

# Macro and Micro Dynamics of Productivity: From Devilish Details to Insights

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## A Online Appendix

### A.1 $\log TFP R^{rr}$ and fundamentals

We derive the properties of  $\log TFP R^{rr}_{is}$  under the more general specification of demand given by  $P_{is}Q_{is} = P_s Q_s^{1-\rho_s} Q_{is}^{\rho_s-1} \xi_{is}^{\rho_s} Q_{is} = P_s Q_s^{1-\rho_s} Q_{is}^{\rho_s} \xi_{is}^{\rho_s}$ , where  $\rho_s = \frac{\sigma_s-1}{\sigma_s}$  as in section I. In this case, log plant level revenues can be written as

$$(A.1) \quad \begin{aligned} \log P_{is} + \log Q_{is} &= \rho_s \log Q_{is} + (1 - \rho_s) \log Q_s + \log P_s + \rho_s \ln \xi_{is} \\ &= \rho_s \left( \sum_j \alpha_{js} \log X_{ijs} + \log \mathcal{A}_{is} \right) + (1 - \rho_s) \log Q_s + \log P_s + \rho_s \ln \xi_{is}. \end{aligned}$$

This permits characterizing  $\log TFP R^{rr}_{is}$  as:

$$(A.2) \quad \begin{aligned} \log P_{is} + \log Q_{is} - \rho_s \sum_j \alpha_{js} \log X_{ijs} &= \log P_{is} + \log Q_{is} - \sum_j \beta_{js} x_{ijs} \\ &= \rho_s \log \mathcal{A}_{is} + \rho_s \ln \xi_{is} + (1 - \rho_s) \log Q_s + \log P_s, \end{aligned}$$

which says that  $\log TFP R^{rr}_{is}$  is a function of  $\log TFP Q_{is}$  ( $\log \mathcal{A}_{is}$ ), demand shocks ( $\log \xi_{is}$ ), and sectoral variables ( $Q_s, P_s$ , and  $\rho_s$ ). In the main text, we abstract from idiosyncratic demand shocks and sectoral variables for transparency. We estimate the  $\beta_{js}$  using a control function approach. In some robustness analysis, we also use the Klette and Griliches (1996) approach to jointly estimate  $\beta_{js}$  and  $\rho_s$  by including a measure of industry-level output as a regressor. This permits us to back out  $\alpha_{js}$  from the combined estimates and provides an alternative method to cost shares for estimating  $\alpha_{js}$ . The advantage of this approach is that it does not impose CRS. The disadvantage of this approach is that, in the absence of data on plant level prices and quantities, this is pushing the data quite hard. Foster et al. (2016) discuss the latter limitations in more depth.

### A.2 AE under NCRS

#### A.2.1 Industry-level prices

Defining  $\tau_{is} = \prod_j (1 + \tau_{is}^j)^{\frac{\alpha_{js}}{\gamma_s}}$ , we have:

$$(A.3) \quad P_s = Q_s^{\frac{1-\gamma_s}{\gamma_s}} \frac{1}{\kappa_s} \left( \sum_i \left( \frac{A_{is}}{\tau_{is}^{\gamma_s}} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \right)^{\frac{\rho_s\gamma_s-1}{\rho_s\gamma_s}}$$

where  $\kappa_s = \left( \rho_s \prod_j \left( \frac{\alpha_{js}}{w_{js}} \right)^{\alpha_{js}/\gamma_s} \right)^{-1}$  is a function of input prices and parameters.

### A.2.2 Industry-level distortions

It can be shown that industry-level distortions can be written as a function of  $TFPR_s$  and a constant:<sup>1</sup>

$$(A.4) \quad \tilde{\tau}_s = \kappa_s TFPR_s = \kappa_s \frac{P_s Q_s}{\prod_j X_{js}^{\alpha_{js}/\gamma_s}}.$$

As noted in Bils, Klenow and Ruane (2020) (hereafter BKR), this can be expressed as a product of sectoral input distortions, which are in turn revenue-weighted harmonic means of plant-level input distortions. Note that  $\tilde{\tau}_s$  can be written as a function of idiosyncratic physical productivities and distortions using  $TFPR_s^{cs} = \sum_i \frac{I_{is}}{\sum_i I_{is}} TFPR_{is}^{cs}$ , where  $I_{is}$  denotes the plant's cost-share based input index.<sup>2</sup> Expressing  $I_{is}$  as a function of  $A_{is}$ ,  $\tau_{is}$  and parameters implies:

$$(A.5) \quad \tilde{\tau}_s = \frac{\sum_i A_{is}^{\frac{\rho_s}{1-\rho_s\gamma_s}} (\tau_{is})^{\frac{-\rho_s\gamma_s}{1-\rho_s\gamma_s}}}{\sum_i A_{is}^{\frac{\rho_s}{1-\rho_s\gamma_s}} (\tau_{is})^{\frac{-1}{1-\rho_s\gamma_s}}} = \frac{S_1}{S_2}.$$

### A.2.3 Industry-level TFP

We define the industry-productivity measure consistent with Hsieh and Klenow (2009) and BKR, where the denominator is the input index weighted by cost shares  $TFP_s = \left( \prod_j X_{js}^{\alpha_{js}/\gamma_s} \right)^{-1} Q_s$ . Multiplying and dividing by  $P_s^{\gamma_s}$  yields:

$$(A.6) \quad TFP_s = \frac{P_s^{\gamma_s} Q_s}{P_s^{\gamma_s} \prod_j X_{js}^{\frac{\alpha_{js}}{\gamma_s}}}.$$

Combining this expression with equation (A.3) yields  $TFP_s = \left( \prod_j X_{js}^{\frac{\alpha_{js}}{\gamma_s}} \right)^{-1} \left( \sum_i \left( \frac{A_{is}}{\tau_{is}^{\gamma_s}} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \right)^{\frac{1-\rho_s\gamma_s}{\rho_s}} \times \kappa_s^{\gamma_s} P_s^{\gamma_s} Q_s^{\gamma_s-1} Q_s$ , which can be rearranged as:

$$(A.7) \quad TFP_s = \left( \sum_i \left( \frac{A_{is}}{\tau_{is}^{\gamma_s}} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \right)^{\frac{1-\rho_s\gamma_s}{\rho_s}} \frac{\left( \frac{\kappa_s P_s Q_s}{\prod_j X_{js}^{\frac{\alpha_{js}}{\gamma_s}}} \right)^{\gamma_s}}{\left( \prod_j X_{js}^{\frac{\alpha_{js}}{\gamma_s}} \right)^{1-\gamma_s}} = \left( \prod_j X_{js}^{\frac{\alpha_{js}}{\gamma_s}} \right)^{\gamma_s-1} \left( \sum_i \left( \frac{A_{is}}{\tau_{is}^{\gamma_s}} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \right)^{\frac{1-\rho_s\gamma_s}{\rho_s}} \tilde{\tau}_s^{\gamma_s}$$

$$= \left( \prod_j X_{js}^{\frac{\alpha_{js}}{\gamma_s}} \right)^{\gamma_s-1} \left( \sum_i A_{is}^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left( \frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{\rho_s\gamma_s}{1-\rho_s\gamma_s}} \right)^{\frac{1-\rho_s\gamma_s}{\rho_s}}$$

This is analogous to the expression obtained in Appendix 1 of HK.

<sup>1</sup>The formula can be obtained by writing  $TFPR_s$  as a geometric average of sectoral marginal revenue product where the weights are based on the cost shares of respective inputs.

<sup>2</sup>Note  $\sum_i \frac{I_{is}}{\sum_i I_{is}} TFPR_{is}^{cs} = \sum_i \frac{I_{is}}{\sum_i I_{is}} \frac{R_{is}}{I_{is}} = \frac{\sum_i R_{is}}{\sum_i I_{is}} = TFPR_s^{cs}$ .

### A.2.4 Maximum of $TFP_s$

This section outlines the solution to the constrained optimization that yields  $AE_s$  under NCRS. Using the notation in (A.5), the Lagrangean of the problem is given by:

(A.8)

$$\mathcal{L}(\tau_{is}) = \left( \sum_i A_{is}^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left( \frac{\tau_{is}}{\tilde{\tau}_s} \right)^{-\frac{\rho_s\gamma_s}{1-\rho_s\gamma_s}} \right)^{\frac{1-\rho_s\gamma_s}{\rho_s}} + \lambda \left( \tilde{\tau}_s - \sum_i \theta_i^I \tau_{is} \right) = \tilde{\tau}_s^\gamma S_1^{\frac{1-\rho_s\gamma_s}{\rho_s}} + \lambda \left( \tilde{\tau}_s - \frac{S_1}{S_2} \right),$$

where  $S_1 = \sum_i A_{is}^{\frac{\rho_s}{1-\rho_s\gamma_s}} \tau_{is}^{-\frac{\rho_s\gamma_s}{1-\rho_s\gamma_s}}$  and  $S_2 = \sum_i A_{is}^{\frac{\rho_s}{1-\rho_s\gamma_s}} \tau_{is}^{-\frac{1}{1-\rho_s\gamma_s}}$ . The first derivative of  $\mathcal{L}(\tau_{is})$  is shown below:

$$(A.9) \quad \frac{\partial \mathcal{L}}{\partial \tau_{is}} = -\gamma_s \tilde{\tau}_s^{\gamma-1} S_1^{\frac{1-\rho_s\gamma_s}{\rho_s}-1} A_{is}^{\frac{\rho_s}{1-\rho_s\gamma_s}} \tau_{is}^{-\frac{1}{1-\rho_s\gamma_s}} + \lambda \frac{\rho_s \gamma_s A_{is}^{\frac{\rho_s}{1-\rho_s\gamma_s}} \tau_{is}^{-\frac{1}{1-\rho_s\gamma_s}} S_2 - S_1 A_{is}^{\frac{\rho_s}{1-\rho_s\gamma_s}} \tau_{is}^{-\frac{1}{1-\rho_s\gamma_s}-1}}{(1-\rho_s\gamma_s) S_2^2}.$$

Summing (A.9) over  $i$  yields a condition that can be solved for  $\lambda$ . Plugging the resulting expression back to (A.9) and rearranging implies  $\tau_{is} = \tilde{\tau}_s$ . Differentiating (A.9) with respect to  $\tau_{is}$  yields  $-2\gamma_s(1-\rho_s\gamma_s)^{-1} S_2^{-2} S_1^{\frac{1-\rho_s\gamma_s}{\rho_s}} A_{is}^{\frac{\rho_s}{1-\rho_s\gamma_s}} \tilde{\tau}_s^{-\frac{2}{1-\rho_s\gamma_s}-2+\gamma_s} \sum_{j \neq i} A_{js}^{\frac{\rho_s}{1-\rho_s\gamma_s}}$ . Since  $0 < A_{is}, \tau_{is}, \rho_s, \gamma_s$ , the sign of the second derivative depends on the sign of  $(1-\rho_s\gamma_s)$ . If  $\rho_s\gamma_s < 1$  then  $\left. \frac{\partial^2 TFP_s}{\partial \tau_{is}^2} \right|_{\tilde{\tau}_s} < 0$ , and therefore  $\tau_{is} = \tilde{\tau}_s$  is a maximum point. However, if  $1 < \rho_s\gamma_s$  then  $0 < \left. \frac{\partial^2 TFP_s}{\partial \tau_{is}^2} \right|_{\tilde{\tau}_s}$ , and  $\tau_{is} = \tilde{\tau}_s$  cannot be a maximum point.

### A.2.5 Aggregate and sectoral production

Aggregate output is assumed to be a Cobb Douglas CRS aggregate of sectoral output. This implies:

$$(A.10) \quad Q = \prod_s Q_s^{\theta_s} = \prod_s \left( A_s \prod_j X_j^{\frac{\alpha_{js}}{\gamma_s}} \right)^{\theta_s}$$

It can then be shown that aggregate output  $Q$  can be written as a product of the geometric averages of industry-level technical efficiencies, revenue shares, cost shares, distortions and inputs:

$$(A.11) \quad Q = \prod_s A_s^{\delta_{s1}} \times \prod_s \theta_s^{\delta_{s1}} \times \prod_s \left[ \prod_j \left( \frac{\alpha_{js}}{\gamma_s} \right)^{\frac{\alpha_{js}}{\gamma_s}} \right]^{\delta_{s1}} \times \prod_s \tau_s^{-\delta_{s1}} \times (1 + \tau^X)^{-\delta_{s2}} \\ \times \prod_{j \neq M} \left[ X_j \left( \sum_s \theta_s \rho_s \alpha_{js} \left( 1 - \frac{\alpha_{Ms}}{\gamma_s} \right) \frac{1 + \tau^j}{1 + \tau_s^j} \right)^{-1} \right]^{\delta_{s3}},$$

where  $\delta_{s1} = \frac{\theta_s}{\sum_s \theta_s (1 - \frac{\alpha_{Ms}}{\gamma_s})}$ ,  $\delta_{s2} = \frac{\sum_s \theta_s \frac{\alpha_{Ms}}{\gamma_s}}{\sum_s \theta_s (1 - \frac{\alpha_{Ms}}{\gamma_s})}$ ,  $\delta_{s3} = \frac{\sum_s \theta_s \frac{\alpha_{js}}{\gamma_s} (1 - \frac{\alpha_{Ms}}{\gamma_s})}{\sum_s \theta_s (1 - \frac{\alpha_{Ms}}{\gamma_s})}$ . Defining  $\tilde{\alpha}_j = \frac{\sum_s \theta_s (\frac{\alpha_{Ms}}{\gamma_s}) \alpha_{js}}{\sum_s \theta_s (\frac{\alpha_{Ms}}{\gamma_s})}$  and aggregate consumption or value added as output less intermediate input  $C = Q - M$ , the expression for aggregate  $TFP$  is given by  $TFP = C / \prod_{j \neq M} X_j^{\tilde{\alpha}_j}$ . Let  $\bar{T}$  denote the part of the expression that depends only on sectoral distortions and parameters. In addition, adjust equation (A.11) for inter-

mediate input use. Then the following expression can be obtained for aggregate  $TFP$ :

$$(A.12) \quad TFP = \bar{T} \times \prod_s^S TFP_s^{\frac{\theta_s}{\sum_s \theta_s \left(1 - \frac{\alpha M_s}{\gamma_s}\right)}}.$$

### A.2.6 Accounting for input demand

It can be shown that if sectoral level inputs with inelastic aggregate supply, so long as *average* sectoral distortions are the same under a new distribution of distortions, sectoral capital and labor are unchanged in the “undistorted” counterfactual. For our analysis, we ignore between sector distortions and assume that average sectoral distortions are the same. Since aggregate labor and capital supply is assumed to be inelastic,  $AE_s$  is given by:

$$(A.13) \quad AE_s = \frac{A_s}{A_s^*} = \left( \sum_i \left( \frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left( \frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{\rho_s\gamma_s}{1-\rho_s\gamma_s}} \right)^{\frac{1-\rho_s\gamma_s}{\rho_s}} \left( \frac{M_s^*}{M_s} \right)^{\frac{\alpha M_s(1-\gamma_s)}{\gamma_s}}.$$

In addition, sectoral intermediate input demand is proportional to total output:  $M_s = \theta_s Q \rho_s \gamma_s (1 - \gamma_s) \frac{1}{1 + \tau_s^M}$ . So long as sectoral average intermediate distortions are held constant, the ratio of undistorted inputs to distorted inputs in industry  $s$  can be expressed as:

$$(A.14) \quad M_s^*/M_s = Q_s^*/Q,$$

where  $Q_s^*$  is the aggregate output under the regime where distortions in sector  $s$  are equalized, holding average distortions constant. Thus, the ratio of aggregate “ $s$ -undistorted” output to realized output is equivalent to the ratio of intermediates. Using equation (A.11), we can obtain an expression for  $Q_s^*$ . On the condition that average distortions are held constant between the actual and counterfactual cases, the only change relative to (A.11) is that the leading term of  $Q_s^*$  is given by  $(A_s^*)^{\frac{\theta_s}{\sum_k \theta_k \left(1 - \frac{\alpha M_k}{\gamma_k}\right)}} \prod_{k \neq s} A_k^{\frac{\theta_k}{\sum_k \theta_k \left(1 - \frac{\alpha M_k}{\gamma_k}\right)}}$ , i.e. only the sectoral productivity of the  $s$ th industry changes. It follows that the ratio in equation (A.14) can be written as a function of allocative efficiency:

$$(A.15) \quad Q_s^*/Q = (A_s^*/A_s)^{\frac{\theta_s}{\sum_k \theta_k \left(1 - \frac{\alpha M_k}{\gamma_k}\right)}} = (AE_s)^{\frac{-\theta_s}{\sum_k \theta_k \left(1 - \frac{\alpha M_k}{\gamma_k}\right)}}.$$

Substituting (A.15) into (A.13), we see that allocative efficiency is a function of  $AE_k^{COV}$  and itself:

$$AE_s = \left( \sum_i \left( \frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left( \frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{\rho_s\gamma_s}{1-\rho_s\gamma_s}} \right)^{\frac{1-\rho_s\gamma_s}{\rho_s}} \times AE_s^{\frac{-\theta_s \alpha M_s (1-\gamma_s)/\gamma_s}{\sum_k \theta_k \left(1 - \frac{\alpha M_k}{\gamma_k}\right)}},$$

which means we can solve for  $AE_s$ :

$$(A.16) \quad AE_s = \left[ \left( \sum_i \left( \frac{A_{is}}{\tilde{A}_s} \right)^{\frac{\rho_s}{1-\rho_s\gamma_s}} \left( \frac{\tau_{is}}{\tilde{\tau}_s} \right)^{\frac{\rho_s\gamma_s}{1-\rho_s\gamma_s}} \right)^{\frac{1-\rho_s\gamma_s}{\rho_s}} \right]^{\frac{\sum_k \theta_k \left(1 - \frac{\alpha M_k}{\gamma_k}\right)}{\sum_k \theta_k \left(1 - \frac{\alpha M_k}{\gamma_k}\right) + \theta_s \alpha M_s \frac{1-\gamma_s}{\gamma_s}}}.$$

As in BKR, the contribution of sectoral  $TFP$  to aggregate  $TFP$  can be written as:

$$TFP \propto \prod_s AE_s^{\frac{\theta_s}{\sum_s \theta_s (1 - \frac{\alpha Ms}{\gamma_s})}} \prod_s (A_s^*)^{\frac{\theta_s}{\sum_s \theta_s (1 - \frac{\alpha Ms}{\gamma_s})}}$$

Thus, we can separate the “undistorted” effect of sectoral  $TFP$  (which incorporates the influence of returns to scale) on aggregate  $TFP$  from the allocative efficiency effect on  $TFP$ . To relate to the variance-covariance term, we would need to aggregate expand the exponent, yielding:

$$(A.17) \quad TFP \propto \prod_s (AE_s^{COV})^{\frac{\theta_s}{\sum_k \theta_k (1 - \frac{\alpha Mk}{\gamma_k}) + \theta_s \frac{\alpha Ms}{\gamma_s} (1 - \gamma_s)}}$$

### A.2.7 Impacts of $\rho_s$ and $\gamma_s$

To understand why  $\rho_s$  and  $\gamma_s$  enter asymmetrically into sectoral  $TFP$  given the following CES aggregator, note we can write plant level output as follows:

$$(A.18) \quad Q_{is} = (\rho_s \gamma_s P_s Q_s^{1-\rho_s})^{\frac{\gamma_s}{1-\rho_s \gamma_s}} \left( \frac{A_{is}}{\tau_{is}^{\gamma_s}} \right)^{\frac{1}{1-\rho_s \gamma_s}} \left[ \prod_j \left( \frac{\alpha_{js}}{w_{js}} \right)^{\alpha_{js}} \right]^{\frac{\gamma_s}{1-\rho_s \gamma_s}}$$

Note here that if the plant does not account for the impact of its decisions on aggregate output and prices (which by assumption it does not), then the elasticity of production with respect to a change in  $A_{is}$  is  $1/(1 - \rho_s \gamma_s)$ . Here we see that both returns to scale  $\gamma_s$  and downward-sloping demand  $\rho_s$  play a role in the impact of shocks on output. Demand parameter  $\rho_s$  impacts output through prices. As  $TFP$  increases, the plant can produce more output, but prices fall in response, dampening the effect of  $TFP$  shocks on output. In the CRS case, with a lower  $\rho_s$  (higher markups), the lower the elasticity. Now consider returns to scale: if  $\gamma_s < 1$ , i.e. DRS, then the elasticity of output with respect to changes in technical efficiency is smaller than the CRS case, and vice versa for increasing returns.

Now consider the aggregator for output in the sector again:

$$(A.19) \quad Q_s = \left( \sum_{i=1}^{N_s} Q_{is}^{\rho_s} \right)^{\frac{1}{\rho_s}}$$

Here note that the elasticity of total  $Q_s$  with respect to plant-level output is the following:

$$(A.20) \quad \epsilon_{Q_s, Q_{is}} = \left( \frac{Q_s}{Q_{is}} \right)^{1-\rho_s}$$

Note that this is independent of any returns to scale at the plant-level. We see  $\rho_s$  impacts output in two ways. First,  $\rho_s$  impacts output through the impact on firm-level decisions, as firms take into account the impact of their choice of output on prices. This effect can be amplified or mitigated by returns to scale  $\neq 1$ . Second, given an increase in output of a plant,  $\rho_s$  dampens the effect of a single plant on sectoral output.

### A.3 Implementing the AE decomposition

In order to assess the overall effect of the two revenue productivity measures on  $AE_s^{COV}$ , we implement equation (12) empirically. This exercise is useful because it helps gauge the relative importance of the mean and dispersion of within-industry distribution. Figures A1(a)-A1(b) show the contributions of the two terms of the right hand side of equation (12), which suggest that the second term contributes more to overall allocative efficiency (see figure A1(c)). The dynamics of  $\log TFPQ_{is}$  and  $\log TFP R_{is}^{cs}$  moments are useful interpreting the findings in the main text: the increasing dispersion and increasing positive correlation yield a negative contribution for the second term in equation (12), which accounts for the majority of the decrease in AE.

### A.4 Full Industry Sample

For the full industry analysis, we compute cost shares for materials, energy, capital, and labor using the full (450) industry sample. For the time invariant estimates, we compute the averages for the SIC and NAICS sample periods separately (i.e., 1972-96 for SIC and 1997-2010 for NAICS). Therefore, we do not need to use concordances between the two classification systems. In addition, unreported results suggest the change in the distribution of the cost shares between the SIC and NAICS periods is small. We find that the location and shape of the cost shares under the full sample are very similar to those of the 50-industry sample (results available upon request). We also implement the DW methodology for  $\rho_s$  for the full industry sample.

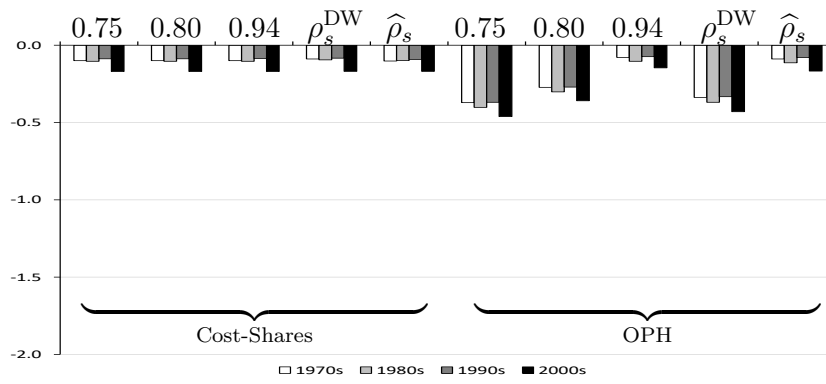
### A.5 Sensitivity Analysis of Aggregation and Time Varying Parameters for 50 industry sample

In this appendix, we consider industry-specific changes in  $\rho_{st}$  using the DW method, and combine those changes with CS (which imposes CRS throughout the time period but we allow cost shares to vary over time) and with OPH (where we permit the revenue curvature parameters to change over time).<sup>3</sup> For this analysis, we consider the original 50-industry sample with two 15-year subperiods: 1972-1986 and 1996-2010. We omit 1987-1995 to highlight potential changes in the parameter distributions. We recognize that combining OPH estimates of revenue elasticities with DW estimates of  $\rho_{st}$  that require CRS is inconsistent with the specification test of CRS under OPH reported above for most industries. However, as discussed in the main text, we think it is instructive to consider the sensitivity of estimates of AE across the range of parameter estimates.

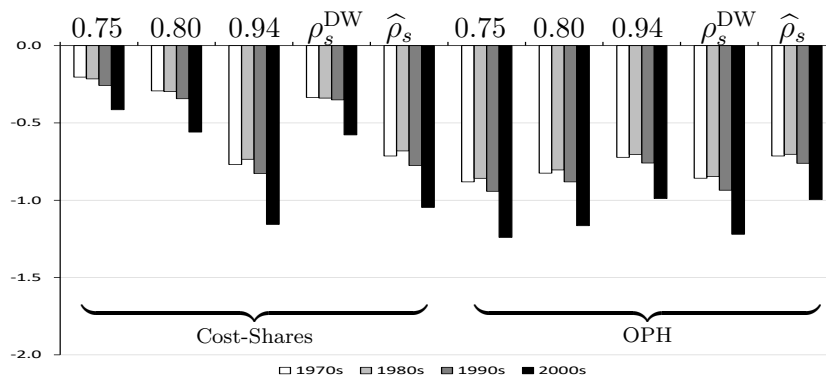
We begin by exploring evidence of changing markups and returns to scale over time for the 50-industry sample. Figure A2(a) shows both the unweighted and revenue-weighted average  $\rho_{st}$  decline from the first sub-period to the second sub-period with the weighted mean indicating a steeper decline. These patterns are broadly consistent with the results reported in the main text for the full

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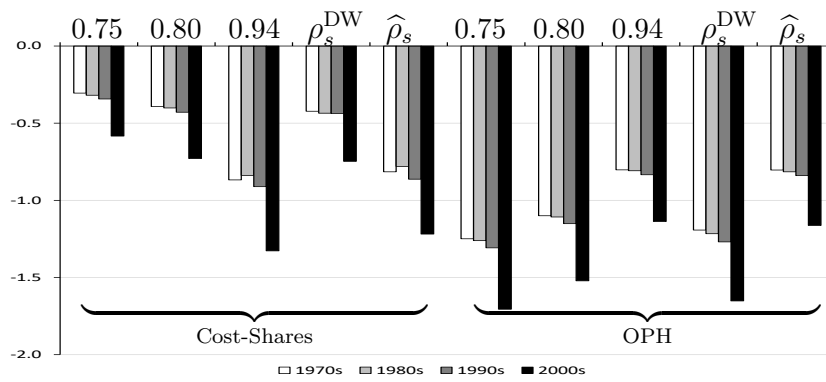
<sup>3</sup>As in section IV.C, if the estimated 4-digit curvature implied by OPH is not below one we use the 2-digit estimate in a sub-period. We do not pursue OPHD in this case since it exploits within industry variation over time and is not well suited to estimate  $\rho_{st}$  in shorter panels.



(a) Term 1



(b) Term 2



(c)  $\ln AE_s^{COV}$

Figure A.1: AE decomposition.

Note: In industries where  $1 < \rho_s \gamma_s$  at the 4-digit level, 2-digit estimates are used.  $\rho_s^{DW}$  denotes industry-specific time series averages calculated as in De Loecker and Warzynski (2012).

sample. Figure A2(b) shows the changes in the average overall revenue curvature ( $\rho_{st}\gamma_{st}$ ). We find evidence that  $\rho_{st}\gamma_{st}$  has declined modestly across the two sub-periods. The decline is only about 0.02 on an unweighted basis and is less than 0.01 using revenue weights. For the rise in markups to be consistent with only modest declines in revenue elasticities, returns to scale might be rising. To show this, we combine the  $\rho_{st}$  from A2(a) with  $\rho_{st}\gamma_{st}$  from A2(b) to show the implied rising  $\gamma_{st}$  in Figure A2(c). This is admittedly speculative but highlights both the potentially offsetting effects of rising markups and returns to scale along with the challenges of estimating time varying markups and returns to scale simultaneously.

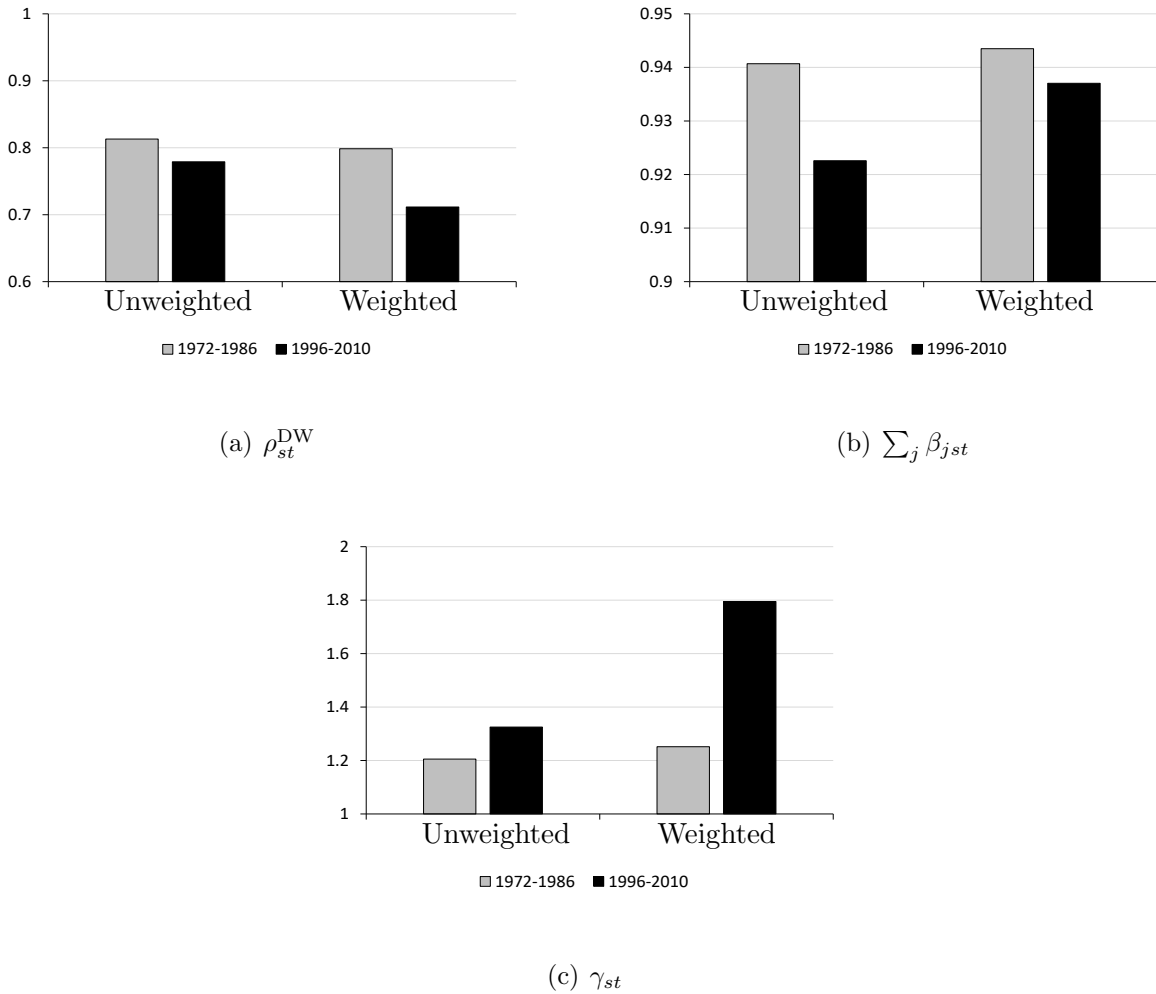


Figure A.2: Means of time-varying parameters for 1972-86 and 1996-2010

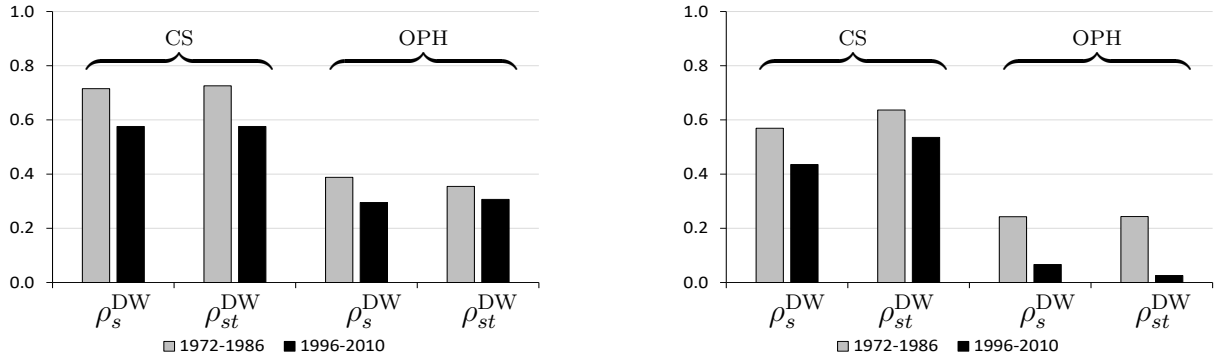
Note: “Unweighted” denotes averages where industries have equal weight. “Weighted” denotes averages where industries are weighted by revenue.

Figures A3(a), A3(b), and A3(c) show the implications for average sectoral AE, fixed-supply-based AE, and roundabout-production-based AE, respectively.<sup>4</sup> In order to illustrate the effect of time-

<sup>4</sup>See equations (14) and (16).

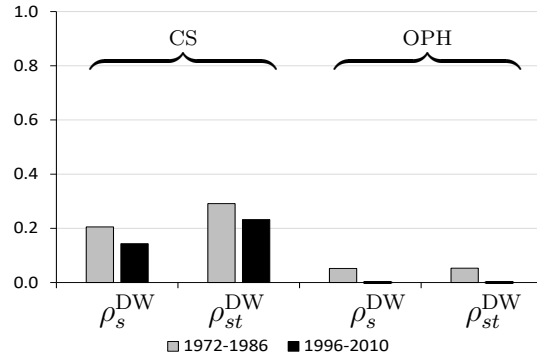


varying parameters, we present pairs of AE estimates under each estimator and assumption, where the first element of the pair repeats results under time invariant parameters as a point of reference. The second element of the pair shows AE under time-varying parameters. It is useful to start with CS-based sectoral average AE. Figure A3(a) shows that the modest decline in the unweighted average  $\rho_{st}$  from A2(a) has little impact on the decline in AE compared to using time-invariant parameters. In contrast, the two bounds in A3(b) and A3(c) for aggregate AE show that the more pronounced decline in weighted average  $\rho_{st}$  has a larger impact. Figure A3(b) shows the fixed-supply assumption yields aggregate AE that mimics the patterns of average AE under CS but yields lower AE levels and declines relative to the time-invariant case. Specifically, the declining average weighted  $\rho_{st}$  mitigates the decline in AE. Decreasing  $\rho_{st}$  in the CS case reduces the decline in AE from 23% to 16% in the fixed supply aggregation and from 30% to 20% in the roundabout production case. These patterns are consistent with the insight from the main text that a decrease in  $\rho_s$  tends to raise AE if returns to scale is held constant. In contrast, OPH implies a different pattern of changes in AE with time varying parameters. Here the decline in average  $\rho_{st}$  is (potentially) accompanied by an increase in  $\gamma_{st}$ , and therefore the mitigating effect on the decline in AE is smaller when using time varying parameters.



(a) Unweighted average,  $N_S^{-1} \sum_s \widehat{AE}_s^{COV}$

(b) Fix input supply:  $AE_s = \prod_s (\widehat{AE}_s^{COV})^{\theta_s}$



(c) Roundabout production:

$$\prod_s (\widehat{AE}_s^{COV})^{\frac{\theta_s}{\sum_{k=1}^S \theta_k (1 - \frac{\alpha M_k}{\gamma_k}) + \theta_s \frac{\alpha M_s}{\gamma_s} (1 - \gamma_s)}}$$

Figure A.3: AE: Aggregation and time-varying parameters

## A.6 Additional Tables

Table A.1: Sample size (number of plant-year observations in thousands) for the specifications shown in Table 3

	OLS	CS	OPH	OPHD
overall growth	424	424	405	405
exit	424	424	405	405
conditional growth	407	407	388	388

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