Online Appendix for

"Time to Repay or Time to Delay? The Effect of Having More Time Before a Payday Loan is Due"

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June 18, 2021

1 Robustness and Heterogeneity of Main Results

In this section, we repeat the baseline analysis as in Table 3 and Figure 6 of Section 4.3.2 in the main text for different groups of borrowers to check for robustness and heterogeneity of our main empirical results. Additionally, we show that our regression results in Table 3 are robust to the exclusion of control variables.

First, we demonstrate the robustness of our main regression results in Table A1. We use the same biweekly sample as in Table 3 but focus only on the first observation for each borrower. Specifications of the underlying regressions are identical to those in Table 3.

Table A1: Regression Results

Biweekly Sample Restricted to First Observations				
	(1)	(2)	(3)	(4)
	Principal paid on first due date	Rolled over some of the loan at the first due date	Number of effective rollovers in loan spell	Total finance charges paid in loan spell
Mean	\$79.02	0.66	3.14	\$218.72

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	(1)	(2)	(3)	(4)
Grace	-1.82	-0.02	-0.38^{***}	-19.04^{**}
	(4.52)	(0.01)	(0.12)	(8.15)
Other Controls	Yes	Yes	Yes	Yes
N	6,778	6,778	6,019	6,019
R^2	0.17	0.11	0.08	0.12

Note: *Grace* is the indicator for having only six days until payday. Data are based on authors' calculations from administrative data from a large payday lender. OLS regressions are shown for four outcomes: Principal paid on first due date calculates the amount of the loan paid by the first due date; Rolled over some of the loan at first due date indicates that the borrower rolled over the loan at the first due date; Number of Effective Rollovers is a variable that counts the number of additional loans in succession by a borrower; and Total Finance is the total finance charged over the loan cycle. Sample is restricted to first observations of borrowers paid biweekly. Controls in all columns include loan size, gender, annual net pay, checking account balance, subprime credit score, and age bins. Dummies for race (White, Black, Hispanic, or other), having paycheck direct deposited, missing control variables, month-year, and each payday loan shop are also included. Columns 3-4 include fewer observations because we did not include loans initiated with less than five pay periods before the end of our sample so as to not artificially truncate these outcomes. Standard errors are clustered at the day the loan was initiated and are reported in parentheses below the coefficients. $***$, $**$, and $*$ designate statistical significance at the 1%, 5%, and 10% level, respectively.

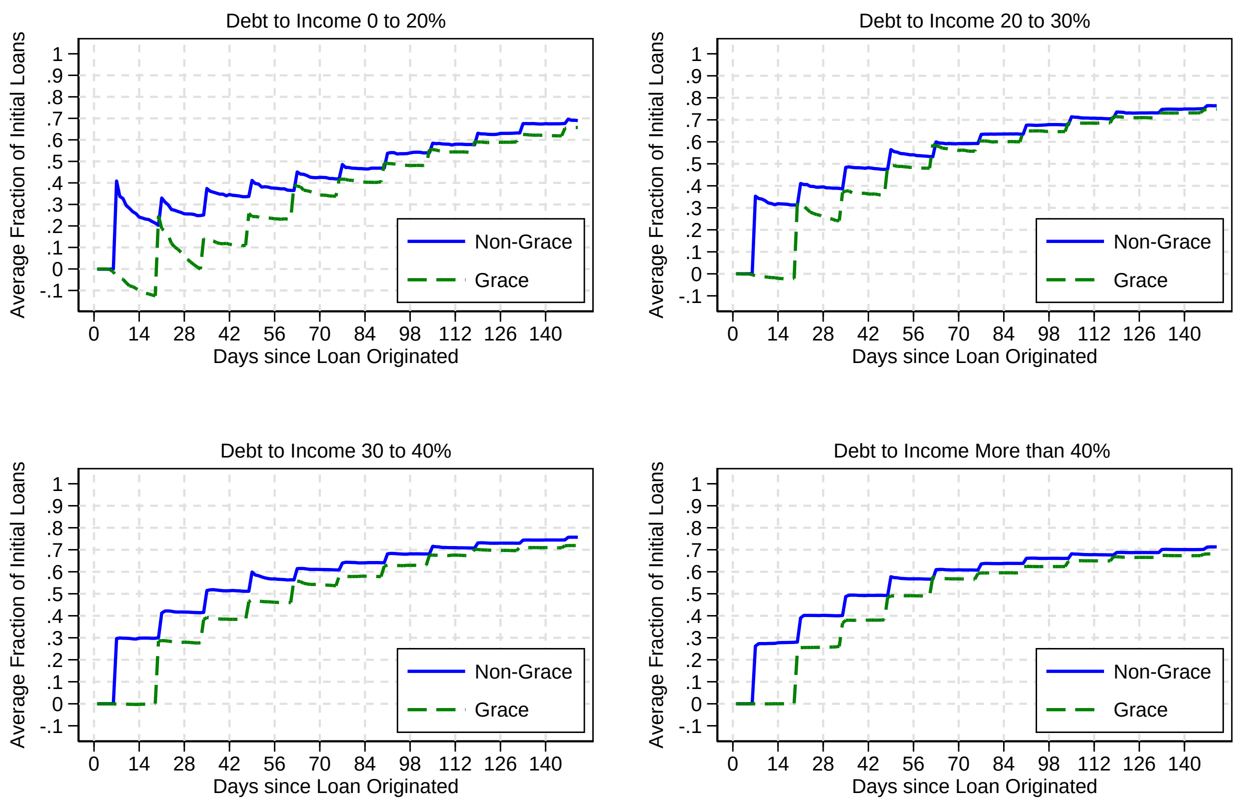
Table A2: Regression Results without Controls

Biweekly Sample (Sample Restricted to Origination Date Six and Seven Days until Payday)				
	(1)	(2)	(3)	(4)
	Principal paid on first due date	Rolled over some of the loan at the first due date	Number of effective rollovers in loan spell	Total finance charges paid in loan spell
Mean	\$88.84	0.64	2.98	\$208.55
Grace	-6.92	-0.00	-0.35^{***}	-17.40^{***}
	(4.23)	(0.01)	(0.09)	(5.54)
Other Controls	No	No	No	No
N	15,491	15,491	14,073	14,073
R^2	0.00	0.00	0.00	0.00

Note: The above table repeats the regression analysis in Table 3 of the main manuscript, excluding all the control variables. Everything else in the regression specification remains unchanged.

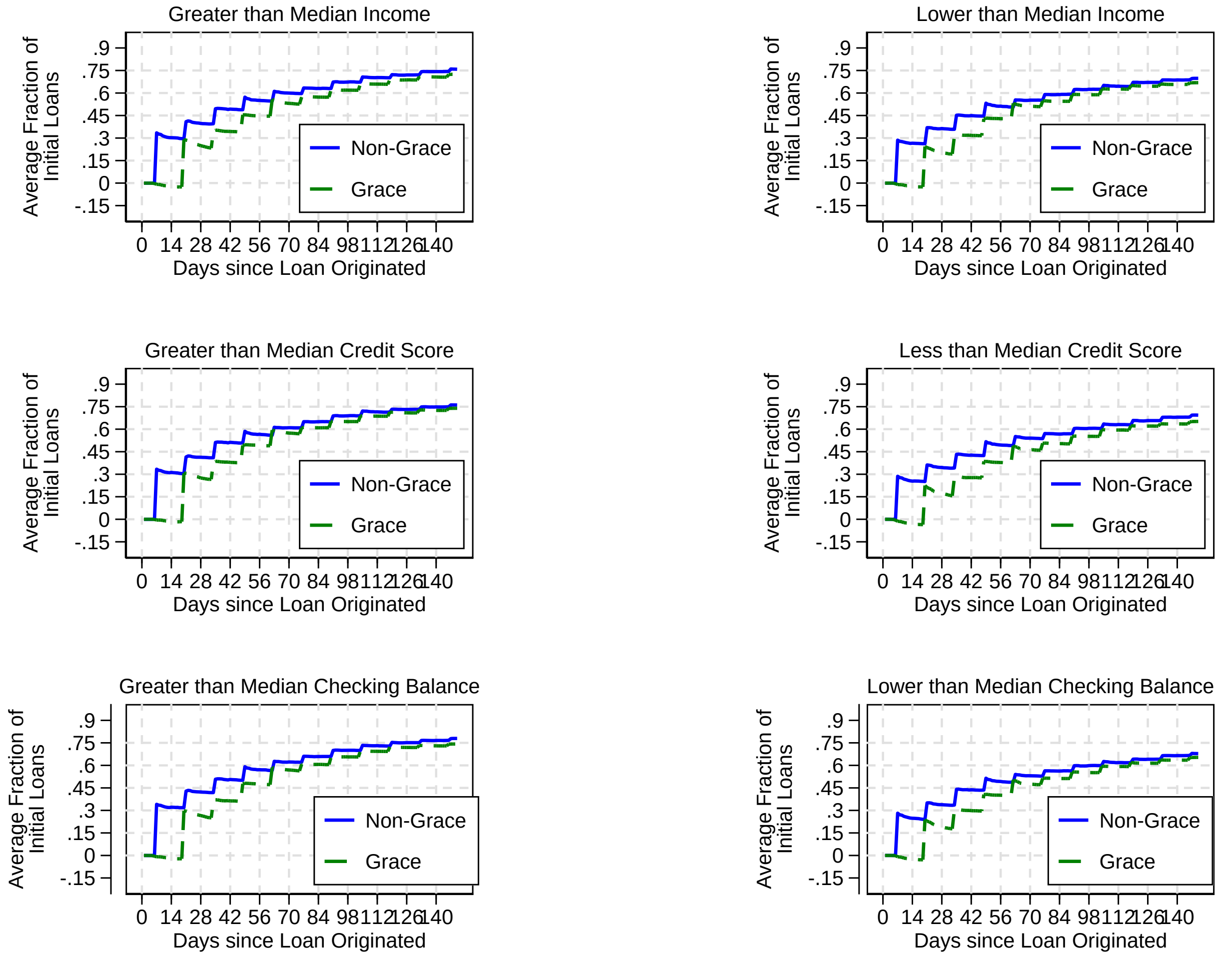
Secondly, we show in Table A2 that our main empirical results concerning the impact of a grace period on a range of repayment behavior are robust when we take out all the control variables in our regression model. Furthermore, the figures below examine the average fraction of initial loan that is repaid every day since the initial loan was taken out. The main purpose here is to investigate whether there is heterogeneity in the response to grace periods across the payday loan borrowers.

Figure A1: Average Fraction of Initial Load Repaid by Debt-to-Income Ratios



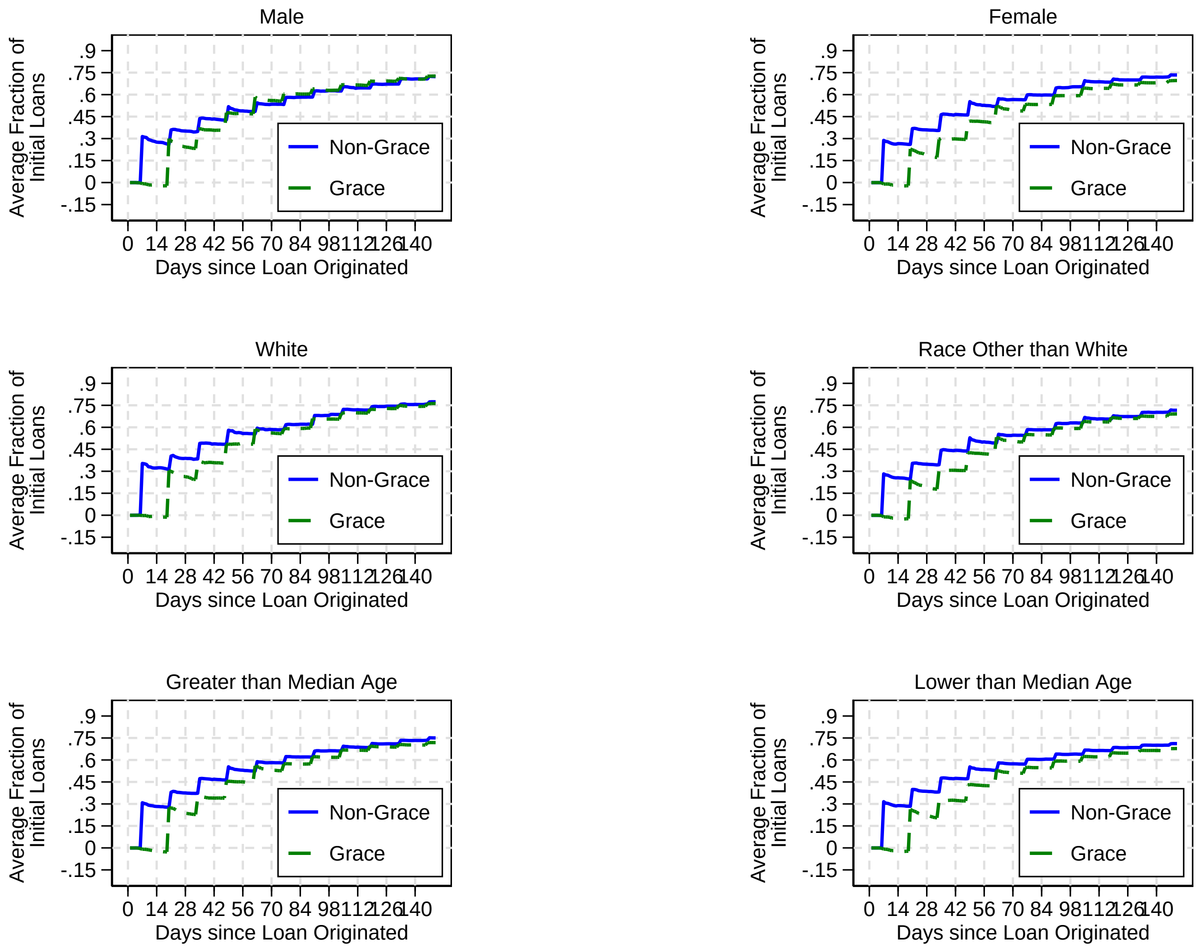
In Figure A1, we split the groups by the payday loan debt-to-income ratio, i.e., the amount of the loan divided by their income for the period. We split the groups into 0 to 20%, 20 to 30%, 30 to 40%, and 40% or more debt-to-income ratio. There are differences in repayment behavior by group: Those who took out lower percentages of debt to income are more likely to take out larger loans. However, the push-off of repayment by those with a grace period in their repayment remains consistent.

Figure A2: Average Fraction of Initial Load Repaid by Financial Conditions



In Figure A2 we categorize groups by financial characteristics: income, credit score, and checking account balance. For each of these groups we split them by above and below median. We again find similar patterns between the grace and non-grace groups as those found in Figure 6.

Figure A3: Average Fraction of Initial Load Repaid by Demographics



Finally, in Figure A3 we split the groups by demographics: gender, race, and age. Again, in each group we find that the grace-period individuals push off their repayment by a pay period. Altogether, we find that repayment differences between those with grace and non-grace loans do not vary by demographic characteristics, and it does not appear our results are driven by a subset of our population.

2 Empirical Analysis for Borrowers Paid Semimonthly

Borrowers paid semimonthly typically receive paychecks on the 15th of the month and either the end of the month or the first of the month. Therefore, borrowers who come in on the 8th of the month will typically receive a seven-day loan, while those who come in on the 9th of the month (six days before their next payday) will get an extra pay cycle to repay their loan. This creates a similar discontinuity to borrowers paid biweekly. There is a similar discontinuity around the 24th of the month, although it is less clean given variation in the number of days in a month and the exact day borrowers are paid. Below we present analogous figures and tables for borrowers paid semimonthly as those shown for borrowers paid biweekly in the main text.

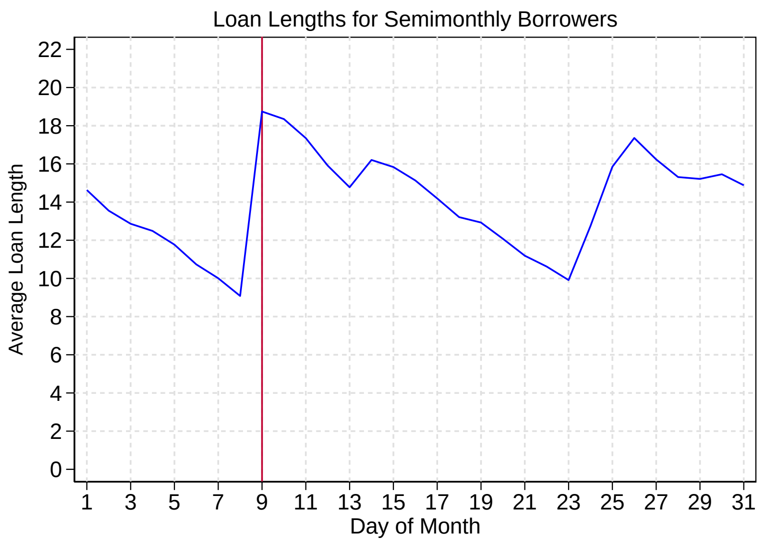
Table A3: Summary Statistics for Borrowers Paid Semimonthly

	Semimonthly Sample	Semimonthly Sample (restricted to the 8th and 9th of the month)
Borrower Characteristics for Initial Loans		
Age	35.98	35.85
	(9.96)	(9.92)
Female	69%	67%
White	25%	29%
Black	40%	40%
Hispanic	33%	30%
Race, other	1%	1%
Homeowner	38%	35%
Direct Deposit	75%	77%
Annualized Net Pay ($)	24,238.66	24,415.79
	(9,622.45)	(9566.44)
Checking Balance ($)	329.10	328.92
	(482.99)	(467.57)
Credit Score ($)	543.75	545.63
	(210.13)	(208.29)
Initial Loan Characteristics		
Principal of Initial Loan ($)	324.87	311.71
	(139.96)	(140.74)
Interest Due on Initial Loan ($)	58.48	56.11
	(25.19)	(25.33)
Initial Loan Duration (days)	13.67	13.66
	(4.61)	(6.66)
Initial Loan Outcomes		
Principal paid on first due date ($)	92.92	92.27
	(159.63)	(160.29)
Rollover on first due date (%)	66%	64%
Number of Effective Rollovers in Loan Spell	2.90	2.82
	(4.27)	(4.26)
Total Finance Charges Paid in Loan Spell ($)	217.34	205.89
	(287.58)	(273.01)
Loan Spell Ended with Default (%)	19%	20%
Total Number of Initial Loans	28,213	2,072
Total Number of Loans (including Rollovers)	110,042	7,920

Note: Means of all variables shown, with standard deviations in parentheses for continuous variables. Data are based on authors' calculations from administrative data from a large payday lender in Texas from

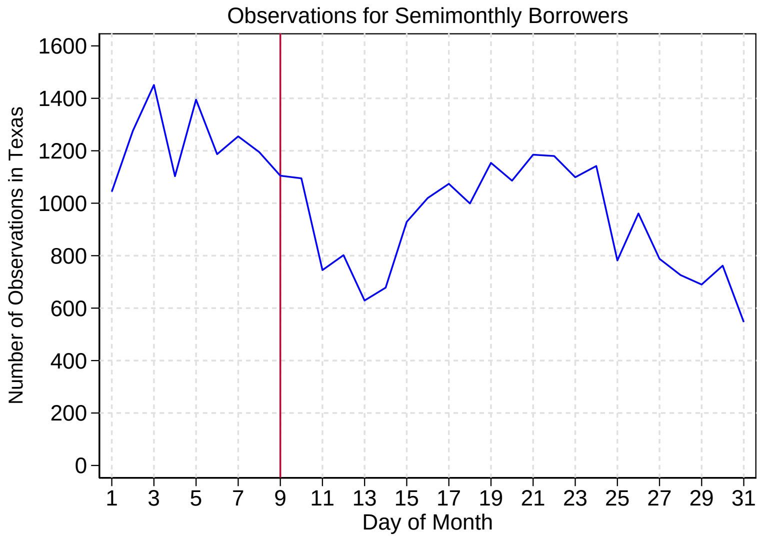
November 2000-August 2004. Initial loans are loans where the borrower did not have a loan outstanding for at least 32 days prior to initiation. Our administrative records do not include demographic information for all borrowers, and we have gender, race, and home ownership information for around 50% of the sample.

Figure A4: Loan Length
Borrowers Paid Semimonthly



Note: Authors' calculations based on payday loan transaction data in Texas from November 2001 to August 2004. The figure reports the average loan length for borrowers paid semimonthly. If workers paid semimonthly arrive at the lender on the 8th day of the month, they will typically receive a loan lasting seven days. If, however, they arrive on the 9th day of the month, there are only six days until their next pay date; hence they will instead have 21 days to repay their loan (six days until next payday plus the 15 days of their next pay cycle). Since there is some variation in exact pay dates (e.g., months when the 15th falls on a Sunday), the observed variation does not exactly match the hypothetical case outlined above. However, there is a clear jump in average loan length between loans originated on the 8th and loans originated on the 9th day of the month. Borrowers obtaining loans on the 8th day of the month have on average 9 days to repay that initial loan, while borrowers on the 9th day have an average of 19 days to repay their loan. Because the number of days in a month varies and some borrowers paid semimonthly get paid at the end of the month rather than the first of the month, the second jump in loan lengths (between the 23rd and 24th of the month) is less precise and therefore we do not use it in our subsequent analyses for borrowers paid semimonthly.

Figure A5: Loan Observations
Borrowers Paid Semi-monthly



Note: Authors' calculations based on payday loan transaction data in Texas from November 2001 until August 2004. The figure reports the number of observations for borrowers paid semimonthly for each day of the month.

Table A4: Control Variables as Outcomes for Borrowers Paid Semimonthly

	(1) Mean	(2) Grace (9th Day of the Month)	(3) Sample Size (Restricted to Origination Date on 8th and 9th Day of Month)
Subprime Credit Score	545.63	-2.46 (9.11)	2,072
Loan Amount	$311.71	-3.06 (6.24)	2,072
Net Pay	$24,415.79	510.57 (421.75)	2,072
Account Balance	$328.92	14.43 (20.83)	2,072
Direct Deposit	0.77	0.004 (0.02)	2,072
Age	35.85	0.39 (0.43)	2,071
Female	0.67	0.01	899

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		(0.03)	
Black/Hispanic	0.70	-0.04	894
		(0.03)	
Homeowner	0.35	-0.02	967
		(0.03)	

Note: Column 2 shows coefficients from individual linear regressions of being in the Grace group on each control variable listed. Data are based on authors' calculations from administrative data from a large payday lender. OLS regressions shown for subprime credit score, loan amount, net pay, account balance, direct deposit indicator, age, female indicator, Black or Hispanic indicator, and homeowner indicator. The sample is restricted to borrowers paid semimonthly with a payday loan origination date on the 8th or 9th day of the month. The sample includes individuals who are missing information on age, gender, race, and home ownership, which is reflected in the changing number of observations in rows six through nine. Standard errors are clustered at the individual level and are reported in parentheses below the coefficients. $***$, $**$, and $*$ designate statistical significance at the 1%, 5%, and 10% level, respectively.

Table A5: Regression Results for Semimonthly Sample

Panel A: Semimonthly Sample (Sample Restricted to Origination Date on 8th and 9th Day of Month)				
	(1)	(2)	(3)	(4)
	Principal paid on first due date	Rolled over some of the loan at the first due date	Number of effective rollovers in loan spell	Total finance charges paid in loan spell
Mean	\$92.27	0.64	2.82	\$205.89
Grace	2.17	-0.05^{**}	-0.35^{**}	-26.18^{***}
	(5.97)	(0.02)	(0.14)	(8.22)
Other Controls	Yes	Yes	Yes	Yes
N	2,072	2,072	1,847	1,847
R^2	0.24	0.16	0.18	0.19
Panel B: First Observations of Semimonthly Sample				
	(1)	(2)	(3)	(4)
	Principal paid on first due date	Rolled over some of the loan at the first due date	Number of effective rollovers in loan spell	Total finance charges paid in loan spell
Mean	\$88.66	0.63	2.86	\$201.46
Grace	1.89	0.00	0.06	1.44
	(10.12)	(0.03)	(0.22)	(15.49)

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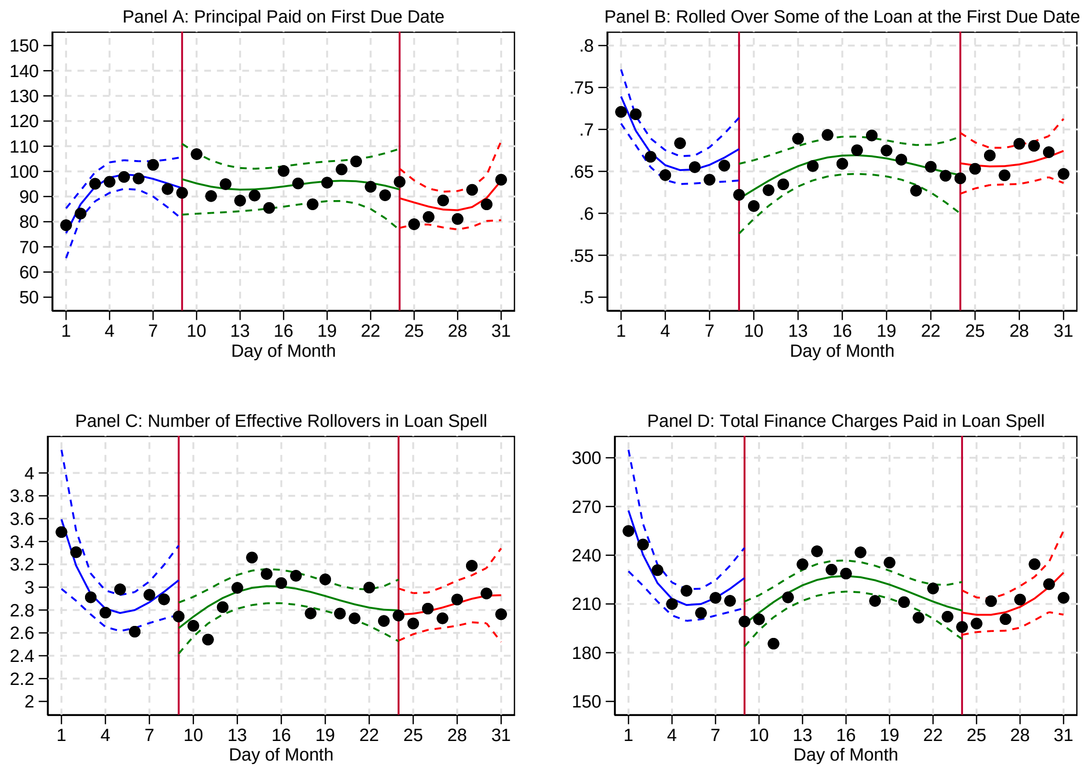
Other Controls	Yes	Yes	Yes	Yes
N	944	944	821	821
R^2	0.36	0.29	0.29	0.29

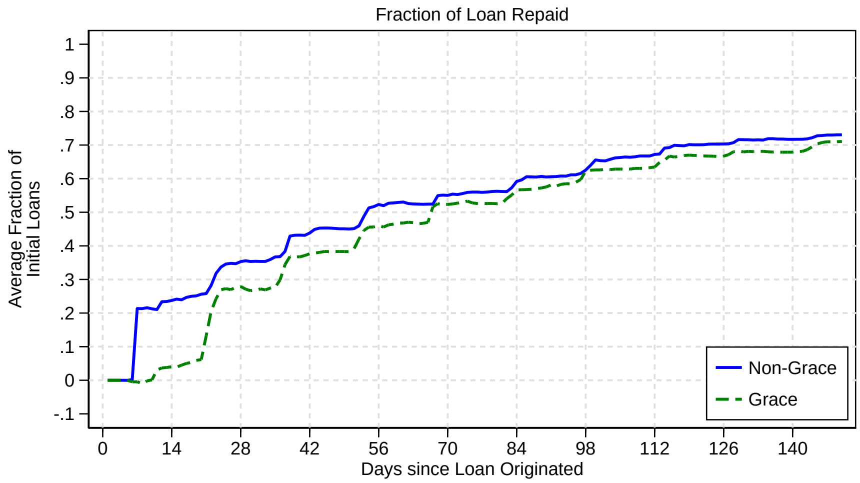
Note: *Grace* is the indicator of having only 6 days until payday. Data are based on authors' calculations from administrative data from a large payday lender. OLS regressions shown for four outcomes: Principal Paid on first due date calculates the amount of the loan paid by the first due date; Rolled over some of the loan at first due date indicates that the borrower rolled over the loan at the first due date; Number of Effective Rollovers is a variable that counts the number of additional loans in succession by a borrower; and Total Finance is the total finance charged over the loan cycle. Panel A includes borrowers paid semimonthly and restricts the sample to loans with an origination date on the 8th or 9th day of the month. Panel B includes the sample in Panel A but only uses the first observation for each borrower. Controls in all columns include loan size, gender, net pay per year, checking account balance, subprime credit score, and age bins. Dummies for race (White, Black, Hispanic, or other), having paycheck direct deposited, missing control variables, month-year, and each payday loan shop are also included. Columns 3-4 include fewer observations because we did not include loans initiated with less than five pay periods before the end of our sample so as to not artificially truncate these outcomes. Standard errors are clustered at the day the loan was initiated and are reported in parentheses below the coefficients. ***, **, and * designate statistical significance at the 1%, 5%, and 10% level, respectively.

Figure A6: Outcomes for Borrowers Paid Semi-monthly



Note: Data are based on authors' calculations based on payday loan transaction data in Texas from November 2001 until August 2004. The vertical lines marks six days until payday, i.e., the day in the pay cycle where the borrower experiences a discontinuous increase in loan length on either the 9th of the month or the 24th of the month. Dots on the graph represent the averages of each outcome (in the figure heading) for each day of the month. The curve shows the predicted outcomes from the regression results of the outcome variable on the day of the month raised to the fifth, as well as an indicator for a borrower taking out a loan after the 9th or 24th of the month. The curve to the left of the line is the predicted outcome without an indicator for six or fewer days until payday. The curve to the right of the line maps the predicted outcomes including the dummy for less than six days until payday. 95% confidence intervals are included in dotted lines.

Figure A7: Average Fraction of Initial Debt Repaid
Borrowers Paid Semimonthly



Note: Authors' calculations based on payday loan transaction data in Texas from November 2001 until August 2004. The figure shows the average fraction of initial debt repaid by days since loan origination. We separate borrowers by the discontinuity in loan lengths. We show borrowers who arrive seven days before their payday and get a seven-day loan ("Non- Grace") and borrowers who arrive six days before their payday and therefore receive a 20-day loan ("Grace").

3 Model Solution Details

We solve the model using recursive methods. To fully characterize optimal decisions of agents in the model, we use a two-step procedure. The first step is to find the solution to a time-consistent (i.e. exponential discounting) version of the agent's dynamic programming problem. The second step is to solve a time-inconsistent version of the agent's problem. The source of time-inconsistency in our model is that agents exhibit quasi-hyperbolic discounting. In addition, we assume that the agents are naive as opposed to sophisticated. Thus in the time-inconsistent problem, agents incorrectly think that their future selves would behave in a time-consistent manner. In the following section, we write out formally the dynamic programming problems of the two-step procedure for the non-grace period case and the grace period case respectively. After that we describe the algorithm used to solve the dynamic programming problems.

3.1 Borrower's Problem—Non-Grace Period Case

In the non-grace borrower's case the time-consistent problem for a day t agent is as follows:

$$V(D^I) = \max_{\{\hat{c}_i^I\}_{i=t}^{T}, \hat{D}^{I+1}} \ln(\hat{c}_i^I) + \sum_{i=t+1}^{T} \delta^{i-t} \ln(\hat{c}_i^I) + \delta^{T+1-t} V(\hat{D}^{I+1}) \tag{A1}$$

subject to

$$\hat{D}^{I+1} \leq \frac{1}{2} y$$
$$\sum_{i=1}^{T} \hat{c}_i^I + rD^I = y - (D^I - \hat{D}^{I+1})$$

T is the terminal day of one pay cycle and we set it to 14 to match a biweekly payday loan repayment schedule. D^0 denotes the initial level of debt. y is the biweekly income. The index for days within a pay cycle is i while I is the index for pay cycles. In addition, δ is the exponential discount factor while β is the quasi-hyperbolic discount factor. The *hat* notation denotes the beliefs of the agent, which are equal to the true values if the agent is time consistent. The solution of the above problem consists of a value function, $V(D^I)$, and a policy function for next period debt, $f(D^I) = D^{I+1}$. They are both time-consistent in the sense that the solutions from different t agents are the same. In other words, the two functions $V(D^I)$ and $f(D^I)$ are not time dependent. Having obtained the solutions of the time consistent problem, the second step is to solve the time-inconsistent problem, which is the one we ultimately focus on in this paper. Note that because agents are assumed to be naive, they incorrectly believe that their future selves would adopt the time-consistent behavior. Through the lens of the time-consistent model, this means that they think they will follow the time-consistent solutions in the future. Formally, we write a day t agent's problem as follows:

$$W(D^I, t) = \max_{c_t^I, \{\hat{c}_i^I\}_{i=t+1}^{14}, \hat{D}^{I+1}} \ln(c_t^I) + \beta \Big\{ \sum_{i=t+1}^{14} \delta^{i-t} \ln(\hat{c}_i^I) + \delta^{15-t} V(\hat{D}^{I+1}) \Big\} \tag{A2}$$

subject to

$$\hat{D}^{I+1} \leq \frac{1}{2} y$$
$$\sum_{i=1}^{t} c_i^I + \sum_{i=t+1}^{14} \hat{c}_i^I + rD^I = y - (D^I - \hat{D}^{I+1})$$

Note that consumption before and including today does not have a *hat* because these are the actual choices made by the agent. However, consumption beyond today has a *hat* due to the naive agent's incorrect beliefs. Given our time convention, the actual level of next cycle's debt, D^{I+1} is determined on day 14. Thus the day 14 agent's problem is

$$W(D^I, 14) = \max_{c_{14}, D^{I+1}} \ln(c^I_{14}) + \beta\delta V(D^{I+1}) \tag{A3}$$

subject to

$$D^{I+1} \leq \frac{1}{2}y$$
$$\sum_{i=1}^{14} c^I_i + rD^I = y - (D^I - D^{I+1})$$

The solution to the above time-inconsistent problem is a set of value functions: $\{W(D^I, t)\}_{t=1}^{14}\}$, and a set of policy functions for next period debt:

$$\left\{\{f(D^I, t) = \hat{D}^{I+1}\}_{t=1}^{13} \text{ and } f(D^I, 14) = D^{I+1}\right\}$$

One thing worth noting is that due to her naive quasi-hyperbolic discounting, the agent revises her expectations each day about the level of debt she will hold next period. Timing is summarized as follows:

- on each day before the 14th day of a pay cycle, an agent makes decisions on her daily consumption and on how much money to leave for tomorrow;
- on the 14th day of a pay cycle, an agent makes decisions on how much to consume for that day and how much to pay down her payday loan principal;
- on the 15th day, an agent receives a new pay check and the next pay cycle begins.

Figure A8 below puts the timing convention in perspective.

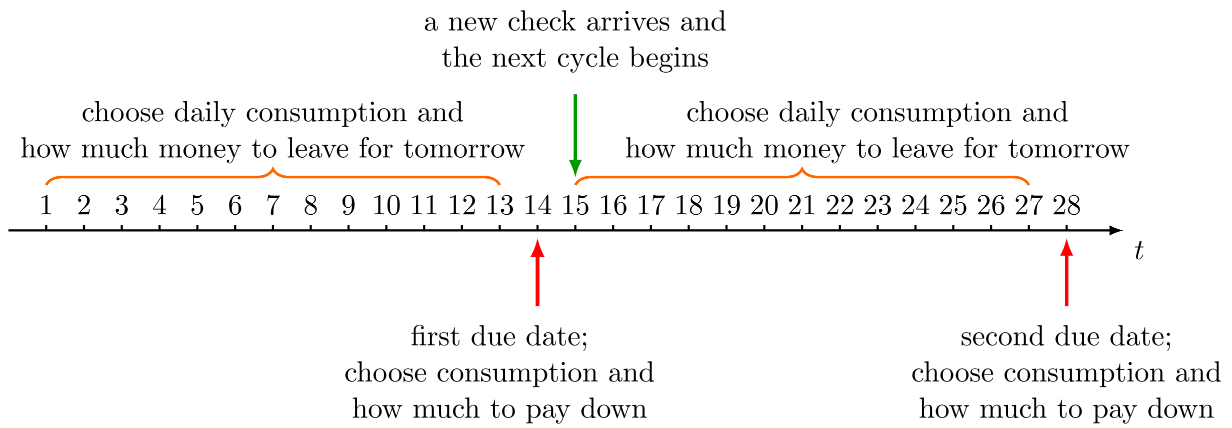


Figure A8: Model Timing—Non-Grace Period Case

3.2 Borrowers' Problem—Grace Period Case

The grace period case differs from the above non-grace period case in that there is no payment required for the first pay cycle. Figure A9 below puts this difference in perspective by outlining the timing convention of the grace period case.

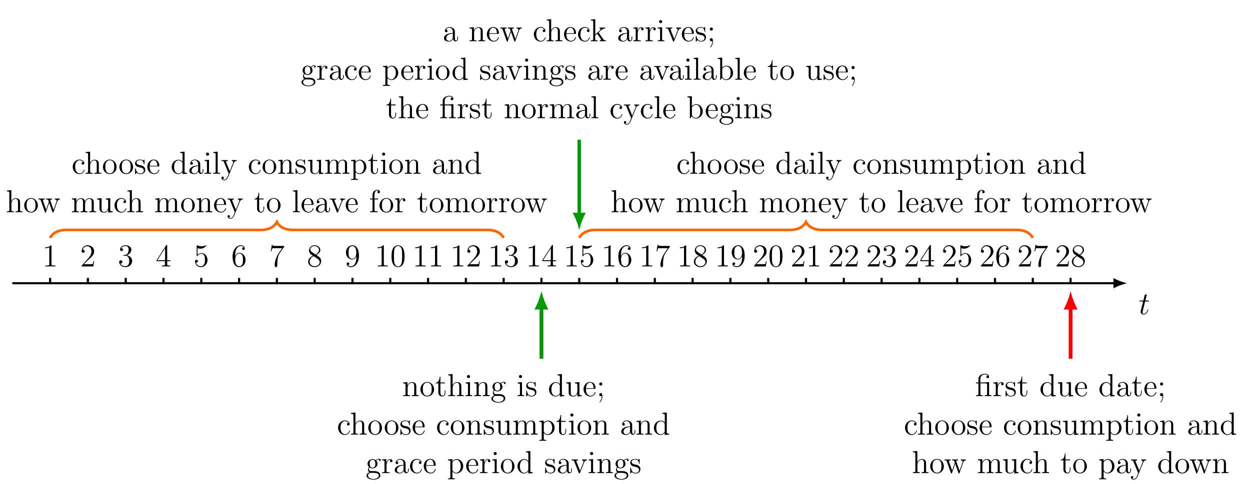


Figure A9: Model Timing—Grace Period Case

To properly reflect this difference in the model, we need to solve two separate Bellman equations for the grace period (i.e. the first 14 days since debt initiation) and the first normal period (i.e. the second 14 days since debt initiation) in addition to the one in the non-grace period case. Each of the two Bellman equations needs to be solved using a two-step procedure that is similar to the one in the non-grace period case. Before writing them down formally, we shall describe what the agent's problems are in these two special cycles. During the grace period the agent makes decisions on her daily consumption and grace-period saving.

Grace-period saving is the money saved for next period. Thus during the first normal cycle, the agent's total disposable income is increased by the amount of grace-period saving while the decisions she needs to make are the same as in any other normal cycle. Formally, the additional dynamic programs for these two special periods are as follows:

3.2.1 Borrower's Problem in the Grace Period

$$W^g(t) = \max_{c_t^g, \{\hat{c}_i^g\}_{i=t+1}^{14}, \hat{G}} \ln(c_t^g) + \beta \Big\{ \sum_{i=t+1}^{14} \delta^{i-t} \ln(\hat{c}_i^g) + \delta^{15-t} V^1(D^0, \hat{G}) \Big\} \tag{A4}$$

subject to

$$\hat{G} \geq 0$$
$$\sum_{i=1}^{t} c_i^g + \sum_{i=t+1}^{14} \hat{c}_i^g + \hat{G} = y$$

where G is the level of grace period saving and $V^1(D^0, G)$ is the time-consistent value function of the agent in the first normal cycle. We highlight that the actual level of grace-period savings is determined on day 14. Hence when $t = 14$, *hat* variables in the above program are replaced with actual values. To obtain the solution to the above problem, we first solve for $V^1(D^0, G)$ in the following way.

$$V^1(D^0, G) = \max_{\{\hat{c}_i^1\}_{i=t}^{14}, \hat{D}^2} \ln(\hat{c}_i^1) + \sum_{i=t+1}^{14} \delta^{i-t} \ln(\hat{c}_i^1) + \delta^{15-t} V(\hat{D}^2) \tag{A5}$$

subject to

$$\hat{D}^2 \leq \frac{1}{2} y$$
$$\sum_{i=1}^{14} \hat{c}_i^1 + rD^0 = y + G - (D^0 - \hat{D}^2)$$

where $V(\cdot)$ on the right-hand side of the above Bellman equation is the time-consistent value function obtained from solving equation (A1). Again, due to naive quasi-hyperbolic discounting, the policy functions for consumption and next-period debt from equation (A5) are not the actual values the agent would choose. To obtain those actual values, we proceed to solve the time-inconsistent problem of the first normal cycle as follows:

$$W^1(D^0, G, t) = \max_{c_t^1, \{\hat{c}_i^1\}_{i=t+1}^{14}, \hat{D}^2} \ln(c_t^1) + \beta \Big\{ \sum_{i=t+1}^{14} \delta^{i-t} \ln(\hat{c}_i^1) + \delta^{15-t} V(\hat{D}^2) \Big\} \tag{A6}$$

subject to

$$\hat{D}^2 \leq \frac{1}{2}y$$
$$\sum_{i=1}^{t} c_i^1 + \sum_{i=t+1}^{14} \hat{c}_i^1 + rD^0 = y + G - (D^0 - \hat{D}^2)$$

Once equation (A4) and equation (A6) are solved, the rest of the problem is exactly the same as in the non-grace period case (i.e. equation (A2)). In the following section, we provide the algorithm for solving equations (A1), (A2), (A4) and (A6).

3.3 Algorithm

We begin by highlighting that one can write the optimal consumption of any day within a cycle as a function of the (perceived) 14th-day's optimal consumption per the first order conditions (FOCs) for daily consumption. These FOCs are characterized by the following two Euler's equations:

$$u'(c_t^I) = \beta\delta u'(\hat{c}_{t+1}^I) \text{ for } t < 14$$
$$u'(c_t^I) = \beta\delta(1+r)u'(\hat{c}_1^{I+1}) \text{ for } t = 14$$

Doing so reduces the number of choice variables in the dynamic programming problem to just the 14th-day's consumption and next-period debt (and grace-period saving in the grace-period case). The following algorithm we use assumes that this simplification has been done.

1. Create a grid for the debt level D^I. In the grace-period case, create another grid for grace-period saving G;
2. Solve equation (A1) on the grid for D^I using the value function iteration method below;
   (a) take a continuous function $V_0(D^I)$ as the initial guess for $V(\cdot)$;
   (b) solve the maximization problem on the right-hand side of equation (A1) using $V_0(D^I)$;
   (c) use the obtained solution to calculate the value function on the left-hand side of equation (A1); call the result $V_1(D^I)$;
   (d) calculate the sup norm between $V_0(D^I)$ and $V_1(D^I)$ over D^I grid;
   (e) if the sup norm is less than some tolerance level, stop; otherwise, update $V_0(D^I)$ using $V_1(D^I)$ and return to step (b);

3. Solve the maximization problem in equation (A2) using $V(D^I)$ on the right-hand side of the Bellman equation;

4. If there is a grace period, solve equation (A5) and equation (A6) using $V(D^I)$ to obtain $V^1(D^0, G)$. Then solve equation (A4) using $V^1(D^0, G)$.

Below we summarize the specifications of the evenly-spaced discrete grids of the state variables used to solve our models. The notations for these state variables are D (payday loan balance), G (savings over grace period), and y (biweekly income).[A1]

Table A6: Grid Specifications

Variable	Min	Max	Increment	No. of Points
D	\$0.00	\$1000.00	\$6.25	161
G	\$0.00	\$800.00	\$3.20	250
y	\$330.00	\$2100.00	\$50.00	36

Note: The upper (lower) bound of D and y grids are chosen to match the largest (smallest) value we observe in our data sample. The upper bound of G is chosen such that there is no one in the model who would ever endogenously decide to save more than that number.

3.4 Income Risks with Awareness

In the modification of our baseline representative model that incorporates income risks, we assume that borrowers in the model are aware of these risks. This assumption means that we need to add another state variable to the above Bellman equations. Specifically, let s^I denote the expense shocks of pay cycle I and the non-grace borrower's problem becomes the following:

$$V(D^I, s^I) = \max_{\{\hat{c}_i^I\}_{i=t}^T, \hat{D}^{I+1}} \ln(\hat{c}_i^I) + \mathbb{E}_s \Big\{ \sum_{i=t+1}^{T} \delta^{i-t} \ln(\hat{c}_i^I) + \delta^{T+1-t} V(\hat{D}^{I+1}, s^{I+1}) \Big\} \tag{A7}$$

subject to

$$\hat{D}^{I+1} \leq \frac{1}{2} y \tag{A8}$$

$$\sum_{i=1}^{T} \hat{c}_i^I + rD^I = y - s^I - (D^I - \hat{D}^{I+1}) \tag{A9}$$

[A1] The grid for y is only used in the calibrated model where we add cross-sectional heterogeneity in income and initial payday loan balance. In the representative models we set $y = \$900$, which is the average biweekly income of our data sample.

There are two key differences in the above from the problem without income risks. First, because we assume awareness, the representative borrower in the model will form expectations over future expense shocks using the probability distribution of the expense shock. This is why we must use the expected continuation value in the Bellman equation above. Secondly, since the model agent is aware of the expense shock, she knows that her disposable income has already been reduced by s^I for cycle I after the expense shock hits. This explains why we must subtract s^I in the budget constraint. While we have only laid out the details of this variant of the baseline model for the non-grace borrowers above, the modifications for the grace borrowers are highly analogous.

To operationalize the addition of income risks as described above, we first postulate that s^I follows an exponential distribution with scale parameter σ and is independently distributed over time. Given the exponential-distribution assumption, the expected expense shock in dollar value is σ. In the numerical exercises of this variant of the baseline model, we assume that $\sigma = \$100$, which means that on average the magnitude of the expense shock is \$100. Furthermore, to make the expense-shock process compatible with the above Bellman equations, we discretize the exponential stochastic process s^I into a five-point process with support [\$50 \$162.5 \$275 \$387.5 \$500] and associated probability of [0.68 0.22 0.07 0.022 0.008] respectively. For a given value of s in the discrete support, the associated probability is calculated using the below formula:

$$Prob(s_i) = \frac{exp(-(\frac{1}{\sigma})s_i)}{\sum_{j=1}^{5} exp(-(\frac{1}{\sigma})s_j)} \tag{A10}$$

4 Model Simulation Details

This section describes in detail our procedure for simulating the calibrated model based on the solutions obtained in the previous section. This simulation is used to generate repayment predictions. Simulations of the representative agent models are similar. Since unlike the calibrated model, we abstract from ex-ante cross-sectional heterogeneity in initial loan balance and biweekly income in all representative agent models in this paper, steps involved in simulating the representative agent models is a subset of the steps involved in the calibrated model simulation. The following description of our simulation procedure assumes a 20-cycle horizon.

1. Construct the cross-sectional joint distribution of initial payday loan balance and biweekly income for the model borrowers in the simulation.

   (a) we simulate 14,073 borrowers in the model to match the number of observations in our biweekly sample used in our empirical analysis;

(b) for each simulated borrower, we take her initial loan balance and biweekly income values observed in her data counterpart as the initial conditions. That is, suppose the observed loan balance and income of the first borrower in our data is \$350 and \$900 respectively; then the first borrower in the simulation will have a initial debt balance D^0 of \$350 and a biweekly income y of \$900;

(c) since we have two discrete grids for D^0 and y in solving the model, when the data numbers are off our grids, we use the closest points on the grids for the simulation;[A2]

2. For each borrower in the simulation, draw a 20-period time series of expense shocks from the exponential distribution with scale parameter σ;

3. Solve the decision problems laid out in Section 3 of this appendix at every possible state on the grids to obtain decision rules of repayment, consumption, and savings over the grace period in the grace-borrower case. Note that in the calibrated model we assume that agents are not aware of the expense shocks, which means that unlike what is in equation (A7), we solve the Bellman equations without the expense shock s^I as a state variable and s^I is no longer a part of the budget constraint;

4. Begin each borrower in the simulation with the initial conditions set up in step 1 and apply the decision rules from step 3 as well as the randomly-drawn expense shocks to simulate each borrower's repayment behavior. In the case of a grace borrower, apply the decision rule of savings over the grace period to obtain money saved during their first pay cycle.

   (a) obtain the total consumption within a 14-day pay cycle for each borrower using the decision rule of daily consumption;

   (b) for each borrower obtain the post-consumption and post-expense shock net disposable income by subtracting 14-day total consumption and the expense shock;

   (c) if the net disposable income is less than the mandatory interest charges given the debt balance, record it as a check bounce and carry the interest charges over to next period;[A3] In the case of a bounced check, repayment is zero;

[A2] We also use this method to deal with debt balance, grace-period savings, and disposable-income numbers that are off the grid in the subsequent steps.

[A3] The particular way we implement this carry-over is to subtract the amount of interest charges from income of next period. We think of this as the payday loan lender drafting whatever amount interest charges that are past due as soon as a new pay check is deposited into a borrower's account.

(d) if the net disposable income is larger or equal to the the mandatory interest charges, apply the decision rule of repayment to obtain debt balance of next period[A4];

(e) in the case of a grace borrower, obtain the savings over the grace period using the decision rules and the drawn expense shock in similar ways as described above; Add grace period savings to disposable income of the second pay cycle;

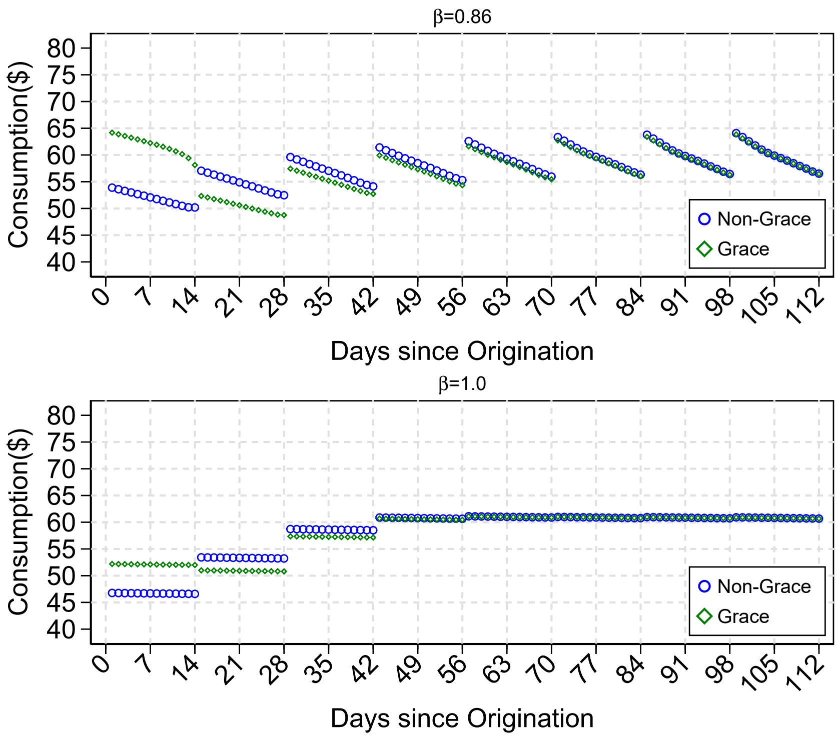
5. Repeat step 4, except for the grace-period saving part, for every subsequent pay cycle after the first one; The only difference is that instead of starting with the initial loan balance like in step 4, the loan balance at the beginning of each subsequent period is determined by the simulated repayment decisions in the previous pay cycle.

4.1 Average Daily Consumption in Model Simulation

One way to see where the welfare results in Section 6.5 of the main manuscript come from is to look at the average consumption of both types of borrowers on a daily basis, as shown in Figure A10 below. First, regardless of whether the borrowers are present focused or not, the grace borrowers consume more than the non-grace borrowers during the first pay cycle because there is no due date for the grace borrowers during this cycle. However, when borrowers are time consistent, the consumption differential between grace and non-grace borrowers is significantly smaller during the first pay cycle. This means that time-consistent grace borrowers save more over the grace period and therefore are able to pay down more of their balance on their first due date and pay less interest charges going forward. Consequently, compared to present-focused non-grace borrowers, present-focused grace borrowers on average have a much lower consumption profile during the second pay cycle, which makes having a grace period less beneficial.

[A4]Note that due to the lack of awareness in solving the decision problem, the decision rule does not account for the existence of the expense shocks. Therefore the expense shock is subtracted from the repayment decision rule to obtain the actual amount of repayment in the simulation.

Figure A10: Calibrated Present-Focus Model Daily Consumption Profiles



4.2 Derivation of the CEV Welfare Measure

As mentioned in Section 6.5 in the main text, we compute the total utility for each borrower n in our simulated sample as follows.

$$U_n = \sum_{t=1}^{280} \delta^t \ln(c_{nt})$$

where t is the index for days of the 20 cycles in our simulation. Based on the definition of of our welfare measure, we derive the following mathematical expression for CEV.

$$U_n^{Grace} = \sum_{t=1}^{280} \delta^t \ln((1+\lambda_n) c_{nt}^{Non-grace})$$

where λ_n is the fraction of daily consumption that the non-grace borrower n is willing to pay for all future days. Since λ_n is a constant over time, we may re-write the above as:

$$
\begin{aligned}
U_n^{Grace} &= \sum_{t=1}^{280} \delta^t \ln(1+\lambda_n) + \sum_{t=1}^{280} \delta^t \ln(c_{nt}^{Non-grace}) \Rightarrow \\
U_n^{Grace} &= \sum_{t=1}^{280} \delta^t \ln(1+\lambda_n) + U_n^{Non-grace} \Rightarrow \\
U_n^{Grace} - U_n^{Non-grace} &= \sum_{t=1}^{280} \delta^t \ln(1+\lambda_n) \Rightarrow \\
U_n^{Grace} - U_n^{Non-grace} &= \ln(1+\lambda_n) \sum_{t=1}^{280} \delta^t \Rightarrow \\
\ln(1+\lambda_n) &= \frac{U_n^{Grace} - U_n^{Non-grace}}{\sum_{t=1}^{280} \delta^t} \Rightarrow \\
1+\lambda_n &= exp\Big(\frac{U_n^{Grace} - U_n^{Non-grace}}{\sum_{t=1}^{280} \delta^t}\Big) \Rightarrow \\
\lambda_n &= exp\Big(\frac{U_n^{Grace} - U_n^{Non-grace}}{\sum_{t=1}^{280} \delta^t}\Big) - 1
\end{aligned}
$$

Note that λ_n is the constant fraction of daily consumption for all days in the 20 cycle. Therefore, the total 20-cycle CEV for borrower n in dollar amount is:

$$
\Lambda_n = \sum_{t=1}^{280} \lambda_n c_{nt}
$$

We then use the simulation sample mean of Λ_n to measure the benefits of having a grace period. That is, our final measure is:

$$
\bar{\Lambda} = \frac{\sum_{n=1}^{N} \Lambda_n}{N}
$$

where N is our sample size.

5 Calibration over δ_y

In this section we calibrate a time-consistent model and compare the calibrated model's predictions on repayment with data observations. The purpose of this exercise is to find out whether there is any plausible parameter values of δ_y and σ such that the model can fit the data well absent present focus. We introduce ex-ante cross-sectional heterogeneity in biweekly income and initial payday loan balance to the baseline neoclassical model. In addition, we also assume away awareness of the expense shocks as we do in the calibration of the present focus model. Table A7 below presents the results of this calibration.

Table A7: Time-Consistent Model Calibration for the Non-Grace Case

Panel A: Targeted Moments
(All Moments are Calculated as Fractions of Initial Debt)

	Data Mean	Model Mean
Non-Grace 1^{st} Cycle Repayment	0.31	0.23
Non-Grace 2^{nd} Cycle Repayment	0.39	0.41
Non-Grace 3^{rd} Cycle Repayment	0.47	0.55

Panel B: Calibrated Parameter Values

Notation	**Definition**	**Value**
δ_y	Yearly Exponential Discount Factor	0.13
σ	Mean of Expense Shock	$85.62

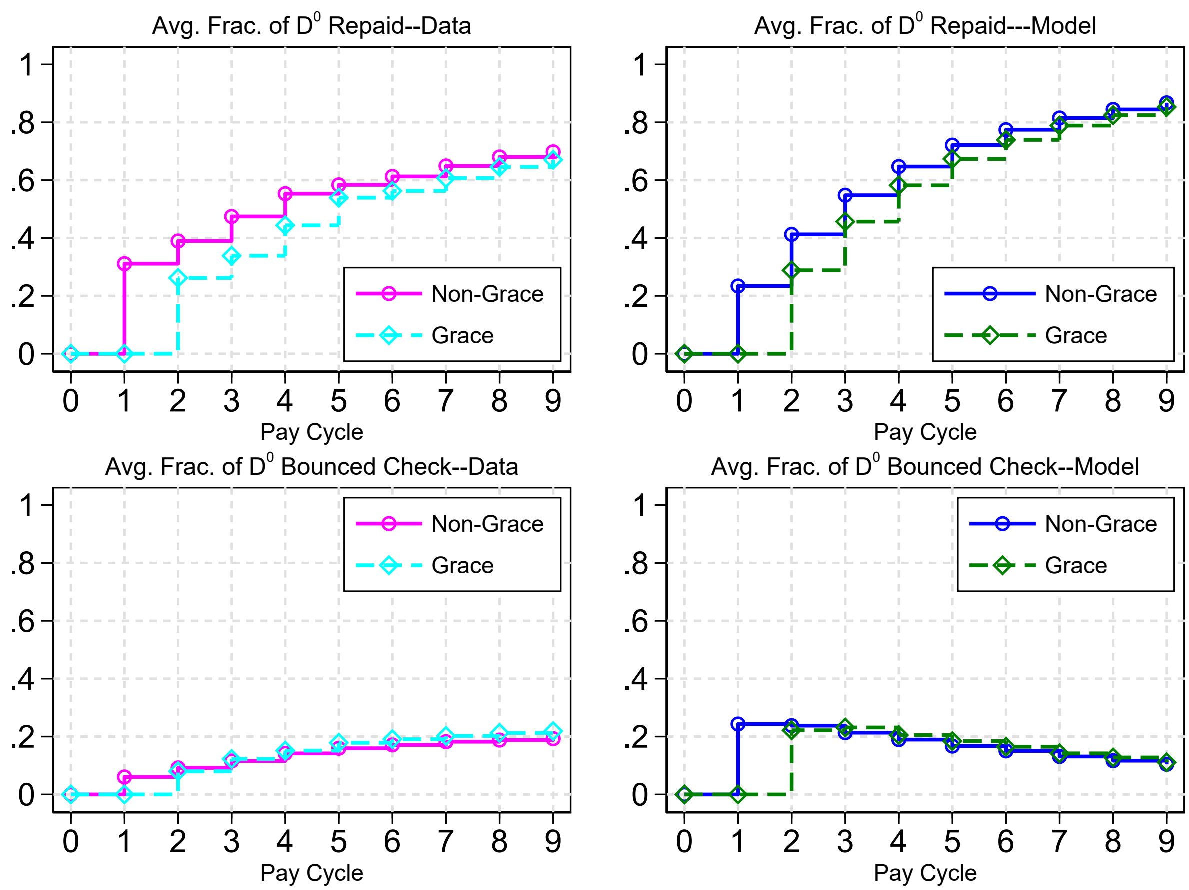
Panel C: Untargeted Moments
(All Moments are Calculated as Fractions of Initial Debt)

	Data Mean	Model Mean
Non-Grace 4^{th} Cycle Repayment	0.55	0.65
Non-Grace 5^{th} Cycle Repayment	0.58	0.72
Non-Grace 6^{th} Cycle Repayment	0.61	0.77
Non-Grace 7^{th} Cycle Repayment	0.65	0.81
Non-Grace 8^{th} Cycle Repayment	0.68	0.84
Non-Grace 9^{th} Cycle Repayment	0.70	0.87
Non-Grace 10^{th} Cycle Repayment	0.72	0.89
Non-Grace 1^{st} Cycle Check Bounced	0.06	0.22
Non-Grace 2^{nd} Cycle Check Bounced	0.09	0.23
Non-Grace 3^{rd} Cycle Check Bounced	0.12	0.20
Non-Grace 4^{th} Cycle Check Bounced	0.14	0.18
Non-Grace 5^{th} Cycle Check Bounced	0.18	0.16
Non-Grace 6^{th} Cycle Check Bounced	0.19	0.14
Non-Grace 7^{th} Cycle Check Bounced	0.20	0.13
Non-Grace 8^{th} Cycle Check Bounced	0.21	0.11
Non-Grace 9^{th} Cycle Check Bounced	0.22	0.10
Non-Grace 10^{th} Cycle Check Bounced	0.22	0.09

Note: Agents are assumed time consistent in this calibration. Repayment is the amount of payday loan principal paid down. In other words, it is the money a borrower repays in addition to the mandatory interest charges. The model averages are computed off a 20-pay-cycle simulation of 14,073 individuals who are heterogeneous in their initial payday loan balance and biweekly income.

Panel A of Table A7 shows the model fit to targeted data moments. As Panel B indicates, the way a time-consistent model tries to capture the targeted data moments is very different from a present-focus model. First, we have to make model agents ultra impatient (i.e. δ_y=0.13) so that they do not payoff the initial balance altogether within a short amount of time, as suggested by the results of the representative agent neoclassical model in Section 3. On top of that, we also need a much higher mean expense shock to prevent non-grace borrowers in the model from paying down a lot more than the data suggests during the first three periods.

Figure A11: Data v.s. Calibrated Time-Consistent Model's Predictions



Note: The model predictions are computed using the estimates of $\delta_y = 0.13$ and $\sigma = \$85.62$ coming out of our calibration. β is fixed at 1.0 so agents are time consistent. The model averages are computed off a 20-pay-cycle simulation of 14,073 individuals who are heterogeneous in their initial payday loan balance and biweekly income. The joint cross-sectional distribution of initial payday loan balance and biweekly income in the simulation is specified according to the observed distribution in our data.

Secondly, the model fit for both targeted as well as untargeted moments are significantly worse than the calibrated present-focus model. Since model borrowers are time consistent, they manage debt balances in ways that are consistent with the goal of achieving a smoother consumption path. This is revealed by the fact that the grace-period borrowers do save over the grace period and then repay more on their first due date, although the salience of this grace-period saving channel is largely suppressed by high expenses shocks (i.e. σ=\$85.62). Additionally, without any present focus, the model agents keep paying down their debt over time such that they end up with much lower balances than the data suggests after nine pay cycles. This channel together with high mean expense shock explains why the check-bounce rate in this model is decreasing in time: agents are more likely to have a bounced check when they get a larger expense shock and their debt balance is relatively high (e.g. before they make the first repayment). As they pay down their balance over time, their interest charges go down so that the same of amount of expense shocks are less likely to make them unable to afford the charges and therefore less likely to have a check bounce. This is in sharp contrast to the data observation where borrowers keep rolling over part of their initial debt for an extended period of time.

6 Heterogeneity in Welfare Results

In this section, we use the present-focus model to explore quantitatively whether the welfare impact of having a grace period is heterogeneous across borrowers who differ in initial debt-to-income ratios. We start off by dividing the borrowers into four groups according to their initial debt-to-income ratios. We then obtain group-specific values of the parameters β and σ by re-calibrating the present-focus model separately for each group of borrowers. We follow the same calibration procedure as in the main text; the only difference here is that the three targeted moments are calculated within each group instead of at the whole-sample level. Table A8 below presents the calibrated parameter values for each debt-to-income group. We note that the degree of present focus needed (i.e. β) to fit the targeted repayment behavior is decreasing in debt-to-income ratio. This is because in the model, as one's debt-to-income ratio increases, the marginal benefit of repaying becomes higher. As a result, high debt-to-income-ratio borrowers are more driven to pay down their balances in the model. Therefore, one would need more present focus to offset the enhanced marginal benefit of repaying as debt-to-income increases.

Table A8: Calibration Results for Different Groups of Borrowers

Panel A: Borrowers with Debt-to-Income ratio $\in (0.2, 0.3]$		
Notation	**Definition**	**Value**

table continues to next page

β	Degree of Naive Present Focus	0.871
σ	Mean of Expense Shock	\$31.13
Panel B: Borrowers with Debt-to-Income ratio $\in (0.3, 0.4]$		
Notation	**Definition**	**Value**
β	Degree of Naive Present Focus	0.858
σ	Mean of Expense Shock	\$31.51
Panel C: Borrowers with Debt-to-Income ratio $\in (0.4, 0.5)$		
Notation	**Definition**	**Value**
β	Degree of Naive Present Focus	0.856
σ	Mean of Expense Shock	\$32.04
Panel D: Borrowers with Debt-to-Income ratio ≥ 0.5		
Notation	**Definition**	**Value**
β	Degree of Naive Present Focus	0.845
σ	Mean of Expense Shock	\$32.04

Note: δ_y is fixed at 0.9 for all of the above calibrations. Unlike what we do in the main text, we leave out tables that detail each calibration process for the benefit of space. However, we follow the same calibration process as in the main text. The only difference here is that moments—including both targeted and untargeted—from the data as well as the model are calculated at the corresponding sub-sample. Detailed information such as model fit to targeted and untargeted moments is available upon request.

As in the main text, we use CEV to gauge the welfare benefit of a grace period for each deb-to-income group. As shown below in Table A9, a grace period is more beneficial for higher debt-to-income borrowers in both the present-focus and time-consistent cases. This is again due to the economic mechanism that marginal benefit of repaying is higher when debt-to-income is higher. Having a grace period provides an opportunity to realize these enhanced benefits for borrowers with a higher debt-to-income ratio. Furthermore, as initial debt-to-income ratio decreases, present-focused borrowers “waste” more of the welfare benefit associated with a grace period relative to time-consistent borrowers.

Table A9: Welfare Results for Different Groups of Borrowers

	(1) Non-grace total interest paid	(2) Grace total interest paid	(3) Interest savings with grace	(4) Welfare benefit of grace (CEV)
Panel A: Borrowers with Debt-to-Income ratio $\in (0.2, 0.3]$				
Present Focused ($\beta = 0.871$)	\$310.51	\$299.63	\$10.88	\$6.10
Time Consistent ($\beta = 1.0$)	\$98.19	\$84.27	\$13.92	\$19.95
Panel B: Borrowers with Debt-to-Income ratio $\in (0.3, 0.4]$				
Present Focused ($\beta = 0.858$)	\$395.01	\$379.52	\$15.49	\$15.86
Time Consistent ($\beta = 1.0$)	\$128.07	\$104.71	\$20.65	\$33.75
Panel C: Borrowers with Debt-to-Income ratio $\in (0.4, 0.5)$				
Present Focused ($\beta = 0.856$)	\$399.03	\$380.40	\$18.64	\$23.24
Time Consistent ($\beta = 1.0$)	\$151.08	\$120.88	\$30.20	\$42.93
Panel D: Borrowers with Debt-to-Income ratio ≥ 0.5				
Present Focused ($\beta = 0.845$)	\$409.24	\$391.08	\$18.16	\$20.27
Time Consistent ($\beta = 1.0$)	\$151.86	\$120.54	\$31.32	\$43.98

Note: All calculations above are based on a 20-cycle simulation of our calibrated model. All statistics reported above are the means of the simulated sample.