ONLINE APPENDIX

of "Sticky Spending, Sequestration, and Government Debt"

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C Derivation of Equations in Section B..

A. Second Period without Sequestration.

The second-period problem can be written as:

$$V_2^I(g_1^I, b_1) = \max_{\{g_2^I \geq \alpha g_1^I\}} \frac{(g_2^I)^{1-\sigma} + \theta(\tau - b_1 - \alpha^2 g_0^O - g_2^I)^{1-\sigma}}{1-\sigma}$$

Letting λ be the multiplier of the constraint, the first-order conditions are:

$$\begin{aligned} g_2^I &: \ (g_2^I)^{-\sigma} + \lambda = \theta(d_2^I)^{-\sigma} = \theta(\tau - b_1 - \alpha^2 g_0^O - g_2^I)^{-\sigma}, \\ \lambda &: \ \lambda(g_2^I - \alpha g_1^I) = 0. \end{aligned}$$

When $\lambda = 0$, it is immediate to compute that:

$$g_2^I = \frac{1}{1+\theta^{1/\sigma}}(\tau - b_1 - \alpha^2 g_0^O)$$

which leads to equation (21). If this satisfies constraint (AC), it is the solution, otherwise:

$$\begin{aligned} g_2^I &= \alpha g_1^I, \\ \lambda &= \theta(d_2^I)^{-\sigma} - (g_2^I)^{-\sigma}. \end{aligned} \tag{47}$$

For any λ the solution implies $d_2^I = \tau - b_1 - \alpha^2 g_0^O - \alpha g_1^I$. Note that by the envelope theorem:

$$\frac{\partial V_2^I(g_1^I, b_1)}{\partial g_1^I} = -\alpha\lambda, \qquad \frac{\partial V_2^I(g_1^I, b_1)}{\partial b_1} = -\theta(d_2^I)^{-\sigma}. \tag{48}$$

Note that the second derivative is the same whether the constraint binds or not.

B. First Period without Sequestration.

Denoting by μ the multiplier of the budget constraint, the first-order necessary conditions are as follows:

$$\begin{aligned} g_1^I &: (g_1^I)^{-\sigma} + q\frac{\partial V_2^I(g_1^I, b_1)}{\partial g_1^I} + (1-q)\alpha^{1-\sigma}(g_1^I)^{-\sigma} = \mu, \\ d_1^I &: \theta(d_1^I)^{-\sigma} = \mu, \\ b_1 &: q\frac{\partial V_2^I(g_1^I, b_1)}{\partial b_1} = -\mu. \end{aligned}$$

Note that using the last two equations together with (48), generates the Euler equation:

$$d_2^I = q^{\frac{1}{\sigma}} d_1^I \tag{49}$$

which is independent on whether λ is positive in the second period.

If in the second period $\lambda = 0$, because of equation (48), the first two optimality conditions deliver equations (50) and (51):

$$d_1^{I*} = \left(\frac{\theta^{1/\sigma}}{(1+(1-q)\alpha^{1-\sigma})^{\frac{1}{\sigma}} + \theta^{1/\sigma}}\right)(\tau + b_1^* - \alpha g_0^O) \tag{50}$$

$$g_1^{I*} = \left(\frac{(1+(1-q)\alpha^{1-\sigma})^{\frac{1}{\sigma}}}{(1+(1-q)\alpha^{1-\sigma})^{\frac{1}{\sigma}} + \theta^{1/\sigma}}\right)(\tau + b_1^* - \alpha g_0^O) \tag{51}$$

Computing the ratio between the two types of spending delivers the first line of expression (24) in Section B.. Replacing d_1^I and d_2^I in equation (49), the level of debt must satisfy:

$$\frac{\tau - b_1^* - \alpha^2 g_0^O}{1+\theta^{1/\sigma}} = q^{\frac{1}{\sigma}}\left(\frac{\tau + b_1^* - \alpha g_0^O}{(1+(1-q)\alpha^{1-\sigma})^{\frac{1}{\sigma}} + \theta^{1/\sigma}}\right)$$

Defining $\Xi_0 = \frac{q^{\frac{1}{\sigma}}(1+\theta^{1/\sigma})}{(1+(1-q)\alpha^{1-\sigma})^{\frac{1}{\sigma}}+\theta^{1/\sigma}} \leq 1$, by simple algebra it follows that:

$$b_1^* = \frac{\tau(1-\Xi_0) - \alpha g_0^O(\alpha - \Xi_0)}{1+\Xi_0}, \quad \text{if } \lambda = 0 \tag{52}$$

When $\lambda > 0$, using (48), we can combine the two first order conditions into:

$$[1 + (1-q)\alpha^{1-\sigma}](g_1^I)^{-\sigma} = q\alpha\lambda + \theta(d_1^I)^{-\sigma}$$

Considering that when $\lambda > 0$, it must be that $g_2^I = \alpha g_1^I$, and replacing equation (47) in the last one:

$$[1+(1-q)\alpha^{1-\sigma}](g_1^I)^{-\sigma} = q\alpha[\theta(d_2^I)^{-\sigma} - (\alpha g_1^I)^{-\sigma}] + \theta(d_1^I)^{-\sigma}$$

$$[1+\alpha^{1-\sigma}](g_1^I)^{-\sigma} = \theta[\alpha q(d_2^I)^{-\sigma} + (d_1^I)^{-\sigma}]$$

Replacing the Euler equation (49), one obtains:

$$[1+\alpha^{1-\sigma}](g_1^I)^{-\sigma} = \theta(1+\alpha)(d_1^I)^{-\sigma}$$

Computing the ratio d_1^I/g_1^I delivers the second line of expression (24) in Section B.. Using the budget constraint, it is straightforward to show that:

$$d_1^I = \frac{\Xi_1}{1+\Xi_1}(\tau + b_1^* - \alpha g_0^O) \quad (53)$$

$$g_1^I = \frac{1}{1+\Xi_1}(\tau + b_1^* - \alpha g_0^O) \quad (54)$$

where $\Xi_1 = \left(\frac{\theta(1+\alpha)}{(1+\alpha^{1-\sigma})}\right)^{\frac{1}{\sigma}}$. Here the insurance effect makes the allocations independent of q. Equation (54) implies that in the second period discretionary spending must be:

$$d_2^I = \tau - b_1^* - \alpha^2 g_0^O - \alpha g_1^I = \tau - b_1^* - \alpha^2 g_0^O - \alpha\frac{1}{1+\Xi_1}(\tau + b_1^* - \alpha g_0^O)$$

As a result:

$$d_2^I = \frac{(\tau - \alpha^2 g_0^O)(1+\Xi_1) - b_1^*(1+\alpha+\Xi_1) - \alpha(\tau - \alpha g_0^O)}{1+\Xi_1} \quad (55)$$

Since the Euler equation is still valid, using equations (53) and the last expression, the optimal level of debt solves:

$$(\tau - \alpha^2 g_0^O)(1+\Xi_1) - b_1^*(1+\alpha+\Xi_1) - \alpha(\tau - \alpha g_0^O) = q^{\frac{1}{\sigma}}\Xi_1(\tau + b_1^* - \alpha g_0^O)$$

As a result:

$$b_1^* = \frac{\tau((1-q^{\frac{1}{\sigma}})\Xi_1 + 1 - \alpha) - \alpha g_0^O \Xi_1(\alpha - q^{\frac{1}{\sigma}})}{(1+q^{\frac{1}{\sigma}})\Xi_1 + 1 + \alpha}, \quad \text{if } \lambda > 0 \quad (56)$$

C. First Period with Sequestration.

If the incumbent chooses a level of debt in which sequestration is triggered, the optimal allocation of spending and debt solves:

$$\begin{aligned} \max_{\{g_1^I, d_1^I, b_1\}} & \left\{ u(g_1^I) + \theta u(d_1^I) + u\left((\tau - b_1)\psi\right) \right\} \\ s.t. \;\; & \tau + b_1 \geq g_1^I + d_1^I + \alpha g_0^O \\ & \psi = g_1^I / (g_1^I + \alpha g_0^O) \end{aligned}$$

Letting μ be the multiplier in the budget constraint, the first order necessary conditions are:

$$\begin{aligned} g_1^I &: \; (g_1^I)^{-\sigma} + ((\tau - b_1)\psi)^{-\sigma} \frac{\alpha g_0^O (\tau - b_1)}{(g_1^I + \alpha g_0^O)^2} = \mu, \\ d_1^I &: \; \theta (d_1^I)^{-\sigma} = \mu, \\ b_1 &: \; \psi((\tau - b_1)\psi)^{-\sigma} = \mu. \end{aligned}$$

Combining the first and the last equations, and using the budget constraint, we obtain:

$$(g_1^I)^{-\sigma} = ((\tau - b_1)\psi)^{-\sigma} \left[\psi - \frac{\alpha g_0^O (\tau - b_1)}{(\tau + b_1 - d_1^I)^2} \right]$$

Note that if either $g_0^O = 0$ or $\alpha = 0$ then we have $\psi = 1$ and $d_1^I = \theta^{\frac{1}{\sigma}} g_1^I$. Then, the Euler equation becomes:

$$\left(\frac{\tau + b_1}{1 + \theta^{\frac{1}{\sigma}}} \right)^{-\sigma} = (\tau - b_1)^{-\sigma}$$

Replacing ψ, one obtains

$$\left(g_1^I + \alpha g_0^O\right)^{-\sigma} = (\tau - b_1)^{-\sigma} \left[\frac{g_1^I}{g_1^I + \alpha g_0^O} - \frac{\alpha g_0^O (\tau - b_1)}{(g_1^I + \alpha g_0^O)^2} \right].$$

D Delayed Sequestration

To study delayed stabilization, we drop Assumption 1 and assume that entitlements are unsustainable already at $t = 1$, by considering the case in which condition (8) holds. We endow the incumbent with the choice of whether to sequester or not. We find conditions under which delay occurs.

To analyze the incumbent's trade-off, it is necessary to determine not only how resources are shared in case of sequestration at $t = 1$, but also how much debt is allowed during a fiscal stabilization. Let b^s be the level of debt allowed during a sequestration. Consistent with the $t = 2$ rule, we assume that upon sequestration current resources are shared in proportion to preexisting entitlements. Thus, at time 1, available resources are given by $\tau + b^s$. Therefore, upon sequestration, spending at time 1 is given by:

$$g_1^O = \frac{g_0^O(\tau + b^s)}{g_0^O + g_0^I} \qquad \text{and} \qquad g_1^I = \frac{g_0^I(\tau + b^s)}{g_0^O + g_0^I} \tag{57}$$

We could make different assumptions about b^s, without changing the main thrust of the results. In what follows, for tractability, we assume that debt is allowed so that in the second period entitlements are exactly sustainable:

$$\alpha\frac{(\tau + b^s)g_0^I}{g_0^I + g_0^0} + \alpha\frac{(\tau + b^s)g_0^O}{g_0^I + g_0^O} = \tau - b^s \tag{58}$$

Using the first-period budget constraint, equation (58) generates $b^s = \tau(1-\alpha)/(1+\alpha)$. For example, $\alpha = 1$ implies $b^s = 0$: upon sequestration τ is shared in each period.

If sequestration occurs, the incumbent cuts spending as discussed above. If, instead, sequestration is delayed, the incumbent must pay existing commitments, and she accumulates debt to spend more on the goods that she values. Besides increasing her current utility, higher spending improves the terms of a future sequestration: future cuts will be more drastic, but they will be relatively more favorable to I.[35]

Denote by V_I^D and V_I^S the incumbent's value from delaying and sequestering, respectively. Ignoring the constant in the utility function, and using the time-1 sequestration rule, we obtain:

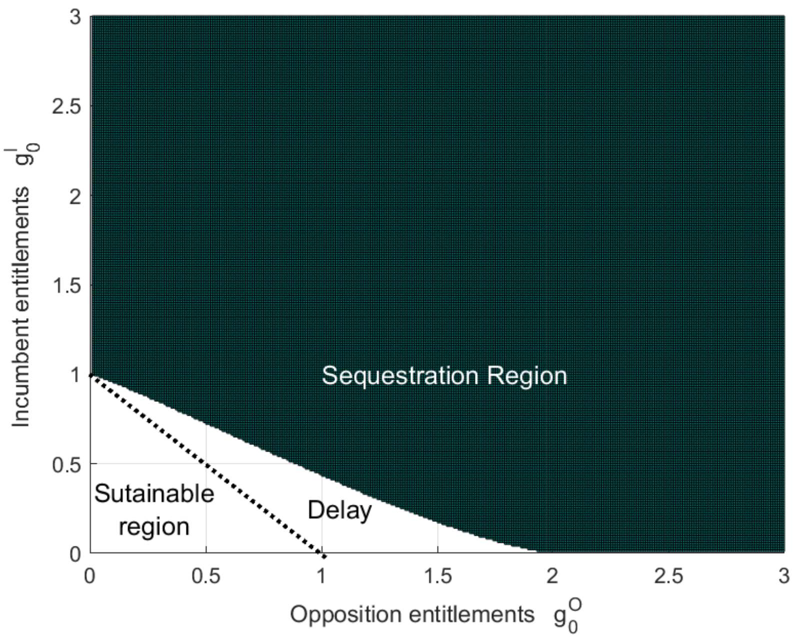
$$V_I^S = (1+\alpha^{1-\sigma})\left(\frac{g_0^I}{g_0^O + g_0^I}\right)^{1-\sigma}\left(\tau + \tau\frac{1-\alpha}{1+\alpha}\right)^{1-\sigma}\frac{1}{1-\sigma} \tag{59}$$

$$V_I^D = \left[(\tau + b_1 - \alpha g_0^O)^{1-\sigma} + \left(\frac{\tau + b_1 - \alpha g_0^O}{\tau + b_1}(\tau - b_1)\right)^{1-\sigma}\right]\frac{1}{1-\sigma} \tag{60}$$

[35] To streamline the analysis we assume that avoiding a sequestration by running more debt is not penalized by financial markets. If we assumed instead that the interest rate increases, the threat of higher borrowing cost would make delay less appealing.

where b_1 solves the first-order condition (12). Both value functions are decreasing in g_0^O, while V_I^S is increasing in g_0^I. A sequestration is triggered at $t = 1$ when $V_I^D \leq V_I^S$.

Figure 10: **Delay and Sequestration Regions**



Throughout this section assume, for simplicity, that $\alpha = 1$. When $g_0^O + g_0^I \leq \tau$, inequality (8) does not hold: precommitments are sustainable, and no sequestration is needed. When, instead, $g_0^O + g_0^I \geq 2\tau$, the government has a solvency problem: a sequestration is unavoidable, because the present value of taxes is larger than existing commitments. Finally, when $\tau < g_0^O + g_0^I < 2\tau$, the incumbent faces a meaningful trade-off. The government is solvent, but the fiscal adjustment must take place, in which case the incumbent can choose whether to do it immediately or leave it to the next government.

We find that a higher ratio g_0^O/g_0^I generally leads to delay. Intuitively, delay is more likely to occur if the party in power has an unfavorable entitlements ratio, which she will try to improve by spending more. As total commitments increase, however, the fiscal space of the incumbent is reduced: after paying existing entitlements, the incumbent would have a small margin for improving the ratio, making delay a less profitable strategy.

In the next proposition, we assume $\alpha = 1$ and characterize the outcome focusing on the region $\mathcal{I} = \left\{(g_0^O, g_0^I) : \tau \leq g_0^O + g_0^I \leq 2\tau\right\}$. Figure 10, illustrates the incentives to delay assuming $\alpha = \tau = 1$ and $\sigma \to 1$.

Proposition 3 *(delayed sequestration) Let* $\alpha = 1$.

(i) If $g_0^O + g_0^I = 2\tau$, then $V_I^S \geq V_I^D$, with strict inequality whenever $g_0^I > 0$.

(ii) If $g_0^O + g_0^I = \tau$, then $V_I^S \leq V_I^D$, with strict inequality whenever $g_0^O > 0$.

(iii) Finally, assume (g_0^O, g_0^I) is such that $\tau < g_0^O + g_0^I < 2\tau$. As $g_0^I/g_0^0 \to \infty$ delay never occurs. As $g_0^I/g_0^0 \to 0$ delay always occurs.

Proof: Assume $\alpha = 1$. Since Proposition 3 focuses on the region $\mathcal{I} = \{(g_0^O, g_0^I) : \tau \leq g_0^O + g_0^I \leq 2\tau\}$, there are three cases to consider.

(i) Assume $g_0^O + g_0^I = 2\tau$. From (59) the value of sequestering is:

$$V_I^S = \frac{2}{1-\sigma}\left(\frac{g_0^I}{g_0^O + g_0^I}\right)^{1-\sigma}(\tau)^{1-\sigma} \tag{61}$$

If, instead, sequestration is delayed, party I pays existing commitments by choosing the maximum possible amount of debt, $b_1 = \tau$. As a result, in the second period, spending will be zero. Therefore, from (60) the value of delaying the sequestration is:

$$V_I^D = \frac{(g_0^I)^{1-\sigma}}{1-\sigma} \tag{62}$$

When $g_0^I = 0$, we have $V_I^S = V_I^D$. When $g_0^I > 0$ we have $V_I^S > V_I^D$ if and only if

$$2\tau^{1-\sigma} > (g_0^O + g_0^I)^{1-\sigma} \tag{63}$$

From $g_0^O + g_0^I = 2\tau$, it follows that $V_I^S > V_I^D$ when $\sigma > 0$. By continuity, $V_I^S > V_I^D$ for $(g_0^O, g_0^I) \in \mathcal{I}$ close to $g_0^O + g_0^I = 2\tau$ and $g_0^I > 0$.

(ii) Assume now $g_0^O + g_0^I = \tau$. We compute the values of sequestering and delaying:

$$V_I^S = 2(g_0^I)^{1-\sigma}\frac{1}{1-\sigma} \tag{64}$$

$$V_I^D = \left[(g_0^I+b_1)^{1-\sigma}+\left(\frac{g_0^I+b_1}{\tau+b_1}(\tau-b_1)\right)^{1-\sigma}\right]\frac{1}{1-\sigma} \tag{65}$$

where b_1 solves the first-order condition under sequestration. Note that the two expressions coincide if b_1 is zero. When $g_0^O=0$, optimal debt is indeed zero, and thus we obtain $V_I^S=V_I^D$. When $g_0^O>0$ it is immediate to verify that optimal debt is strictly positive, implying that $V_I^S<V_I^D$. By continuity, $V_I^S<V_I^D$ for $(g_0^O,g_0^I)\in\mathcal{I}$ close to $g_0^O+g_0^I=\tau$ and $g_0^O>0$.

(iii) Finally, assume $g_0^O+g_0^I<2\tau$. The values of sequestering and delaying are:

$$V_I^S = 2\left(\frac{g_0^I}{g_0^O+g_0^I}\right)^{1-\sigma}(\tau)^{1-\sigma} \tag{66}$$

$$V_I^D = (\tau+b_1-g_0^O)^{1-\sigma}+\left(\frac{\tau+b_1-g_0^O}{\tau+b_1}(\tau-b_1)\right)^{1-\sigma} \tag{67}$$

When $(g_0^O,g_0^I)\in\mathcal{I}$, the ratio g_0^I/g_0^0 goes to $+\infty$ when g_0^0 goes to zero. In this case, $V_I^S>V_I^D$ since party I can implement her preferred allocation under commitment by triggering a sequestration at $t=1$. Finally, $(g_0^O,g_0^I)\in\mathcal{I}$ the ratio g_0^I/g_0^0 goes to zero, when g_0^I goes to zero. In this case, V_I^S converges to zero and, consequently, $V_I^D>V_I^S$. $\square$

E Infinite Horizon

A. The Model

Suppose time is discrete and runs forever, $t=0,1,...,\infty$. Analogously to the two-period economy, the budget constraint is

$$g_t^O + g_t^I + (1+r)b_t = \tau + b_{t+1}. \tag{68}$$

We denote total spending at time t by $g_t \equiv g_t^O + g_t^I$. When considering the infinite horizon model, it is necessary to redefine the sequestration threshold. We assume that sequestration is automatically triggered when the present value of future revenues net of liabilities is smaller than the present value of future spending. In Section A..4, we will also consider the scenario in which sequestration is not automatic and the incumbent may choose to delay sequestration.

Given total entitlements αg_{t-1} at time t, the budget is sustainable if

$$\sum_{j=t}^{\infty} \frac{\alpha g_{j-1}}{(1+r)^{j-t}} \leq \sum_{j=t}^{\infty} \frac{\tau}{(1+r)^{j-t}} - (1+r)b_t \tag{69}$$

$$\alpha \sum_{j=0}^{\infty} \frac{\alpha^j g_{t-1}}{(1+r)^j} \leq (1+r)\frac{\tau}{r} - (1+r)b_t \tag{70}$$

The sequestration threshold takes into account that entitlements depreciate over time when $\alpha < 1$. The previous equation can be written as

$$\alpha g_{t-1} \frac{1+r}{1+r-\alpha} \leq (1+r)\frac{\tau}{r} - (1+r)b_t \tag{71}$$

As a result, spending is sustainable if

$$\underbrace{\frac{\alpha(g_{t-1}^i + g_{t-1}^{-i})}{1+r-\alpha}}_{PV\ of\ spending:} \leq \underbrace{\frac{\tau}{r} - b_t}_{PV\ of\ resources} \tag{72}$$

It is noteworthy that when $\alpha = 0$ the constraint coincides with the standard natural debt limit, indicating that sustainability would never be a concern.

The next step is to characterize the sequestration threshold implied by the current decisions. From (68), selecting (g_t, b_{t+1}) at time t implies a sequestration in the following period if:

$$\alpha[\tau + b_{t+1} - (1+r)b_t] \geq (1+r-\alpha)\left(\frac{\tau}{r} - b_{t+1}\right) \tag{73}$$

or

$$b_{t+1} \geq \tilde{b}_{t+1}(b_t) \equiv (1-\alpha)\frac{\tau}{r} + \alpha b_t \tag{74}$$

When there is initial debt, the sequestration threshold depends on the level of debt. Notice the stark similarity with the sequestration threshold derived in the two-period model. In the baseline economy, we assumed that $b_0 = 0$, which generates a sequestration threshold as in (74). Had we allowed for initial debt, the sequestration threshold in the two-period economy would be similar to (74), with b_t replaced by b_0.

A..1 Debt after Sequestration

To understand the consequences of sequestration, it is crucial to compute the debt trajectory after sequestration. Upon consolidation the implied level of debt is

$$b^s_{t+1} = \alpha g_{t-1}\nu_t + (1+r)b_t - \tau \tag{75}$$

where, analogously to the two-period model, the proportion ν_t is chosen to ensure that the spending path is sustainable:

$$\nu_t = \frac{\frac{\tau}{r} - b_t}{\frac{\alpha g_{t-1}}{1+r-\alpha}}$$

Therefore, upon sequestration, entitlements are cut as follows:

$$g_t = \alpha g_{t-1}\nu_t = \alpha g_{t-1}\frac{\frac{\tau}{r} - b_t}{\frac{\alpha g_{t-1}}{1+r-\alpha}} = (1+r-\alpha)\left(\frac{\tau}{r} - b_t\right). \tag{76}$$

Replacing equation (76) in (75), the law of motion of debt after sequestration can be expressed as:

$$b^s_{t+1} = (1+r-\alpha)\left(\frac{\tau}{r} - b_t\right) + (1+r)b_t - \tau.$$

After canceling out terms, we obtain

$$b^s_{t+1} = \tilde{b}_{t+1}(b_t).$$

This is an important result because it implies that after sequestration, the economy remains on the sequestration threshold forever.

A..2 Sequestration Value Functions

Since sequestration is an absorbing state, the computation of its continuation value is greatly simplified. We begin by computing an average value function, and then we explain how to assign that value to each party. Note that

$$b^s_{t+1} = (1-\alpha)\left(\frac{\tau}{r} - b^s_t\right) + b^s_t \quad \Rightarrow \quad \left(\frac{\tau}{r} - b^s_{t+1}\right) = \alpha\left(\frac{\tau}{r} - b^s_t\right). \tag{77}$$

We guess (and later verify) that the sequestration value function depends on total wealth:

$$V^s(b_t) = Au\left(\frac{\tau}{r} - b_t\right) \tag{78}$$

where $A \neq 0$. Given the spending solution in equation (76), the value function must satisfy the following equation:

$$V^s(b) = u\left(g_t\right) + \beta V^s(b')$$

$$Au\left(\frac{\tau}{r} - b\right) = u\left((1 + r - \alpha)\left(\frac{\tau}{r} - b\right)\right) + \beta Au\left(\alpha\left(\frac{\tau}{r} - b\right)\right).$$

Assuming

$$u(g_t) = \frac{(g_t)^{1-\sigma}}{1-\sigma},$$

we can solve for the constant A, obtaining

$$A = \frac{(1 + r - \alpha)^{1-\sigma}}{1 - \beta\alpha^{1-\sigma}}.$$

The sequestration value function V^s represents the value of a deterministic decaying stream of spending, calculated based on the total wealth. However, party i only consumes a fraction $\psi_i = \frac{g^i}{g^i + g^{-i}}$ of it. Since this share remains constant over time, it is straightforward to modify the previous calculation to derive the sequestration value function for agent i:

$$V^s(b; \psi_i) = \frac{(1 + r - \alpha)^{1-\sigma} u\left(\psi_i\left(\frac{\tau}{r} - b\right)\right)}{1 - \beta\alpha^{1-\sigma}}. \tag{79}$$

It is important to note that after sequestration, ψ_i remains constant. It is only ex-ante, before sequestration, that the incumbent can modify it:

$$\frac{\partial \psi_i}{\partial g^i} \geq 0 \quad (> 0 \text{ only if } g^{-i} > 0)$$

As in the two-period model, the dilution effect only appears if the party out of power has some previously accumulated entitlements.

A..3 Equilibrium

We introduce some notation before defining the equilibrium. The value functions depend on three state variables. The first state, common to both agents, is the level of debt denoted by b. The second and third variables represent the entitlements of the incumbent and of the opposition, respectively. Denote by x the entitlements of each party in the value function ("my" entitlements) and by y the entitlements of the other party ("other party's" entitlements). Given that preferences are fully polarized, the incumbent provides the opposition with entitlements $g^{-i} = \alpha y$. We denote the incumbent's preferred good as g.

We define an indicator function that takes the value 1 when commitments are sustainable and 0 otherwise:

$$\Phi = \begin{cases} 1 & \text{if } \frac{\alpha(x+y)}{1+r-\alpha} \leq \frac{\tau}{r} - b \\ 0 & \text{otherwise} \end{cases}$$

Definition. An equilibrium is $V^I(x, y, b)$, $V^O(x, y, b)$, $V^s(b; \psi_i)$ and $b^{*'}(x, y, b)$ such that:

1) The incumbent solves:

$$\begin{aligned} V(x, y, b) \quad &= \quad \max_{\{g, b'\}} \Bigg\{ u(g) + \beta\Phi\big[qV^I(g, \alpha y, b') + (1-q)V^O(\alpha y, g, b')\big] \\ &\qquad\qquad + \beta(1-\Phi)V^s\left(b'; \frac{g}{\alpha y + g}\right)\Bigg\} \\ s.t. \quad &\quad g \leq \tau - (1+r)b + b' - \alpha y \\ with \quad &\quad \Phi = 1 \quad \text{if } b^{*'} \leq \tilde{b}(b) \end{aligned}$$

Then, the incumbent's value function satisfies:

$$V^I(x, y, b) = \Phi V(x, y, b) + (1-\Phi)V^s\left(\tilde{b}(b); \frac{x}{x+y}\right)$$

2) $b^{*'}(x, y, b)$ achieves the maximum in the previous problem.

3) The opposition's value function satisfies:

$$
\begin{aligned}
V^O(x,y,b) &= \Phi\left\{u(\alpha x) + \beta\Phi[qV^O(\alpha x, g^*, b^{*'}) + (1-q)V^I(g^*, \alpha x, b^{*'})] \right.\\
&\left. +\beta(1-\Phi)V^s\left(b^{*'}; \frac{\alpha x}{\alpha x + g}\right)\right\} + (1-\Phi)V^s\left(\tilde{b}(b); \frac{x}{x+y}\right)\\
\textit{with} \quad & \Phi = 1 \quad \text{if } b^{*'} \leq \tilde{b}(b)
\end{aligned}
$$

Recall the choice of notation for x and y. For instance, x in the incumbent's value function represents the incumbent's entitlements, while the x in the opposition's value function represents the opposition's entitlements. Also, it is important to note that in this equilibrium, the incumbent is not required to meet her own entitlement level. In other words, in equilibrium, the incumbent might choose to set $g < \alpha x$. This simplifying assumption helps to isolate the impact of the opposition's entitlement only.

A..4 Endogenous Sequestration

In the previous notion of equilibrium, we assumed that the incumbent has no choice when $\Phi = 0$, leading to automatic sequestration. Now, we will consider the possibility that the incumbent can postpone sequestration by borrowing more. The equilibrium remains similar to the previous section with some small modifications.

Definition. An equilibrium with *endogenous sequestration* is $V^I(x,y,b)$, $V^O(x,y,b)$, $V^s(\psi_i, b)$, $b^{*'}(x,y,b)$ and $\Phi^*(x,y,b)$ such that:

1) Conditionally on not sequestering, the incumbent solves:

$$
\begin{aligned}
V(x,y,b) &= \max_{\{g,b'\}} \left\{ u(g) + \beta\left[qV^I(g, \alpha y, b') + (1-q)V^O(\alpha y, g, b')\right] \right\}\\
s.t. \quad & g \leq \tau - (1+r)b + b' - \alpha y
\end{aligned}
$$

Then, the incumbent's value function satisfies:

$$
V^I(x,y,b) = \max\left\{V(x,y,b), V^s\left(\tilde{b}(b); \frac{x}{x+y}\right)\right\}
$$

2) $b^{*'}(x,y,b)$ achieves the maximum in the previous problem.

3) $\Phi^*(x,y,b) = 1$ if $V(x,y,b) \geq V^s\left(\tilde{b}(b); \frac{x}{x+y}\right)$, and zero otherwise.

4) The opposition's value function satisfies:

$$\begin{aligned} V^O(x,y,b) &= \Phi^*(y,x,b)\left\{u(\alpha x) + \beta[qV^O(\alpha x, g^*, b^{*'}) + (1-q)V^I(g^*, \alpha x, b^{*'})]\right\} \\ &+ (1 - \Phi^*(y,x,b))V^s\left(\tilde{b}(b); \frac{x}{x+y}\right) \end{aligned}$$

In terms of notation, there is an important distinction between this setting and the previous one. In Section A..3, we explicitly model Φ, while in the above definition, Φ^* does not appear in the continuation value. There are two ways to explain this. First, the inclusion of Φ in Section A..3 is unnecessary: all the relevant information provided by Φ is already captured in the value functions. We chose to include it merely to establish an analogy with the two-period model. In the delayed equilibrium, however, since Φ^* is endogenous and more challenging to characterize, such an approach would only introduce additional notation and confusion. Consequently, the problem is written emphasizing the determination of Φ^* at each point.

B. Computation

B..1 Taste Shocks

It is well known that computing Markov-perfect equilibria in political games can be problematic. As shown by Chatterjee and Eyigungor (2012b), one of the main issues encountered when solving such problems is the potential lack of concavity in the value functions. A common approach, which is also adopted here, is to slightly perturb the choices of the agent by introducing small, independent, and identically distributed shocks. These shocks may apply to fundamentals, as in Chatterjee and Eyigungor (2012b), or directly to the agent's payoff, as in Chatterjee and Eyigungor (2020). This randomization over options with payoff of comparable value greatly facilitates the computation of the model, resulting in smooth value functions and policy functions, and inducing near-monotone convergence through standard value function iteration. Following Gordon (2019), we use functional forms and assumptions commonly employed in discrete choice methods. We perturb the incumbent's problem by augmenting it with choice-specific taste shocks extracted from a Gumbel distribution. To streamline the notation, let $s = \{x, y, b\}$ be the state variables. For each s define the maximized value:

$$M(s) = \max_{\{s' \ is \ feasible\}} V^I(s')$$

In each period, after observing the taste shock $\epsilon_{s'}$ for each potential policy, the incumbent makes a decision. She solves the following problem:

$$\tilde{V}^I(s; \{\epsilon_{s'}\}_{s'}) = \max_{s'} \left\{V^I(s') + \rho\epsilon_{s'}\right\}$$

Following McFadden (1973), it can be shown that the ex-ante probability of selecting a specific option s_i is given by:

$$Pr(s' = s_i|s) = \frac{\exp[V^I(s_i)/\rho]}{\sum_j \exp[V^I(s_j)/\rho]} = \frac{\exp[(V^I(s_i) - M(s))/\rho]}{\sum_j \exp[(V^I(s_j) - M(s))/\rho]} \tag{80}$$

From Rust (1987) and Balog et al. (2017), the expected value for the incumbent, prior to observing the taste shocks, is:

$$V^I(s) = M(s) + \rho \log\left[\sum_j \exp([V^I(s_j) - M(s)]/\rho)\right] \tag{81}$$

In turn, the ex-ante expected value for the opposition party is given by:

$$V^O(s) = \sum_j \left[Pr(s' = s_j|s)(u(s_j) + \beta[qV^O(s_j) + (1-q)V^I(s_j)])\right] \tag{82}$$

where s_j should be interpreted as the implied future state variables resulting from the optimal choices of b, and ρ represents the extent of exposure to taste-shock risk. Thus, as $\rho \to 0$, the economy converges to the baseline setup.

B..2 Numerical Solution

As shown in the baseline model, the solution to the problem may feature jumps, and the optimum may lie on a non-differentiable point. Thus, using the first-order necessary condition to find the equilibrium would be inappropriate. As a result, we solve the model economy using value function iteration. A few considerations are in order. Recall that there are three state variables: x, y, and b. If we assume a grid for b, the implied values for x and y (taking into account the "depreciation" α) may not belong to a predetermined discrete set for x and y. Although, with a sufficiently fine grid, one could look for the nearest point, the problem extends to the spending choice since the implied value for g (through the budget constraint) due to different combinations of y and b' may not lie in x. Appealing to the nearest points in this case would result in violations of the budget constraint. As a result, it is not possible to compute the solutions appealing only to values on a grid. Thus, we perform some interpolations during the calculations. The numerical algorithm proceeds as follows:

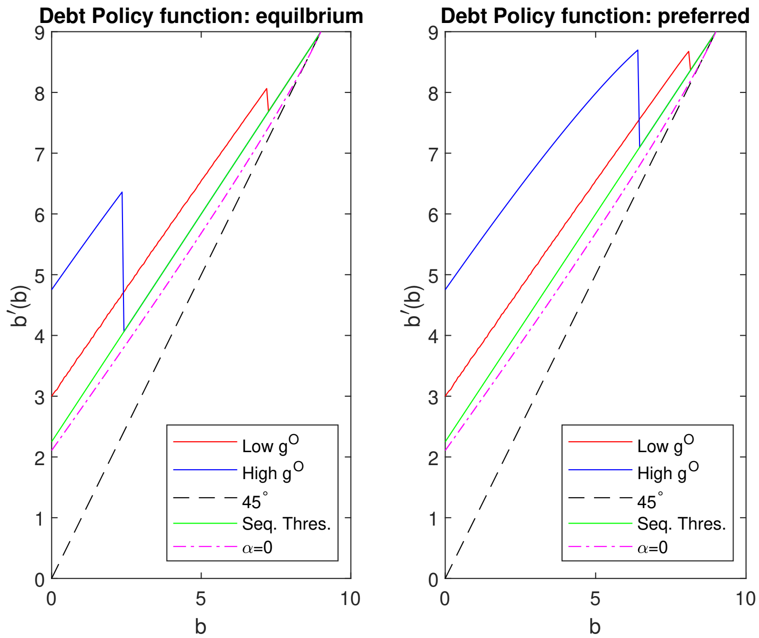
1) Fix a grid for x, y and b. Guess initial values for $V_0^I(x, y, b)$ and $V_0^O(x, y, b)$ and compute the sequestration value V^s for all combinations of x, y and b.

2) Compute all the possible combinations of spending g using the budget constraint: $g = \tau + (1 + r)b - b' - \alpha y$ and evaluate the continuation value functions. This step requires interpolation. For instance, the payoff for each g requires computing $V^I(g, \alpha y, b')$. The computation is performed by using linear interpolation (Matlab built-in function).

3) Given the state s, find the optimal b'. This delivers the value $M(s)$ described in section B..1. Then, we can compute the probabilities over future states using equation (80). Armed with these values, we can compute the new value functions $V_1^I(x, y, b)$ and $V_1^O(x, y, b)$. This step varies whether we are computing the equilibrium with exogenous or endogenous sequestration. With exogenous sequestration, we use equation (74) to generate the new value, with $V^i = V^s$ whenever $\Phi = 0$, for $i = I, O$.

4) If $V_1^I(x, y, b)$ and $V_1^O(x, y, b)$ are sufficiently close to $V_0^I(x, y, b)$ and $V_0^O(x, y, b)$, a solution has been found. Otherwise, start again in step 2 updating $V_0^i(x, y, b) = \chi V_1^i(x, y, b) + (1 - \chi)V_0^i(x, y, b)$, for some $\chi \in (0, 1)$ and $i = I, O$.

Figure 11: **Policy Function: Baseline Model** ($\alpha = 0.75$)



B..3 Quantitative Results

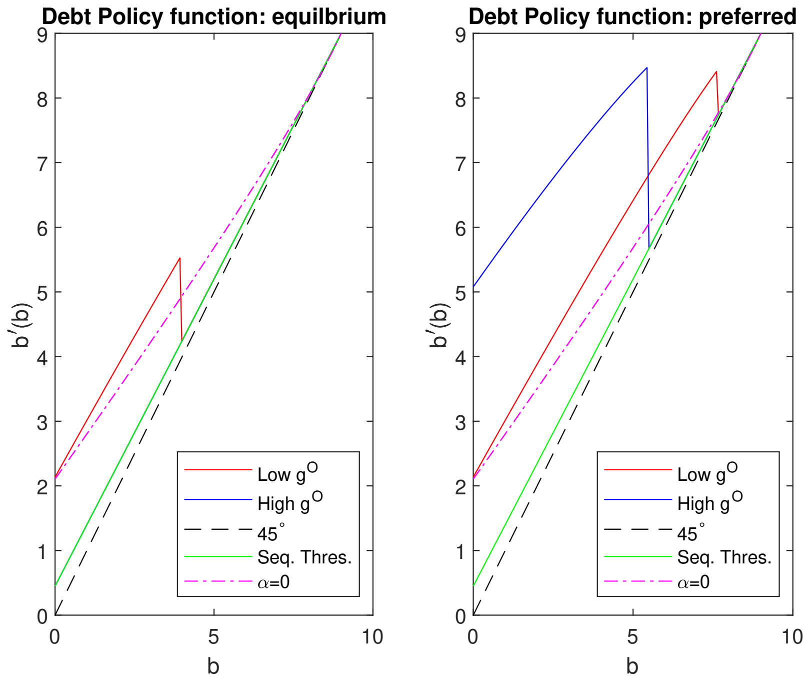
Next, we present some quantitative results. For this exercise, we make the following assumptions: $\rho = 0.3$, $\tau = 1$, $q = \sigma = 1/2$. The grids consist of 150 points for the variable b and 151 points for the entitlement's states (x and y).

The left panel of Figure 11 illustrates the debt policy function in the model with automatic sequestration. The blue line represents debt accumulation when the opposition's entitlement is relatively high, while the red line depicts the same but with lower opposition entitlements. The sequestration threshold, equation (74), is given by the green line. Additionally, the dashed-dotted magenta lines represent the solution of the canonical model where $\alpha = 0$. The right panel of Figure 11 illustrates the "preferred" level of debt if sequestration were not automatic and serves as an illustration of the incentives to delay sequestration by one period.

The main takeaway from the left Panel of Figure 11 is that the infinite horizon economy produces similar predictions to the two-period model. In this specific calibration, the debt level is consistently higher than the optimal level (i.e., it is above the $45°$ line) and also higher than the debt level in the canonical model with $\alpha = 0$. The increase in debt is particularly pronounced when the opposition's entitlements (g^O) are high. When g^O reaches a sufficiently large value, and there is enough fiscal space (low b), the incumbent government chooses to significantly increase spending to induce a future sequestration and dilute the opposition's entitlements. The larger g^O, the stronger the temptation to dilute, resulting in a larger increase in debt. Once the dilution occurs, the economy remains at the sequestration threshold forever (green line). Note that if both the initial endowments and the level of debt are high, there is no fiscal space available, leading to an automatic sequestration from the outset. This explains why, for certain combinations of g^O and b, the initial debt choice coincides with the sequestration threshold.

Another important takeaway from Figure 11 is that despite the varying speeds, the debt paths always exhibit increasing debt levels and ultimately converge to the upper bound (namely, the natural debt limit). This holds not only for the specific parameterization presented here but for all combinations of parameters as long as $q \in (0, 1)$. The different parameters may impact the rate at which the economy converges to this asymptotic limit, but they do not alter the limit itself.

Figure 12: **Policy Function: Baseline Model** ($\alpha = 0.95$)



The fact that in Figure 11 the debt level is consistently higher than in the canonical model is due to the chosen calibration. As shown in the left Panel of Figure 12, when a larger value of α is used, the sequestration threshold falls below the debt choice of the canonical model. Consequently, the economy can exhibit less debt accumulation than the canonical model.

There could be situations where the rule calls for automatic sequestration, but the incumbent is unwilling to implement it. As an illustration, the right panels of Figures 11 and 12 present the "preferred" policy functions: the debt choices the incumbent would make if she didn't have to sequester. It is evident that the policy functions in the left and right panels do not overlap. In particular, in the right panel, the area of dilution is larger than in the left panel. This extended area captures the temptation to avoid automatic sequestrations and delay them to the future. Note that the right panels of Figures 11 and 12 do not represent the equilibrium debt rule under endogenous sequestration. For that, we need to properly incorporate the expectations of future delays into the incumbent's decision-making process as discussed in Section A..4 (see Figure 15 below).

Figure 13 shows two sample paths of the baseline economy with automatic sequestrations. Panel A corresponds to a path where, given the initially large opposition's entitlements, there is a sequestration in the initial period. Panel B corresponds to a path where such initial sequestration is not needed. Comparing the debt paths, we can see that they are not substantially different. Although it may not be clear from the figures, the path without initial sequestration generates a slightly higher level of debt. The differences are not larger because in the case without initial sequestration, sequestration occurs in the following period.

In the bottom of each panel, we have plotted a binary variable that equals one when the economy enters a state of automatic sequestration (blue line) and zero otherwise. The dashed line ("Preferred") in each figure reflects the incumbent party's willingness to remain in the sequestration state. It is worth noting that when there is a change of power, there are instances where the incumbent would prefer to exit the sequestration state by not implementing it. However, in the baseline model, such actions are not allowed, but they would play a role in the model with endogenous sequestration.

Figure 13: **Time Path: Baseline Model** ($\alpha = 0.75$)

Panel A: Initial sequestration (g_o^O large) Panel B: No initial sequestration (g_o^O small)

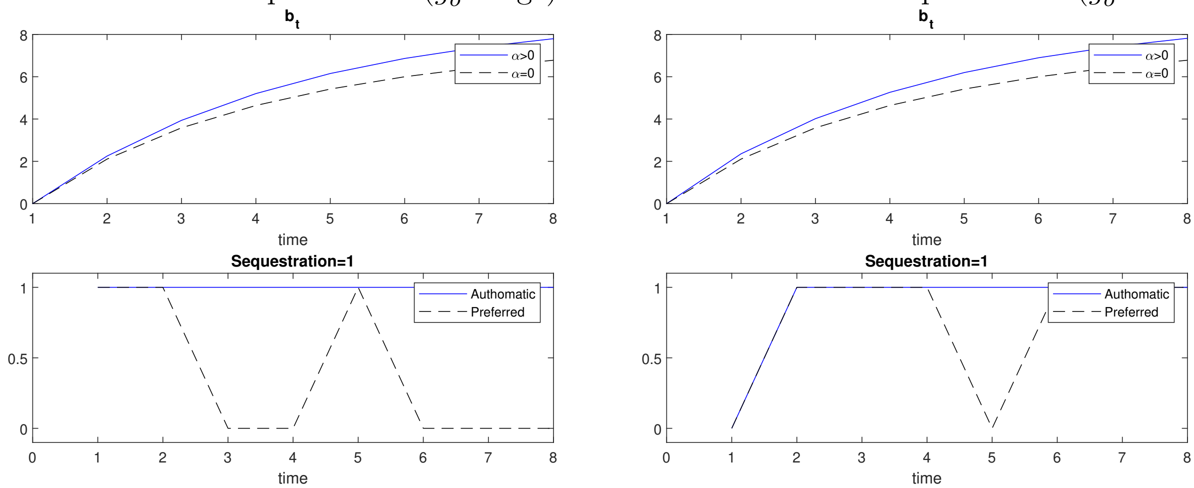
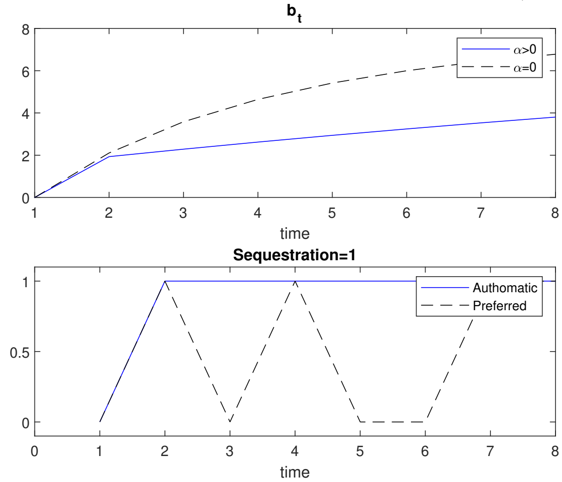


Figure 14: **Time Path: Baseline Model** ($\alpha = 0.95$)

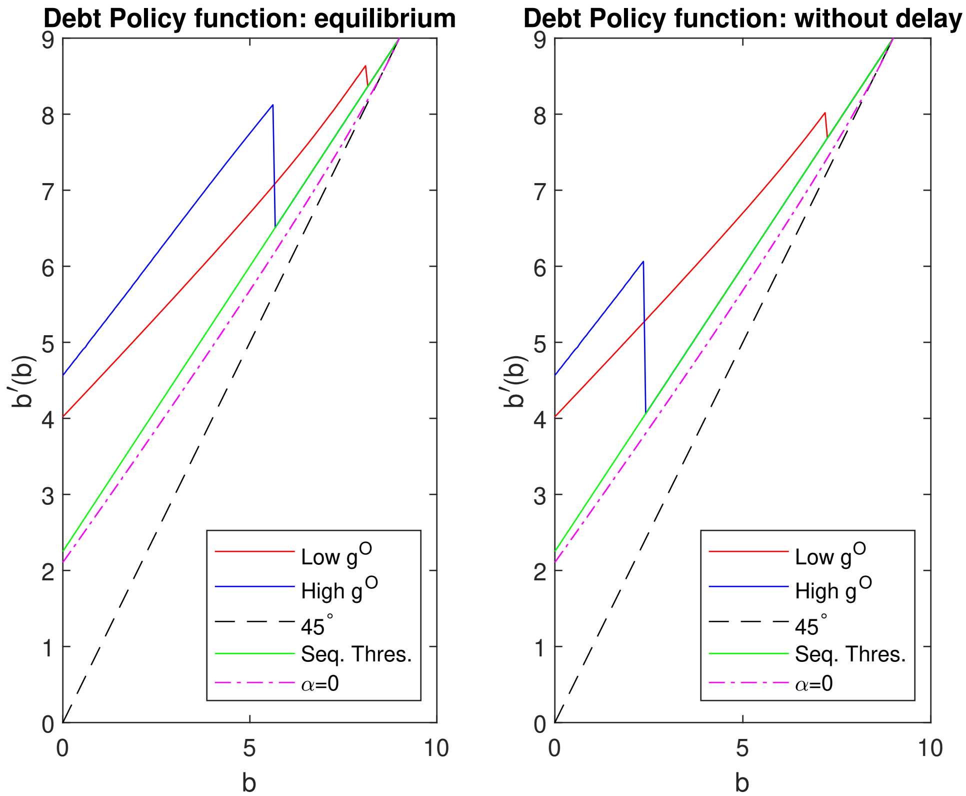


No initial sequestration ($g_o^O > 0$ small)

In Figure 14, we plot a sample path, comparable to Panel B of Figure 13, but with higher persistence: $\alpha = 0.95$. The main difference now is that upon sequestration, debt grows substantially less than in the previous calibration and less than it would be observed in the canonical model ($\alpha = 0$) for the same initial conditions and history of the shocks. Because of equation (74), a larger α makes the future implied debt closer to the current debt, reducing debt growth.

So far, we have presented results for the model with automatic sequestrations. Now, we turn to the results for the model with endogenous sequestration, as discussed in Section A..4. The policy functions are depicted in Figure 15. Analogous to Figure 11, the figure on the left corresponds to the equilibrium, while the figure on the right corresponds to a thought experiment where the incumbent is forced to sequester even when she does not want to. The parameterization in these two figures is the same. The patterns are qualitatively similar from comparing Figures 15 and 11. The new finding is that sequestrations are sometimes delayed. Even when the incumbent internalizes the possibility of future incumbents delaying sequestrations, delays still happen. And, as discussed in Section D, when the level of debt relative to entitlements is sufficiently high, sequestration takes place anyway. Although the qualitative patterns are similar, quantitatively they are not. Observing Figures 15 and 11, one can see that the level of debt is higher than in the case of automatic sequestration. This happens because the incumbent understands that the future incumbent may also delay sequestration, increasing spending to improve her relative entitlement's position and obtain a larger share in forthcoming sequestrations. Anticipating this, the incumbent drastically increases spending today to reduce to a minimum the fiscal capacity and, if possible, leaves no choice for the future incumbent but to sequester. This potential path can be seen in Figure 16, which is analogous to Figure 13. Looking at Panel A, we can see that instead of implementing the sequestration in the initial period, as in Figure 13, the incumbent now delays it and drastically increases spending. The increase in debt is so large that the future incumbent falls into the no-delay area, leading to the implementation of sequestration. Another interesting new pattern is that when there is endogenous sequestration, the economy does not enter an indefinite sequestration state. On the equilibrium path, incumbents (generally new ones) occasionally avoid sequestration and increase spending to improve their entitlements' position. To observe this, compare the dashed black lines and the blue lines in the lower right corner of each panel.

Figure 15: **Policy Function: Model with Endogenous Sequestration** ($\alpha = 0.75$)

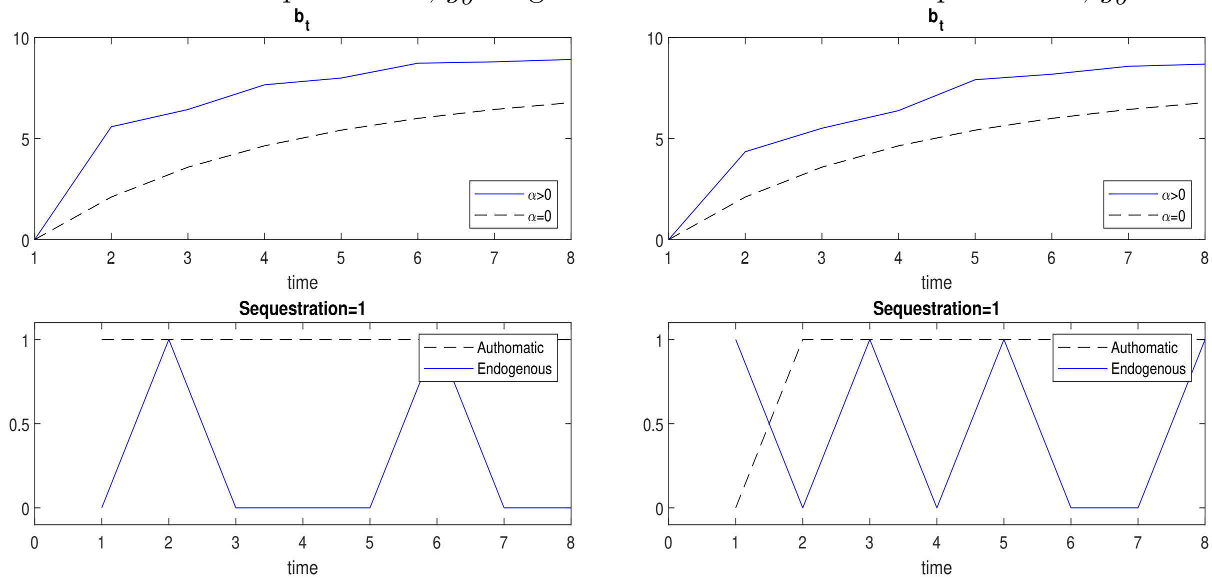


F Equal Split between Nondefense and Defense

The baseline model assumes that all spending programs are reduced by the same percentage. Section II. shows that this assumption is broadly verified during deliberate fiscal consolidations. Automatic fiscal adjustments, however, are subject to more complex rules. For example, US sequestration exempts certain programs and divides cuts equally between defense and non-defense programs (see Section A.). To simplify the exposition, in this section, we model the requirement that the cuts be divided equally between defense and non-defense programs. We will examine exemptions in Section G. We show that modeling these features maintains the thrust of our results. In particular, when equally splitting the sequestration burden between defense and nondefense, the dilution effect is preserved and, if anything, exacerbated. In the next section, we demonstrate that sequestration exceptions also maintain the dilution channel and lead to a non-monotonic relationship between debt and stickiness.

Figure 16: **Time Path: Endogenous Sequestration**

Panel A: Initial sequestration, g_o^O large — Panel B: No initial sequestration, g_o^O small



There are two goods, I and O, which we will now interpret as non-defense and defense spending. The model is the same as in Section IV., but we modify the sequestration formula. Suppose that commitments inherited from period 1 are not sustainable in the second period: $\alpha g_1^I + \alpha g_1^O > \tau - b_1$. After sequestration, the budget constraint of the government is given by

$$\underbrace{\nu_I \alpha g_1^I}_{=g_2^I} + \underbrace{\nu_O \alpha g_1^O}_{=g_2^O} = \tau - b_1, \tag{83}$$

where $\nu_I \in [0, 1)$ and $\nu_O \in [0, 1)$ denote the proportions of past commitments that are maintained upon sequestration. The smaller ν_I and ν_O, the more drastic the sequestration. Instead of imposing $\nu_I = \nu_O$ as in Section IV., we modify the sequestration formula by requiring that cuts are evenly divided between defense and non-defense programs: that is,

$$(1 - \nu_I)\alpha g_1^I = (1 - \nu_O)\alpha g_1^O \tag{84}$$

The first thing to note is that the equal split requirement tends to favor the larger program, whether nondefense or defense. For instance, when $g_1^I > g_1^O$, the sequestration formula results in $\nu_I > \nu_O$, which leads to a comparatively smaller percentage reduction for the larger program I. We will see that this exacerbates the dilution problem by creating a stronger incentive to increase spending.

The second thing to note is that it is not always possible to find interior solutions for ν_I and ν_O that simultaneously satisfy (83) and (84). In some cases, one of the constraints $v_I \geq 0, v_O \geq 0$ might be binding. To illustrate this point, consider the case where inherited commitments for good O are small (i.e., $\alpha g_1^O \approx 0$), while commitments for good I are much larger. In this case, equation (84) results in $(1-\nu_I)\alpha g_1^I \approx 0$, implying that ν_I must be close to one. However, setting $\nu_I \approx 1$ and $\nu_0 = 0$ may not satisfy (83) when g_1^I is sufficiently large. If reducing program O to zero does not make total commitments sustainable, we assume that program I is further cut to restore feasibility, even if it means departing from an equal split.[36] The following Lemma characterizes the solution after sequestration.

Lemma F.1 (Equal Split Sequestration) *After sequestration, spending is given by*

$$g_2^I = \begin{cases} \left(\frac{\alpha(g_1^I - g_1^O)}{2(\tau - b_1)} + \frac{1}{2}\right)(\tau - b_1), & \text{if } \alpha|g_1^I - g_1^O| \leq \tau - b_1 \\ 0, & \text{if } \alpha(g_1^O - g_1^I) > \tau - b_1 \\ \tau - b_1, & \text{if } \alpha(g_1^I - g_1^O) > \tau - b_1 \end{cases}$$

$$g_2^O = \begin{cases} \left(\frac{\alpha(g_1^O - g_1^I)}{2(\tau - b_1)} + \frac{1}{2}\right)(\tau - b_1), & \text{if } \alpha|g_1^I - g_1^O| \leq \tau - b_1 \\ 0, & \text{if } \alpha(g_1^I - g_1^O) > \tau - b_1 \\ \tau - b_1, & \text{if } \alpha(g_1^O - g_1^I) > \tau - b_1 \end{cases}$$

Proof: There are three cases to consider. The first and easiest one is when $g_1^I = g_1^O = g$. Equation (84) implies that $\nu_O = \nu_I$. By inspecting (83), it is immediately apparent that it is possible to find a value $\nu_O = \nu_I \in [0, 1)$ that restores feasibility. In particular, the solution that simultaneously satisfies both (83) and (84) is given by $\nu_O = \nu_I = (\tau - b_1)/(2g\alpha)$, which implies that $g_2^I = g_2^O = (\tau - b_1)/2$, as stated in the Lemma.

[36] The possibility that either $v_I = 0$ or $v_O = 0$ might be binding is a theoretical possibility. However, given that the size of US defense and non-defense programs is significantly greater than the total cuts required by sequestration ($109.3 billion annually), constraints $v_I, v_O \geq 0$ are non-binding in practice.

Second, suppose that $g_1^I > g_1^O$. According to equation (84), g_1^O is cut proportionally more than g_1^I, which implies that $\nu_0 < \nu_I$. Among all pairs (ν_0, ν_I) that satisfy (84), the combination that results in the largest spending cut is when $\nu^O = 0$ and $\nu_I = 1 - \frac{g_1^O}{g_1^I}$. These cuts satisfy the government budget constrain if

$$\alpha \left(1 - \frac{g_1^O}{g_1^I}\right) g_1^I \leq \tau - b_1$$

We thus conclude that when

$$\alpha(g_1^I - g_1^O) \leq \tau - b_1 \tag{85}$$

there exists an interior solution that simultaneously satisfies both (83) and (84), which is given by

$$\nu_O = \frac{\alpha(g_1^O - g_1^I) + \tau - b_1}{2\alpha g_1^O} \tag{86}$$

$$\nu_I = \frac{\alpha(g_1^I - g_1^O) + \tau - b_1}{2\alpha g_1^I} \tag{87}$$

Using (86) and (87), spending after sequestration is given by:

$$g_2^I = \underbrace{\left(\frac{\alpha(g_1^I - g_1^O)}{2(\tau - b_1)} + \frac{1}{2}\right)}_{\psi_I}(\tau - b_1) \tag{88}$$

$$g_2^O = \underbrace{\left(\frac{\alpha(g_1^O - g_1^I)}{2(\tau - b_1)} + \frac{1}{2}\right)}_{1-\psi_I}(\tau - b_1) \tag{89}$$

as stated in the Lemma. Using (2), we can rewrite (88) and (89) as

$$g_2^I = \frac{\tau(1+\alpha) - 2\alpha g^O - b_1(1-\alpha)}{2}$$

$$g_2^O = \frac{\tau(1+\alpha) - 2\alpha g^I - b_1(1-\alpha)}{2} \tag{90}$$

Notice that when $\alpha = 1$, spending in the first period is a "free-lunch" for the incumbent: her spending in the second period is constant and equal to $\tau - g_1^O$.

When (85) does not hold, the equal split requirement has to be modified to satisfy the budget constraint. We set $\nu^O = 0$ and program I is further cut to restore feasibility: $\nu^I = (\tau - b_1)/(\alpha g_1^I)$. Then, g_2^I and g_2^O coincide, respectively, with $\tau - b_1$ and 0 as stated in the Lemma.

Third, suppose $g_1^I < g_1^O$. According to equation (84), this implies that $\nu_0 > \nu_I$. Among all pairs (ν_0, ν_I) that satisfy (84), the combination that results in the largest spending cut is when $\nu^I = 0$ and $\nu_O = 1 - \frac{g_1^I}{g_1^O}$. These cuts generate sufficient savings if

$$\alpha \left(1 - \frac{g_1^I}{g_1^O}\right) g_1^O \leq \tau - b_1 \tag{91}$$

We thus conclude that when

$$\alpha(g_1^O - g_1^I) \leq \tau - b_1 \tag{92}$$

there exists an interior solution that simultaneously satisfies both (84) and (83), which is given by (86) and (87). When instead (92) does not hold, the solution is at the corner: $\nu^I = 0$ and $\nu^O = \frac{\tau - b_1}{\alpha g_1^O}$ so that $g_2^I = 0$ and $g_2^O = \tau - b_1$, respectively, as stated in the Lemma. Finally, note that inequalities (85) and (92) can be written as

$$\alpha |g_1^I - g_1^O| \leq \tau - b_1. \tag{93}$$

When (93) holds, the solution is interior, given by (88) and (89), as stated in the Lemma. □

To write down the incumbent's problem in the first period, as per usual, we denote by Φ an indicator function that takes the value one when sequestration occurs, and we denote by ψ_I the share of available resources appropriated by the incumbent in case of sequestration, as derived in Lemma F.1

$$\begin{aligned} \max_{\{g_1^O \geq \alpha g_0^O, g_1^I \geq \alpha g_0^I, b_1\}} & u(\tau + b_1 - g_1^O) + (1 - \Phi)\left[qu\left(\tau - b_1 - \alpha g_1^O\right) + (1-q)u(\alpha(\tau + b_1 - g_1^O))\right] \\ & + \Phi u\left(\psi_I(\tau - b_1)\right) \end{aligned} \tag{94}$$

In the next Proposition, we provide analytical results when $\alpha = 1$.

Proposition 4 (Debt under equal split) *Assume $g_0^O < \tau$ and $\alpha = 1$. Optimal debt is $b_1^* = g_1^O$*

Proof: Recall that when $\alpha = 1$, there is no sequestration in the second period if $b_1^* \leq 0$. When $b_1^* = 0$, there is perfect consumption smoothing: $g_1^I = g_2^I = \tau - g_1^O$, generating a utility of $2u(\tau - g_1^O)$.

Let's assume that the initial incumbent's entitlements are such that $\alpha(g_1^O - g_1^I) \leq \tau - b_1$. Then, using F.1 the optimal policy solves the following problem:

$$\max_{b_1} \left\{ u(\tau - g_1^O + b_1) + u((g_1^I - g_1^O)/2 + (\tau - b_1)/2) \right\}$$

Using the first-period budget constrain,

$$\max_{b_1} \left\{ u(\tau - g_1^O + b_1) + u((\tau - g_1^O + b_1 - g_1^O)/2 + (\tau - b_1)/2) \right\}$$

$$\max_{b_1} \left\{ u(\tau - g_1^O + b_1) + u(\tau - g_1^O) \right\}$$

Note that the present value of utility is strictly increasing in b_1 as long as $b_1 \leq g_1^O$. From Lemma F.1 it is simple to show that any debt level above

$$\tau \frac{1-\alpha}{1+\alpha} + 2\alpha g_1^O \frac{1}{1\alpha}$$

would generate a sequestration on the incumbent's own entitlements. Thus, the incumbent chooses $b_1 = g_1^O$, completely diluting the opposition's entitlements.

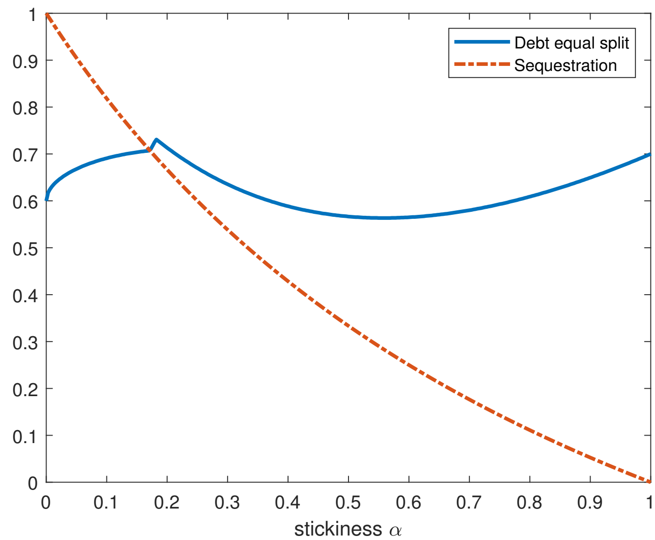
It is straightforward that the sequestration strategy $b_1 = g_1^O$ generates a larger payoff since:

$$u(\tau) + u(\tau - g_1^O) > 2u(\tau - g_1^O)$$

Here, the incumbent gives up spending-smoothing to increase its total spending and dilute the opposition's entitlements. □

Figure 17 illustrates debt under equal-split sequestration (we assume $g_0^O = 0.7$, $\sigma = 0.5$ and $q = 0.5$), showing that the dilution effect is exacerbated compared to the baseline model. For small values of α, the sequestration threshold would require a too-high debt level; as a result, sequestration does not occur, and the debt levels are the same as in the baseline model. For higher values of α, debt is above the sequestration threshold and is equal to

Figure 17: **Debt under Equal Split**



$$b_1^* = \tau\frac{1-\alpha}{1+\alpha} + 2\alpha g_1^O \frac{1}{1+\alpha}. \tag{95}$$

Using the government budget constraint, this level of debt guarantees that $g_1^I - g_1^O = \tau - b_1$. From Lemma F.1, it then follows that $g_2^I = \tau - b_1$ and the entitlements of the opposition are completely diluted. Note that as α goes to one, debt converges to $g_1^O = 0.7$, as stated in Proposition 4.

G Sequestration Exemptions

Certain programs, such as Social Security and Medicaid, are exempted from automatic sequestration. To assess the impact of these exemptions on politicians' incentives, we consider a model consisting of two programs that are exempt from sequestration and two programs that are not exempt. With sequestration exceptions, we continue to have a non-monotonic relationship between debt and stickiness, and the dilution effect is maintained. This section will also examine how the ratio between exempted and non-exempted programs is distorted compared to the first-best.

Assume that party $j = I, O$ utility is:

$$u_j(e^j, e^{-j}, g^j, g^{-j}) = \frac{(g^j)^{1-\sigma} - 1}{1-\sigma} + \theta \frac{(e^j)^{1-\sigma} - 1}{1-\sigma}$$

where $\theta > 0$. We assume that party I (party O) only values good g^I and e^I (g^O and e^O). All programs are sticky in the sense that, absent consolidation, the incumbent must maintain a proportion α of the opposition's previous spending. While goods e^O and e^I are exempted or prioritized during sequestration, goods g^I and g^O denote non-exempt spending programs. Given that budget feasibility cannot be ignored, this does not mean that e^O and e^I will never be cut; however, they will be given priority over non-exempt spending programs in the event of fiscal consolidation.

The budget constraints are:

$$g_1^O + g_1^I + e_1^I + e_1^O \leq \tau + b_1, \tag{96}$$

$$g_2^O + g_2^I + e_2^I + e_2^O \leq \tau - b_1.$$

As in the baseline model, assume that entitlements from period 0 are sustainable. Three potential scenarios could occur at $t = 2$: no sequestration, a "mild" sequestration where non-exempt programs are partially cut, and a severe sequestration, where non-exempt programs are entirely cut and also the exempted programs are affected.

There is no sequestration if total past commitments are sustainable:

$$\alpha(g_1^O + g_1^I + e_1^I + e_1^O) \leq \tau - b_1. \qquad \text{(no sequestration)}$$

A mild sequestration occurs if total commitments are unsustainable, but exempt programs remain sustainable:

$$\alpha(g_1^O + g_1^I + e_1^I + e_1^O) > \tau - b_1 \quad \wedge \quad \alpha(e_1^I + e_1^O) \leq \tau - b_1 \qquad \text{(mild sequestration)}$$

In a mild sequestration, cuts only apply to non-exempted spending. To keep the analysis tractable, in this section we do not impose the 50-50 split requirement studied in Section F. As in the baseline model, non-exempted programs are reduced by the same percentage. This implies that available resources are shared proportionally between the two nonexempt programs with weights ψ^M and $1 - \psi^M$:

$$g_2^I = (\tau - b_1 - \alpha e_1^I - \alpha e_1^O) \underbrace{\left(\frac{\alpha g_1^I}{\alpha g_1^I + \alpha g_1^O}\right)}_{\equiv \psi^M} \quad \text{and} \quad {g_2}^O = (\tau - b_1 - \alpha e_1^I - \alpha e_1^O) \underbrace{\left(\frac{\alpha g_1^O}{\alpha g_1^I + \alpha g_1^O}\right)}_{\equiv 1-\psi^M} \qquad (97)$$

$$e_2^I = \alpha e_1^I \quad \text{and} \quad e_2^O = \alpha e_1^O \qquad (98)$$

The last scenario is that of severe sequestration, which occurs if the exempted commitments alone are not sustainable:

$$\alpha(e_1^I + e_1^O) > \tau - b_1. \qquad \text{(severe sequestration)}$$

In this case, non-exempt programs are entirely cut and available resources are shared proportionally between the two exempt programs with weights ψ^S and $1 - \psi^S$:

$$e_2^I = (\tau - b_1) \underbrace{\left(\frac{\alpha e_1^I}{\alpha e_1^I + \alpha e_1^O}\right)}_{\equiv \psi^S} \quad \text{and} \quad {e_2}^O = (\tau - b_1) \underbrace{\left(\frac{\alpha e_1^O}{\alpha e_1^I + \alpha e_1^O}\right)}_{\equiv 1-\psi^S} \qquad (99)$$

$$g_2^I = 0 \quad \text{and} \quad g_O^I = 0 \qquad (100)$$

As in the baseline model the debt choice is characterized by thresholds that determine whether a sequestration takes place and which type. In this setting, there would be two thresholds.

Using (96) there is no sequestration if:

$$\alpha(\tau + b_1) \leq \tau - b_1 \Rightarrow b_1 \leq \frac{\tau(1-\alpha)}{1+\alpha}. \tag{101}$$

There is a mild sequestration if:

$$\frac{\tau(1-\alpha)}{1+\alpha} \leq b_1 \leq \frac{\tau(1-\alpha)}{1+\alpha} + \alpha\frac{(g_1^O + g_1^I)}{1+\alpha}.$$

A drastic sequestration occurs when:

$$b_1 \geq \frac{\tau(1-\alpha)}{1+\alpha} + \alpha\frac{(g_1^O + g_1^I)}{1+\alpha}.$$

We solve for the equilibrium backwards. Without sequestration, the incumbent at $t = 2$ retains some discretion to allocate resources after satisfying past commitments. In this case, let $V_2^I(b_1; g_1^O, e_1^O)$ be the time-2 value function of party I if she stays in power:

$$\begin{aligned} V_2^I(b_1, g_1^I, e_1^I) &= \max_{e_2^I, g_2^I} u(g_2^I) + \theta u(e_2^I) \\ \alpha g_1^O + g_2^I + \alpha e_1^O + e_2^I &\leq \tau - b_1 \\ g_2^I \geq \alpha g_1^I & \\ e_2^I \geq \alpha e_1^I & \end{aligned}$$

To simplify the exposition, we will assume that the constraints $g_2^I \geq \alpha g_1^I$ and $g_2^I \geq \alpha g_1^I$ are not binding, which will be the case when α is small. (In the simulations, we will include both constraints) Then,

$$\frac{\partial V_2^I(\cdot)}{\partial b_1} = -u'(g_2^I) \tag{102}$$

The indicator function Z takes a value of one when sequestration is severe and zero when it is mild. As before, Φ takes a value of one when sequestration (regardless of its severity) occurs. Recall $\psi^M = \frac{g_1^I}{g_1^I + g_1^O}$.

$$\max_{g_1^I, e_1^I, b_1} \Big\{ u(g_1^I) + \theta u(e_1^I) + (1-\Phi)\Big[qV_2^I(b_1, g_1^I, e_1^I) + (1-q)\big(u(\alpha g_1^I) + \theta u(\alpha e_1^I)\big)\Big]$$

$$\Phi\Big[Z\theta u((\tau - b_1)\psi^S) + (1-Z)\big(u((\tau - b_1 - \alpha(e_1^I + e_1^O))\psi^M) + \theta u(\alpha e_1^I)\big)\Big]\Big\}$$

$$\text{s.t. } g_1^I + \alpha e_0^O + e_1^I + \alpha g_0^O \leq \tau + b_1$$

Let μ be the multiplier associated with the budget constraint. Then, the first-order conditions are:

$$u'(g_1^I) + (1-\Phi)(1-q)\alpha^{1-\sigma}u'(g_1^I) + \Phi(1-Z)u'((\tau - b_1 - \alpha(e_1^I + e_1^O))\psi^M)(\tau - b_1 - \alpha(e_1^I + e_1^O))\frac{\partial \psi^M}{\partial g_1^I} = \mu$$

$$\theta u'(e_1^I) + [(1-\Phi)(1-q) + \Phi(1-Z)]\alpha^{1-\sigma}\theta u'(e_1^I) + \Phi Z\theta u'((\tau - b_1)\psi^S)(\tau - b_1)\frac{\partial \psi^S}{\partial e_1^I}$$

$$-\alpha(1-Z)\Phi\psi^M u'((\tau - b_1 - \alpha(e_1^I + e_1^O))\psi^M) = \mu$$

$$(1-\Phi)q\frac{\partial V_2^I(\cdot)}{\partial b_1} - \Phi Z\theta u'((\tau - b_1)\psi^S)\psi^S - \Phi(1-Z)u'((\tau - b_1 - \alpha(e_1^I + e_1^O)\psi^M)\psi^M = -\mu$$

Suppose first that $\Phi = 0$, then we have:

$$u'(g_1^I)[1 + (1-q)\alpha^{1-\sigma}] = \mu$$

$$\theta u'(e_1^I)[1 + (1-q)\alpha^{1-\sigma}] = \mu$$

$$q\frac{\partial V_2^I(\cdot)}{\partial b_1} = -\mu$$

Note that the first two equations imply that *the ratio of the two goods is undistorted relative to the first-best*: the allocation satisfies $e_1^I = g_1^I\theta^{\frac{1}{\sigma}}$.

Regarding debt accumulation, using the envelope condition (102) the debt allocation satisfies:

$$u'(g_1^I)[1+(1-q)\alpha^{1-\sigma}] = qu'(g_2^I)$$

This condition is similar to the one in the baseline model. Moreover, since the two types of spending are directly proportional to each other, one can demonstrate following the same procedure as in Proposition 1, that debt increases with α in this scenario, as long as the constraint $g_2^I \geq \alpha g_1^I, e_2^I \geq \alpha e_1^I$ are not binding. This would be the case if, for instance, α is close to zero.

Now suppose that a severe sequestration occurs, $\Phi = 1$ and $Z = 1$. Then, the first-order conditions are:

$$u'(g_1^I) = \mu \tag{103}$$

$$\theta u'(e_1^I) + \theta u'((\tau - b_1)\psi^S)(\tau - b_1)\frac{\partial \psi^S}{\partial e_1^I} = \mu \tag{104}$$

$$\theta u'((\tau - b_1)\psi^S)\psi^S = \mu \tag{105}$$

Replacing (105) into (104), we have:

$$u'(e_1^I) = u'((\tau - b_1)\psi^S)\psi^S \left[1 - \frac{\partial \psi^S}{\partial e_1^I}\frac{(\tau - b_1)}{\psi^S}\right] \tag{106}$$

From (103) and (105), we obtain

$$\theta u'((\tau - b_1)\psi^S)\psi^S = u'(g_1^I) \tag{107}$$

Note that equations (106) and (107) imply that:

$$\frac{\theta u'(e_1^I)}{u'(g_1^I)} = \left[1 - \frac{\partial \psi^S}{\partial e_1^I}\frac{(\tau - b_1)}{\psi^S}\right] = \left[1 - \frac{\alpha e_0^O}{e_1^I}\frac{(\tau - b_1)}{(e_1^I + \alpha e_0^O)}\right] \leq 1$$

The exempted good is over-provided as long as $e_0^O \alpha > 0$ to dilute the exempted programs of the opposition. The extent of the distortion diminishes as the incumbent increases the level of debt. Since achieving severe sequestration requires significant debt build-ups, this distortion will likely be minimal in this area.

Finally, if a mild sequestration occurs, $\Phi = 1$ and $Z = 0$, then the first-order conditions are:

$$u'(g_1^I) + u'((\tau - b_1 - \alpha(e_1^I + e_1^O))\psi^M)(\tau - b_1 - \alpha(e_1^I + e_1^O))\frac{\partial \psi^M}{\partial g_1^I} = \mu \tag{108}$$

$$\theta u'(e_1^I) + \alpha^{1-\sigma}\theta u'(e_1^I) - \alpha\psi^M u'((\tau - b_1 - \alpha(e_1^I + e_1^O))\psi^M) = \mu \tag{109}$$

$$u'((\tau - b_1 - \alpha(e_1^I + e_1^O)\psi^M)\psi^M = \mu \tag{110}$$

From (109) and (110), we obtain:

$$\theta u'(e_1^I) = u'((\tau - b_1 - \alpha(e_1^I + e_1^O)\psi^M)\psi^M \underbrace{\frac{1+\alpha}{1+\alpha^{1-\sigma}}}_{\leq 1 \quad when \ \alpha \in [0,1]} \tag{111}$$

Using (110) and (108),

$$u'(g_1^I) = u'((\tau - b_1 - \alpha(e_1^I + e_1^O))\psi^M)\psi^M \left[1 - \frac{(\tau - b_1 - \alpha(e_1^I + e_1^O))}{\psi^M}\frac{\partial \psi^M}{\partial g_1^I}\right]$$

As a result, by equation (111), we can write the last conditions as:

$$\frac{\theta u'(e_1^I)}{u'(g_1^I)} = \frac{\frac{1+\alpha}{1+\alpha^{1-\sigma}}}{\left[1 - \frac{(\tau - b_1 - \alpha(e_1^I + e_1^O))}{\psi^M}\frac{\partial \psi^M}{\partial g_1^I}\right]} \tag{112}$$

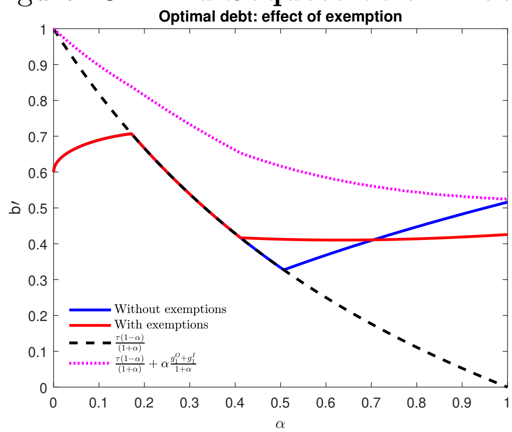
Since the right-hand side of Equation (112) may be above or below 1, it follows that, depending on the parameters, the exempted good may be overprovided or underprovided compared to the first-best scenario. When σ is close to zero (implying $\frac{1+\alpha}{1+\alpha^{1-\sigma}}$ is close to 1), *the exempted good is underprovided*. Intuitively, when σ is close to zero, the incumbent has stronger incentives to overspend on non-exempted goods to dilute the non-exempted goods of the opposition. On the other hand, when σ is higher, the incumbent may prioritize exempted goods since they are not subject to cuts in a sequestration, providing better insurance against the loss of power.

In summary, we have three areas. In the first area, there is no sequestration, and debt increases with persistence. However, the ratio between exempted and non-exempted goods is efficient. In the second area, there is mild sequestration, and the exempted goods are under-provided (at least for low level of σ). In the third area, there is severe sequestration, and the exempted goods are over-provided.

Figures 18 to 19 illustrate the debt solution for different values of the opposition's entitlements e_0^O and g_0^O. We set $\theta = 1$ so that the first best allocation features an e/g ratio equal to 1: the incumbent should spend the same amount on both programs. The two configurations of parameters lead, respectively, to mild-only and mild and severe sequestrations. The dotted magenta curve represents the drastic sequestration threshold, while the dashed black one represents the standard sequestration threshold. The blue curve illustrates what the solution would be if there were no exemptions, while the red line is the solution with exempted programs. As in the baseline model, whenever there is no sequestration, debt increases with persistence if α is small, and decreases with persistence for intermediate values of α, when the dilution strategy is not used. The first difference with the baseline model can be see in Figure 18, where $e_0^O = 0$ so that there is no severe sequestration: the spending dilution effect starts to be used for smaller values of α than in the baseline model, but when used the dilution effect is milder. This can be seen on the crossing of the red and blue lines. As opposed to the baseline model, debt can still be decreasing in persistence when the dilution effect is operating. This happens for two reasons. First, because the exempted programs reduce the amount of resources available for non-exempt good, it raises the marginal utility of future spending. Second, precisely because the non-exempted goods are a smaller share of total spending, the gains from diluting are reduced. In Figure 19 we depict the configuration when also $e_0^O > 0$. This generates the possibility that the budget is unsustainable even when all the non-exempt programs are eliminated, triggering the possibility of sequestering exempted programs. This generates discontinuity in the red line at the highest levels of persistence. The intuition for result is akin to the model with discretionary spending, thus we defer the reader to the respective section.

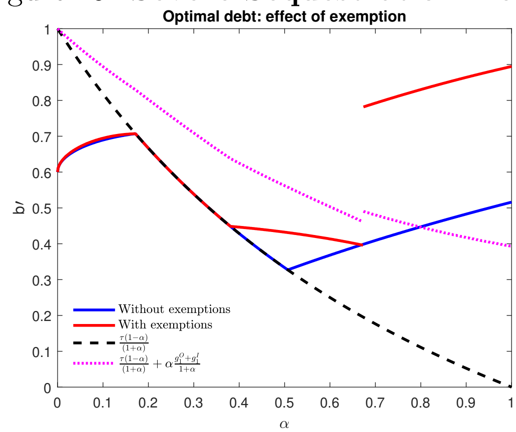
Figures 20 to 21 illustrate the corresponding spending ratios between programs. Here two patterns emerge: one of over-provision and another of under-provision of exempted goods. Figure 20 shows the e_1/g_1 ratio corresponding to the debt accumulation depicted in Figure 18, while analogously Figure 21 complements Figure 19. In both figures the blue continuous line is the ratio e_1^I/g_1^I, while the black dashed line corresponds to the first-best ratio, equal to 1 since $\theta = 1$. The program that it is marginally used to dilute entitlements is over-provided. For instance, when the debt choice intends to generate a mild sequestration, the non-exempt programs have an "extra dynamic value," and thus, are over-provided. Counter intuitively, when the incumbent expects a mild sequestration rather than allocating more spending to the protected (exempted) programs, it over-spends in the unprotected one. This effect is what it generates decreasing section of the blue line in both figures. However, when there are initial entitlements of the exempt programs, if they are sufficiently large and the persistence is also high, the incumbents over-spends in the exempt programs to dilute the exempted opposition's preferred programs. This can be seen in Figure 21 when the blue line jumps from below 1 to a ratio substantially above 1 when $\alpha > 0.7$.

Figure 18: **Mild-Sequestration Debt**



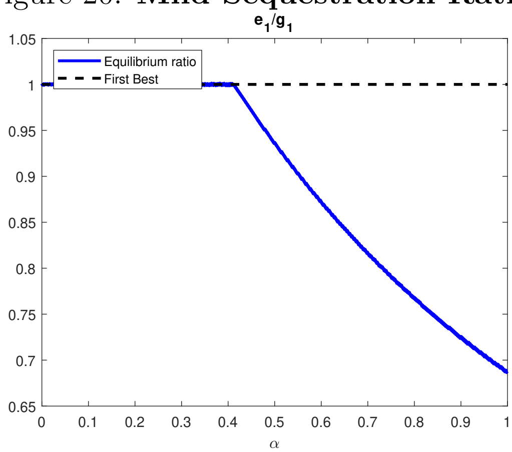
Parameters: $g_0^O = 0.5$ and $e_0^O = 0$, $q = \sigma = 1/2$, $\theta = 1$ and $\tau = 1$.

Figure 19: **Severe-Sequestration Debt**



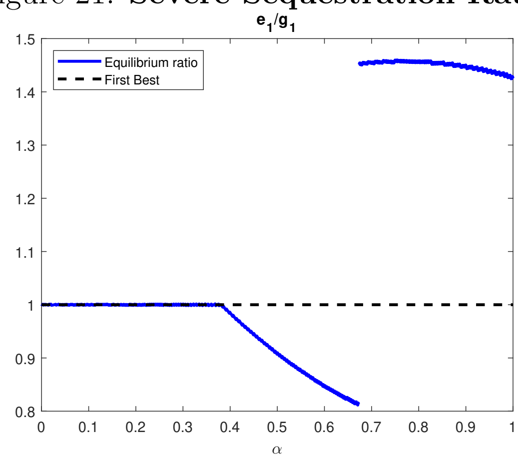
Parameters: $g_0^O = 0.5$ and $e_0^O = 0.7$, $q = \sigma = 1/2$, $\theta = 1$ and $\tau = 1$.

Figure 20: **Mild-Sequestration Ratio**



Parameters: $g_0^O = 0.5$ and $e_0^O = 0$, $q = \sigma = 1/2$, $\theta = 1$ and $\tau = 1$.

Figure 21: **Severe-Sequestration Ratio**



Parameters: $g_0^O = 0.5$ and $e_0^O = 0.7$, $q = \sigma = 1/2$, $\theta = 1$ and $\tau = 1$.

H Polarization

In this section, we analyze the implications of varying degrees of preference misalignment between the two parties.

A. Baseline Model

Consider first the baseline model of Section IV. with no discretionary spending, and assume that the per-period utility of political party $j = I, O$ is given by:

$$u_j(g^j, g^{-j}) = \frac{(g^j)^{1-\sigma}}{1-\sigma} + \omega \frac{(g^{-j})^{1-\sigma}}{1-\sigma}$$

where $\omega \in [0,1]$. For simplicity and without any loss of generality, we omit the -1 constant in the numerator of the CES utility. The baseline model with full polarization is represented by $\omega = 0$, which means that the incumbent assigns no value whatsoever to the opposition's preferred good. In the opposite extreme case where $\omega = 1$, there is no polarization as both parties share the same preferences.

We solve the equilibrium backward. When there is no sequestration, the incumbent at $t = 2$ retains some discretion to allocate resources after satisfying past commitments. Let $V_2^j(g_1^j, g_1^{-j}, b_1)$ be the time-2 value function of party j if she is *in power* and there is no sequestration:

$$V_2^j(g_1^j, g_1^{-j}, b_1) = \max_{\{g_2^j, g_2^{-j}\}} \left\{ \frac{(g_2^j)^{1-\sigma}}{1-\sigma} + \omega \frac{(g_2^{-j})^{1-\sigma}}{1-\sigma} \right\} \tag{113}$$

$$s.t. \qquad \tau - b_1 - g_2^j + g_2^{-j} \geq 0 \tag{114}$$

$$g_2^j \geq \alpha g_1^j \tag{115}$$

$$g_2^{-j} \geq \alpha g_1^{-j} \tag{116}$$

where (115) and (116) are the period-2 "sticky constraints".

Solving the analogous problem for the other party, one can compute party $-j$'s spending rules $\tilde{g}_2^{*j}$ and $\tilde{g}_2^{*-j}$.

For example, if both sticky constraints are not binding in party $-j$'s problem in period 2, we have:

$$\tilde{g}_2^{*j} = \frac{\omega^{1/\sigma}(\tau - b_1)}{1 + \omega^{1/\sigma}}$$

$$\tilde{g}_2^{*-j} = \frac{(\tau - b_1)}{1 + \omega^{1/\sigma}}$$

Once the spending rules of party $-j$ are computed, the time-2 value function of party j, if she is *out of power* and if there is no sequestration, can be written as follows:

$$\tilde{V}_2^j(g_1^j, g_1^{-j}, b_1) = \frac{(\tilde{g}_2^{*j})^{1-\sigma}}{1-\sigma} + \omega \frac{(\tilde{g}_2^{*-j})^{1-\sigma}}{1-\sigma} \tag{117}$$

Lastly, the time-2 value function of party j in the case of sequestration is given by

$$V_2^{S,j}(g_1^j, g_1^{-j}, b_1) = \frac{((\tau - b_1)\psi_j)^{1-\sigma}}{1-\sigma} + \omega \frac{((\tau - b_1)(1-\psi_j))^{1-\sigma}}{1-\sigma} = \frac{(\tau - b_1)^{1-\sigma}}{1-\sigma}\Psi \tag{118}$$

where $\psi_j = g_1^j/(g_1^j + g_1^{-j})$ and $\Psi = \psi_j^{1-\sigma} + \omega(1-\psi_j)^{1-\sigma}$.

We now move to the first period. Let $\Phi = 1$ denote the indicator for sequestration at $t = 2$. The problem of the period-1 incumbent can be written as follows:

$$V_1^j(g_0^j, g_0^{-j}, b_0) = \max_{\{g_1^j, g_1^{-j}, b_1 \leq \tau\}} \Bigg\{ \frac{(g_1^j)^{1-\sigma} + \omega(g_1^{-j})^{1-\sigma}}{1-\sigma} + (1-\Phi)\left[qV_2^j(g_1^j, g_1^{-j}, b_1) + (1-q)\tilde{V}_2^j(g_1^j, g_1^{-j}, b_1)\right]$$

$$+\Phi V_2^{S,j}(g_1^j, g_1^{-j}, b_1)\Bigg\}$$

$$s.t. \quad g_1^j + g_1^{-j} \leq \tau + b_1$$

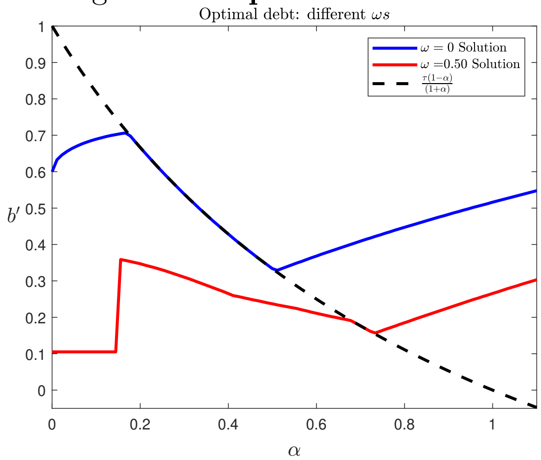
$$g_1^j \geq \alpha g_0^j \tag{119}$$

$$g_1^{-j} \geq \alpha g_0^{-j} \tag{120}$$

The algebra of the problem is standard, but can be somewhat intricate due to the numerous possible cases, contingent on whether the sticky constraints for either the incumbent or the opposition in either period 1 or period 2, may be slack. Rather than presenting the algebra, we will discuss the numerical solutions. In Figure 22, we display equilibrium debt as a function of stickiness in the full-polarization model (blue line) and in the medium polarization case, $\omega = 1/2$, (red line). Figure 23 illustrates spending on goods j and $-j$ as a function of stickiness.

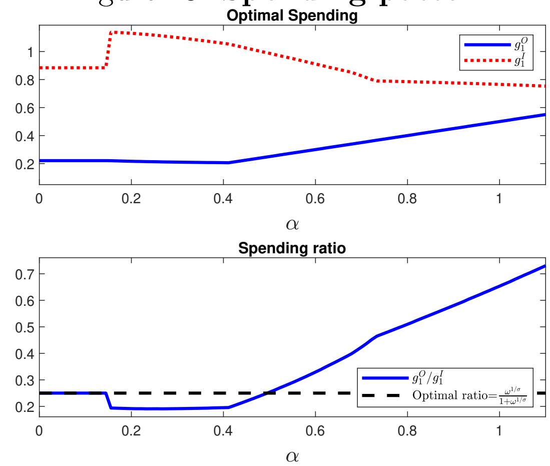
The first thing to notice in Figure 22 is that debt is lower when there is less polarization, which aligns with intuition: when governments disagree less, there are weaker incentives to use spending and debt to manipulate future governments. Moreover, note from Figures 22 and 23 that when α is sufficiently small, debt and spending do not depend on α. The reason for this is that in this parameter range, the sticky constraints (115) and (116) (as well as their corresponding counterparts for the opposition) are not binding. As a result, spending is not a payoff-relevant state variable in the second period – only debt is payoff-relevant. Because stickiness plays no role, debt and spending on both goods remain constant with respect to α, and the model has the same solution as Tabellini and Alesina (1990). The threshold for α below which the sticky constraints are not binding is approximately 0.15 when $\omega = 1/2$ and, more generally, depends on the level of polarization. In cases of high polarization, incumbent j would prefer to select a very small g^{-j}, which explains why the sticky constraint is more likely to be binding even for low values of α, reducing the flat region. In the limit, when there is full polarization, the sticky constraints are always binding (when $\alpha > 0$), and the flat part disappears.

Figure 22: **Equilibrium debt**



Parameters: $g_0^O = 0.5$, $q = \sigma = 1/2$, and $\tau = 1$.

Figure 23: **Spending pattern**

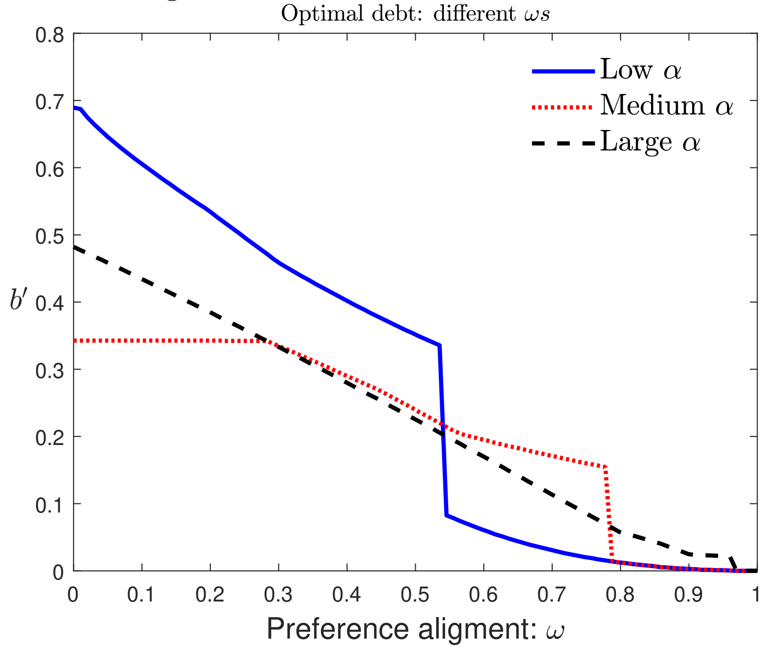


Parameters: $g_0^O = 0.5$, $q = \sigma = 1/2$, $\omega = 0.5$ and $\tau = 1$.

As α increases, the sticky constraint $g_2^j \geq \alpha g_1^j$ becomes binding for incumbent $-j$ in period 2. This implies that by increasing spending on good j in the first period, incumbent j compels the subsequent government to allocate more funds to good j. Spending on g_1^j thus generates an additional "strategic" benefit for the current incumbent. Conversely, spending on g_1^{-j} does not yield such an additional benefit because party $-j$ will spend on goods g^{-j} anyway if she goes to power in period 2. Consequently, when $\alpha \approx 0.15$, spending on g_1^j jumps to a higher value, while g_1^{-j} is continuously affected. As α further increases, this reduces the variability in expected future spending, leading to decreased expenditure on both goods and debt. In Figure 23, it may seem that g_1^{-j} remains constant with respect to α, but this is a visual effect caused by the relatively small scale of $\frac{\partial g_1^{-j}}{\partial \alpha}$ in comparison to $\frac{\partial g_1^j}{\partial \alpha}$.

As α continues to increase (approximately after $\alpha = 0.4$ in Figures 22 and 23), the first-period sticky constraint $g_1^{-j} \geq \alpha g_0^{-j}$ begins to bind for incumbent j in period 1. This is why spending on the opposition's good is increasing for $\alpha > 0.4$ in Figure 23. Moreover, for the same reasons as before, debt is decreasing with ω, as shown in Figure 22. Eventually, as α continues to rise, sequestration will be triggered, and dilution will occur.

Figure 24: **Debt and Polarization**

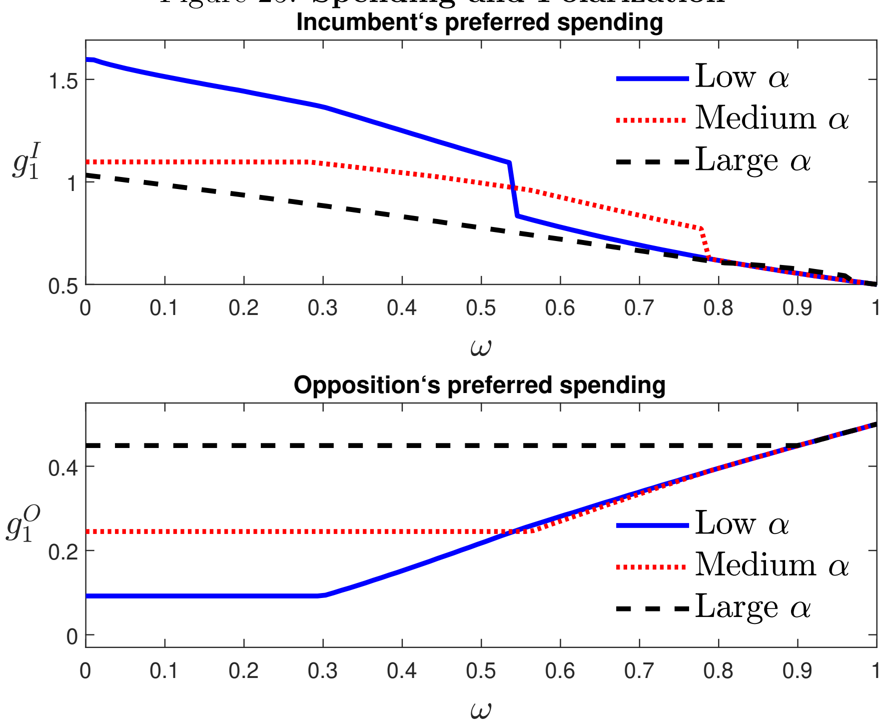


Parameters: $g_0^O = 0.5$, $q = \sigma = 1/2$, and $\tau = 1$. Low $\alpha = 0.184$, medium $\alpha = 0.49$, large $\alpha = 0.9$

For empirical purposes, we analyze how debt accumulation changes in ω. Figure 24 illustrates the equilibrium debt as preferences become more aligned (as ω transitions from 0 to 1) across different degrees of spending persistence. In general, debt decreases as polarization diminishes, in line with the standard Tabellini and Alesina (1990) model. However, additional patterns emerge. Note that debt can exhibit continuous changes, sudden jumps, or remain constant. For example, when ω is high (indicating low polarization), incremental changes in polarization lead to modest shifts in debt accumulation. However, as polarization continues to increase (and ω decreases), debt accumulation might sharply rise (as seen in the blue continuous and red dotted lines) or remain constant (as observed in the flat region of the red dotted line), depending on the stickiness of spending and whether or not sequestration occurs.

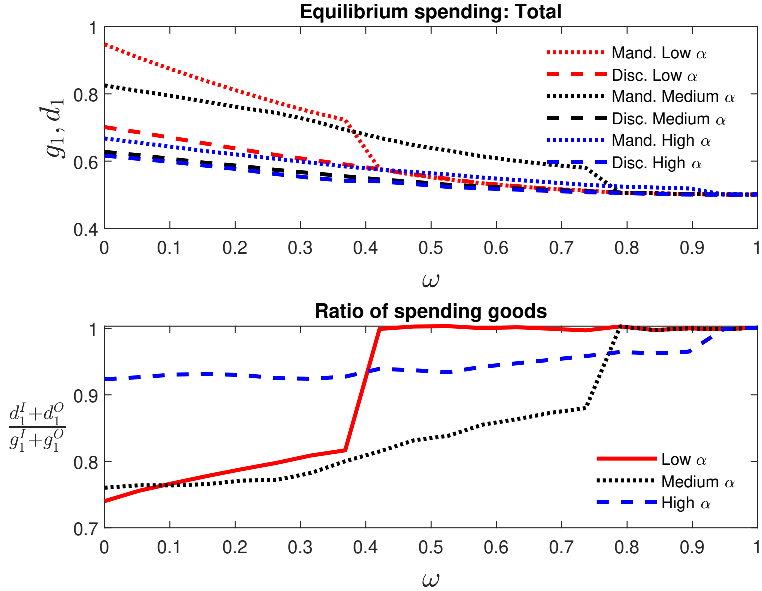
In Figure 25, we illustrate the corresponding spending trends. As polarization increases, the incumbent allocates more funds to her favored good while reducing the allocation to the opposition's preferred good. However, the extent to which these adjustments can be made depends on the level of persistence.

Figure 25: **Spending and Polarization**



Parameters: $g_0^O = 0.5$, $q = \sigma = 1/2$, and $\tau = 1$. Low $\alpha = 0.184$, medium $\alpha = 0.49$, large $\alpha = 0.9$

Figure 26: **Mandatory vs. Discretionary Spending and Polarization**



Parameters: $g_0^O = 0$, $q = \sigma = 1/2$, $\theta = 1$, and $\tau = 1$. Low $\alpha = 0.184$, medium $\alpha = 0.49$, large $\alpha = 0.9$

B. Discretionary and Mandatory Spending

We now describe the model with partial polarization and discretionary spending, extending the model of Section B.. The utility function for party $j = I, O$ is represented by:

$$u_j(g^j, g^{-j}, d^j, d^{-j}) = \frac{(g^j)^{1-\sigma} + \omega(g^{-j})^{1-\sigma}}{1-\sigma} + \theta\frac{(d^j)^{1-\sigma} + \omega(d^{-j})^{1-\sigma}}{1-\sigma}$$

where $\omega \in [0, 1]$ and $\theta \geq 0$. The rest of the model is identical to the one outlined in Section B.. In particular, goods d are nonsticky and can be cut at no cost by the current government, while programs g are inertial with stickiness equal to $\alpha \geq 0$. Moreover, goods g are prioritized in case of sequestration. As a result of these assumptions, goods d do not constitute a payoff-relevant state, while goods g might be payoff-relevant depending on whether the sticky constraints are binding. Figure 26 depicts spending on sticky and nonsticky goods as a function of polarization ω for various levels of the stickiness parameter. The results indicate that, in addition to increasing spending, higher polarization (lower ω) tends to lead parties to prioritize sticky programs, which allow to manipulate subsequent governments. In fact, the ratio of total discretionary spending divided by mandatory spending $(d^j + d^O)/(g^I + g^O)$ gets smaller as ω decreases.

I Consolidation Episodes

We rely on Alesina et al. (2019) to identify episodes of fiscal consolidation and we utilize data on government expenditure by functional classification (COFOG) to examine which functions are primarily reduced during consolidations.

To determine the starting year and ending year of fiscal adjustment episodes, we use the table from Appendix 3 of Alesina et al. (2016), which provides information on the consolidation measures, as a proportion of GDP.[37] Considering that our model emphasizes fiscal consolidation through spending cuts, we exclude consolidation episodes that only involve adjustments on the revenue side.[38] Due to the more limited availability of COFOG (Classification of the Functions of Government) data compared to the aggregated expenditure data used by Alesina et al. (2019), the number of consolidation episodes that can be included in our analysis is reduced (see the "Cofog Data" column in Table 1).

From Appendix 3 of Alesina et al. (2016) and the detailed description in the file appendix_description.pdf associated to Alesina et al. (2019), and relying on the database of political institutions (DPI, see Beck et al., 2000), we identify the name of the majority party promoting fiscal consolidation ("name party start") and its political ideology ("party start"). Similarly, we find the name of the party government in power in the final year of the consolidation ("name party end") and determine its ideology ("party end").[39] In Table 1, column "Political Color," we define the political color of a consolidation as "Left" ("Right") if the consolidation is initiated and carried out until its conclusion by a left-wing (right-wing) party. If there is a majority change during the consolidation, we categorize it as a "majority change." The three US fiscal consolidations were executed during years of divided governments; we have taken a conservative approach and refrained from assigning a political color to these consolidations.

[37]See also the data_structuring.xlsx file provided by Alesina et al. (2019).

[38]For instance, we exclude the 1979 French consolidation episode from our analysis since it only involved a tax increase without any accompanying expenditure cuts. Moreover, we adjust the length of a consolidation episode if there are years when government spending remains unchanged. For example, according to Alesina et al. (2016), Australia underwent a period of fiscal consolidation from 1993 to 1999. However, we start this episode in 1995 since Alesina et al. (2016) indicates no spending adjustments in the first two years.

[39]We made two adjustments compared to Beck et al., 2000 to better take into account the party that implemented the consolidation episode. The UK episode from 2010 to 2014 is defined as conservative ("The Conservative [UK] government implemented a program of budget cuts" on p. 123 of Alesina et al. (2019)), while the Portuguese episodes of 2002 and 2005-2007 are classified, respectively, as right-wing and left-wing consolidations – the first was implemented by conservative Prime Minister Barroso and the latter by left-wing Prime Minister Sócrates.

Figure 27: **Left-Wing Consolidations**

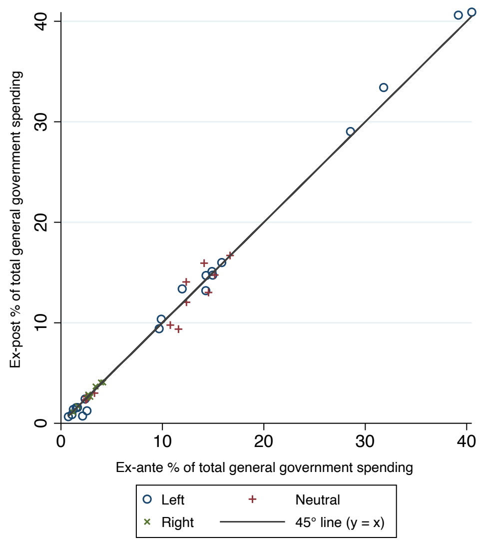
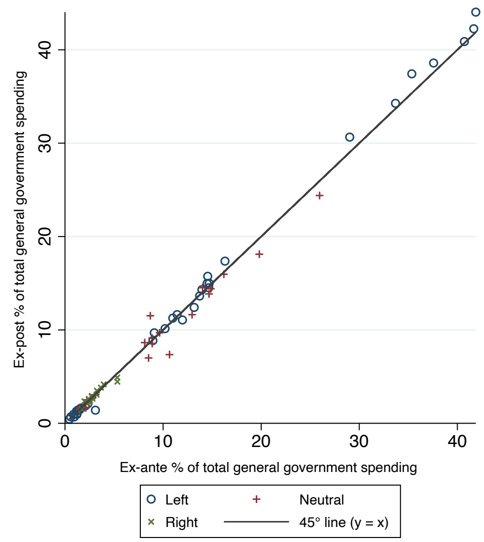


Figure 28: **Right-Wing Consolidations**



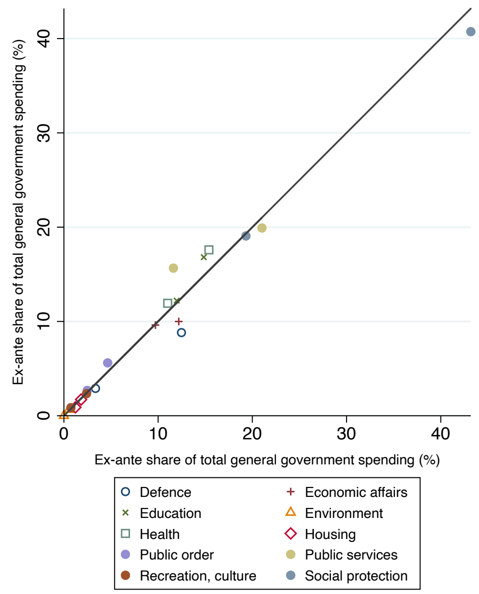
In Figures 27 and 28, we examine consolidations carried out by left-leaning and right-leaning governments, respectively. As in Figure 1, the x-axis represents the function share in the year preceding the start of the consolidation, while the y-axis represents the function share in the final year of the consolidation. As discussed in Section B. of the main text, we categorize the ten functions into left-wing, right-wing, and neutral functions based on party-issue ownership. With the caveat that our analysis is based on a limited number of episodes, we do not find clear evidence indicating that right-wing parties prioritize cutting left programs, represented by red crosses, over right programs, represented by blue crosses, (and vice-versa). Cuts are proportional, and observations lie on the 45-degree line.

Finally, we consider the dataset of 13 large consolidations by Blöchliger et al. (2012). Among these consolidations, we have spending data by function for only two of them: Finland (1993-2000) and the United States (1993-1998). For both episodes, ex-post and ex-ante shares align along the 45-degree line. See Figure 29.

Table 1: **Fiscal Consolidation Episodes**

Location	Start Year	End Year	Party Start	Name Party Start	Party End	Name Party End	Political Color	COFOG Data
AUS	1985	1988	Left	ALP	Left	ALP	Left	No
AUS	1996	1999	Left	ALP	Right	LPA	Majority change	No
AUT	1980	1981	Left	SPO	Left	SPO	Left	No
AUT	1984	1984	Left	SPO	Left	SPO	Left	No
AUT	1996	1997	Left	SPO	Left	SPO	Left	Yes
AUT	2001	2002	Right	OVP	Right	OVP	Right	Yes
AUT	2011	2014	Left	SPO	Left	SPO	Left	Yes
BEL	1982	1987	Right	CVP	Right	CVP	Right	No
BEL	1990	1990	Right	CVP	Right	CVP	Right	No
BEL	1992	1994	Right	CVP	Right	CVP	Right	No
BEL	1996	1997	Right	CVP	Right	CVP	Right	Yes
BEL	2010	2014	Right	CD&V	Left	PS	Majority change	Yes
CAN	1985	1997	Right	PCP	Left	LPC	Majority change	No
CAN	2010	2014	Right	CPC	Right	CPC	Right	No
DEU	1982	1984	Left	SPD	Right	CDU	Majority change	No
DEU	1991	1995	Right	CDU	Right	CDU	Right	No
DEU	1997	2000	Right	CDU	Left	SPD	Majority change	Yes
DEU	2004	2007	Left	SPD	Right	CDU	Majority change	Yes
DEU	2011	2013	Right	CDU	Right	CDU	Right	Yes
DNK	1983	1985	Right	KF	Right	KF	Right	No
DNK	1995	1995	Left	SD	Left	SD	Left	No
DNK	2010	2013	Right	V	Left	SD	Majority change	Yes
ESP	1984	1984	Left	PSOE	Left	PSOE	Left	No
ESP	1989	1989	Left	PSOE	Left	PSOE	Left	No
ESP	1992	1997	Left	PSOE	Right	PP	Majority change	No
ESP	2010	2014	Left	PSOE	Right	PP	Majority change	Yes
FIN	1992	1997	Center	KESK	Left	SSDP	Majority change	Yes
FIN	2012	2014	Right	KOK	Right	KOK	Right	Yes
FRA	1987	1989	Right	RPR	Left	PS	Majority change	No
FRA	1991	1991	Left	PS	Left	PS	Left	No
FRA	1995	1997	Right	RPR	Right	RPR	Right	No
FRA	2010	2014	Right	UMP	Left	PS	Majority change	Yes
GBR	1979	1982	Right	Conservative	Right	Conservative	Right	No
GBR	1994	1999	Right	Conservative	Left	Labour	Majority change	No
GBR	2010	2014	Right	Conservative	Right	Conservative	Right	Yes
IRL	1982	1983	Right	Fine Gael	Right	Fine Gael	Right	No
IRL	1987	1988	Center	Fianna Fail	Center	Fianna Fail	Center	No
IRL	2008	2014	Center	Fianna Fail	Right	Fine Gael	Majority change	Yes
ITA	1991	1998	Center	DC	Center	Ulivo Alliance	Center	No
ITA	2004	2007	Right	Casa delle Liberta	Left	L'Unione	Majority change	Yes
ITA	2009	2009	Right	PdL	Right	PdL	Right	Yes
ITA	2011	2014	Right	PdL	N/A	independent	Majority change	Yes
JPN	1982	1983	Right	LDP	Right	LDP	Right	No
JPN	1997	1998	Right	LDP	Right	LDP	Right	No
JPN	2003	2006	Right	LDP	Right	LDP	Right	No
PRT	1983	1983	Center	AD (PSD+CDS)	Center	AD (PSD+CDS)	Center	No
PRT	2000	2000	Left	PS	Left	PS	Left	Yes
PRT	2002	2002	Right	PSD	Right	PSD	Right	Yes
PRT	2005	2007	Left	PS	Left	PS	Left	Yes
PRT	2010	2014	Left	PS	Right	PSD	Majority change	Yes
SWE	1984	1984	Left	Social Dem	Left	Social Dem	Left	No
SWE	1993	1998	Right	Moderate	Left	Social Dem	Majority change	No
USA	1988	1988	Right	Republicans	Right	Republicans	Divided government	Yes
USA	1990	1998	Right	Republicans	Left	Democrats	Divided government	Yes
USA	2011	2013	Left	Democrats	Left	Democrats	Divided government	Yes

Figure 29: **Finland (1993-2000) and US (1993-1998) consolidations from Blöchliger et al. (2012)**



Note: The dark gray line corresponds to the 45° line (y = x).

J Spending Persistence

Table 2: **First-Order Autocorrelation: OMB Function Share**

Variable name	Party Ownership	Spending Type	AC	% Total Budget in 2022
050 National Defense	Republican	Discretionary	0.90	12.74
150 International Affairs	Neutral	Discretionary	0.29	1.53
250 General Science, Space, and Technology	Neutral	Discretionary	0.84	0.63
270 Energy	Neutral/Republican	Discretionary	0.51	1.00
300 Natural Resources and Environment	Neutral	Discretionary	0.50	2.54
350 Agriculture	Democrat	Mandatory	0.65	0.61
370 Commerce and Housing Credit	Undetermined	Mandatory	0.01	0.91
400 Transportation	Neutral	Discretionary	0.64	2.48
450 Community and Regional Development	Neutral	Discretionary	0.30	0.66
500 Education, Training, Employment, and Social Services	Democrat	Discretionary	0.50	9.17
550 Health	Democrat	Mandatory	0.93	13.08
570 Medicare	Democrat	Mandatory	0.90	12.43
600 Income Security	Democrat	Mandatory	0.40	12.80
650 Social Security	Democrat	Mandatory	0.69	18.73
700 Veterans Benefits and Services	Neutral	Discretionary and Mandatory	0.84	4.10
750 Administration of Justice	Republican	Discretionary	0.92	1.23
800 General Government	Neutral	Discretionary	0.51	1.70

data sources:
Budget Data: OMB Historical Tables, Table 5.1 https://www.whitehouse.gov/omb/budget/historical-tables/
Party Ownership: Epp et al. (2014b), Supplementary Material A.
Spending Type: based on OMB Historical Tables, Tables 8.5 and 8.7.

K Debt Default

We present a simple extension of the baseline model that allows for debt default. We find conditions under which the incumbent might default on debt rather than reducing entitlements.

To have a meaningful trade-off, we assume that there are costs associated with debt default. When a country defaults on debt, we assume that the tax revenue becomes $(1-\phi)\tau$, where $\phi \in [0,1]$ due to trade sanctions and other economic disruptions, which may lower output (Bolton and Jeanne, 2009 and Arellano, 2008 make a similar assumption).

When entitlements are cut, there are two associated costs. First, as in the baseline model, there is a standard utility cost, as the two parties derive utility from spending. Second, we assume that cutting entitlements by an amount S reduces tax revenues by λS, where $\lambda \in [0,1)$. This additional cost captures the potential extra costs that may arise from reducing spending, beyond the direct utility cost. One way to rationalize $\lambda > 0$ is to assume that past entitlements serve as reference points, and reducing them may result in decreased "tax-morale", increased tax evasion, and ultimately, a reduction in tax revenue. The more drastic the cut, the higher this cost. Assuming $\lambda = 0$ would shut down this channel without changing the main results of this section.

By S^D and S^N, we denote the entitlement cut under, respectively, debt default and no debt default. Let R denote the available resources in the second period:

$$R = \begin{cases} (1-\phi)\tau - \lambda S^D, & \text{if there is default on debt} \\ \tau - b_1 - \lambda S^N, & \text{if there is no default on debt} \end{cases} \tag{121}$$

where b_1 is inherited debt and S^j, with $j = N, D$ is the entitlement cut, which is positive if there is sequestration and zero otherwise:

$$S^j = \max\{\alpha(g_1^I + g_1^O) - R, 0\} \tag{122}$$

Notice from (121) and (122) that the entitlement cut depends on R, which in turn depends on the extent of the entitlements cut. This implies that cutting entitlements reduces tax revenue, resulting in even more drastic spending cuts, which further erodes tax morale and leads to larger cuts. As we show below, if $\lambda < 1$, this "fiscal" multiplier is finite. While this multiplier is a feature of our setting, it is entirely non-essential to the main results and we can set $\lambda = 0$. By replacing (122) into (121), using the government budget constraint in the first period (2), and solving for the fixed point, we obtain:

$$S^D = \max\{\frac{(\tau + b_1)\alpha - (1-\phi)\tau)}{1-\lambda}, 0\} \tag{123}$$

$$S^N = \max\{\frac{(\tau + b_1)\alpha - (\tau - b_1)}{1-\lambda}, 0\} \tag{124}$$

The baseline model in the paper is a special case of this model in the case where there are no additional costs associated with cutting entitlements ($\lambda = 0$) and defaulting on debt is infinitely costly ($\phi = 1$), making it an unviable option.

We now outline the timeline of events. The first period follows the same structure as the baseline model of Section IV. because there are no default decisions to be made. This is because we assumed that the initial debt is zero and that the entitlements inherited from period 0 are sustainable. In period 2, given inherited debt and entitlements, the incumbent makes a decision to either default or not default on debt, and chooses the allocation of the available resources R, as defined in (121), (123), and (124). As in the baseline model, when inherited entitlements are sustainable, the incumbent needs to pay the opposition's entitlements αg^O, and keeps the remainder. If instead $R < (\tau + b_1)\alpha$, the incumbent has no fiscal space and sequestration occurs with proportional cuts as in the baseline model.

The remainder of this section proceeds as follows. First, we determine whether entitlement cuts are necessary in the event of no debt default and debt default, respectively. Next, we show that the incumbent will default when the debt is above a certain threshold, thereby imposing a limit on the amount of debt that can be issued in period 1 (Lemma K.1). Finally, we solve for the equilibrium debt and compare it to the baseline model's debt level.

When a nation does not default on its debt, entitlements are sustainable when

$$\tau - b \geq (g_1^O + g_1^I)\alpha \tag{125}$$

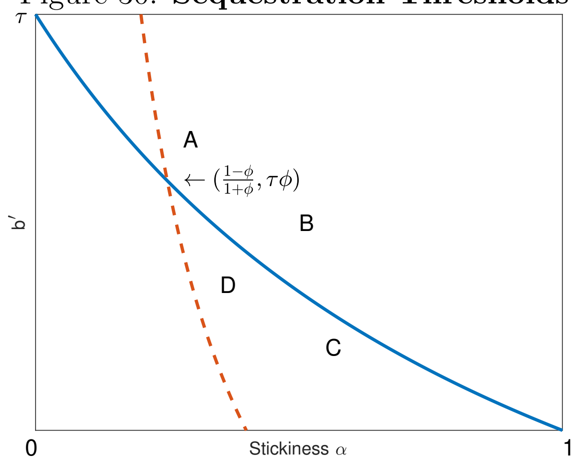
Therefore, using (2), we derive an identical threshold to that of the baseline model:

$$S^N = 0 \iff b \leq \frac{\tau(1-\alpha)}{1+\alpha} \tag{126}$$

When instead a nation defaults on its debt, entitlements are sustainable when

$$(1-\phi)\tau \geq (g_1^O + g_1^I)\alpha \tag{127}$$

Figure 30: **Sequestration Thresholds**



$$S^D = 0 \quad \Longleftrightarrow \quad b \leq \frac{(1-\phi)\tau - \alpha\tau}{\alpha} \tag{128}$$

Note that defaulting on debt does not necessarily result in sustainable entitlements. We obtain $S^D = 0$ when default carries a small cost or when past entitlements (and consequently debt) are low.

In Figure 30, the green line corresponds to the sequestration threshold as defined in (126), while the red line represents the sequestration threshold under default, defined in (128). Both lines are drawn in the space (α, b) where $\alpha \in [0,1]$ and $b \in [0, \tau]$. The red and green curves intersect at $\alpha = (1-\phi)/(1+\phi)$ and $b = \tau\phi$. The space is partitioned into four regions: A, B, C and D. In region A, entitlements become sustainable after a debt default, but they are unsustainable when there is no debt default, i.e., $S^N > 0$ and $S^D = 0$. In region B, entitlements are not sustainable regardless of the decision to default on the debt, i.e., $S^D > 0$ and $S^N > 0$. In region C, entitlements are sustainable without defaulting on the debt, but they become unsustainable when there is a default, i.e., $S^D > 0$ and $S^N = 0$. Intuitively, in region C, the level of debt is low, which reduces the potential benefits of defaulting, while the cost of default is proportional to the tax revenue, which is why defaulting leads to entitlement cuts. Last, in set D, sequestration is never necessary, i.e., $S^D = S^N = 0$.

It is straightforward that in period 2, the incumbent defaults on debt if and only if debt default increases the resources available at her disposal:

$$(1-\phi)\tau - \lambda S^D > \tau - b_1 - \lambda S^N, \tag{129}$$

Notice that the composition of entitlements does not play a role in the decision to default. What matters are the total entitlements, which determine b_1.

In the first period, investors realize the incumbent defaults on debt when (129) holds, placing an upper limit on the amount of debt that can be issued at t=1. As stated in the Lemma below, the maximum credible level of debt decreases with ϕ, and, quite surprisingly, does not depend on either λ or α. This maximum level of debt is exactly at the intersection of the two curves in Figure 30.

Lemma K.1 (Maximum Debt) *In period 1, there exists an upper bound on the debt that can be borrowed:*

$$b_1 \leq \phi\tau \tag{130}$$

Proof: To show that any debt above $\phi\tau$ is not credibly repaid, we need to study the incentives to default in the four regions of Figure 30. In set D, both inequalities (128) and (126) are satisfied, which implies that $S^D, S^N = 0$. By (129), the two parties choose to default on debt when

$$(\tau - b_1) < (1-\phi)\tau. \tag{131}$$

Since the above condition can be rewritten as (130), we have shown that in set D, the maximum credible amount of debt is indeed $\phi\tau$. Second, for every point in set B, both inequalities (128) and (126) are not satisfied, which implies that S^D and S^N are both strictly positive. Debt default is chosen when

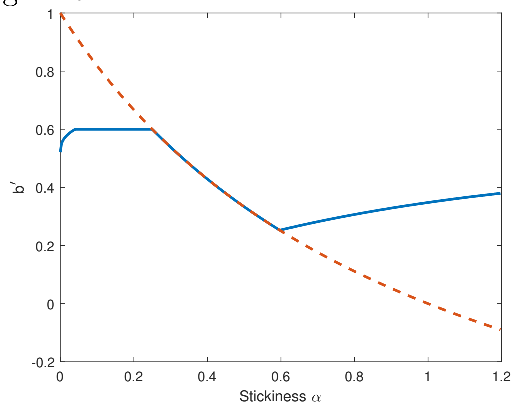
$$(\tau-b_1)-\max\left\{\frac{\lambda}{1-\lambda}[(\tau+b_1)\alpha-(\tau-b_1)],0\right\}< (1-\phi)\tau-\max\left\{\frac{\lambda}{1-\lambda}[(\tau+b_1)\alpha-(1-\phi)\tau)],0\right\}, \tag{132}$$

which can also be expressed as inequality (130). Third, in set A, we have $S^N > 0$ and $S^D = 0$. Consequently, debt default is chosen if

$$(\tau - b_1) - \max\left\{\frac{\lambda}{1-\lambda}[\alpha(\tau + b_1) - (\tau - b_1)], 0\right\}< (1-\phi)\tau \tag{133}$$

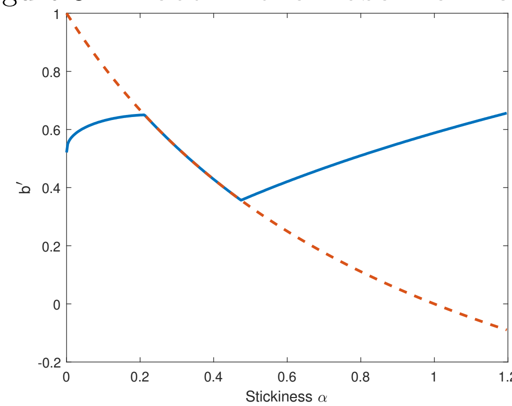
Because under debt default no sequestration is needed, the option to default in region A is more attractive compared to the payoff of defaulting in region B. Consequently, the debt default threshold will be lower than $\phi\tau$. Since all points in region A are above $\phi\tau$, we can

Figure 31: **Debt in the Default Model**



parameters: $\phi = 0.6$ and $\lambda = 0.2$

Figure 32: **Debt in the Baseline Model**



parameters: $\phi = 1$ and $\lambda = 0$

conclude that debt is not credible at any point in set A. Finally, in region C the incentives to default are weaker than the incentives to default in region B, because a debt default leads to a sequestration, unlike in the case of no debt default. Consequently, the debt default threshold will be higher than $\phi\tau$. Since all points in region C are below $\phi\tau$, we can conclude that debt is credible at any point in set C. $\square$

Moving to the first period, Party I's dynamic problem can be written as:

$$\max_{\{g_1^O, g_1^I, b_1\}} \Big\{ u(g_1^I) + (1-\Phi)\left[qu\left(\tau - b_1 - \alpha g_1^O\right) + (1-q)u(\alpha g_1^I)\right] + \\ \Phi u\left(\psi\left[(\tau - b_1)\frac{1}{1+\lambda} - \frac{\lambda}{1-\lambda}(\tau + b_1)\alpha)\right]\right)\Big\} \tag{134}$$

subject to (2), (4) and $b_1 \leq \phi\tau$.

The incumbent's problem is similar to the one in the baseline model, but there are two differences: the maximum level of debt is $\phi\tau$ instead of τ, and available resources upon sequestration are reduced due to the "tax-morale" effect of cutting entitlements. We simulate the optimal level of debt as a function of stickiness.[40] After comparing the baseline model (Figure 32) with the current model (Figure 31), we observe that there is now a maximum level of debt ($\phi\tau = 0.6$) and the slope of the dilution channel is smaller, indicating a higher cost of sequestration. However, the overall conclusion remains unchanged: the relationship between debt and stickiness is not monotonic, and there is a dilution effect when spending is sufficiently sticky.

[40] We assume $\sigma = 0.6$, $g_0^O = 0.7$, $q = 1/2$, $\tau = 1$, $\phi = 0.6$ and $\lambda = 0.2$.

L The Role of the Intertemporal Elasticity of Substitution

The paper assumes that $\sigma \in (0,1)$. When political polarization is extreme, this assumption is not critical and helps avoid addressing issues related to the potentially unbounded utility function when spending is zero.[41] In this section, we generalize the baseline model of Section IV. with full polarization by presenting results for values of σ smaller and higher than 1.[42]

The top panels of Figure 33 show equilibrium debt as a function of α when $\sigma = 0.25$ and $\sigma = 0.99$, while the lower panels illustrate the patterns when $\sigma > 1$, showing the $\sigma = 1.25$ and $\sigma = 2$ cases. All the other parameters are the same: $g_0^O = 0.5$, $q = 1/2$, and $\tau = 1$.

From the top panels, it is evident that a higher σ (i.e., a lower intertemporal elasticity of substitution, IES) makes it less likely that the incumbent triggers a sequestration, and it flattens the debt accumulation curves whenever the sequestration is not used. Still, as long as sequestration does not occur (small α), debt is increasing in persistence.

The lower panels of Figure 33 show that the patterns are qualitatively similar when σ is above 1. There still exists an area characterized by low persistence where sequestration isn't triggered, an intermediate area where debt coincides with the sequestration threshold and a high-stickiness area where sequestration occurs along the equilibrium path. The main difference is that debt decreases with persistence in the low α region where sequestration isn't triggered. Moreover, as α approaches zero, the incumbent commits all available resources, present and future, in the initial period.

To understand the different patterns when α is small, recall that the first-order condition (11) is given by

$$(\tau + b_1 - \alpha g_0^O)^{-\sigma} + (1-q)\alpha^{1-\sigma}(\tau + b_1 - \alpha g_0^O)^{-\sigma} = q(\tau - b_1 - \alpha^2 g_0^O)^{-\sigma}, \qquad (135)$$

where the left-hand side represents the marginal benefit of present spending. Note that when $\sigma > 1$, the factor $\alpha^{1-\sigma}$ multiplying the second term on the left-hand side tends to infinity as α approaches zero, and this factor *decreases* as α increases. In contrast, for $\sigma \in (0,1)$, this factor is zero when $\alpha = 0$, and it *increases* as α rises. Another way to understand the role of

[41] When $\sigma > 1$, CES utility is not well-defined when $\alpha = 0$, because when out of power, a party experiences zero spending.

[42] As discussed in Tabellini and Alesina (1990), the assumption $\sigma \in (0,1)$ becomes less innocuous when preferences are not fully polarized. When σ is larger than one, the incumbent may leave a surplus for its successor to incentivize increased spending on the incumbent's preferred goods. This is why Tabellini and Alesina (1990) assumes $\sigma \in (0,1)$—see condition (c) in their paper.

σ is to recognize that stickiness has two effects on spending incentives. On one hand, higher α makes spending more effective in constraining future governments, inducing less spending and debt. On the other hand, with a higher α, a given target level of future spending is achieved with less spending. The parameter σ determines which effect dominates, explaining the different slopes in the two cases.

Finally, as σ increases, the dilution area would get smaller. This is fairly intuitive. Triggering a sequestration requires inducing sizeable intertemporal spending variations. Consequently, the lower the IES, the less attractive the dilution strategy becomes.

Figure 33: **Debt: the Role of IES**