

Online Appendix B: Applications

For "Combinatorial Auctions in Practice," by Ignacio Palacios-Huerta, David C. Parkes, and Richard Steinberg, *Journal of Economic Literature* 2024, 62 (2).

In the taxonomy we use for summarizing the design of CAs across the various applications, each application is specified by *auction type*, which consists of three components: (i) *payment* rule (FP | VCG | CORE), (ii) *auction design* (SB | MR | CCS), and (iii) *cross-cutting features* {F1, F2, F3}. The notation takes the form: *payment-design(features)*. Here are two illustrative auction types:

- FP-SB(F1,F2,F3): a first-price sealed-bid CA with package bids, non-package expressiveness, and business rules.
- CORE-CCA(F1,F3): a CCA with package bids and business rules, employing a Vickrey-nearest core-selecting payment rule.

Below, we survey applications of CAs in practice, first for reverse auctions, then for forward auctions.

B.1 Reverse Combinatorial Auctions

PUBLIC SECTOR

Sourcing of Bus Services

- **Sourcing of School Bus Services in Manchester, UK** (Letchford 1996)

Organization: Greater Manchester Passenger Transport Executive (GMPTE)(1993-)²⁹

Auction Type: FP-SB(F1,F2,F3).

Description: Annual tendering for school bus contracts, with one contract per vehicle, for the 38 weeks of the normal school year, from Monday to Friday. Winner determination via integer programming.

- (F1) *Group bids*, where a discount is offered if certain contracts are awarded to the bidder simultaneously.
- (F2) Bidders free to impose almost any restriction onto the basic bid price schema.
- (F3) Quality-of-service constraints.

Outcome: In the first three years of operations, there were around 300 contracts per year. The auction was reported to be a success, but quantitative measures were unavailable (Letchford 1996). GMPTE was superseded by Transport for Greater Manchester in 2001, which continues to accept group bids.

- **Sourcing of Municipal Bus Services in London, UK** (Cantillon & Pesendorfer 2006)

Organization: London Regional Transport (LRT) (1995-01).

Auction Type: FP-SB(F1,F3).

Description: Annual tendering of gross-cost contracts, with revenue from fares going to LRT. Contracts for five years, representing 15-20% of the eight hundred route, 3.5M passengers/day network. Auctions of 1 to 21 routes conducted in sequence, usually for routes in a single geographic area.

²⁹Letchford (1996) incorrectly refers to the GMPTE as the "Greater Manchester Public [*sic*] Transport Executive". We thank Adam Letchford for pointing out this correction to us.

- (F1) Package bids, OR language.
- (F3) Quality-of-service adjustments.

Outcome: The total value of auctioned routes approximately \$900M/year. For larger auctions with seven or more routes, most bidders submitted at least one package bid, with discounts relative to bids on individual routes of 5-8%.

- **Sourcing of Municipal Bus Services in Helsinki, Finland** (Tukiainen 2008) (J. Tukiainen, personal communication, 2017)

Organization: City of Helsinki Intra-city Bus Services, Finland (1997-2009).

Auction Type: FP-SB(F1,F3).

Description: Annual tendering of gross-cost, typically five-year contracts representing some portion of the city network, conducted through a sequence of smaller auctions. Bidders could specify a bid price for a route by specifying the unit cost of service per km, per hour, and per vehicle-day.

- (F1) Package bids, OR language.
- (F3) Quality-of-service adjustments.

Outcome: Sixty-four contracts, some covering multiple routes, auctioned between 1997 and 2005. Package bids in a sequence of seven auctions for the entire network in a sequence of auctions between 1997-01, with around 9% of bids placed on packages. (When the responsibility to run the auctions for bus transit moved from Helsinki to a larger regional unit in 2010, the use of CAs ceased. At the time of this writing, the use of CAs is rare in Finnish public procurement, perhaps because in many cases the government unit conducting the auctions has not adopted software that can accommodate CAs.)

- **Sourcing of Municipal and Regional Bus Services in Sweden** (Lunander & Lundberg 2012a)

Organizations: Local governments in Sweden (2003-04).

Auction Type: FP-SB(F1,F2,F3); one auction FP-MR(F1,F2,F3).

Description: Tendering of gross-cost contracts. One auction was multi-round with the chance to adjust bids based on feedback about ranking relative to other bids. Winner determination via integer programming.

- (F1) Package bids, OR language.
- (F2) Maximum distance by vehicle type.
- (F3) Quality-of-service adjustments.

Outcome: Between 30-85% of the contracts were allocated to package bids. The package discount relative to item bids was between 1.7% and 4.95%. Constraints on maximum distance were not used.

Sourcing of School Meal Services

- **Sourcing of School Meal Services to Public Schools in Chile** (Epstein et al. 2002, Catalán et al. 2009, Kim et al. 2014, Olivares et al. 2012)

Organizations: National School Assistance and Scholarship Board (JUNAEB) and pre-school boards JUNJI and INTEGRA (1997-present)

Auction Type: FP-SB(F1,F3).

Description: Three-year contracts auctioned for 100 geographical areas, with tenders on around 1/3 each year. Winner determination via integer programming. Scenario navigation allows the bid taker to vary projected demand.

- (F1) Package bids, XOR language.
- (F2) Allocation to a bidder limited by financial capacity. Bids specified through prices on different meals at different nutritional quality levels, linked to price and food indexes, and with price adjustments based on realized demand.
- (F3) Constraints on the total number of meals and geographic areas each bidder can supply. Constraints on the number of winners in each region.

Outcome: Typical winning packages on three areas. Electronic bid submission in 2004 saw number of bids increase from 43,000 to 190,000. Approx. \$500M annual spend from 1997 to 2017, with meals served to around 2.5M children each year. Credited with savings of around 20%, i.e., \$100M/year.

Sourcing of Miscellaneous Government Services

- **Sourcing of Domestic Flight Routes in Sweden** (Lunander & Lundberg 2012a)

Organization: Swedish National Public Transport Agency (2003).

Auction Type: FP-SB(F1,F2,F3).

Description: Tendering for flight service contracts, with regulated ticket prices and with revenue from fares collected by winning bidders. Winner determination via integer programming.

- (F1) Package bids, OR language.
- (F2) Maximum number of awarded contracts.
- (F3) Quality-of-service constraints.

Outcome: Three of six flight contracts were allocated to a package bid, with the discount from the package bid at a cost saving of 0.3% relative to bids on individual contracts. Constraints on maximum number of contracts were not used.

- **Sourcing of Vehicle Fleets in Ireland** (A. Holland, Keelvar, personal communication, 2017).

Organization: Cork City Council, Ireland (2005), auction technology developed by University College Cork.

Auction Type: FP-SB(F1,F3).

Description: Tendering for 360 vehicles across 16 categories that included both light and heavy equipment. Winner determination via integer programming.

- (F1) Package bids.
- (F3) Quality-of-service adjustments.

Outcome: The auction was reported to have resulted in comparative prices for similar specification vehicles at 5-10% lower prices than Dublin City Council. Niche suppliers also received contracts for specialized vehicles, and thus public sector objectives for supplier inclusion were also reached.

- **Sourcing in Sweden of Cleaning of Schools and Offices, of Road Resurfacing, and of Elderly Care** (Lunander & Lundberg 2013, 2012*a,b*)

Organizations: Local governments, the National Social Insurance Agency, and the Swedish Road Administration (2005-11).

Auction Type: FP-SB(F1,F2,F3).

Description: Sourcing of various services, with bids selected to maximize total value based on quality-adjusted scores for bids. Winner determination via integer programming.

- (F1) Package bids, OR language. (Sometimes on restricted packages, perhaps size-restricted; must also bid on items in a package, sometimes with total price on items close to package bid.)
- (F2) Capacity constraints on the total value (or total quantity) of allocated business, and the total number of winning bids.
- (F3) Minimum quality requirements. Quality-of-service adjustments.

Outcome: Package bids for cleaning services at a 2-9% discount from item bids, and tended to be selected. A single package bid won in a nationwide cleaning services auction for forty-two contracts in 2007, at an estimated cost saving relative to item bids of 6%.

- **Sourcing of Snow Removal in Denmark** (A. Lunander and F. Ygge, Coupa (formerly Trade Extensions), personal communication, 2017).

Organization: Danish National Road Administration (*Vejdirektoratet*) (2013), technology developed by Coupa.

Auction Type: FP-SB(F1,F2,F3).

Description: Sourcing of snow removal services for roads for the coming three years, comprising 97 separate geographic contracts, covering all governmental roads in Denmark. Winner determination via integer programming.

- (F1) Package bids, specified as a discount on a set of bids on items.
- (F2) Maximum number of awarded contracts. Conditional discounts that triggered based on a quantity threshold.
- (F3) Miscellaneous business rules.

Outcome: A total of 103 suppliers submitted bids, 75% of whom submitted bids on three or fewer contracts, with only five using package bids. Two suppliers used a discount on packages of size two or more. Contracts awarded to 51 suppliers.

Sourcing of Electricity Generation Capacity

- **Sourcing of Forward Capacity for Electricity Distribution Companies in Chile** (Moreno et al. 2010, Maurer & Barroso 2011, Mastropietro et al. 2014)

Organizations: Chilectra, Chilquinta, EMEL, CGE, SAESA (Chilean electricity distribution companies) (2006-10).

Auction Type: FP-SB(F1,F2,F3).

Description: Three auctions in total, with an average capacity auctioned of 28 KW hours/year for years 2010-2025, and multiple distributors per event. The government determines an inflation-indexed price for the amount of reserved capacity.

- (F1) Bids are peak-load generation capacity to each distributor, along with a price-indexed amount per MWh.
- (F2) Maximum on the allocated generation capacity.
- (F3) Price caps, while also seeking to balance the allocation across distribution companies.

Outcome: There were high price differences among awarded contracts and distribution areas along with generally high prices and poor coverage. There was difficulty in defining the criteria to balance the dual objectives of cost minimization and demand coverage maximization (Moreno et al. 2010).

PRIVATE SECTOR

Sourcing of Transportation Services

- **Sourcing of Truckload, Rail, Inter-modal and Ocean Transportation Logistics at Global 1000 companies** (Sheffi 2004, Caplice & Sheffi 2006, Elmaghraby & Keskinocak 2003, Caplice & Sheffi 2003)

Organizations: Walmart, Compaq, Staples, The Limited, K-Mart, Ford, among others (1992-02); technology developed by Logistics.com (acquired by Manhattan Associates, Inc. in 2002).

Auction Type: FP-SB(F1,F2,F3).

Description: Sourcing contracts for truckload, rail, inter-modal and ocean logistics. Bids selected to maximize total value subject to business rules. Winner determination via integer programming.

- (F1) Package bids. (E.g., for truckload transportation, bids on a specified fraction of the total volume in each of several lanes.)
- (F2) Capacity constraints. (E.g., for truckload transportation, a limit on the total number of vehicles used in a particular allocation.)
- (F3) Capacity limits on winners. Preferences for minority-owned businesses. Quality-of-service based adjustments. Other business rules (e.g., for truckload transportation, maximum and minimum number of allocated bidders on sets of lanes).

Outcome: E.g., for truckload transportation, in excess of \$8B from 1988-2002, with a combined savings of over \$500M and typical savings reported of 3-15%. More than half of the truckload auctions allowed package bids.

Less than 10% of lanes were bid in packages, perhaps because of the difficulty to utilize back-haul synergies operationally.

- **Sourcing of Truckload Transportation at Sears Logistics Services, Inc.** (Ledyard et al. 2002)

Organization: Sears Logistics Services, Inc. (1993-02).

Auction Type: FP-MR(F1).

Description: Buy 3-year contracts for truckload services on shipping lanes. An auction was held in 1993 on 800 trucking lanes, followed by five auctions over the next three years, covering an additional 500 lanes. Provisional winners reported each round. Winner determination via integer programming.

- (F1) Package bids, OR language, at most 5000 bids per bidder in each round.

Outcome: Reported savings over the first six auctions was \$85M on a \$587M spend. Some rounds included more than 2000 package bids, and typically around 30% of the package bids in the final round were successful.

- **Sourcing of Truckload Transportation at Mars, Inc.** (Hohner et al. 2003, Bichler et al. 2006)

Organization: Mars, Inc., Worldwide (2000-2002); technology developed by IBM, Inc.

Auction Type: FP-MR(F1,F2,F3).

Description: Sourcing and procurement of indirect materials (e.g., in-store display boxes) and direct materials (e.g., sugar). Winner determination via integer programming.

(F1) Package bids, OR language.

(F2) Discount schedules.

(F3) Maximum and minimum number of winners. Maximum and minimum on the total quantity allocated to a single bidder.

Outcome: 60 CAs were conducted by end of 2002, credited with providing return-on-investment in less than a year. When package bids were enabled, typically less than 5% of bids were on packages. When non-package expressiveness was enabled, typically this was used by more than 80% of bids.

- **Sourcing of Air Transport at Motorola, Inc.** (Metty et al. 2005)

Organization: Motorola, Inc. (2002-2003); technology developed by Emptoris, Inc., acquired by IBM in 2012.

Auction Type: FP-MR(F2,F3).

Description: Winner determination via integer programming.

(F2) Discount schedules. Also, conditional discounts that triggered based on a quantity threshold.

(F3) Maximum and minimum number of winners. Preferred number of winners. Quality-of-service constraints.

Outcome: As of 2003, \$600M saving on volume over \$16B for Materials and air transport. Auctions credited with facilitating faster more focused negotiation, and providing more transparency for suppliers. Around 80% of spend went through CAs, compared with 20% going through simple, non-combinatorial reverse auctions.

Industrial Procurement and Sourcing of Direct and Indirect Materials

- **Sourcing and Procurement of Direct and Indirect Materials at Mars, Inc.** (Hohner et al. 2003, Bichler et al. 2006)

Organization: Mars, Inc., Worldwide (2000-2002); technology developed by IBM, Inc.

Auction Type: FP-MR(F1,F2,F3).

Description: Sourcing and procurement of indirect materials (e.g., in-store display boxes) and direct materials (e.g., sugar). Winner determination via integer programming.

(F1) Package bids, OR language.

(F2) Discount schedules.

- (F3) Maximum and minimum number of winners and on the total quantity allocated to a single bidder.

Outcome: 60 CAs were conducted by end of 2002, credited with providing return-on-investment in less than a year. When package bids were enabled, typically less than 5% of bids were on packages. When non-package expressiveness was enabled, typically this was used by more than 80% of bids.

- **Sourcing of Materials at Motorola, Inc.** (Metty et al. 2005)

Organization: Motorola, Inc. (2002-2003); technology developed by Emptoris, Inc., acquired by IBM in 2012.

Auction Type: FP-MR(F2,F3).

Description: Sourcing events for direct materials (e.g., cables, displays, semiconductors), and indirect materials (e.g., paper, printed materials). Winner determination via integer programming. CAs were used to run the sourcing events for “over 50% of [Motorola’s] annual spending.”

- (F2) Discount schedules. Also, conditional discounts that triggered based on a quantity threshold.
- (F3) Maximum and minimum number of winners and of preferred winners. Quality-of-service constraints.

Outcome: As of 2003, \$600M saving on volume over \$16B for materials and air transport. Auctions credited with facilitating faster more focused negotiation, and providing more transparency for suppliers. Around 80% of spend went through CAs, compared with 20% going through simple, non-combinatorial reverse auctions.

- **Sourcing and Procurement of Materials at Global 1000 companies** (Sandholm et al. 2006, Sandholm 2013).

Organizations: More than sixty companies, mostly in the Global 1000, including Proctor & Gamble, Walmart and Target (2000-present); Jaggaer (rebranded from SciQuest), technology and products developed at CombineNet.

Auction Type: FP-MR(F2,F3).

Description: Sourcing and procurement for transportation, direct materials, packaging, indirect materials, chemicals, technology, services, health care and telecommunications. Activity rules sometimes used. Winner determination via integer programming.

- (F2) Discount schedules. Also, conditional discounts that triggered based on a quantity threshold.
- (F3) Maximum and minimum number of winners. Preferences to existing suppliers and minority-owned businesses. Quality-of-service constraints. Scenario navigation by the bid taker.

Outcome: As of 2006, \$35B spend had been sourced in events ranging in size from \$2M to \$1.6B, involving over 12,000 bidders, and with reported year-on-year cost savings of 11.1%. Non-package forms of expressive bidding were used more often than package bids.

Sourcing of Miscellaneous Non-Industrial Services

- **Sourcing of Roadside Assistance Services in the UK** (A. Holland, Keelvar, personal communication, 2017).

Organization: Direct Line Insurance (2012), auction technology developed by Keelvar Systems Ltd.

Auction Type: FP-SB(F1,F3).

Description: Tendering for roadside assistance services for broken down vehicles. Winner determination via integer programming.

(F1) Package bids.

(F3) A quality-of-service constraints.

Outcome: The speed of contract award was reported to have improved by a factor of four, supplier consolidation goals were achieved, and suppliers reported winning bundles of contiguous regions that permitted improved operational efficiency.

B.2 Forward Combinatorial Auctions

PUBLIC SECTOR, NON-SPECTRUM APPLICATIONS

Wholesale Electricity Markets

- **Sale of Electricity Generation Capacity in Virtual Power Plant auctions**

Organization: EDF (Electricité de France); Electrabel, Belgium; Nuon, Netherlands; Elsam, Denmark; Endesa and Iberdrola, Spain; REN and EDP, Portugal; Texas Capacity Auctions, Texas, U.S.; E.ON and RWE, Germany; Annual Capacity Auctions, Illinois, U.S.; Penelec, Pennsylvania, U.S.

Auction Type: FP-CCA(F1)

Description: Multiple products, i.e., generating capacity, are sold in a single auction, each representing base load or peak load and different contract duration, which is typically six months to two years. A typical CCA design for these auctions has the following features: feedback is provided regarding excess demand for each product across rounds, the auction closes when there is no excess demand, and the activity rules adopt to constrain bids in terms of bids placed in previous rounds, thus preventing bidders from jumping into the bidding towards the end of the auction. Intra-round bids are also adopted, to allow a bidder to express his demand at all price vectors along a price segment between the start-of-round and end-of-round prices. The idea is to allow larger price changes between rounds, and to enable fewer rounds and faster auctions.

(F1) Bidders can place a bid on a single package in each round, the bid specifying demand for a particular quantity of each product at current prices.

Outcome: The first VPP auction was used by EDF (France) in 2001 to divest 6 GW of generating capacity following an acquisition; quarterly auctions have been held by EDF since then, typically allocating capacity to more than fifteen bidders and with an average contract volume of US \$430 million per auction, and auctions completing in less than ten days and within a single day. Almost all capacity is typically sold, and any unsold capacity added to the next auction in the sequence. Electrabel (Belgium) has run quarterly VPP auctions since 2003 with a typical contract volume of \$130m. Eslam (Denmark) and E.ON and RWE (Germany) have run quarterly auctions, and Endesa and Iberdrola (Spain) run semi-annual auctions. Texas Capacity Auctions has been running VPP auctions five times a year since 2001.

Sale of Government Plant

- **Liberty Yard Auction (1922)** (U.S. Congress 1925, Cassady, Jr. 1967)

Organization: U.S. Shipping Board Emergency Fleet Corp.

Auction Name, Date: Liberty Yard Auction

Auction Type: FP-SB(F1).

Description: Sale of 70 residence sites comprising the Liberty Plant, Alameda, California

(F1) Package bids; specifically, entirety bids only. (Typical auction designs for government real estate allow for a first stage of package bids on all items, and a second stage with an auction on individual items, with these bidders winning if the sum total bid exceeded the best package bid. Package bids on all items remain a feature of real estate auctions.)

Outcome: An entirety bid of \$350,000 won on appeal.

Rental of Floor Space

- **Configuration and Rental of Floor Space in Amsterdam (2011)** (Goossens et al. 2014)

Organization: Stadgenoot (2011).

Auction Name: Furore Solids Auction; IJburg Solids Auction

Auction Type: FP-MR(F1,F3).

Description: To configure and rent space in “Solids,” buildings in which the tenants determine the allocation of space within the building. A Solid is divided into lots, which tenants use as “building blocks” to specify spaces. Winner determination via integer programming. Two auctions were held: May 2011, for a single building in the west of Amsterdam, called Furore, with 7000 square meters of floor space partitioned into 125 lots over seven floors; various constraints reduced the number of valid packages to 1,214; and June 2011, for two buildings in IJburg, a new planned neighborhood in the east of Amsterdam. (Further information on the June 2011 auction is unavailable.)

(F1) Package bids, OR-of-XOR language. Package bids subject to restrictions, including composed of adjacent lots on a single floor, and having access to the central hallway and utility shaft.

(F3) Minimum percentage of floor area required for each of the three different bidder types (residential, commercial and social). Unsold lots must not be too fragmented.

Outcome: May 2011 auction: 100 bidders placed 700 bids, with 30 winning bidders and 95% of space allocated. Monthly rental revenue generated was between €80,000 and €120,000 (\$100,000 and \$160,000)(F. Spieksma, personal communication, 2014 & 2017).

PUBLIC SECTOR, SALE OF WIRELESS SPECTRUM LICENSES

Here are some technical terms used in this section that did not arise in the main text: *GSM*, Global System for Mobile Communication, a wireless technology originally developed for Europe, but now widely used in other parts of the world; *E-GSM*, Extended GSM, a spectrum band that includes an extension of an additional 10 MHz at the lower end of the GSM 900 MHz band; *paired spectrum*, two associated blocks of spectrum, one used to transmit in one direction, and the other used to simultaneously transmit in the opposite direction, via a technology called *FDD*, Frequency Division Duplexing; *TDD* Time Division Duplexing, a technology for unpaired spectrum that makes use of a single frequency band, with transmitting and receiving occurring at different times; *MVNO*,

Mobile Virtual Network Operator, a company that does not own any radio spectrum but nevertheless provides mobile telephone service.

Sealed-Bid Combinatorial Auction

- **Sale of 900 MHz AND Sale of 1.8 GHz Spectrum in Norway (2001)** (NKOM 2001 a,b)

Organization: Norwegian Communications Authority (Nkom)

Auction Name, Date: 900 MHz Auction (Auction #1), October 2001.

Auction Name, Date: 1800 MHz (Auction #2), December 2001.

Auction Type: FP-SB(F1,F3).

Description: 900 MHz band divided into six 2x1.15 MHz lots comprising 11 E-GSM channels.

1.8 GHz band divided into seven lots comprising 32 GSM channels.

(F1) Bidders could submit one bid on a combination of two or more lots, and any number of bids on individual lots where the price offered for each lot may vary.

(F3) Spectrum caps.

Outcome of 900 MHz Auction: Three bidders won two adjacent lots each.

Total of lump sum payments: NOK 11.5M (\$1.3M). Total of annual fees: NOK 13.8M (\$1.5M).

Outcome of 1800 MHz: Two bidders won one license each, leaving five lots unsold.

Total of lump sum payments: NOK 56,800 (\$6,369). Total of annual fees: NOK 12.8M (\$1.4M).

- **Sale of 3.5 GHz Spectrum in Nigeria (2002)** (Koboldt et al. 2003)

Organization: Nigerian Communications Commission (NCC)

Auction Name, Date: Fixed Wireless Access Auction, June 2002.

Auction Type: FP-SB(F1).

Description: 80 licenses in the 3.5 GHz band, with two or three licenses available in each of 37 regions.

(F1) Package bids, XOR language.

Outcome: Around 40% of bids were on packages. 67 of the 80 licenses allocated. The contested licenses initially generated \$3.78B Nigerian naira (\$38M), but five of the twenty-five winners defaulted.

- **Sale of 400 MHz Spectrum in the UK (2006)**

Organization: Ofcom

Auction Name, Date: Auction of 412-414 MHz paired with 422-424 MHz, October 2006.

Auction Type: FP-SB(F1)

Description: 4 MHz of paired spectrum in four frequency lots, hence fifteen possible packages.

(F1) Package bids, OR language.

(F3) Spectrum caps.

Outcome: There were five bidders. One bidder outbid all the others by a considerable margin and won all four lots for a bid of £1,500,025 (\$2.3M).

- **Sale of 26 GHz Spectrum in Ireland (2008)** (COMREG 2008)

Organization: Commission for Communications Regulation (ComReg)

Auction Name, Date: 26 GHz Auction, June 2008.

Auction Type: CORE-SB(F1,F3).

Description: 18 lots of 2x28 MHz, in 27 possible packages. Auction design allowed market to determine split between point-to-point (P2P) and point-to-multipoint (PMP) applications. (This was one of the first auctions that allowed the market to determine how bands are to be divided between competing technologies.) Complex pricing rule involved adjusting all bids at once with a sum-of-squares adjustment penalty.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: Multiple winners in both P2P and PMP categories. Thus, a second stage, sealed-bid round was required to determine frequency assignments to bidders. Total revenue: €1,100,000 (US \$1.7M).

- **Sale of 3.4-3.6 GHz AND 3.6-3.8 GHz Spectrum in Portugal (2009–2010)** (ANACOM 2010)

Organization: ANACOM

Auction Name, Date: Broadband Wireless Access (BWA) Auction, October 2009–April 2010.

Auction Type: CORE-SB(F1).

Description: 36 2x28 MHz licenses. Two-stage auction. Variation on CORE payment rule. Auction design allowed market to determine how spectrum bands are to be divided between FDD and TDD technologies.

(F1) Each bidder could submit up to 100 package bids, OR language.

Outcome: There were three bidders; two were allocated licenses. 50% of the lots were allocated. Total revenue: €3,400,000 (\$4.6M).

- **Sale of 2.6 GHz Spectrum (2011) AND 800 MHz Spectrum in France (2011)** (DotEcon 2012)

Organization: ARCEP (French Regulatory Authority for Electronic Communications and Post)

Auction Name, Date: 2.6 GHz Spectrum Auction, September 2011.

Auction Name, Date: 800 MHz Spectrum Auction, December 2011.

Auction Type: FP-SB(F1,F3).

Description: 2.6 GHz auction: Two lots of 2x20 MHz and two blocks of 2x5 MHz. (The 2.6 GHz spectrum band, 2500 to 2690 MHz, is also known as the 2.5 GHz spectrum band (Marsden et al. 2010).) 800 MHz auction: Two lots of 2x10 MHz and two 2x5 MHz lots. Licensees to pay annual fees of 1% of revenue from allocated spectrum.

(F1) Package bids, XOR language.

(F3) 2.6 GHz auction: Bidders agreeing to provide MVNO access had a multiplier applied to their bid. 800 MHz auction: Bidders agreeing to provide MVNO access and/or commit to accelerated rollout schedule had a multiplier applied to their bid.

Outcome: 2.6 MHz auction: 70 MHz of paired spectrum awarded. All four bidders won licenses. Total revenue: €936M (\$854M). 800 MHz auction: 30 MHz of paired spectrum awarded. Three of the four bidders won licenses. Total revenue: €2.649B (\$3.353B).

- **Sale of 800 MHz, 900 MHz and 1.8 GHz Spectrum in Norway (2013)** (NKOM 2013)

Organization: Norwegian Post and Telecommunications Authority

Auction Name, Date: 800, 900 and 1800 MHz Auction (Auction #14), December 2013.

Auction Type: FP-SB(F1,F3).

Description: Spectrum in the 800MHz, 900MHz and 1800MHz bands. Specifically: 2x30MHz in the 800MHz band, 2x15 MHz in the 900MHz band and 2x55MHz in the 1800MHz band.

(F1) Package bids, XOR language.

(F3) One of the lots (A2) required to be in the winning allocation. Spectrum caps.

Outcome: Three of the four bidders won licenses. Two each allocated 2x10 MHz in 800MHz band, 2x5 MHz in 900MHz band, 2x10 MHz in 1800MHz band. The third was allocated 2x10 MHz in 800MHz band, 2x5 MHz in 900MHz band, 2x20 MHz in 1800MHz band. Total revenue: NOK 1.8B (\$292M).

- **Sale of 26 GHz Spectrum in Ireland (2018)** (COMREG 2008)

Organization: Commission for Communications Regulation (ComReg)

Auction Name, Date: 26 GHz Auction, April–June 2018.

Auction Type: FB-SB(F1,F3).

Description: Spectrum offered for 19 2x28 MHz blocks of spectrum in the range 24.275–25.277 GHz paired with 25.753-26.285 GHz. The auction was in two-stages. A sealed-bid combinatorial auction was first run to assign the spectrum as frequency-generic lots, with prices set on the basis of opportunity cost. Bidders were restricted to winning no more than five 2x28 MHz blocks.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: There were three winners: Vodafone Ireland, Three Ireland, and Meteor Mobil Communications, each of which won five 2x28 MHz blocks, which was the maximum possible. The duration of each of the licenses is ten years. Lump sum payments: Three and Meteor €£350K each; Vodafone: €550 (higher due to an additional payment for being awarded a specific, requested frequency range). Annual fees: All three winners had a requirement to pay approx. €1.25 million each over the 10-year license duration.

Multi-Round Combinatorial Auction

- **Sale of 900 MHz Spectrum in the US (2003)** (FCC 2003)

Organization: Federal Communications Commission (FCC)

Auction Name, Date: Regional Narrowband PCS Auction (Auction 51), September 2003.

Auction Type: FP-MR(F1,F3).

Description: Six regional licenses in Channels 16 and 17. Channel 16: 901.8125–901.8250 MHz & 930.65–930.70 MHz. Channel 17: 901.8250–901.8375 MHz & 930.70–930.75 MHz.

(F1) Package bids, OR language (maximum 12 packages per bidder).

- (F3) Bidding credits for small and very small businesses, or consortia of either, corresponded to a discount on the winning bid. (In this paper, revenue is reported net of bidding credits, if any.)

Outcome: There were two bidders, with three rounds of bidding over two days, with only a single bid: A package bid for the five items consisting of the five regions for Channel 16. Total revenue: \$134,250.

- **Sale of 700 MHz Spectrum in the US (2008)** (FCC 2007, Bazelon 2009, Brusco et al. 2009, Cramton et al. 2011)

Organization: Federal Communications Commission (FCC) (2008)

Auction Name, Date: 700 MHz Auction (Auction 73), January–March 2008.

Auction Type: FP-MR(F1,F3)

Description: Five blocks of licenses: A, B, C, D, and E. Combinatorial bidding on the twelve licenses of 22 MHz of spectrum in block C using hierarchical package bidding involving three pre-defined packages.

- (F1) Package bids, OR language.

- (F3) Bidding credits for small and very small businesses, or consortia of either. This corresponded to a discount on the winning bid.

Outcome: There were 214 bidders. There were 261 rounds over 38 days, where 101 bidders won 1090 of the 1099 licenses. Only a small number of package bids were submitted, only one of which was in the final allocation. C Block raised \$5B (with reserve price \$4.5B). Total revenue: \$19.12 billion.

Combinatorial Clock Auction

- **Sale of 800 MHz and 1.9 GHz Spectrum in Trinidad and Tobago (2005)** (TATT 2005)

Organization: Telecommunications Authority of Trinidad and Tobago (TATT)

Auction Name, Date: Spectrum Auction, June 2005.

Auction Type: FP-CCA(F1,F3). A first-price two-stage CCA, consisting of a clock stage and an assignment stage, with no supplementary bid stage. (In the specification of design features for spectrum auctions, F1, F2, and F3, we will ignore the assignment stage for the purpose of taxonomy and just focus on bidding languages for the main part of the auction.)

Description: 10 lots of 5 MHz spectrum. 800 MHz band: 2 lots in of 2x5 MHz. 1.9 GHz band: 8 lots of 2x5 MHz. In Stage 2, the bidders could choose to bid for 3-lot, 4-lot, or 5-lot packages.

- (F1) Package bids in the second stage, XOR language.

- (F3) At most two winners, plus additional conditions on packages accepted.

Outcome: There were two bidders. Both won licenses. One firm won a package of 5 lots at US \$15,756,003. The other firm won a package of 3 lots at \$9,300,007. Total revenue: \$25.1M.

- **Sale of 12 GHz, 700 MHz, and 28 GHz Spectrum in Trinidad and Tobago (2007)** (TATT 2007)

Organization: Telecommunications Authority of Trinidad and Tobago (TATT)

Auction Name, Date: BWA Auction, October 2007.

Auction Type: FP-CCA(F1,F3). Similar structure to Trinidad and Tobago Spectrum Auction

of 2005.

Description: Spectrum in the Lower 700 MHz (698-746 MHz), 12 GHz, and 28 GHz bands.

(F1) Package bids in the second stage, XOR language.

(F3) At most two winners, plus additional conditions on packages accepted.

Outcome: There were three bidders. All won licenses. In the 12 GHz band, one bidder won all twelve blocks at TT\$650,000 per block p.a. each. In the Lower 70 MHz band, one bidder won three blocks at TT\$ 170,000 per block p.a. In the 28 GHz band, there was no winner, and thus no spectrum was allocated. Total revenue: TT\$8,331,000 (\$1.3M) p.a.

- **Sale of 10-40 GHz Spectrum (2008) AND 1.4 GHz Spectrum in the UK (2008)** (Cramton 2013, Ausubel et al. 2006)

Organization: Ofcom.

Auction Name, Date: 10-40 GHz Auction, February 2008.

Auction Name, Date: L-Band Auction, May 2008.

Auction Type: CORE-CCA(F1).

Description: The 10-40 GHz band was divided into 27 lots; the L-band (1.4 GHz) was divided into 17 lots. The L-band auction design allowed the market to determine how bands are to be divided up between high power vs. low power technologies.

(F1) Package bids, XOR language.

Outcome: 10-40 GHz auction: Each of the ten bidders were allocated spectrum. Total revenue: £1,430,000 (\$2.8M). L-Band auction: One of the eight bidders won all the lots. Total revenue: £8,330,00 (\$16M).

- **Sale of 2.3 GHz, 2.6, and 700 MHz Spectrum in Trinidad and Tobago (2009)** (TATT 2009)

Organization: Telecommunications Authority of Trinidad and Tobago (TATT)

Auction Name, Date: Second BWA Auction, April 2009.

Auction Type: FP-CCA(F1,F3). Similar structure to Trinidad and Tobago BWA Auction of 2007.

Description: Lots in the 2.3 GHz and 2.6 GHz bands, as well as spectrum remaining in the 700 MHz band left over from the first BWA auction.

(F1) Package bids in the second stage, XOR language.

(F3) At most two winners, plus additional conditions on packages accepted.

Outcome: There were three bidders. All won licenses. One won 1 lot in the Lower 700 MHz band, 4 lots in the 2.3 GHz band. The second won 4 lots in the Lower 700 MHz band, 10 lots in the 2.6 MHz band. The third won 8 lots in the 2.6 MHz band. Revenue: Lower 700 MHz band TT\$1,000,000; 2.3 GHz band TT\$80,000; 2.6 GHz band TT\$3,600,000. Total revenue: TT\$4,680,000 (\$0.74M) p.a.

- **Sale of 2.6 GHz Spectrum in the Netherlands (2010)** (Marsden et al. 2010)

Organization: Ministry of Economic Affairs

Auction Name, Date: 2.6 GHz Spectrum Auction, April 2010.

Auction Type: CORE-CCA(F1, F3).

Description: 37 blocks of 5 MHz to be awarded as paired or unpaired spectrum, or as unpaired guard blocks. The actual number of lots in each category to be determined by the auction. Also, one lot of unpaired spectrum at 2010-2019.7 MHz.

(F1) Package bids. Restrictions on some lots being contiguous. XOR language.

(F3) Spectrum caps.

Outcome: All five bidders who participated were awarded spectrum, with the amounts of frequency awarded per bidder the maximum permitted according to the caps imposed by the government. The total revenue from the auction was €2,627,000 (\$3.3M).

- **Sale of 2.5 GHz Spectrum in Denmark (2010)** (Marsden et al. 2010)

Organization: National IT and Telecom Agency

Auction Name, Date: 2.5 GHz Auction, May 2010.

Auction Type: CORE-CCA(F1,F3).

Description: 14 lots of 2x5 MHz and 9 lots of 5 MHz.

(F1) Each bidder could submit a single package bid in each clock round, together with multiple supplementary package bids; XOR language.

(F3) Spectrum caps.

Outcome: There were four winners. One won 2 blocks of paired and 5 blocks of unpaired spectrum; a second won 4 blocks of paired and no unpaired spectrum, and the third and fourth each won 4 blocks of paired and 2 blocks of unpaired spectrum. Revenue of DKK 1.01 billion (\$175M).

- **Sale of 2.6 GHz Spectrum in Austria (2010)** (Marsden et al. 2010, DotEcon 2012)

Organization: Austrian Regulatory Authority for Broadcasting and Telecommunications (RTR).

Auction Name, Date: 2.6 GHz Auction, September 2010.

Auction Type: CORE-CCA(F1, F3).

Description: 14 lots of 2x5 MHz and 10 lots of 5 MHz. Auction design was essentially the same as the Danish (2010) design.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: The auction had four winners, with revenue €39.5M (\$53M).

- **Sale of 1.8 GHz Spectrum in Sweden (2011)** (PTS 2011)

Organization: PTS (National Post & Telecom Agency)

Auction Name, Date: 1800 MHz Auction, October 2011.

Auction Type: VCG-CCA(F1).

Description: 2x70 MHz of spectrum divided into 14 lots of 2x5 MHz each. Two-stage CCA consisting of: (1) Clock Stage; and (2) Assignment Stage with winner and price determination for specific lots, employing VCG pricing rule—not Vickrey-nearest core-selecting pricing rule.

(F1) Package bids, XOR language.

Outcome: There were three bidders, two winners, each winning 7 lots. Total revenue: SEK 1.35B (\$204.5M).

- **Sale of Multi-Band Spectrum in Switzerland (2012)**

Organizations: Swiss Federal Communications Commission (ComCom) and Swiss Federal Office of Communications (BAKOM)

Auction Name, Date: Auction of Mobile Radio Frequencies, February 2012.

Auction Type: CORE-CCA(F1, F3). *Description:* Spectrum in the 800 MHz, 900 MHz, 1.8 GHz, and 2.1 GHz, and 2.6 GHz bands.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: The three eligible bidders each won licenses, generating revenues of CHF 996.3M (\$1.085B).

- **Sale of 800 MHz Spectrum in Denmark (2012)** (DotEcon 2012)

Organization: Danish Business Authority

Auction Name, Date: 800 MHz Spectrum Auction, June 2012.

Auction Type: CORE-CCA(F1,F3).

Description: 2x30 MHz divided into five lots: one lot of 2x10 MHz, four lots of 2x5 MHz.

(F1) Package bids, XOR language.

(F3) Coverage obligations.

Outcome: Two of the three bidders won licenses. One won 2x10 MHz without coverage obligation; the other the remaining 2x20 MHz with the coverage obligation. Total revenue: DKK 793.2M (\$130M).

- **Sale of Multi-Band Spectrum in Romania (2012)** (ANCOM 2012, Marinescu 2012)

Organization: ANCOM

Auction Name, Date: Spectrum Auction, September 2012.

Auction Type: FP-CCA(F1, F3).

Description: Spectrum in the 800 MHz, 900 MHz, 1.8 GHz and 2.6 GHz bands, available in 2x5 MHz lots, other than unpaired 2.6 GHz spectrum, where there were three 15 MHz lots. No supplementary bids round for winner determination, with one further round only for any undersell.

(F1) Package bids, XOR language.

(F3) Spectrum caps. Coverage obligations.

Outcome: All five bidders were awarded spectrum. Of the 545 MHz auctioned, 485 MHz was awarded. Total revenue: €6.82M (\$891M).

- **Sale of Multi-Band Spectrum in the Netherlands (2012)** (DotEcon 2012)

Organization: Agentschap Telecom

Auction Name, Date: Multiband Spectrum Auction, October–December 2012.

Auction Type: CORE-CCA(F1, F3).

Description: Spectrum in the 800 MHz, 900 MHz, 1.8 GHz, 1.9 GHz, and 2.6 GHz bands. 41 licenses.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: Four of the five bidders won licenses. All licenses were sold. Total revenue: €3.80b (\$4.85B).

- **Sale of 800 MHz, 900 MHz, and 1.8 GHz Spectrum in Ireland (2012)** (COMREG 2012)

Organization: ComReg

Auction Name, Date: Multi-Band Spectrum Auction, November 2012.

Auction Type: CORE-CCA(F1, F3).

Description: Spectrum in the 800 MHz, 900 MHz, 1.8 GHz bands. Existing license holders could relinquish existing GSM-only licenses and simultaneously receive a liberalized license for same spectrum if this exchange is included in their winning bid. Also, licenses could be short-term or long-term.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: There were four bidders, all of whom were allocated spectrum, paying €481.7m (\$631m) upfront and the remaining €372.95m (\$284.7m) paid in annual fees over the 17 year license period. Total revenue: \$1.088B.

- **Sale of 800 MHz and 2.6 GHz Spectrum in the UK (2013)** (Morse 2014)

Organization: Ofcom

Auction Name, Date: 4G Spectrum Auction

Auction Type: CORE-CCA(F1,F3)

Description: 250 MHz of spectrum. 800 MHz band: One 2x10 MHz lot plus four 2x5 MHz lots. 2.6 GHz band: Paired spectrum in 2x5 MHz lots, unpaired spectrum in 5 MHz lots. Eligibility-point based activity rule.

(F1) Package bids, XOR language.

(F3) Spectrum caps. Coverage obligations.

Outcome: Five of the seven bidders were allocated spectrum. Total revenue: £2.4B (\$4.55B).

- **Sale of 700 MHz and 2.6 GHz Spectrum in Australia (2013)** (Ausubel & Baranov 2014b)

Organization: ACMA (Australian Communications and Media Authority)

Auction Name, Date: Digital Dividend Spectrum Auction, May 2013.

Auction Type: CORE-CCA(F1, F3)

Description: 700 MHz band: 9 generic lots of 2x5 MHz. 2.6 GHz band: 14 generic lots of 2x5 MHz across 11 geographic regions. Variation of CORE payment rule minimized a weighted Euclidean distance to the Vickrey payments.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: All three bidders won licenses, with a total revenue of AUS\$1.965 billion (US\$2.02 billion).

- **Sale of 800 MHz, 900 MHz, and 1.8 GHz Spectrum in Austria (2013)** (Mochon & Saez 2017)

Organization: (RTR) Austrian Regulatory Authority for Broadcasting and Telecommunications
Auction Name, Date: Multiband Auction, October 2013.

Auction Type: CORE-CCA(F1,F3).

Description: Paired spectrum in the 800, 900, and 1.8 GHz bands. There were 28 blocks of spectrum for auction: 6 blocks of 800 MHz, one of which had increased coverage obligations; 7 blocks of 900 MHz, and 15 blocks of 1.8 GHz.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: Three of the four bidders were allocated spectrum. However, two filed complaints in the high court against the auction results. On 4 December 2014, the Austrian Constitutional Court ruled confirming the official decisions of the multiband auction. Revenue raised €2B (\$2.8B).

- **Sale of 800 MHz, 1.8 GHz and 2.6 GHz Spectrum in Slovakia (2014)** (TUSR 2013)

Organization: Telecommunications Regulatory Authority of the Slovak Republic

Auction Name, Date: 800, 1800 and 2600 MHz Spectrum Auction, January 2014.

Auction Type: CORE-CCA(F1).

Description: 800 MHz band: 6 blocks of 2x5.0 MHz. 1.8 GHz band: 3 blocks of 2x5.0 MHz, 1 block of 2x2.2 MHz, 1 block of 2x1.2 MHz, 1 block of 2x1.0 MHz, 1 block of 2x0.6 MHz, 1 block of 2x0.4 MHz. 2.6 GHz band: 14 blocks of 2x5.0 MHz, 10 blocks of 5.0 MHz.

(F1) Package bids, OR language.

Outcome: Four companies won licenses. Total revenue: €163.9M (\$222M).

- **Sale of 700 MHz Spectrum in Canada (2014)** (Taylor 2015)

Organization: Industry Canada

Auction Name, Date: 700 MHz Spectrum Auction, February 2014.

Auction Type: CORE-CCA(F1,F3).

Description: 68 MHz of spectrum: 3 blocks of 2x6 MHz, 2 blocks of 2x5 MHz, 2 blocks of 6 MHz, covering 14 license areas, 98 licenses in total. Auction design similar to UK 4G auction (2013). However, here there were revealed-preference based activity rules, which were not in the UK design.

(F1) Package bids, OR language.

(F3) Spectrum caps.

Outcome: There were ten bidders and eight winners, with 97 of the 98 licenses awarded. Total revenue: CA\$5.27B (\$4.8B).

- **Sale of Multi-Band Spectrum in Slovenia (2014)** (Mochon & Saez 2017)

Organization: AKOS (Agency for Communication Networks and Services of the Republic of Slovenia)

Auction Name, Date: Multi-Band Spectrum Auction, April 2014.

Auction Type: CORE-CCA(F1,F3).

Description: Licenses offered in the 800 MHz, 900 MHz, 1.8 GHz, 2.1 GHz, and 2.6 GHz bands.

(F1) Package bids, XOR language.

(F3) Reserved spectrum for new entrants or existing operators with market share at most 15%. Coverage obligations.

Outcome: All three bidders were awarded spectrum. Total revenue: €148,741,000 (\$206M).

- **Sale of 700 MHz Spectrum in New Zealand (2014)** (MBIE 2013)

Organization: Ministry of Business, Innovation & Employment

Auction Name, Date: 700 MHz Auction (Auction 12), October 2013–June 2014.

Auction Type: FP-CCA(F1,F3).

Description: Nine lots of 2x5 MHz in the 700 MHz band. Supplementary bids round to be held only if aggregate number of bids at end of clock stage does not exceed the number of lots available, in which case a supplementary bids round may be used to offer any remaining lots unsold in the clock rounds.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: Eight lots were sold in the clock stage, with the 9th lot sold in a run-off between two bidders. No supplementary bids round was held. Three bidders were allocated spectrum. Total revenue: NZ\$270,100,174 (\$234M).

- **Sale of 2.6 GHz Spectrum in Canada (2015)**

Organization: Industry Canada

Auction Name, Date: 2500 MHz Auction, April–May 2015.

Auction Type: CORE-CCA(F1,F3).

Description: 318 blocks of spectrum in the 2.6 GHz band across 61 regions of Canada. This was the first CCA to allow OR bidding.

(F1) Package bids, XOR language and OR language.

(F3) Spectrum caps.

Outcome: Total revenue: C\$755M (\$624M).

- **Sale of 1.7 GHz and 2.1 GHz Spectrum in Mexico (2016)** (IWS 2015)

Organization: IFT (Ifetel, the Federal Telecommunications Institute)

Auction Name, Date: AWS Spectrum Auction, February 2016.

Auction Type: CORE-CA(F1, F3).

Description: 80 MHz of 2x5 MHz spectrum across two AWS bands for the deployment of 4G networks: 30 MHz for AWS-1 (1710-1725 MHz/2110-2125 MHz); 50 MHz for AWS-3 (1755-1780 MHz/2155-2180 MHz). Allowed for replanning of entire AWS band so that bidders received contiguous spectrum.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: There were two winners. One paid 31.0B pesos (\$1.7B); the other paid 12.7B pesos (\$698M). Unassigned at conclusion: AWS-3 bands 1755-1760 MHz and 2155-2160 MHz. Total revenue: 43.7B Mexican pesos (\$2.4B).

- **Sale of 800 MHz, 900 MHz, 1.8 GHz, 2 GH and 2.6 GHz Spectrum in Montenegro (2016)** (EKIP 2016)

Organization: Agency for Electronic Communications and Postal Services

Auction Name, Date: Auction for Spectrum of Mobile Networks, July–August 2016.

Auction Type: CORE-CCA(F1,F3).

Description: 625 MHz of spectrum in 5 bands. 800 MHz: 6 lots of 2x5 MHz for 15 years. 900 MHz: 5 lots of 2x5 MHz for 15 years. 1800 MHz: 11 lots of 2x5 MHz for 15 years, 4 lots for 10 years. 2 GHz: 3 lots of 2x5 MHz for 15 years, 9 lots of 2x5 MHz for 10 years, 4 lots of 5 MHz for 15 years, 3 lots of 5 MHz for 10 years. 2.6 GHz: 14 lots of 2x5 MHz for 15 years, 10 lots of 5 MHz for 15 years.

(F1) Package bids, XOR language.

(F3) Spectrum caps. Reserved spectrum for existing operators in bands already in use and for new operators in newly available bands.

Outcome: There were three bidders, all of whom won spectrum. The clock stage ran 50 rounds. There was a supplementary bids round. The assignment stage ran 4 rounds. The auction raised €50,650,000 (\$57M). However, unallocated spectrum with a total reserve price of €8,305,000 (\$9M) was left unsold.

- **Sale of 1.8 GHz Spectrum in Denmark (2016)** (DEA 2016)

Organization: Danish Energy Agency

Auction Name, Date: 1800 MHz Auction, September 2016.

Auction Type: FP-CCA(F1, F3).

Description: 2x64.9 MHz paired frequencies (1720.1–1785.0 MHz paired with 1815.1–1880.0 MHz). The format is a CMRA (and thus, first price, with no proxy stage).

(F1) Package bids, XOR language.

(F3) Spectrum caps. Coverage obligation.

Outcome: There were three winners, and raised DKK 1.025 billion (\$154M).

- **Sale of 3.6 GHz Spectrum in Ireland (2017)** (COMREG 2017)

Organization: ComReg

Auction Name, Date: 3.6 GHz Band Spectrum Auction, May 2017.

Auction Type: CORE-CCA(F1, F3).

Description: 350 MHz of spectrum in the 3.6 GHz band in four rural regions (Borders Midland and West, South West, East, South East) and five cities and their suburbs (Dublin, Cork, Limerick, Galway, Waterford). 66 lots were offered: one 25 MHz frequency-specific lot and sixty-five 5 MHz generic lots.

(F1) Package bids, XOR language.

(F3) Spectrum caps.

Outcome: There were five winning bidders. The first won 25 MHz in each of the rural regions, and 60 MHz in each of the cities. The second won 60 MHz in the rural regions. The third won 80 MHz in the rural regions and 85 MHz in the cities. The fourth won 100 MHz nationally. The fifth won 85 MHz in the rural regions and 105 MHz in the cities. Total revenue: €60,466,312 (\$67.7M) in total up front fees plus €17,707,200 (\$19.8M) in total spectrum usage fees paid annually for 15 years.

- **Sale of 700 MHz, 900 MHz and 2300MHz Spectrum in Denmark (2019)**

Organization: ComReg.

Auction Name, Date: 700 MHz, 900 MHz and 2300 MHz, March–April 2019.

Auction Type: FP-CCA(F1, F3).

Description: All three Danish Mobile Network Operators participated in the auction. TDC paid DKK 1.6 billion for 2x10 MHz of paired spectrum in the 700 MHz band, 20 MHz of paired spectrum of 700 MHz supplemental downlink spectrum, 2x10 MHz in the 900 MHz band, and 60 MHz in the 2.3 GHz band. Hi3G paid DKK 485.2 million for 2x10 MHz in each of the 700 MHz and 900 MHz bands. TT-Network paid DKK 107.6 million for 2x5 MHz in the 700 MHz band and 2x10 MHz in the 900 MHz band. The remaining spectrum was assigned through a CMRA format, where an additional assignment stage allowed bidders to express preferences over different locations within each band to establish the specific frequencies assigned.

(F1) Package bids, XOR language.

(F3) Spectrum caps. Regional coverage obligation.

Outcome: All three bidders were winners. The auction raised in excess of DKK 2.2 billion (\$385.5 million).

- **Sale of Microwave Bands, 10, 13, 18, 23, 28, 32, 38 GHz in Norway (2020)**

Organization: Nkom.

Auction Name, Date: Auction of Fixed Link Microwave Bands, May 2020.

Auction Type: FP-CCA(F1, F3).

Description: The CMRA was used for the assignment of frequency-generic lots grouped into 24 lot categories, with bidders able to place multiple package bids in each round, and optimization used in each round to seek an assignment that included all active bidders; otherwise, the prices were increased to resolve demand conflicts. The specific frequencies were assigned in a subsequent assignment stage and with an additional sealed-bid process, with the objective of maximizing contiguity of holdings.

(F1) Package bids, XOR language.

Outcome: The auction raised NOK 34.3 million (\$4.2 million).

PRIVATE SECTOR

Bankruptcy and Real Estate Sales

- **Sale of General Property** (McLean 1955, Shakers 2012)

Organization: Various (from prior to 1955, to the present).

Auction Type: FP-SB(F1); followed by single-item auctions.

Description: A common design allows for a first stage of sealed-bid package bidding (on all items only), followed by a second stage with ascending-price auctions on individual items. A variation changes the order of the two stages. Package bids on all items remain a feature of bankruptcy auctions.

(F1) Package bids, entirety bids only.

Outcome: Early examples include the sale of a machine shop (a package bid of \$60,000 losing to a single bidder with bids on individual lots totalling \$95,150), the sale of prefab houses (a package bid of \$325,000 losing to a combination of bids on individual lots), and the sale of machinery and equipment of a manufacturing plant (a package bid of \$285,000 winning on appeal).

Contextual Display Advertising on the Internet

- **Contextual Display Ads** (Edelman et al. 2007, Varian & Harris 2014)

Organization: Major search engines and social media platforms (2000-present).

Auction Type: VCG-SB(F1,F2,F3)

Description: Ads auctioned to appear next to content when viewed in web browsers and on mobile devices. A typical auction is for multiple items, each item representing a different position on the page. The VCG payment rule has been adopted by Facebook (2010) and Google (2012) for contextual display ads. Auction designs vary across platforms, but can include:

(F1) Bids for multiple positions on a page to allow a more prominent ad.

(F2) Bids state a per-click willingness-to-pay, used to impute an XOR bid on each of various positions on the page via estimated click-through rates.

(F3) Quality thresholds prevent low quality or inappropriate ads from winning the auction.