Online Appendix for “Taxing Our Wealth”

Florian Scheuer
University of Zurich

Joel Slemrod
University of Michigan

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A model of medium-term wealth taxation

In this Appendix, we present a parsimonious model to characterize optimal wealth taxes in the presence of pre-existing wealth inequality, as discussed in Section 6.3 of the paper. Suppose individuals have a fixed labor productivity $\theta \sim F(\theta)$. They live for two periods $t = 0, 1$: They work, consume and save in the first period and consume out of their (after-tax) savings in the second (which can be interpreted as retirement). Their preferences are

$$u(c_0) - h(y_0/\theta) + \beta u(c_1),$$

where c_t is consumption in period t, y_0 is the labor income earned in period 0, and β is a common discount factor. Importantly, these preferences satisfy the Atkinson-Stiglitz assumptions: (i) the disutility of labor h is separable from the utility of consumption, and (ii) the subutility of consumption $u(c_0) + \beta u(c_1)$ is common across individuals.[1]

There is a linear savings technology with gross rate of return R (again common across individuals). Individual θ is born with wealth $k_0(\theta)$ in period 0. Notice that the initial wealth inequality is perfectly correlated with labor productivity θ. This is assumed to conceptually weaken the scope of a wealth tax: If the correlation were imperfect, so that there could be individuals with the same labor income but very different initial wealth levels, we would be in a situation with two-dimensional heterogeneity and a labor income tax alone would be insufficient to achieve redistribution across these two dimensions. Here, this well-known argument for a wealth tax does not apply.

Because we assume that no direct tax on initial wealth is possible, the remaining policy instruments are a tax on first-period labor income $T_y(y_0)$ and a tax on second-period wealth

[1] With heterogeneity in individuals’ savings propensities, governed by β, in addition to heterogeneity in labor skills θ, it is well known that the Atkinson-Stiglitz theorem does not apply and there is a case for savings taxes. We rule this out to stack the cards against a wealth tax.

$T_k(Rk_1)$, which is equivalent to a tax on capital income in this model. Individuals' budget constraints are

$$c_0 = y_0 - T_y(y_0) + k_0 - k_1 \tag{1}$$

$$c_1 = Rk_1 - T_k(Rk_1). \tag{2}$$

If $k_0(\theta)$ was common across all θ, the Atkinson-Stiglitz theorem would imply that, at any Pareto optimum, $T_k(Rk_1) = 0$: We should only use the labor income tax $T_y(y_0)$ to achieve redistribution. Here we ask what happens when $k_0(\theta)$ varies across individuals: Is the labor tax T_y still sufficient to deal with both initial wealth inequality and labor income inequality, or should we (also) use the tax on future wealth accumulation T_k despite its distortionary effects?[2] Proposition 1 shows that the latter is optimal.

Proposition 1. *In any Pareto optimum, the optimal marginal wealth tax schedule satisfies*

$$T_k'(Rk_1(\theta)) = \frac{T_y'(y_0(\theta))}{1 - T_y'(y_0(\theta))} \left[\frac{\sigma(\theta)}{\alpha(\theta)\eta(\theta)} \left(1 + \frac{1}{\varepsilon(\theta)} \right) - 1 \right]^{-1} \tag{3}$$

where σ denotes the intertemporal elasticity of substitution, ε the Frisch elasticity of labor supply, $\alpha = k_0/c_0$ measures the share of period-0 consumption financed out of initial wealth, and $\eta(\theta) \equiv k_0'(\theta)\theta/k_0(\theta)$ the elasticity of initial wealth with respect to labor productivity.

To gain intuition for the formula, consider first the case with no initial wealth inequality, so $\eta = 0$. Hence, we return to the Atkinson-Stiglitz benchmark with $T_k'(Rk_1) = 0$ and all desired redistribution is achieved through the labor income tax. The same is true when the intertemporal substitution elasticity σ is infinite (the saving distortions induced by a wealth tax explode) or when the Frisch elasticity ε is zero (inelastic labor supply implies that the labor tax is lump-sum, so there is no need for an additional wealth tax).

Second, more generally, formula (3) links the shapes of the wealth and labor income tax schedules at any optimum (as both are driven by the redistributive motives of the government). But the term in square brackets introduces a wedge between the two. For instance, suppose σ and ε are fixed parameters.[3] Then any variation in this term is determined by how $\alpha\eta$ varies across the distribution. If $\alpha\eta$, which summarizes the importance of initial wealth relative to labor income inequality, is increasing towards the top, the wealth tax should be more progressive than the labor income tax, and vice versa.[4]

[2] Boadway et al. (2000) and Cremer et al. (2001, 2003) and observe that the Atkinson-Stiglitz uniform commodity taxation result breaks down when individuals have heterogeneous endowments and consider optimal linear capital income taxes in a four-type model.

[3] This would be the case when $u(c)$ exhibits constant relative risk aversion and the disutility of labor is iso-elastic.

[4] It can also be shown that $T_k'(Rk_1) > 0$ under standard redistributive motives.

The parameters in formula (3) can all be connected to empirical statistics; notably, η can be backed out from the joint distribution of wealth and income. For example, suppose the marginal tax rates on wealth and labor income converge to the constants t_k and t_y at the top, respectively, and the same holds for the intertemporal substitution elasticity σ, the Frisch elasticity ε and the importance of initial wealth as captured by the statistic α. Then we have the following corollary of Proposition 1:

Corollary 1. *In any Pareto optimum, the top marginal wealth tax rate is*

$$t_k = \frac{t_y}{1-t_y}\left[\frac{\sigma\rho_k + \varepsilon\rho_y}{\varepsilon\alpha\rho_y} - 1\right]^{-1}$$

where t_y is the top marginal income tax rate and ρ_k and ρ_y are the Pareto tail coefficients of the wealth and labor income distribution, respectively.

It is straightforward to calibrate this formula. For instance, suppose $\sigma = 1$, so $u(c) = \log(c)$. Because $\alpha \leq 1 + \beta$, we can use the formula to obtain an upper bound to the marginal wealth tax rate. Empirical estimates suggest $\rho_k \approx 1.4$ and $\rho_y \approx 1.6$. Moreover, suppose $\varepsilon = .3$ and $t_y = 50\%$. Interpreting the period length in this two-period model as roughly 30 years and assuming a yearly interest rate of 3%, we have $\beta = .97^{30} = .4$. This implies

$$t_k \leq \frac{.5}{1-.5}\left[\frac{1 \times 1.4 + .3 \times 1.6}{.3 \times 1.4 \times 1.6} - 1\right]^{-1} = 56\%$$

over the 30-year horizon, or an annual wealth tax of at most

$$t_k^{annual} \leq 1 - (1 - .56)^{1/30} = 2.7\%.$$

References

Boadway, R., M. Marchand and P. Pestieau (2000), “Redistribution with Unobservable Bequests: A Case for Taxing Capital Income,” *Scandinavian Journal of Economics* 102, 253-267

Cremer, H., P. Pestieau and J.-C. Rochet (2001), “Direct versus Indirect Taxation: The Design of the Tax Structure Revisited,” *International Economic Review* 42, 781-799

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Proof of Proposition 1

Because initial wealth k_0 cannot be directly targeted, this amounts to the assumption that both θ and k_0 are unobservable to the government. In other words, lump-sum instruments based on labor productivities or initial wealth are unavailable. This means that the government cannot directly control $c_0(\theta)$: instead, it can only determine $\hat{c}_0(\theta) \equiv c_0(\theta) - k_0(\theta)$. Intuitively, the government can give individuals a transfer in $t = 0$ of the amount $\hat{c}_0$, but their actual consumption will then be given by $c_0 = \hat{c}_0 + k_0$, which is unobservable.

By the revelation principle, any allocation $(\hat{c}_0(\theta), c_1(\theta), y_0(\theta))$ that is attainable through some tax system must therefore satisfy the incentive compatibility constraints

$$u(\hat{c}_0(\theta) + k_0(\theta)) - h(y_0(\theta)/\theta) + \beta u(c_1(\theta)) \geq u(\hat{c}_0(\theta') + k_0(\theta)) - h(y_0(\theta')/\theta) + \beta u(c_1(\theta')) \tag{4}$$

for all θ, θ'. The aggregate resource constraint is

$$\int \hat{c}_0(\theta) dF(\theta) + \frac{1}{R} \int c_1(\theta) dF(\theta) \leq \int y_0(\theta) dF(\theta). \tag{5}$$

The government maximizes

$$\int g(\theta) \left[u(\hat{c}_0(\theta) + k_0(\theta)) - h(y_0(\theta)/\theta) + \beta u(c_1(\theta)) \right] dF(\theta)$$

using some Pareto weights $g(\theta)$.

Define

$$V(\theta) \equiv u(\hat{c}_0(\theta) + k_0(\theta)) - h(y_0(\theta)/\theta) + \beta u(c_1(\theta)).$$

The necessary envelope condition corresponding to (4) is

$$V'(\theta) = u'(\hat{c}_0(\theta) + k_0(\theta)) k_0'(\theta) + h' \left(\frac{y_0(\theta)}{\theta} \right) \frac{y_0(\theta)}{\theta^2}. \tag{6}$$

It is useful to formulate the planning problem in terms of $(V(\theta), c_1(\theta), y_0(\theta))$ using

$$\hat{c}_0(\theta) = \Phi \left[V(\theta) + h(y_0(\theta)/\theta) - \beta u(c_1(\theta)) \right] - k_0(\theta)$$

where $\Phi(u)$ denotes the inverse function of $u(c)$. This allows us to write the Pareto problem as follows:

$$\max_{(V(\theta), c_1(\theta), y_0(\theta))} \int g(\theta) V(\theta) dF(\theta)$$

s.t.

$$V'(\theta) = u'\left[\Phi\left(V(\theta) + h\left(\frac{y_0(\theta)}{\theta}\right) - \beta u(c_1(\theta))\right)\right]k_0'(\theta) + h'\left(\frac{y_0(\theta)}{\theta}\right)\frac{y_0(\theta)}{\theta^2}$$

and

$$\int \Phi\left[V(\theta) + h\left(\frac{y_0(\theta)}{\theta}\right) - \beta u(c_1(\theta))\right]dF(\theta) + \frac{1}{R}\int c_1(\theta)dF(\theta) \leq \int y_0(\theta)dF(\theta) + \int k_0(\theta)dF(\theta).$$

After integration by parts (and dropping boundary terms), the corresponding Lagrangian becomes

$$\begin{aligned}
\mathcal{L} \;=\; & \int g(\theta)V(\theta)dF(\theta) - \int \mu'(\theta)V(\theta)d\theta \\
& - \int \mu(\theta)\left\{u'\left[\Phi\left(V(\theta) + h\left(\frac{y_0(\theta)}{\theta}\right) - \beta u(c_1(\theta))\right)\right]k_0'(\theta) + h'\left(\frac{y_0(\theta)}{\theta}\right)\frac{y_0(\theta)}{\theta^2}\right\}d\theta \\
& +\lambda\left\{\int y_0(\theta)dF(\theta) + \int k_0(\theta)dF(\theta)\right\} \\
& -\lambda\left\{\int \Phi\left[V(\theta) + h\left(\frac{y_0(\theta)}{\theta}\right) - \beta u(c_1(\theta))\right]dF(\theta) + \frac{1}{R}\int c_1(\theta)dF(\theta)\right\} \qquad (7)
\end{aligned}$$

where $\mu(\theta)$ and λ denote the multipliers on the incentive and resource constraints, respectively. The first-order condition for $c_1(\theta)$ is

$$\mu(\theta)\frac{u''(c_0(\theta))}{u'(c_0(\theta))}\beta u'(c_1(\theta))k_0'(\theta) - \lambda f(\theta)\left[\frac{1}{R} - \beta\frac{u'(c_1(\theta))}{u'(c_0(\theta))}\right] = 0$$

where we used $\Phi'(u) = 1/u'(c)$. Define the savings wedge as

$$T_k'(Rk_1) \equiv 1 - \frac{u'(c_0)}{\beta R u'(c_1)}.$$

Substituting this yields

$$T_k'(Rk_1(\theta)) = -\frac{\mu(\theta)}{\lambda f(\theta)}u''(c_0(\theta))k_0'(\theta). \qquad (8)$$

This already reveals that there is a savings wedge at the optimum whenever $k_0'(\theta) \neq 0$ and $\mu(\theta) \neq 0$.

The first-order condition for $y_0(\theta)$ is (dropping arguments to simplify notation)

$$-\mu\left[\frac{u''(c_0)}{u'(c_0)}h'\left(\frac{y_0}{\theta}\right)\frac{1}{\theta}k_0'(\theta) + h''\left(\frac{y_0}{\theta}\right)\frac{y_0}{\theta^3} + h'\left(\frac{y_0}{\theta}\right)\frac{1}{\theta^2}\right] + \lambda f\left[1 - \frac{h'(y_0/\theta)}{\theta u'(c_0)}\right] = 0.$$

Rearranging,

$$\mu = \frac{\lambda f \left[1 - \frac{h'(y_0/\theta)}{\theta u'(c_0)}\right]}{\frac{h'(y_0/\theta)}{\theta u'(c_0)} \left[u''(c_0)k_0'(\theta) + \frac{u'(c_0)}{\theta}\left(\frac{h''(y_0/\theta)}{h'(y_0/\theta)}\frac{y_0}{\theta} + 1\right)\right]}.$$

Define the labor wedge as

$$T_y'(y_0) \equiv 1 - \frac{h'(y_0/\theta)}{\theta u'(c_0)}.$$

Moreover, the Frisch elasticity is

$$\varepsilon \equiv \left.\frac{dy_0}{d(1-T_y')}\right|_{u'(c_0)} \frac{1-T_y'}{y_0} = \frac{h'(y_0/\theta)\theta}{h''\left(y_0/\theta\right) y_0}.$$

Substituting both yields

$$\mu = \frac{\lambda f T_y'(y_0)}{(1-T_y'(y_0))\left[u''(c_0)k_0'(\theta) + \frac{u'(c_0)}{\theta}\left(\frac{1}{\varepsilon}+1\right)\right]}.$$

Finally, substituting this in (8) delivers

$$T_k'(Rk_1(\theta)) = -\frac{T_y'(y_0)}{1-T_y'(y_0)} \frac{1}{1 + \frac{u'(c_0)}{\theta u''(c_0)k_0'(\theta)}\left(\frac{1}{\varepsilon}+1\right)}.$$

The intertemporal elasticity of substitution is

$$\sigma = -\frac{u'(c)}{cu''(c)},$$

so we can write this as

$$T_k'(Rk_1(\theta)) = \frac{T_y'(y_0)}{1-T_y'(y_0)} \frac{1}{\frac{\sigma c_0}{\theta k_0'(\theta)}\left(\frac{1}{\varepsilon}+1\right) - 1}.$$

Together with the definition of $\alpha(\theta) = k_0(\theta)/c_0(\theta)$ and $\eta(\theta) \equiv k_0'(\theta)\theta/k_0(\theta)$, this delivers the result in (3).

Proof of Corollary 1

The individuals' Euler equation is

$$u'(c_0) = \beta R(1-t_k)u'\left((1-t_k)R\left((1-t_y)y_0 + k_0 - c_0\right)\right).$$

With a constant intertemporal elasticity of substitution σ, this can be written as

$$c_0 = \gamma \left((1 - t_y) y_0 + k_0 - c_0 \right)$$

with

$$\gamma = \beta^{-\sigma} R^{1-\sigma} (1 - t_k)^{1-\sigma}.$$

Rearranging yields

$$\frac{1+\gamma}{\gamma} = (1 - t_y) \frac{y_0}{c_0} + \frac{k_0}{c_0}$$

and hence $\alpha \leq 1 + 1/\gamma$.

With a constant Frisch elasticity ε, the first-order condition for y_0 is

$$y_0 = (1 - t_y)^{\varepsilon} \theta^{1+\varepsilon} c_0^{-\frac{\varepsilon}{\sigma}}.$$

If $\alpha(\theta) \to \alpha > 0$ and $\eta(\theta) \to \eta$ as θ grows large, then for high-earners

$$y_0(\theta) \to \kappa_y \theta^{1+\varepsilon(1-\eta/\sigma)}$$

for some constant κ_y. Second-period wealth is

$$Rk_1(\theta) = R \left((1 - t_y) y_0(\theta) + k_0(\theta) - c_0(\theta) \right),$$

which under the same conditions becomes

$$Rk_1(\theta) \to \tilde{\kappa}_y \theta^{1+\varepsilon(1-\eta/\sigma)} + \kappa_k \theta^{\eta}$$

for some constants $\tilde{\kappa}_y$ and κ_k. Because the Pareto tail coefficient of the empirical income distribution ρ_y is typically higher than that of the empirical wealth distribution ρ_k (because wealth is more unequally distributed than income) and in this model both are driven by the same underlying skill parameter θ, this implies that the Pareto tail coefficient of the distribution of θ must satisfy both

$$\rho_\theta = \left(1 + \varepsilon \left(1 - \frac{\eta}{\sigma} \right) \right) \rho_y \quad \text{and} \quad \rho_\theta = \eta \rho_k,$$

which can be used to solve for η:

$$\eta = \frac{\sigma(1+\varepsilon)\rho_y}{\sigma\rho_k + \varepsilon\rho_y}.$$

Substituting this in (3) delivers the sufficient-statistics formula in Corollary 1.