Online Appendix to "Data, Competition, and Digital Platforms," by Dirk Bergemann and Alessandro Bonatti

A Platform Size and Competition

Having examined the informational sources of the platform's bargaining power, we now return to our baseline setting of Section 4 to study the role of the size of the platform and the competition among the sellers. We first investigate how the market share of the platform λ affects the welfare and distribution of the social surplus. We then analyze how an increase in competition in terms of the number of competing sellers affects the welfare outcomes on and off the platform.

Platform Size The opportunity cost of serving consumers off the platform increases as the platform becomes (exogenously) larger. Intuitively, the information rents of the off-platform consumers must also be paid to a mass λ of on-platform consumers. This should lead to further distortions in the off-platform quality levels. We formalize this intuition in Proposition 10.

Proposition 10 (Platform Size)

1. *The equilibrium quality levels $\widehat{q}_j^*(\theta_j)$ are decreasing in λ for all $\theta_j < \overline{\theta}$, and the information rents $\widehat{U}_j^*(\theta_j)$ are decreasing in λ for all θ_j.*

2. *For every $\theta_j < \overline{\theta}$, there exists $\bar{\lambda} < 1$ such that $\widehat{q}_j^*(\theta_j) = 0$ for all $\lambda \geq \bar{\lambda}$.*

Proof of Proposition 10. These results are obtained by differentiating expression (19) for the equilibrium quality off-platform with respect to λ. In particular, whenever it is strictly positive, the equilibrium $\widehat{q}_j^*(\theta_j)$ is strictly decreasing in λ. Because the equilibrium quality provision is equal to the marginal information rent, the comparative statics of quality $\widehat{q}_j^*$ immediately extend to the information rent $\widehat{U}_j^*$. ■

In Figure 7, we illustrate how the off-platform quality provision changes as the market share λ of the platform increases.

The on-platform allocation remains unchanged and is given by the socially efficient quality provision. However, as the platform grows larger, each seller attempts to minimize the information rents on the platform and in turn renders the menu off the platform less attractive. Thus, for every value θ_j, the equilibrium quality-match off the platform $\widehat{q}_j^*(\theta_j)$

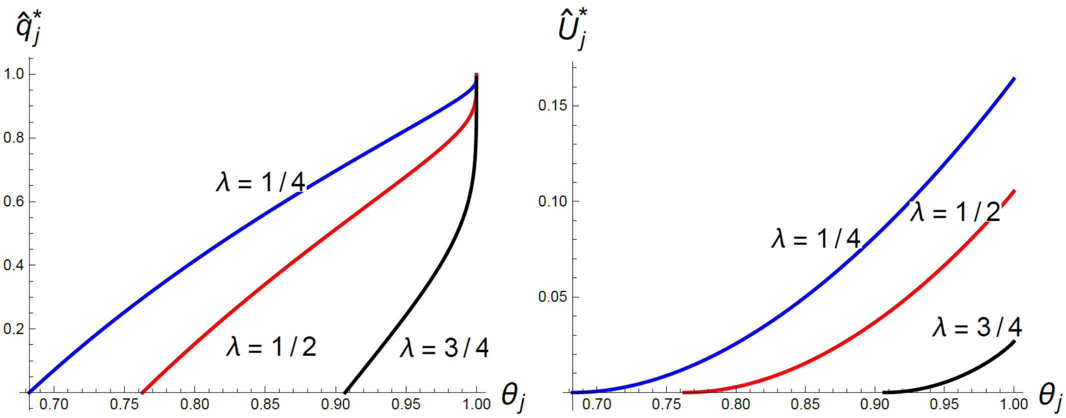

FIGURE 7: Off-Platform Menus, $J = 3, G(m_j) = m_j, F(\theta_j) = \text{Beta}(\theta_j, 1/4, 1/4)$.

decreases, the price per unit of quality increases, and the consumer surplus $\widehat{U}^*_j(\theta_j)$ off the platform decreases as the size of the platform increases.

Importantly, Proposition 10 offers an ex-post welfare comparison, i.e., based on the consumer's realized value profile θ. An ex-ante comparison, which is stated in expectation over all value profiles, must also account for the positive welfare effect described of shifting consumers from the off-platform to the on-platform market, as captured in (22).

Number of Sellers As the number of sellers increases, a larger number J of draws for each value θ_j improves the distributions F^J and G^J in the likelihood-ratio order. This leads to lower information rents. In the limit, a seller will know that every consumer who shops on their site, or receives their ads, has a value near $\overline{\theta}$ with probability close to 1, and therefore information rents vanish. This result is a direct implication of the Diamond (1971) model adapted to our setting. We illustrate this result for the benchmark case of an off-platform market only (i.e., $\lambda = 0$) in Figure 8.

Relative to the Diamond (1971) model, quality distortions decrease faster in our setting for lower values and slower for higher values. This effect is due to the interaction of showrooming and the different distributions of values. In particular, one can show that the additional distortion term in the equilibrium quality (21), i.e.,

$$\frac{\lambda}{1-\lambda}\frac{1-F^J(\theta_j)}{JG^{J-1}(\theta_j)g(\theta_j)}$$

is decreasing in J when θ_j is close to $\overline{\theta}$. Thus, for a small number of sellers, high values of θ_j receive a higher quality as J increases while low values of θ_j receive a lower quality.

As Figure 9 illustrates, this effect may not be sufficient to generate a larger rent for any value. Furthermore, Proposition 11 shows that as J grows large, every value's quality

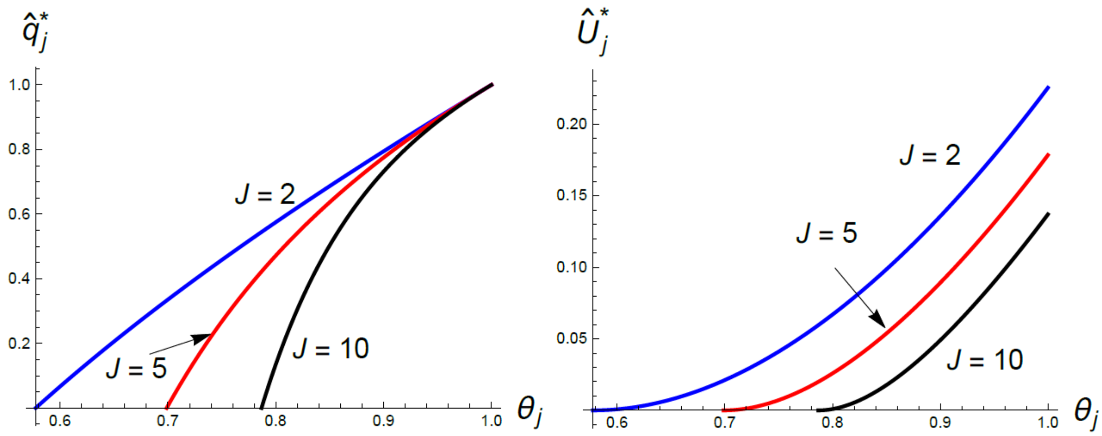

FIGURE 8: Off-Platform Menus, $\lambda = 0, G(m_j) = m_j, F(\theta_j) = \text{Beta}(\theta_j, 1/4, 1/4)$.

allocation eventually decreases in the number of sellers.

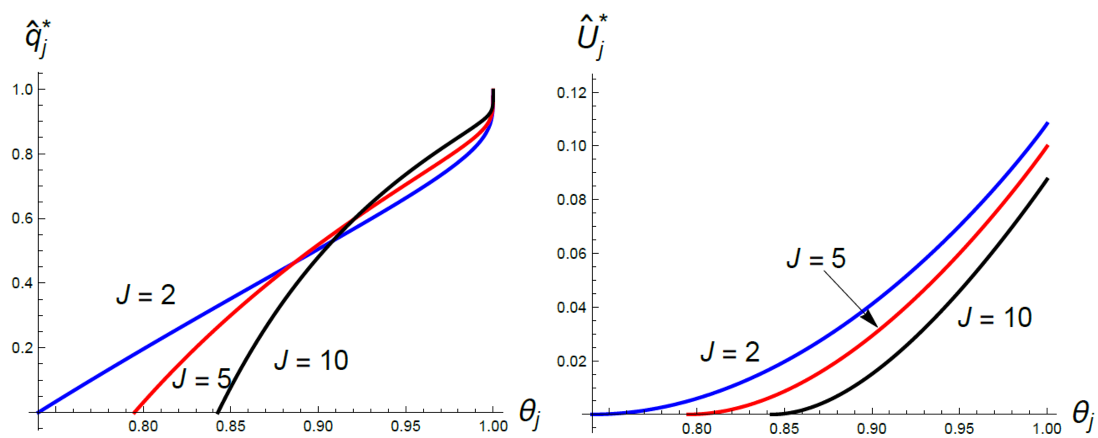

FIGURE 9: Off-Platform Menus, $\lambda = 1/2, G(m_j) = m_j, F(\theta_j) = \text{Beta}(\theta_, 1/4, 1/4)$.

Proposition 11 (Number of Sellers)

1. *For every $\theta_j < \overline{\theta}$, the equilibrium quality $\widehat{q}_j^*(\theta_j)$ and information rent $\widehat{U}_j^*(\theta_j)$ are decreasing in J if J is large enough.*

2. *For every $\theta_j < \overline{\theta}$, there exists $\widehat{J}$ such that $\widehat{q}_j^*(\theta_j) = 0$ for all $J \geq \widehat{J}$.*

Proof of Proposition 11. These results are obtained by differentiating expression (19) with respect to J. Whenever it is strictly positive, the equilibrium $\widehat{q}_j^*(\theta_j)$ is strictly decreasing in J for J large enough. Because the equilibrium quality provision is equal to the marginal information rent, the comparative statics of quality $\widehat{q}_j^*$ immediately extend to the information rent $\widehat{U}_j^*$. ■

Importantly Proposition 11 also offers an ex-post welfare comparison, i.e., based on the consumer's realized value profile θ. An ex-ante comparison would also account for the positive welfare effect associated with selecting the highest-order statistic $\max_j \theta_j$ from a larger set J.

However, a consequence of Propositions 10 and 11 is that the expected consumer surplus on- and off-platform is eventually decreasing in both λ and J. Indeed, as J grows without bound (or as $\lambda \to 1$), the platform captures the entire (first best) social surplus it creates. Intuitively, the consumers have no information rents (as the highest value component is converging in probability to 1), and therefore sellers need not distort the off-platform menus when they participate in the platform's mechanism. Figure 10 illustrates the platform revenue, consumer surplus, seller surplus (i.e., the outside option $\overline{\Pi}_j$), and the profit generated on the platform for various numbers of sellers.

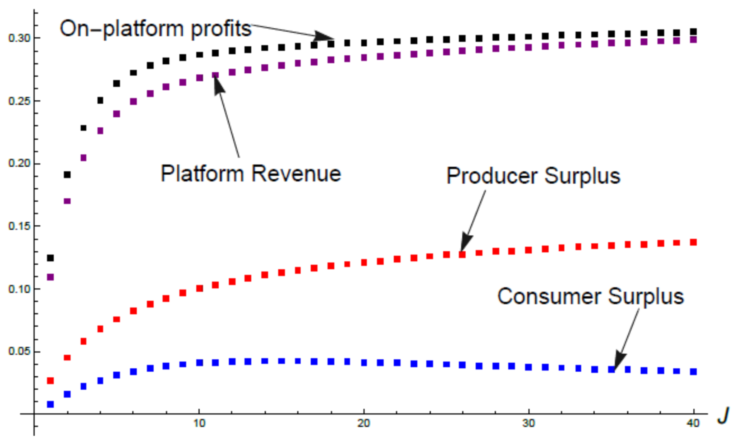

FIGURE 10: Surplus Levels, $\lambda = 2/3, G(m_j) = m_j, F(\theta_j) = \text{Beta}(\theta_j, 1/3, 1/3)$.

B Sales Fees

Consider a single seller and a retail platform. The seller faces a distribution of consumer values F on the platform and G off the platform. For every product sold on the platform, the seller must pay a linear commission τ on the price of that product. Throughout this section, we assume a slight strengthening of the monotonicity of the virtual values in (16). In particular, we require that

$$\theta - \frac{1 - \lambda F(\theta) - (1-\lambda) G(\theta) - \lambda\tau (F(x) - F(\theta))}{(1-\lambda) g(\theta)} \tag{44}$$

is strictly increasing in θ whenever positive. For the case of $J = 1$, which we analyze here, condition (44) reduces to (16) when $\tau = 0$.

B.1 Laissez Faire Solution

We begin by considering the problem of setting on platform quality and price levels (q, p) first. Thus, we fix the quality and indirect utility schedule offered off platform $(\hat{q}, \hat{U})$. Because the consumer can showroom, the seller obtains the following profits on-platform from each type θ:

$$\pi(\theta, q, U) = \begin{cases} (1-\tau)(\theta q - U) - q^2/2 & \text{if } U \geq \hat{U}, \\ \theta\hat{q} - \hat{q}^2/2 - \hat{U} & \text{if } U < \hat{U}. \end{cases}$$

It is then intuitive that, for any rent function U, the optimal quality level $q(\theta)$ maximizes profits $\pi(\theta, q, U)$. Furthermore, the seller will lead consumer θ to either (a) actively showroom (i.e., not offer them a good, or offer them a product at an excessively high price), or (b) buy on the platform, in which case it is optimal for the showrooming constraint to bind as in the baseline model. In the latter case, the consumer obtains the same utility on and off platform, $U = \hat{U}$.

Claim 1 *The seller offers quality level $q(\theta) = (1-\tau)\theta$ and sets $U(\theta) = \hat{U}(\theta)$ for all types that purchase on platform.*

To establish this claim, simply maximize profits above with respect to q. In this case, quality provision is inefficient because the sales fee τ acts like a tax on quality investment. The candidate on-platform profit level on type θ is therefore given by

$$\pi(\theta, \hat{U}) = (1-\tau)^2 \theta^2/2 - (1-\tau)\hat{U}(\theta).$$

Armed with this characterization, we can write the seller's problem as choosing the off-platform quality $\hat{q}$ and whether each type θ showrooms or not. Denote the latter decision by $h(\theta) = 1$ (consumer θ showrooms) and $h(\theta) = 0$ (consumer θ buys on the platform).

Claim 2 *The optimal showrooming policy is a cutoff policy: there exists $x \in (0, 1)$ such that all $\theta \geq x$ showroom and all $\theta < x$ buy on platform.*

To establish this claim, consider the difference in profit levels

$$\Delta(\theta) := (1-\tau)^2 \theta^2/2 - (1-\tau)\hat{U}(\theta) - (\theta\hat{q}(\theta) - \hat{q}(\theta)^2/2 - \hat{U}(\theta)).$$

Differentiating with respect to θ, and using the characterization of incentive compatibility off-platform $\hat{U}'(\theta) = \hat{q}(\theta)$, we obtain

$$\Delta'(\theta) = (1-\tau)\left((1-\tau)\theta - \hat{q}(\theta)\right) - \hat{q}'(\theta)\left(\theta - \hat{q}(\theta)\right) - \hat{q}(\theta).$$

Under the monotonicity assumption on the virtual values, we have $\hat{q}'(\theta) > 1$. Furthermore, using $\tau > 0$, we have

$$\Delta'(\theta) < -\hat{q}(\theta) < 0.$$

Finally, taking limits in $\Delta(\theta)$ for $\theta \to \{0,1\}$, and using the property that $\hat{q}(\overline{\theta}) = 1$ yields the finding that the cutoff x is interior.

Thus, the seller will choose to let high-value consumers showroom and to serve low values on the platform. This is of course intuitive, because commissions are costlier on high quality, high-price items. We can therefore write the seller's profits as

$$\begin{aligned} V \;=\; & \int_0^x \left(\theta\hat{q}(\theta) - \hat{q}(\theta)^2/2 - \hat{U}(\theta)\right)(1-\lambda)\,dG(\theta) \\ & + \int_0^x \left((1-\tau)^2\theta^2/2 - (1-\tau)\hat{U}(\theta)\right)\lambda dF(\theta) \\ & + \int_x^1 \left(\theta\hat{q}(\theta) - \hat{q}(\theta)^2/2 - \hat{U}(\theta)\right)\left((1-\lambda)\,dG(\theta) + \lambda dF(\theta)\right), \end{aligned}$$

where

$$\hat{U}(\theta) = \int_0^\theta \hat{q}(s)\,ds.$$

Proposition 12 (Variable Fees, No Clauses)
The optimal quality provision off platform is piecewise continuous, with an upward jump at a cutoff x. On-platform consumers with values $\theta \le x$ buy at the constrained-efficient quality level $q(\theta) = (1-\tau)\theta$, and on-platform consumers with values $\theta > x$ buy off platform.

Proof of Proposition 12. To establish this result, we rewrite the seller's dynamic optimization problem as choosing controls $h, \hat{q}$ with state $\hat{U}$. We can write the Hamiltonian $H(\theta, \hat{q}, \hat{U}, \gamma)$ as follows:

$$H = (\theta\hat{q} - \hat{q}^2/2 - \hat{U})\left((1-\lambda)g + h\lambda f\right) + \left((1-\tau)^2\theta^2/2 - (1-\tau)\hat{U}\right)(1-h)\lambda f + \gamma\hat{q}.$$

By the Maximum Principle, the necessary conditions for this problem are

$$(\theta - \hat{q})\left((1-\lambda) g + h\lambda f\right) + \gamma = 0$$
$$\theta\hat{q} - \hat{q}^2/2 - \hat{U} > (1-\tau)^2 \theta^2/2 - (1-\tau)\hat{U} \Rightarrow h = 1$$
$$(1-\lambda) g + h\lambda f + (1-\tau)\lambda f = \dot{\gamma}$$
$$\gamma(\overline{\theta}) = 0.$$

Using the cutoff result, we therefore have the following solution for the costate variable

$$\gamma(\theta) = (1-\lambda) G(\theta) + \lambda F(\theta) - \left(1 - \lambda\tau \int_\theta^1 (1-h)\, dF(s)\right)$$

which implies the following structure for the optimal quality off platform:

$$\hat{q}(\theta) = \theta - \begin{cases} \frac{1-\lambda F(\theta)-(1-\lambda)G(\theta)-\lambda\tau(F(x)-F(\theta))}{(1-\lambda)g(\theta)} & \text{for } \theta \leq x \\ \frac{1-\lambda F(\theta)-(1-\lambda)G(\theta)}{(1-\lambda)g(\theta)+\lambda f(\theta)} & \text{for } \theta > x. \end{cases}$$

This ends the proof. ■

Therefore, the optimal quality provision is separate for on- and off-platform consumers with values lower than x. It then jumps up (for off-platform consumers) and it attracts high-θ on-platform consumers too.

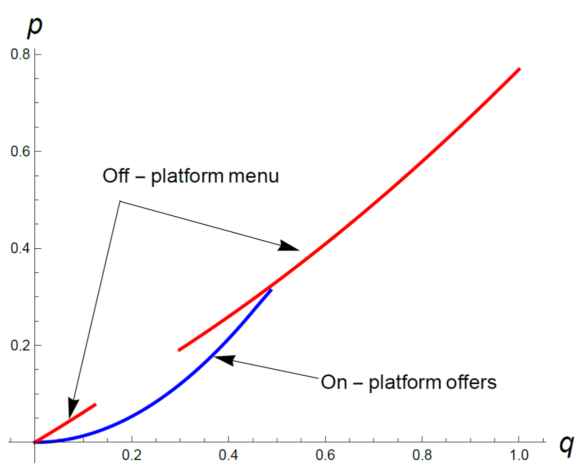

FIGURE 11: Optimal Menus without Price-Parity Clause

B.2 Price-Parity Clauses

Now suppose the seller imposes a price-parity clause that forces every on-platform consumer to buy the good advertised on the platform. In terms of the construction above, this policy is equivalent to setting $h = 0$ for all θ. Thus, we obtain the following result.

Proposition 13 (Variable Fees, Price Parity)
With a price-parity clause, the optimal quality provision on- and off-platform is given by

$$\begin{aligned} q(\theta) &= (1-\tau)\theta, \\ \hat{q}(\theta) &= \theta - \frac{1-\lambda\tau-\lambda(1-\tau)F(\theta)-(1-\lambda)G(\theta)}{(1-\lambda)g(\theta)}, \end{aligned}$$

respectively, and showrooming binds for all types:

$$U(\theta) = \hat{U}(\theta).$$

Despite the fact that the off-platform quality schedule is now continuous, note the similar properties of this allocation: the on-platform menu offers discounts relative to the off-platform one, but its quality range is limited. In other words, it is still difficult (in fact suboptimal) to sell high quality goods. As a result, high types purchase worse products at far lower prices on platform than off platform. Note that this is the opposite of what happens with fixed fees in the baseline analysis.

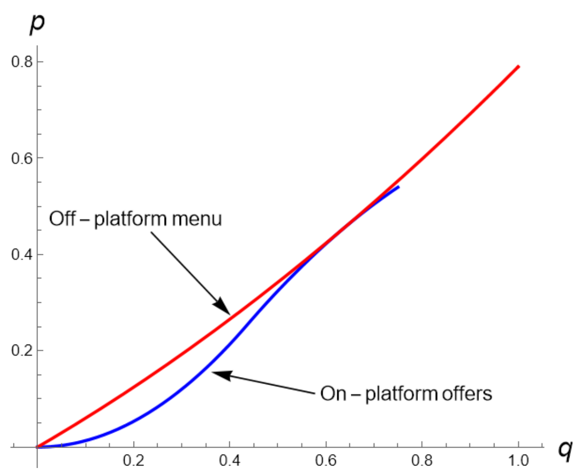

Figure 12: Optimal Menus with Price-Parity Clause