Personalized Pricing and Competition

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Online Appendix

A Omitted Details and Proofs in Sections 2 and 3

A.1 Primitive conditions for Assumption 1

We report some primitive conditions for Assumption 1. We use the following notation

$$G_2(x|y) \equiv \frac{\partial G(x|y)}{\partial y} \ . \tag{22}$$

Lemma 8. *(i) If the joint density $\tilde{f}$ is log-concave, then $1 - H_z(x)$ is log-concave in x. (ii) $\phi(z) = \frac{1-H_z(0)}{h_z(0)}$ is non-increasing in z if (a) $G_2(v|v) \geq 0$ and $f'(v) \geq 0$, or (b) $\tilde{f}$ is log-concave and $\frac{G_2(v|v)}{G(v|v)}$ is non-increasing in v. (In particular, condition (b) holds in the IID case with a log-concave f.)*

Proof. (i) Note that

$$1 - H_z(x) = \int_{A_x} \tilde{f}(\mathbf{v}) d\mathbf{v} \ ,$$

where $A_x = \{\mathbf{v} : v_i - \max_{j \neq i}\{z, v_j\} > x\}$. To prove $1 - H_z(x)$ is log-concave in x, according to the Prékopa-Borell Theorem (see, e.g., Caplin and Nalebuff, 1991), it suffices to show that, for any $\lambda \in [0, 1]$, we have

$$\lambda A_{x_0} + (1 - \lambda) A_{x_1} \subset A_{\lambda x_0 + (1-\lambda) x_1} \ , \tag{23}$$

where the former is the Minkowski average of A_{x_0} and A_{x_1}. Let $\mathbf{v}^0 \in A_{x_0}$ and $\mathbf{v}^1 \in A_{x_1}$, i.e.,

$$v_i^0 > z + x_0 \ \text{ and } \ v_i^0 > v_j^0 + x_0 \ \text{ for any } j \neq i \ ,$$

and

$$v_i^1 > z + x_1 \ \text{ and } \ v_i^1 > v_j^1 + x_1 \ \text{ for any } j \neq i \ .$$

These immediately imply that

$$v_i^\lambda > z + \lambda x_0 + (1 - \lambda) x_1 \ \text{ and } \ v_i^\lambda > v_j^\lambda + \lambda x_0 + (1 - \lambda) x_1 \ \text{ for any } j \neq i \ ,$$

where $v_i^\lambda = \lambda v_i^0 + (1 - \lambda) v_i^1$. Hence, we have $\mathbf{v}^\lambda \in A_{\lambda x_0 + (1-\lambda) x_1}$, and so (23) holds.

(ii) Recall that

$$\phi(z) = \frac{\int_z^{\overline{v}} G(v|v)dF(v)}{G(z|z)f(z) + \int_z^{\overline{v}} g(v|v)dF(v)} .$$

For $z \le \underline{v}$, $\phi(z)$ is a constant and so is non-increasing. In the following, we focus on $z > \underline{v}$. Using $\frac{dG(v|v)}{dv} = g(v|v) + G_2(v|v)$, one can check that $\phi'(z) \le 0$ if and only if

$$G(z|z)f(z) + \int_z^{\overline{v}} g(v|v)dF(v) + \left(\frac{f'(z)}{f(z)} + \frac{G_2(z|z)}{G(z|z)}\right) \int_z^{\overline{v}} G(v|v)dF(v) \ge 0 . \tag{24}$$

This must be true if condition (a) holds. To see condition (b), notice that the log-concavity of the joint density $\tilde{f}$ implies log-concavity of the marginal density f and so $\frac{f'(z)}{f(z)} \ge \frac{f'(v)}{f(v)}$ for $v \ge z$. Therefore, a sufficient condition for (24) is

$$G(z|z)f(z) + \int_z^{\overline{v}} g(v|v)dF(v) + \int_z^{\overline{v}} G(v|v)f'(v)dv + \frac{G_2(z|z)}{G(z|z)} \int_z^{\overline{v}} G(v|v)dF(v) \ge 0 .$$

Applying integration by parts to the third term, we can rewrite the above condition as

$$f(\overline{v}) - \int_z^{\overline{v}} G_2(v|v)dF(v) + \frac{G_2(z|z)}{G(z|z)} \int_z^{\overline{v}} G(v|v)dF(v) \ge 0 .$$

A sufficient condition for this to hold is that

$$\frac{G_2(z|z)}{G(z|z)} \ge \frac{G_2(v|v)}{G(v|v)}$$

for any $v \in [z, \overline{v}]$. This is true if $\frac{G_2(v|v)}{G(v|v)}$ is non-increasing in v. $\square$

A.2 Proofs of the lemmas in Proposition 3

Proof of Lemma 4. From the equilibrium price condition $p - c = \phi(p)$, we derive $p'(c) = \frac{1}{1-\phi'(p)}$. Since $\phi'(\cdot) \le 0$ under Assumption 1, we must have $p'(c) \le 1$. Since $p \to \overline{v}$ as $c \to \overline{v}$, it suffices to examine $-\phi'(\overline{v})$.

Recall that

$$\phi(p) = \frac{\int_p^{\overline{v}} G(v|v)dF(v)}{h_p(0)} ,$$

where $h_p(0) = G(p|p)f(p) + \int_p^{\overline{v}} g(v|v)dF(v)$. Then

$$-\phi'(p) = \frac{G(p|p)f(p)}{h_p(0)} + \frac{\int_p^{\overline{v}} G(v|v)dF(v) \times \frac{\partial h_p(0)}{\partial p}}{h_p(0)^2} , \tag{25}$$

where $\frac{\partial h_p(0)}{\partial p} = G_2(p|p)f(p) + G(p|p)f'(p)$.

(i) Suppose first $f(\overline{v}) > 0$. Then $\lim_{p \to \overline{v}} h_p(0) = f(\overline{v}) > 0$, and so as $p \to \overline{v}$, the second term in (25) equals 0 and $-\phi'(\overline{v}) = \frac{f(\overline{v})}{f(\overline{v})} = 1$. Therefore, $p'(\overline{v}) = \frac{1}{2}$.

(ii) Suppose $f(\overline{v}) = 0$ and $f'(\overline{v}) < 0$. Then in the limit the first term in (25) equals

$$\lim_{p \to \overline{v}} \frac{G(p|p)f(p)}{h_p(0)} = \frac{1}{1 + \lim_{p \to \overline{v}} \frac{\int_p^{\overline{v}} g(v|v)dF(v)}{G(p|p)f(p)}} = 1 \ . \tag{26}$$

Here the first equality is from dividing both the numerator and denominator by $G(p|p)f(p)$, and the second equality is because L'hôpital's rule implies that

$$\lim_{p \to \overline{v}} \frac{\int_p^{\overline{v}} g(v|v)dF(v)}{G(p|p)f(p)} = \lim_{p \to \overline{v}} \frac{-g(p|p)f(p)}{g(p|p)f(p) + G_2(p|p)f(p) + G(p|p)f'(p)} = 0 \ ,$$

where we have used $f(\overline{v}) = 0$, $f'(\overline{v}) < 0$, and $G_2(\overline{v}|\overline{v}) = 0$ (which is from the fact that $G(\overline{v}|y) = 1$ for any y). On the other hand, in the limit the second term in (25) equals $-\frac{1}{2}$. This is because

$$\lim_{p \to \overline{v}} \frac{h_p(0)^2}{\int_p^{\overline{v}} G(v|v)dF(v)} = -2 \lim_{p \to \overline{v}} \frac{h_p(0)\frac{\partial h_p(0)}{\partial p}}{G(p|p)f(p)} = -2 \lim_{p \to \overline{v}} \frac{\partial h_p(0)}{\partial p} \ , \tag{27}$$

where the first step is from L'hôpital's rule and the second step uses (26). The claim then follows since $\lim_{p \to \overline{v}} \frac{\partial h_p(0)}{\partial p} \neq 0$ in this case. Therefore, $-\phi'(\overline{v}) = \frac{1}{2}$ and so $p'(\overline{v}) = \frac{2}{3}$.

(iii) The last possibility is $f(\overline{v}) = 0$ and $f'(\overline{v}) = 0$. We focus on the case when $f(v)$ is log-concave. Notice that $\frac{\partial h_p(0)}{\partial p} = f(p)[G_2(p|p) + G(p|p)\frac{f'(p)}{f(p)}]$. Given $f(\overline{v}) = 0$, $f'(v) < 0$ when v is sufficiently large; meanwhile, given f is log-concave, $\frac{f'}{f}$ is decreasing and so $\frac{f'(p)}{f(p)}$ must be strictly negative for p sufficiently close to $\overline{v}$. This, together with $G_2(\overline{v}|\overline{v}) = 0$, implies that $\frac{\partial h_p(0)}{\partial p}$ must be negative when p is sufficiently large. Therefore, for sufficiently large p, the second term in (25) is negative and so

$$-\phi'(p) \leq \frac{G(p|p)f(p)}{h_p(0)} \leq 1 \ ,$$

where the second inequality uses the expression for $h_p(0)$. This leads to $-\phi'(\overline{v}) \leq 1$ and so $p'(\overline{v}) \geq \frac{1}{2}$. □

Proof of Lemma 5 . Notice that

$$\frac{\phi(p)f(p)}{1 - F(p)} = \frac{\frac{\int_p^{\overline{v}} G(v|v)dF(v)}{1-F(p)}}{G(p|p) + \frac{\int_p^{\overline{v}} g(v|v)dF(v)}{f(p)}} \ .$$

Concerning the numerator, we have

$$\lim_{p \to \overline{v}} \frac{\int_p^{\overline{v}} G(v|v)dF(v)}{1 - F(p)} = \lim_{p \to \overline{v}} \frac{G(p|p)f(p)}{f(p)} = 1 \ ,$$

where the first equality is from L'hôpital's rule. Concerning the denominator, it obviously converges to 1 if $f(\overline{v}) > 0$. If $f(\overline{v}) = 0$ but we focus on the IID case, $\int_p^{\overline{v}} g(v)dF(v) = -G(p)f(p) - \int_p^{\overline{v}} G(v)f'(v)dv$ by integration by parts. The denominator then becomes $-\int_p^{\overline{v}} G(v)f'(v)dv/f(p)$, which converges to 1 as well by L'hôpital's rule. $\square$

Proof of Lemma 6. Given $f(v)$ is log-concave, we can rewrite it as $f(v) = e^{\mu(v)}$ with $\mu(v)$ being a concave function. (Given $f(\overline{v}) = 0$, $\mu(v)$ must be decreasing as well when v is sufficiently large and $\lim_{v\to\overline{v}} \mu(v) = -\infty$.) Then $f(c)/f(p) = e^{\mu(c)-\mu(p)}$, and so it suffices to show that

$$2p'(\overline{v}) - 1 \le \lim_{c\to\overline{v}} [\mu(c) - \mu(p)] \le 2 - \frac{1}{p'(\overline{v})} \ .$$

First, given $\mu(\cdot)$ is concave, we have

$$\mu(c) - \mu(p) \le -(p-c)\mu'(p) \ . \tag{28}$$

From the equilibrium price condition in the IID case, we have

$$\frac{(p-c)[F(p)^{n-1}f(p) + \int_p^{\overline{v}} f(v)dF(v)^{n-1}]}{[1-F(p)^n]/n} = 1$$

for any c. In the limit of $c \to \overline{v}$, both the numerator and denominator must go to zero. Therefore, L'hôpital's rule implies that

$$\lim_{c\to\overline{v}} \frac{(p-c)F(p)^{n-1}f'(p)p'(c) + [p'(c)-1][F(p)^{n-1}f(p) + \int_p^{\overline{v}} f(v)dF(v)^{n-1}]}{-F(p)^{n-1}f(p)p'(c)} = 1 \ ,$$

or

$$\lim_{c\to\overline{v}} -(p-c)\frac{f'(p)}{f(p)} + \lim_{c\to\overline{v}} \frac{1-p'(c)}{p'(c)}[1 + \frac{\int_p^{\overline{v}} f(v)dF(v)^{n-1}}{F(p)^{n-1}f(p)}] = 1 \ . \tag{29}$$

Notice that

$$\begin{aligned}\lim_{p\to\overline{v}} \frac{\int_p^{\overline{v}} f(v)dF(v)^{n-1}}{F(p)^{n-1}f(p)} &= \lim_{p\to\overline{v}} \frac{-(n-1)F(p)^{n-2}f(p)^2}{(n-1)F(p)^{n-2}f(p)^2 + F(p)^{n-1}f'(p)} \\ &= \lim_{p\to\overline{v}} \frac{-(n-1)f(p)}{(n-1)f(p) + F(p)f'(p)/f(p)} \\ &= 0 \ ,\end{aligned}$$

where the first equality is from L'hôpital's rule and the last equality is from $f(\overline{v}) = 0$ and the fact that under our conditions, $f'(p)/f(p)$ must be negative and bounded away from zero when p is sufficiently close to $\overline{v}$. (Given $f(\overline{v}) = 0$, $f'/f < 0$ for sufficiently large p; given$f$ is log-concave, f'/f is decreasing.) Then,

$$\lim_{c\to\overline{v}} -(p-c)\frac{f'(p)}{f(p)} = \lim_{c\to\overline{v}} -(p-c)\mu'(p) = 1 - \frac{1-p'(\overline{v})}{p'(\overline{v})} = 2 - \frac{1}{p'(\overline{v})} \ , \tag{30}$$

where the first equality used $f(p) = e^{\mu(p)}$ and the second is from (29). Together with (28), this implies $\lim_{c\to\overline{v}}[\mu(c) - \mu(p)] \leq 2 - \frac{1}{p'(\overline{v})}$.

Second, given $\mu(\cdot)$ is concave, we also have

$$\mu(c) - \mu(p) \geq -(p - c)\mu'(c) \ . \tag{31}$$

Notice that

$$\lim_{c\to\overline{v}} -(p - c)\mu'(c) = \lim_{c\to\overline{v}} -(p - c)\frac{f'(c)}{f(c)} = \lim_{c\to\overline{v}} -(p - c)\frac{f'(p)}{f(p)}\frac{f(p)}{f'(p)}\frac{f'(c)}{f(c)} \ ,$$

and

$$\lim_{c\to\overline{v}} \frac{f(p)}{f'(p)}\frac{f'(c)}{f(c)} = \lim_{c\to\overline{v}} \frac{f(p)}{f(c)} \times \lim_{c\to\overline{v}} \frac{f'(c)}{f'(p)} = \lim_{c\to\overline{v}} \frac{f(p)}{f(c)} \times p'(\overline{v}) \lim_{c\to\overline{v}} \frac{f(c)}{f(p)} = p'(\overline{v}) \ ,$$

where the manipulation in the first step is legitimate because $f(p)/f(c) < 1$ given $f(v)$ is decreasing at least for large v and $\lim_{c\to\overline{v}} \frac{f'(c)}{f'(p)} = p'(\overline{v}) \lim_{c\to\overline{v}} \frac{f(c)}{f(p)}$ is also finite as shown in the first step. Together with (30), these results imply that

$$\lim_{c\to\overline{v}} -(p - c)\mu'(c) = 2p'(\overline{v}) - 1 \ .$$

This, together with (31), proves $\lim_{c\to\overline{v}}[\mu(c) - \mu(p)] \geq 2p'(\overline{v}) - 1$. □

Proof of Lemma 7. Notice that $F_{(n)}(v)$, the CDF of the valuation for the best product, can also be written as

$$F_{(n)}(v) = \int_{[\underline{v},v]^n} \tilde{f}(\mathbf{v})d\mathbf{v} \ .$$

Then

$$f_{(n)}(v) = n\int_{[\underline{v},v]^{n-1}} \tilde{f}(v, \mathbf{v}_{-i})d\mathbf{v}_{-i} \ ,$$

where we have used the exchangeability of $\tilde{f}$. Therefore, $f_{(n)}(\overline{v}) = nf(\overline{v})$ by using the definition of the marginal density. □

A.3 Proof of Proposition 4

For notational simplicity, we normalize $\underline{v}$ to 0 and consider $F(v) = 1 - (1 - av)^{\frac{1}{a}}$ on $[0, \frac{1}{a}]$, where $a \in (0, 1]$. (Full market coverage under uniform pricing, i.e., $F(p) = 0$, can then be achieved as long as we allow c to be negative.) This distribution converges to the exponential as $a \to 0$ and becomes the uniform at $a = 1$. Notice that $f(v) = (1-av)^{\frac{1}{a}-1} = [1 - F(v)]^{1-a}$ is log-concave.

Let us first deal with the uniform price. Using the above relationship between f and $1-F$, we can rewrite the denominator of (9) as a function of $F(p)$:

$$
\begin{aligned}
&F(p)^{n-1}f(p)+(n-1)\int_p^{\frac{1}{a}}F(v)^{n-2}f(v)dF(v)\\
&=F(p)^{n-1}[1-F(p)]^{1-a}+(n-1)\int_p^{\frac{1}{a}}F(v)^{n-2}[1-F(v)]^{1-a}dF(v)\\
&=F(p)^{n-1}[1-F(p)]^{1-a}+(n-1)\int_{F(p)}^{1}x^{n-2}(1-x)^{1-a}dx\ .
\end{aligned}
$$

Unless $n=2$ or $a=1$, this does not have an elementary analytical expression. In the following, we focus on $n=2$. Then one can check that the above expression simplifies to

$$\frac{1}{2-a}[1-F(p)]^{1-a}[1+(1-a)F(p)]\ ,$$

and the uniform price in the duopoly case solves

$$p-c=(1-\frac{a}{2})[1-F(p)]^a\frac{1+F(p)}{1+(1-a)F(p)}=(1-\frac{a}{2})\frac{[1-F(p)]^a}{1-a+\frac{a}{1+F(p)}}\ .$$

(Note that this pricing formula also works for $a=0$, in which case $p=1+c$.) It does not have an explicit solution unless $a=1$. In the following, we will heavily use the following notation:

$$y\equiv 1-F(p)=(1-ap)^{\frac{1}{a}}\ , \tag{32}$$

and

$$z\equiv\frac{1-\frac{a}{2}}{\frac{1-a}{a}+\frac{1}{2-y}}\ . \tag{33}$$

Note that $y\in[0,1]$ as we allow the cost condition to cover from full to zero market coverage, and z is decreasing in $y\in[0,1]$ and lies in the range of $[a(1-\frac{a}{2}),a]$. Then we can write the uniform pricing formula as

$$p-c=\frac{y^a}{a}z\ . \tag{34}$$

From (32) we solve $p=(1-y^a)/a$. Substituting this into (34), we derive

$$1-ac=y^a(1+z)\ . \tag{35}$$

Consumer surplus. Consumer surplus in the duopoly case is respectively

$$V_U=\int_p^{\frac{1}{a}}[1-F(v)^2]dv=\int_p^{\frac{1}{a}}[2(1-av)^{\frac{1}{a}}-(1-av)^{\frac{2}{a}}]dv=(1-ap)^{\frac{1}{a}+1}\left[\frac{2}{1+a}-\frac{(1-ap)^{\frac{1}{a}}}{2+a}\right]\ ,$$

and

$$V_D = \int_c^{\frac{1}{a}} [1-F(v)]^2 dv = \int_c^{\frac{1}{a}} (1-av)^{\frac{2}{a}} dv = \frac{(1-ac)^{\frac{2}{a}+1}}{2+a} \ .$$

Using the notation y and (35), one can rewrite $V_D = V_U$ as

$$\frac{y^{2+a}}{2+a}(1+z)^{\frac{2}{a}+1} = y^{1+a}\left(\frac{2}{1+a} - \frac{y}{2+a}\right) \ ,$$

which simplifies to

$$(1+z)^{\frac{2}{a}+1} = \frac{2(2+a)}{1+a}\frac{1}{y} - 1 \ , \tag{36}$$

or

$$y(z) = \frac{2(2+a)}{1+a}\frac{1}{1+(1+z)^{\frac{2}{a}+1}} \tag{37}$$

where

$$y(z) = 2 - \frac{1}{\frac{1-a/2}{z} - \frac{1-a}{a}} \tag{38}$$

is solved from (33). We now show that (37) has a unique solution $z \in (a(1-\frac{a}{2}), a)$.

We first show that the solution exists. When $z = a(1-\frac{a}{2})$, $y(z) = 1$ is strictly greater than the righthand side of (37). To see that, notice that

$$\left[1 + a\left(1 - \frac{a}{2}\right)\right]^{\frac{2}{a}+1} > 1 + a\left(1-\frac{a}{2}\right)\left(\frac{2}{a}+1\right) = 3 - \frac{a^2}{2} \ ,$$

where the inequality used the fact that $(1+x)^k > 1+kx$ for any $x > 0$ and $k > 1$ since $(1+x)^k$ is convex in x. Then the righthand side of (37) is less than

$$\frac{2(2+a)}{(1+a)(4-a^2/2)} < 1 \ ,$$

where the inequality is equivalent to $4 > a + a^2$ and this must be true for $a \in (0,1]$. On the other hand, when $z = a$, $y(z) = 0$ must be less than the righthand side of (37) since the latter must be strictly positive.

For the uniqueness, we further show that $y(z)$ is concave while the righthand side of (37) is convex in z. One can check that

$$y''(z) = \frac{(a-2)\frac{1-a}{a}}{\left(1-\frac{a}{2}-\frac{1-a}{a}z\right)^3} \leq 0$$

since $1 - \frac{a}{2} - \frac{1-a}{a}z > 0$ given $z \leq a$, so $y(z)$ is concave. On the other hand, one can show that $1/[1+(1+z)^k]$ is strictly convex in z if and only if $k - 1 < (k+1)(1+z)^k$. This is obviously true given $z > 0$ and $k = \frac{2}{a} + 1 > 0$. Therefore, the righthand side of (37) is strictly convex.

Profit. Using the notation y and z, we can derive industry profit in the duopoly case:

$$\Pi_U = (p-c)[1-F(p)^2] = \frac{z}{a}(2-y)y^{a+1}$$

where we used (34) and $1-F(p)^2 = y(2-y)$, and

$$\begin{aligned}\Pi_D = \int_c^{\frac{1}{a}} \frac{1-F(v)}{f(v)} dF(v)^2 &= 2\int_c^{\frac{1}{a}} [1-F(v)]^a F(v) dF(v) \\ &= \frac{2}{a+1}(1-ac)^{1+\frac{1}{a}} - \frac{2}{a+2}(1-ac)^{1+\frac{2}{a}} \\ &= y^{a+1}\left[\frac{2}{a+1}(1+z)^{\frac{1}{a}+1} - \frac{2y}{a+2}(1+z)^{\frac{2}{a}+1}\right]\end{aligned}$$

where the second equality used $f(v) = [1-F(v)]^{1-a}$ and the last equality used (35). Then one can simplify $\Pi_U = \Pi_D$ to

$$\frac{\frac{a}{a+1}(1+z)^{\frac{1}{a}+1} - z}{\frac{a}{a+2}(1+z)^{\frac{2}{a}+1} - \frac{z}{2}} = y(z) \ , \tag{39}$$

where $y(z)$ is defined in (38).

We first show that (39) must have a solution $z \in (a(1-\frac{a}{2}), a)$. Recall that $y(z)$ decreases in z from 1 to 0 as z varies in its range. On the other hand, the lefthand side of (39) is strictly positive, because

$$\frac{a}{a+1}(1+z)^{\frac{1}{a}+1} - z > \frac{a}{a+1}(1+z(\frac{1}{a}+1)) - z = \frac{a}{a+1} > 0$$

and

$$\frac{a}{a+2}(1+z)^{\frac{2}{a}+1} - \frac{z}{2} > \frac{a}{a+2}(1+z(\frac{2}{a}+1)) - \frac{z}{2} = \frac{a}{a+2} + \frac{z}{2} > 0$$

where the first inequality in each line used the fact that $(1+x)^k > 1+kx$ for any $x > 0$ and $k > 1$. Meanwhile, the lefthand side of (39) is also strictly less than 1. Given $z > 0$, a sufficient condition for that is

$$\frac{a}{a+1}(1+z)^{\frac{1}{a}+1} \leq \frac{a}{a+2}(1+z)^{\frac{2}{a}+1} \Leftrightarrow \frac{a+2}{a+1} \leq (1+z)^{\frac{1}{a}} \ ,$$

and this must be true since

$$(1+z)^{\frac{1}{a}} \geq 1 + \frac{z}{a} \geq 2 - \frac{a}{2} \geq \frac{a+2}{a+1}$$

where the last inequality is equivalent to $a \geq a^2$. Therefore, we deduce that the lefthand side of (39) is strictly less than the righthand side at $z = a(1-\frac{a}{2})$, whereas the opposite is true at $z = a$.

To prove the uniqueness, we show that the derivative of the lefthand side of (39) is everywhere greater than $y'(z)$. The derivative of the lefthand side is

$$\frac{(1+z)^{\frac{1}{a}}-1}{\frac{a}{a+2}(1+z)^{\frac{2}{a}+1}-\frac{z}{2}}-\frac{[\frac{a}{a+1}(1+z)^{\frac{1}{a}+1}-z][(1+z)^{\frac{2}{a}}-\frac{1}{2}]}{[\frac{a}{a+2}(1+z)^{\frac{2}{a}+1}-\frac{z}{2}]^2}.$$

Since the lefthand side of (39) is less than 1, this implies that the above derivative is strictly greater than

$$\begin{aligned}\frac{(1+z)^{\frac{1}{a}}-1}{\frac{a}{a+2}(1+z)^{\frac{2}{a}+1}-\frac{z}{2}}-\frac{(1+z)^{\frac{2}{a}}-\frac{1}{2}}{\frac{a}{a+2}(1+z)^{\frac{2}{a}+1}-\frac{z}{2}} &= \frac{(1+z)^{\frac{1}{a}}-(1+z)^{\frac{2}{a}}-\frac{1}{2}}{\frac{a}{a+2}(1+z)^{\frac{2}{a}+1}-\frac{z}{2}} \\ &\geq \frac{\frac{1}{2}+\frac{z}{a}-(1+z)^{\frac{2}{a}}}{\frac{a}{a+2}(1+z)^{\frac{2}{a}+1}-\frac{z}{2}} \\ &> \frac{\frac{z}{a}-(1+z)^{\frac{2}{a}}}{\frac{a}{a+2}(1+z)^{\frac{2}{a}+1}-\frac{z}{2}}\end{aligned}$$

where the first inequality is because $(1+z)^{\frac{1}{a}} \geq 1+\frac{z}{a}$. On the other hand,

$$y'(z)=-\frac{1-\frac{a}{2}}{\left(1-\frac{a}{2}-\frac{1-a}{a}z\right)^2} \leq -\frac{1-\frac{a}{2}}{\left(1-\frac{a}{2}-\frac{1-a}{a}a(1-\frac{a}{2})\right)^2}=-\frac{1}{a^2(1-\frac{a}{2})}$$

since $z \geq a(1-\frac{a}{2})$ and $1-\frac{a}{2}-\frac{1-a}{a}z$ is positive in the range of z. Therefore, it suffices to show that

$$\frac{(1+z)^{\frac{2}{a}}-\frac{z}{a}}{\frac{a}{a+2}(1+z)^{\frac{2}{a}+1}-\frac{z}{2}} \leq \frac{1}{a^2(1-\frac{a}{2})} \ .$$

Since $a(1-\frac{a}{2}) \leq \frac{1}{2}$, the above inequality holds if

$$\frac{a(1+z)^{\frac{2}{a}}-z}{\frac{2a}{a+2}(1+z)^{\frac{2}{a}+1}-z} \leq 1 \Leftrightarrow (1+z)^{\frac{2}{a}} \leq \frac{2}{a+2}(1+z)^{\frac{2}{a}+1} \Leftrightarrow z \geq \frac{a}{2} \ ,$$

which must be true given $z \geq a(1-\frac{a}{2})$ and $a \leq 1$.

A.4 Proof of Proposition 5

If the market becomes fully covered under uniform pricing when n exceeds a threshold (which can happen if $c < \underline{v}$), then our result is obvious. In the following, we focus on the case where the market is not fully covered for any n.

In the IID case industry profit under uniform pricing can be written as

$$\Pi_U = \frac{[1-F(p)^n]^2/n}{F(p)^{n-1}f(p)+\int_p^{\overline{v}} f(v)dF(v)^{n-1}} \ .$$

Under the log-concavity condition, the uniform price p is decreasing in n, and so $F(p)^n$ must be of order $o(\frac{1}{n})$, i.e., $\lim_{n\to\infty}\frac{F(p)^n}{1/n}=0$. Meanwhile, Theorem 1 in Gabaix et al. (2016), which approximates the Perloff-Salop price, shows that as $n\to\infty$,

$$\int_{\underline{v}}^{\overline{v}} f(v)dF(v)^{n-1}\sim f(F^{-1}(1-\frac{1}{n}))\cdot\Gamma(2+\gamma)\ ,$$

where $\Gamma(\cdot)$ is the Gamma function. (Notice that $\Gamma(x)$ decreases first and then increases in $x\in[1,2]$, and it is strictly positive but no greater than 1 in that range, and $\Gamma(1)=\Gamma(2)=1$.) At the same time, notice that $\int_{\underline{v}}^{p} f(v)dF(v)^{n-1}<F(p)^{n-1}\times\max_{v\in[\underline{v},p]}f(v)$, so it must be of order $o(\frac{1}{n})$ given f is finite. Therefore, as $n\to\infty$, we have

$$\Pi_U\sim\frac{[1-o(\frac{1}{n})]^2}{o(\frac{1}{n})/(\frac{1}{n})+nf(F^{-1}(1-\frac{1}{n}))\cdot\Gamma(2+\gamma)}\ .$$

Since the price is decreasing in n, Π_U must be finite for any n. This implies that $\lim_{n\to\infty}nf(F^{-1}(1-\frac{1}{n}))>0$. Therefore, when n is large, those $o(\frac{1}{n})$ terms can be safely ignored. This yields

$$\Pi_U\sim\frac{1}{nf(F^{-1}(1-\frac{1}{n}))\cdot\Gamma(2+\gamma)}\ . \tag{40}$$

Industry profit under personalized pricing is

$$\Pi_D=\int_c^{\overline{v}}\frac{1-F(v)}{f(v)}dF(v)^n=\int_{F(c)}^1\frac{1-t}{f(F^{-1}(t))}dt^n\ .$$

Proposition 2 in Gabaix et al. (2016) has shown that, as $n\to\infty$,

$$\mathbb{E}[v_{n:n}-v_{n-1:n}]=\int_0^1\frac{1-t}{f(F^{-1}(t))}dt^n\sim\frac{\Gamma(1-\gamma)}{nf(F^{-1}(1-\frac{1}{n}))}\ .$$

Notice that

$$\Pi_D=\mathbb{E}[v_{n:n}-v_{n-1:n}]-\int_0^{F(c)}\frac{1-t}{f(F^{-1}(t))}dt^n\ .$$

The second term equals $\frac{1-\tilde{t}}{f(F^{-1}(\tilde{t}))}F(c)^n$ for some $\tilde{t}\in(0,F(c))$, and so is of order $o(\frac{1}{n})$ and can be safely ignored when n is large. Therefore, for large n, we have

$$\Pi_D\sim\frac{\Gamma(1-\gamma)}{nf(F^{-1}(1-\frac{1}{n}))}\ . \tag{41}$$

Comparing (40) and (41), we can claim that when n is sufficiently large, personalized pricing reduces profit (and so improves consumer surplus) if

$$\Gamma(1-\gamma)\Gamma(2+\gamma)<1\ ,$$

which is true when $\gamma\in(-1,0)$. (Notice that the equality holds when $\gamma=-1$ or 0. In these cases, unfortunately, the approximations are not precise enough to generate meaningful comparison results in the limit.)

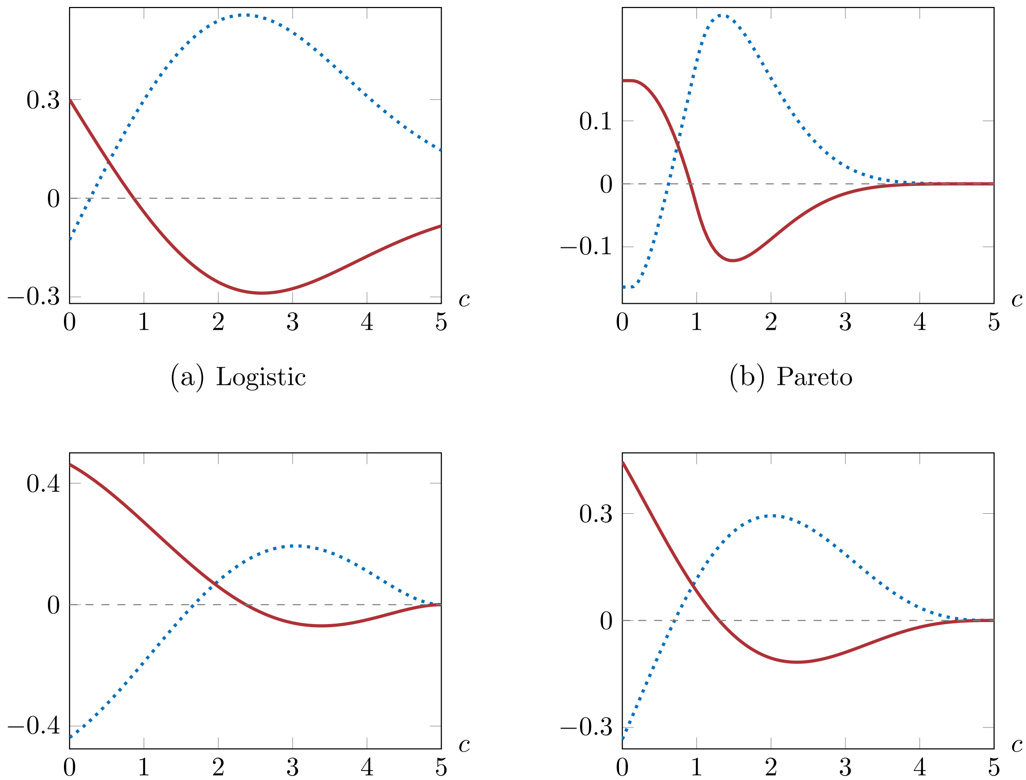
A.5 Further numerical simulations

In this section we report more numerical examples to show the robustness of the cut-off results reported in Sections 3.2.2 and 3.2.3.

The impact of personalized pricing as c changes: duopoly with other distributions. Figure 7 below plots the impact of personalized pricing on profit and consumer surplus for different values of c when $n = 2$, and product valuations are IID from four new distributions (Logistic, Pareto, Power, and Beta). In the case of the Logistic distribution, we use $F(v) = \frac{1}{1+e^{3-v}}$, which has support $(-\infty, \infty)$. In the case of the Pareto distribution, we use $F(v) = 1 - \left[1 - \frac{1}{4}(v-1)\right]^4$, which has a decreasing density on its support $[1, 5]$. In the case of the Power distribution, we use $F(v) = \left(\frac{v}{5}\right)^2$, which has an increasing density on its support $[0, 5]$. In the case of the Beta distribution, we use $F(v) = \frac{6}{125}\left(\frac{5}{2}v^2 - \frac{1}{3}v^3\right)$, which has a symmetric hump-shaped density on its support $[0, 5]$. (Note that in the latter three cases $\overline{v} = 5$, which is the same as in the uniform example in the main text, and so as c approaches 5 the effect of personalized pricing on both profit and consumer surplus equals zero.) Meanwhile Figure 8 below plots the impact of personalized pricing on profit and consumer surplus for different values of c when product valuations are bivariate Normal with mean 2, variance 1, and the correlation coefficient is $\rho = \{-3/4, -1/4, 1/4, 3/4\}$. The impact of personalized pricing in each of these examples is qualitatively the same as in the examples reported in the main text.

The impact of personalized pricing as c changes: beyond duopoly. Figures 9 and 10 below examine the impact of personalized pricing for different values of c when $n = 3$ and $n = 4$ respectively, using the four distributions used in the main text (Exponential, Extreme value, Normal, and Uniform) as well as the four new distributions used in Figure 7, all for the IID case. The impact of personalized pricing is qualitatively the same as in the duopoly examples.

The impact of personalized pricing as n changes: other distributions. Finally, Figure 11 below reports the impact of personalized pricing for different values of n for the four new distributions in the IID case. It is qualitatively the same as in the previous four examples in the main text.



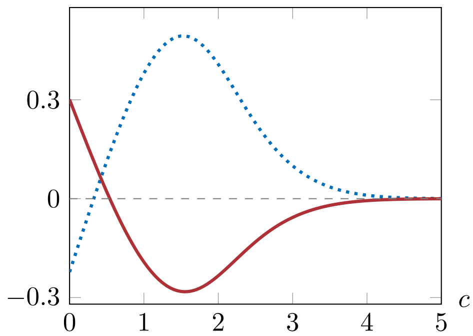
(a) Logistic

(b) Pareto

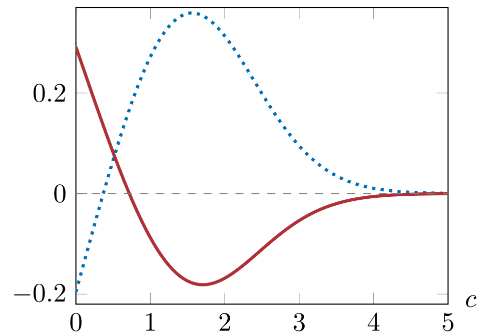
(c) Power

(d) Beta

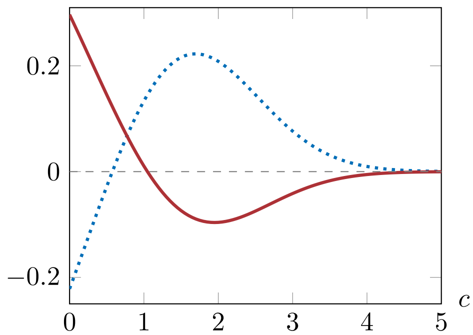
Figure 7: The impact of personalized pricing when $n = 2$, for different values of c
(The dotted and solid lines represent, respectively, the change in industry profit and consumer surplus.)



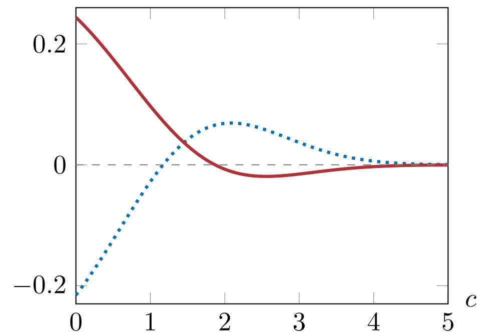
(a) $\rho = -3/4$



(b) $\rho = -1/4$



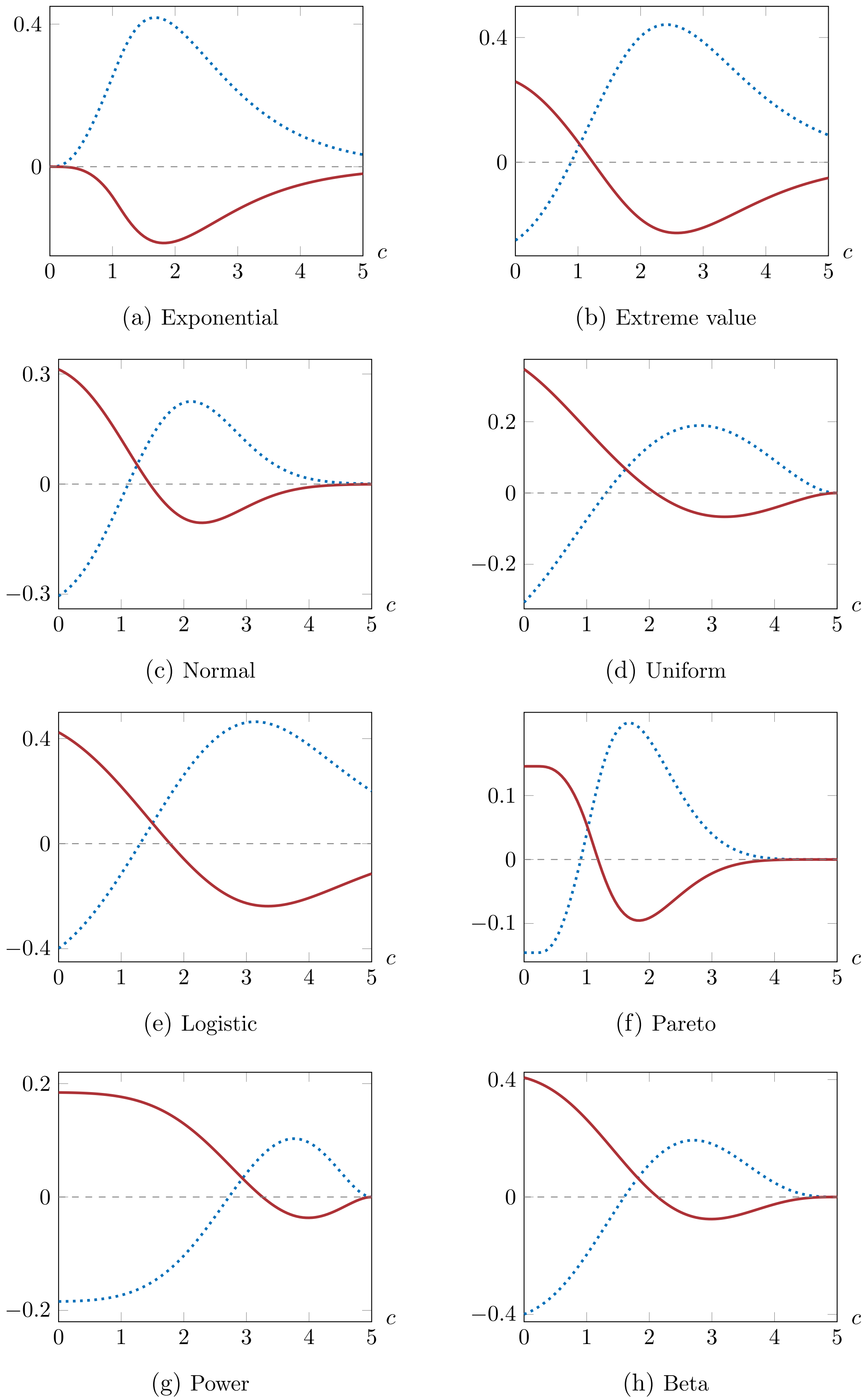
(c) $\rho = 1/4$



(d) $\rho = 3/4$

Figure 8: The impact of personalized pricing when valuations are bivariate Normal, for different values of c

(The dotted and solid lines represent, respectively, the change in industry profit and consumer surplus.)



(a) Exponential (b) Extreme value

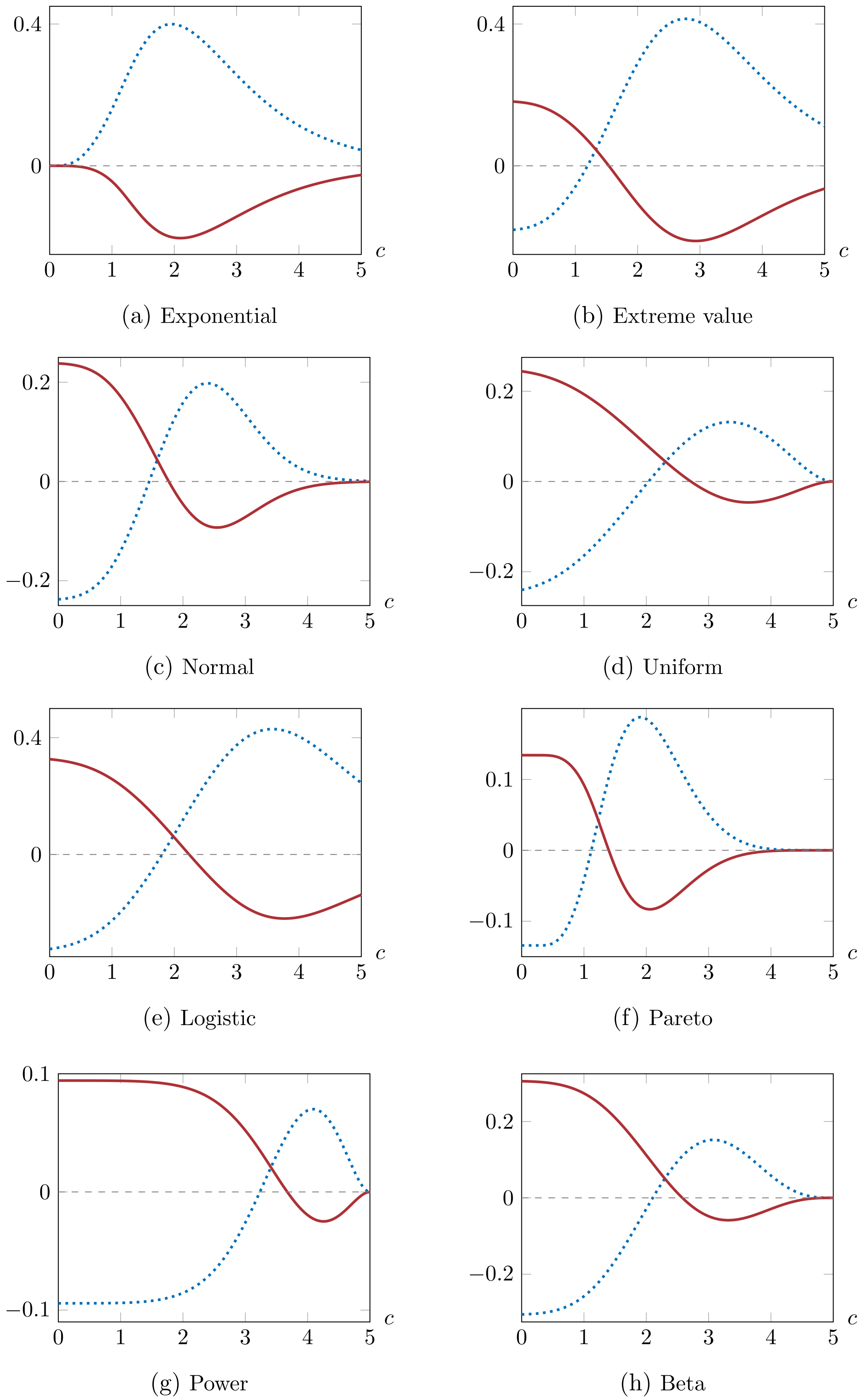
(c) Normal (d) Uniform

(e) Logistic (f) Pareto

(g) Power (h) Beta

Figure 9: The impact of personalized pricing when $n = 3$, for different values of c

(The dotted and solid lines represent, respectively, the change in industry profit and consumer surplus.)



(a) Exponential (b) Extreme value (c) Normal (d) Uniform (e) Logistic (f) Pareto (g) Power (h) Beta

Figure 10: The impact of personalized pricing when $n = 4$, for different values of c

(The dotted and solid lines represent, respectively, the change in industry profit and consumer surplus.)

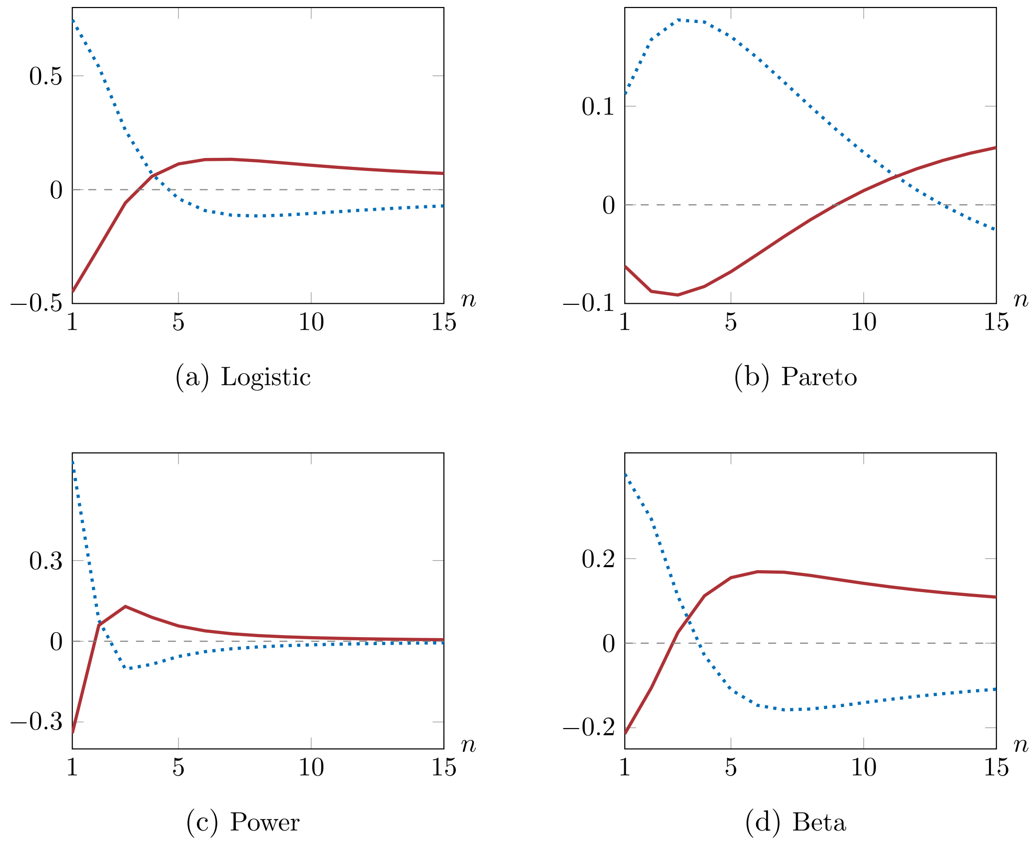


Figure 11: The impact of personalized pricing when $c = 2$, for different values of n
(The dotted and solid lines represent, respectively, the change in industry profit and consumer surplus.)

A.6 Details on valuation correlation and dispersion

Here we report the details of the impact of product differentiation discussed in Section 3.3.

A.6.1 Valuation correlation

One way to capture product differentiation is the degree of valuation correlation across products when the marginal distribution is fixed. Intuitively, products become less differentiated when their valuations become more positively correlated, which should lessen the impact of personalized pricing as price competition in either regime becomes fiercer. A general investigation into this issue is, however, challenging. Here we consider a duopoly example, where valuations are drawn from a bivariate Normal distribution with mean μ, variance σ, and correlation coefficient $\rho \in (-1, 1)$.

First, suppose the market is fully covered.[35] Here the effect of personalized pricing can be derived analytically. Specifically, note that $v_1 - v_2$ is normally distributed with mean 0, variance $\tau^2 \equiv 2\sigma^2(1-\rho)$, and density function $h(x) = \frac{1}{\sqrt{2\pi}\tau} e^{-\frac{x^2}{2\tau^2}}$. Hence profit under uniform pricing is

$$\Pi_U = \frac{1}{2h(0)} = \sigma\sqrt{\pi(1-\rho)},$$

while profit under discriminatory pricing is

$$\Pi_D = 2\int_0^\infty xh(x)dx = -2\tau^2\int_0^\infty h'(x)dx = 2\tau^2 h(0) = 2\sigma\sqrt{\frac{1-\rho}{\pi}},$$

where the second equality uses the fact that $xh(x) = -\tau^2 h'(x)$. It then follows that $\Pi_D - \Pi_U \propto -\sigma\sqrt{1-\rho} < 0$. Therefore, in this example, as product valuations become more correlated (i.e., as ρ increases), profit falls under both uniform and discriminatory pricing, and the impact of personalized pricing on profit also becomes smaller. (Given the assumption of a covered market, the impact on consumer surplus also gets smaller.) In the limit case of perfect positive correlation (i.e., as $\rho \to 1$), personalized pricing has no impact on profit or consumer surplus because firms' products become homogeneous.

Second, suppose the market is not fully covered. In this case only numerical progress can be made. Figure 12 shows how the impact of personalized pricing varies with ρ, for the case where $\mu = 2$, $\sigma = 1$, and $c = 1$. The impact of ρ is subtler than in the full-coverage case, given that it simultaneously affects both market coverage for given prices as well as the equilibrium prices themselves in the two regimes. However, heuristically, when ρ is very low, consumer preferences for the two goods tend to be quite polarized; relatively

[35]Strictly speaking, full market coverage is impossible because the lower support of the valuation distribution is unbounded. However the market is almost covered if μ is large enough for a given c.

many consumers are in the "monopoly" and "pay more" regions, so personalized pricing tends to harm consumers and benefit firms. Instead, when ρ is high, consumer preferences for the two goods tend to be more aligned; relatively many consumers are in the "pay less" region, so personalized pricing can now benefit consumers and harm firms. (Given the impact changes sign, it cannot be monontonic in ρ.) Finally, in the limit as $\rho \to 1$, prices under both regimes tend to marginal cost, so the impact of personalized pricing on consumers and firms vanishes.

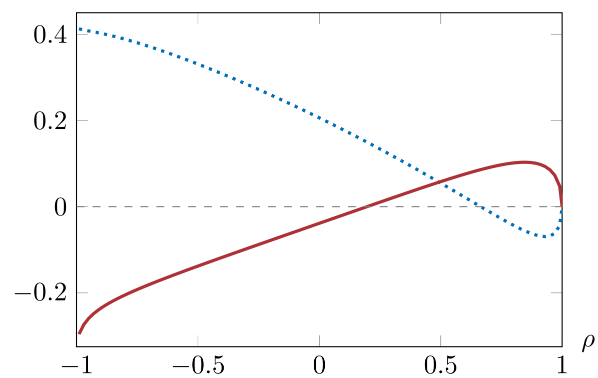


Figure 12: The impact of personalized pricing when valuations are bivariate Normal, for different values of ρ

(The dotted and sold lines represent, respectively, the change in industry profit and consumer surplus.)

A.6.2 Valuation dispersion

Another possible way to capture product differentiation is the dispersion of the (marginal) valuation distribution when the correlation structure is fixed. In the previous example with the Normal distribution and full coverage, it is clear that a higher variance leads to higher profit in both regimes and a greater impact of personalized pricing. If we keep the assumption of full market coverage, the same insight also applies when dispersion/product differentiation is captured by scaling the match utility variable in the form of $\tilde{v}_i = \mu + \theta(v_i - \mu)$, where μ is the mean of v_i and $\theta > 0$ is a constant. (When θ increases, $\tilde{v}_i$ becomes more dispersed though its mean remains unchanged.) Then $\tilde{F}(v) = F(\frac{v-\mu}{\theta} + \mu)$ and $\tilde{G}(v|v) = G(\frac{v-\mu}{\theta} + \mu|\frac{v-\mu}{\theta} + \mu)$. Under uniform pricing, it is easy to verify from (8) that $\tilde{p} - c = \theta(p - c)$ by changing the integral variable. Since the market is fully covered, this implies that $\tilde{\Pi}_U = \theta\Pi_U$ and also

$$\tilde{V}_U = \mathbb{E}[\tilde{v}_{n:n}] - \tilde{p} = \mu + \theta(\mathbb{E}[v_{n:n}] - \mu) - \theta p + (\theta - 1)c = \theta V_U + (1 - \theta)(\mu - c) \ .$$

Under personalized pricing and full coverage, we have $\tilde{p}(v_i, \mathbf{v}_{-i}) - c = \theta(p(v_i, \mathbf{v}_{-i}) - c)$ whenever firm i wins a consumer. Then $\tilde{\Pi}_D = \theta \Pi_D$ and

$$\tilde{V}_D = \mathbb{E}[\tilde{v}_{n-1:n}] - c = \mu + \theta(\mathbb{E}[v_{n-1:n}] - \mu) - c = \theta V_D + (1-\theta)(\mu - c) \ .$$

Therefore the impact on both profit $\tilde{\Pi}_D - \tilde{\Pi}_U = \theta(\Pi_D - \Pi_U)$ and consumer surplus $\tilde{V}_D - \tilde{V}_U = \theta(V_D - V_U)$ increases in θ.

We can also use the Normal example to investigate the impact of valuation dispersion when the market is not fully covered. In particular, Figure 13 shows how the impact of personalized pricing varies with σ, for the case where $\mu = 2$, $\rho = 0$, and $c = 1$. Again, the impact of σ is subtle, affecting both market coverage for given prices as well as the equilibrium prices themselves. However, heuristically, when σ is relatively large, there are relatively more consumers with very low valuations; coverage is low, so personalized pricing harms consumers and benefits firms. Instead, when σ is low, consumers tend to have similar (above-cost) product valuations; coverage is high, so personalized pricing now benefits consumers and harms firms. (Again, given the impact changes sign, it cannot be monotonic in σ.) Finally, in the limit as $\sigma \to 0$, products become homogeneous so the impact of personalized pricing disappears.

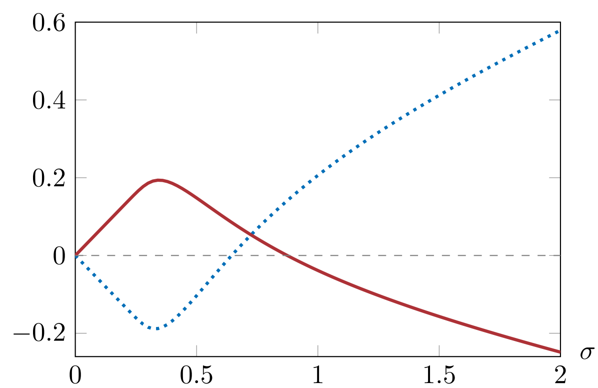


Figure 13: The impact of personalized pricing when valuations are IID Normal, for different values of σ

(The dotted and sold lines represent, respectively, the change in industry profit and consumer surplus.)

We also report an analytical example where the market is not fully covered. Consider the IID case with an exponential distribution $F(v) = 1 - e^{-\frac{1}{\lambda}(v - \underline{v})}$. Here the mean is $\underline{v} + \lambda$ and the variance is λ^2. If we let $\underline{v} = a - \lambda$, then a higher λ increases the dispersion but does not change the mean. One can check that the uniform price is $p = c + \lambda$ (which is independent of the way we set up the valuation lower bound). The market is then partially covered under uniform pricing if $c + \lambda > \underline{v} = a - \lambda$ or $c > a - 2\lambda$. (As expected, partial coverage becomes more likely as the valuation becomes more dispersed.) Industry profit under uniform pricing is $\Pi_U = \lambda[1 - F(c+\lambda)^n]$, and that under personalized pricing is $\Pi_D = \lambda[1 - F(c)^n]$. It is clear that the impact on profit is $\Pi_D - \Pi_U = \lambda[F(c+\lambda)^n - F(c)^n] > 0$, which increases in λ. The impact on consumer surplus is $V_D - V_U = \int_c^{c+\lambda}(v-c)dF(v)^n - (\Pi_D - \Pi_U) < 0$, where the first term is the impact on total welfare when moving from uniform to personalized pricing. One can check that $V_D - V_U$ decreases in λ, i.e., the magnitude of the impact of personalized pricing on consumer surplus is also greater when λ is higher.

A.7 Details on alternative information structures

Here we report the details of the discussion on alternative information structures in Section 3.3. Consider the case where firms observe only a consumer's valuation for *their own* product, i.e., each firm i observes only v_i. For convenience, we refer to this case as "partial" discrimination, and the case where each firm observes a consumer's valuations for all products as "full" discrimination. We focus on the case with symmetrically informed firms.

Under partial discrimination a firm offers a price $p(v)$ to a consumer who has valuation v for its product. It turns out that $p(v)$ can be derived in the same way as a bid in a standard first-price auction. This is because we can interpret a firm as "bidding" surplus of $v - p(v)$ to a consumer who has valuation v for its product, and then the consumer picking the best (non-negative) "bid."

Formally, recall that $G(\cdot|v_i)$ and $g(\cdot|v_i)$ are respectively the CDF and density function of $\max_{j\neq i}\{v_j\}$ conditional on v_i, and define

$$b(z) \equiv \int_c^z [1 - L(x|z)]dx \quad \text{with} \quad L(x|z) = \exp\left(-\int_x^z \frac{g(t|t)}{G(t|t)}dt\right). \tag{42}$$

(Here $b(\cdot)$ is the equilibrium bidding function in a standard first-price auction with interdependent values; see, e.g., Milgrom and Weber, 1982.) We impose the following regularity condition to ensure that $b(v)$ is monotonically increasing:

Assumption 2. *For all $z \in [\underline{v}, \overline{v}]$, $[v - b(z) - c]\frac{g(z|v)}{G(z|v)}$ increases in v whenever it is positive.*

This assumption holds if $\frac{g(z|v)}{G(z|v)}$ increases in v, which is satisfied when product valuations are IID or positively affiliated in the sense of Milgrom and Weber (1982). The assumption can also hold when $\frac{g(z|v)}{G(z|v)}$ decreases in v, provided it does not decrease too quickly.

Lemma 9. *Suppose Assumption 2 holds. Under partial discrimination there exists a symmetric equilibrium in which each firm uses the price schedule $p(v) = v - b(v)$, where $b(v)$ is defined in equation (42) and it is strictly increasing in v.*

Proof. The proof largely follows the literature on auctions with interdependent values. We look for a symmetric equilibrium where $b(v) = v - p(v)$ is the equilibrium surplus bidding function and $b(v)$ increases monotonically in v. When a firm observes consumer valuation v but deviates and bids according to valuation z, its expected profit is

$$[v - b(z) - c]G(z|v) \ . \tag{43}$$

The derivative with respect to z is

$$-\, b'(z)G(z|v) + [v - b(z) - c]g(z|v) \ . \tag{44}$$

The deviation profit (43) should be maximized at $z = v$ in symmetric equilibrium, and so the first-order condition is

$$-b'(v)G(v|v) + [v - b(v) - c]g(v|v) = 0 \ ,$$

from which we derive a differential equation

$$b'(v) = [v - b(v) - c]\frac{g(v|v)}{G(v|v)} \ . \tag{45}$$

The natural boundary condition is $b(c) = 0$, which allows us to solve for

$$b(v) = \int_c^v (x - c)dL(x|v) = \int_c^v [1 - L(x|v)]dx \ ,$$

where $L(x|v)$ is defined in (42). Notice that $b'(v) = -\int_c^v \frac{\partial L(x|v)}{\partial v}dx > 0$ (where we have used the facts that $L(v|v) = 1$ and $L(x|v)$ decreases in v), so $b(v)$ is indeed increasing. To check that the first-order condition is sufficient, substitute (45) into (44) to get

$$\begin{aligned} G(z|v)&\left(-b'(z) + [v - b(z) - c]\frac{g(z|v)}{G(z|v)}\right) \\ &= G(z|v)\left([v - b(z) - c]\frac{g(z|v)}{G(z|v)} - [z - b(z) - c]\frac{g(z|z)}{G(z|z)}\right) \ . \end{aligned}$$

Under Assumption 2 this is positive for $z < v$ and negative for $z > v$, and hence the first-order condition is indeed sufficient for defining the equilibrium. □

This lemma implies that a consumer buys her best product provided its valuation exceeds marginal cost. This is because the surplus $b(v) = v - p(v)$ offered by a firm with valuation v is positive if and only if $v > c$, and is also strictly increasing in v.

Proposition 6. *Suppose Assumption 2 holds. Comparing partial and full discrimination:*
(i) Total welfare is the same under both regimes.
(ii) When valuations are IID, profit and consumer surplus are the same in both regimes.
(iii) Otherwise firms earn more (less) under partial discrimination if $\frac{g(z|v)}{G(z|v)}$ *increases (decreases) in* v*, and the opposite is true for consumer surplus.*

Proof. For part (i), note that under partial discrimination $b(v)$ is strictly positive and strictly increasing in v whenever $v > c$, and so a consumer buys the best-matched product whenever its valuation exceeds marginal cost. The same is true under full discrimination, so the two regimes yield the same total welfare.

For parts (ii) and (iii) let us compare profit. (The comparison of consumer surplus is just the opposite.) When a firm wins a consumer with valuation v, its profit is

$$p(v) - c = v - b(v) - c = \int_c^v L(x|v)dx \ . \tag{46}$$

Recall that as derived in footnote 19 the counterpart under full discrimination is

$$\frac{\int_c^v G(x|v)dx}{G(v|v)} \ . \tag{47}$$

Suppose first that $\frac{g(z|v)}{G(z|v)}$ increases in v. Then

$$\begin{aligned} L(x|v) = \exp\left(-\int_x^v \frac{g(t|t)}{G(t|t)}dt\right) &\geq \exp\left(-\int_x^v \frac{g(t|v)}{G(t|v)}dt\right) \\ &= \exp\left(-\int_x^v [\ln G(t|v)]'dt\right) = \frac{G(x|v)}{G(v|v)} \ . \end{aligned}$$

Therefore, (46) is greater than (47), i.e., firms earn more under partial discrimination. The opposite is true if $\frac{g(z|v)}{G(z|v)}$ decreases in v. In the IID case, $\frac{g(z|v)}{G(z|v)}$ is independent of v, so the equivalence result follows. □

The intuition for these results is as follows. For part (i), total welfare is the same under both partial and full discrimination because in both regimes consumers buy the best product conditional on its valuation exceeding marginal cost. Parts (ii) and (iii) exploit the connection with auction theory. In particular, notice that under *full* discrimination competition is the same as in a second-price auction—because the winning firm earns a profit equal to the difference between the highest and second-highest valuations (including the outside option). The well-known revenue equivalence theorem then implies that, when valuations are IID, firm and consumer payoffs are the same under both full and partial discrimination.[36] Meanwhile the theory of auctions with interdependent valuations (e.g., Milgrom and Weber, 1982) implies that bidders (i.e., firms) are better off and

[36] Note, however, that partial and full discrimination affect different consumers differently. A consumer's payment is determined by her valuation for the best product under partial discrimination, but by the gap between the highest two valuations under full discrimination. Hence consumers with very weak (respectively, strong) preferences tend to prefer full (respectively, partial) discrimination.

the auctioneer (i.e., consumers) are worse off with partial information if valuations are positively affiliated, while the reverse is true if they are negatively affiliated.[37]

Proposition 6 implies that with IID valuations, all our earlier results about the impact of full discrimination carry over to partial discrimination. However, outside the IID case, the correlation structure of product valuations matters for whether consumers prefer firms to have more or less information about their tastes.

Although beyond the scope of the current paper, it would be interesting to also consider other information structures, and investigate how the welfare impact of price discrimination changes with the amount of information that firms have access to. This is, however, a challenging question because the space of information structures is large and the pricing equilibrium under some information structures is complicated to characterize. One approach is to consider a special class of information structures. For example, in the linear Hotelling model, some papers (e.g., Liu and Serfes, 2004; Bounie, Dubus, and Waelbroeck, 2021) have studied the class of interval information structures and shown that providing firms with finer information has a non-monotonic impact on profit and consumer surplus. (This non-monotonic relationship is also suggested by our comparison between partial and full discrimination.) Another approach is to explore the welfare limits when arbitrary information structures are feasible. For example, Bergemann, Brooks, and Morris (2015) and Elliott, Galeotti, Koh, and Li (2021) have studied this issue in respectively the monopoly and competition cases.[38] Our approach, by focusing on simpler information structures, is arguably more suitable for evaluating policies which simply either allow or ban the use of consumer data.

[37]We note that while the auctions literature focuses on the case of positive affiliation, negative affiliation may be reasonable in our context, e.g., if products differ in characteristics space, and consumers have different preferences over different characteristics.

[38]The information structures available to firms might also be constrained by communication incentives between firms and consumers. See, e.g., Ali, Lewis, and Vasserman (2023), and Ichihashi and Smolin (2022) for some recent research in this direction.

B Omitted Details and Proofs for Section 4

B.1 Asymmetrically informed firms

In this section we report the omitted details on the extension of asymmetrically informed firms in the main text. Let us first explain the technical issue in the case of $0 < k < n$ mentioned in the main text. In this mixed case, if all firms set prices simultaneously, there is no pure-strategy pricing equilibrium. This is because in any hypothetical pure-strategy equilibrium, each uniform-pricing firm has a positive measure of consumers who are indifferent between it and some personalized-pricing firm but buy from the latter. If the uniform-pricing firm slightly reduces its price, it wins these consumers and so its demand increases discontinuously. This discontinuity in demand leads to non-existence of pure-strategy pricing equilibrium, and the mixed-strategy equilibrium is rather complicated to characterize. This issue is well known in the literature, and the usual approach to avoid it is to consider a sequential pricing game where the firms that can do personalized pricing move after seeing other firms' prices.This timing captures the idea that firms with lots of data often also have better pricing technology and so can adjust prices more frequently.

B.1.1 Equilibrium analysis of the "mixed" regime

We first derive the equilibrium prices in the mixed regime. (All proofs can be found in the next section.) Consider $0 < k < n$ and let p denote the equilibrium price of each firm that cannot price discriminate.

Personalized prices. For any given uniform price vector $\mathbf{p}_U = (p_{k+1}, ..., p_n)$ offered by firms $k+1$ to n that move first, firms 1 to k compete as in Section 2.2, except that now consumers have an outside option $v_0 \equiv \max\{0, v_{k+1} - p_{k+1}, ..., v_n - p_n\}$. Let

$$\hat{x}_{\mathbf{p}_U,c} \equiv v_1 - c - \max\{v_0, v_2 - c, ..., v_k - c\}$$

be the advantage of firm 1 relative to a consumer's other alternatives (including the outside option) when all firms that can price discriminate charge a price equal to marginal cost. Therefore in equilibrium firm 1, say, offers personalized prices

$$p_1(v_1, ..., v_n; \mathbf{p}_U) = \begin{cases} c + \hat{x}_{\mathbf{p}_U,c} & \text{if } \hat{x}_{\mathbf{p}_U,c} > 0 \\ c & \text{otherwise} \end{cases},$$

and wins a consumer if and only if $\hat{x}_{\mathbf{p}_U,c} > 0$.

In the equilibrium we are looking for, firms $k+1$ to n offer the same uniform price p, i.e., $\mathbf{p}_U = (p, ...p)$, where p will be solved for later. In that case, we simplify the notation

$\hat{x}_{\mathbf{p}_U,c}$ to $\hat{x}_{p,c}$, and let $\hat{H}_{p,c}$ be its CDF. Since firm 1 wins a consumer if and only if $\hat{x}_{p,c} > 0$, its equilibrium profit is

$$\hat{\pi}_D = \int_0^\infty x d\hat{H}_{p,c}(x) = \int_0^\infty [1 - \hat{H}_{p,c}(x)]dx \ . \tag{48}$$

Whenever $p > c$, $\hat{x}_{p,c}$ exceeds $x_c = v_1 - c - \max_{j>1}\{0, v_j - c\}$ (which was introduced in Section 2.2) in the sense of first order stochastic dominance. Therefore comparing with equation (13) from earlier, each personalized-pricing firm here in the mixed regime earns more than in the regime where all firms price discriminate (i.e., $\hat{\pi}_D > \frac{1}{n}\Pi_D$).

The uniform price. We now solve for the equilibrium uniform price p charged by firms $k+1$ to n. To do this, it is useful to define the random variable

$$\tilde{x}_{p,c} \equiv v_n - p - \max\{0, v_1 - c, ..., v_k - c, v_{k+1} - p, ..., v_{n-1} - p\} \ , \tag{49}$$

which is the advantage of firm n relative to a consumer's other alternatives (including the outside option) when all firms charge their lowest possible equilibrium price (i.e., c for firms that do personalized pricing, and p for all the others). Let $\tilde{H}_{p,c}$ and $\tilde{h}_{p,c}$ be respectively the CDF and density function of $\tilde{x}_{p,c}$.

Suppose firm n unilaterally deviates to price p_n. Using (49) its deviation demand is

$$\Pr[v_n - p_n > \max\{0, v_1 - c, ..., v_k - c, v_{k+1} - p, ..., v_{n-1} - p\}] = 1 - \tilde{H}_{p,c}(p_n - p) \ , \tag{50}$$

and its deviation profit by $(p_n - c)[1 - \tilde{H}_{p,c}(p_n - p)]$. To understand (50), notice that if, say, $v_n - p_n < v_1 - c$, firm n cannot win the consumer because firm 1 can offer her more surplus with a personalized price close to marginal cost. Therefore, to calculate firm n's deviation demand we should set the price of each firm that can price discriminate to c. This implies that the existence of firms that can do personalized pricing intensifies competition for firms that cannot.

To ensure the firm's problem is well-behaved we make the following assumption:

Assumption 3. *$1 - \tilde{H}_{p,c}(x)$ is log-concave in x, and $\frac{1-\tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)}$ is non-decreasing in c and non-increasing in both p and k.*

In the next section we show that this assumption holds in the IID case with a log-concave f. Under this assumption, the equilibrium uniform price p solves

$$p - c = \frac{1 - \tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)} \ , \tag{51}$$

and each firm that does uniform pricing earns profit

$$\tilde{\pi}_U = (p - c)[1 - \tilde{H}_{p,c}(0)] = \frac{[1 - \tilde{H}_{p,c}(0)]^2}{\tilde{h}_{p,c}(0)} \ . \tag{52}$$

Lemma 10. *Suppose Assumption 3 holds. The equilibrium uniform price p uniquely solves* (51)*, and it is lower than in the uniform pricing regime and decreasing in k.*

Intuitively, as more firms are able to personalize their price, their ability to poach consumers with low personalized offers induces the uniform-pricing firms to cut their price. Since the equilibrium uniform price p is lower than in the case where all firms set a uniform price, for any given deviation price p_n, firm n's deviation demand in this mixed regime is smaller than in the regime of uniform pricing. This implies that each uniform-pricing firm earns less in this mixed regime than in the regime of uniform pricing (i.e., $\tilde{\pi}_U < \frac{1}{n}\Pi_U$). Another implication is the following result.

Corollary 1. *Suppose Assumption 3 holds. The profit of a personalized-pricing firm $\hat{\pi}_D$ is decreasing in the number of firms k that can personalize prices.*

Proof. Note that since the uniform price p decreases in k (from Lemma 10), it follows that $\hat{x}_{p,c}$ decreases in k in the sense of First Order Stochastic Dominance. Equation (48) then implies that $\hat{\pi}_D$ decreases in k. □

Intuitively, suppose one more firm gains the ability to do discriminatory pricing. A firm that could already personalize prices faces more competition from this firm (which can now compete selectively for consumers with very low prices) and also from firms which still use uniform pricing (because they react by cutting their price).

Welfare measures. Industry profit in the mixed regime is

$$\Pi_M = k\hat{\pi}_D + (n-k)\tilde{\pi}_U \ . \tag{53}$$

Expected consumer surplus is

$$V_M = \mathbb{E}[\max\{0, v_{k-1:k} - c, v_{n-k:n-k} - p\}] \ , \tag{54}$$

where $v_{k-1:k}$ denotes the second best among $\{v_1, ..., v_k\}$ and $v_{n-k:n-k}$ denotes the best among $\{v_{k+1}, ..., v_n\}$. (If $k = 1$ the $v_{k-1:k} - c$ term vanishes.) To see this, recall that $v_0 = \max\{0, v_{n-k:n-k} - p\}$ is a consumer's outside option when the first k firms compete for her by offering personalized prices. If $\max\{v_1 - c, ..., v_k - c\} \geq v_0$, the consumer buys from one of the first k firms, in which case her surplus is $\max\{v_0, v_{k-1:k} - c\}$. (If the second best among the k firms is worse than v_0, the consumer's surplus is v_0; otherwise it is $v_{k-1:k} - c$.) Otherwise, she takes the outside option v_0. Therefore, the expected consumer surplus is $\mathbb{E}[\max\{v_0, v_{k-1:k} - c\}]$, which leads to the expression for V_M in equation (54). Finally, total welfare in the mixed case is simply $W_M = \Pi_M + V_M$.

B.1.2 Comparison of regimes

When the production cost c is sufficiently high we can compare the three regimes analytically. In particular, we can show that if $f(\overline{v}) > 0$, there exists a $\hat{c}$ such that when $c > \hat{c}$ we have $\Pi_U < \Pi_M < \Pi_D$, $V_D < V_M < V_U$, and $W_U < W_M < W_D$ i.e., for each of the three welfare measures the mixed regime is ranked in the middle.[39] Intuitively, recall that when c is sufficiently large, each firm approximately acts like a monopolist. As a result, as more firms are able to personalize prices, profit and welfare increase but consumers become worse off.

When valuations are IID exponential, it turns out that any firm that cannot price discriminate charges $1+c$ regardless of k. This simple pricing result enables us to compare the three regimes analytically for any level of c.

Proposition 7. *Suppose valuations are IID exponential. Then:*
(i) There exists a $\widetilde{c}_\Pi$ such that for $c < \widetilde{c}_\Pi$ we have $\Pi_U \leq \Pi_M = \Pi_D$, and for $c > \widetilde{c}_\Pi$ we have $\Pi_U < \Pi_M < \Pi_D$.
(ii) There exists a $\widetilde{c}_V$ such that for $c < \widetilde{c}_V$ we have $V_M < V_D \leq V_U$, and for $c > \widetilde{c}_V$ we have $V_D < V_M < V_U$.
(iii) There exists a $\widetilde{c}_W$ such that for $c < \widetilde{c}_W$ we have $W_M < W_U \leq W_D$, and for $c > \widetilde{c}_W$ we have $W_U < W_M < W_D$.

When c is relatively high the mixed case is intermediate for profit, welfare, and consumer surplus. When c is relatively low, however, the mixed case is the worst for both total welfare and consumer surplus. Intuitively, in the mixed case firms that are able to personalize can "poach" some consumers for whom they are not the consumer's favorite product via a low personalized price. This harms match efficiency and can also make consumers worse off in aggregate. As reported in the main text, numerical simulations suggest that qualitatively similar patterns emerge when product valuations are IID and drawn from other distributions. Figure 6 in the main text already illustrated this for consumer surplus, in the duopoly case when valuations are IID Extreme value and Normal. Figure 14 depicts industry profit and total welfare for these two distributions in the duopoly case; notice that consistent with the above discussion, for low c the mixed regime is (strictly) best for industry profit and worst for total welfare.

[39] More precisely, when $c = \overline{v} - \varepsilon$ where ε is small, for any $0 \leq k \leq n$ we can show $\Pi_M \approx kf(\overline{v})\frac{\varepsilon^2}{2} + (n-k)f(\overline{v})\frac{\varepsilon^2}{4}$ and $V_M \approx (n-k)f(\overline{v})\frac{\varepsilon^2}{8}$. (When $k = n$, $V_M = V_D$ is of less than second order.) It is then clear that profit and total welfare increase in k while consumer surplus decreases in k.

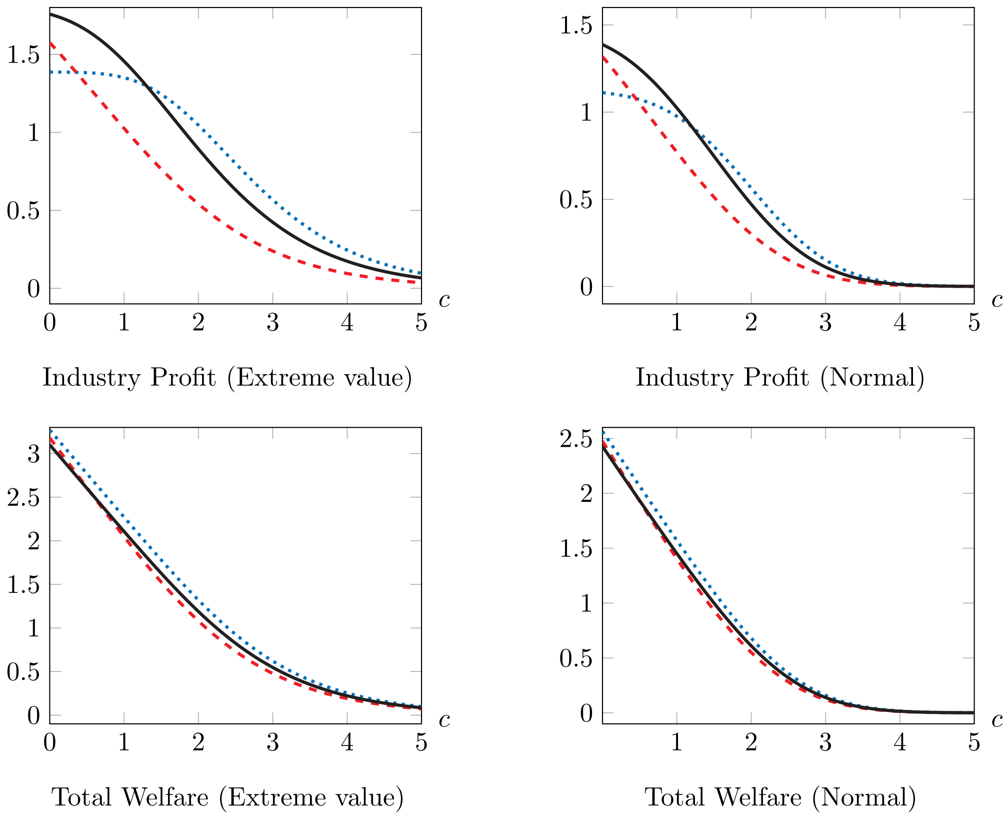


Figure 14: Asymmetric case versus symmetric cases, when $n = 2$ for different values of c
(The solid, dashed, and dotted curves are respectively the mixed, uniform, and discriminatory cases.)

B.1.3 Omitted proofs

All the omitted proofs for the case of asymmetrically informed firms can be found in this section. We begin with some preliminary results.

Lemma 11. *Suppose valuations are IID. Then*

$$\frac{1 - \tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)} = \frac{\int_p^{\overline{v}} F(v - p + c)^k F(v)^{n-k-1} dF(v)}{f(\overline{v})F(\overline{v} - p + c)^k - \int_p^{\overline{v}} F(v - p + c)^k F(v)^{n-k-1} df(v)} \ . \tag{55}$$

Proof. Since valuations are IID we can write

$$1 - \tilde{H}_{p,c}(x) = \int_{p+x}^{\overline{v}} F(v - p + c - x)^k F(v - x)^{n-k-1} dF(v) \ , \tag{56}$$

and hence we can write that

$$\frac{1 - \tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)} = \frac{\int_p^{\overline{v}} F(v - p + c)^k F(v)^{n-k-1} dF(v)}{F(c)^k F(p)^{n-k-1} f(p) + \int_p^{\overline{v}} f(v) d[F(v - p + c)^k F(v)^{n-k-1}]} \ . \tag{57}$$

Integrating the denominator by parts then gives the stated expression. $\square$

Lemma 12. *Suppose valuations are IID and f is log-concave. Then Assumption 3 holds.*

Proof. Firstly, log-concavity of $1 - \tilde{H}_{p,c}(x)$ can be established using very similar steps as in the proof of Lemma 8, and so we omit the details.

Secondly, define $\lambda(v) = F(v-p+c)^k F(v)^{n-k-1} f(v)$. After some manipulations, one can show that (55) is non-decreasing in c if and only if the following is weakly positive:

$$\begin{aligned}
&\int_p^{\overline{v}} \lambda(v)dv \int_p^{\overline{v}} \lambda(v)\frac{f(v-p+c)}{F(v-p+c)}\frac{f'(v)}{f(v)}dv - \int_p^{\overline{v}} \lambda(v)\frac{f'(v)}{f(v)}dv \int_p^{\overline{v}} \lambda(v)\frac{f(v-p+c)}{F(v-p+c)}dv \\
&+\lambda(\overline{v}) \int_p^{\overline{v}} \lambda(v) \left[\frac{f(v-p+c)}{F(v-p+c)} - \frac{f(\overline{v}-p+c)}{F(\overline{v}-p+c)}\right] dv \ . \qquad (58)
\end{aligned}$$

The first line of (58) can be written as

$$\Delta = \int_p^{\overline{v}} \int_p^{\overline{v}} \lambda(v)\lambda(w)\frac{f(w-p+c)}{F(w-p+c)} \left[\frac{f'(w)}{f(w)} - \frac{f'(v)}{f(v)}\right] dvdw \ ,$$

or, alternatively, after changing the order of integration, as

$$\Delta = \int_p^{\overline{v}} \int_p^{\overline{v}} \lambda(v)\lambda(w)\frac{f(v-p+c)}{F(v-p+c)} \left[\frac{f'(v)}{f(v)} - \frac{f'(w)}{f(w)}\right] dvdw \ .$$

Summing these last two equations together, we obtain that

$$2\Delta = \int_p^{\overline{v}} \int_p^{\overline{v}} \lambda(v)\lambda(w) \left[\frac{f(w-p+c)}{F(w-p+c)} - \frac{f(v-p+c)}{F(v-p+c)}\right] \left[\frac{f'(w)}{f(w)} - \frac{f'(v)}{f(v)}\right] dvdw \geq 0 \ ,$$

where the inequality follows because f logconcave implies that f/F and f'/f are both decreasing. The second line of (58) is also weakly positive due to f/F being decreasing. Hence (55) is indeed non-decreasing in c.

Thirdly, again using the definition of $\lambda(v) = F(v-p+c)^k F(v)^{n-k-1} f(v)$, one can show that (55) is non-increasing in p if and only if the following is weakly negative:

$$\begin{aligned}
&-k\left[\int_p^{\overline{v}} \lambda(v)dv \int_p^{\overline{v}} \lambda(v)\frac{f(v-p+c)}{F(v-p+c)}\frac{f'(v)}{f(v)}dv - \int_p^{\overline{v}} \lambda(v)\frac{f'(v)}{f(v)}dv \int_p^{\overline{v}} \lambda(v)\frac{f(v-p+c)}{F(v-p+c)}dv\right] \\
&+F(c)^k F(p)^{n-1-k} f(p) \int_p^{\overline{v}} \lambda(v) \left[\frac{f'(v)}{f(v)} - \frac{f'(p)}{f(p)}\right] dv \\
&+\lambda(\overline{v}) \left\{\int_p^{\overline{v}} \lambda(v)k \left[\frac{f(\overline{v}-p+c)}{F(\overline{v}-p+c)} - \frac{f(v-p+c)}{F(v-p+c)}\right] dv - F(c)^k F(p)^{n-k-1} f(p)\right\} \ . \qquad (59)
\end{aligned}$$

Using the previous step of the proof, the first line of (59) is negative. Meanwhile the second and third lines of (59) are also negative because f logconcave implies that f'/f and f/F are decreasing. Hence (55) is indeed non-increasing in p.

Finally, to prove that (55) is non-increasing in k, define

$$\tilde{x}_{p,c,q} \equiv v_n - p - \max\{0, v_1 - q, v_2 - c, ..., v_k - c, v_{k+1} - p, ..., v_{n-1} - p\} \ ,$$

as the advantage of product n when product 1 is priced at q, products $2, \ldots, k$ are priced at c, and products $k+1, \ldots, n$ are priced at p. Letting $\tilde{H}_{p,c,q}(x)$ denote its CDF, we have

$$1 - \tilde{H}_{p,c,q}(x) = \int_{p+x}^{\overline{v}} F(v-p+c-x)^{k-1} F(v-p+q-x) F(v-x)^{n-k-1} dF(v) \ .$$

After some manipulations we can then write

$$\frac{1-\tilde{H}_{p,c,q}(0)}{\tilde{h}_{p,c,q}(0)} = \frac{\int_p^{\overline{v}} F(v-p+c)^{k-1} F(v-p+q) F(v)^{n-k-1} dF(v)}{f(\overline{v}) F(\overline{v}-p+c)^{k-1} F(\overline{v}-p+q) - \int_p^{\overline{v}} F(v-p+c)^{k-1} F(v-p+q) F(v)^{n-k-1} df(v)} \ . \tag{60}$$

Note that when $q = c$ this degenerates to (55) with k firms doing personalized pricing, and when $q = p$ it degenerates to (55) but with $k-1$ firms doing personalized pricing. Therefore to prove that (55) is non-increasing in k, it is sufficient to prove that (60) is increasing in q. One can show that this is true if and only if the following is positive:

$$\int_p^{\overline{v}} \tilde{\lambda}(v) dv \int_p^{\overline{v}} \tilde{\lambda}(v) \frac{f(v-p+q)}{F(v-p+q)} \frac{f'(v)}{f(v)} dv - \int_p^{\overline{v}} \tilde{\lambda}(v) \frac{f'(v)}{f(v)} dv \int_p^{\overline{v}} \tilde{\lambda}(v) \frac{f(v-p+q)}{F(v-p+q)} dv$$
$$+ \tilde{\lambda}(\overline{v}) \int_p^{\overline{v}} \tilde{\lambda}(v) \left[\frac{f(v-p+q)}{F(v-p+q)} - \frac{f(\overline{v}-p+q)}{F(\overline{v}-p+q)} \right] dv \ , \tag{61}$$

where we define $\tilde{\lambda}(v) = F(v-p+c)^{k-1} F(v-p+q) F(v)^{n-k-1} f(v)$. However notice that (61) is the same as (58), just with c replaced by q and $\lambda(v)$ replaced by $\tilde{\lambda}(v)$. Therefore using the same steps as in the second part of the proof, it is easy to show that (61) is positive. Hence (55) is indeed non-increasing in k as claimed. $\square$

Lemma 13. *Suppose valuations are IID standard exponential. Then*

$$\frac{1-\tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)} = 1 \quad \textit{and} \quad \frac{1-\hat{H}_{p,c}(0)}{\hat{h}_{p,c}(0)} = 1 \ . \tag{62}$$

Proof. The first equation follows from equation (55), and the fact that with the standard exponential $f(\overline{v}) = 0$ and $df(v) = -dF(v)$.

To derive the second equation, first note that

$$1 - \hat{H}_{p,c}(x) = \int_{c+x}^{\overline{v}} F(v-x)^{k-1} F(v-x+p-c)^{n-k} dF(v) \ , \tag{63}$$

and hence

$$\begin{aligned}
\hat{h}_{p,c}(x) &= F(c)^{k-1}F(p)^{n-k}f(c+x) + \int_{c+x}^{\overline{v}} f(v)d\left[F(v-x)^{k-1}F(v-x+p-c)^{n-k}\right] \\
&= F(\overline{v}-x)^{k-1}F(\overline{v}-x+p-c)^{n-k}f(\overline{v}) - \int_{c+x}^{\overline{v}} F(v-x)^{k-1}F(v-x+p-c)^{n-k}df(v) \\
&= 1-\hat{H}_{p,c}(x)\ ,
\end{aligned}$$

where the second line uses integration by parts, and the third line again uses $f(\overline{v}) = 0$ and $df(v) = -dF(v)$. Therefore $[1-\hat{H}_{p,c}(0)]/\hat{h}_{p,c}(0) = 1$. □

We now prove the remaining results.

Proof of Lemma 10. Since $1-\tilde{H}_{p,c}(x)$ is log-concave in x, a uniform-pricing firm's profit is quasiconcave in its price. Hence the first-order condition (51) is sufficient to determine the equilibrium uniform price. Equation (51) has a unique solution because its lefthand side is strictly increasing in p while its righthand side is non-increasing in p. Moreover this solution is decreasing in k because the lefthand side of (51) is independent of k while the righthand side is non-increasing in k.

Finally, we prove that for any $0 < k < n$ the uniform price p is lower than in the uniform-pricing regime. To this end, let p_U denote the equilibrium price when all firms do uniform pricing. Towards a contradiction, suppose that $p > p_U$. Notice that

$$\frac{1-\tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)} \leq \frac{1-\tilde{H}_{p,p}(0)}{\tilde{h}_{p,p}(0)} = \frac{1-H_p(0)}{h_p(0)} \leq \frac{1-H_{p_U}(0)}{h_{p_U}(0)}\ , \tag{64}$$

where the first inequality follows because $p > c$ and $\frac{1-\tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)}$ is non-decreasing in c (from Assumption 3), the equality follows from inspection of (9) and (57), and the second inequality follows from the supposition that $p > p_U$ and because $\frac{1-H_p(0)}{h_p(0)}$ is non-increasing in p (from Assumption 1). However we also know from equations (6) and (51) that

$$p_U - c = \frac{1-H_{p_U}(0)}{h_{p_U}(0)} \quad \text{and} \quad p-c = \frac{1-\tilde{H}_{p,c}(0)}{\tilde{h}_{p,c}(0)}\ .$$

Combined with equation (64) this implies that $p \leq p_U$. But this is a contradiction to our original supposition that $p > p_U$. □

Proof of Proposition 7. As a preliminary step, we note that $p = c+1$. This follows from the first-order condition (51), and Lemma 13 which shows that the righthand side of the first-order condition equals 1.

Consider industry profit. Using equation (52) and $p = c + 1$ we can write

$$\tilde{\pi}_U = (p - c)[1 - \tilde{H}_{p,c}(0)] = 1 - \tilde{H}_{p,c}(0) \ .$$

Using equation (48) and Lemma 13 we can also write

$$\hat{\pi}_D = \int_0^\infty [1 - \hat{H}_{p,c}(x)]dx = \int_0^\infty \left[\frac{1 - \hat{H}_{p,c}(x)}{\hat{h}_{p,c}(x)}\right] d\hat{H}_{p,c}(x) = 1 - \hat{H}_{p,c}(0) \ .$$

Hence using equation (53) we can write industry profit as

$$\begin{aligned}
\Pi_M &= k[1 - \hat{H}_{p,c}(0)] + (n-k)[1 - \tilde{H}_{p,c}(0)] \\
&= k \int_c^\infty F(v)^{k-1} F(v+1)^{n-k} dF(v) + (n-k) \int_{1+c}^\infty F(v)^{n-k-1} F(v-1)^k dF(v) \\
&= k \int_c^\infty F(v)^{k-1} F(v+1)^{n-k} dF(v) + (n-k) \int_c^\infty F(v+1)^{n-k-1} F(v)^k dF(v+1) \\
&= \int_c^\infty dF(v+1)^{n-k} F(v)^k = 1 - F(c+1)^{n-k} F(c)^k \ ,
\end{aligned}$$

where the second line uses equations (56) and (63) as well as $p = c+1$, and the third line uses a change of variables. Recall from page 15 in the main text that $\Pi_D = 1 - F(c)^n$ and $\Pi_U = 1 - F(c+1)^n$. Hence when $c \leq 0$ then $\Pi_M = \Pi_D$ but otherwise $\Pi_M < \Pi_D$. Similarly when $c \leq -1$ then $\Pi_M = \Pi_U$ but otherwise $\Pi_M > \Pi_U$. Item (i) then follows.

Now consider welfare. Clearly $W_D > W_M$ because under the discriminatory regime each consumer buys the product i with the highest value of $v_i - c$ conditional on it being positive, which is not the case in the mixed regime. Similarly when the market is fully covered in the uniform regime (i.e., when $c \leq -1$) $W_D = W_U$ because every consumer buys the product with the highest v_i, but otherwise $W_D > W_U$ due to some consumers with a valuation above cost being excluded from the market in the uniform regime.

Now compare W_U and W_M. Note that when $c \leq -1$ then under both regimes the uniform price is $1 + c \leq 0$ and hence the market is covered; it is immediate then that $W_U > W_M$ because under the uniform pricing regime each consumer buys the product i with the highest value of $v_i - c$, whereas this is not the case in the mixed regime. In the remainder of this part of the proof consider $c > -1$. Letting $F_{j:j}$ denote the CDF of the highest of j random variables, it is convenient to write

$$\begin{aligned}
W_M &= \int_p^\infty (v-c) F_{k:k}(v-p+c) dF_{n-k:n-k}(v) + \int_c^\infty (v-c) F_{n-k:n-k}(v+p-c) dF_{k:k}(v) \\
&= \int_{c+1}^\infty (v-c) F_{k:k}(v-1) dF_{n-k:n-k}(v) + \int_c^\infty (v-c) F_{n-k:n-k}(v+1) dF_{k:k}(v) \\
&= \int_c^\infty (v-c+1) F_{k:k}(v) dF_{n-k:n-k}(v+1) + \int_c^\infty (v-c) F_{n-k:n-k}(v+1) dF_{k:k}(v) \\
&= \int_c^\infty F_{k:k}(v) dF_{n-k:n-k}(v+1) + \int_c^\infty [1 - F_{n-k:n-k}(v+1) F_{k:k}(v)] dv \ ,
\end{aligned}$$

where the second line uses $p = c + 1$, the third line uses a change of variables, and the fourth line integrates by parts. Also note that $W_U = \int_{1+c}^{\infty}(v - c)dF(v)^n$. After some simplifications one can then write that

$$\begin{aligned} \frac{d}{dc}(W_U - W_M) &= -nF(c+1)^{n-1}f(c+1) - F(c+1)^{n-k}F(c)^k \\ &\quad +F(1+c)^n + F(c)^k(n-k)F(c+1)^{n-k-1}f(c+1) \ . \end{aligned} \tag{65}$$

When $c \in (-1, 0]$ we have that $F(c) = 0$, and hence using the fact that $f(c + 1) = 1 - F(c + 1)$, (65) simplifies to

$$\begin{aligned} -nF(c+1)^{n-1}f(c+1) + F(c+1)^n &= -nF(c+1)^{n-1}[1 - F(c+1)] + F(c+1)^n \\ &\propto -n[1 - F(c+1)] + F(c+1) \\ &\leq -2[1 - F(1)] + F(1) < 0 \ , \end{aligned}$$

i.e., for $c \in (-1, 0]$, $W_U - W_M$ is decreasing in c. Otherwise, for $c > 0$, we have that

$$\begin{aligned} \frac{d}{dc}(W_U - W_M) &\propto \frac{-nF(c+1)^k f(c+1)}{F(c)^k} + \frac{F(c+1)^{k+1}}{F(c)^k} + (n-k)f(c+1) - F(c+1) \\ &= \frac{-nX^k(1-X)}{[1 - e(1-X)]^k} + \frac{X^{k+1}}{[1 - e(1-X)]^k} + (n-k)(1-X) - X \ , \end{aligned} \tag{66}$$

where the second line uses $f(c+1) = 1 - F(c+1)$ and $F(c) = 1 - e[1 - F(c+1)]$, and then defines $X \equiv F(c + 1)$. Notice that $X \in (1 - e^{-1}, 1)$. It is straightforward (but lengthy) to show that (66) is negative as $X \to 1 - e^{-1}$, zero as $X \to 1$, concave, and decreasing in X around $X = 1$. We can therefore conclude that $W_U - W_M$ is quasiconvex in c, and increasing in c for sufficiently high c. Given that $W_U > W_M$ for $c \leq -1$, and $\lim_{c\to\infty}(W_U - W_M) = 0$. Item (iii) then follows.

Finally, consider consumer surplus. Using equation (54) we can write

$$\begin{aligned} V_M &= \int_p^{\infty}(v-p)F_{k-1:k}(v-p+c)dF_{n-k:n-k}(v) + \int_c^{\infty}(v-c)F_{n-k:n-k}(v+p-c)dF_{k-1:k}(v) \\ &= \int_c^{\infty}(v-c)F_{k-1:k}(v)dF_{n-k:n-k}(v+1) + \int_c^{\infty}(v-c)F_{n-k:n-k}(v+1)dF_{k-1:k}(v) \\ &= \int_c^{\infty}(v-c)dF_{k-1:k}(v)F_{n-k:n-k}(v+1) \\ &= \int_c^{\infty}[1 - F_{k-1:k}(v)F_{n-k:n-k}(v+1)]dv \end{aligned} \tag{67}$$

where the second line uses $p = c + 1$ and changes the variable of integration in the first part, and the fourth line integrates by parts.

We start by proving that $V_M < V_U$. It is straightforward to see that $V_M < V_U$ when $c \leq -1$; this follows because we have just proved that for this range of c, $W_M < W_U$ while

$\Pi_M = \Pi_U$. Now consider $c > -1$ and note that $V_U = \int_c^\infty [1 - F(v+1)^n]dv$. Hence

$$\frac{d}{dc}(V_U - V_M) = F(c+1)^n - F_{k-1:k}(c)F_{n-k:n-k}(c+1) \ . \tag{68}$$

When $c \in (-1, 0]$ we have that (68) is strictly positive because $F_{k-1:k}(c) = 0$ for this range of c. When $c > 0$ we can rewrite (68) as

$$\begin{aligned} \frac{d}{dc}(V_U - V_M) &= F(c+1)^n - \{F(c)^k + k[1-F(c)]F(c)^{k-1}\}F(c+1)^{n-k} \\ &\propto \frac{F(c+1)^k}{F(c)^{k-1}} - F(c) - k[1-F(c)] \\ &= \frac{[1-e^{-1}(1-Y)]^k}{Y^{k-1}} - Y - k(1-Y) \ , \end{aligned} \tag{69}$$

where the final line uses $F(c+1) = 1 - e^{-1}[1-F(c)]$ and defines $Y = F(c)$. Note that $Y \in (0, 1)$. It is straightforward to show that (69) is positive as $Y \to 0$, is zero at $Y = 1$, is convex in Y, and strictly increasing in Y as $Y \to 1$. We therefore conclude that $V_U - V_M$ is quasiconcave in c. However we also know that $V_U > V_M$ for $c \leq -1$, and we know that $V_U - V_M = 0$ as $c \to \infty$. Hence $V_U > V_M$ for all values of c.

We now prove the relationship between V_M and V_D. It is straightforward to see that $V_M < V_D$ when $c \leq 0$; this follows because we have just proved that for this range of c, $W_M < W_D$ while $\Pi_M = \Pi_D$. Now consider $c > 0$ and note that

$$V_D = \int_c^\infty [1 - F_{(n-1)}(v)]dv = \int_c^\infty [1 - F(v)^n - n[1-F(v)]F(v)^{n-1}]dv \ . \tag{70}$$

Hence we can write that

$$\begin{aligned} \frac{d}{dc}(V_D - V_M) &= F(c)^n + n[1-F(c)]F(c)^{n-1} - F_{k-1:k}(c)F_{n-k:n-k}(c+1) \\ &= F(c)^n + n[1-F(c)]F(c)^{n-1} - [F(c)^k + k[1-F(c)]F(c)^{k-1}]F(c+1)^{n-k} \\ &\propto F(c) + n[1-F(c)] - [F(c) + k[1-F(c)]]\left(\frac{F(c+1)}{F(c)}\right)^{n-k} \\ &= Z + n(1-Z) - [Z + k(1-Z)]\left(\frac{1-e^{-1}(1-Z)}{Z}\right)^{n-k} \ , \end{aligned} \tag{71}$$

where the final line uses $F(c+1) = 1 - e^{-1}[1-F(c)]$ and defines $Z = F(c)$. Note that $Z \in (0, 1)$. It is straightforward to show that (71) is negative as $Z \to 0$, is zero at $Z = 1$, is concave in Z, and strictly decreasing in Z as $Z \to 1$. We therefore conclude that $V_D - V_M$ is quasiconvex in c and increasing in c for large enough c. However we also know that $V_D > V_M$ for $c \leq 0$, and we know that $V_D - V_M = 0$ as $c \to \infty$. Hence there exists a critical $c > 0$ such that $V_D < V_M$ for cost above this critical value. We also know from earlier that $V_U \geq V_D$ with strict inequality for all $c > -1$. Item (ii) then follows. $\square$

B.2 Free entry and endogenous market structure

In this section we report the omitted details on the discussion of the impact of personalized pricing in a free-entry market.

Let us first study the case with personalized pricing. As explained earlier, firms engage in asymmetric Bertrand competition. Therefore the profit on each consumer is simply the difference between her best and second-best product valuations, adjusted for the marginal cost. Hence, with n firms in the market, each firm's profit can be expressed as

$$\tfrac{1}{n}\Pi_D = \tfrac{1}{n}\mathbb{E}[\max\{c, v_{n:n}\} - \max\{c, v_{n-1:n}\}] \ . \tag{72}$$

On the other hand, the increase in match efficiency when the number of firms goes from $n-1$ to n is

$$\mathbb{E}[\max\{c, v_{n:n}\}] - \mathbb{E}[\max\{c, \hat{v}_{n-1:n-1}\}] \ , \tag{73}$$

where $\hat{v}_{n-1:n-1}$ denotes the best match among the original $n-1$ products. (We use $\hat{v}_i$ to denote the valuation for product $i \le n-1$ when there are only $n-1$ firms in the market.) To determine whether there is too much or too little entry relative to the social optimum (under the usual assumption that the social planner is unable to control firms' pricing behavior once they enter the market), it then suffices to compare (72) and (73). It turns out that they are actually equal to each other under the assumption stated in the main text that the entry of a new firm does not affect consumers' valuations for existing products.

Lemma 14. *If the entry of a new firm does not affect consumers' valuations for existing products, the free-entry equilibrium under personalized pricing is unique and it is also socially optimal.*

Proof. Notice that

$$\begin{aligned}\mathbb{E}[\max\{c, v_{n:n}\}] = \tfrac{1}{n}&\mathbb{E}[\max\{c, v_n\}|v_n > \max\{v_1, ..., v_{n-1}\}] \\ &+ (1 - \tfrac{1}{n})\mathbb{E}[\max\{c, v_1, ..., v_{n-1}\}|v_n < \max\{v_1, ..., v_{n-1}\}] \ ,\end{aligned}$$

and with the stated assumption we also have

$$\begin{aligned}\mathbb{E}[\max\{c, \hat{v}_{n-1:n-1}\}] = \tfrac{1}{n}&\mathbb{E}[\max\{c, v_1, ..., v_{n-1}\}|v_n > \max\{v_1, ..., v_{n-1}\}] \\ &+ (1 - \tfrac{1}{n})\mathbb{E}[\max\{c, v_1, ..., v_{n-1}\}|v_n < \max\{v_1, ..., v_{n-1}\}] \ .\end{aligned}$$

Therefore, the match efficiency improvement in (73) is equal to

$$\tfrac{1}{n}\mathbb{E}[\max\{c, v_n\} - \max\{c, v_1, ..., v_{n-1}\}|v_n > \max\{v_1, ..., v_{n-1}\}] \ ,$$

which is just equal to (72).

We need to further show that both the free-entry equilibrium and the socially optimal solution are unique. (Otherwise, a free-entry equilibrium could differ from a socially optimal solution due to a selection issue.) It suffices to show that (72) is decreasing in n. To see that, it is more convenient to use the expression for Π_D in (13). Under the stated assumption, $x_c = v_i - \max_{j \neq i}\{c, v_j\}$ must become smaller in the sense of first-order stochastic dominance as one more firm is added, and so $1 - H_c(x)$ decreases in n for any x. This implies that $\frac{1}{n}\Pi_D$ decreases in n. □

The intuition for this result is simple and has been explained in the main text.

Now consider the case with uniform pricing. Let n^* denote the socially optimal number of firms. A simple corollary of Lemma 14 is the following:

Corollary 2. *Suppose that the entry of a new firm does not affect consumers' valuations for existing products, and each firm's profit under uniform pricing decreases in n.*[40]
(i) Entry under uniform pricing is excessive if $\Pi_U > \Pi_D$ at $n = n^*$, but insufficient if $\Pi_U < \Pi_D$ at $n = n^*$.
(ii) Uniform pricing thus leads to excessive entry if the market is fully covered at $n = n^*$.

Part (i) of the corollary is simply because, as we have just shown, a new entrant's profit under personalized pricing is equal to the (expected) increase in match efficiency due to its entry. To understand part (ii), recall from Proposition 1 that with full coverage it is always the case that $\Pi_U > \Pi_D$.

Finally, if we follow many papers in the literature and assume away integer constraints on the number of firms in the industry, Lemma 14 also implies the following:

Proposition 8. *Suppose there is no integer constraint. Then, compared to uniform pricing, personalized pricing benefits consumers in the long run.*

In a free-entry market firms earn zero profit (after accounting for the fixed entry cost) in both pricing regimes. Therefore since total welfare is maximized under personalized pricing, so is aggregate consumer surplus.

Remark. We emphasize that Proposition 8 may not hold anymore when the number of firms in the industry is restricted to be an integer. The reason is that although personalized pricing maximizes total welfare, with integer constraints it may also lead to higher industry profit, in which case consumer surplus can be lower compared to under uniform

[40] The assumption that a firm's profit under uniform pricing decreases in n ensures uniqueness of the free-entry equilibrium. It is easy to see that this assumption must hold if the equilibrium uniform price decreases in n (which, as shown in Zhou, 2017, is true in the IID case with a log-concave f).

pricing. For instance, it is possible to construct examples where there is a monopoly (i.e., exactly one firm enters) in both regimes, such that consumers are worse off with personalized pricing.[41] Similarly, it is possible to construct examples where the same number $n \geq 2$ firms enter under each regime, in which case the long-run effect of personalized pricing coincides with the short-run effect (and so, following earlier analysis, depends, e.g., on market coverage).

On the other hand, with integer constraints, personalized pricing can also benefit consumers in a free-entry market through a new channel—namely by creating markets, much like can happen under third-degree price discrimination. Specifically, recall from Proposition 3 and surrounding discussion that when c is high, firms are approximately like local monopolists, and so for any fixed n each firm earns more under personalized pricing. There then exist fixed entry costs (say, slightly above the monopoly profit under uniform pricing) such that no firm enters under uniform pricing, but two or more firms enter under personalized pricing, resulting in strictly positive consumer surplus.

Finally, we note that if one firm enters a market first, and if there are economies of scale in collecting and processing data, the firm may have a data advantage relative to later entrants. Anticipating this, other firms may choose not to enter the market at all. In that case, the personalized pricing regime leads to a natural monopoly. Since this discussion does not apply to the uniform pricing regime, it provides an additional reason why consumers may be worse off with personalized pricing.

[41] Formally, let κ be the entry cost, and let $\pi_D(n)$ and $\pi_U(n)$ denote per-firm profits under the two pricing regimes when n firms enter. Suppose $\pi_D(n)$ and $\pi_U(n)$ both decrease in n (as in, e.g., the IID case described in the previous footnote). Exactly one firm enters in each regime if $\pi_D(1) \geq \kappa > \pi_D(2)$ and $\pi_U(1) \geq \kappa > \pi_U(2)$. A necessary and sufficient condition for these two sets of inequalities to hold simultaneously for some κ is that $\pi_D(2) < \pi_U(1)$, which in turn holds, e.g., when the market is fully covered under duopolistic uniform pricing (in which case $\pi_D(2) < \pi_U(2)$ according to Proposition 1).