

# Forward Guidance and Heterogeneous Beliefs

## Online Appendix

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## 1 Additional Evidence

### 1.1 Average expectations did not react much in the US SPF

Figure 1 displays the evolution of the average of individual short term interest rate forecasts 1 quarter, 1 year and 2 years ahead. Three specific subperiods are highlighted: 2008Q4-2011Q2 which corresponds to the time when the US economy reached the ZLB and the Fed conducted its “open-ended” forward guidance; 2011Q3-2012Q3 which corresponds to the “date-based” forward guidance period; and 2012Q4-2013Q2 which corresponds to a “state-based” forward guidance policy.

This figure shows that 1-quarter ahead short-term interest rate forecasts reached levels close to zero in 2009 that is when the US economy hit the ZLB. 1-year and 2-year ahead short-term interest rate forecasts were already low when date-based forward guidance policy started, but they went further down during that period to finally reach levels close to zero and comparable to the 1-quarter ahead forecasts.

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As Figure 2 illustrates, over the same date-based forward guidance period, 1-quarter, 1-year and 2-year ahead consumption growth (resp. inflation) forecasts only slightly decreased (resp. increased).<sup>1</sup> This makes a preliminary fact.

**Fact 0.** *Date-based forward guidance was coincident with a drop in the mean forecasts of the short-term nominal interest rates to historically low (and close to zero) levels up to 2 years, a limited increase in the average forecast of inflation and a limited decrease in the average forecast of consumption growth.*

These patterns are reminiscent of results stressed in previous studies documenting the reaction of macroeconomic expectations to various forward guidance announcements (???): such policy lowered expected future short-term interest rates but the reaction of inflation, output or consumption growth were much smaller and sometimes negative. One reading is that forecasters had a Delphic interpretation of forward guidance: announcements of future low interest rates were interpreted as signalling worse future macroeconomic conditions. We show that this is consistent with agents interpreting differently the same announcement.

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<sup>1</sup>1-year ahead and 2-year ahead forecasts provided here are not directly comparable. The former correspond to an annualized quarter-over-quarter percentage change expected in four quarters. The later correspond to an annual average percentage change expected over the next calendar year.

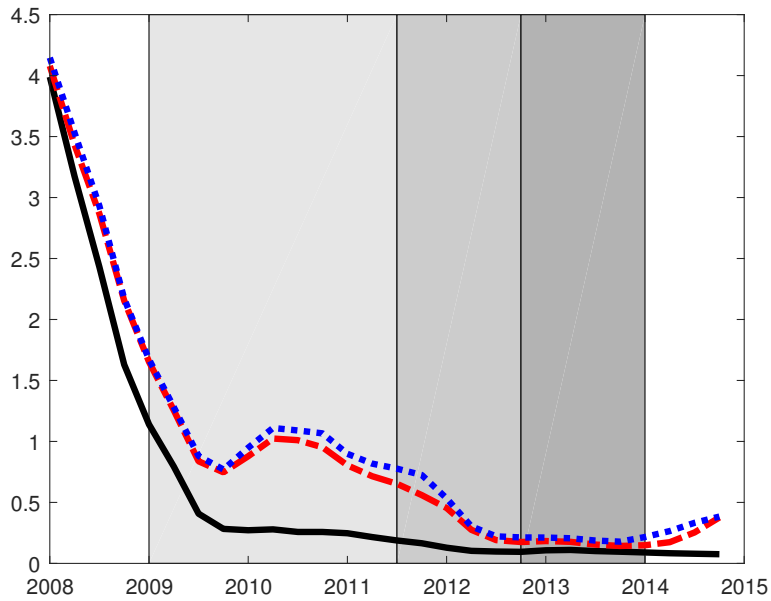
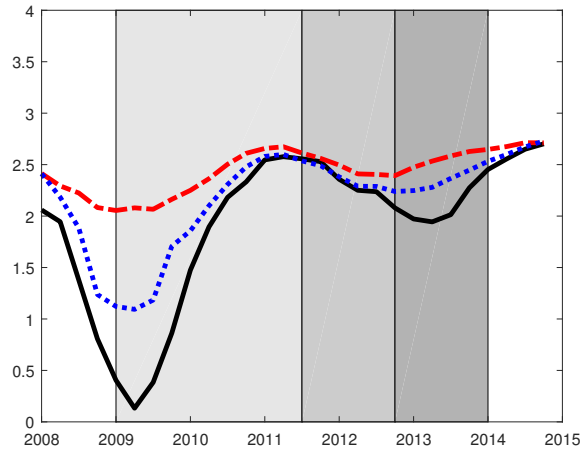


Figure 1: Average of individual short-term interest rate forecasts.

The chart displays the evolution of a moving average over the last 4 quarters of the average of individual forecasts of the 1-quarter (plain line), 1-year (dashed/dotted line), and 2-year (dotted line) ahead individual mean point forecasts for 3-month T-Bill interest rate. 1-year ahead forecasts are 'fixed horizon' forecasts and correspond to the quarterly average (annualized) rate expected in four quarters. 2-year ahead forecasts are 'fixed date' forecasts and correspond to the annual average rate expected over the next calendar year. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "date-based" forward guidance and the "state-contingent" forward guidance.

(a) Consumption



(b) Inflation

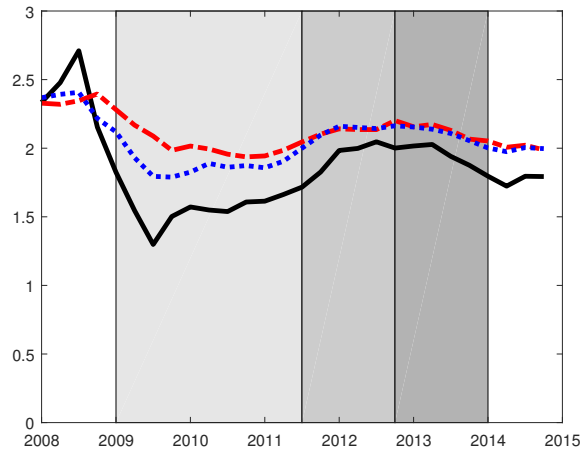


Figure 2: Average of individual consumption growth and inflation forecasts.

The figure shows the evolution of a moving average over the last 4 quarters of the average of individual forecasts of 1-quarter (plain line), 1-year (dashed/dotted line), and 2-year (dotted line) ahead individual mean point forecasts for real consumption growth and CPI inflation. 1-year ahead forecasts are 'fixed horizon' forecasts and correspond to annualized quarter-over-quarter percent changes of the real personal consumption expenditure and the consumption price index level expected in four quarters. 2-year ahead forecasts are 'fixed date' forecasts and correspond to the annual average percent changes in the real personal consumption expenditure and consumption price index expected over the next calendar year. The shaded areas correspond to the periods of the ZLB and "open-date" forward guidance, "date-based" forward guidance and the "state-contingent" forward guidance.

## 1.2 Household survey

We also investigate how expectations of US households evolved when date-based forward guidance was conducted by exploiting the Michigan Survey of Consumers. Although data characteristics prevents us to conduct exactly the same analysis than for the SPF, individual households' expectations observed in Michigan Survey of Consumers feature comparable patterns over the same period and show that the heterogeneity in expectations translated in heterogeneous decisions.

More specifically, we analyse households' expectations about *(i)* the evolution of interest rates over the next 12 months (increase, stay constant, decrease), *(ii)* the evolution of prices over the next 12 months (average inflation rate), *(iii)* whether it is a good time to buy durables (good, neutral, bad) and *(iv)* the expected overall aggregate business conditions over the next 12 months (good, neutral, bad).

Each month, about 500 households are surveyed. The sample is designed to be representative of the US population. 60% of individuals that are first time respondents to the survey. Due to this repeated cross-section structure it is not possible to compute revisions of forecasts between 2 subsequent survey rounds for the whole sample of household surveyed. 40% of households that are interviewed twice but with a 6 months period between the two interviews. In the appendix, we analyse individual revisions based on this subsample. Moreover, several questions asked to households call for qualitative rather than quantitative answers. These data limitations prevent us to conduct an analysis as detailed as the one we can conduct with the SPF. The other way around, the question on current durable good consumption in the Michigan survey as no equivalent in the SPF. As ? emphasize, consumption of durable goods follows total consumption so that answers to this question is a good proxy for the current total consumption decisions.

We start with households' expectations of future interest rates. Figure 3 plots the share of respondents anticipating that interest rates will stay constant over the next 12 months over a 2002-2014 sample. The chart underlines that this share jumped to levels above 50% during the date-based forward guidance period.<sup>2</sup> So the majority of households in the

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<sup>2</sup>This surge in the proportion of households expecting that interest rate will not increase is mainly driven by households expecting interest rates will stay constant which reaches an all-time high (above 50%) during

Michigan Survey of Consumers interpreted forward guidance announcements as indicating that interest rate will not increase (at least) over the next year. This complements the results of ?. They show that, in normal times, some households in the Michigan survey understand monetary policy: they adjust their interest rate expectations in a way that is consistent with a Taylor rule and their views on the macroeconomic outlook. We find that a substantial share of households reports interest rate forecasts consistent with the date-based forward guidance policy implemented at the ZLB.

In a second step, we analyze the heterogeneity of expectations across surveyed households. We split the sample of respondents expecting stable or lower interest rates into three categories: optimists if they expect better aggregate business conditions and have inflation expectations above the average; pessimists if they expect worsening business conditions and have inflation expectations below average; and others. Table 1 reports the average expectations of each of these groups observed in the month following the three date-based forward guidance announcements of August 2011, January 2012 and September 2012. The results reveal that among households who anticipated stable or lower interest rates, the ones who expected higher inflation and better economic conditions also considered that the time was more favorable to purchase durable goods. By contrast, pessimists expect lower inflation and a smaller fraction among them consider that it is time to purchase durable goods.

Note that observing a fraction of optimistic households who declare to consume more when they anticipate higher inflation and better economic conditions does not contradict the results in ? who find that, *on average*, during the ZLB period, households who report higher inflation expectations in the Michigan survey are likely to consume less. Moreover, in the Appendix, we also report results that are very similar to theirs over the date-based forward guidance episode. Namely, we drop the expected inflation criteria in the definition of optimistic households and consider the larger group of agents who expect an improvement in future activity. The results show that this broader class of optimistic households is more likely to purchase durable goods but also expect lower inflation than the average household. These optimists behave like in ?'s model of forward guidance in which such policy increases

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the date-based forward guidance. This contrasts with the 2009 episode where the high share of households foreseeing that interest rate will not increase is mostly due to a large majority of people expecting a drop in future interest rates.

consumption today by generating expectations of a boom in activity tomorrow. Finally while this broader class of optimists by definition accounts for a larger fraction of the sample of households surveyed, it does not represent the whole sample: again optimists coexisted with pessimists during the period of date-based forward guidance.

Table 1: Average of forecasts across groups of households.

	Optimists	Pessimists	Pessimists and others
Averages observed in 2011m9			
Fraction of respondents	5%	50%	95%
Good times for durable	.50	.27	.25
Inflation	6.64	1.77	3.51
Averages observed in 2012m2			
Fraction of respondents	13%	28%	87%
Good times for durable	.55	.30	.36
Inflation	5.50	1.37	3.10
Averages observed in 2012m10			
Fraction of respondents	15%	30%	85%
Good times for durable	.46	.24	.29
Inflation	7.34	1.95	3.37

This table computes the cross-sectional mean for current durable consumption (qualitative answers) and expected inflation over next 12 months (quantitative answers) when forecasters are sorted according to their expected business conditions and nominal interest rate over next 12 months. Pessimists expected lower inflation than the cross-sectional mean and had a negative view of the business/financial conditions over the next 12 months. Others include all households except optimists. All forecasters considered expect constant or decreasing nominal interest rates over the next 12 months.

Overall, during the period of date-based forward guidance, among households expecting interest rates not to increase over the next 12 months, the ones anticipating better economic conditions and higher inflation were more likely to purchase durable goods than the ones anticipating worse economic conditions and lower inflation.

**Evidence with a broader definition of optimists households** Only a few households in the Michigan survey have expectations that are consistent with what New-Keynesian models predict the impact of forward guidance policy should be: they foresee stable or lower interest rates, more inflation, a boom in future activity and therefore want to consume more today. That their number is quite limited is consistent with the evidence in ?. These authors showed using data from the Michigan survey that, on average, when the US economy was at

the ZLB, households who expected higher inflation expectations also considered the time as less favorable to consume.

Figure 2 showed that forward guidance does not require that agents expect higher future inflation (hence lower future real interest rates) to have a positive impact on consumption today. Such policy can be effective if agents only expect a future boom. This suggests to conduct the same analysis than before but with a broader definition of optimistic and pessimistic households that does not depend on their inflation expectations. More specifically we sort households into two categories only: optimists if they expect better future conditions and pessimists otherwise. Table 2 shows the average of macro expectations of these two groups observed at the dates following date-based forward guidance announcements. Three comments can be made. First, two views (an optimistic and a pessimistic ones) about the macroeconomic outlook prevailed within the group of households who foresaw stable or lower interest rates. Second, there is a now substantial number, sometimes a majority, of optimistic households, who have expectations consistent with the effects of forward guidance as emphasized in Figure 2: after the forward guidance announcements they expected better future economic conditions and were likely to consume more today. Third, and again consistent with Figure 2's results aforementioned, households who expected better economic conditions in the future and consumed more after date-based forward guidance, were also expecting lower inflation on average.



Table 2: Average of forecasts across groups of households.

	Optimists	Pessimists
Averages observed in 2011m9		
Fraction of respondents	14%	86%
Good times for durable	.64	.21
Inflation	2.95	3.90
Averages observed in 2012m2		
Fraction of respondents	60%	40%
Good times for durable	.56	.19
Inflation	2.45	3.99
Averages observed in 2012m10		
Fraction of respondents	68%	32%
Good times for durable	.48	.17
Inflation	2.81	4.69

This table computes the cross-sectional mean for current durable consumption (qualitative answers) and expected inflation over next 12 months (quantitative answers) when forecasters are sorted according to their expected business conditions and nominal interest rate over next 12 months. Optimistic forecasters had a positive view of the business/financial conditions over the next 12 months. Pessimists had a negative view of the same business/financial conditions. All forecasters considered expect constant or decreasing nominal interest rates over the next 12 months.

**Evidence using revisions in households' expectations** Some households in the Michigan survey are sampled twice with a 6 month interval. We exploit this panel dimension to control for individuals' fixed effects. Table 3 below shows that optimists households have both higher revision of inflation expectations and willingness to buy durable goods after FG announcements.

Table 3: Average of forecasts revisions across groups of households surveyed twice.

	Optimists	Pessimists
Averages observed in 2011m9		
Fraction of respondents	4%	58%
Good times for durable	.00	-.19
Inflation	-1.75	-2.71
Averages observed in 2012m2		
Fraction of respondents	14%	28%
Good times for durable	.32	.25
Inflation	-1.24	-2.29
Averages observed in 2012m10		
Fraction of respondents	9%	31%
Good times for durable	.11	.03
Inflation	3.25	-1.55

This table computes the cross-sectional mean for revisions in current durable consumption (qualitative answers) and revisions in expected inflation over next 12 months (quantitative answers) for forecasters surveyed twice (with a 6 months interval). Forecasters are sorted according to their expected business conditions and nominal interest rate over next 12 months (at the time of the second survey). Pessimists expected lower inflation than the cross-sectional mean and had a negative view of the business/financial conditions over the next 12 months. Others include all households except optimists. All forecasters considered expect constant or decreasing nominal interest rates over the next 12 months. All forecasters considered expect constant or decreasing nominal interest rates over the next 12 months.

### 1.3 Patterns in various measures of macroeconomic uncertainty

In this paragraph, we investigate whether forward guidance has had effects on another channel than just the first moments namely a reduction in uncertainty. Figure 4 plots three different recent measures of uncertainty between 2002 and 2016: the CBOE financial market volatility index (VIX), the macroeconomic uncertainty measure developed by ? (JLN), the economic policy uncertainty measure developed by ? (BBD). Figure 5 shows two additional measures that are derived from subjective probability distribution observed in the survey of professional forecasters: the probability of a drop in the level of real GDP in 4 quarters (REC, directly observed in the SPF) and the conditional variance of inflation 4 quarters ahead (VIN, derived from individual probabilistic assessments of inflation scenarios in the SPF, see ? for details). A first observation is that, consistent with e.g. ?, macroeconomic uncertainty increased as the economy hit the ZLB and the usual monetary policy stabilisation instrument has been lost. Yet, when the Fed switched to date-based forward guidance, there is no clear common pattern in the three measures of uncertainty. The index by ? remained almost unaffected, while economic policy uncertainty measure and the VIX both peaked around the time of the first announcement. In sum, this evidence is not consistent with a systematic reduction of uncertainty due to date-based forward guidance announcements.<sup>3</sup>

We also checked that the fact that date-based forward guidance is associated with an increase in disagreement about medium-run forecasts of consumption growth and inflation, illustrated in Figure ??, does not primarily result from variations in macroeconomic uncertainty.

We regressed the disagreement about 2-year ahead forecasts of consumption (resp. inflation) on the disagreement about 2-year ahead forecasts of short-term nominal interest rates estimated on a pre-crisis sample, controlling for the disagreement about 1-quarter ahead consumption and inflation forecasts as previously, as well as for four different measures of uncertainty: the JLN measure of macroeconomic uncertainty, the BBD measure of economic policy uncertainty, and the 2 SPF based measures REC and VIN.

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<sup>3</sup>Note, in contrast, that state-contingent forward guidance has had a large negative impact on the economic policy uncertainty measure by ?. As a result, the reduction in uncertainty channel seems to be more relevant for state-contingent than for date-based forward guidance.

Figures 6 and 7 display the residuals from these regressions. They show that the beginning of the date-based forward guidance policy is again a striking outlier: controlling for fundamental uncertainty, disagreement about future inflation should have been significantly lower given how much agents agreed on future short-term interest rates. So, changes in uncertainty are not the main explanation for why the normal time correlation between disagreement about future interest rates and disagreement about future fundamentals disappears at the time of forward guidance.

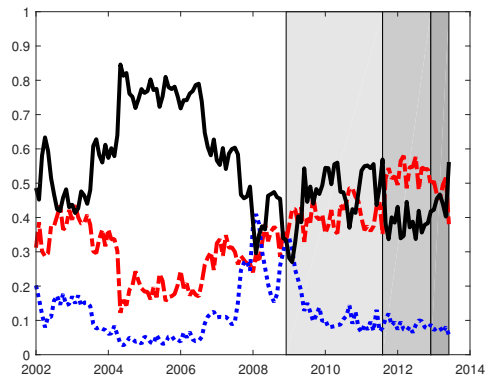


Figure 3: Interest rate expectations in the Michigan survey of households.

The chart displays the evolution of the share of respondents to the survey who thought that over the next 12 months, interest rates will increase (solid line), stay constant (dashed line) or decline (dotted line).

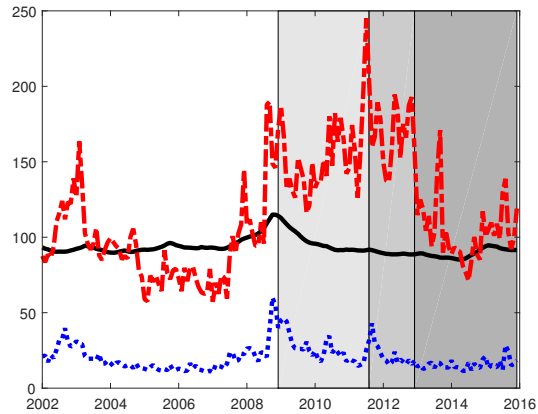


Figure 4: Measures of uncertainty.

The chart displays the evolution of 3 different measures of uncertainty: the CBOE financial market volatility index (VIX, blue line), the macroeconomic uncertainty measure developed by ? (JLN, dark line), the economic policy uncertainty measure developed by ? (BBD, red line).

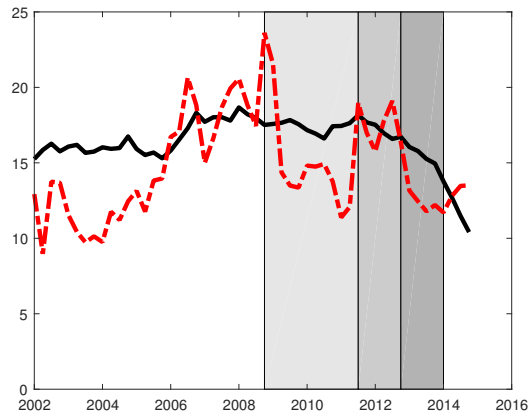
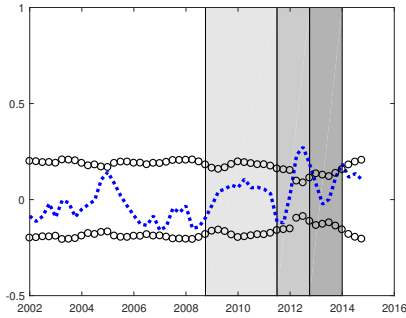


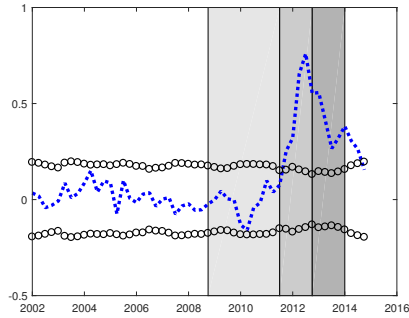
Figure 5: Survey based measures of uncertainty.

The chart displays the evolution of 2 different measures of uncertainty based on survey of professional forecasters: the cross-sectional average of the probability of a recession (REC, dashed line) and the cross-sectional average of the conditional variance of inflation 1-year ahead derived from the individual subjective probability distribution forecasts computed for instance in ? (VIN, plain line).

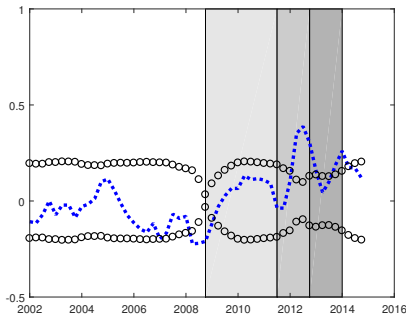
(a) Consumption - 2 years ahead - EPU



(b) Inflation - 2 years ahead - EPU



(c) Consumption - 2 years ahead - JLN



(d) Inflation - 2 years ahead - JLN

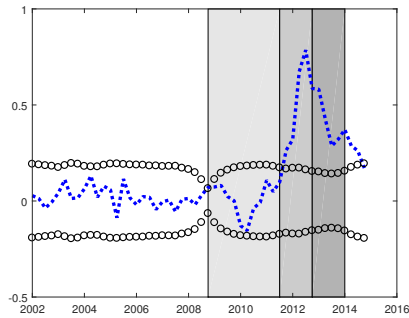
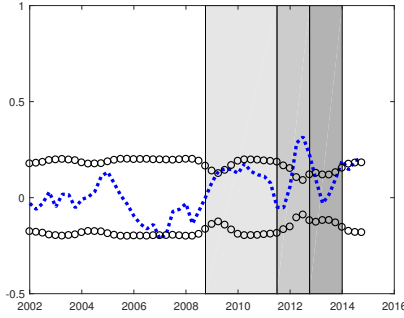


Figure 6: Excess disagreement about future consumption and inflation, controlled by uncertainty.

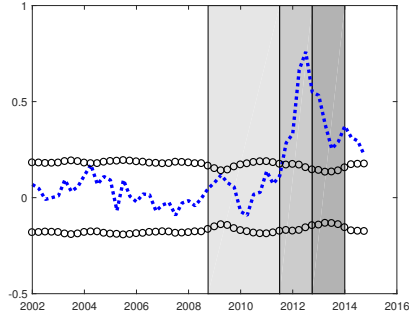
The Figure plots the residuals of a regression of the (log) disagreement on 2-year ahead consumption growth (inflation) forecasts on the (log) disagreement on 2-year ahead short-term interest rate, disagreement on 1-quarter ahead consumption growth (inflation) forecast and an uncertainty measure: either JLN (developed by ?) or EPU (economic policy uncertainty measure developed by ?). The regression is estimated on a pre-crisis sample (1982Q2-2008Q4). Circles give the bands of a 95% confidence interval that take into account autocorrelation and heteroscedasticity of the residuals. The shaded areas correspond to the periods of the ZLB and “open-date” forward guidance, “date-based” forward guidance and the “state-contingent” forward guidance.



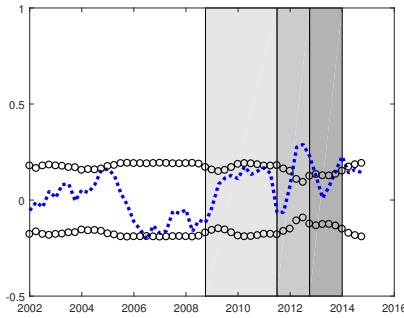
(a) Consumption - 2 years ahead - REC



(b) Inflation - 2 years ahead - REC



(c) Consumption - 2 years ahead - VIN



(d) Inflation - 2 years ahead - VIN

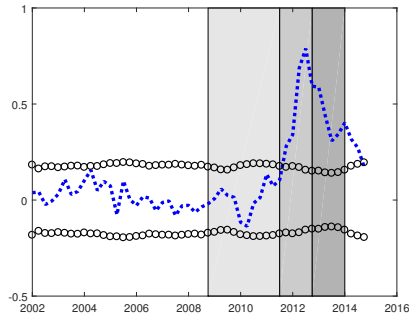


Figure 7: Excess disagreement about future consumption and inflation, controlled by uncertainty.

The Figure plots the residuals of a regression of the (log) disagreement on 2-year ahead consumption growth (inflation) forecasts on the (log) disagreement on 2-year ahead short-term interest rate, disagreement on 1-quarter ahead consumption growth (inflation) forecast and an uncertainty measure derived from SPF data: either REC (the average of individual assessments of a probability of a recession 4 quarters ahead) or VIN (the conditional variance of inflation forecast -year ahead derived from individual subjective probabilistic assessment of inflation scenarios in 1 year; see ? for details). The regression is estimated on a pre-crisis sample (1982Q2-2008Q4). Circles give the bands of a 95% confidence interval that take into account autocorrelation and heteroscedasticity of the residuals. The shaded areas correspond to the periods of the ZLB and “open-date” forward guidance, “date-based” forward guidance and the “state-contingent” forward guidance.

## 2 Model Derivation

In this appendix, we micro-found the linear model that we use in the core of this paper.

### 2.1 Environment

The economy is populated by a continuum of households, firms and the central bank. Time is discrete and indexed by  $t \in \{0, \dots, \infty\}$ .

**Household.** The household family is constituted by a continuum of agents of mass one indexed by  $i \in [0, 1]$ . Each agent decides how much to work, consume and save in order to maximally contribute to the household welfare:

$$U = \int_0^1 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left( \frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{i,t}^{1+\psi}}{1+\psi} \right) di, \quad (1)$$

where  $C_{i,t}$  and  $L_{i,t}$  are respectively consumption and labor supply of agent  $i$  in period  $t$ . The parameter  $\beta \in (0, 1)$  is a discount factor, the parameter  $\gamma > 0$  is the inverse of the inter-temporal elasticity of substitution, and the parameter  $\psi \geq 0$  is the inverse of the Frisch elasticity of labor supply. The variable  $\xi_t$  is a preference shock discussed below.

Each agent manages a portfolio representing a fraction of the household wealth. Between periods  $t$  and  $t + 1$ , agent  $i$  deals with the following flow budget constraint:

$$B_{i,t} = R_{t-1}B_{i,t-1} + W_tL_{i,t} + D_t - P_tC_{i,t} + Z_{i,t}, \quad (2)$$

where  $B_{i,t}$  are bond holdings of the agent between periods  $t - 1$  and  $t$ ,  $R_{t-1}$  is the gross nominal interest rate on bond holdings between periods  $t - 1$  and  $t$ ,  $W_t$  is the nominal wage rate in period  $t$ ,  $D_t$  is the difference between nominal profits received and nominal lump-sum taxes paid, by each agent in period  $t$  (we assume here diffuse ownership), and  $P_t$  is the price of the final good in period  $t$ . The agent can borrow (formally, bond holdings can be negative), but the household is not allowed to run a Ponzi scheme. Finally, the term  $Z_{i,t}$  denotes a nominal intra-household transfer by agent  $i$ .

**Intra-Household risk sharing.** Each period is divided into three stages. In the first stage, current shocks hit and agents observe them. At this stage agents form their beliefs on the state of the world. In the second stage of each period, agents can implement a feasible transfer plan in which each agent  $i \in [0, 1]$  at date  $t$  contributes by an amount  $Z_{i,t}$  and such that:

$$\int_0^1 Z_{i,t} di = 0. \quad (3)$$

only if every agent agrees on it. Without loss of generality,<sup>4</sup> we assume that when no unanimity is reached, then no transfers are made; in such a case each agent owns the wealth resulting from her own portfolio management. Let us therefore introduce the following formal definition.

**Definition 1.** *An implementable transfer plan at time  $t$  is a feasible transfer plan  $\{\hat{Z}_{i,t}\}_{i=0}^1$  such that*

$$E_{t,i}[U_t|\{\hat{Z}_{i,t}\}_{i=0}^1] \geq E_{t,i}[U_t|\{Z_{i,t}\}_{i=0}^1],$$

for each  $i \in [0, 1]$  and each feasible transfer plan  $\{Z_{i,t}\}_{i=0}^1$ .

In the last stage, once intra-household wealth transfers are carried out, each agent decides on her own labor supply and consumption, based on their own individual beliefs and taking other agents' decisions as given. The crucial assumption we are making here is that agents cannot commit on future transfers: each period they decide under discretion. We also assume that the whole mechanism is common knowledge.

**Firms.** Production is implemented in the context of a standard monopolistic competition environment. The final good is produced by competitive firms using the technology:  $Y_t = (\int Y_{j,t}^{(\theta-1)/\theta} dj)^{\theta/(\theta-1)}$ .  $Y_t$  denotes output of the final good and  $Y_{j,t}$  denotes input of intermediate good  $j$ . The parameter  $\theta$  is the elasticity of substitution between intermediate goods. Final good firms have perfect information and fully flexible prices. Profit maximization of firms producing final goods implies the following demand function for intermediate

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<sup>4</sup>To explain why is without loss of generality, we need to introduce a bit more structure. See footnote 5 below.

good  $j$ :

$$Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^\theta Y_t, \quad (4)$$

where  $P_{j,t}$  is the price of intermediate good  $j$  and  $P_t$  is the price of the final good. Furthermore, the zero profit condition of firms producing final goods implies  $P_t = (\int P_{j,t}^{1-\theta} dj)^{1/(1-\theta)}$ . Each intermediate good  $j$  is produced by a monopolist using the linear technology:

$$Y_{j,t} = L_{j,t}, \quad (5)$$

where  $Y_{j,t}$  is output and  $L_{j,t}$  is labor input of this monopolist.

Monopolists producing intermediate goods are subject to a price-setting friction as in ?. Each monopolist can optimize its price with probability  $1 - \chi$  in any given period. Finally, we assume that firms' stocks are held by households in equal shares.

## 2.2 Intra-household risk-sharing

In this subsection, we derive our result on endogenous risk sharing. Disagreement has major consequences for the dynamics of intra-family transfers. At the second stage of each period, agents need to decide on the wealth transfers. In the absence of disagreement, this would optimally result in an even distribution of wealth. Yet, the type of policy will be revealed only at a future date, let us say  $\hat{T}$  (in the simplified version of our model at  $p_{cb} \rightarrow 1$  such date is  $T_o$ ). Before that date, agents have different beliefs on the future course of the economy and so on which transfer plan maximizes family welfare; this prevents transfers from happening before the truth unfolds. In any case, all agents anticipate that they will share their wealth in the future as soon as they have evidence on which they cannot disagree any longer. This implies that no transfer plans can be implemented before date  $\hat{T}$ .

The following proposition states this formally.

**Proposition 1.** *Consider the case of heterogeneous beliefs before  $\hat{T}$ , then the only equilibrium sequence of implementable plans of transfers  $\{\{Z_{i,t}^*\}_0^1\}_{t=0}^\infty$  is the one providing for  $\{Z_{i,t}^*\}_0^1 = 0$  at each  $t \neq \hat{T}$  and  $\{Z_{i,\hat{T}}^*\}_0^1$  such that*

$$U_c(C_{i,t}) = U_c(C_{j,t}) \text{ for } t = \hat{T}, \quad (6)$$

namely, the marginal utility of consumption is equal across agents at the time  $\hat{T}$  when heterogeneity in beliefs vanishes, which implies  $B_{i,t} = B_{j,t}, \forall (i, j) \in [0, 1]^2$  for  $t \geq \hat{T}$ .

*Proof.* The proof is organized in five steps. *First step.* Consider an economy with homogeneous agents at the date  $T_{zlb} + 1$  just after the end of the zero-rate period (no matter how it gets fixed), so that the steady state can be restored. Because of Ricardian equivalence holds, the present value of their life-utility is the same irrespective of the stock of bonds they hold at that time, which is a legacy of the realized states of the world. Therefore, because of the permanent income hypothesis, the level of homogenous individual consumption  $C_{T_{zlb}+1} = \bar{C}$  is pin down only by the forward evolution of the economy that will remain at steady state. *Second step.* At time  $\hat{T}$  (given the nature of our shock  $\hat{T} \leq T_{zlb} + 1$ ), as soon as agents become homogeneous, they would agree on a plan of transfers  $\{Z_{i,t}^*\}_0^1$  such that  $B_{o,t} = B_{p,t}$ , that is, their stock bonds is equalized. In fact, as a consequence, consumption is equalized and so  $U_{C_{o,T}} = U_{C_{p,T}}$ , that is, social welfare is maximized. After that period, irrespective of whether or not the economy is already at steady state (preference shock does not hit), individual consumption will converge to  $C_{T_{zlb}+1} = \bar{C}$  because of what argued in the first step. *Third Step.* Consider now the sequence of transfers  $\{\{Z_{i,t}^*\}_0^1\}_{t=0}^\infty$ , since step two and three are common knowledge, there is only one equilibrium consumption path associated to each state of the world as described in the proposition. *Fourth step.* Different transfers plans, which modify agents' path of consumption, imply, because of the permanent income hypothesis, different level of consumption at steady state. Given that agents anticipate step 2, no plan of this kind can be implemented. In other words, agents anticipate that at time  $t$  they will agree to equalize their wealth so that  $\bar{C}$  will be their steady state consumption that in turn determines the unique consumption path described at step three. *Fifth step.* Among all the transfer plans that can engineer an equalization in the stock of bonds at time  $\hat{T}$  onwards,  $\{Z_{i,t}^*\}_0^1$  is the only one that is implementable because before time  $T$  agents disagree on the actual transfer that will equalize bonds holding at time  $\hat{T}$  as they expect different real interest rates paths, after time  $\hat{T}$  they agree on no transfers.  $\square$

As no transfers are made during the period of the trap, the two types of agents then consume according to their beliefs, managing the share of wealth that they hold at the beginning of the trap.

It is worth to remark that proposition 1 relies on the assumption that households cannot commit to future transfers. As a consequence, agents of each type anticipate that, whatever their financial position, intra-household wealth will be equalized at a future date, when the truth will eventually unfold. Before that date, intra-family transfers, even if they were

implemented,<sup>5</sup> cannot change agents' perceptions of their permanent income, and so cannot affect current consumption-saving choices. In other words, as they expect wealth to be equalized in the future – even though not at the same level – but anticipate different paths of real interest rates, pessimists and optimists select different paths of consumption. If different transfers are implemented, pessimists and optimists both modify their portfolio choices, keeping consumption paths unmodified and anticipating future transfers.

Finally, once we obtain 1, we can log-linearize our model around the unique steady state where the ZLB is not binding.

### 2.3 Aggregate Behavior and the New-Keynesian Phillips Curve

Following standard steps, we can write down the log-linearized versions of optimality conditions as:

$$c_{i,t} = -\frac{1}{\gamma} (E_{i,t}\xi_{t+1} - \xi_t + r_t - E_{i,t}\pi_{t+1}) + E_{i,t}c_{i,t+1}, \quad (7)$$

$$\gamma c_{i,t} + \psi l_{i,t} = w_t - p_t \quad (8)$$

Notice that that  $\xi_t < 0$  in the trap and  $\xi_t = 0$  out of the trap. This means that an exit from the trap, say at time  $t + 1$ , implies  $\xi = E_{i,t}\xi_{t+1} - \xi_t > 0$ . So, the term  $\xi = E_{i,t}\xi_{t+1} - \xi_t$  is positive at the time of reverting to normal times and equals 0 otherwise. As a result, the Euler equation (7) implies that consumption decreases at the beginning of the liquidity trap before it gradually increases during the trap.

**Aggregate behavior.** Assuming that  $\xi$  can be anticipated a period in advance and by solving forward, we obtain that individual consumption equals:

$$c_{i,t} = -\frac{1}{\gamma} E_{i,t} \left[ \sum_{\tau=t}^{\infty} (r_{\tau} - \pi_{\tau+1} + \xi_{\tau+1} - \xi_{\tau}) \right]$$

---

<sup>5</sup> With different opinions about which plan achieves the first best, agents cannot implement any transfer plan. However, this rule has the mere role of selecting a unique feasible plan when agents disagree. That is, another backup rule would not change the results. For example, we could have equally assumed that, when agents disagree, a dictator decides on their transfers. Given that the dictator cannot enforce future transfers (no commitment), agents commonly know that, from some future date onward, they will agree again, and so, their wealth will be equalized. In this case, the dictator's transfers cannot affect the perceived permanent income of an agent, and so cannot change agents consumption-saving plans.

and aggregate consumption equals:

$$c_t = -\frac{1}{\gamma} E_t \left[ \sum_{\tau=t}^{\infty} (r_{\tau} - \pi_{\tau+1})_{\tau+1} + \xi_{\tau+1} - \xi_{\tau} \right]$$

Notice that as long as agents do not disagree on the size of the shock (this is the case as they observe it), but only on the future date on which it will unfold, it enters as a fix wedge in the IS curve. This wedge will disappear only at the optimistic date when agents will discover the truth.

**New-Keynesian Phillips Curve.** The optimal price setting for producer  $j$  is given by:

$$x_{j,t} = (1 - \chi\beta) E_t^j \left[ \sum_{\tau=t}^{\infty} (\chi\beta)^{\tau-t} w_{\tau} \right]$$

as standard in the sticky price literature. Aggregating over producers yields:

$$x_t = (1 - \chi\beta) w_t + \chi\beta \int E_{i,t} x_{i,t+1} di,$$

which is a standard reaction. We obtain the New-keynesian Phillips Curve in the presence of heterogeneous beliefs as follows. By defining  $\Delta_t \equiv \int E_{i,t} x_{i,t+1} di - E_t x_{t+1}$ , we can write  $x_t$  recursively as:

$$x_t = (1 - \chi\beta) w_t + \chi\beta E_t x_{t+1} + \chi\beta \Delta_t$$

At the same time,  $x_t = \frac{p_t - \chi p_{t-1}}{1 - \chi}$  and so, we can write

$$p_t - \chi p_{t-1} = (1 - \chi) (1 - \chi\beta) w_t + \chi\beta E_t (p_{t+1} - \chi p_t) + (1 - \chi) \chi\beta \Delta_t$$

Thus, by noticing that  $\pi_t = p_t - p_{t-1}$ , we obtain:

$$\pi_t = \frac{(1 - \chi) (1 - \chi\beta)}{\alpha} (w_t - p_t) + \beta E_t \pi_{t+1} + (1 - \chi) \beta \Delta_t$$

By definition,  $\Delta_t \equiv \int E_{i,t} x_{i,t+1} di - E_t x_{t+1}$  and  $x_{i,t}$  is a function of current and future wages ( $w_{\tau s}$ ). As a result, we can rewrite  $\Delta_t$  as follows:

$$\Delta_t = (1 - \chi\beta) \sum_{\tau=0}^{\infty} (\chi\beta)^\tau \int E_{i,t} \left( w_{t+\tau+1} - \int E_{i,t+1} [w_{t+\tau+1}] di \right) di$$

which equals 0 in this case, yielding the New Keynesian Phillips Curve

$$\pi_t = \frac{(1 - \chi)(1 - \chi\beta)}{\chi} (w_t - p_t) + \beta E_t \pi_{t+1}, \quad (9)$$

which is identical to the one under homogeneous beliefs. This result crucially relies on the assumption that producers observe all current variables, wage included, and that there is a unique labor market. As a result, it is common knowledge that there will be no aggregate forecast error on the wage neither at present nor at a future date, which makes  $\Delta$  nil.

## 3 Optimal Policy

### 3.1 The welfare function

To determine optimal policy, the central bank's problem is to maximize the expected utility of agents:

$$U = \int_0^1 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left( \frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{i,t}^{1+\psi}}{1+\psi} \right) di, \quad (10)$$

where  $C_{i,t}$  and  $L_{i,t}$  are respectively consumption and labor supply of agent  $i$  in period  $t$ . The parameter  $\beta \in (0, 1)$  is a discount factor, the parameter  $\gamma > 0$  is the inverse of the inter-temporal elasticity of substitution, and the parameter  $\psi \geq 0$  is the inverse of the Frisch elasticity of labor supply. The variable  $\xi_t$  is a preference shock as discussed above.

We show that proceeding similarly to ? (page 87), we can approximate the per period utility of each agent around a steady state as:

$$\mathbb{W}_i \equiv \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t U_{i,t} - U \right] \simeq \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t U_c C \left( c_{i,t} + \frac{1-\gamma}{2} c_{i,t}^2 \right) + U_l L \left( l_{i,t} + \frac{1+\psi}{2} l_{i,t}^2 \right) \right]. \quad (11)$$

For the sake of notational convenience, let us denote by  $i = p$  the pessimist type and by



$i = o$  the optimist type. The next step is to use the fact  $L_t = Y_t \int (P_{j,t}/P_t)^{-\theta} di$  to derive

$$(1 - \alpha)l_{o,t} + \alpha l_{p,t} = (1 - \alpha)c_{o,t} + \alpha c_{p,t} + d_t \quad (12)$$

where the last price dispersion term is derived from as a direct implication of the Calvo assumption (for a proof see ? p.399) as being proportional to the square of inflation  $\pi_t^2$ . Given the first order condition on labor supply, and in particular because of the assumption of homogeneous labor market (8), we have that  $\gamma c_{o,t} + \psi l_{o,t} = \gamma c_{p,t} + \psi l_{p,t}$  that is:

$$l_{p,t} - l_{o,t} = -\frac{\gamma}{\psi} (c_{p,t} - c_{o,t}). \quad (13)$$

Therefore we can rewrite

$$\begin{aligned} l_{o,t} + \alpha(l_{p,t} - l_{o,t}) &= c_{o,t} + \alpha(c_{p,t} - c_{o,t}) + d_t, \\ l_{p,t} + (1 - \alpha)(l_{o,t} - l_{p,t}) &= c_{p,t} + (1 - \alpha)(c_{o,t} - c_{p,t}) + d_t, \end{aligned}$$

or

$$\begin{aligned} l_{o,t} &= c_{o,t} + \alpha \left(1 + \frac{\gamma}{\psi}\right) (c_{p,t} - c_{o,t}) + d_t, \\ l_{p,t} &= c_{p,t} + (1 - \alpha) \left(1 + \frac{\gamma}{\psi}\right) (c_{o,t} - c_{p,t}) + d_t. \end{aligned}$$

In the special case  $\gamma = \psi$  we can show that

$$\alpha l_{p,t}^2 + (1 - \alpha)l_{o,t}^2 = \alpha c_{p,t}^2 + (1 - \alpha)c_{o,t}^2$$

since

$$\alpha (c_{p,t} + 2(1 - \alpha)(c_{o,t} - c_{p,t}))^2 + (1 - \alpha)(c_{o,t} + 2\alpha(c_{p,t} - c_{o,t}))^2 = (1 - \alpha)c_{o,t}^2 + \alpha c_{p,t}^2.$$

Therefore, as in ?, we get

$$\mathbb{W}_i = -\varpi \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left( (1 + \psi)l_{i,t}^2 - (1 - \gamma)c_{i,t}^2 + \theta\pi_t^2 \right) \right] \quad (14)$$

where  $\varpi$  is a positive constant, so that finally social welfare can be approximated by  $\mathbb{W} =$

$\int_0^1 \mathbb{W}_i di$ . In the special case,  $\gamma = \psi$ , (14) becomes

$$\tilde{\mathbb{W}}_i = -\varpi\theta^{-1}\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t(\lambda c_{i,t}^2 + \pi_t^2)\right] \quad (15)$$

with  $\lambda = 2\gamma/\theta$ , which is identical to the case with homogeneous agents.

### 3.2 Proof of Proposition ??

To enlighten the main intuition behind the proof, we firstly only consider a one-period trap that hits at time 0, in the case  $\lambda = 0$ . Let us then denote by  $FG(k) = \sum_{t>0}\beta^t\pi_t^2$  when there is  $k$  of periods of Odyssean forward guidance.  $FG(k)$  is increasing in  $k$  and does not depend on  $\alpha$ . The last two properties are general all the periods after the end of the trap, irrespective of its length and the value  $\lambda$ . The reason is that for  $t > T$  agents will not disagree and anticipate that at time 0. For the sake of notational convenience, let us denote by  $i = p$  the pessimist type and by  $i = o$  the optimist type.

Inflation and consumption at time 0 are given by

$$\begin{aligned}\pi_0 &= (\alpha(\beta\pi_{p,1} + \kappa c_{p,0}) + (1 - \alpha)(\beta\pi_{o,1} + \kappa c_{o,0})), \\ c_0 &= \alpha c_{p,0} + (1 - \alpha)c_{o,0} \\ c_{o,0} &= c_{o,1} - \gamma^{-1}(\rho_l - \pi_{o,1}) \\ c_{p,0} &= c_{p,1} - \gamma^{-1}(\rho_l - \pi_{p,1})\end{aligned}$$

where  $\pi_{i,1}$  is a short notation for the expectation of agent  $i$  about inflation at time 1 and  $\rho_l = -\log R - \xi < 0$ . Bear in mind that  $c_{i,1}$  and  $\pi_{i,1}$  do not depend on  $\alpha$  as agents consistently expect homogeneous beliefs are restored after that date.

Let us investigate the conditions for which for  $k > k'$  we can have  $\pi_0^2(k) + FG(k) \leq \pi_0^2(k') + FG(k')$ , i.e. forward guidance for  $k$  period is not less efficient of a forward guidance for  $k'$  periods. First note that

$$\frac{\partial c_0(k)}{\partial \alpha} = c_{p,0}(k) - c_{o,0}(k) = \gamma^{-1}(\pi_{p,1}(k') - \pi_{o,1}(k')) + c_{p,1}(k') - c_{o,1}(k')$$

as  $\partial c_{p,0}/\partial\alpha = \partial c_{o,0}/\partial\alpha = 0$ , and so

$$\begin{aligned}\frac{\partial c_0(k)}{\partial\alpha} &< \frac{\partial c_0(k')}{\partial\alpha} \\ \frac{\partial\pi_0(k)}{\partial\alpha} &= \beta(\pi_{p,1}(k) - \pi_{o,1}(k)) + \kappa(c_{p,0}(k) - c_{o,0}(k)) < \frac{\partial\pi_0(k')}{\partial\alpha} < 0.\end{aligned}$$

given the facts:

- i)  $\pi_{p,1}(k) < \pi_{p,1}(k') < \pi_{p,1}(0) = 0$ ,  $\pi_{o,1}(k) > \pi_{o,1}(k') > \pi_{o,1}(0)$ ,
- ii)  $c_{p,1}(k) < c_{p,1}(k') < c_{p,1}(0) = 0$ ,  $c_{o,1}(k) > c_{o,1}(k') > c_{o,1}(0)$ .

The derivative of  $\Pi(k, k', \alpha) = \pi_0^2(k) - \pi_0^2(k')$  with respect to  $\alpha$  is:

$$\frac{\partial\Pi(k, k', \alpha)}{\partial\alpha} = 2\left(\pi_0(k)\frac{\partial\pi_0(k)}{\partial\alpha} - \pi_0(k')\frac{\partial\pi_0(k')}{\partial\alpha}\right) \quad (16)$$

whereas,  $\Phi(k, k') = FG(k') - FG(k) < 0$ . By substitution we get:

$$\begin{aligned}\pi_0\frac{\partial\pi_0}{\partial\alpha} &= \alpha\left((\beta + \kappa\gamma^{-1})(\pi_{p,1} - \pi_{o,1}) + \kappa(c_{p,1} - c_{o,1})\right)^2 + \\ &+ \left((\beta + \kappa\gamma^{-1})\pi_{o,1} + \kappa c_{o,1} - \gamma^{-1}\rho_l\right)\left((\beta + \kappa\gamma^{-1})(\pi_{p,1} - \pi_{o,1}) + \kappa(c_{p,1} - c_{o,1})\right)\end{aligned}$$

where the term

$$\left((\beta + \kappa\gamma^{-1})(\pi_{p,1}(k) - \pi_{o,1}(k)) + \kappa(c_{p,1}(k) - c_{o,1}(k))\right),$$

is smaller than

$$\left((\beta + \kappa\gamma^{-1})(\pi_{p,1}(k') - \pi_{o,1}(k')) + \kappa(c_{p,1}(k') - c_{o,1}(k'))\right),$$

for the facts i) and ii) above. As a result, when  $\alpha = 0$ , the derivative  $\partial\Pi(k, k')/\partial\alpha$  is negative. In addition,  $\partial\Pi(k, k')/\partial\alpha$  is a linear and increasing function of  $\alpha$ .

Therefore, let us consider a situation in which  $\Pi(k, k', \alpha) > \Phi(k, k')$  - i.e. forward guidance for  $k'$  is preferred to  $k$ , with  $k > k'$ , in the absence of pessimists. As  $\alpha$  increases in the range  $(0, 1)$ , the inequality can switch sign either never or twice, given that by construction  $\Pi(k, k', 1) > 0$  (all agents are Delphic). In particular, the upper threshold  $\bar{\alpha}$  is such that  $\Pi(1, 0, \bar{\alpha}) = \Phi(1, 0)$ .

Let us go back now to the case  $\lambda > 0$ . In this case the relevant inequality becomes

$$(1 - \alpha) (c_{o,0}^2(k) - c_{o,0}^2(k')) + \alpha (c_{p,0}^2(k) - c_{p,0}^2(k')) + \Pi(k, k', \alpha) \leq \hat{\Phi}(k, k')$$

where  $\hat{\Phi}(k, k')$ , which preserves the properties of  $\Phi(k, k')$ , has been extended accordingly. As before with  $k > k'$ , we have facts i) and ii). To show that the additional term

$$c_{o,0}^2(k) - c_{o,0}^2(k') + \alpha (c_{p,0}^2(k) - c_{o,0}^2(k)) - \alpha (c_{p,0}^2(k') - c_{o,0}^2(k'))$$

is also increasing in  $\alpha$ , notice that  $0 > c_{p,0}(k') > c_{p,0}(k)$  and  $c_{o,0}(k) > c_{o,0}(k') > 0$  implies

$$c_{o,0}(k) > c_{p,0}(k') + c_{o,0}(k')$$

so that

$$(c_{p,0}(k) + c_{o,0}(k)) (c_{p,0}(k) - c_{o,0}(k)) > (c_{p,0}(k') + c_{o,0}(k')) (c_{p,0}(k') - c_{o,0}(k'))$$

can be easily shown given that  $c_{p,0}(k) - c_{o,0}(k) > c_{p,0}(k') - c_{o,0}(k')$  from facts ii). Nevertheless, the additional term is positive at  $\alpha = 0$ . This implies that whereas all the qualitative features of our analysis equally hold considering  $\lambda > 0$ , a longer forward guidance are *ceteris paribus* more efficient at low  $\alpha$ .

Let us look at how the reasoning can be extended to multiple periods in the liquidity trap. Without loss of generality, let us go back to the simple case  $\lambda = 0$ . We add a period  $t = -1$  that takes place just before period 0, then the reasoning can be extended recursively. We have then to compare:

$$1/\beta\pi_{-1}^2(k) - 1/\beta\pi_{-1}^2(k') + \Pi(k, k', \alpha) \leq \Phi(k, k').$$

Notice that the additional term is typically positive, so *ceteris paribus*, with a longer trap a longer forward is needed for low  $\alpha$ . The derivative with respect to  $\alpha$  of the additional terms  $\Pi_{-1}(k, k', \alpha)$  is:

$$\frac{\partial \Pi_{-1}(k, k', \alpha)}{\partial \alpha} = \beta^{-1} \left( 2\pi_{-1}(k) \frac{\partial \pi_{-1}(k)}{\partial \alpha} - 2\pi_{-1}(k') \frac{\partial \pi_{-1}(k')}{\partial \alpha} \right),$$

which has the same structure than (16) and can be expressed similarly as a linear combination of future actual aggregate consumption and inflation. In particular, we can show

$$\begin{aligned}\frac{\partial c_{-1}(k)}{\partial \alpha} &= \frac{\partial c_0(k)}{\partial \alpha} + \gamma^{-1} \frac{\partial \pi_0(k)}{\partial \alpha} < \frac{\partial c_{-1}(k')}{\partial \alpha} < 0 \\ \frac{\partial \pi_{-1}(k)}{\partial \alpha} &= \kappa \frac{\partial c_{-1}(k)}{\partial \alpha} + \beta \frac{\partial \pi_0(k)}{\partial \alpha} < \frac{\partial \pi_{-1}(k')}{\partial \alpha} < 0\end{aligned}$$

using previous relations. Therefore,  $\partial \pi_{-1}^2(k)/\partial \alpha$  is a linear downward sloping function of  $\alpha$ . Given this result, we can then extend recursively the analysis to an arbitrarily number of periods.