Online Appendix

A Framework for Economic Growth with Capital-Embodied Technical Change

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Proposition 2 *(Unbalanced growth path). Under condition 2, an equilibrium exists with a capital share* $s_{K_t} = \beta_t Z_t^{-1}$ *and explicitly determined paths of output, consumption, investment, the wage, the interest rate, and all other prices and quantities in the economy.*

Proof. On the production side firm optimization presents solutions at each point t for intermediate outputs and prices ($y_t(i)$ and $p_t(i)$), factor inputs ($l_t(i)$ and $x_t(i)$), aggregate GDP and investment (Y_t and I_t), and wages (w_t). On the consumption side, we have $c_t = w_t$. The Euler equation is therefore $r_t = \omega + \theta g_{w_t}$. The interest rate is then determined directly by taking the growth rate of w_t from (20). The interest rate is

$$r_t = \omega + \theta \frac{\rho - 1}{\rho} \left(\frac{\beta_t Z_t^{-1}}{1 - \beta_t Z_t^{-1}} (g_{\beta_t} - g_{Z_t}) - g_{1-\beta_t} \right) \tag{53}$$

The transversality condition requires $\lim_{t\to\infty} \psi X_t \exp - \int_0^t r_s ds = 0$. From the Euler equation, the integral can be solved from the path of wages, and the transversality condition can be written as

$$\lim_{t\to\infty} e^{-(\omega - n - (1-\theta)\overline{g}_t)t} \frac{\beta_t Z_t^{-1}}{\left(1 - \beta_t Z_t^{-1}\right)^{\theta}} = 0$$

where $\overline{g}_t$ is the mean growth rate in per-capita income from time 0 to time t, and we define $\overline{g}$ as the mean growth rate over all future time. Under condition 2, we have $\omega - n > (1-\theta)\overline{g}$ and the transversality condition is therefore satisfied.[40]

The economy thus has a well-defined equilibrium, with quantities and prices following explicitly defined paths as given in the relevant equations of Section I. The capital share is $s_{K_t} = \beta_t Z_t^{-1}$ from (17). The path of GDP is given in (19). The path of wages is given by (20), which also determines per-capita consumption. And the interest rate is (53). All other prices and quantities are found in

[40] Note also that we require that the capital share ($\beta_t Z_t^{-1}$) be less than 1 as $t \to \infty$. This is a very weak condition because the technology adoption condition already implies $Z_t \geq 1$, as discussed in Section IF, and as long as capital productivities advance beyond this minimum floor, so that $Z_\infty > 1$, the capital share will be less than 1 in the limit.

Section I, given either directly in terms of exogenous parameters there, or indirectly by replacing the GDP or wage term with its explicit solution. ■

Corollary 2: *(Labor share and growth dynamics). The labor share is decreasing with time if $g_{\beta_t} > g_{Z_t}$, constant if $g_{\beta_t} = g_{Z_t}$ and increasing with time if $g_{\beta_t} < g_{Z_t}$. The growth rate in income per-capita increases in g_{β_t} and g_{Z_t}.*

Proof. The labor share results follow by inspection of (18). For the growth dynamics, consider GDP as given by (19). Take logs and differentiate with respect to time. Then rearrange terms to write

$$g_{y_t} = -\frac{1}{\rho}\left(\frac{\beta_t Z_t^{-1}}{1-\beta_t Z_t^{-1}}\right) g_{Z_t} - \frac{1}{\rho}\left((1-\rho)\frac{\beta_t}{1-\beta_t} - \frac{\beta_t Z_t^{-1}}{1-\beta_t Z_t^{-1}}\right) g_{\beta_t} \tag{54}$$

Note that $\rho < 0$. Further note that $\beta_t Z_t^{-1} < 1$. Therefore we see that g_{y_t} is increasing in g_{Z_t}. Noting again that $\rho < 0$, we see that g_{y_t} is increasing in g_{β_t} if

$$\frac{\beta_t}{1-\beta_t} - \frac{\beta_t Z_t^{-1}}{1-\beta_t Z_t^{-1}} > 0 \tag{55}$$

The left hand side terms can be combined into the expression

$$\frac{\beta_t(1-Z_t^{-1})}{(1-\beta_t)(1-\beta_t Z_t^{-1})} > 0 \tag{56}$$

This is positive so long as $Z_t > 1$, which is guaranteed by (22), an implication of the adoption condition (21). ■

Corollary 3: *(Automation-led growth with constant wages) Let automation proceed at some rate $q^h > 0$ where newly automated technologies have productivity level z_t^{min}, the lowest level of productivity where they will still be adopted. Let prior automated technologies see no productivity improvement. Then wages remain constant. Income per-capita grows at rate q^h, and the labor share falls at rate q^h. The technology index Z_t declines at rate $q^h\frac{1-Z_t}{\beta_t} < 0$.*

Proof. Consider first the growth rate in Z_t. Take the definition of Z_t as the harmonic average of the $z_t(i)$, as in (6). Differentiating (6) with respect to time and using Leibniz's rule gives

$$g_{Z_t} = g_{\beta_t} - Z_t \frac{g_{\beta_t}}{z_t(\beta_t)} + \frac{Z_t}{\beta_t}\int_0^{\beta_t} \frac{1}{z_t(i)} g_{z_t(i)} di \tag{57}$$

Following the technological pathways defined in the Corollary, we have

1. $g_{1-\beta_t} = -q^h$, which is equivalently $g_{\beta_t} = ((1-\beta_t)/\beta_t)q^h$;

2. $z_t(\beta_t) = z_t^{min} = (1-\beta_t)/(1-\beta_t Z_t^{-1})$;

3. $g_{z_t(i)} = 0$ for all $i < \beta_t$ (no vertical progress).

Under these conditions, the growth rate of Z_t in (58) becomes

$$g_{Z_t} = \frac{1-Z_t}{\beta_t} q^h \tag{58}$$

which is less than zero because $Z_t > 1$.

The labor share is $s_{L_t} = 1 - \beta_t Z_t^{-1}$. Taking logs and differentiating with respect to time, we have

$$g_{s_{L_t}} = \frac{\beta_t Z_t^{-1}}{1-\beta_t Z_t^{-1}} \left(g_{\beta_t} - g_{Z_t} \right) \tag{59}$$

Using the above technology paths for β_t and Z_t this simplifies to $g_{s_{L_t}} = -q^h$.

The wage is given by (20). Taking logs and differentiating with respect to time gives

$$g_{w_t} = \frac{\rho - 1}{\rho} \left(g_{s_{L_t}} - g_{1-\beta_t} \right) \tag{60}$$

Using the results above, we have $g_{s_{L_t}} = -q^h = g_{1-\beta_t}$ and so $g_{w_t} = 0$.

Finally, given that wages are constant and that the labor share of income, $s_{L_t} = \frac{w_t L_t}{Y_t}$, falls at rate q^h, it follows directly that Y_t/L_t grows at rate q^h. ■

Corollary 4: *(Sectoral advance). Holding other sector technology levels fixed, the GDP share of a sector will decline with its automation level, β_t^j, or capital productivity level, Z_t^j. The labor share of income within the sector will decrease in the automation level, β_t^j, but increase in the capital productivity level, Z_t^j.*

Proof. From (27), write the GDP share of the sector as

$$\Phi^j = \beta_t^j \left(Z_t^j \right)^{-1} + \left(1 - \sum_i \beta_t^i \left(Z_t^i \right)^{-1} \right) \frac{u_t^j - \beta_t^j}{1 - \sum_i \beta_t^i} \tag{61}$$

Differentiate with respect to Z_t^j. After simplification, this can be written as

$$\frac{\partial \Phi^j}{\partial Z_t^j} = -\beta_t^j \left(Z_t^j\right)^{-2} \left(\frac{\sum_{i \neq j} u_t^i - \beta_t^i}{1 - \beta_t}\right) \tag{62}$$

which by inspection is less than zero.

Next differentiate (61) with respect to β_t^j. After simplification, this can be written as

$$\frac{\partial \Phi^j}{\partial \beta_t^j} = (Z_t^j)^{-1} \left(\frac{s_{L_t} Z_t^j}{1 - \beta_t} - 1\right) \left(\frac{u_t^j - \beta_t^j}{1 - \beta_t} - 1\right) \tag{63}$$

This is also less than zero. To see this, note that $\frac{u_t^j - \beta_t^j}{1-\beta_t} \leq 1$ and $\frac{s_{L_t} Z_t^j}{1-\beta_t} \geq 1$. The former is true by inspection, as the measure of non-automated tasks in the sector, $u_t^j - \beta_t^j$, must be weakly less than the measure of non-automated tasks in the economy overall, $1 - \beta_t$. To show the latter, which requires that $s_{L_t} Z_t^j \geq 1 - \beta_t$, we can rewrite this expression using the definition of $s_{L_t} = 1 - \beta_t Z_t^{-1}$ to produce the equivalent condition

$$1 \geq \beta_t Z_t^{-1} \left(\frac{Z_t^j - Z_t}{Z_t^j - 1}\right) \tag{64}$$

which holds because $\beta_t Z_t^{-1} \leq 1$ and $Z_t \geq 1$. Thus the sector's GDP share is increasing in Z_t^j and β_t^j.

Turning to the income share within a sector, we find that

$$\frac{\partial s_{L_t}^j}{\partial \beta_t^j} \leq 0 \tag{65}$$

This follows by inspection of (28), since we have β_t^j increasing and the GDP share of the sector falling.

Differentiating the sectoral labor share of income in (28) by Z_t^j we find that

$$\frac{\partial s_{L_t}^j}{\partial Z_t^j} = -\beta_t^j \frac{1}{(Z_t^j)^2} \frac{1}{(\Phi^j)^2} \frac{\partial \Phi^j}{\partial Z_t^j} \geq 0 \tag{66}$$

where the sign follows by inspection, using the above result showing $\frac{\partial \Phi^j}{\partial Z_t^j} \leq 0$. ■

As a related result, on the growth path of the economy, technological advance may proceed in all sectors simultaneously. We may therefore also consider a variant of this corollary that emphasizes the relative evolution of sectors along a balanced growth path. To do so, we can define the relative technology state of different sectors. Specifically, define the relative automation rate in sector j as $\eta_t^j = (\beta_t^j/u_t^j)/\beta_t$.[41] Similarly, define the relative capital productivity in sector j as $\varphi_t^j = Z_t^j/Z_t$. Thus a sector with $\eta_t^j > 1$ is relatively highly automated compared to the economy at large, and a sector with $\varphi_t^j > 1$ has relatively advanced capital productivity.

With these definitions, we can sum up sector-specific tasks and write the GDP share and capital share for a given sector in the following, relative technology form,

$$\Phi^j = u_t^j \left(\frac{\eta_t^j}{\varphi_t^j} s_{K_t} + \frac{1 - \eta_t^j \beta_t}{1 - \beta_t} (1 - s_{K_t}) \right) \tag{67}$$

$$s_{K_t}^j = s_{K_t} \left(s_{K_t} + \varphi_t^j \frac{1/\eta_t^j - \beta_t}{1 - \beta_t} (1 - s_{K_t}) \right)^{-1} \tag{68}$$

We can then consider structural change in the economy along a balanced growth path, as follows.

Corollary 4a: *(Structural change). Along a balanced growth path, an increase in the relative productivity, φ_t^j, of the sector's capital inputs or relative automation level, η_t^j, will cause the sector's GDP share to decline. An increase in the sector's relative automation, η_t^j, will cause its labor share to decline while an increase in the relative productivity, φ_t^j, of the sector's capital inputs will cause its labor share to rise.*

Proof. Consider the within-sector capital share, as in (68). Consider dynamics where the overall capital share in the economy is fixed (i.e., the economy is on a balanced growth path). Focus on a particular sector j. By inspection of (68), the capital share in that sector will rise if its relative automation rate, η_t^j, increases, and the capital share in that sector will fall if its relative productivity level, φ_t^j, increases.

Next consider the sector's share of GDP, as in (67). By inspection, an increase in the sector's relative capital productivity level, φ_t^j, will cause the GDP share of that sector to decline. The effect

[41] This is the relative automation rate in that β_t^j/u_t^j is the share of tasks in the sector that are automated, which is compared to β_t, the share of all tasks that are automated.

of higher relative automation cannot be seen by inspection, however. Differentiate the sectoral GDP share by its relative automation rate, holding the economy wide technology indices fixed. We have

$$\frac{\partial \Phi^j}{\partial \eta_t^j} = u^j \left(\frac{1}{\varphi_t^j} s_{K_t} - \frac{\beta_t}{1-\beta_t} s_{L_t} \right) \tag{69}$$

Thus the GDP share of the sector is declining in its relative automation rate if the term on parentheses is negative. Recalling the definition $\varphi_t^j = Z_t^j / Z_t$, and that $s_{K_t} = 1 - s_{L_t} = \beta_t Z_t^{-1}$ we can write

$$\frac{\partial \Phi^j}{\partial \eta_t^j} < 0 \text{ iff } Z_t^j > \frac{1-\beta_t}{1-\beta_t Z_t^{-1}} \tag{70}$$

Now recall from (21) that

$$z_t^{\min} \geq \frac{1-\beta_t}{1-\beta_t Z_t^{-1}} \tag{71}$$

Since the harmonic average Z_t^j must exceed $z_t^{\min}$, the above condition must hold. ■

Corollary 5: *(Extreme technological advance). Let a fraction α of the automated tasks have the same distribution of $z_t(i)$ as the other automated tasks. Holding other technologies constant, take $z_t(i) \to \infty$ for this fraction α of automated tasks. The capital share will decline by α percent. Income per capita will increase by $\Delta \ln(y_t) = -\frac{1}{\rho} \ln \left(1 + \alpha \frac{s_{K_t}}{s_{L_t}}\right)$.*

Proof. Consider a fraction α of the automated tasks. Define the harmonic average of the $z_t(i)$ for this fraction of tasks as $Z_{t,\alpha}$. Define the harmonic average of the $z_t(i)$ for the remaining $1-\alpha$ fraction of tasks as $Z_{t,1-\alpha}$. Therefore we can write

$$Z_t = \left(\alpha Z_{t,\alpha}^{-1} + (1-\alpha) Z_{t,1-\alpha}^{-1} \right)^{-1} \tag{72}$$

For simplicity, consider the initial state at time t where the harmonic average is the same for the fraction α of tasks as for the all automated tasks. Then $Z_{t,1-\alpha} = Z_{t,\alpha} = Z_t$. Now, for the fraction α of automated tasks, let the technology level $z_t(i) \to \infty$ at time $t = t'$, holding β_t and the other $z_t(i)$ fixed. The index $Z_{t'}$ becomes

$$Z_{t'} = \frac{1}{1-\alpha} Z_{t',1-\alpha} = \frac{1}{1-\alpha} Z_t \tag{73}$$

and the capital share becomes

$$s_{K_{t'}} = \beta_{t'} Z_{t'}^{-1} = (1-\alpha)\beta_t Z_t^{-1} = (1-\alpha) s_{K_t} \tag{74}$$

Hence the capital share falls by α percent.

For income per capita, consider GDP given by (19). With β_t fixed, the change in income per capita is

$$\frac{y_{t'}}{y_t} = \left(\frac{1-\beta_{t'}Z_{t'}^{-1}}{1-\beta_t Z_t^{-1}}\right)^{-1/\rho} = \left(\frac{1-(1-\alpha)s_{K_t}}{s_L}\right)^{-1/\rho} \tag{75}$$

Taking logs and using $1 - s_{K_t} = s_{L_t}$ produces the result in the corollary. ■

Lemma 1: *The vertical and horizontal hazard rates of innovation, q^v and q^h, are both constants on a BGP and have the ratio $\frac{q^h}{q^v} = \left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}$. Aggregate R&D expenditure in the vertical direction and in the horizontal direction are both constant shares of GDP.*

Proof. By (37) and (38), we can write the equilibrium vertical rate of innovation as

$$q^v = \xi^{v\frac{1}{\alpha}} \mu q^{v\frac{\alpha-1}{\alpha}} - (r-g) \tag{76}$$

Similarly, from the horizontal side, (40) and (41) imply

$$q^v = \xi^{h\frac{1}{\alpha}} \mu q^{h\frac{\alpha-1}{\alpha}} - (r-g)\,. \tag{77}$$

Combining these produces the ratio $\frac{q^h}{q^v} = \left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}$, as was to be shown.

Further, recall that the ratio $z_t(i)d_t^v(i)/Y_t$ is a constant for vertical lines. Define this constant as χ^v. We can then write R&D expenditure on a given line as $d_t^v(i) = \chi^v Y_t/z_t(i)$. Total vertical research investment across the automated lines then adds up as

$$\frac{D_t^v}{Y_t} = \chi^v \beta_t Z_t^{-1}. \tag{78}$$

which will be a constant share of GDP on the BGP.

Similarly, for horizontal lines, the BGP features $z_t^h d_t^h(i) = \chi^h Y_t$, where χ^h is a constant. Recalling that the initial quality of any newly automated line is (39), the R&D effort will then be the same across these horizontal lines. The aggregate investment in the horizontal research sector then adds up as

$$\frac{D_t^h}{Y_t} = \chi^h h^{-1}, \tag{79}$$

so that horizontal R&D expenditure is also a constant share of GDP, as was to be shown. ■

Lemma 2: *All automated technologies will be adopted on the BGP if* $h \geq \gamma^{-1}\frac{1}{\phi}\left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}$.

Proof. The adoption condition is that the price of using an automated technology, $p_t(i) = \psi\left[\gamma z_t(i)\right]^{\frac{1-\rho}{\rho}}$, is less than the price when using labor, $\hat{p}_t(i) = w_t/A$. This will be satisfied for all automated sectors if it is satisfied for the automated sector with the lowest productivity. The lowest productivity level will be the one for the marginally automated technology, which has productivity z_t^h. Thus we require

$$w_t/A \geq \psi\left[\gamma z_t^h\right]^{\frac{1-\rho}{\rho}}$$

Using the initial productivity for z_t^h, (39), and the equilibrium wage via the labor share, (45), this becomes

$$(1-\gamma^{-1}\beta_t Z_t^{-1})\frac{Y_t}{L} \geq \psi A h^{\frac{1-\rho}{\rho}}(1-\beta_t)^{\frac{1-\rho}{\rho}}$$

Using GDP, (46), this simplifies as

$$1-\gamma^{-1}\beta_t Z_t^{-1} \geq h^{-1}$$

where $\gamma^{-1}\beta_t Z_t^{-1}$ is the capital share. Using the result for the capital share, (43), this produces the statement in the Lemma. ■

Proposition 3: *The balanced growth path exists and is unique if* $\theta \geq 1$.

Proof. We will first consider the existence and uniqueness of the horizontal innovation rate, q^h. Using the system of four equations ((8), (23), (76), and (42)) we substitute out the other endogenous variables and write an implicit function for q^h in terms of the exogenous parameters. This expression is

$$q^h\left(\left(\frac{\xi^v}{\xi^h}\right)^{\frac{1}{1-\alpha}} + (\theta-1)\frac{\rho-1}{\rho}\right) = \frac{\xi^{h\frac{1}{\alpha}}\mu}{q^{h\frac{1-\alpha}{\alpha}}} - \omega \tag{80}$$

The left hand side of this equation is a linear function of q^h. Note that $\theta \geq 1$ is a sufficient condition for the expression in parentheses to be positive. Therefore this function starts at the origin and rises monotonically in q^h and without bound as $q^h \to \infty$. Meanwhile, the right hand side of this equation is a function that declines in q^h. The function is unbounded at $q^h = 0$ and declines monotonically, crossing zero for some positive q^h. Therefore there is a single crossing property in these two functions at some unique positive value of q^h.

Next, note that a unique positive value of q^h implies a unique, positive q^v (via (42)) and a unique, positive g (via (23). A unique r is then determined uniquely from the Euler equation (8) (and we must also have $r > g$ with $\theta \geq 1$). Therefore there exists a set of values $\{q^v, q^h, r, g\}$ that are unique and create a balanced growth path equilibrium. ■

Proposition 4: *(Endogenous growth comparative statics). The labor share on a balanced growth path increases with ξ^v, h, ϕ, and γ and decreases with ξ^h. The growth rate on a balanced growth path increases with ξ^h and μ, decreases with ξ^v, and is unchanging in h.*

Proof. Write the labor share as

$$s^L = 1 - \frac{\gamma^{-1}}{h\phi}\left(\frac{\xi^h}{\xi^v}\right)^{\frac{1}{1-\alpha}}$$

The comparative statics for the labor share with respect to ξ^v, h, ϕ, and ξ^h follow by inspection.

For the growth rate, note that it is linear and monotonic in q^h, from (23). So we will consider the comparative statics in terms of the behavior of q^h. In particular, turn again to (80) and the single crossing property analyzed in the proof of Proposition 3. Consider the intersection point of the increasing function of q^h on the left hand side and the decreasing function of q^h on the right hand side of (3).

By inspection, an increase in ξ^h decreases the slope on the left hand side and shifts rightward the function on the right hand side. Both forces cause the equilibrium q^h to rise. By inspection, an increase in ξ^v increases the slope on the left hand side (while the function on the right hand side does not change), causing the equilibrium q^h to fall. By inspection, a rise in the markup, μ, cause the right hand side function to shift rightward (while the function on the left hand side does not change), causing the equilibrium q^h to rise. By inspection, changes in h have no effect on q^h. (In fact, h does not appear in the four equation system and thus has no influence on the growth rate, innovation rates, or interest rate.) ■

Data Analysis

The paper provides two illustrative empirical applications of the model. We discuss the data sets and estimation strategies in further detail here.

Income and Growth Dynamics

In Figure 4, we use standard data series from the Bureau of Labor Statistics for U.S. labor productivity (output her hour) and the U.S. labor share of income. These data were drawn from FRED, with links to both data series provided in the references to this paper (BLS 2021a, 2021b). Both series run from 1947-2020.

The two technology paths are then estimated using (18) and (19). To pin down the technology paths we need one initial condition and we set $Z_0 = 1.5$ for the first year of the data series. Given this initial condition, the initial automation rate is pinned down by the initial capital share, $\beta_0 = Z_0 s_{K_0}$. We then set $\rho = -1$ and pin down νA given the observed initial output per worker level, Y_0/L_0, thus normalizing the output per worker measure. Having normalized this measure, one can then proceed in each period to estimate the two unknowns, β_t and Z_t, from the two equations (18) and (19) and the observed output per hour and labor share data series.

Figure 4 presents the technology pathways in their limits form as $1 - \beta_t$ and $1/s - Z_t$. This is useful visually because (in logs) a common, constant slope then appears as a balanced growth path, allowing one to see a BGP and deviations from a BGP more easily. On a balanced growth path, the limit of Z_t is $c = 1/s$; i.e., the inverse of the capital share. For visualization purposes, we take $c = 1/\min[s_{K_t}]$.

Sectoral Dynamics

In Figure 5, we provide data on sectoral GDP share and labor shares of income. The raw data come from Mendieta-Munoz et al. (2020), who calculate output and labor compensation shares for 14 sectors (leaving out the public sector and housing). We consider agriculture, manufacturing, and the remaining sectors as one group (see main text).

To estimate the path of the sectoral technology parameters, β_t^j and Z_t^j, we can use (27) and (28). However, we also need information on the task shares, u_t^j, and these are not determined within the model. These task shares may also be evolving to some extent with time. For example, information services may replace certain manufactured goods as the leading technology for performing certain tasks (as when Internet search services replace dictionaries, phone books, etc.).

To provide some external grounding for the task shares, we use SIC and NAICS codes. The idea is to estimate the task share using the given industrial categorization scheme. Specifically, we count the number of six-digit subsectors in the 2012 NAICS, grouped according to their two-digit definitions (11 for agriculture, 31-33 for manufacturing). We drop the public sector, consistent with the Mendieta-Munoz et al. (2020) data, and group the remaining six-digit industries as "other". We similarly apportion industries in the 1987 SIC classification system. The NAICS results produce $\{u_{2012}^{agr}, u_{2012}^{man}, u_{2012}^{oth}\} = \{.0618, .3514, .5868\}$. The SIC results produce $\{u_{1987}^{agr}, u_{1987}^{man}, u_{1987}^{oth}\} = \{.0592, .4192, .5216\}$. We see that both schemes agree quite closely on the share of different subsectors that constitute agriculture. Over time, however, we see that the share of manufacturing subsectors appears by this measure to have declined compared to services. For illustration purposes, we take this shift as substantive (as information services do seem, e.g., to have led to the creative destruction in some manufactured goods used for some tasks), and we assign $\{u_t^{agr}, u_t^{man}, u_t^{oth}\}$ as linear trends that match the SIC and NAICS measures in the appropriate years.