# Online Appendix to "Credit Spreads, Financial Crises, and Macroprudential Policy"[*]

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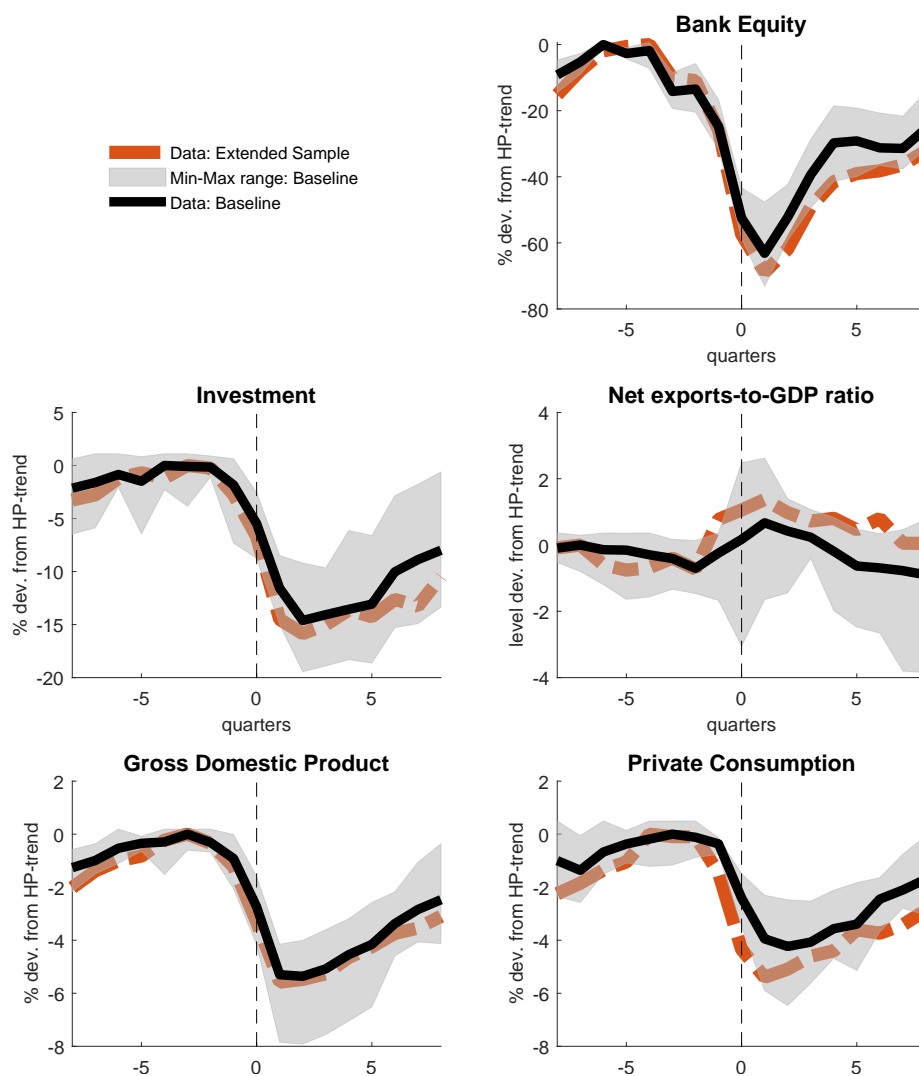Federal Reserve Board

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# A    Crisis Event Study for the Extended Sample

Figure A.1: Average Financial Crisis: Extended Sample



**Note**: All series are HP detrended with a smoothing parameter 1600, and normalized at their respective pre-crisis peaks. The sample includes Austria (2008), Belgium (2008), Denmark (2008), France (2008), Germany (2008), Greece (2008), Iceland (2008), Ireland (2008), Italy (2008), Japan (1997), Luxembourg (2008), Netherlands (2008), Norway (1991), Portugal (2008), Spain (2008), Switzerland (2008), UK (2007), US (1988, 2007), Korea (1997), Mexico (1994), and Turkey (2000). Years in the parenthesis refer to the financial crises identified by Laeven and Valencia (2012). The event window includes 8 quarters before and after the event. Data sources: Haver Analytics, authors' calculations.

# B Household's Optimality Conditions

$$
\begin{aligned}
U_1(C_t, H_t) &= U_{C,t} & (1) \\
-U_2(C_t, H_t) &= U_{C,t}W_t & (2) \\
\mathbb{E}_t(\Lambda_{t,t+1}R_t) &= 1 & (3)
\end{aligned}
$$

The household's stochastic discount factor is defined as

$$
\Lambda_{t,t+1} = \beta \frac{U_{C,t+1}}{U_{C,t}} \tag{4}
$$

where $U_{C,t}$ is the marginal utility of consumption.

## C   Details on Working Capital Loans

We follow the timing assumptions in Neumeyer and Perri (2005). Within each period $t$ there are two subperiods: $t^-$ and $t^+$. Period-$t$ shocks are revealed in $t^-$. Times $t^+$ and $(t+1)^-$ are arbitrarily close. As in Neumeyer and Perri (2005), final goods producers hire labor and capital at time $t^-$, and output becomes available in $t^+$. Firms need to set aside fraction $\Upsilon$ of the wage bill in $t^-$ (before production takes place), and so they need to borrow this amount. In contrast to Neumeyer and Perri (2005), we assume firms cannot borrow directly from international financial markets (or from domestic households), but need to borrow from domestic banks. The interest charged by banks on working capital loans taken out in $t^-$ is denoted $R_{L,t}$. These loans are repaid at time $t^+$, after final goods become available.

In period $t^-$, banks borrow $d_{W,t}$ in the international noncontingent debt market, at interest rate $R_{t-1}$. Noncontingent bonds issued at time $t^-$ or at time $(t-1)^+$ mature at time $t^+$. Risky loans for investment financing, $s_{t-1}$, also mature at $t^+$, once nonfinancial firms' output becomes available. Equity issued in the previous period also becomes available at time $t^+$. Accordingly, banks do not have internal funds at time $t^-$, and so working capital loans—denoted $s_{W,t}$—are financed entirely by noncontingent debt: $s_{W,t} = d_{W,t}$. The net proceeds to the bank from working capital lending are therefore given by $(R_{L,t} - R_{t-1}) s_{W,t}$. These proceeds become available to the bank at time $t^+$. All told, the bank's period-$t$ budget constraint (assuming the bank does not default) is given by equation (4) in the main text.

We assume that at the beginning of period $t$, the banker makes a decision on whether he or she will default or not in that period. Further, we assume that if the banker defaults, he or she does so on both working capital loans and investment loans (i.e. the banker cannot decide to default on one type of loan but not the other). A defaulting banker is able to go undetected through the end of period $t$, at which point it is forced into bankruptcy.

Suppose the banker chooses to default in period $t$. The banker will default *both* on its debt position $d_{W,t}$ intended for working capital loans (a position which opens in $t^-$ and closes in $t^+$) *and* on its debt position $d_t$ issued to finance investment loans (a position which opens in $t^+$ and closes in $(t+1)^+$). We allow the recovery rates for working capital assets and for physical capital assets are to differ in principle: we assume a defaulting banker can abscond fraction $\omega$ of resources to finance working capital loans and fraction $\theta$ of resources to finance investment loans, with $0 < \omega, \theta < 1$. Consider first working capital loans. After borrowing $d_{W,t}$ in $t^-$, a defaulting banker is able to set these funds aside (rather than lending them out in the working capital market and repaying creditors in $t^+$), and will be able to retain $\omega d_{W,t}$ at the end of period $t$ after bankruptcy proceedings. As a consequence, the defaulting

banker forgoes the net revenue from the working capital position (equal to $(R_{L,t} - R_{t-1})d_{W,t}$) which would otherwise have been available in period $t^+$ to finance investment loans.[1] Thus, in period $t^+$ the resources available to a banker that has decided to default are $n_t + d_t$ (compared to $n_t + d_t + (R_{L,t} - R_{t-1})d_{W,t}$ for a non-defaulting banker). After issuing debt $d_t$, the banker then diverts the existing funds, and is able to keep only $\theta(n_t + d_t)$ after bankruptcy proceedings. Therefore, the total payoff obtained by a defaulting banker is given by

$$\theta \left( d_t + n_t \right) + \omega d_{W,t} = \theta \left[ Q_t s_t - (R_{L,t} - R_{t-1})s_{W,t} \right] + \omega s_{W,t}, \tag{5}$$

where we have used the balance sheet identity (5).

---

[1] Put differently, the budget constraint facing a defaulting banker is not equation (4), and is instead given by $Q_t s_t + R_{t-1} d_{t-1} \leq R_{K,t} Q_{t-1} s_{t-1} + d_t + e_{t-1}$.

# D   Banker's Problem

Under our functional form assumption for $C(e_t, n_t)$, the banker's problem is:

$$\alpha_t n_t = \max_{s_t, s_{W,t}, x_t} \mu_t Q_t s_t + \nu_t \Delta_{L,t} s_{W,t} + \nu_t n_t + \sigma n_t (\nu_{e,t} x_t - \frac{\kappa}{2} x_t^2) \tag{6}$$

subject to

$$\mu_t Q_t s_t + \nu_t \Delta_{L,t} s_{W,t} + \nu_t n_t + \sigma n_t (\nu_{e,t} x_t - \frac{\kappa}{2} x_t^2) \geq \theta \left[ Q_t s_t - \Delta_{L,t} s_{W,t} \right] + \omega s_{W,t} \tag{7}$$

The first order condition for $x_t$ is $\nu_{e,t} = \kappa x_t$. Imposing this condition, the bank's problem becomes:

$$\alpha_t n_t = \max_{s_t, s_{W,t}} \mu_t Q_t s_t + \nu_t \Delta_{L,t} s_{W,t} + (\nu_t + \frac{\sigma \kappa}{2} x_t^2) n_t \tag{8}$$

subject to

$$\mu_t Q_t s_t + \nu_t \Delta_{L,t} s_{W,t} + (\nu_t + \frac{\sigma \kappa}{2} x_t^2) n_t \geq \theta \left[ Q_t s_t - \Delta_{L,t} s_{W,t} \right] + \omega s_{W,t} \tag{9}$$

Then the Lagrangian can be written as:

$$\mathcal{L}_t = (1 + \lambda_t) \left[ \mu_t Q_t s_t + \nu_t \Delta_{L,t} s_{W,t} + (\nu_t + \frac{\sigma \kappa}{2} x_t^2) n_t \right] - \lambda_t \left[ \theta Q_t s_t + (\omega - \theta \Delta_{L,t}) s_{W,t} \right] \tag{10}$$

where $\lambda_t$ is the multiplier on the incentive constraint.

# E   Resource Constraint and Balance of Payments

Aggregating the bank's budget constraint across banks and combining it with the household's budget constraint along with the market clearing condition for claims on capital $(S_t = K_t)$, we obtain:

$$Q_t K_t + R_{t-1} B_{t-1}^* + C_t + \sigma \frac{\kappa}{2} x_t^2 N_t \leq W_t H_t + R_{K,t} Q_{t-1} K_{t-1}$$
$$+ (R_{L,t} - R_{t-1}) S_{W,t} + Q_{t-1} K_{t-1} + B_t^* + \Pi_t^F + \Pi_t^C \tag{11}$$

The last two terms, $\Pi_t^F$ and $\Pi_t^C$, are the profits of final goods firms and capital producers, respectively. They are given by their respective budget constraints:

$$Y_t + Q_t (1 - \delta) e^{\psi_t} K_{t-1} = \Pi_t^F + W_t H_t (1 + \Upsilon(R_{L,t} - 1)) + R_{K,t} Q_{t-1} K_{t-1} \tag{12}$$

$$\Pi_t^C = Q_t I_t - \left[ 1 + f \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \tag{13}$$

Using these expressions, we can derive the resource constraint and the balance of payments equation for the economy as the following:

$$Y_t = C_t + \left[ 1 + f \left( \frac{I_t}{I_{t-1}} \right) \right] I_t + \sigma \frac{\kappa}{2} x_t^2 N_t + N X_t \tag{14}$$

$$R_{t-1} B_{t-1}^* - B_t^* + (R_{t-1} - 1) \Upsilon W_t H_t = N X_t \tag{15}$$

# F Full Set of Equilibrium Conditions

$$Y_t = C_t + \left[ 1 + f\left(\frac{I_t}{I_{t-1}}\right) \right] I_t + \sigma \frac{\kappa}{2} x_t^2 N_t + NX_t \tag{16}$$

$$NX_t = R_{t-1} B_{t-1}^* - B_t^* + (R_{t-1} - 1)\Upsilon W_t H_t \tag{17}$$

$$K_t = I_t + (1-\delta)e^{\psi_t} K_{t-1} \tag{18}$$

$$Q_t = 1 + f\left(\frac{I_t}{I_{t-1}}\right) + \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right) - E_t \Lambda_{t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right) \tag{19}$$

$$1 = \mathbb{E}_t\left(\Lambda_{t+1}\right) R_t \tag{20}$$

$$\Lambda_t = \beta \left( \frac{C_t - \chi \frac{H_t^{1+\epsilon}}{1+\epsilon}}{C_{t-1} - \chi \frac{H_{t-1}^{1+\epsilon}}{1+\epsilon}} \right)^{-\gamma} \tag{21}$$

$$R_{K,t} = e^{\psi_t} \frac{\alpha \frac{Y_t}{e^{\psi_t} K_{t-1}} + (1-\delta)Q_t}{Q_{t-1}} \tag{22}$$

$$Y_t = A_t (e^{\psi_t} K_{t-1})^\eta H_t^{1-\eta} \tag{23}$$

$$\mu_t = \mathbb{E}_t\left[\Lambda_{t+1}(1 - \sigma + \sigma\alpha_{t+1})\left(R_{K,t+1} - R_t\right)\right] \tag{24}$$

$$\nu_t = \mathbb{E}_t\left[\Lambda_{t+1}(1 - \sigma + \sigma\alpha_{t+1})\right] R_t \tag{25}$$

$$\nu_{e,t} = \mathbb{E}_t\left[\Lambda_{t+1}\left(\alpha_{t+1} - 1\right)\right] \tag{26}$$

$$\alpha_t = \mu_t \phi_t + \nu_t + \sigma \frac{\kappa}{2} x_t^2 \tag{27}$$

$$N_t = \sigma[(R_{K,t} - R_{t-1}) Q_{t-1} K_{t-1} + x_{t-1} N_{t-1} \tag{28}$$
$$+ R_{t-1}\left(N_{t-1} + \Delta_{L,t-1}\Upsilon W_{t-1} H_{t-1}\right)] + (1-\sigma)\xi Q_{t-1} K_{t-1}$$

$$W_t = \chi H_t^\epsilon \tag{29}$$

$$\frac{Y_t}{H_t} = \frac{W_t\left[1 + \Upsilon\left(R_{L,t} - 1\right)\right]}{(1-\eta)} \tag{30}$$

$$R_t = \frac{1}{\beta} + \varphi\left(e^{\frac{B_t^*}{Y_t} - \bar{b}} - 1\right) + e^{R_t^* - 1} - 1 \tag{31}$$

$$\phi_t = \frac{\nu_t + \sigma \frac{\kappa}{2} x_t^2}{\theta - \mu_t} \tag{32}$$

$$x_t = \frac{\nu_{e,t}}{\kappa} \tag{33}$$

$$R_{L,t} = R_{t-1} + \frac{\mu_t}{\mu_t + \nu_t} \tag{34}$$

If the constraint binds, we have $Q_t S_t + (1 - \Delta_{L,t})\Upsilon W_t H_t = \phi_t N_t$ and $(\mu_t > 0)$. If it does not bind, we have $\mu_t = 0$ (and $Q_t S_t + (1 - \Delta_{L,t})\Upsilon W_t H_t < \phi_t N_t$). This condition can be summarized as

$$\mu_t \cdot \left[\phi_t N_t - Q_t K_t + (1 - \Delta_{L,t})\Upsilon W_t H_t\right] = 0 \tag{35}$$

An equilibrium is a stochastic sequence for the 9 quantity variables $Y_t, C_t, I_t, NX_t, B_t^*, H_t,$ $K_t, N_t, x_t$, the 5 prices $R_{K,t}, Q_t, R_t, R_{L,t}, W_t$, the 5 banking sector coefficients $\mu_t, \nu_t, \nu_{e,t}, \alpha_t, \phi_t,$ and the household's stochastic discount factor $\Lambda_t$ such that equations (16)-(35) are satisfied, given exogenous stochastic sequences for $A_t, \psi_t,$ and $R_t^*$.

# G    Solution Method

Let $\overline{K}_t \equiv e^{\psi_t} K_{t-1}$ denote the effective amount of physical capital at the beginning of period $t$ (after the capital quality shock is realized), and define $\overline{B}_{t-1} \equiv R_{t-1} B^*_{t-1}$ to be the stock of external debt plus interest. Let also $\overline{N}_{t-1}$ refer to the predetermined part of aggregate net worth (i.e., the component of net worth that does not depend on time-$t$ variables like $Q_t$), given by the following:

$$\overline{N}_{t-1} = \sigma \left[ x_{t-1} N_{t-1} + R_{t-1} \left( \underbrace{N_{t-1} + \Delta_{L,t} \Upsilon W_{t-1} H_{t-1} - Q_{t-1} K_{t-1}}_{=-D_{t-1}} \right) \right] + (1-\sigma)\xi Q_{t-1} K_{t-1}$$

Note that $\overline{N}_{t-1}$ is equal to aggregate new equity issued by surviving banks $(\sigma x_{t-1} N_{t-1})$, plus startup transfers to entering banks $((1-\sigma)\xi Q_{t-1} K_{t-1})$, minus the total stock of debt (with interest) carried over by surviving banks $(\sigma R_{t-1} D_{t-1})$. Given our calibration the latter term will always be large relative to the first two, so that $\overline{N}_{t-1} < 0$.

Given these definitions, let $\mathcal{S}_t$ denote the model's aggregate state vector, given by seven variables:

$$\mathcal{S}_t \equiv \left\{ \overline{K}_t, -\overline{N}_{t-1}, \overline{B}_{t-1}, R_{t-1}, I_{t-1}, R^*_t, A_t \right\}$$

We use the negative of $\overline{N}_{t-1}$ so that $\mathcal{S}_t > 0$. Following the parameterized expectations approach (PEA henceforth), our solution method relies on using parametric functions to approximate the model's one-step-ahead expectations. To this end, define the following objects, capturing the model's expectations as a function of the aggregate state:

$$\varepsilon_1(\mathcal{S}_t) \equiv \mathbb{E}_t \left( U_{C,t+1} \right) R_t \tag{36}$$

$$\varepsilon_2(\mathcal{S}_t) \equiv \mathbb{E}_t \left[ U_{C,t+1} \left( 1 - \sigma + \sigma \alpha_{t+1} \right) \left( \frac{\alpha Y_{t+1}}{e^{\psi_{t+1}} K_t} + (1-\delta) Q_{t+1} \right) \right] \tag{37}$$

$$\varepsilon_3(\mathcal{S}_t) \equiv \mathbb{E}_t \left[ U_{C,t+1} \left( 1 - \sigma + \sigma \alpha_{t+1} \right) \right] R_t \tag{38}$$

$$\varepsilon_4(\mathcal{S}_t) \equiv \mathbb{E}_t \left[ U_{C,t+1} \left( \nu_{t+1} + \mu_{t+1} \phi_{t+1} + \frac{\sigma \kappa}{2} x_{t+1}^2 \right) \right] \tag{39}$$

$$\varepsilon_5(\mathcal{S}_t) \equiv \mathbb{E}_t \left( U_{C,t+1} \right) \tag{40}$$

$$\varepsilon_6(\mathcal{S}_t) \equiv \mathbb{E}_t \left[ U_{C,t+1} \left( \frac{I_{t+1}}{I_t} \right)^3 \right] \tag{41}$$

$$\varepsilon_7(\mathcal{S}_t) \equiv \mathbb{E}_t \left[ U_{C,t+1} \left( \frac{I_{t+1}}{I_t} \right)^2 \right] \tag{42}$$

Our choice of expectations reflects two main considerations: First, it should be possible to approximate them accurately with a parameterized function of the state. Second, they should facilitate the solving of the model. We have found that the expectations above satisfy both criteria successfully: as we illustrate below, they are in general smooth functions of the state; in addition, it is straightforward to show that knowing $\mathcal{S}_t$, and therefore $\{\varepsilon_i(\mathcal{S}_t)\}_{i=1}^7$, all time-$t$ endogenous variables (as well as the evolution of the state vector) can be retrieved in closed form from (16)-(35) when the constraint does not bind, and the system (16)-(35) can be collapsed to just one nonlinear equation in one unknown ($Q_t$ or $\mu_t$) when the constraint binds.

The problem is then to find the set of functions of the aggregate state $\varepsilon_i(\mathcal{S}_t) : \mathbb{R}^7 \to \mathbb{R}$ for $i = 1, ...7$ that determine the expectations (36)-(42). To approximate the $\varepsilon_i$ functions, we use third-order polynomials for the log of the conditional expectations in the log of the state, following Den Haan (2007). That is, letting $P_3(s; \varrho)$ stand for the third-order polynomial in the vector $s$ with coefficients $\varrho$, we approximate the functions as $\varepsilon_i(\mathcal{S}_t) \approx e^{P_3(\log(\mathcal{S}_t); \varrho_i)}$. We have found that the exponential-log formulation enhances accuracy significantly compared to a standard polynomial. All told, we need to find the 120 coefficients in $\varrho_i$ for each $i = 1, ..., 7$. Our algorithm adapts PEA by using quadrature to compute expectations, as advised by Judd, Maliar and Maliar (2011) and Den Haan (2007). As these authors point out, this approach significantly enhances accuracy and speed. We continue to use simulation-based PEA: also as pointed out by Judd, Maliar and Maliar (2011) and Den Haan (2007),

a virtue of this approach (as opposed to solving the model on a pre-specified grid) is that one needs to solve the model only in points in the state space that are actually visited in equilibrium.

Let $\boldsymbol{\varrho} \equiv (\varrho_i)_{i=1}^7$. Our algorithm proceeds as follows:

0. Let $\boldsymbol{\varrho}^0$ be the initial set of coefficients in this step. Simulate the model for 5,000 periods by solving the system (16)-(35) characterizing equilibrium, given a sequence of realizations of the exogenous innovations (and setting $\mathcal{S}_0$ at the steady-state value of the state vector). To do so, at each period $t$ we first solve the system assuming that the constraint does not bind (implying $\mu_t = 0$). We then check if bank leverage is above the maximum allowed by the constraint. If it is not, we proceed; if it is, we again solve the system, this time imposing that the constraint binds.

   - At each $t$, after solving for the equilibrium, we approximate the set of one-step-ahead conditional expectations (36)-(42) by Gauss-Hermite quadrature.

1. Obtain a new set of coefficients $\boldsymbol{\varrho}^1$ by regressing the (log of the) conditional expectations obtained in the previous step on $P_3 (\log(\mathcal{S}_t); \cdot)$, where $\mathcal{S}_t$ is the state vector from the simulation in the previous step.

2. Compare $\boldsymbol{\varrho}^1$ with $\boldsymbol{\varrho}^0$. If they are close enough, stop. If not, update $\boldsymbol{\varrho}^0$ by setting $\boldsymbol{\varrho}^0 = \lambda_\varrho \boldsymbol{\varrho}^1 + (1 - \lambda_\varrho)\boldsymbol{\varrho}^0$ and go back to step 0. We have found a value of 2/3 for the "smoothing" parameter $\lambda_\varrho$ to work well in our setting.

To initialize our algorithm we need an initial value for $\boldsymbol{\varrho}^0$. To this purpose, we first simulate the model using the OccBin toolkit developed by Guerrieri and Iacoviello (2015), and regress the resulting conditional expectations on the state vector to initialize $\boldsymbol{\varrho}^0$. This approach has proved very helpful to obtain fast convergence of our algorithm.

# H Euler Residuals and Expectation Functions Fit

Following Judd (1992), we provide a check on the accuracy of our solution method by computing Euler equation errors. Moving from the Euler equation for consumption, we define the Euler equation error (as a fraction of units of consumption) as

$$err_t = \left| \frac{[\beta \mathbb{E}_t(U_{C,t+1})R_t]^{-\frac{1}{\gamma}} + \frac{\chi}{1+\epsilon}H_t^{1+\epsilon} - C_t}{C_t} \right| \tag{43}$$

Above, we again approximate $\mathbb{E}_t(U_{C,t+1})$ by Gauss-Hermite quadrature. Figure A.2 shows a histogram of the errors for a given simulation. We express the errors in decimal log scale, as is common in the literature. The Euler errors are reasonably small, and comparable to those found in the literature. The average error is -5.45. To interpret, recall that under the decimal log scale a value of, say, -5 indicates an error sized at \$1 per \$100,000 of consumption.

Figure A.2: Histogram of Euler Residuals
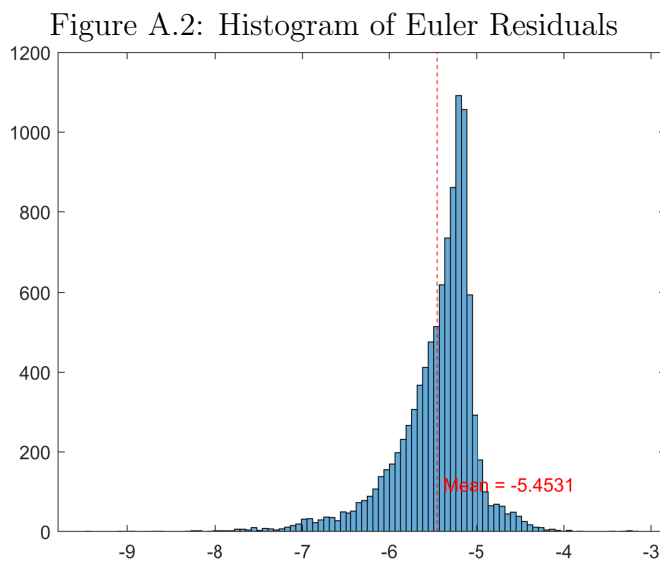


**Note**: Histogram of Euler equation errors in model simulation.

Table A.1: $R^2$ of Expectation Regressions

| $\mathbb{E}_{1,t}$ | $\mathbb{E}_{2,t}$ | $\mathbb{E}_{3,t}$ | $\mathbb{E}_{4,t}$ | $\mathbb{E}_{5,t}$ | $\mathbb{E}_{6,t}$ | $\mathbb{E}_{7,t}$ |
|---|---|---|---|---|---|---|
| 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

As a second accuracy check, we verify that the expectations are approximated accurately by examining goodness-of-fit measures of the regression in step 1 of our algorithm (see the

previous section). Table A.1 provides the values of $R^2$ for each of the expectations (36)-(42). The $R^2$'s are extremely close to unity (the smallest one equals 0.99996), indicating a good fit of the parameterized expectation functions.

# I  Model Policy and Expectation Functions

Figures A.3 through A.6 show different slices of the model's policy functions for a set of the model's endogenous variables. Note that moving toward adverse values of the state (for example, toward lower values of $\overline{K}_t$ in Figure A.3, or lower values of $\overline{N}_{t-1}$ in Figure A.4, or higher values of $\overline{B}_{t-1}$ in Figure A.5) net worth declines and eventually the constraint starts binding: $\mu_t$ turns positive, and the declines in net worth, Tobin's $Q$, investment and output turn steeper. Note also that as $\overline{K}_t$ declines (or $\overline{N}_{t-1}$ declines, or $\overline{B}_{t-1}$ increases), leverage, $\phi_t$, increases, and so does equity issuance, $x_t$. Similar observations apply when one varies the remaining states (not shown). In addition, the constrained region is not only characterized by very low values of $\overline{K}_t$ or $\overline{N}_{t-1}$, but also by a combination of relatively low values of both, as shown by Figure A.6. Overall, note that the model's policy functions tend to be highly nonlinear, displaying sharp kinks when the constraint starts binding.

By contrast, the model's expectation functions tend to be much smoother. As an example, Figures A.7 and A.8 show the model's one-step-ahead conditional expectations (36)-(42) as functions of the state variables $\overline{K}_t$ and $\overline{N}_{t-1}$. Note that the conditional expectations (shown by the blue solid line) in general do not display the kinks exhibited by the policy functions. Note also that our polynomial approximation (the orange dashed line) tracks very well the actual expectations, in line with the very high $R^2$'s reported in the previous section.

# Figure A.3: Model Policy Functions (I): $\overline{K}_t = e^{\psi_t} K_{t-1}$
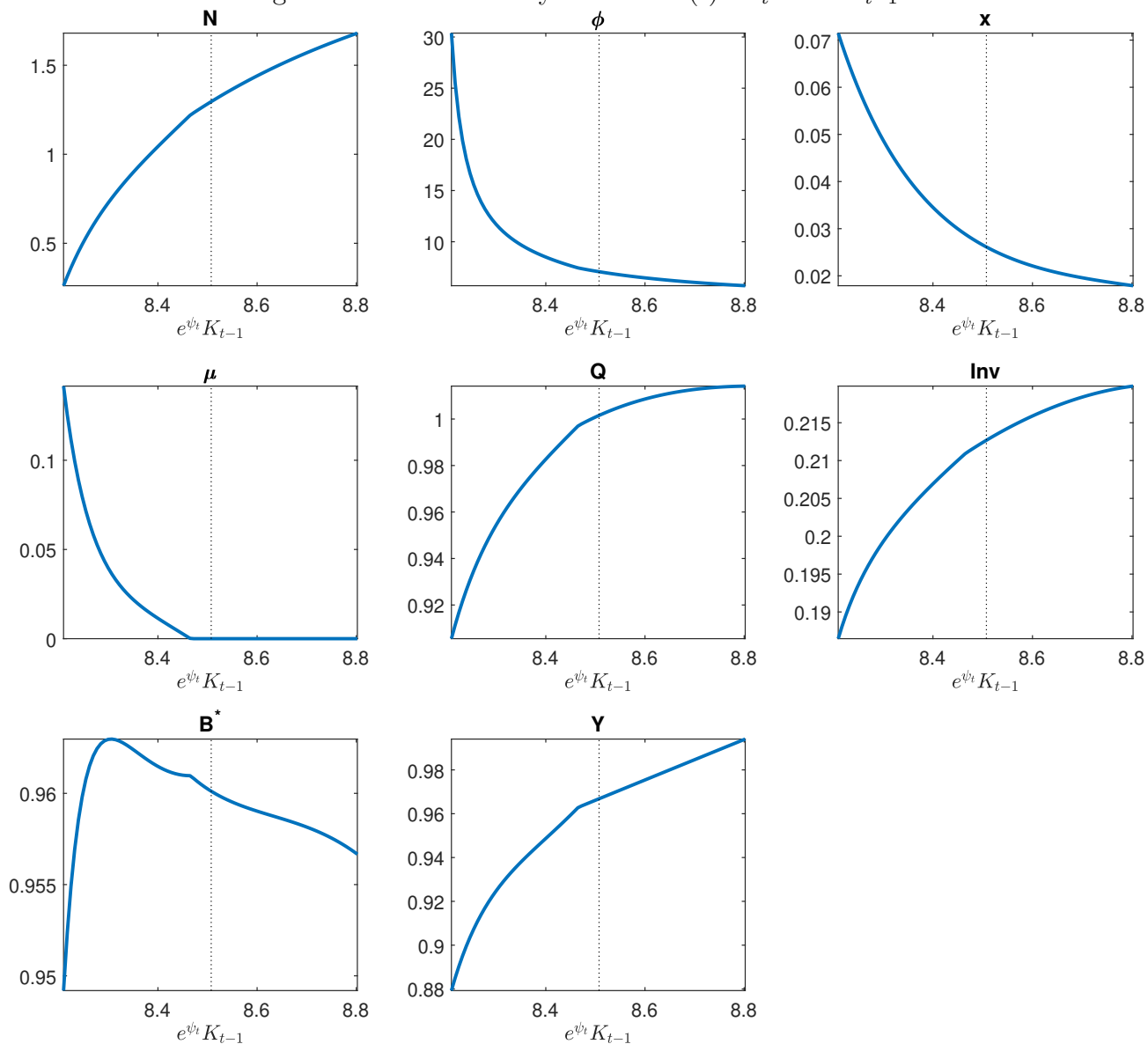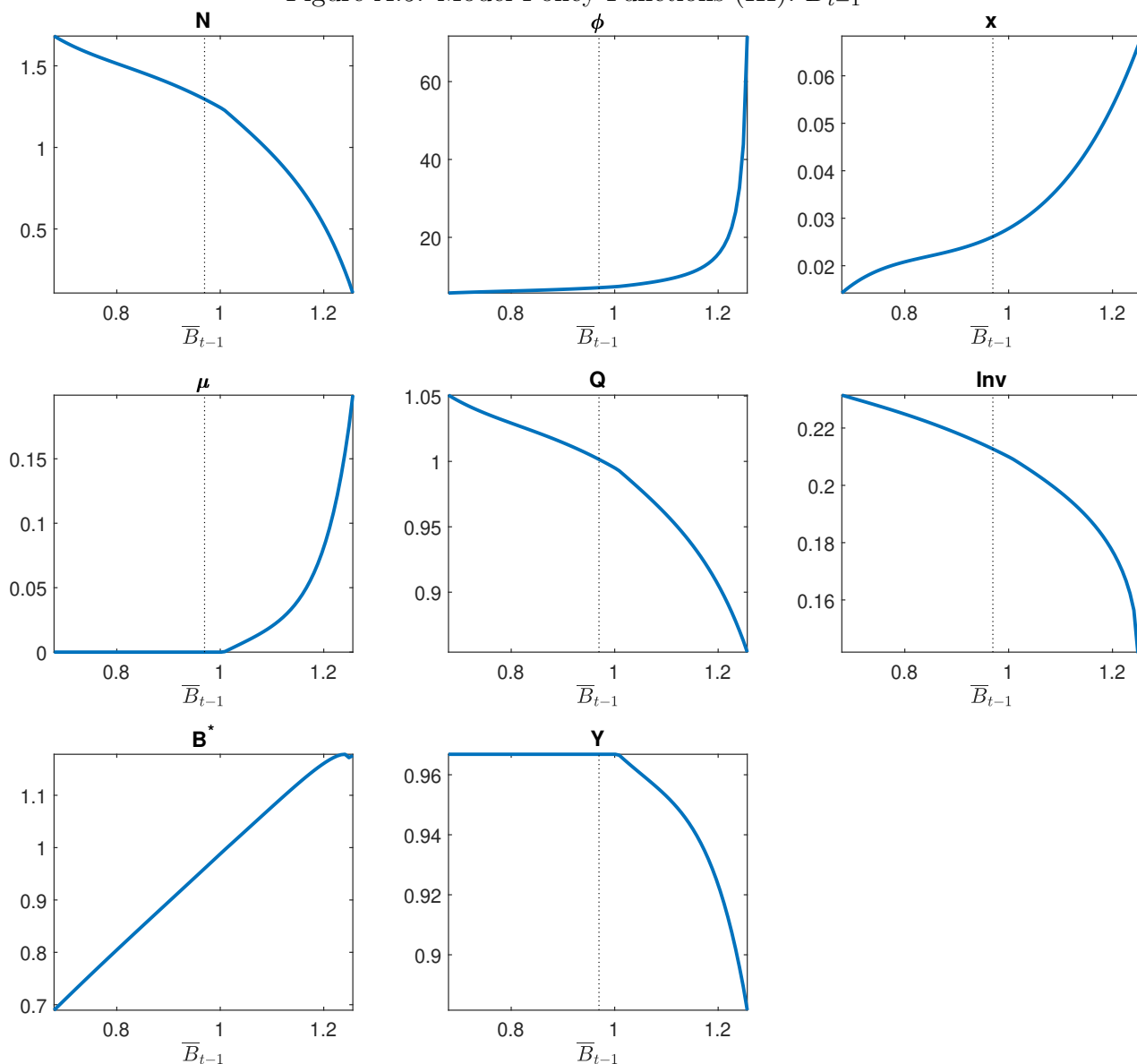


**Note**: Model endogenous variables as a function of state variable $\overline{K}_t$. All other states are kept at their risk-adjusted-steady-state value. Dotted vertical line indicates risk-adjusted-steady-state value of $\overline{K}_t$.

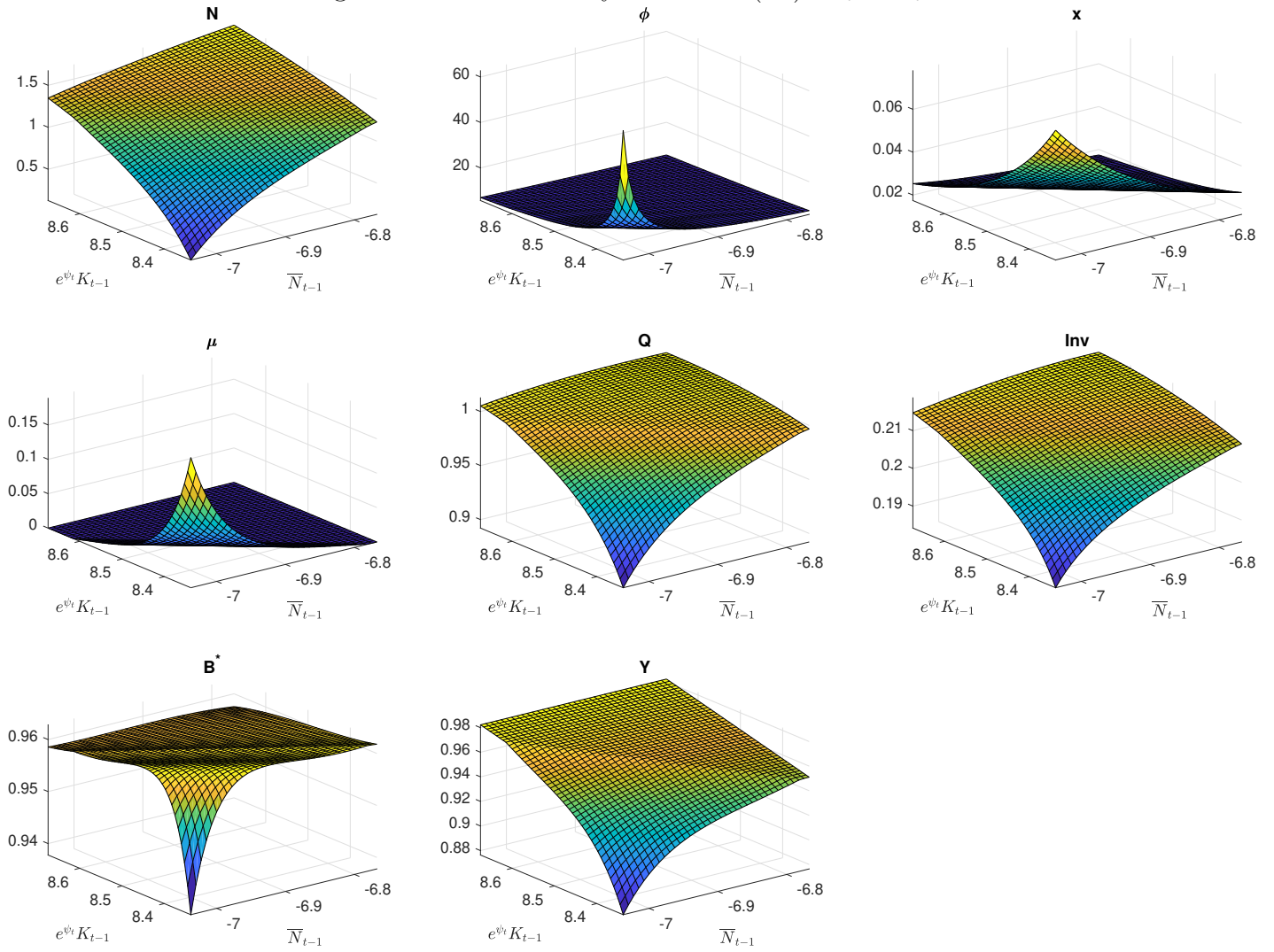Figure A.4: Model Policy Functions (II): $\overline{N}_{t-1}$
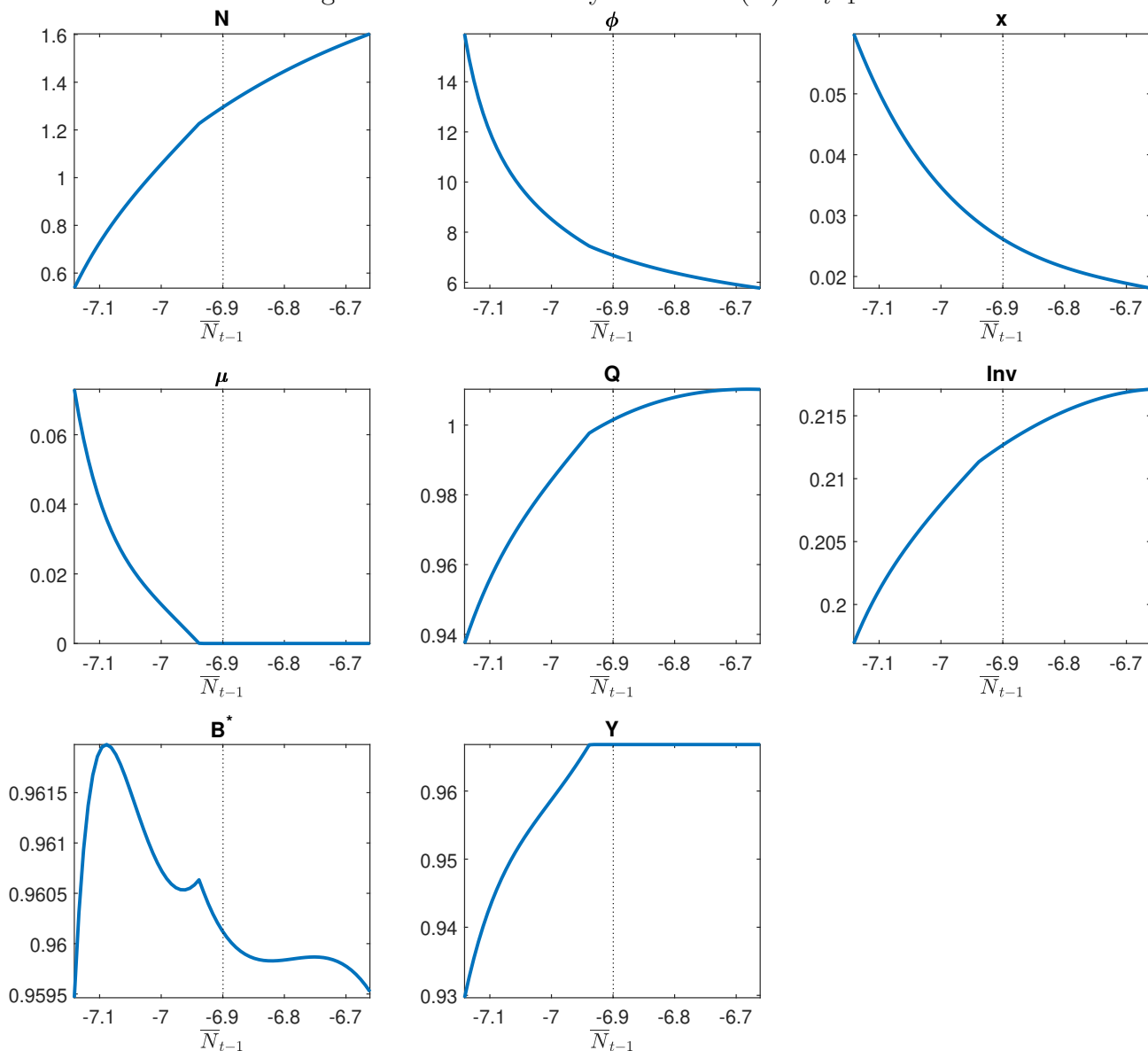
**Note**: Model endogenous variables as a function of state variable $\overline{N}_{t-1}$. All other states are kept at their risk-adjusted-steady-state value. Dotted vertical line indicates risk-adjusted-steady-state value of $\overline{N}_{t-1}$.

Figure A.5: Model Policy Functions (III): $\overline{B}_{t-1}$



**Note**: Model endogenous variables as a function of state variable $\overline{B}_{t-1}$. All other states are kept at their risk-adjusted-steady-state value. Dotted vertical line indicates risk-adjusted-steady-state value of $\overline{B}_{t-1}$.

Figure A.6: Model Policy Functions (IV): $\overline{K}_t \times \overline{N}_{t-1}$



**Note**: Model endogenous variables as a function of $\overline{K}_t$ and $\overline{N}_{t-1}$. All other states kept at risk-adjusted-steady-state value.

Figure A.7: Model Expectation Functions (I): $\overline{K}_t = e^{\psi_t} K_{t-1}$
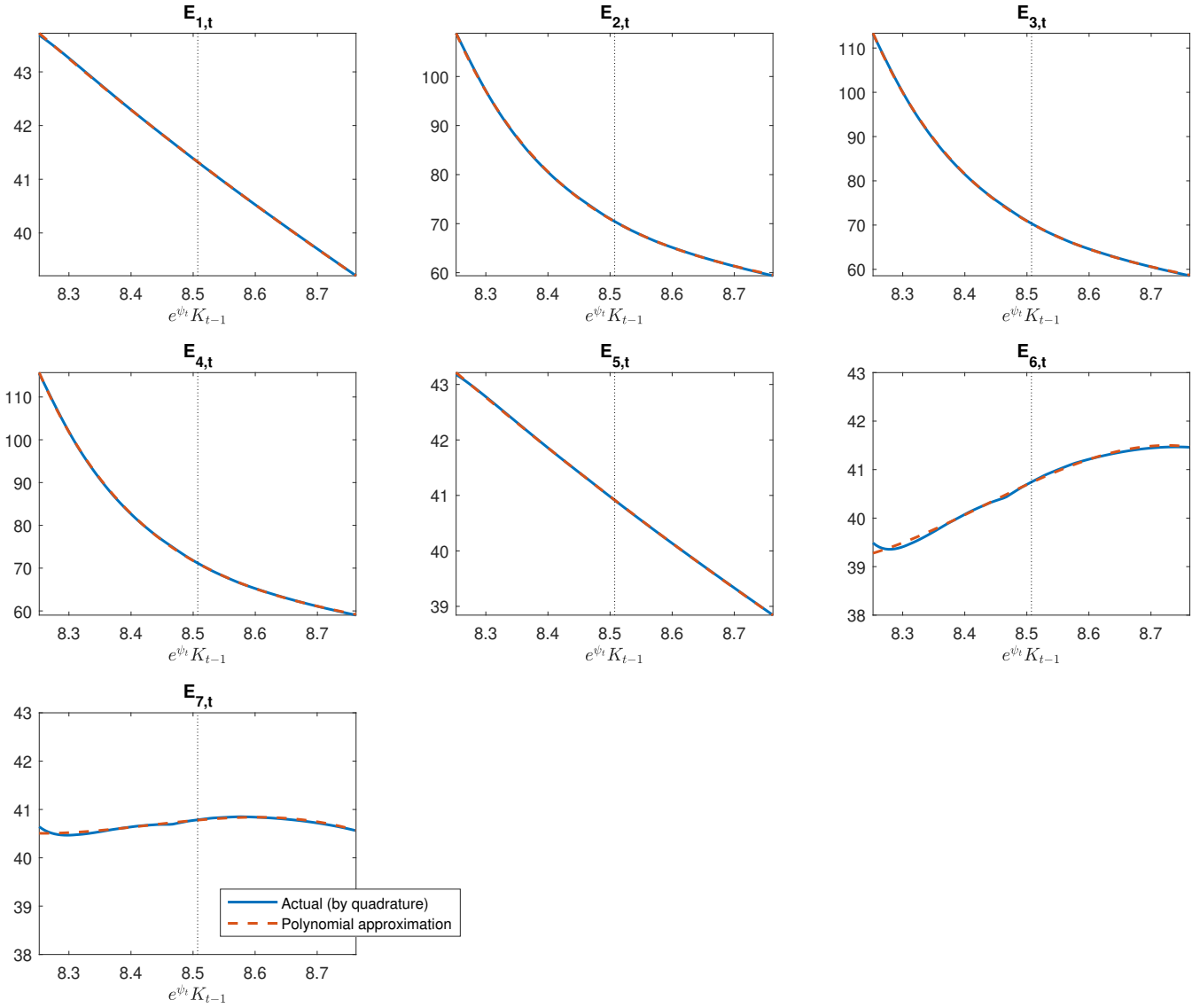
**Note**: Model expectations as a function of state variable $\overline{K}_t$. All other states kept at risk-adjusted-steady-state value. Dotted vertical line indicates risk-adjusted-steady-state value of $\overline{K}_t$.

Figure A.8: Model Expectation Functions (II): $\overline{N}_{t-1}$
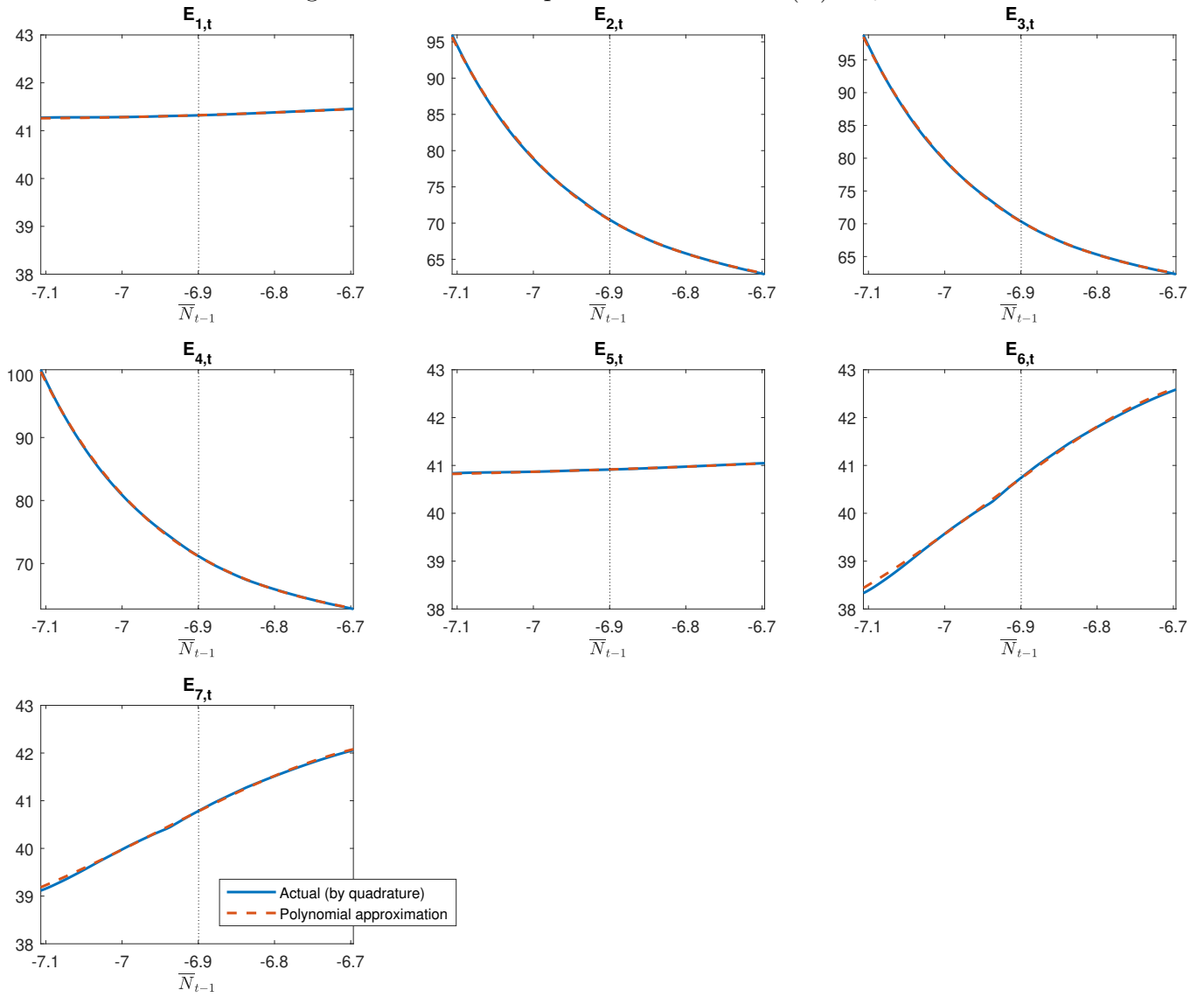
**Note**: Model expectations as a function of state variable $\overline{N}_{t-1}$. All other states kept at risk-adjusted-steady-state value. Dotted vertical line indicates risk-adjusted-steady-state value of $\overline{N}_{t-1}$.

# J  Role of Banker Exit

This section discusses our assumption of allowing for random banker exit ($\sigma < 1$), even though the model also features costly equity issuance. To this end, Figure A.9 shows the behavior of key financial variables as $\sigma \to 1$ in the economy's deterministic steady state.

Focusing first on the blue line, note that as $\sigma$ approaches unity, leverage falls rapidly. The economy approaches the unconstrained region, leading both $\mu$ and the credit spread to converge to zero. This leads the value of a unit of bank equity ($\Omega$) to approach unity (the value it would take in a model without financing frictions). For this reason, banks have no incentive to issue equity, and thus $x$ approaches zero. In short, the economy becomes largely unconstrained, rendering financing frictions irrelevant and equity issuance unnecessary.

Comparing the blue with the red and yellow lines show why the presence of the equity issuance cost alone cannot recover a role for financing frictions as long as $\sigma$ takes a value close to unity. Note that when $\sigma \approx 1$, the steady-state values of $x, \mu, \Omega$, leverage, and the credit spread are largely insensitive to the value $\kappa$ (the parameter governing the equity issuance cost). The reason is that since banks are largely unconstrained, they have little incentive to issue equity, regardless of its issuance cost. Thus, allowing for $\sigma < 1$ is key for the model to produce instances in which the constraint binds.

In addition, Figure A.9 also shows how $\sigma < 1$ is helpful in the model's calibration: unlike when $\sigma \approx 1$, for values of $\sigma$ below unity we do observe meaningful variation in both the equity issuance rate and the leverage ratio as we vary $\kappa$. Our parameterization exploits this property, which permits the model to generate realistic values for the leverage ratio and for the equity issuance rate.

This reasoning may help explain why related literature preserves the assumption that $\sigma < 1$, even in cases in which some form of equity issuance cost is allowed for (see, for example, Gertler et al. 2012 or Gertler et al. 2019). Other approaches found in the literature involve the presence of a debt subsidy—an approach taken in Jermann and Quadrini (2012), for example, who also allow for a convex cost of dividend payouts. Our modeling approach relies on $\sigma < 1$ to facilitate comparison with the related literature.

Figure A.9: Role of $\sigma < 1$
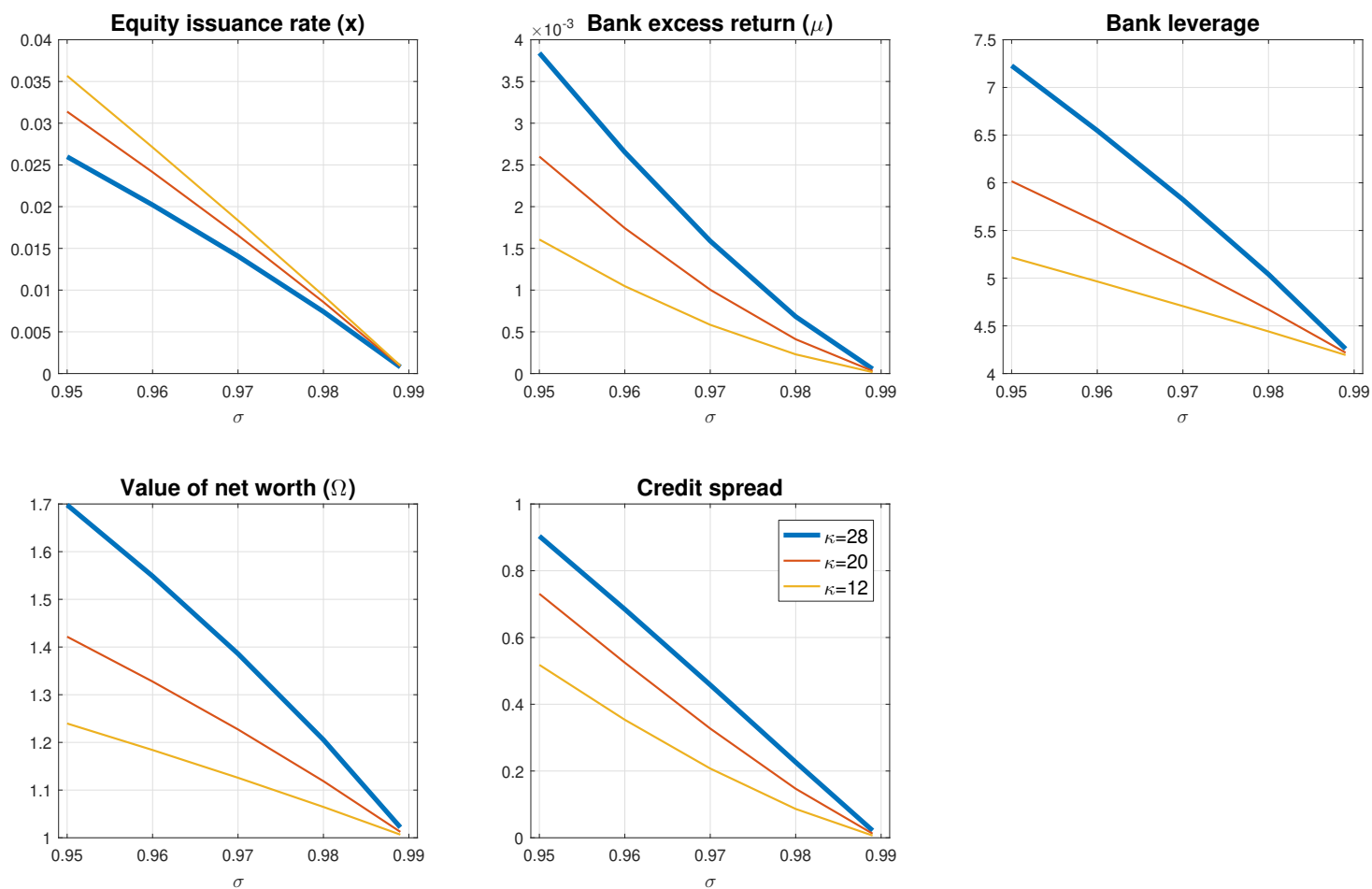


**Note**: steady-state value of selected variables as a function of parameters $\sigma$ and $\kappa$.

# K  Sensitivity Analysis: Long-Run Moments

Table A.2: Comparing Alternative Models and Data: Business Cycle Moments

| | $g^Y$ | $g^C$ | $g^I$ | $\Delta NX$/GDP | $g^N$ | Spread |
|---|---|---|---|---|---|---|
| **Standard Deviation** | | | | | | |
| Baseline Model | 0.92 | 1.02 | 2.56 | 0.48 | 10.53 | 0.95 |
| Baseline Model w/ | | | | | | |
| $\Upsilon = 0$ | 0.71 | 0.69 | 2.35 | 0.44 | 11.25 | 0.98 |
| $\epsilon = 1/2$ | 1.01 | 0.87 | 2.39 | 0.56 | 10.68 | 0.97 |
| Data | 0.91 | 0.99 | 2.41 | 0.63 | 8.24 | 0.74 |
| | [0.66, 1.07] | [0.81, 1.24] | [1.77, 3.12] | [0.30, 0.86] | [6.31, 10.23] | [0.48, 0.94] |
| **Correlation with $g^Y$** | | | | | | |
| Baseline Model | - | 0.98 | 0.62 | -0.06 | 0.71 | -0.54 |
| Baseline Model w/ | | | | | | |
| $\Upsilon = 0$ | - | 0.99 | 0.44 | 0.15 | 0.45 | -0.41 |
| $\epsilon = 1/2$ | - | 0.96 | 0.52 | 0.44 | 0.62 | -0.39 |
| Data | - | 0.69 | 0.65 | -0.11 | 0.19 | -0.55 |
| | - | [0.59, 1.00] | [0.46, 0.77] | [-0.58, 0.24 ] | [-0.00,0.34] | [-0.69,-0.40] |

*Note:* $g^Y$ , $g^C$, $g^I$, $g^N$ denote the growth rates of output, consumption, investment, the banking sector equity, respectively, and $\Delta NX$ denotes the first difference of net exports. The domestic corporate spread is denoted by Spread, and is expressed in annual terms. Except for *NX*, all variables are measured in logs. The data are expressed in units of the GDP deflator. Data moments are calculated as the simple average across all the countries in our sample (Italy, Spain, Germany, France, UK and the US). Square brackets denote the min-max range for each moment across the full sample of countries. Data sources: Haver Analytics, Gilchrist and Mojon (2014), Bank of England, Gilchrist and Zakrajsek (2012), authors' calculations.

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