Figure 1: Trade War Tariffs and Export Growth


Notes: The panels show binscatter plots of the regression in (8), $\Delta \ln X=\alpha+\beta \Delta \ln T+\varepsilon$. This is a regression of bystanders' export growth (on the $y$-axes) against changes in tariffs due to the trade war (on the $x$-axes). Panel A is bystanders' exports to the US $\left(X_{i \omega}^{U S}\right)$ against the US tariffs $\left(T_{C H, \omega}^{U S}\right)$. Panels B is bystanders' exports to China ( $X_{i \omega}^{C H}$ ) against the China tariffs $\left(T_{U S, \omega}^{C H}\right)$. Panels C and D show bystanders' exports to $R W\left(X_{i \omega}^{R W}\right)$ against the US $\left(T_{C H, \omega}^{U S}\right)$ and China tariffs $\left(T_{U S, \omega}^{C H}\right)$, respectively. Also shown are the binscatters of the regressions with exports prior to the trade war from 2015-17. Below each panel are OLS coefficients, with standard errors clustered by product shown in parentheses. Panels A and B of Table A. 2 report the corresponding regression tables.

Figure 2: Relative Export Growth in Targeted Products across Countries


Notes: The figure plots changes in predicted exports to the world in taxed relative to untaxed products using (11):

$$
\Delta \widehat{\ln X_{i}^{W} D}=\sum_{\omega} \sum_{n=U S, C H, R W} \lambda_{i \omega}^{n}\left(\widehat{\beta_{1 i \omega}^{n}} \Delta \ln T_{C H, \omega}^{U S}+\widehat{\beta_{2 i \omega}^{n}} \Delta \ln T_{U S, \omega}^{C H}+\widehat{\beta_{3 i \omega}^{n}} \ln T_{i, \omega}^{U S}+\widehat{\beta_{4 i \omega}^{n}} \Delta \ln T_{i, \omega}^{C H}\right) .
$$

The $\beta^{\prime}$ s are estimated from the specification (10):

$$
\begin{gathered}
\Delta \ln X_{i \omega}^{n}=\beta_{1 i \omega}^{n} \Delta \ln T_{C H, \omega}^{U S}+\beta_{2 i \omega}^{n} \Delta \ln T_{U S, \omega}^{C H}+\beta_{3 i \omega}^{n} \Delta \ln T_{i, \omega}^{U S}+\beta_{4 i \omega}^{n} \Delta \ln T_{i, \omega}^{C H} \\
+\alpha_{i j(\omega)}^{n}+\Omega^{n} S I Z E_{i \omega}+\pi^{n} \Delta \ln X_{i \omega, t-1}^{n}+\epsilon_{i \omega}^{n}
\end{gathered}
$$

Bootstrapped error bars denote $90 \%$ confidence intervals. These bands are constructed by implementing (10) on 50 bootstrap samples and calculating countries' predicted exports using (11).

Figure 3: Decomposing Relative Exports by Heterogenous Response Type


Notes: Figure reports alternative predictions for exports to the world constructed using (11):

$$
\Delta \widehat{\ln X_{i}^{W} D}=\sum_{\omega} \sum_{n=U S, C H, R W} \lambda_{i \omega}^{n}\left(\widehat{\beta_{1 i \omega}^{n}} \Delta \ln T_{C H, \omega}^{U S}+\widehat{\beta_{2 i \omega}^{n}} \Delta \ln T_{U S, \omega}^{C H}+\widehat{\beta_{3 i \omega}^{n}} \ln T_{i, \omega}^{U S}+\widehat{\beta_{4 i \omega}^{n}} \Delta \ln T_{i, \omega}^{C H}\right)
$$

where the $\beta^{\prime}$ 's are estimated under alternative configurations of the heterogeneity in tariff responses. The first series (grey) constructs predicted exports assuming a homogenous response to the tariffs across countries. The next three series emphasize each of the three components of the full heterogenous response: sectoral $\left(\widehat{\beta_{z i \omega}^{n}}=\widehat{\beta_{z j(\omega)}^{n}}\right)$, size $\left(\widehat{\beta_{z i \omega}^{n}}=\right.$ $\left.\widehat{\Gamma_{z}^{n}} S I Z E_{z i \omega}\right)$, and country $\left(\widehat{\beta_{z i \omega}^{n}}=\widehat{\beta_{i}^{n}}\right.$ ). The 45-degree line (black) is the benchmark full heterogeneity series.

## Figure 4: Supply and Demand Forces



Notes: The figure plots the tariff responses to the US-China tariffs, $\widehat{\beta_{i \omega}^{n}}=\sum_{\omega} \lambda_{X i \omega}^{n} \widehat{\beta_{z i \omega}^{n}}$ using the taxonomy in Table A.1. Panel A plots $\left(\widehat{\beta_{1 i}^{U S}}, \widehat{\beta_{1 i}^{R W}}\right)$. Panel B plots $\left(\widehat{\beta_{2 i}^{C H}}, \widehat{\beta_{2 i}^{R W}}\right)$. Countries noted in blue operate in the same quadrant in both figures. Countries in red operate along downward-sloping supplies in both figures. Countries in green operate along upward-sloping supplies in both figures.

## Online Appendix: The US-China Trade War and Global Reallocations

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## A Appendix Figures and Tables

Figure A.1: Pre-War Export Baskets


Notes: Figure reports countries' pre-war export shares by sector. Agriculture includes products in HS code chapters 1-24; Apparel includes chapters 41-43 and 50-67; Chemicals includes chapters 28-38; Machinery includes chapters 84-85; Materials includes chapters 39-40, 44-49, and 68-71; Metals includes chapters 72-83; Minerals includes chapters 25-27; Transport includes chapters 86-89; and Miscellaneous includes chapters 90-99.

Figure A.2: Tariff Changes
Panel A: US Tariff Changes


Panel B: China Tariff Changes


Notes: Figure reports the set of tariff changes imposed by the US (Panel A) and China (Panel B), by sector. The tariff changes are scaled by total time in effect over the two year window. For example, if the US raised tariffs on a product from China in September 2018 by 10\%, the scaled tariff change over the two year window would be $6.66 \%=(16 / 24) *$ $10 \%$. If the tariff of a product went up $25 \%$ in September 2019, the scaled tariff change would be $4.16 \%(=(4 / 24) * 25 \%)$. The black dots indicate the median tariff increase, the boxes denote the 25 th and 75 th percentiles, and whiskers show the 10th and 90th percentiles.

Figure A.3: Trade War Tariffs and Export Changes for USA and CHN


Notes: The panels show binscatter plots of the regression in (8), $\Delta \ln X=\alpha+\beta \Delta \ln T+\varepsilon$, for US and China exports. Panel A is China's exports to the US $\left(X_{C H, \omega}^{U S}\right)$ against the US tariffs $\left(T_{C H, \omega}^{U S}\right)$. Panel B is US exports to China ( $X_{U S, \omega}^{C H}$ ) against the China tariffs $\left(T_{U S, \omega}^{C H}\right)$. Also shown are the binscatters of the regressions with exports prior to the trade war from 2015-17. Below each panel are OLS coefficients.

Figure A.4: Trade War Tariffs and Export Changes, with Fixed Effects


Notes: The panels show binscatter plots of the regression in (8), $\Delta \ln X=\alpha_{i j}+\beta \Delta \ln T+\varepsilon$. This is a regression of bystanders' export growth (on the y-axes) against changes in tariffs due to the trade war (on the x-axes), controlling for country-sector fixed effects. Panel A is bystanders' exports to the US $\left(X_{i \omega}^{U S}\right)$ against the US tariffs $\left(T_{C H, \omega}^{U S}\right)$. Panels B is bystanders' exports to China $\left(X_{i \omega}^{C H}\right)$ against the China tariffs $\left(T_{U S, \omega}^{C H}\right)$. Panels C and D show bystanders' exports to $R W$ ( $X_{i \omega}^{R W}$ ) against the US $\left(T_{C H, \omega}^{U S}\right)$ and China tariffs $\left(T_{U S, \omega}^{C H}\right)$, respectively. Also shown are the binscatters of the regressions with exports prior to the trade war from 2015-17. Below each panel are OLS coefficients (standard errors clustered by product). Panels C and D of Table A. 2 report the regression coefficients.

## Table A.1: Parameter Regions Implied by Export Responses to US Tariffs on China

|  | Country $i^{\prime}$ 's Export Response to US $\left(\beta_{1 i \omega}^{U S}\right)$ |  |
| :--- | :---: | :---: | :---: |
|  | Decrease | Increase |

Notes: Table shows the parameter regions implied by the export response of country to the US and to the rest of the world (RW) when the US increases tariffs on China. $\sigma_{C H i}$ represents the demand substitution between Chinese and country $i$ 's goods, while $b_{i}$ represents the inverse supply elasticity in country $i$. A similar taxonomy applies for China's tariffs on the US, in which case the responses would reveal substitutability with the US ( $\sigma_{U S i}$ instead of $\sigma_{C H i}$ ).

Table A.2: Regressions Corresponding to Figure 1 and Figure A. 4

Panel A: Pre-Period, Without Controls


Notes: Panels A and B report the regression output corresponding to Figure 1, and Panels C and D report the regression output corresponding to Figure A.4. Standard errors clustered by product shown in parentheses.

## B Model Appendix

## B. 1 Microfoundation of the Supply Side

We present a microfoundation for the supply curve in (3). We assume that, in country $i$ and sector $j$, a quantity $K_{T i}^{j}$ of a bundle of inputs and primary factors is used to produce tradeable goods in sector $j$ This sector-specific input supply could be determined endogenously through domestic or international mobility or be taken as given under the assumption of no factor mobility; however,
we do not need to take a stand for our empirical analysis.
This factor supply consists of a continuum of heterogeneous units, with each unit $k$ having productivity $z_{i \omega}^{0} e_{\omega}^{k}$. The term $z_{i \omega}^{0}$ is common to all inputs in $\omega$. It depends on an exogenous country-product specific component of productivity $Z_{i \omega}$ and, through scale economies, on the amount of inputs $K_{i \omega}$ allocated to the product:

$$
\begin{equation*}
z_{i \omega}^{0}=Z_{i \omega} K_{i \omega}^{\gamma_{i}^{j}}, \tag{B.13}
\end{equation*}
$$

where $\gamma_{i}^{j}$ is a country-sector specific scale elasticity. In turn, the term $e_{\omega}^{k}$ is specific to each unit with CDF from an iid Frechet distribution:

$$
\begin{equation*}
\operatorname{Pr}\left(e_{\omega}^{k}<x\right)=\exp \left(-x^{-\varepsilon_{i}^{j}}\right), \tag{B.14}
\end{equation*}
$$

where the parameter $\varepsilon_{i}^{j}$ is also country-specific and determines factor mobility across products in response to changes in factor returns.

Each unit of factors $k$ in sector $j$ chooses a product $\omega$ in that sector and, conditional on the product, a bundle of intermediate inputs $x$ with sector-specific intensity $\alpha_{j}^{I}$ and unit $\operatorname{cost} c_{i j}^{I}$, to maximize its returns $\pi_{i}^{k}$ :

$$
\begin{equation*}
\pi_{i}^{k} \equiv \max _{\omega} \max _{x}\left(p_{i \omega} z_{i \omega}^{0} e_{\omega}^{k}\right)^{1-\alpha_{j}^{I}} x^{\alpha_{j}^{I}}-c_{i j}^{I} x \tag{B.15}
\end{equation*}
$$

where $p_{i \omega}$ is the price received by producers of $\omega$ in country $i$. The input bundle used by each product combines output from other sectors. For our empirical analysis, we impose that $c_{i j \omega}^{I}$ does not vary across products within a sector, but may vary across sectors. This corresponds to the standard assumption of sector-level input-output matrixes. Maximizing out inputs $x$, the problem in (B.15) is equivalent to:

$$
\begin{equation*}
\pi_{i}^{k} \equiv \max _{\omega} p_{i \omega} z_{i \omega} e_{\omega}^{k}, \tag{B.16}
\end{equation*}
$$

where $z_{i \omega} \equiv\left(c_{i j}^{I} / \alpha_{j}^{I}\right)^{\frac{\alpha_{j}^{I}}{\alpha_{j}^{I}-1}} z_{i \omega}^{0}$ captures productivity and input costs of product $\omega$. From the solution to (B.16), the supply of inputs to product $\omega$ in sector $j$ of country $i$ is

$$
\begin{equation*}
K_{i \omega}=K_{T i}^{j}\left(\frac{p_{i \omega} z_{i \omega}}{r_{T i}^{j}}\right)^{\varepsilon_{i}^{j}} \tag{B.17}
\end{equation*}
$$

where $r_{T i}^{j}$ are the average factor returns in sector $j$ of country $i$. The distributional assumption in (B.14) implies that the average factor return by product is equalized across products within a sector, and therefore the total sales $X_{i \omega}$ vary within a sector only with the size of each product: $X_{i \omega}=r_{T i}^{j} K_{i \omega}$. Combining this property with (B.13) and (B.17) we obtain (3) in the text, where the inverse supply elasticity (defined as the elasticity of price with respect to total sales) is

$$
\begin{equation*}
b_{i}^{j}=\frac{1}{\varepsilon_{i}^{j}}-\gamma_{i}^{j}, \tag{B.18}
\end{equation*}
$$

the supply shifter is

$$
\begin{equation*}
A_{i}^{j} \equiv\left(c_{i j}^{I} / \alpha_{j}^{I}\right)^{\frac{\alpha_{j}^{I}}{\alpha_{j}^{I}-1}}\left(K_{T i}^{j}\right)^{\frac{1}{b_{i}^{j} \varepsilon_{i}^{j}}}\left(r_{T i j}^{j}\right)^{1-\frac{1}{b_{i}^{J}}} \tag{B.19}
\end{equation*}
$$

and the exogenous component of productivity is $Z_{i \omega} \equiv\left(Z_{i \omega}^{0}\right)^{\frac{1}{b_{i}^{j}}}$. The supply curve is upward-sloping as long as scale economies are not too strong $\left(\gamma_{i}^{j} \varepsilon_{i}^{j}<1\right)$. The average returns to inputs in the sector $r_{T i}^{j}$ must be such that the factor market clears within each sector, $\sum_{\omega \in \Omega^{j}} K_{i \omega}=$ $K_{T i}^{j}$, implying:

$$
\begin{equation*}
r_{T i}^{j}=\left(\sum_{\omega \in \Omega^{j}}\left(p_{i \omega} z_{i \omega}\right)^{\varepsilon_{i}^{j}}\right)^{\frac{1}{\varepsilon_{i}^{j}}} . \tag{B.20}
\end{equation*}
$$

Combining (B.13), (B.17), and (B.20), we obtain a function $r_{T i}^{j}$ as an implicit function of the goods prices $\left\{p_{i \omega}\right\}_{\omega \in \Omega^{j}}$ and the aggregate factor supply $K_{T i}^{j}$ in sector $j$.

## B. 2 Proof of Proposition 1

As a preliminary step, we derive some equilibrium equations in changes. In what follows, let $\hat{X} \equiv \frac{\Delta X}{X}$ denote the infinitesimal change in the log of variable $X$, where $\Delta X=X^{\prime}-X$ is the difference in the value of $X$ between a counterfactual and an initial equilibrium. Given tariff shocks $\left\{\hat{T}_{i \omega}^{n}\right\}$, to a first order approximation, the equilibrium consists of changes in tradeable prices $\left\{\hat{p}_{i \omega}\right\}$ such that
i) from (3), price changes are given by

$$
\begin{equation*}
\hat{p}_{i \omega}=b_{i}^{j} \hat{X}_{i \omega}-b_{i}^{j} \hat{A}_{i}^{j} ; \tag{B.21}
\end{equation*}
$$

ii) from (4), the changes in total sales are consistent with goods market clearing,

$$
\begin{equation*}
\hat{X}_{i \omega}=\sum_{n \in \mathcal{I}} \lambda_{i \omega}^{n}\left(\hat{s}_{i \omega}^{n}+\hat{E}^{n}-\hat{T}_{i \omega}^{n}\right), \tag{B.22}
\end{equation*}
$$

where $\lambda_{i \omega}^{n} \equiv \frac{X_{i \omega}^{n}}{X_{i \omega}}$ is the share of sales to $n$ in total sales of product $\omega$ from $i$, and where from (1) and (2), the changes in expenditure shares are

$$
\begin{equation*}
\hat{s}_{i \omega}^{n}=\frac{1}{s_{i \omega}^{n}} \sum_{i^{\prime} \in \mathcal{I}} \sigma_{i^{\prime} i}^{j}\left(\hat{T}_{i^{\prime} \omega}^{n}+\hat{p}_{i^{\prime} \omega}\right) . \tag{B.23}
\end{equation*}
$$

Take exporter $i \neq U S, C H$ and suppose that the US and China impose tariffs on each other and on other countries. From the market clearing condition (B.22) and the definition of expenditure shares (B.23), the total sales of $\omega$ from $i$ change around an initial equilibrium according to

$$
\begin{align*}
\hat{X}_{i \omega}= & \tilde{\lambda}_{i \omega}^{C H} \sigma_{U S i}^{j} \hat{T}_{U S, \omega}^{C H}+\tilde{\lambda}_{i \omega}^{U S} \sigma_{C H i}^{j} \hat{T}_{C H, \omega}^{U S} \\
& +\sigma_{i i}^{j} \hat{p}_{i \omega} \sum_{n \in \mathcal{I}} \tilde{\lambda}_{i \omega}^{n} \\
& +\hat{T}_{i \omega}^{\text {other }}+\sigma_{R W}^{j} \sum_{n \in \mathcal{I}} \sum_{i^{\prime} \neq i} \tilde{\lambda}_{i \omega}^{n} \hat{p}_{i^{\prime} \omega}+\sum_{n \in \mathcal{I}} \lambda_{i \omega}^{n} \hat{E}^{n}, \tag{B.24}
\end{align*}
$$

where we have imposed the restriction that $\sigma_{R W}^{j}=\sigma_{i^{\prime} i}^{j}$ for $i^{\prime}, i \neq U S, C H$ and $i^{\prime} \neq i$ and where, to shorten notation, we have defined $\tilde{\lambda}_{i \omega}^{n} \equiv \frac{\lambda_{i \omega}^{n}}{s_{i \omega}^{n}}=\frac{E_{\omega}^{n}}{X_{i \omega}}$.

The two terms in the first line of (B.24) capture the direct impact of US and Chinese tariffs country $i$ 's exports these two markets. For example, the first of these terms says that a bigger Chinese tariff on the US reallocates Chinese demand to country $i$ if country $i$ and the US are
substitutes ( $\sigma_{U S i}>0$ ); in percentage, this reallocation is larger the bigger is Chinese expenditure in product $\omega$ (a larger $E_{\omega}^{C H}$ ) or the smaller are the initial sales of $\omega$ from $i$ (a smaller $X_{i \omega}$ ). The second line of (B.24) is the change in sales due to the change in variety $i \omega$ 's price. Finally, in the third line of (B.24), $\hat{T}_{i \omega}^{\text {other }}$ captures the impact on country $i$ of US and China tariffs imposed on countries other than each other,

$$
\begin{equation*}
\hat{T}_{i \omega}^{\text {other }}=\sum_{n=U S, C H}\left(\sigma_{i i}^{j} \tilde{\lambda}_{i \omega}^{n}-\lambda_{i \omega}^{n}\right) \hat{T}_{i \omega}^{n}+\sigma_{R W}^{j} \sum_{i^{\prime} \neq C H, U S, i}\left(\tilde{\lambda}_{i \omega}^{C H} \hat{T}_{i^{\prime} \omega}^{C H}+\tilde{\lambda}_{i \omega}^{U S} \hat{T}_{i^{\prime} \omega}^{U S}\right) . \tag{B.25}
\end{equation*}
$$

The remaining terms in the third line capture changes in prices of other varieties and in aggregate expenditures.

Combining (B.24) with the inverse supply (B.21) and solving for $\hat{p}_{i \omega}$ we obtain the price change of variety $i \omega$ :

$$
\begin{align*}
\hat{p}_{i \omega}= & \frac{b_{i}^{j}}{1-b_{i}^{j} \sigma_{i i}^{j} \sum_{n \in \mathcal{I}} \tilde{\lambda}_{i \omega}^{n}}\left(\tilde{\lambda}_{i \omega}^{U S} \sigma_{C H i}^{j} \hat{T}_{C H, \omega}^{U S}+\tilde{\lambda}_{i \omega}^{C H} \sigma_{U S i}^{j} \hat{T}_{U S, \omega}^{C H}+\hat{T}_{i \omega}^{\text {other }}+\sum_{n \in \mathcal{I}} \sum_{i^{\prime} \neq i} \tilde{\lambda}_{i \omega}^{n} \sigma_{i^{\prime} i}^{j} \hat{p}_{i^{\prime} \omega}+\sum_{n \in \mathcal{I}} \lambda_{i \omega}^{n} \hat{E}^{n}\right) \\
& -\frac{b_{i}^{j} \hat{A}_{i}^{j}}{1-b_{i}^{j} \sigma_{i i}^{j} \sum_{n} \tilde{\lambda}_{i \omega}^{n}} . \tag{B.26}
\end{align*}
$$

Consider now the change in sales from $i$ to a specific destination $n$ :

$$
\begin{equation*}
\hat{X}_{i \omega}^{n}=\hat{E}^{n}+\hat{s}_{i \omega}^{n}-\hat{T}_{i \omega}^{n} \tag{B.27}
\end{equation*}
$$

Combining (B.23), (B.26), and (B.27) with this expression we obtain:

$$
\begin{align*}
\hat{X}_{i \omega}^{n}= & \left(1_{n=U S}+\frac{b_{i}^{j} \sigma_{i i}^{j} \tilde{\lambda}_{i \omega}^{U S}}{1-b_{i}^{j} \sigma_{i i}^{j} \sum_{n^{\prime} \in \mathcal{I}} \tilde{\lambda}_{i \omega}^{n^{\prime}}}\right) \frac{\sigma_{C H i}^{j}}{s_{i \omega}^{n}} \hat{T}_{C H, \omega}^{U S}+1_{n=U S}\left(\left(\frac{\sigma_{i i}^{j}}{s_{i \omega}^{U S}}-1\right) \hat{T}_{i \omega}^{U S}+\frac{\sigma_{R W}^{j}}{s_{i \omega}^{U S}} \sum_{i^{\prime} \neq i, C H, U S} \hat{T}_{i^{\prime} \omega}^{U S}\right) \\
& +\left(1_{n=C H}+\frac{b_{i}^{j} \sigma_{i i}^{j} \tilde{\lambda}_{i \omega}^{C H}}{1-b_{i}^{j} \sigma_{i i}^{j} \sum_{n^{\prime} \in \mathcal{I}} \tilde{\lambda}_{i \omega}^{n^{\prime}}}\right) \frac{\sigma_{U S S}^{j}}{s_{i \omega}^{n}} \hat{T}_{U S, \omega}^{C H}+1_{n=C H}\left(\left(\frac{\sigma_{i i}^{j}}{s_{i \omega}^{C H}}-1\right) \hat{T}_{i \omega}^{C H}+\frac{\sigma_{R W}^{j}}{s_{i \omega}^{C H}} \sum_{i^{\prime} \neq i, C H, U S} \hat{T}_{i^{\prime} \omega}^{C H}\right) \\
& +\frac{1}{s_{i \omega}^{n}} \frac{b_{i}^{j} \sigma_{i i}^{j}}{1-b_{i}^{j} \sigma_{i i}^{j} \sum_{n^{\prime} \in \mathcal{I}} \tilde{\lambda}_{i \omega}^{n^{\prime}}} \hat{T}_{i \omega}^{\text {other }}+\eta_{i \omega}^{n} \tag{B.28}
\end{align*}
$$

where $\eta_{i \omega}^{n}$ is defined in (7) in the main text.Using (B.25) and rearranging terms in (B.28) we obtain

$$
\begin{align*}
\hat{X}_{i \omega}^{n}= & \underbrace{\left(1_{n=U S}+\frac{b_{i}^{j} \sigma_{i i}^{j} \tilde{\lambda}_{i \omega}^{U S}}{1-b_{i}^{j} \sigma_{i i}^{j} \sum_{n^{\prime} \in \mathcal{I}} \tilde{\lambda}_{i \omega}^{n^{\prime}}}\right) \frac{\sigma_{C H i}^{j}}{s_{i \omega}^{n}}}_{\beta_{1 i \omega}^{n}} \hat{T}_{C H, \omega}^{U S}+\underbrace{\left(1_{n=C H}+\frac{b_{i}^{j} \sigma_{i i}^{j} \tilde{\lambda}_{i \omega}^{C H}}{1-b_{i}^{j} \sigma_{i i}^{j} \sum_{n^{\prime} \in \mathcal{I}} \tilde{\lambda}_{i \omega}^{n^{\prime}}}\right) \frac{\sigma_{U S S}^{j}}{s_{i \omega}^{n}}}_{\beta_{2 i \omega}^{n}} \hat{T}_{U S, \omega}^{C H} \\
& +\sum_{n^{\prime}=U S, C H}\left(1_{n=n^{\prime}}\left(\frac{\sigma_{i i}^{j}}{s_{i \omega}^{n^{\prime}}}-1\right)+\frac{b_{i}^{j} \sigma_{i i \omega}^{j} \tilde{\lambda}_{i \omega}^{n^{\prime}}}{1-b_{i}^{j} \sigma_{i i}^{j} \sum_{m \in \mathcal{I}} \tilde{\lambda}_{i \omega}^{m}} \frac{\sigma_{i i}^{j}-s_{i \omega}^{n^{\prime}}}{s_{i \omega}^{n}}\right) \hat{T}_{i \omega}^{n^{\prime}} \\
& +\sum_{n^{\prime}=U S, C H}\left(1_{n=n^{\prime}}+\frac{b_{i}^{j} \sigma_{i i}^{j} \tilde{\lambda}_{i \omega}^{n^{\prime}}}{1-b_{i}^{j} \sigma_{i i}^{j} \sum_{m \in \mathcal{I}} \tilde{\lambda}_{i \omega}^{m}}\right) \frac{\sigma_{R W}^{j}}{s_{i \omega}^{n}} \sum_{j \neq C H, U S, i} \hat{T}_{j \omega}^{n^{\prime}} \\
& +\eta_{i \omega}^{n} . \tag{B.29}
\end{align*}
$$

Using the definition of $\tilde{\lambda}_{i \omega}^{n}$ and the fact that $s_{i \omega}^{n} \equiv \frac{T_{i \omega}^{n} X_{\sum_{\omega}^{n}}^{n}}{E_{\omega}^{n}}$, we can write equation (B.29) as (5), where

$$
\begin{align*}
& \beta_{1 i \omega}^{n}=\left(1_{n=U S}+\frac{E_{\omega}^{U S}}{E_{\omega}} \frac{\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega} / E_{\omega}}}{1-\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega} / E_{\omega}}}\right) \frac{\sigma_{C H i}^{j}}{s_{i \omega}^{n}},  \tag{B.30}\\
& \beta_{2 i \omega}^{n}=\left(1_{n=C H}+\frac{E_{\omega}^{C H}}{E_{\omega}} \frac{\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega} / E_{\omega}}}{1-\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega} / E_{\omega}}}\right) \frac{\sigma_{U S S i}^{j}}{s_{i \omega}^{n}},  \tag{B.31}\\
& \beta_{3 i \omega}^{n}=1_{n=U S}\left(\frac{\sigma_{i i}^{j}}{s_{i \omega}^{U S}}-1\right)+\frac{E_{\omega}^{U S}}{E_{\omega}} \frac{b_{i}^{j}}{\frac{b_{i}^{j}\left(\sigma_{i i}^{j}-s_{i \omega}^{U S}\right)}{X_{i \omega} / E_{\omega}}} 1-\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega} / E_{\omega}}  \tag{B.32}\\
& \sigma_{i i \omega}^{j}  \tag{B.33}\\
& s_{i \omega}^{n} \tag{B.34}
\end{align*},
$$

and where $\eta_{i \omega}^{n}$ is given by (7).

## B. 3 Proof of Proposition 2

Focus on (6). Using $X_{i \omega}^{n}=\frac{E_{\omega}^{n} s_{i \omega}^{n}}{T_{i \omega}^{n}}$ we can write $\beta_{1 i \omega}^{n} \equiv\left(1_{n=U S}+\frac{b_{i}^{j} \sigma_{i \omega}^{j} E_{\omega}^{U S}}{X_{i \omega}-b_{i}^{j} \sigma_{i i}^{j} E_{\omega}}\right) \frac{\sigma_{C H i}^{j}}{s_{i \omega}^{n}}$.To the US, $\beta_{1 i \omega}^{U S} \equiv\left(\frac{1-\left(1-\frac{E_{\omega}^{U S}}{E_{\omega}}\right) \frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega}}{ }_{1-\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega} / E_{\omega}}}^{X_{i \omega} / E_{\omega}}}{}\right) \frac{\sigma_{C H i}^{j}}{S_{i \omega}^{H}}$. Hence, if $\sigma_{C H i}^{j}>0$ then $\beta_{1 i \omega}^{U S}>0$ if $\min \left(\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega}^{i / E_{\omega}}}\right)<1$ or if $1<\left(1-E_{\omega}^{U S} / E_{\omega}\right) \frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega} / E_{\omega}}$, i.e. iff $\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega} / E_{\omega}} \in(-\infty, 1] \cup\left[\frac{1}{1-E_{\omega}^{U S} / E_{\omega}}, \infty\right)$, and $\beta_{1 i \omega}^{U S}<0$ otherwise. Similarly, to RW, $\beta_{1 i \omega}^{R W} \equiv\left(\frac{\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega}^{j} / E_{\omega}}}{1-\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega} / E_{\omega}}}\right) \frac{E_{\omega}^{U S}}{E_{\omega}} \frac{\sigma_{C H i}^{j}}{S_{i \omega}^{n}}$. Hence, conditional on $\sigma_{C H i}^{j}>0, \beta_{1 i \omega}^{R W}<0$ if $\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega} / E_{\omega}}<0$ or $\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega} / E_{\omega}}$; hence, $\beta_{1 i \omega}^{R W}>0$ whenever $\frac{b_{i}^{j} \sigma_{i i}^{j}}{X_{i \omega} / E_{\omega}} \in(0,1)$, and $\beta_{1 i \omega}^{R W}<0$ otherwise.

