# Online Appendix: Repression and Repertoires

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## Example

Suppose $B(e) = e^m$, $0 < m < 1$, and $C(e) = c^l$, $l > m$, and set $e_{max} = 2 > \tilde{e} = 1$. Then, $C'(\bar{e}) = B'(\bar{e})$ implies $(\bar{e})^{l-m} = m/l$. Moreover, using integration by parts, equation (3) can be written as:

$$\int_{p=0}^{1} e^*(p)dp = \bar{e} + \int_{e=\bar{e}}^{e_{max}} \frac{B'(e)}{C'(e)} de.$$

Thus,

$$\begin{aligned}
\theta^{**}_{\infty} &= \bar{e} + \int_{e=\bar{e}}^{e_{max}} \frac{m}{l}\, e^{m-l} de = \bar{e} + \left[\frac{m}{l}\frac{(e)^{m-l+1}}{m-l+1}\right]_{e=\bar{e}}^{e=e_{max}} \\
&= \bar{e} + \frac{m}{l}\frac{(e_{max})^{m+1-l}}{m+1-l} - \frac{m}{l}\frac{(\bar{e})^{m+1-l}}{m+1-l} \\
&= (\bar{e})^{m+1-l}\,\frac{m}{l} + \frac{m}{l}\frac{(e_{max})^{m+1-l}}{m+1-l} - \frac{m}{l}\frac{(\bar{e})^{m+1-l}}{m+1-l} \quad \text{(substituting for } \bar{e}) \\
&= (e_{max})^{m+1-l}\,\frac{m}{l}\frac{1}{m+1-l} + (\bar{e})^{m+1-l}\,\frac{m}{l}\left(1 - \frac{1}{m+1-l}\right). \qquad (17)
\end{aligned}$$

If the opposition leader restricts efforts to a single effort level $e$, from (6), we have

$$\hat{e} = \arg\max_{e\in[0,e_{max}]} \begin{cases} e & ; e \le \tilde{e} \\ e^{m-l+1} & ; e \ge \tilde{e} \end{cases} = \begin{cases} \tilde{e} & ; m+1 < l \\ [\tilde{e}, e_{max}] & ; m+1 = l \\ e_{max} & ; m+1 > l. \end{cases}$$

Thus, recalling that $\tilde{e} = 1$,

$$\theta^{**}_1 = \begin{cases} \tilde{e} & ; m+1 \le l \\ (e_{max})^{m+1-l} & ; m+1 \ge l. \end{cases} \qquad (18)$$

First, consider the comparison between $\theta^{**}_1$ and $\tilde{\theta}^{**}_{\infty}$ from Proposition 2. Mirroring the calculations leading to equation (17),

$$\begin{aligned}
\tilde{\theta}^{**}_{\infty} &= \tilde{e} + \int_{e=\tilde{e}}^{e_{max}} \frac{m}{l}\, e^{m-l} de = \tilde{e} + \left[\frac{m}{l}\frac{(e)^{m-l+1}}{m-l+1}\right]_{e=\tilde{e}}^{e=e_{max}} \\
&= \tilde{e} + \frac{m}{l}\frac{(e_{max})^{m+1-l}}{m+1-l} - \frac{m}{l}\frac{(\tilde{e})^{m+1-l}}{m+1-l}.
\end{aligned}$$

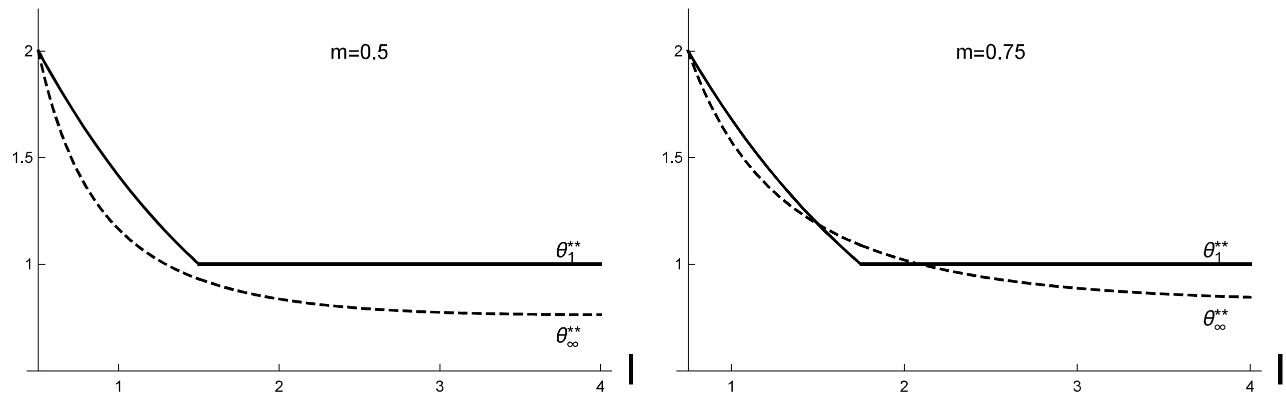


Figure 1: $\theta_1^{**}$ and $\theta_\infty^{**}$ as a function of $l$ for two values of $m$. When $l \leq 1$, so that $C(e)$ is concave, $\theta_1^{**} > \theta_\infty^{**}$. When $l > 1$, so that $C(e)$ is strictly convex, we can have $\theta_1^{**} < \theta_\infty^{**}$. Parameters: $B(e) = e^m,\ 0 < m < 1,\ C(e) = c^l,\ l > m$, and $e_{max} = 2 > \tilde{e} = 1$.

Thus, recognizing that $\tilde{e} = 1$,

$$\theta_1^{**} - \tilde{\theta}_\infty^{**} = \begin{cases} -\int_{e=\tilde{e}}^{e_{max}} \frac{m}{l}\ e^{m-l} de & ; m+1 < l \\ (e_{max})^{m+1-l}\ \left(1 - \frac{m}{l}\frac{1}{m+1-l}\right) - \left(1 - \frac{m}{l}\frac{1}{m+1-l}\right) & ; m+1 > l. \\ = \left[(e_{max})^{m+1-l} - 1\right]\ \frac{(l-m)}{l(m+1-l)}\ (1-l), & \end{cases}$$

as prescribed by Proposition 2.

Now, consider the comparison of $\theta_1^{**}$ and $\theta_\infty^{**}$. If $m+1 > l$, then from equations (17) and (18),

$$\begin{aligned} \theta_1^{**} - \theta_\infty^{**} &= (e_{max})^{m+1-l} \left(1 - \frac{m}{l}\frac{1}{m+1-l}\right) - (\bar{e})^{m+1-l}\ \frac{m}{l}\ \left(1 - \frac{1}{m+1-l}\right) \\ &= \frac{l-m}{l(m+1-l)}\ \left[(e_{max})^{m+1-l}\ (1-l) + (\bar{e})^{m+1-l}\ m\right] \end{aligned}$$

If $C(e)$ is concave, so that $l \leq 1$, then $\theta_1^{**} - \theta_\infty^{**} > 0$ as prescribed by Proposition 3. This result also reflects that even when $l > 1$, when efforts are not restricted to be greater than $\tilde{e}$, convexity is not sufficient to deliver $\theta_1^{**} < \theta_\infty^{**}$.

If $m + 1 < l$, then from equations (17) and (18),

$$\theta_1^{**} - \theta_\infty^{**} = [\tilde{e} - \bar{e}] - \left[(e_{max})^{m+1-l} - (\bar{e})^{m+1-l}\right]\ \frac{m}{l}\frac{1}{m+1-l}.$$

Figure 1 illustrates.