# Online Appendix
# Trading on Sunspots

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## A.1 Proof of proposition 6

**Proof.** According to proposition 5, only equilibrium H can be destroyed and, thus, we need to analyze its existence. Since it exists under financial autarky, we have

$$U(z_i + 1 + \alpha) - \kappa - U(z_i) \geqslant 0. \tag{A.1}$$

At $\pi^l = 0$ the payoff $n_1^h = n_2^h = 0$, and equilibrium H continues to exist. At $\pi^l > 0$, the gain from exerting effort in equilibrium H is

$$\Delta_i \equiv U(z_i + 1 + \alpha + n_i^h) - \kappa - U(z_i + n_i^h). \tag{A.2}$$

Since $n_1^h \geqslant 0$, $\Delta_1 \geqslant 0$. Hence, we only need to analyze the incentives of the endowment-rich individuals.

The derivative of the gain $\Delta_2$ is

$$\frac{d\Delta_2}{d\pi^l} = \left[U'(z_2 + 1 + \alpha + n_2^h) - U'(z_2 + n_2^h)\right]\frac{dn_2^h}{d\pi^l} < 0.$$

It is negative because the term in the square brackets is negative and the derivative of $n_2^h$ is positive, *i.e.*, the payoff to the endowment-rich individual is decreasing with $\pi^l$, as explained in section I.B. The incentives for the wealthy to exert effort decrease, as the probability of equilibrium L increases. This means that there exists a upper bound on $\pi^l$, proving the first statement of this proposition.

The upper bound is informative if the rich do not have incentives to work at $\pi^l = 1$. Using equation (16), consumption of the rich in state $h$, when it occurs with probability 0, is $\tilde{c}_2 = z_2/\bar{z} \cdot (\bar{z} + 1 + \alpha)$. If the rich stop working, their consumption declines by $1 + \alpha$. Hence, work incentives of the rich are destroyed when

$$U\left(\frac{z_2}{\bar{z}}(\bar{z} + 1 + \alpha)\right) - \kappa < U\left(\frac{z_2}{\bar{z}}(\bar{z} + 1 + \alpha) - 1 - \alpha\right).$$

The above inequality can be rewritten to resemble the condition for the rich to exert effort in financial autarky, equation (7), which is this proposition's assumption,

$$U\left(z_2 + \frac{z_2}{\bar{z}}(1 + \alpha)\right) - \kappa < U\left(z_2 + \left(\frac{z_2}{\bar{z}} - 1\right) \cdot (1 + \alpha)\right). \tag{A.3}$$

If $z_2$ equalled $\bar{z}$, the above inequality would contradict equation (7). However, it is possible that both (A.3) and equation (7) hold when $z_2 > \bar{z}$. In particular, take $z_2 = z_{max}$ such that equation (7) holds at equality. Then, (A.3) holds strictly because $U\left(z_2 + \frac{z_2}{\bar{z}}(1+\alpha)\right) - U\left(z_2 + \left(\frac{z_2}{\bar{z}} - 1\right)(1+\alpha)\right)$ is decreasing in $z_2$. Because $U$ is continuous, the two inequalities hold strictly for $z_2$ near $z_{max}$. This proves that the set of parameters for which $\bar{\pi}^l < 1$ is non-empty. ■

## A.2 Proof that welfare decreases in $\pi^l$

**Lemma 1** *Assume that $U = \log$. If both the L and H equilibria exist, welfare level $W_i$ decreases in $\pi^l$ for $i = 1, 2$.*

**Proof.** The following is true for any utility function:

$$\begin{aligned}\frac{dW_1}{d\pi^L} =& \underbrace{U(z_1 + n_1^L) - U(z_1 + \alpha + 1 + n_1^H)}_{negative} \\ &+ \underbrace{\pi^L u'(z_1 + n_1^L)\frac{dn_1^L}{d\pi^L} + (1-\pi^L)U'(z_1 + \alpha + 1 + n_1^H)\frac{dn_1^H}{d\pi^L}}_{both\ terms\ are\ negative} < 0.\end{aligned}$$

Letting $U(c) = \log(c)$ one obtains:

$$\begin{aligned}\frac{dW_2}{d\pi^L} =& \underbrace{U(z_2 + n_2^L) - U(z_2 + \alpha + 1 + n_2^H)}_{negative} \\ &+ \underbrace{\pi^L u'(z_2 + n_2^L)\frac{dn_2^L}{d\pi^L} + (1-\pi^L)U'(z_2 + \alpha + 1 + n_2^H)\frac{dn_2^H}{d\pi^L}}_{both\ terms\ are\ positive} \\ =& U(z_2 + n_2^L) - U(z_2 + \alpha + 1 + n_2^H) \\ &+ \pi^L\pi^H f_1\Delta_z(\alpha+1)\left[\frac{U'(z_2 + n_2^L)}{\bar{z} + \alpha + 1} + \frac{U'(z_2 + \alpha + 1 + n_2^H)}{\bar{z}}\right],\end{aligned}$$

where the last equality relies on the optimal portfolios derived in equations (22a) and (22b). Then by the concavity of $U$ and the fact that $U'(z_2 + \alpha + 1 + n_2^H)/U'(z_2 + n_2^L) = (\bar{z} + \alpha + 1)/\bar{z}$ we get

$$\begin{aligned}\frac{dW_2}{d\pi^L} \leqslant& -U'(z_2 + a + 1 + n_2^H)(\alpha + 1) \\ &+ \pi^L\pi^H f_1\Delta_z(\alpha+1)\left[\frac{U'(z_2 + n_2^L)}{\bar{z} + \alpha + 1} + \frac{U'(z_2 + \alpha + 1 + n_2^H)}{\bar{z}}\right] \\ =& -U'(z_2 + a + 1 + n_2^H)(\alpha+1) + 2\pi^L\pi^H f_1\Delta_z(\alpha+1)U'(z_2 + \alpha + 1 + n_2^H)/\bar{z} \\ =& U'(z_2 + a + 1 + n_2^H)(\alpha+1)[-1 + 2\pi^L\pi^H f_1\Delta_z/\bar{z}] < 0.\end{aligned}$$

■

## A.3 Proof that equilibria cannot be created

Suppose that without financial markets only equilibrium L or equilibrium H exists. Could the other equilibrium be "created" by opening financial markets? The following proposition states that it is not the case.

**Proposition 7** *Opening financial markets cannot create equilibrium H (L) equilibrium if only equilibrium L (H) existed under financial autarky.*

**Proof.** *Suppose only the L equilibrium exists under financial autarky.* In this case, one or both of the inequalities holds so that at least one type is not willing to work

$$U(z_1 + \alpha + 1) - \kappa < U(z_1),$$
$$U(z_2 + \alpha + 1) - \kappa < U(z_2).$$

When financial markets open, the incentives of the endowment-rich to work decrease because $n_2^h \geqslant 0$. Hence, the H equilibrium continues to be non-viable. Mathematically, the concavity of $U$ implies that the gain from working is negative:

$$U(z_2 + \alpha + 1 + n_2^h) - \kappa - U(z_2 + n_2^h) \leqslant U(z_2 + \alpha + 1) - \kappa - U(z_2) < 0.$$

*Suppose only equilibrium H exists under financial autarky.* In this case, at least one of the inequalities below holds

$$U(z_1 + \alpha) - \kappa > U(z_1),$$
$$U(z_2 + \alpha) - \kappa > U(z_2).$$

When financial markets open, the incentives of the endowment-rich to work increase because $n_2^l \leqslant 0$. Hence, the L equilibrium continues to be non-viable. Mathematically, the concavity of $U$ implies that the gain from working is positive

$$U(z_2 + \alpha + n_2^l) - \kappa - U(z_2 + n_2^l) \geqslant U(z_2 + \alpha) - \kappa - U(z_2) > 0.$$

■

## A.4 Proofs for the infinite horizon model

### A.4.1 Proof of Lemma 8

We start with optimization problem of agent $i$, as stated in equation 8. The envelope theorem implies that $V'(n) = U'(c)$, and so the first-order condition w.r.t. $n_j^{j\prime}$ yields the consumption Euler equation

$$Q^{sj} = \beta \pi^j \frac{U'\left(c_i^{\prime j}\right)}{U'\left(c_i^s\right)}. \tag{A.4}$$

Because each agent faces the same asset prices, the marginal utilities of the two types must grow at the same rate

$$\frac{U'\left(c_1'^{\,j}\right)}{U'\left(c_1^s\right)} = \frac{U'\left(c_2'^{\,j}\right)}{U'\left(c_2^s\right)}, \quad \forall\,(s,j)\,.$$

Homotheticity of $U$ then implies that the consumption growth of both types must also be the same, and must equal the growth of the aggregate supply of goods in all state pairs:

$$\frac{c_1'^{\,j}}{c_1} = \frac{c_2'^{\,j}}{c_2} = \frac{\bar{z}+y^j}{\bar{z}+y^s}, \quad \forall(s,j). \tag{A.5}$$

Individual consumption must then be proportional to the aggregate supply of goods:

$$c_i = \phi_i(\bar{z}+y^s), \quad i \in \{1,2\}\,. \tag{A.6}$$

Equations (A.4) and (A.6) imply (32).

Expected marginal utilities at $t=1$ (when $z$ is received for the first time) are equated to costs $Q^{0j}$ and the envelope condition then yields

$$\frac{\pi^h U'(c_i^h)}{\pi^l U'(c_i^l)} = \frac{\pi^h}{\pi^l}\left(\frac{\bar{z}+y^h}{\bar{z}+y^l}\right)^{-\gamma} = \frac{Q^{0h}}{Q^{0l}}.$$

Because there is no consumption in the opening period, one price at $t=0$ needs to be normalized. We thus set

$$Q^{0l} = \pi^l, \quad \text{and} \quad Q^{0h} = \pi^h D, \tag{A.7}$$

so that

$$D = \frac{Q^{0h}}{\pi^h} = \left(\frac{\bar{z}+\alpha+1}{\bar{z}}\right)^{-\gamma}. \tag{A.8}$$

Then for $t=0$, security prices are:

$$Q^{0l} = \pi^l, \tag{A.9a}$$
$$Q^{0h} = \pi^h D. \tag{A.9b}$$

For $t \geqslant 1$, security prices are state-dependent but not time-dependent. The present discounted value of aggregate income, that depends on state $s$, solves the following system of equations

$$I^0 = \pi^l I^l + \pi^h D I^h, \tag{A.10a}$$
$$I^l = \bar{z} + \beta\pi^l I^l + \beta\pi^h D I^h, \tag{A.10b}$$
$$I^h = \bar{z} + y^h + \beta\pi^l I^l/D + \beta\pi^h I^h. \tag{A.10c}$$

The solution is

$$I^0 = [\pi^l\bar{z} + \pi^h(\bar{z}+y^h)D]/(1-\beta), \tag{A.11a}$$
$$I^l = [(1-\beta\pi^h)\bar{z} + \beta\pi^h(\bar{z}+y^h)D]/(1-\beta), \tag{A.11b}$$
$$I^h = [\beta\pi^l\bar{z}/D + (1-\beta\pi^l)(\bar{z}+y^h)]/(1-\beta). \tag{A.11c}$$

Derived in the same way, present discounted value of individual income is

$$I_i^0 = [\pi^l z_i + \pi^h (z_i + y^h) D]/(1-\beta), \tag{A.12a}$$
$$I_i^l = [(1-\beta\pi^h) z_i + \beta\pi^h (z_i + y^h) D]/(1-\beta), \tag{A.12b}$$
$$I_i^h = [\beta\pi^l z_i / D + (1-\beta\pi^l)(z_i + y^h)]/(1-\beta). \tag{A.12c}$$

The consumption share of an agent $i$ is

$$\phi_i = I_i^0 / I^0 = \frac{\pi^l z_i + \pi^h (z_i + y^h) D}{\pi^l \bar{z} + \pi^h (\bar{z} + y^h) D}. \tag{A.13}$$

*Choice of x.*—Equilibrium H exists if for $i \in \{1,2\}$

$$\Delta V_i^h \equiv \max_{n'} \left\{ U(\alpha + 1 + z_i + n - \sum_{j\in\{l,h\}} Q^{hj} n'^j) - \kappa + \beta \sum_{j\in\{l,h\}} \pi^j V_i^j(n'^j) \right\} - \max_{n'} \left\{ U(z_i + n - \sum_{j\in\{l,h\}} Q^{hj} n'^j) + \beta \sum_{j\in\{l,h\}} \pi^j V_i^j(n'^j) \right\} \geqslant 0, \tag{A.14a}$$

and equilibrium L exists if for $i \in \{1,2\}$

$$\Delta V_i^l \equiv \max_{n'} \left\{ U(\alpha + z_i + n - \sum_{j\in\{l,h\}} Q^{hj} n'^j) - \kappa + \beta \sum_{j\in\{l,h\}} \pi^j V_i^j(n'^j) \right\} - \max_{n'} \left\{ U(z_i + n - \sum_{j\in\{l,h\}} Q^{hj} n'^j) + \beta \sum_{j\in\{l,h\}} \pi^j V_i^j(n'^j) \right\} \leq 0. \tag{A.14b}$$

The optimal $n'$ will generally change if the agent deviates from the equilibrium choice of $x$. We can obtain a sufficient condition if the equilibrium portfolio choices remain feasible following the deviation. Feasibility can be an issue if the deviation is from $x = 1$ to $x = 0$ that entails a loss of income. We will show in equation (A.17) that if the rich deviate, they can still hold their pre-deviation portfolio and still have strictly positive consumption. In that case, we have the following upper bound on the return to exerting effort

$$\Delta V_2^h \leqslant U\left( z_2 + y^h + n_2^h - \sum_{j\in\{l,h\}} Q^{hj} {n_2'}^j \right) - \kappa - U(z_2 + n_2^h - \sum_{j\in\{l,h\}} Q^{hj} {n_2'}^j)$$
$$= U(z_2 + y^h + b_2^h) - U(z_2 + b_2^h) - \kappa. \tag{A.15}$$

### A.4.2 Derivation of net portfolio payoff

By the budget constraint 27, net portfolio payoff must equal net consumption

$$b_i^s \equiv n_i^s - \sum_j Q^{sj} {n_i'}^j = c_i^s - (z_i + y^s). \tag{A.16}$$

Under the assumptions of lemma 8, net consumption of agent $i$ in state $s = h$ is

$$c_i^h - (z_i + y^h) = \frac{\pi^l (z_i - \bar{z}) y^h}{\pi^l \bar{z} + \pi^h (\bar{z} + y^h) D} = \begin{cases} < 0, & i = 1 \\ > 0, & i = 2 \end{cases} . \tag{A.17}$$

That is, the endowment-rich type 2 agent consumes more than his or her income in state $s = h$ or, equivalently, receives a net financial transfer from an endowment-poor type 1 agent.

Intuitively, consumption of the richer type-2 agent is more volatile. He or she suffers from consumption volatility less because since $U''' > 0$, period utility is flatter at higher levels of consumption. ∎

### A.4.3 Proof of proposition 9

Suppose that $\Delta A_i^h \geqslant 0, \forall i$. We will show that it is possible to chose $\kappa$ so that $\Delta V_2^h < 0$.

By equation (A.15), we have $\Delta V_2^h \leqslant U(z_2 + y^h + b_2^h) - \kappa - U(z_2 + b_2^h)$, where $b_2^h > 0$ is the optimal consumption of wealth of type-2 agent in state $h$. Because $b_2^h > 0$ and $U$ is strictly concave, we have

$$U(z_2 + y^h + b_2^h) - U(z_2 + b_2^h) < U(z_2 + y^h) - U(z_2).$$

Define $\kappa = 0.5[U(z_2 + y^h + b_2^h) - U(z_2 + b_2^h) + U(z_2 + y^h) - U(z_2)] > 0$, for which we have

$$U(z_2 + y^h + b_2^h) - U(z_2 + b_2^h) < \kappa < U(z_2 + y^h) - U(z_2),$$

or, equivlently,

$$U(z_2 + y^h + b_2^h) - \kappa - U(z_2 + b_2^h) < 0 < U(z_2 + y^h) - \kappa - U(z_2). \tag{A.18}$$

The left and right inequality imply, respectively, that $\Delta V_2^h < 0$ and $\Delta A_2^h > 0$. Because type-2 agent prefers not to work when $s = h$, equilibrium H is destroyed after opening financial markets.

Note that, because all expressions are continuous functions of the parameters, there must exist an open set containing the identified parameters and for which the proposition's statement holds. This proves the proposition.

### A.4.4 Proof of corollary 10

Since all expressions are continuous functions of the parameters, there must exist an open set containing the identified parameters and for which the proposition's statement holds. As shown above,

$$U(z_2 + y^h + b_2^h) - U(z_2 + b_2^h) < \kappa < U(z_2 + y^h) - U(z_2).$$

Since $b_2^h$ in Eq. (A.16) increases with $\pi^l$, $U(z_2 + y^h + b_2^h) - U(z_2 + b_2^h)$ decreases as $\pi^l$ increases. Thus, there must exist a value $\hat{\pi}$ such that the left inequality holds for all $\pi^l < \hat{\pi}$ but that it reverses for $\pi^l \geqslant \hat{\pi}$.

For log utility $U = \log$, we solve $\log(z_2 + y^h + b_2^h) - \log(z_2 + b_2^h) = \kappa$ for $\hat{\pi}^l$ as follows

$$\begin{aligned}\log(z_2 + y^h + b_2^h) - \log(z_2 + b_2^h) = \log(1 + y^h/(z_2 + b_2^h)) &= \kappa \\ \log(1 + y^h/(z_2 + b_2^h)) &= \kappa \\ \delta y^h/(z_2 + b_2^h) &= 1 \\ \delta y^h - z_2 &= b_2^h\end{aligned}$$

where $\delta$ is defined in equation (23). In equation (A.8) we see that when $\gamma = 1$, $D = \bar{z}/(\bar{z} + y^h)$. Using (A.17) and the fact that $b_2^h$ is increasing in $\pi^l$ we then get the following upper bound for $\bar{\pi}$:

$$\hat{\pi} = \frac{(\bar{z} + y^h)D}{\frac{(z_2 - \bar{z})y^h}{\delta y^h - z_2} - (\bar{z} - (\bar{z} + y^h)D)} = \frac{\bar{z}\left(\delta y^h - z_2\right)}{(z_2 - \bar{z})y^h}.$$

Since $y^h = \alpha + 1$, $\hat{\pi} = \bar{\pi}^l$ as defined in equation (24).

### A.4.5 Equilibrium L cannot be destroyed

Suppose that equilibrium L exists under financial autarky, *i.e.*,

$$\kappa > u(z_i + \alpha) - u(z_i), \quad \forall i.$$

This implies that for any $b > 0$ we have

$$u(z_i + b) - u(z_i + \alpha + b) + \kappa > 0, \quad \forall i. \tag{A.19}$$

Let $E$ to denote expectation over state $s$, and let $\hat{m}$ denote the optimal portfolio chosen by an agent choosing to deviate in equilibrium L. Then

$$\begin{aligned}&\max_m \{u(z_i + n - Qm) + \beta E[V^s(m^s)]\} \\ &\quad - \max_m \{u(z_i + \alpha + n - Qm) - \kappa + \beta E[V^s(m^s)]\} \\ &\quad > u(z_i + n - Q \cdot \hat{m}) - u(z_i + \alpha + n - Q \cdot \hat{m}) + \kappa > 0.\end{aligned}$$

The last inequality follows from (A.19) if $n - Q \cdot \hat{m} > 0$, which we establish next.

Independently of whether an agent follows the equilibrium actions or deviates, his or her consumption at the same aggregate rate as anyone else's. Thus, the present discounted value of one's consumption is a constant fraction of the present discounted value of the aggregate income, $PY$. Then,

$$\begin{aligned}n + PY_i &= \phi_i PY \\ n + PY_i + \alpha &= \hat{\phi}_i PY\end{aligned}$$

and a deviating agent consumes a larger fraction of the aggregate income than his or her non-deviating counterpart

$$\hat{\phi}_i = \phi_i + \alpha/PY. \tag{A.20}$$

Next

$$n - Q \cdot \hat{m} = \hat{\phi}_i \bar{z} - z_i - \alpha = \phi_i \bar{z} - z_i + \alpha(\bar{z}/PY - 1) \to_{\beta \to 0} \phi_i \bar{z} - z_i. \tag{A.21}$$