg\max_a \left[u(0)F(a/2) + u(a)(1 - F(a/2))\right],$$

and so if $a_U < 1$ then it solves the first-order condition

$$2u'(a)\left[1 - F(a/2)\right] - f(a/2)\left[u(a) - u(0)\right] = 0. \tag{D.1}$$

Any influential cheap-talk equilibrium outcome can be characterized by a threshold type $v_I \in (0,1)$ such that types $v < v_I$ pool on the "veto threat" message, and types $v > v_I$ pool on the "acquiesce" message. Since type v_I must be indifferent between sending the two messages, and she will accept either proposal from the Proposer, it holds that

$$v_I = \frac{1 + a_I}{2}.$$

It follows that a_I is an influential equilibrium proposal if and only if

$$a_I \in \arg\max_a \left[u(0) \frac{F(a/2)}{F((1+a_I)/2)} + u(a) \left(1 - \frac{F(a/2)}{F((1+a_I)/2)} \right) \right].$$

The first-order condition is that function (3) in the main text equals zero, i.e., $a_I \in (0,1)$ solves

$$2u'(a) \left[F((1+a)/2) - F(a/2) \right] - f(a/2) \left[u(a) - u(0) \right] = 0. \tag{D.2}$$

Note that at $a = 0$, the LHS is strictly positive. Hence, if the LHS is strictly downcrossing on $(0,1)$, then (D.2) has at most one solution in that domain; if there is a solution, then (D.2)'s LHS is strictly positive (resp., strictly negative) to its left (resp., right); furthermore, it can be verified that the solution then identifies an influential equilibrium. Note that if there is no solution to (D.2) on $(0,1)$ then there is no influential equilibrium.

Turning to optimal interval delegation, recall from Section 4 that the threshold is a zero of the function (2), i.e., $c^* \in (0,1)$ solves

$$2u'(a) \left[F(a) - F(a/2) \right] - f(a/2) \left[u(a) - u(0) \right] = 0. \tag{D.3}$$

If the LHS is strictly downcrossing on $(0,1)$, then on that domain c^* is the unique solution to (D.3) and (D.3)'s LHS is strictly positive (resp., strictly negative) to the solution's left (resp., right).

For any $a \in (0,1)$ the LHS of (D.1) is strictly larger than the LHS of (D.2), which in turn is strictly larger than the LHS of (D.3). If there is no solution in $(0,1)$ to (D.2), then its LHS is always strictly positive, and hence there are neither any influential equilibria nor any noninfluential equilibria with $a_U < 1$, and we are done. So assume at least one solution in $(0,1)$ to (D.2). Let

$$\underline{a}_2 := \inf\{a \in (0,1) : \text{ (D.2)'s LHS } \leq 0\},$$
$$\overline{a}_2 := \sup\{a \in (0,1) : \text{ (D.2)'s LHS } \geq 0\},$$

and analogously define $\underline{a}_3$ and $\overline{a}_3$ using (D.3)'s LHS. The aforementioned ordering of the

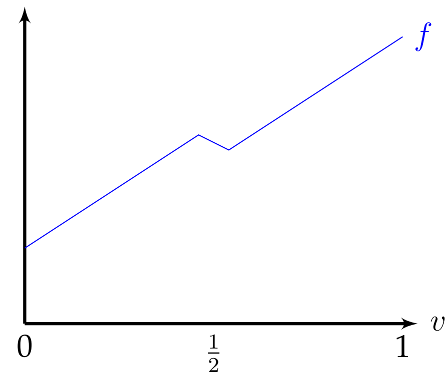

Figure 1 – A density under which no compromise is the optimal delegation set when Proposer has a linear loss function, but it is worse than some stochastic mechanism.

equations' LHS, (D.2)'s LHS being strictly positive at 0, and continuity combine to imply $0 \leq \underline{a}_3 < \underline{a}_2 < a_U$, and $\overline{a}_3 \leq \overline{a}_2$ with a strict inequality if either $\overline{a}_2 < 1$ or $\overline{a}_3 < 1$. Furthermore, $a_I \in [\underline{a}_2, \overline{a}_2]$ and $c^* \in [\underline{a}_3, \overline{a}_3]$.

If the LHS of (D.2) is strictly downcrossing on $(0, 1)$, then by the properties noted right after (D.2), $a_I = \underline{a}_2 = \overline{a}_2 < 1$ and hence $c^* < \min\{a_I, a_U\}$. If the LHS of (D.3) is strictly downcrossing on $(0, 1)$, then by the properties noted right after (D.3), $c^* = \underline{a}_3 = \overline{a}_3 < 1$ and hence $c^* < \min\{a_I, a_U\}$.

E. Stochastic Mechanisms can be Optimal

Example E.1. Suppose Proposer has a linear loss function, $\underline{v} = 0$, $\overline{v} = 1$, and $f(v)$ is strictly increasing except on $(1/2-\delta, 1/2+\delta)$, where it is strictly decreasing. Assume $|f'(v)|$ is constant (on $[0, 1]$).[1] Take $\delta > 0$ to be small. See Figure 1.

Recall that if δ were 0, then no compromise (i.e., the singleton menu $\{1\}$) would be optimal by Proposition 2 or the discussion preceding it. It can be verified that no compromise remains an optimal delegation set for small $\delta > 0$. We argue below that Proposer can obtain a strictly higher payoff, however, by adding a stochastic option ℓ that has expected value $1/2$ and is chosen only by types in $(1/2 - \delta, 1/2 + \delta)$.

The stochastic option ℓ provides action $1 - \frac{1}{2p}$ with probability p and action 1 with probability $1 - p$. For any $p \in (0, 1)$, this lottery has expected value $1/2$. Moreover, when $p = \frac{1}{2-4\delta}$, quadratic loss implies that type $1/2 - \delta$ is indifferent between ℓ and action 0 while type $1/2 + \delta$

[1] As it is nondifferentiable at two points, this density violates our maintained assumption of continuous differentiability. But the example could straightforwardly be modified to satisfy that assumption.

is indifferent between ℓ and 1. Consequently, any type in $[0, 1/2 - \delta)$ strictly prefers 0 to both ℓ and 1; any type in $(1/2-\delta, 1/2+\delta)$ strictly prefers ℓ to both 0 and 1; and any type in $(1/2+\delta, 1]$ strictly prefers 1 to both ℓ and 0.

Therefore, offering the menu $\{\ell, 1\}$ rather than $\{1\}$ changes the induced expected action from 0 to $1/2$ when $v \in (1/2 - \delta, 1/2)$ and from 1 to $1/2$ when $v \in (1/2, 1/2 + \delta)$. Since $f(v)$ is strictly decreasing on $(1/2 - \delta, 1/2 + \delta)$, Proposer is strictly better off. Note that if one were to replace ℓ with a deterministic option that provides ℓ's expected action $1/2$, then all types in $(1/4, 3/4)$ would strictly prefer to choose that option over both 0 and 1. So the menu $\{1/2, 1\}$ is strictly worse than not only $\{\ell, 1\}$ but also just $\{1\}$.[2] $\diamond$

References

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MILGROM, P. AND C. SHANNON (1994): "Monotone Comparative Statics," *Econometrica*, 157–180.

[2] Vis-à-vis Lemma A.1 and its proof that involves replacing a stochastic mechanism with its "averaged" deterministic counterpart: in this example the deterministic mechanism that solves problem (R) cannot be incentive compatible. In particular, it is not a mechanism corresponding to any delegation set.