Online Appendix

Fewer but Better: Sudden Stops, Firm Entry, and Financial Selection

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A. Empirical Analysis

A1. ENIA: Data Cleaning

The Encuesta Nacional Industrial Anual (ENIA, Annual National Industrial Survey) conducted the by the INE covers all manufacturing plants in Chile with more than 10 workers. Our version extends from 1995 to 2007.

We eliminate observations with one or more of the following inconsistencies, with original variable names provided in parenthesis: negative electricity consumption (*elecons*), worked days less than or equal to 0 (*diatra*), gross value of the production less than value added (*vpn*<*va*), value added less than 0 (*va*), remuneration of workers equal to 0 (*rem-pag*), size equal to 0 (*tamano*), ISIC code less than 3000 (bad coding in *sector*), and sales income less than income from exports (*ingtot*<*ingexp*). Finally, we include in the analysis 22 of the 29 three-digit industries. We exclude commodity-related industries (353 and 354 for petroleum, 371 and 372 for metals). We also drop industries where revenue productivity cannot be reliable estimated (one or the sum of the input elasticities are outside the unit circle, typically due to the lack of observations); this is the case for 361 (pottery), 323 (leather),

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and 314 (tobacco). After all the cleaning procedures, the sample has 85% of the firms-year observations and 90% of the workers. The most important drop is copper related (371 and 372), implying a combined loss of 2.3% of observations and 5.6% of workers).

A2. Variable Construction and Other Controls

We calculate entry rates at year t at the industry level for each cohort, dividing the number of new plants in year t by the average of the total plants in years t and $t-1$. The variable used to build the productivity used in Table 2 in the main text is value added. We define capital as the end-of-period value of land, machinery, buildings and vehicles (*salter+salmaq+saledi+salveh*). We deflate monetary variables using three-digit industry level deflators provided by the INE. The revenue (*ingtot-revval-reviva*) used to calculate the Herfindahl-Hirschman concentration index (HHI) excludes nonmanufactured products (re-selling products and their tax shield); the costs include wages and exclude the costs and taxes associated with non-manufactured products (*costot-mrevval-mreviva+rempag*). The index of manufacturing production (22866EY.ZF...), the unemployment rate (22867R..ZF...), and the producer price and wholesale price index (PPI/WPI, 22863...ZF...) are taken from the IFS database. The labor cost index is from the Chilean central bank.

For each three-digit industry (denoted by s) we separately estimate the following production function:

$$\log y_{it} = d_t^s + \beta^{sl}\log l_{it} + \beta^{sk}\log k_{it} + \log z_{it} + \varepsilon_{it},$$

where y_{it} is real value added for firm i in year t, d_t^s is a time-fixed effect, l_{it} is total workers, and k_{it} is real capital stock. The coefficient β^{sl} denotes the industry-specific elasticity of value added with respect to labor and β^{sk} denotes the elasticity of value added with respect to capital. We estimate these elasticities using the methodology described in Wooldridge (2009). Using the estimated elasticities $\hat{\beta}^{sl}$ and $\hat{\beta}^{sk}$, we calculate firm productivity as:

$$\log z_{it} = \log y_{it} - \hat{\beta}^{sl}\log l_{it} - \hat{\beta}^{sk}\log k_{it}$$

Table I shows the estimated elasticities. The sum of the elasticities is always less than one.

Table I
Estimated Elasticities by Industry

Industry	$\hat{\beta}^l$	$\hat{\beta}^k$	$\hat{\beta}^l + \hat{\beta}^k$
311	0.49	0.12	0.60
312	0.81	0.05	0.85
313	0.27	0.11	0.38
321	0.71	0.07	0.78
322	0.68	0.09	0.78
324	0.70	0.17	0.87
331	0.54	0.17	0.71
332	0.66	0.11	0.77
341	0.46	0.12	0.57
342	0.55	0.13	0.68
351	0.44	0.18	0.62
352	0.59	0.05	0.64
355	0.67	0.04	0.71
356	0.44	0.13	0.57
362	0.68	0.07	0.74
369	0.61	0.14	0.75
381	0.70	0.11	0.81
382	0.70	0.04	0.74
383	0.58	0.11	0.69
384	0.63	0.05	0.68
385	0.35	0.06	0.42
390	0.62	0.12	0.73

Notes: For each three-digit industry (denoted by s) we separately estimate the production function $\log y_{it} = d_t^s + \beta^{sl}\log l_{it} + \beta^{sk}\log k_{it} + \log z_{it} + \varepsilon_{it}$, where y_{it} is real value added for firm i in year t, d_t^s is a time fixed effect, l_{it} is total workers and k_{it} is real capital stock. The coefficient β^{sl} denotes the industry-specific elasticity of value added with respect to labor and β^{sk} denotes the elasticity of value added with respect to capital. We estimate these elasticities using the methodology described in Wooldridge (2009), an extension of Levinsohn and Petrin (2003).

A3. Macro Data

In this subsection, we present the sources of the macroeconomic data used in this paper and the behavior of the aggregated time series during the crisis. To start, Chile is a small economy both in terms of population and aggregate output. It has also experienced spectacular growth, which led it to be the first OECD member in South America (2010). Its trade and debt ratio justify the small open economy framework adopted in this paper. In particular, while its trade-to-GDP ratio is quite high, according to the *World Trade Organization* database, in 2011 Chile had 0.45% of the world's exports and 0.41% of the world's imports. Chile is also the 7^{th} freest economy in the world (2013 *International Economic Freedom Ranking*).

The main source of data for the quantitative analysis in Section **??** is the Central Bank of Chile, from whose database we obtained real GDP, real gross fixed capital information, and real consumption series. In order to be able to cover pre-crisis years, we used the versions in millions of 2003 Chilean Pesos, spanning between 1996:Q1 and 2011:Q2. To conduct the empirical analysis in **??**, we used additional data from the International Finance Statistics (IFS) database from the International Monetary Fund (IMF). From that source, we obtained exchange rate (228..RF.ZF...), financial accounts (22878BJ DZF...), direct investment abroad (22878BDDZF...), direct investment in Chile (22878BEDZF...), net errors and omissions (22878CADZF...), exports (22890C..ZF...), and imports (22898C..ZF...). We use employment data from the Instituto Nacional de Estadística (INE, National institute of Statistics) of Chile and hours worked per week from the *Encuesta de Ocupación y Desocupación* from the Economics Department of *Universidad de Chile*. The interest rate is the average observed real interest rate for commercial loans with a maturity of 3 to 12 months; this data is provided by the Chilean Central Bank online.

A4. Working Capital in the Data

In order to discipline the working capital parameter in the model, we use firm-level information on interest payments (*intgas*) and total cost of production (*totcost*) from ENIA. We link these variables to their model counterparts using the following relationship:

$$\eta\left(R(s^{t-1})-1\right)(\text{production cost}) = \text{interest spending} \Rightarrow$$
$$\eta = \frac{\text{interest spending}}{(\text{production cost})\left(R(s^{t-1})-1\right)},$$

where R is the Chilean real interest rate. We derive this ratio at the firm level. The value of η is roughly 50% before the crisis period when calculated as the simple average across firms. When firm-specific values for η are weighted by the employment size of firms, the average value increases to 70% for the same period. Taking an average value of these two estimates, we use $\eta = 60\%$ in our baseline calibration. Online Appendix B2.1 presents a robustness analysis for different values of η.

A5. Hausman and Taylor (1981)

The method can be summarized as a four-step procedure. First, a fixed-effects regression delivers consistent estimators, $\hat{\beta}_1$ and $\hat{\beta}_2$, that are used to retrieve estimators $\hat{u}_{i,t}$ and $\hat{\sigma}_u$. The second step is an instrumental variables (IV) regression with $\hat{u}_{i,t}$ as dependent variable, Z^1 and Z^2 as independent variables, and Z^1 and X^1 as instruments; this delivers a consistent estimator for $\tilde{\sigma}$ (the dispersion of the residual). Third, an estimator for the variance of the unobserved fixed-effect component can be built as $\hat{\sigma}^2_\mu = \tilde{\sigma}^2 - \frac{\hat{\sigma}^2_u}{T}$, in order to form the usual generalized least squares (GLS) correction. Finally, the GLS correction is used to transform the original equation and estimate all the coefficients simultaneously in equation (**??**), using an IV procedure where the instruments are given by Z^1, the mean of X^1, and the deviations from the mean of X^1 and X^2. After every estimation, we perform the Sargan-Hansen test to assess the validity of the instrumental variables procedure.

Table II presents the details of the regression results from the main text. In our regressions we use as time-variant exogenous variables ($X^1_{i,t}$) four macroeconomic aggregates: an index of manufacturing production, the unemployment rate, an index of wholesale producer prices, and an index of the cost of labor.[1] The coefficients associated with these variables are stable across the productivity regressions. The signs of the significant coefficients suggest that productivity is higher when production is high, and inflation in producer prices are low. Higher labor cost are associated with higher productivity (this could be due to a selection effect, given that more productive firms can afford higher labor costs). There are four potentially endogenous post-entry controls ($X^2_{i,t}$): electricity consumption, number of workers, capital stock, and the age of the plant. We use five geographic regions and two-digit industry controls as time-invariant exogenous variables (Z^1_i). Besides the coefficients of interest, we include the initial capital stock of the plant. To control for competition at the moment of entry, we also include the Herfindahl-Hirschman concentration index of the industry at the particular region in the year of entry among the time-invariant endogenous variables (Z^2_i). In line with the firm dynamics literature, larger entrants are more profitable and more productive than smaller entrants.

[1] Because this method relies on $X^1_{i,t}$ to build instruments, and because they are all aggregate variables, we cannot include year dummies, which are perfectly correlated with our instruments.

Table II
Hausman and Taylor

	(1) $A_{i,t}$	(2) $A_{i,t}$	(3) $A_{i,t}$	(4) $A_{i,t}$
During Crisis	0.637***	0.618**		0.586**
	(0.227)	(0.275)		(0.245)
After Crisis		0.0893		0.0603
		(0.143)		(0.144)
Avg entry$_{j,0}$			-5.923***	
			(1.552)	
log Manu Prod$_t$	1.251***	1.250***	1.224***	1.090***
	(0.126)	(0.126)	(0.126)	(0.130)
Unemp Rate$_t$	-0.196	-0.198	-0.207	-0.190
	(0.556)	(0.557)	(0.555)	(0.488)
log PPI/WPI$_t$	-2.360***	-2.358***	-2.353***	-2.356***
	(0.165)	(0.165)	(0.165)	(0.164)
log L Cost$_t$	3.127***	3.099***	2.923***	3.258***
	(0.646)	(0.648)	(0.639)	(0.671)
log age$_{i,t}$	0.00636	0.00798	0.0224	0.0139
	(0.0219)	(0.0217)	(0.0210)	(0.0208)
HHI$_{r,j,0}$	27.21**	19.86	9.870	26.76
	(13.30)	(21.26)	(11.22)	(20.86)
log K$_{i,t_0}$	0.796***	0.726***	0.561***	0.726***
	(0.136)	(0.190)	(0.0657)	(0.206)
log Elec Con$_{i,t}$				0.0462***
				(0.00790)
log L$_{i,t}$				0.0638**
				(0.0258)
log K$_{i,t}$				-0.0576***
				(0.00943)
Ind. Control	Yes	Yes	Yes	Yes
Region Control	Yes	Yes	Yes	Yes
Constant	Yes	Yes	Yes	Yes
Observations	17646	17646	17646	17484
Sargan-Hansen (p)	0.495	0.242	0.0137	0.205

Standard errors in parentheses (bootstrapped (250), clustered by firm)
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

A6. Cox Estimation

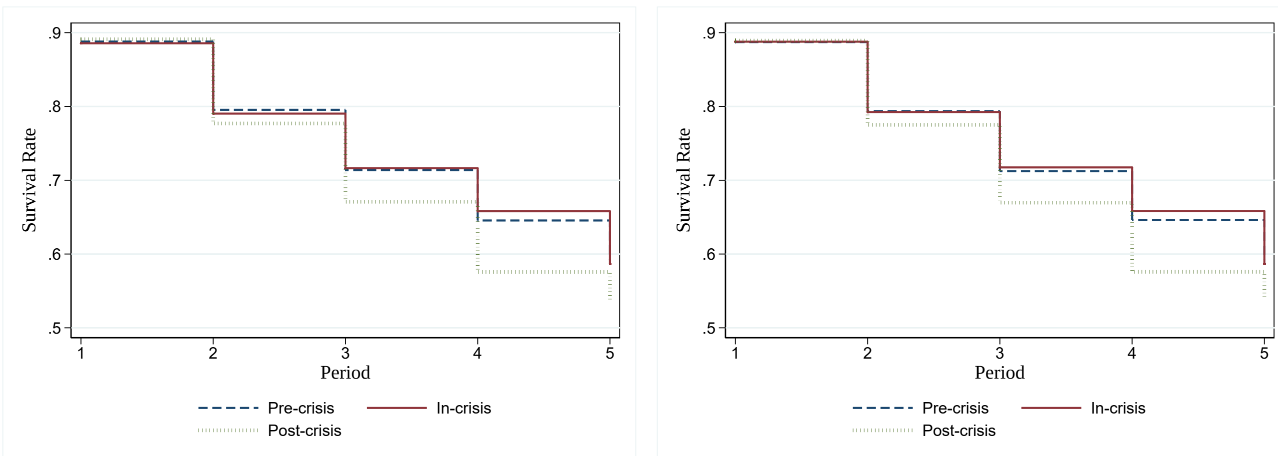
This section shows that the higher profitability of the cohorts born during the sudden stop is not due to *ex-post* selection. In particular, we perform the following stratified proportional hazard estimation to show that firms born during the crisis are not more likely to die at any horizon:

$$h_{r,c}\left(t|\boldsymbol{X}_i\right) \quad = \quad h_{0,r,c}\left(t\right)\exp\left[\boldsymbol{X}_i \cdot \boldsymbol{\beta}\right].$$

The two strata are geographical region (r) and time period (c). This means that the baseline hazard $h_{r,c}$ varies across these two dimensions. We divide Chile into five geographical regions. The time periods correspond to the *pre-crisis*, *crisis*, and *post-crisis* period of the second specification in the Hausman and Taylor estimation of Section III in the main text. The Cox-Snell test cannot reject the proportional hazard structure with 95% confidence. Sub-index t refers to time, while i refers to a plant and j to an industry. Table III shows the estimates of the common covariates.

Bigger plants have less probability of exiting (for both electricity consumption and number of workers), while the initial size increases the probability of exiting (for number of workers and electricity consumption). The specification controls for the industry cycle (using the average profitability of the industry $\bar{P}_{j,t}$ or the average productivity $\bar{A}_{j,t}$) and industry-specific effects. Figure I plots the survival rates at different horizons for cohorts born during the three different time periods in the central zone of Chile. We pick this zone because it concentrates most of the plants in the sample; the main message does not change when considering the other four regions.

Importantly, firms born during the crisis do not exit more than other cohorts. Moreover, they even seem stronger in this dimension in that, until year 6, they have a higher predicted survival probability than firms born either before or after the episode. Hence, *ex-post* selection does not explain the higher profitability of cohorts born during the sudden stop.



(A) Profitability

(B) Productivity

FIGURE I
Survival Rates, Cox Proportional Hazard Model

Notes: The figures plot average survival rates at different horizons for cohorts born during three different time periods in the central zone of Chile. The survival rates are estimated by a Cox proportional hazard model. The left panel is based on a model that uses profitability as an explanatory variable, whereas the estimated model for the right panel includes productivity.

Table III
Proportional Hazard

	(1)	(2)
$\ln(L_{i,t})$	-0.631***	-0.628***
	(0.0713)	(0.0713)
$\ln(L_{i,0})$	0.548***	0.548***
	(0.0721)	(0.0721)
$\ln(elec_{i,t})$	-0.0742***	-0.0761***
	(0.0266)	(0.0267)
$\ln(elec_{i,0})$	0.0546**	0.0549**
	(0.0253)	(0.0254)
$\ln(K_{i,t})$	-0.0338	-0.0328
	(0.0249)	(0.0249)
$\ln(K_{i,0})$	-0.0295	-0.0304
	(0.0238)	(0.0238)
$\mathrm{P}_{j,t}$	0.342	
	(0.261)	
$\mathrm{A}_{j,t}$		0.185**
		(0.0861)
$\mathrm{HHI}_{j,t}$	0.110	0.131
	(0.368)	(0.367)
Ind. Control	Yes	Yes
Observations	15149	15149
Plants	2981	2981
Exits	1758	1758
Hazard assumption test (p)	0.366	0.371

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

B. Quantitative Analysis

B1. Business Cycle Analysis

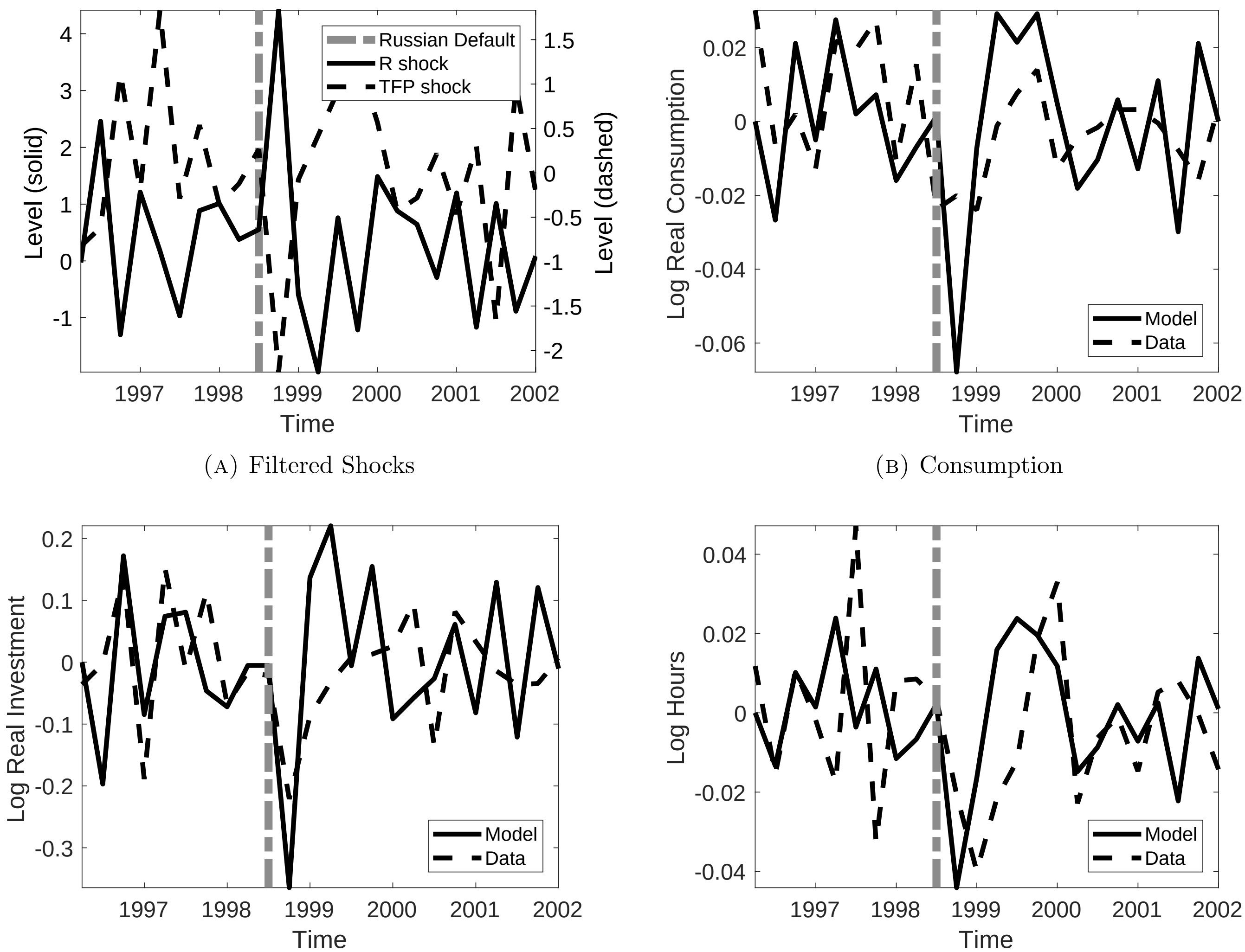
The Chilean crisis is characterized by an increase of 80 basis points in the quarterly interest rate between the beginning of the Asian crisis and the Russian default as well as by 4.5% drop in quarterly output. Using these series, we smooth out the interest and productivity innovations, and Figure IIa shows these filtered series, which we then use to mimic the crisis in the model. The sudden stop is explained by a negative exogenous productivity shock and a simultaneous positive interest rate shock.

Feeding the filtered innovations into the model allows us to use the Chilean crisis to evaluate the business cycle behavior of the model. Figure II compares the model-implied path for the log differences of consumption, investment, and hours with data counterparts. The graphs establish that the model tracks well the behavior of the macro aggregates during the period.

Table IV shows the variance decomposition of the macro aggregates. The calibrated model is consistent with the evidence in Neumeyer and Perri (2005) and Uribe and Yue (2006), where interest rate fluctuations explain one-third of the fluctuations in Argentinian output. Because Chilean spreads are less volatile than Argentinian spreads, it is natural that interest rate fluctuations play a lower role with respect to Chilean output.

Figure III shows the impulse response functions to a one standard deviation shock for the main macro variables.

As illustrated in figures IIIa and IIIb, the responses of output, labor, consumption and investment (right axis) are aligned with the literature. Consumption responds more on impact to interest rate shocks than output, but output responds more than consumption to stationary productivity shocks. As such, with a more volatile interest rate, the model would generate less smoothing in consumption. Importantly, none of the variables will return to its original long-run trend. In fact, for the case of the interest rate shock, the new path for these variables is permanently 0.1% lower. This hysteresis arises because of the permanent loss in the level of productivity, as shown in Figure IIIa.



(A) Filtered Shocks

(B) Consumption

(C) Investment

(D) Hours

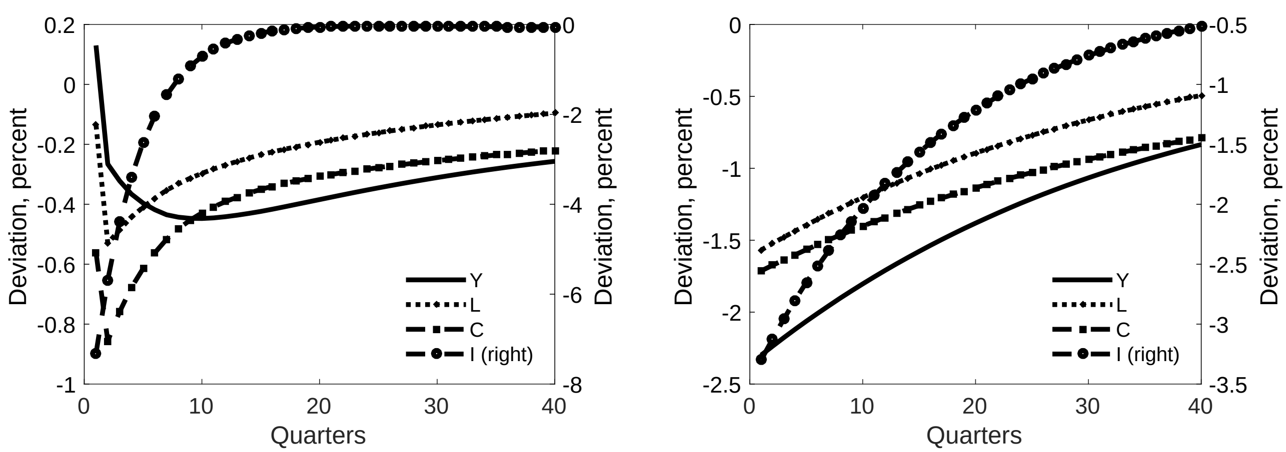
Figure II
Filtered Shocks and Non-targeted Variables during Crisis

Notes: The figure shows the smoothed shocks fed into the model and compares the evolution of model-generated aggregate variables in response to these shocks to the data. The top-left panel reports the level of smoothed shock series during Chilean sudden stop. The aggregate exogenous shocks across the business cycle are smoothed out using demeaned log differences of aggregate output and interest rate series over 1996:Q1–2011:Q2. Panels b, c, and d report consumption, investment, and hours worked, respectively. The variables are presented in deviations of demeaned log series.

Table IV
Variance Decomposition

	TFP	R
c	0.944	0.056
y	0.969	0.031
L	0.945	0.055
inv	0.453	0.547
a	0.208	0.792
entry	0.262	0.738
ih	0.192	0.808
il	0.281	0.719
vh	0.869	0.131
vl	0.884	0.116

Notes: Consumption, output, investment, and firm values are normalized by endogenous productivity.



(A) Macro Aggregates' Response to R

(B) Macro Aggregates' Response to Z

Figure III
IRFs to R (left panel) and TFP (right panel) shocks

Notes: The left panel shows the impulse responses of output, consumption, hours worked, and investment (secondary axis) to a one standard deviation interest rate shock, whereas the right panel shows the impulse responses of the same aggregate variables to a one standard deviation TFP shock.

Finally, Figure IV shows that the exit rate in the model is also consistent with the data. The exit rate is flatter and less volatile than the entry rate.[2]

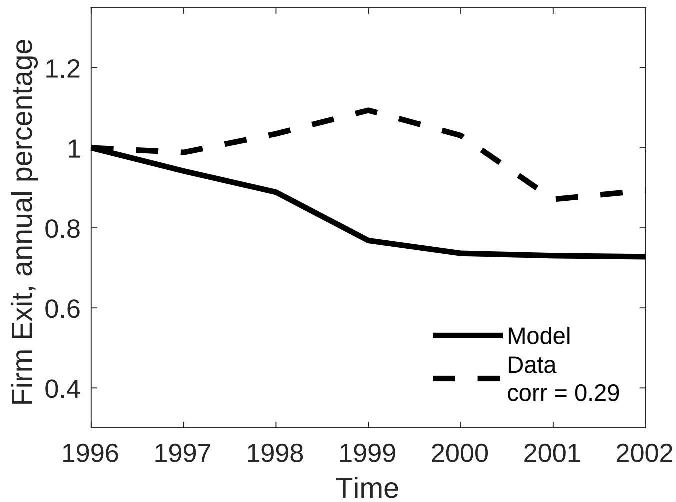


FIGURE IV
Non-Targeted Exit Dynamics

B2. Sensitivity Analysis

In this section we discuss the sensitivity of key model-generated moments to the parameters of the model. We focus on three moments (entry rate, share of high-type entrants, permanent productivity loss) and eight parameters (step sizes, $\{\sigma^L, \sigma^H\}$; scarcity, ν; entry cost, κ; parameters of expansion cost function, $\{\varphi, \xi\}$; and working capital constraint parameters, $\{\eta_i, \eta_i\}$). In this exercise, we follow a procedure described in Daruich (2018). Specifically, we draw 100,000 quasi-random Sobol points from an eight-dimensional hypercube, which is defined by a $\pm 1\%$ interval around the calibrated value of each parameter.[3] Then, at each point, we compute the model moments over the business cycle.[4] When doing so, we introduce to the model the shocks that we filtered in the baseline model using the Chilean series of manufacturing output and real interest rate (shown in Figure IIa).[5]

[2] Incumbent dynamics and aggregate risk are critical for the model to deliver this asymmetric behavior. Without incumbent expansion, the entry and exit rates are necessarily the same, even outside the balanced growth path.

[3] Therefore, if the calibrated value of a certain parameter is x, the interval we consider is $(0.99x, 1.01x)$.

[4] This procedure yields a different BGP at each point.

[5] In this way, we standardize the shock process used and thus avoid additional variation that would stem from filtering shocks again at each point.

We compute the peak deviation in the entry rate and its composition from their respective (recomputed) BGP values. For the permanent loss, we compute the log-deviation in endogenous productivity five years after the sudden stop (similar to the exercise in Section **??**). Finally, we divide the vector for each parameter into 20 quintiles and compute the 25^{th}, 50^{th}, and 75^{th} percentiles of each moment in each quintile. Figures V-VII plot the variation in each moment along quintiles of different parameters, with blue circles denoting 25^{th} and 75^{th} percentiles of the moment in each quintile, while red circles denote the median value. In a sense, the slope of the median highlights the sensitivity of the moment to the specific parameter, while the difference between highest and lowest quartiles of the moment value in each parameter quintile indicates the relative importance of the other parameters.

A few observations stand out. Step-size parameters have a sizeable effect on all moments around their calibrated value (the calibrated value corresponds to the higher bound of 10^{th} quintile). Second, entrant compositions and permanent productivity loss are strongly sensitive to entry and expansion cost parameters. They are also sensitive to step-size parameters. Third, the scarcity parameter has some effect on the entry rate, but its effect on other moments is relatively muted. Finally, working capital constraint has a negligible effect on the dynamics of the model.

We conclude this section by providing further evidence on the limited role the working capital constraint plays in our mechanism in subsection B2.1.

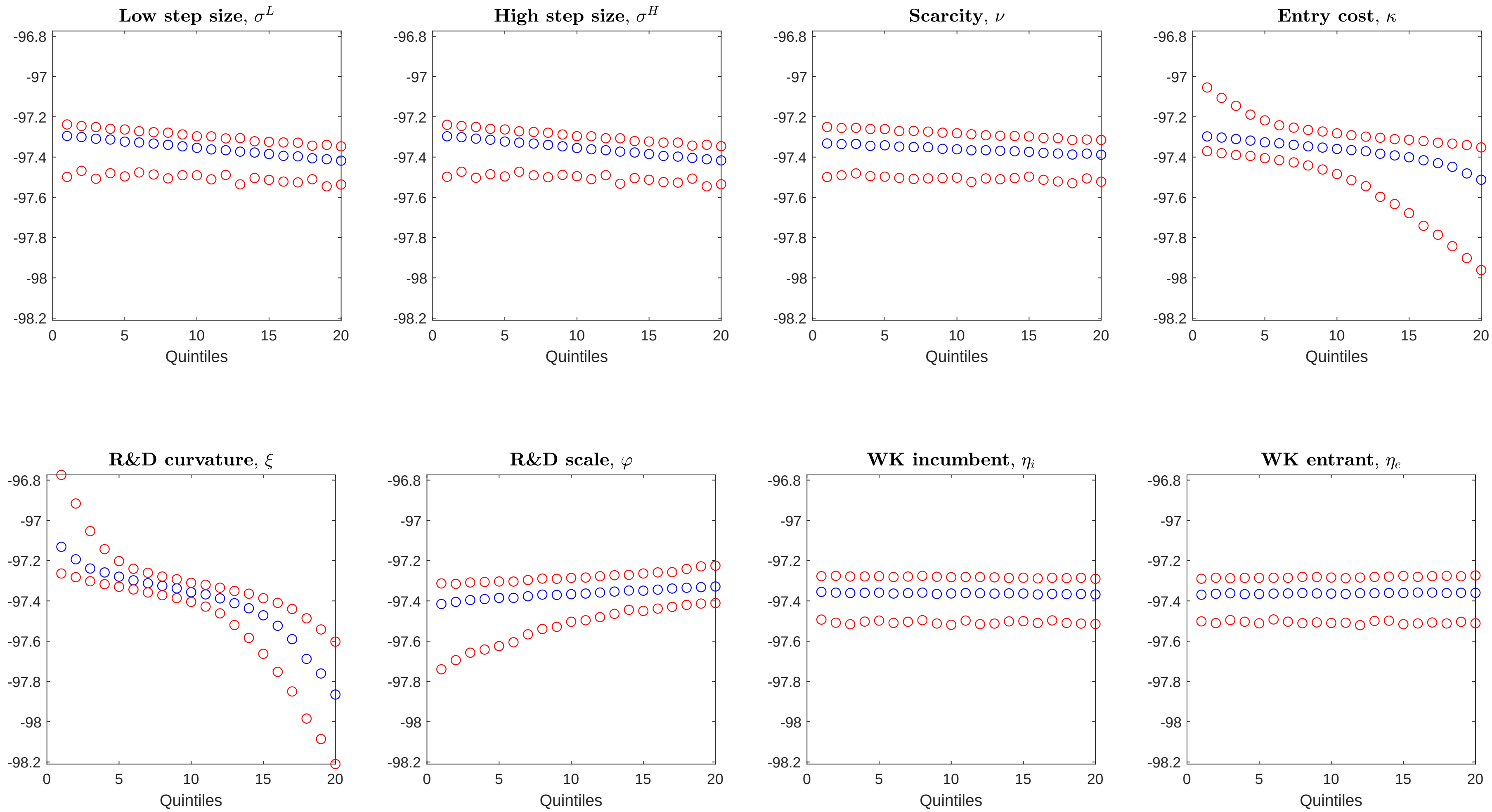


FIGURE V
Sensitivity of entry

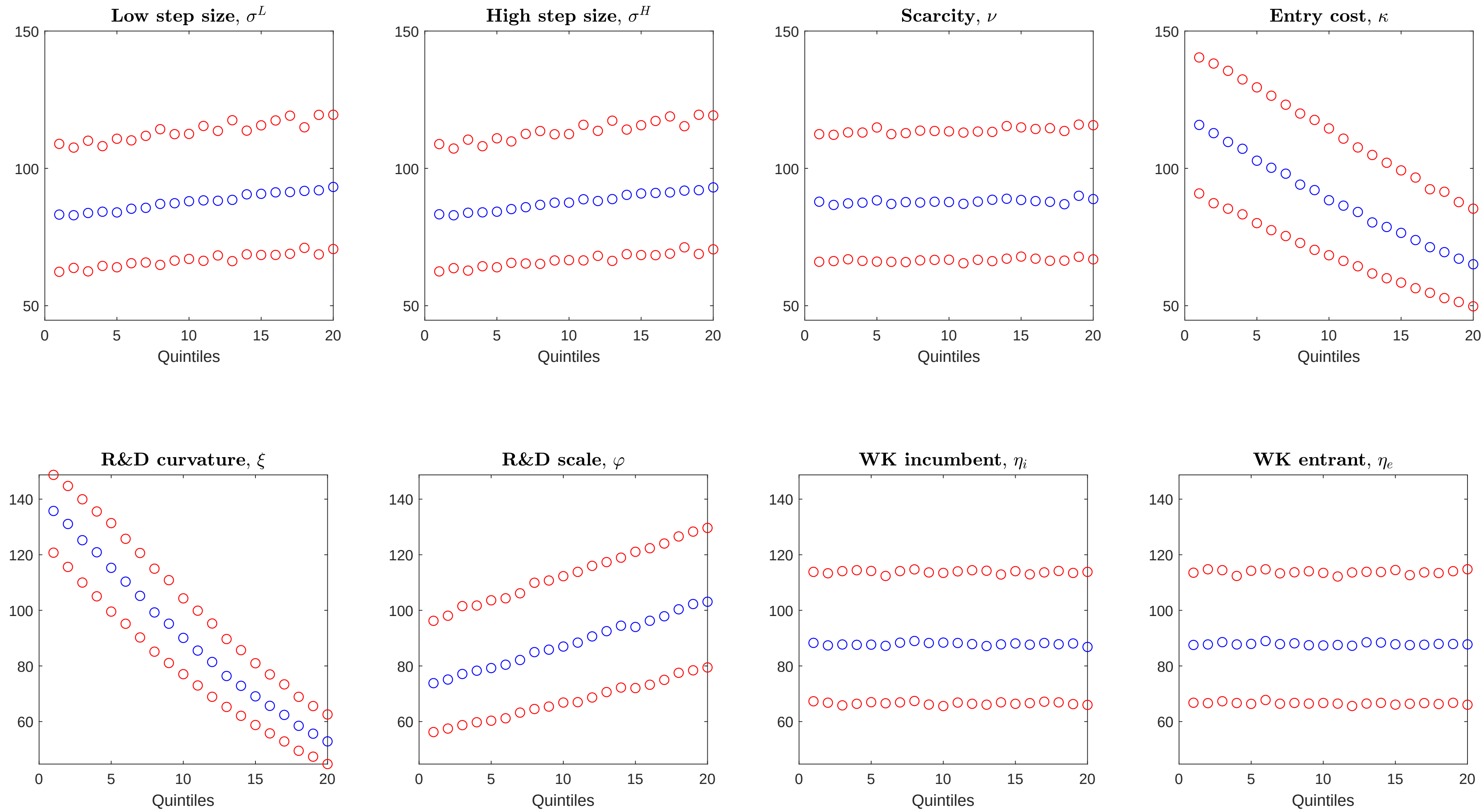


FIGURE VI
Sensitivity of entrant composition

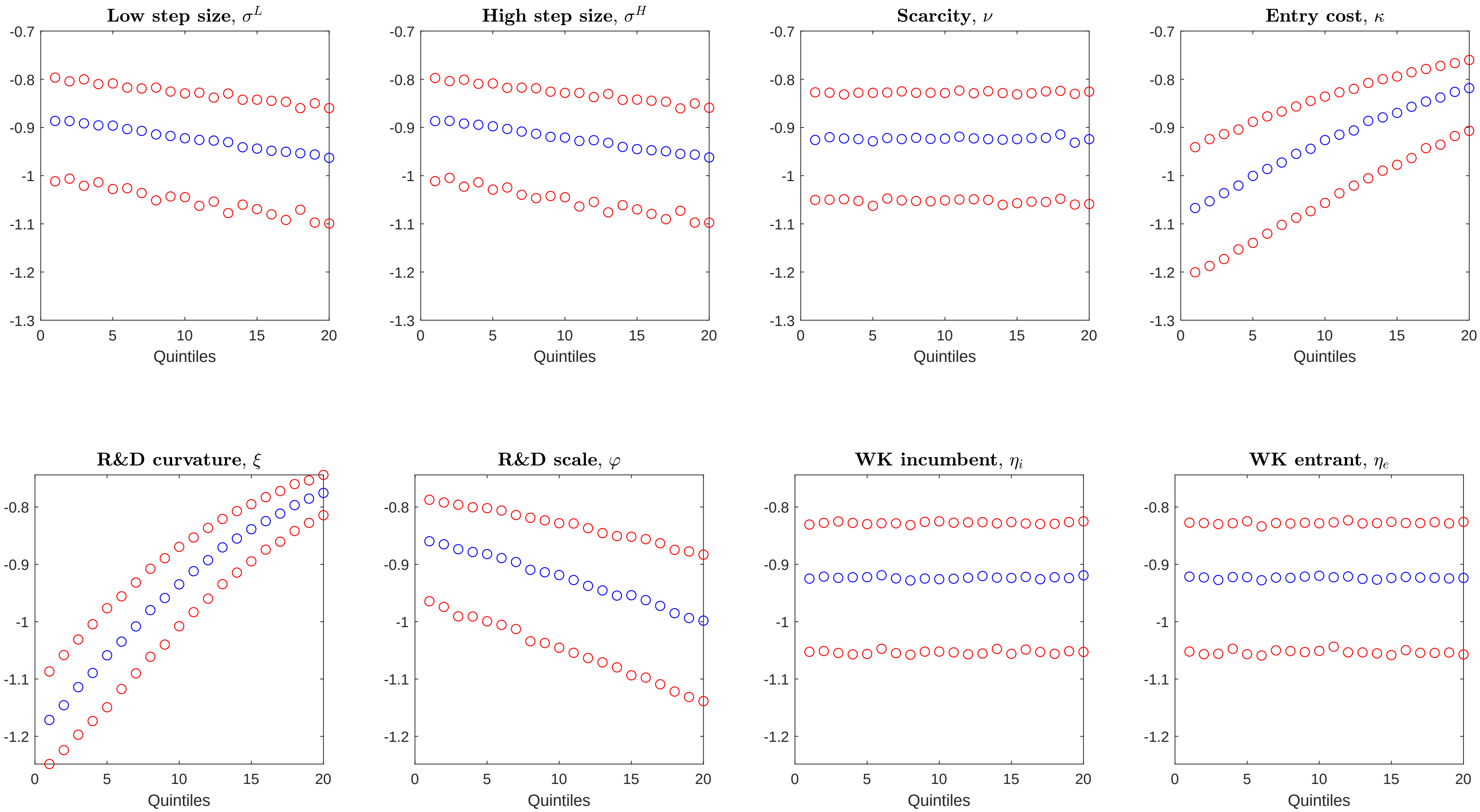
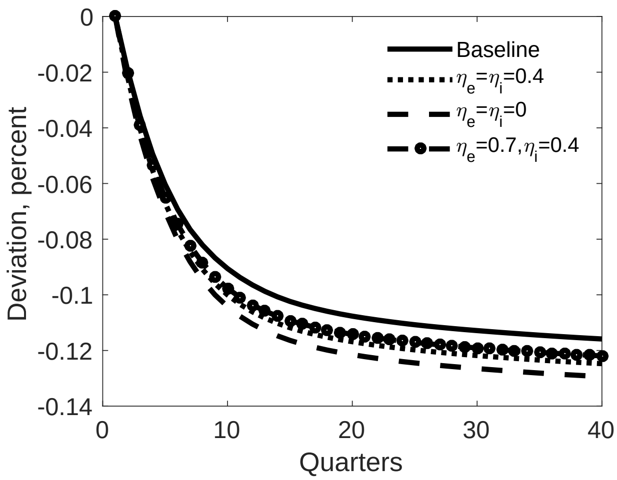


FIGURE VII
Sensitivity of permanent productivity loss

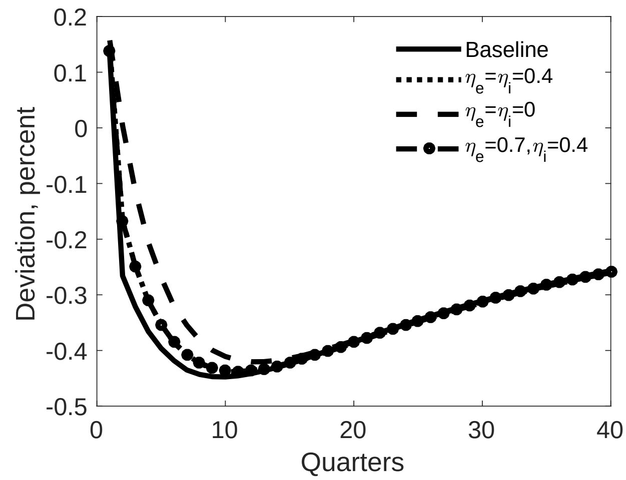
B2.1. A Further Look at Working Capital Constraint

To explore the role of the working capital constraint, Figure VIII compares the baseline calibration of $\eta = 0.6$ to several alternatives. In particular, the dotted line represents an economy with a slightly lower level of working capital needs ($\eta = 0.4$), the dashed line is an economy with no working capital constraint ($\eta = 0$), and the dashed and dotted line represents an economy where the financial intermediary (entrants) face a tighter working capital constraint than the one faced by incumbents ($\eta_e = 0.7$ and $\eta_i = 0.4$). In line with Arellano, Bai and Zhang (2012), the latter economy captures the fact that entrants are likely to be more financially constrained than incumbent firms.

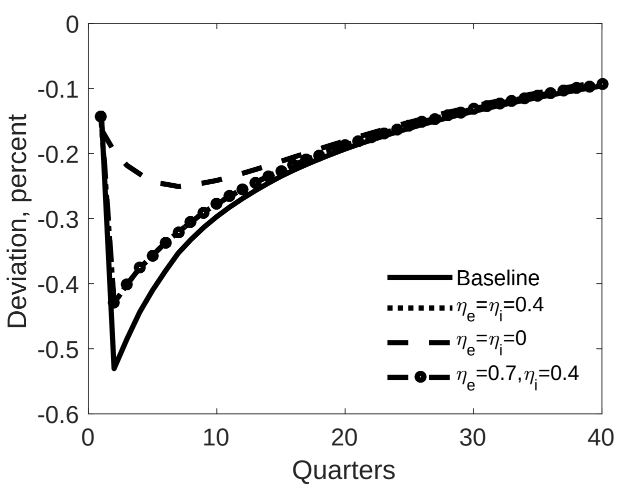
Figure VIIIa shows that the endogenous productivity component reacts very similarly in every economy to interest rate shocks. Because interest rates are the main driver of endogenous productivity, this similarity implies that our quantification of the permanent productivity loss of the Chilean sudden stop does not depends on the value of η. In fact, Figure VIIId shows that every economy predicts the same long-run productivity loss. In contrast, because the working capital channel makes stationary interest rate shocks behave like productivity shocks, we do see a difference in the short-run behavior of output in Figure VIIIb and employment in Figure VIIIc. In line with Neumeyer and Perri (2005), the larger the working capital channel is, the stronger the real short-run effects of interest rate shocks are. Interestingly, the economy where entrants are more constrained than incumbents behaves very similarly to the economy where entrants and incumbents are equally constrained. This similarity is driven by the fact that, because entrants have only one product, they therefore account for a very small portion of the economy-wide labor. Finally, this outcome illustrates that the permanent productivity loss of sudden stop is not driven by the working capital constraint but by the effect that the interest rate has on innovation. This effect is driven by the pass-through of interest rate shocks to the value of varieties triggered by fluctuations in the stochastic discount factor.



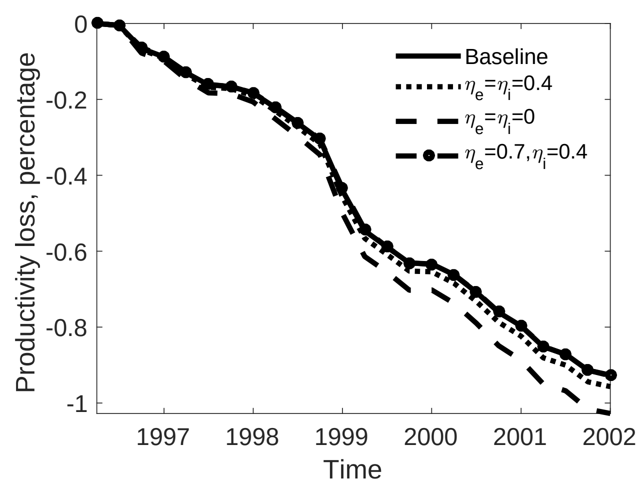
(A) Productivity Response to R



(B) Output Response to R



(C) Labor Response to R



(D) Prodcutivity loss

Figure VIII
Impulse response functions to R and long-run productivity cost of the crisis

Notes: The top-left panel shows the impulse response of endogenous productivity growth to a one standard deviation interest rate shock in models with different levels of working capital parameters. The top-right and the bottom-left panels do the same for output and hours worked, respectively. The bottom-right panel shows the percentage loss in the endogenous productivity component over the business cycle with respect to its path along the balance growth path, again across models with different levels of working capital parameters.

B3. Alternative Models

B3.1. Model without Heterogeneity (NH)

The model with no heterogeneity eliminates firm types, keeping the expansion decision of firms. This transformation is equivalent to setting $\sigma = \sigma^L = \sigma^H$ in the original model. The following two changes convert the baseline set of equations to the set of equations needed to characterize NH:

1. Any generic variable $\boldsymbol{x}^d$ has a single value; and
2. composition variables in the economy are set to unity, i.e., $\mu = \tilde{\mu} = 1$.

The problem of the financial intermediary is linear and simplifies to a zero expected profit condition:

$$\mathbb{E}\left[m(s^{t+1})\left(1 + a(s^t)\right)\bar{v}^L(s^{t+1})|s^t\right] = \left(1 + \eta\left(R\left(s^{t-1}\right) - 1\right)\right)w(s^t)\kappa. \tag{1}$$

B3.1.1 Calibration

We want to assess the permanent productivity loss estimated by a model with no heterogeneity. Therefore, we recalibrate the model to match a subset of the original moments. NH has only one step size and no scarcity parameter (ν); therefore, we drop the mean and the standard deviation of the size distribution from the targets and recalibrate the model. The measure of firms is fixed to the calibrated value in the baseline model. Table V shows the results of this exercise:

TABLE V
INTERNALLY CALIBRATED PARAMETERS

Parameter	Symbol	Value	Main identification	Target	Model
Labor disutility level	Θ	27.47%	Working time	33.00%	33.00%
Entry Cost	κ	4.62%	Entry rate	10.80%	10.78%
Step Size	σ	7.06%	Annual GDP Growth	3.00%	3.00%
Mass of Varieties	λ	7.62	Mass of Firms	1.00	1.00
Expansion Cost scale	φ	22.12%	Mean of firm employment distribution	7.62	7.61
Stdev TFP	σ_z	1.22%	Quarterly output volatility (HP filtered)	3.00%	3.00%
Capital adjustment cost	ϕ	9.33	Quarterly investment volatility (HP filtered)	9.56%	9.56%

B3.2. Model without Heterogeneity and Firm Dynamics (NDNH)

The NDNH economy goes one step further and eliminates the expansion decision of firms. In this sense, every firm has only one product, and firms remain in operation until they are replaced by an entrant. Therefore, NDNH is equivalent to NH without ι decision.

B3.2.1 Normalized System of Equations

The following set of equations represents all the equations of NDNH that differ from the baseline economy.

Final Good Producer

$$y(s^t) = \exp\left(z(s^t)\right) \cdot \left(l_p(s^t)\right)^{\alpha} \left(k(s^{t-1})\right)^{1-\alpha} \tag{2}$$

$$k(s^{t-1}) = \frac{(1-\alpha)y(s^t)}{r(s^t)} \tag{3}$$

Intermediate Good Producers

$$l_p(s^t) = \frac{\frac{\alpha}{\Lambda}y(s^t)}{w(s^t)(1+\sigma)\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)} \tag{4}$$

$$\pi_j(s^t) = \frac{\alpha}{\Lambda}\frac{\sigma}{(1+\sigma)}y(s^t) \tag{5}$$

$$\bar{v}(s^t) = \pi(s^t) + \mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\left(1-\Delta(s^t)\right)\bar{v}(s^{t+1})|s^t\right] \tag{6}$$

Financial Intermediary

$$\left(1+\eta\left(R\left(s^{t-1}\right)-1\right)\right)w(s^t)\kappa = \mathbb{E}\left[m(s^{t+1})\left(1+a(s^t)\right)\bar{v}(s^{t+1})|s^t\right] \tag{7}$$

$$\tilde{\mu}(s^t) = 1 \tag{8}$$

Aggregate Variables

$$a(s^t) = (1+\sigma)^{\frac{M(s^t)}{\Lambda}} - 1 \tag{9}$$

$$\mu(s^t) = 1 \tag{10}$$

$$\Delta(s^t) = \frac{M(s^t)}{\Lambda} \tag{11}$$

$$t(s^t) = \pi(s^t) - \left(1 + \eta\left(R\left(s^{t-1}\right) - 1\right)\right) M(s^t)\kappa w(s^t) \tag{12}$$

$$nx(s^t) = y(s^t) - c(s^t) - i(s^t) - \frac{\psi}{2} y(s^t) \left(\frac{b(s^t)}{y(s^t)}(1 + a(s^t)) - \bar{b}(1+\bar{g})\right)^2 \tag{13}$$

$$d(s^t) = b(s^{t-1}) - \eta w(s^t) l(s^t) \tag{14}$$

$$l(s^t) = l_p(s^t) + \kappa M(s^t) \tag{15}$$

B3.2.2 Calibration

Compared with NH we drop φ and the share of labor of the 10% largest firms. Table VI presents the result.

Table VI
Internally Calibrated Parameters

Parameter	Symbol	Value	Main identification	Target	Model
Labor disutility level	Θ	24.21%	Working time	33.0%	33.0%
Entry Cost	κ	38.48%	Entry rate	10.8%	10.8%
Step Size	σ	32.92%	Annual GDP Growth	3.00%	3.00%
Stdev TFP	σ_z	0.99%	Quarterly output volatility (HP filtered)	3.00%	3.00%
Capital adjustment cost	ϕ	8.89	Quarterly investment volatility (HP filtered)	9.56%	9.56%

Compared with NH, the unique step size is five times larger. This result is due to the fact that the same entry rate needs to trigger the same growth rate but without incumbent dynamics. We can think of the step size in NDNH as a summary of all the innovations that an average entrant on NH would perform during its life cycle.

B3.3. Model with Exogenous Growth

The economy with exogenous growth is characterized by the same set of equations as the baseline. However, expansion rates (ι^d) and entry mass (M) are taken as parameters, and they are set to the balanced growth path. Thus, the equations that correspond to those variables are dropped from the system. Therefore, by construction, the parameters that determine the BGP of Exo are the same as the baseline calibration. The two remaining parameters—the capital adjustment cost ϕ and the standard deviation of the TFP shocks σ_z—are again calibrated using the business cycle properties the model and take the values $\phi = 9.40$ and $\sigma_z = 1.20\%$. Of note, this model is practically analogous to the economy of Neumeyer and Perri (2005).

B4. Robustness

In this section, we discuss the robustness of our main results under different model specifications. First, we analyze a version in which R&D is conducted using capital instead of labor. Second, we look at a version in which the entry cost is not linear but convex. Third, we consider a version in which we assume that we observe only a subset of firms in the model economy, reflecting the truncation in the data. Table VII summarizes the calibration results for these three alternative specifications. Table VIII presents the long-run loss and consumption-equivalent welfare changes in response to a 100-basis-point shock to the interest rate in each version. We will refer to these tables when discussing each version in more detail below. To summarize briefly, the results in this section show that the main findings in the baseline version go through under these robustness specifications.

Table VII
Internally Calibrated Parameters

Parameter	Symbol	K in R&D	Quadratic entry	Truncated	Main identification	Target	K in R&D	Quadratic entry	Truncated
Mass of Varieties	λ	7.06	6.96	$--$	Mass of firms	1.00	1.01	1.12	$--$
Unaccounted employment	λ	$--$	$--$	7.11	Unaccounted employment	33.00%	$--$	$--$	33.43%
Labor disutility level	Θ	30.56%	30.67%	29.17%	Working time	33.00%	31.27%	31.92%	33.00%
Entry Cost	κ	31.06%	85.08%	5.00%	Entry rate	10.80%	12.78%	13.88%	7.84%
Step Size H	σ^H	6.36%	6.56%	7.97%	Annual GDP Growth	3.00%	3.01%	2.81%	3.10%
Step Size L	σ^L	6.27%	6.20%	7.35%	Mean firm employment	7.62	7.00	6.23	7.53
Scarcity	ν	55.57	54.62	73.28	Stdev firm employment	13.29	12.84	12.38	13.30
Expansion Cost scale	φ	13.35%	20.99%	30.97%	L-share of top 10% firms	48.30%	52.26%	55.58%	50.14%
Stdev TFP	σ_z	1.17%	1.22%	1.22%	Output volatility	3.00%	3.00%	3.00%	3.00%
Capital adjustment cost	ϕ	9.34	9.52	9.28	Investment volatility	9.56%	9.56%	9.56%	9.56%

Table VIII
Long-run output and welfare cost of a 100 bps R shock

	Baseline	K in R&D	Quadratic entry	Truncated
LRC	-0.24%	-0.33%	-0.19%	-0.21%
LRC rel. to Baseline	100%	135%	78%	88%
CEQ	-0.15%	-0.17%	-0.14%	-0.13%
CEQ rel. to Baseline	100%	113%	93%	91%

Notes: LRC and CEQ stand for long-run cost and consumption equivalent welfare cost, respectively. A negative x% for LRC means that the endogenous productivity is x% lower than the un-shocked path in the corresponding model 1200 periods after the shock hits. A negative x% for CEQ implies that the representative household in the shocked economy would have the same welfare if she consumed x% less in the un-shocked economy. The "K in R&D" model refers to a version in which entry costs and productivity-enhancing investment by incumbents are quoted in physical capital instead of labor. The "Quadratic entry" model refers to a version in which the entry cost is quadratic in entrant mass. The "Truncated" model refers to a version in which entry is defined as expansion of a firm that has one product line, to account for the employment cutoff in the data.

B4.1. Capital Input in R&D

In this exercise, we analyze a version where productivity enhancing investments require capital input instead of labor, bringing the specification of investment closer to the standard small open economy model. Specifically, enacting a new project requires $\kappa A(s^t)$ units of capital. Therefore, the cost of enacting a measure of $M(s^t)$ projects is given by

$$cost(M(s^t)) = r(s^t)\kappa A(s^t)M(s^t).$$

Similarly, the cost of expansion per product line at rate ι for a d-type incumbent is given by

$$cost(\iota^d(s^t)) = \varphi r(s^t)A(s^t)\iota^d(s^t)^{\xi}.$$

We recalibrate the model, and the results are shown in column 1 of Table VII. As highlighted in column 4, the match is quite successful only with the entry rate being somewhat above the target. The BGP capital to output ratio, which is not targeted in any calibration exercise, is 2.7 in this specification compared with 2.4 in the baseline case. We then assess the implications of the model for the long-run loss in productivity and consumption-equivalent welfare loss in response to a 100 basis points interest rate shock, similar to the analysis in Section IV.D in the main text. As shown in column 2 of Table VIII, the findings are in line with the baseline, only with some larger long-run loss in this version. Therefore, we conclude that our findings are robust to this alternative specification.

B4.2. Quadratic Cost of Entry

In this specification, we consider a quadratic cost for entry, which makes the entry cost and incumbent R&D cost functions to have the same convexity. We assume the following specification for entry cost:

$$cost(M(s^t)) = \frac{\kappa}{2} M(s^t)^2 \bar{W}(s^t).$$

The calibration results for this version are shown in column 2 of Table VII. As shown in column 5, the match is fairly good, with the calibration missing the mean and the standard deviation of the employment distribution somewhat on the downside, while the entry rate is above the target. Turning to Table VIII, column 3 reveals that in this case, the implied long-run loss is a bit smaller than in the baseline. This result would be expected, as the convexity in the entry cost damps the adjustment in the entry margin, limiting the permanent loss. However, the welfare loss generated by the interest rate shock is very similar to that in the baseline. The reason is that a smaller drop in entry relative to the baseline means fewer resources being diverted to production from entry activity. But overall, the implications of this specification again echo our baseline findings.

B4.3. Employment Cutoff in Data and Truncation

In ENIA, we observe firms that have at least 10 workers. To reflect this truncation in the data, we consider a version of the model in which we assume that in the model economy we observe only firms that have at least two product lines. As such, we truncate our model economy in a similar way.

While, in contrast to the previous two exercises, this specification does not alter the equations that define the equilibrium; it changes how we compute some of the moments in the model. In particular while aggregate moments such as the growth rate and working time are computed based on the whole economy, entry rate and moments regarding the employment distribution are computed based on the truncated sample of firms. Crucially, we define entry in this economy as obtaining at least two product lines. Moreover, in line with the empirical analysis, we carefully keep record of firms that lose product lines and shrink to only one line in order to not recount them as entry in case they successfully expand again. In other words, we distinguish first-time entry from reentry.

The recalibrated values of the model parameters are listed in column 3 of Table VII.

Of note, in the recalibration, we replace the moment "mass of firms" with the share of employment that is not employed in the truncated model economy. The empirical counterpart of this moment is roughly a third of the labor force. In this way, we discipline the size of the truncated economy based on the data. The overall match to the data is quite good, except for a somewhat low rate of entry. Also, the last column of Table VIII reveals again that the main findings of the baseline model are intact in this version. Therefore, we choose to proceed with the baseline model, refraining from additional complexity that this specification creates.

B5. Firm Size and Crises

Sedláček and Sterk (2017) use U.S. census data to document the cyclicality of job creation by startups and the persistence of these differences over the life cycle (at five years in the baseline result). Because our mechanism suggests that, on average, better cohorts with proportionally more H-type firms—which have higher growth potential—enter during downturns, our model may appear to contradict these findings, making it worth discussing our model's implications in this regard. To shed light on this point, Figure IX shows the employment levels (in percentage deviations from the respective value of the first cohort) of startup and five-year-old cohorts, where the time series is shifted back to the year of their birth.[6]

Our baseline economy predicts a cyclical variation in the size of startup cohorts, and this feature persists even at age five, echoing the findings in Sedláček and Sterk (2017).[7] Expansion rates (ι^d) are common to every firm of type d regardless of its size or age. Moreover, ι^d is procyclical. Therefore, a d type firm with age T will be larger in expectation if most of those T years were expansions. However, firms born in 1997 spend most of their early years during crisis, and therefore, are the smallest at age five, according to the model.

Potentially, the composition effect could be strong enough to overcome this force. In fact, high-type firms always expand faster than low-type firms; then, if cohorts born during crises have more high-type firms they could end up larger on average. However, the compo-

[6]The exercise replicates Figure 1 in Sedláček and Sterk (2017).

[7]When we perform the empirical analysis on equation (37) in the main text using labor growth rate as the dependent variable we see that firms born during crises do not grow more quickly. Interestingly, when we use physical investment as a measure of growth, we see that firms born during the crisis accumulate capital more quickly. Because this analysis is beyond the scope of this paper, those regressions are available upon request.

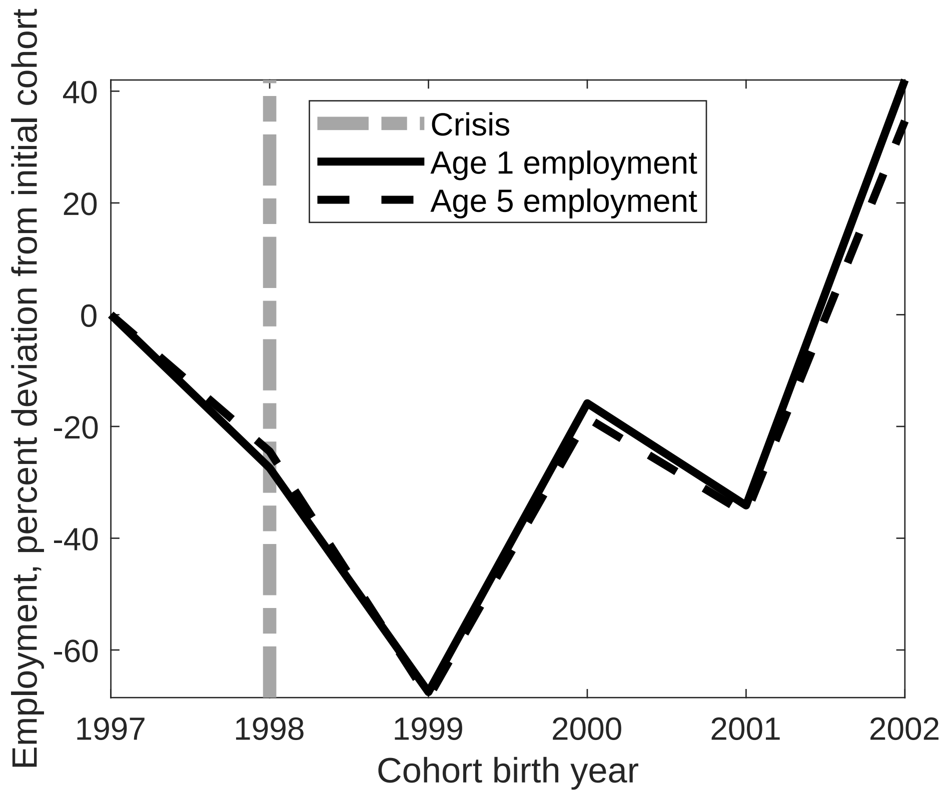


Figure IX
Average Cohort Employment 5 Years after Entry (relative to first year)

sition of cohorts born during booms and downturns is different mostly at very young ages, and as the cohorts get older, the proportion of high types rises for both type of cohorts because low-type firms are scrapped more quickly. This relationship reduces the compositional differences at birth across cohorts over time and drives the result in Figure IX, as the exercise considers cohorts at already five years into their life cycles. Therefore, this model can generate the basic premise documented by Sedláček and Sterk (2017) for the U.S. economy. Future research should explore a closed economy version of our economy and compare it to the U.S. firm-level dynamics.

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