ONLINE APPENDIX

The Cyclical Behavior of Unemployment and Wages under Information Frictions

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A Proofs

A.1 Proof of Lemma 1

If all agents in the economy have complete and full information, the following strategy profiles constitute the unique sub-game perfect Nash equilibrium of the wage determination game:

- *Worker: To accept only wage offers greater than or equal to x^* (first stage), and to demand a wage equal to y^* (second stage).*
- *Firm: To offer x^* (first stage) and to accept only wage demands that are less than or equal to y^* (second stage).*

where x^* and y^* are such that $\overrightarrow{J}_j(x^*) = (1-\vartheta)\cdot S_j$ and $\overrightarrow{J}_j(y^*) = 0$.

Proof. I begin at the third stage of the game (i.e., when the worker makes an offer). At this stage, the firm will accept any wage demand y as long as $\overrightarrow{J}_j(y) \geq 0$. Hence, the worker will demand a wage y^* such that $\overrightarrow{J}_j(y^*) = 0$ and she keeps all the match surplus. Thus, at the second stage (i.e., when the worker has to accept or reject the firm's offer), the worker knows that if she rejects this offer, her expected payoff at the third stage will be $\vartheta \cdot S_j$. Therefore, she will only accept wage offers that are greater than or equal to x^* where $\overrightarrow{W}_j(x^*) - U = \vartheta \cdot S_j$. Finally, at the first stage of the game (i.e., when the firm makes an initial offer), the firm anticipates a payoff of zero if it

[1] The views expressed in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of anyone else associated with the Federal Reserve System.

makes an offer less than x^* and a payoff of $\overrightarrow{J}_j(x)$ if $x \geq x^*$. Hence, the firm offers exactly x^* to the worker, and she accepts it. $\square$

A.2 Proof of Lemma 2

Suppose that agents are information constrained as described in section 2.3. If there is an equilibrium in which a firm's strategy is to reveal the aggregate state of the economy, the best strategy for the firm is the same strategy described in Lemma 1

Proof. As we are considering the equilibrium of the game, if firms are following a revealing strategy, workers know it and behave rationally. As a consequence, workers can perfectly infer the current state of the economy based on the firm's wage offer.

Hence, a worker knows that she will receive, in expectation, $\vartheta \cdot S_j$ if she rejects a firm's offer. Therefore, the optimal strategy for workers is the following:

- To infer the current level of the aggregate productivity based on firm's offer x: $a = x^{-1}(a)$
- To accept only wage offers greater than or equal to x^* where:

$$\begin{aligned} \overrightarrow{W}_j(x^*) - U &= \vartheta \cdot S_j \\ \overrightarrow{J}_j(x^*) &= 0 \end{aligned}$$

- To demand a wage equal to y^* if she has the chance such that:

$$\overrightarrow{W}_j(y^*) - U = S_j$$

Now, given the workers' strategy, the firm anticipates a payoff of zero if it makes an offer less than x^* and a payoff of $\overrightarrow{J}_j(x) - U$ if $x \geq x^*$. Given that $\overrightarrow{J}_j(x)$ is strictly decreasing in x, the optimal strategy for firms, *assuming that they follow a revealing strategy* is the following:

- To offer x^*.
- To accept only wage demands that are less than or equal to y^*.

As a consequence, if there exists an equilibrium in which firms reveal the true state of the economy, in equilibrium firms offer exactly x^* and workers will accept it. In other words, workers rationally believe that if a firm extends a wage offer x, it has to be the case that $x = x^*$. $\square$

A.3 Proof of Lemma 3

If agents in the economy are information constrained as described in section 2.3, then in equilibrium, firms do not follow a strategy in which they perfectly reveal the true state of the economy.

Proof. Suppose not. By Lemma A.2, if there is an equilibrium in which firms reveal the true state of the economy, firms always offer $x = x^*$ and workers accept all wage offers (x) because they rationally believe that x is always equal to x^*. However, in order for these strategies to be an equilibrium, firms cannot have incentives to deviate.

Suppose that firms deviate to a strategy in which they offer $\tilde{x} = 0.5x^*$. Workers will accept this offer because they believe $\tilde{x} = x^*$, and firms will be better off because $J_j(\tilde{x}) > J_j(x^*)$. Therefore, there is not an equilibrium in which firms reveal the true state of the economy. $\square$

A.4 Proof of Lemma 4

If agents in the economy are information constrained as described in section 2.3, the following strategy profiles constitute a Perfect Bayesian Nash equilibrium that satisfies the Intuitive Criterion:

- *Worker: To accept only wage offers greater than or equal to x^{**} (first stage), and to demand a wage equal to y^{**} (second stage).*
- *Firm: To offer x^{**} (first stage), and to accept only wage demands that are less than or equal to $\tilde{y}^{**}$.*

where x^{**} and y^{**} are such that $E_{\mathcal{I}_h}\left[\overrightarrow{J}_j(x^{**})\right] = (1-\vartheta)E_{\mathcal{I}_h}\left[S_j\right]$ and $E_{\mathcal{I}_h}\left[\overrightarrow{J}_j(y^{**})\right] = 0$.

Proof. I begin at the third stage of the game (i.e., when the worker gets to make an offer). At this stage, the firm will accept any wage demand y as long as its expected value is greater than or equal to zero. Given the firm's strategy, the firm's offer does not reveal its information. Therefore, the worker will demand a wage y^{**} such that, given her information set, the firm's value is zero. Thus, at the second stage (i.e., when the worker has to accept or reject the firm's offer), the worker knows that if she rejects this offer, her expected payoff at the third stage will be $\vartheta \cdot E_{\mathcal{I}_h}[S_j]$. Therefore, she will only accept wage offers that are greater than or equal to x^{**}. Finally, at the first stage of the game (i.e., when the firm makes an offer), the firm anticipates a payoff of zero if it makes an offer less than x^{**} and a payoff of $\overrightarrow{J}_j(x) \geq 0$ if $x \geq x^{**}$. Hence, the firm offers exactly x^{**} to the worker and she accepts it.

To prove that this equilibrium satisfies the intuitive criterion, define Θ as the set of all possible realizations of $\{a, n\}$. Then, for a given information set $\mathcal{I}_h$, define $\hat{\Theta}(w) \subseteq \Theta$ as the set of pairs $\{a, n\}$ for which a wage offer w is not equilibrium dominated for firm j. Hence:

$$\hat{\Theta}(w > x^{**}) = \emptyset \tag{17}$$

$$\hat{\Theta}(w < x^{**}) = \Theta \tag{18}$$

Any wage offer above the equilibrium wage x^{**} is always equilibrium dominated. Given that a worker is willing to work for a wage x^{**}, offering a higher wage will only reduce the firm's profits.

However, wage offers below the equilibrium wage are not equilibrium dominated for any realization of $\{a, n\}$, because firm's profits will increase if a worker is willing to work for a lower wage. As a consequence, if a worker receives a wage offer below the equilibrium wage, she cannot extract more information from that offer. Regardless of the realization of a and n, offering a lower wage could always be a profitable deviation for firms. Formally:

$$Pr(a = x|\mathcal{I}_h, w < x^{**}) = Pr(a = x|\mathcal{I}_h) \quad \forall \quad x \in \Theta \tag{19}$$

$$E[a|\mathcal{I}_h, w < x^{**}] = E[a|\mathcal{I}_h] \tag{20}$$

Conditional on receiving a wage offer below the equilibrium wage, the worker's expectations do not change.

□

B Computation

To compute the solution to this model numerically, it is important to find and determine a law of motion for the economy, based on which the household forms expectations and makes decisions. This task may not be simple for a large vector Ω, given a distribution of firms and workers. Another challenge is that the vector of state variables should incorporate a set of variables that capture agents' beliefs, which could include prior beliefs about Ω or lags of the vector of state variables. This could not only increase significantly the size of the vector of state variables, but also may require the computation of higher order expectations (expectations of agents' expectations).

Hence, I propose a procedure that combines the solution method for heterogeneous agent models developed by Reiter (2009) and the Kalman filter. In particular, the law of motion for the aggregate economy will be linear (Reiter method), which will allow me to form expectations using the Kalman filter. I show that I can keep track of agents' expectations by keeping track of the last $\mathcal{T}$ realizations of the aggregate shocks in the economy, where $\mathcal{T}$ is a "large" integer.[2] Hence, *to solve the model with noisy signals, you only need to include the last $\mathcal{T}$ realizations of the aggregate shocks as state variables*.[3]

in more details, the Reiter method solves heterogeneous agent models by taking a first-order approximation of the model around the deterministic steady state of the economy.[4] Assume that the following system of equations describes the equilibrium of the economy:

$$f(\Omega, \Omega', \Upsilon, \Upsilon', \mathbb{E}) = 0 \tag{21}$$

where Υ is the vector of endogenous variables of the economy and $\mathbb{E}$ is the vector of exogenous shocks. The Reiter method then finds the solution in three steps:

1. A finite representation of the economy is provided by discretizing the distribution of agents.

2. The deterministic steady state of the economy is found by imposing $\mathbb{E} = 0$ and

[2] $\mathcal{T}$ is large enough if $\tilde{\mathcal{T}} >> \mathcal{T}$ generates almost the same results. In my calibration, I find that $\mathcal{T}$=100 was more than enough.

[3] Notice that vector Ω in the equilibrium definition in section 2.6 included the last $\mathcal{T}$ realization of the aggregate shocks.

[4] For a detailed application of the Reiter method, see Costain and Nakov (2011).

finding the solution to:

$$f^* = f(\Omega^*, \Omega^*, \Upsilon^*, \Upsilon^*, 0) = 0 \tag{22}$$

3. The model is linearized numerically around the steady state, which yields the system of linear equations:

$$f_1^*(\Omega - \Omega^*) + f_2^*(\Omega' - \Omega^*) + f_3^*(\Upsilon - \Upsilon^*) + f_4^*(\Upsilon' - \Upsilon^*) + f_5^*\mathbb{E} = 0 \tag{23}$$

where f_i^* is the partial derivative of (22) with respect to its *i-th* argument. This system is solved using a standard method such as Sims (2002) or Klein (2000).

Hence, the Reiter method induces a law of motion for the economy of the form:

$$\Omega' = \mathbb{F}\Omega + \mathbb{E} \tag{24}$$

$$\Upsilon = \mathbb{G}\Omega \tag{25}$$

where $\mathbb{F}$ and $\mathbb{G}$ are matrices of coefficients. Therefore, the law of motion for the economy is described by $\lambda = \{\mathbb{F}, \mathbb{G}\}$. The challenge for a model with information frictions comes from the fact that the law of motion λ is derived from a perceived law of motion λ^h, which in equilibrium has to be equal to the actual law of motion λ.

I exploit the linearity of the Reiter method and proceed as follows:[5]

1. Define a tolerance level.

2. Guess a linear law of motion for the economy $\lambda^{h\{1\}} = \{\mathbb{F}^{h\{1\}}, \mathbb{G}^{h\{1\}}\}$. A good initial guess may be the law of motion of the model under full information.

3. Let the household form expectations based on this guess and the Kalman filter, which is explained in B.1.

4. Find the solution of the model using the Reiter method, which is given by a new law of motion $\lambda^{\{1\}} = \{\mathbb{F}^{\{1\}}, \mathbb{G}^{\{1\}}\}$.

[5]The linearity of the model makes the model tractable as I can compute expectations based on a linear filter. Otherwise, I would need to use nonlinear filters (such as the particle filter), which would substantially increase the complexity of the problem for a large vector Ω.

5. If the maximum difference between $\lambda^{h\{1\}}$ and $\lambda^{\{1\}}$ is less than the predetermined tolerance level, stop and conclude that $\lambda^{h\{1\}} = \lambda$. Otherwise, update the household's perceived law of motion as follows:

$$\lambda^{h\{n+1\}} = d \cdot \lambda^{h\{n\}} + (1-d) \cdot \lambda^{\{n\}}; \qquad 0 < d < 1 \tag{26}$$

   where d is a fraction that determines how smoothly the guess is updated.

6. Go back to step 3.

B.1 Computing Expectations

In this subsection, I show how to compute workers expectations given a vector of state variables $\Omega = \{k, \{h_j\}_{j=0}^{1}, a^{\mathcal{T}}, n^{\mathcal{T}}\}$, and a linear law of motion for the economy as in (24) and (25). Reiter (2009) shows how to find the aggregate law of motion for the economy for heterogeneous agent models. In this subsection, I focus on how to compute workers' expectations, which is the novel part of my paper. In particular, *I show that we only need to keep track of the last $\mathcal{T}$ realization of the exogenous state variables in order to compute expectations. Therefore, we do not need to include as a state variable agents' beliefs or the realization of vector Ω, $\mathcal{T}$ periods ago.* This represents a significant gain in efficiency because the dimensionality of the problem does not significantly increase. Solving numerically this model, I found that setting $\mathcal{T} = 100$ was more than enough, meaning that a value of $\tilde{\mathcal{T}} >> \mathcal{T}$ did not represent any significant difference. However, the optimal value for $\mathcal{T}$ is a function of how informative the signal is. For example, when ρ_a and ρ_n are close, it becomes more difficult to distinguish between TFP and noise shocks. Agents' expectations take longer to converge to the true values, and a larger value of $\mathcal{T}$ is needed.

This note is based on the Kalman filter and follows the notation of Hamilton (1994). I start with some definitions. Then, I show how to compute expectations regarding the vector of state variables. I conclude by computing expectations about endogenous variables and forecasting economic conditions.

B.1.1 Preliminary Definitions

Define $e_t = [a_t \quad n_t]'$ as the vector of exogenous state variables, which evolves according to:

$$e_{t+1} = \mathcal{F} e_t + \mathcal{V}_t \tag{27}$$

$$\mathcal{F} = \begin{bmatrix} \rho_a & 0 \\ 0 & \rho_n \end{bmatrix} \tag{28}$$

$$E\left[\mathcal{V}_t \mathcal{V}_t'\right] = \mathcal{Q} \tag{29}$$

$$= \begin{bmatrix} \varsigma_a^2 & 0 \\ 0 & \varsigma_n^2 \end{bmatrix} \tag{30}$$

The vector of states variables Ω can be partitioned as $\Omega_t = \begin{bmatrix} \tilde{\Omega}_t & e_t \end{bmatrix}'$ where $\tilde{\Omega}$ is the vector of endogenous predetermined state variables. Then, the dynamics for Ω and Υ (vector of non-predetermined variables) are given by:

$$\Omega_{t+1} = \mathbb{F}\Omega_t + \mathbb{E}_t \tag{31}$$

$$\Upsilon_t = \mathbb{J}\Omega_t \tag{32}$$

$$\mathbb{E}_t = [\varnothing \quad \mathcal{V}_t] \tag{33}$$

$$\mathbb{F} = \begin{bmatrix} \mathbb{F}_{\tilde{\Omega}} & \mathbb{F}_e \\ \varnothing & \mathcal{F} \end{bmatrix} \tag{34}$$

$$E\left[\mathbb{E}_t \mathbb{E}_t'\right] = \mathcal{R} \tag{35}$$

$$= \begin{bmatrix} \varnothing & \varnothing \\ \varnothing & \mathcal{Q} \end{bmatrix} \tag{36}$$

Hence, assuming that vector Ω was perfectly observed at time $t - \mathcal{T}$, we only need to form expectations about e to infer agents' beliefs regarding other variables in the economy.

B.1.2 Computing Expectations about e

Forming beliefs about e is a classic signal extraction problem. In this section, I show that the expectation of e_t conditional on information available at time x, which is denoted by $e_{t|x}$ can be expressed as a linear combination of the last $\mathcal{T} - 1$ shocks.

In this case, the signal is given by $\hat{a} = a + n$, and the evolution of conditional expectations ($e_{t|x}$ is given by:

$$
\begin{aligned}
a_t &= H' e_t && (37)\\
e_{t|t} &= \mathcal{F} e_{t|t-1} + B_t \left(a_t - H' \mathcal{F} e_{t|t-1} \right) && (38)
\end{aligned}
$$

where matrices H and B_t:

$$
\begin{aligned}
H &= \begin{bmatrix} 1 & 1 \end{bmatrix}' && (39)\\
B_t &= P_{t|t-1} H \left(H' P_{t|t-1} H \right)^{-1} && (40)\\
P_{t|t-1} &= \mathcal{F} P_{t|t} \mathcal{F}' + \mathcal{Q} && (41)\\
P_{t|t} &= P_{t|t-1} - P_{t|t-1} H \left(H' P_{t|t-1} H \right)^{-1} P_{t|t-1} && (42)\\
P_{t-\mathcal{T}|t-\mathcal{T}} &= \varnothing && (43)
\end{aligned}
$$

It can be verified that:

$$
\begin{aligned}
e_{t|t} &= e_t + \sum_{j=0}^{\mathcal{T}-1} C_{t-j} \mathcal{V}_{t-j} && (44)\\
C_t &= \left(B_t H' - I \right) && (45)\\
C_{t-j} &= -C_{t-j+1} \mathcal{F} \left(B_{t-j} H' - I \right) \quad 0 < j <= \mathcal{T} && (46)
\end{aligned}
$$

B.1.3 Computing Expectations about $\tilde{\Omega}$

We can rewrite the law of motion for $\tilde{\Omega}_t$ as follows:

$$
\begin{aligned}
\tilde{\Omega}_t &= \mathbb{F}_{\tilde{\Omega}} \tilde{\Omega}_{t-1} + \mathbb{F}_e e_{t-1} && (47)\\
&= \mathbb{F}_{\tilde{\Omega}}^{\mathcal{T}} \tilde{\Omega}_{t-\mathcal{T}} + \mathbb{F}_{\tilde{\Omega}}^{\mathcal{T}-1} \mathbb{F}_e e_{t-\mathcal{T}} + \sum_{j=0}^{\mathcal{T}-2} \mathbb{F}_{\tilde{\Omega}}^{j} \mathbb{F}_e e_{t-1-j} && (48)
\end{aligned}
$$

Given that workers perfectly know $\tilde{\Omega}_{t-\mathcal{T}}$ and $e_{t-\mathcal{T}}$, the expected value of $\tilde{\Omega}$ given information available at time t is given by:

$$
\tilde{\Omega}_{t|t} = \mathbb{F}_{\tilde{\Omega}}^{\mathcal{T}} \tilde{\Omega}_{t-\mathcal{T}} + \mathbb{F}_{\tilde{\Omega}}^{\mathcal{T}-1} \mathbb{F}_e e_{t-\mathcal{T}} + \sum_{j=0}^{\mathcal{T}-2} \mathbb{F}_{\tilde{\Omega}}^{j} \mathbb{F}_e e_{t-1-j|t-1-j} \qquad (49)
$$

Combining, (48), (49), and (44), we get:

$$\tilde{\Omega}_{t|t} = \tilde{\Omega}_t + \sum_{j=0}^{\mathcal{T}-2} \mathbb{F}_{\tilde{\Omega}}^{j} \mathbb{F}_e \tilde{\mathcal{V}}^{t-1-j} \tag{50}$$

$$\tilde{\mathcal{V}}^{t-1-j} = \sum_{i=0}^{\mathcal{T}} C_{t-1-j-i} \mathcal{V}_{t-1-j-i} \tag{51}$$

According to (44) and (51), I only need to keep track of the last $\mathcal{T}$ realization of e instead of keeping track of the whole vector of expectations $\tilde{\Omega}_{t|t}$ and $e_{t|t}$ to compute workers expectations.[6]

B.1.4 Expectations about Υ and Forecast

It is now straightforward to define the expectations of Υ given information available at time t.

$$\Upsilon_{t|t} = \mathbb{G}\Omega_{t|t} \tag{52}$$

$$\Omega_{t|t} = \begin{bmatrix} \tilde{\Omega}_{t|t} & e_{t|t} \end{bmatrix}' \tag{53}$$

Hence, partitioning matrix $\mathbb{G} = \begin{bmatrix} \mathbb{G}_{\tilde{\Omega}} & \mathbb{G}_e \end{bmatrix}$, the forecast f periods ahead is given by:

$$\begin{bmatrix} \tilde{\Omega}_{t+f|t} \\ e_{t+f|t} \\ \Upsilon_{t+f|t} \end{bmatrix} = \begin{bmatrix} \mathbb{F}_{\tilde{\Omega}} & \mathbb{F}_e & \varnothing \\ \varnothing & \mathcal{F} & \varnothing \\ \mathbb{G}_{\tilde{\Omega}} & \mathbb{G}_e & \varnothing \end{bmatrix}^f \begin{bmatrix} \tilde{\Omega}_{t|t} \\ e_{t|t} \\ \Upsilon_{t|t} \end{bmatrix} \tag{54}$$

[6] Notice that you can also keep track of $\mathcal{V}$ directly.

C Data Details

I assess the model's predictions using quarterly data for the United States for the period 1979 to 2015. I present business cycle statistics for the quarterly time series (seasonally adjusted) of unemployment, vacancies, output, consumption, investment, aggregate TFP, and real wages (deflated by PCE) for new employees, job changers, and all workers. I take the quarterly average of series that are available monthly. Following Shimer (2005), all variables are HP-filtered in logs with a smoothing parameter of 10^5.[7]

Unemployment is the total number of unemployed people from the CPS. Vacancies is an index constructed based on the composite help-wanted index computed by Barnichon (2010) and the series of Job openings from the Job Openings and Labor Turnover Survey (JOLTS).[8] Output is real output in the nonfarm business sector. Aggregate productivity is measured as the Solow residual, which is available and updated on the Federal Reserve Bank of San Francisco's website.[9] Consumption consists of non-durable goods and services.[10] Finally, investment is real gross private domestic investment. I include investment as a variable of interest because the effect of the information friction on investment plays an important role in my model.

Given the debate about the cyclicality of wages, I use the CPS, CPS-MORG, and IPUMS-CPS (Flood et al., 2015) microdata to construct the average hourly wage for three group of workers: All workers, new employees, and job changers. In order to compute these wages, I follow Muller (2012) and Haefke et al. (2013) who also used the CPS microdata to construct similar series. The series presented in this paper are the coefficients of time fixed effects in Mincer equations controlling for education, a fourth order polynomial in experience, gender, race, marital status, state, 10 occupation

[7] In general my results are not very sensitive to this parameter.

[8] The vacancies index is equal to the Barnichon's index until the last quarter 2020. From 2001, my index is equal to Job Openings from JOLTS normalized such that its value in 2001Q1 is equal to the Barnichon's index.

[9] In the business cycle tables, I use the Solow residual (cumulative sum of dtfp series devided by 400 and starting at 1 in the first quarter of 1947). For the wage elasticities, I use the utilization adjusted series for TFP (cumulative sum of dtfp_util divided by 400 and starting at 1 in the first quarter of 1947).

[10] Consumption is real personal consumption expedition on nondurable goods and services. To construct the consumption series for nondurable goods, I use data from the 1990, 2000, and 2015 vintages. As not all vintages are available for all quarters of my sample, I begin with the level of the 1990 vintage in 1947 Q1 and then project forward by applying the one-year quarter-on-quarter growth rate of the most recent vintage year available. I construct a series for consumption of services in a similar way.

dummies and 14 industry dummies. Since 1994, the CPS has asked individuals whether they still work at the same job as in the previous month, making it possible to identify job changers. However, it is not possible to identify job-to-job transitions prior to that year. Hence, the sample period for the average wage for job changers is 1994 to 2015. Figure 6 plots these wage series. Figure 6 suggests that workers who were previously unemployed usually get employed at low-paying jobs and move up the job ladder. Notice that these differences in wages cannot be explained by education, experience, gender, race, marital status, state, occupation, or industry, as those variables were included in the Mincer equation.

C.1 Data sources:

Unemployment: U.S. Bureau of Labor Statistics, Unemployment Level [UNEMPLOY], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/UNEMPLOY, August 10, 2016.

Vacancies: Barnichon, Regis. 2010. "Building a composite Help-Wanted Index", retrieved from
https://sites.google.com/site/regisbarnichon/data, August 10, 2016

Job Openings: U.S. Bureau of Labor Statistics, Job Openings [JTS10000000JOL], retrieved from https://data.bls.gov/timeseries/JTS10000000JOL, August 10, 2016.

Output: U.S. Bureau of Economic Analysis, Real Gross Domestic Product (DISCONTINUED) [GDPC96], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/GDPC96, August 10, 2016.

Consumption of nondurable goods: U.S. Bureau of Economic Analysis, Real Personal Consumption Expenditures: Nondurable Goods [PCNDGC96], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/PCNDGC96, August 10, 2016.

Consumption of Services: U.S. Bureau of Economic Analysis, Real Personal Consumption Expenditures: Services [PCESVC96], retrieved from FRED, Federal Reserve Bank of St. Louis;

https://fred.stlouisfed.org/series/PCESVC96, August 10, 2016.

Investment: U.S. Bureau of Economic Analysis, Real Gross Private Domestic Investment (DISCONTINUED) [GPDIC96], retrieved from FRED, Federal Reserve Bank of St. Louis;
https://fred.stlouisfed.org/series/GPDIC96, August 10, 2016.

PCE inflation: U.S. Bureau of Economic Analysis, Personal Consumption Expenditures: Chain-type Price Index [PCEPI], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PCEPI, August 10, 2016.

Aggregate productivity: Federal Reserve Bank of San Francisco, Total Factor Productivity, retrieved from:
https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/, August 10, 2016

Labor productivity: U.S. Bureau of Labor Statistics, Labor Productivity (output per hour) [PRS85006093], retrieved from https://data.bls.gov/timeseries/PRS85006093, August 10, 2016.

D Wages

In this online appendix, I discuss in more details the construction of my wages series, the computation of the empirical wages responses to changes in aggregate productivity, and the discrepancy between my empirical results and those of Gertler et al. (2019).

D.1 Constructing Wage Series for New Hires

I use the Current Population Survey (CPS), Current Population Survey Merged Outgoing Rotation Groups (CPS-MORG), and IPUMS-CPS (Flood, Kind, Ruggles & Warren, 2015) microdata to construct wage series adjusted for workers characteristics. The CPS is the main labor force survey for the United States, and it is the primary source of labor force statistics such as the national unemployment rate. The CPS consists of a rotating panel where households and their members are surveyed for four consecutive months, not surveyed for the following eight months, and then interviewed again for another four consecutive months. The CPS includes individual information such as employment status, sex, education, race, and state. However, individual earnings and hours worked are collected only in the fourth and eighth interviews. In addition, since 1994, individuals have been asked if they still work in the same job reported in the previous month, making it possible to identify job changers. IPUMS-CPS is a project from the University of Minnesota that integrates, disseminates, and harmonizes CPS microdata. IPUMS-CPS is free and includes, among many other variables, harmonized series for education, occupation, and industry and a unique person and household ID, which makes it easier to link individuals across samples and facilitates data analysis.

My empirical model is based on the following Mincer equation for the wage of individual i at time t (w_{it}):

$$log(w_{it}) = x_{it}'\beta_x + \left(\sum_{j=1}^{T} \alpha_j^a \cdot D_j + \alpha_j^{nhu} \cdot D_j \cdot D_{it,nhu} + \alpha_j^{nhc} \cdot D_j \cdot D_{it,nhc}\right) + e_{it} \tag{55}$$

x_{it} is a vector of individual characteristics, and β_x, $\{\alpha_j^a,\ \alpha_j^{nhu},\ \alpha_j^{nhc}\}_{j=1}^T$ are coefficients. D_j is a time dummy equal to 1 if $j = t$ and 0 otherwise. $D_{it,nhu}$ is a dummy variable equal to 1 if worker i spent time in unemployment during the past three months and 0 otherwise. $D_{it,nhc}$ is a dummy variable equal to 1 if worker i was previously employed at another firm during the past three months and has not been unemployed while switching jobs. Hence, the average (log) wage for all workers (w^a), new employees (w^u), and job changers (w^c) are given by:

$$w_t^a = \alpha_t \tag{56}$$

$$w_t^u = \alpha_t + \alpha_t^{nhu} \tag{57}$$

$$w_t^c = \alpha_t + \alpha_t^{nhc} \tag{58}$$

The hourly wage rate is constructed by dividing weekly earnings by weekly hours. Following Schmitt (2003), top-coded weekly earnings are imputed assuming a log-normal cross-sectional distribution for earnings. Following Haefke et al. (2013) I drop hourly wage rates below the 0.25th and above the 99.75th percentiles each month. In order to uniquely identify workers in the CPS files, I use the IMPUMS-CPS ID variables: CPSID and CPSIDP.[11]

Vector x_{it} includes a fourth-order polynomial in experience, gender, race, marital status, state, 10 occupation dummies, and 14 industry dummies. For occupation, industry and education, I use harmonized variables OCC1950, IND1950, and EDUC provided by IPUMS-CPS. Experience is defined as age minus years of education minus 6. Following the literature, individual i's weight is the product of the individual's weight reported by the BLS and hours worked.

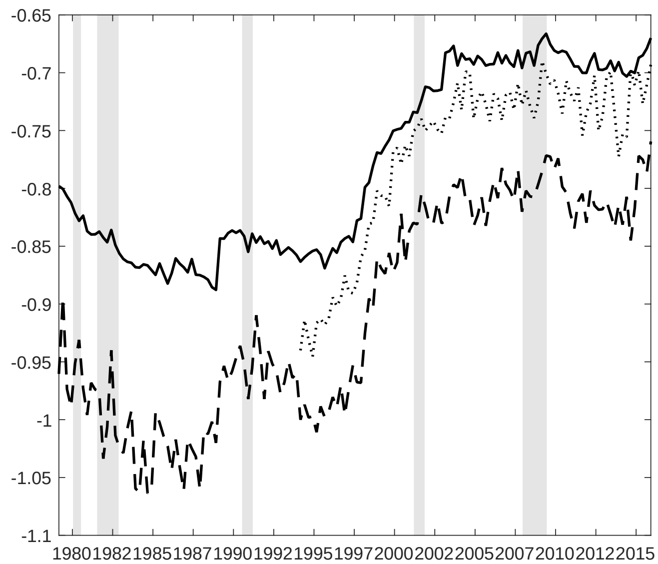
Due to sample design, it is not possible to match individuals between July 1985 and December 1985 and between June 1995 and November 1995. Hence, with the exception of the average wage for all workers, wage series have a missing value in those months. To compute business cycle statistics for these wage series, I impute the missing months using the average wage for all workers. Figure 6 plots the wage series that result from this methodology.

D.2 Dynamic Wage Responses

Figure 2 reports the empirical wages responses to an aggregate productivity shock. Those responses were computed based on local projections as in Jordà (2005). In particular, for each group of workers $x = \{a, u, c\}$ and for each time horizon $h =$

[11] I follow IPUMS-CPS recommendations, and I drop a few observations for which changes in sex or race are reported and for individuals whose age changes more than 2 years between samples.

Figure 6: Average (Log) Real Hourly Wages
1979:Q1-2015:Q4



Note: This figure plots the average log real hourly wages for all workers (solid line), new employees (dashed line), and job changers (dotted line). These series are the coefficient of the time fixed effect in a Mincer equation using CPS and IPUMS-CPS microdata. Wages are deflated using the PCE price index with 2009=100. Shaded areas represent NBER recession dates. The sample period is 1979-2015.

$0, 1...35$, I regress the log-differentiated wage series $log(w^x_{t+h}) - log(w^x_{t-1})$ on current labor productivity (log-differentiated) while controlling for four lags of the log-differentiated wages and labor productivity:

$$\begin{aligned} log(w^x_{t+h}) - log(w^x_{t-1}) =& \alpha^x_h + \beta^x_h \Delta log(p_t) \\ &+ \left[\sum_{l=1}^{4} \phi^x_{h,l} \Delta log(p_{t-l}) + \rho^x_{h,l} \Delta log(w_{t-l}) \right] + e^x_{h,t+h} \end{aligned} \tag{59}$$

As in the case of the contemporaneous elasticities reported in Table 1, I instrument $\Delta log(p_t)$ with the first difference of the utilization-adjusted TFP. Hence, the (log) wage response for group x at horizon h to a 1% increase in labor productivity is given by β^x_h.

D.3 Discrepancy with Gertler, Huckfeldt, and Trigari (2019)

The empirical evidence presented by Gertler, Huckfeldt, and Trigari (2019) suggests that wages for new employees (new hires from non-employment) are as cyclical as wages for job stayers and that wages for job changer are more pro-cyclical than wages for stayers. These results look at odds with the evidence presented in this paper, which suggest that wages for job changers *and* new employees are more pro-cyclical than wages for job stayers.

I show in this online appendix that after estimating a similar equation as in Gertler et al. (2019) for a similar sample and a similar time period using the CPS microdata, I obtain results comparable to theirs: Only wages for job changers are more pro-cyclical than wages for job stayers. However, when I re-estimate the same equation but for the sample and time period used in this paper, I obtain results similar to those presented in the main text: Both wages for job changers and new employees are more pro-cyclical than wages for job stayers, even though wages for job changers tend to be more pro-cyclical than wages for new employees.

I document possible sources for this "discrepancy" in results and investigate which source plays the most important role. I conclude that the main (and almost exclusive) source for the discrepancy in results is the time period. Additionally, I show that wage elasticities with respect to productivity are significantly less sensitive to the sample period than wage semi-elasticities with respect to unemployment. However, I find that wage semi-elasticities with respect to unemployment tend to suggest that wages for job changers are more pro-cyclical than wages for new employees regardless of the sample period. Hence, the sample period and the cyclical indicator matter.

D.3.1 Empirical Approaches

Let me begin by describing the main differences between the empirical approach discussed in this paper and the one used in Gertler et al. (2019). Then, I will assess their contribution to the discrepancy between our results.

Data source The main data source for wages in this paper is the Current Population Survey (CPS), which I described before. The main data source for Gertler, et al. (2019) is the Survey of Income and Program Participation (SIPP), which they discuss in detail in section 2 of their paper. These two data sources are available at a monthly frequency. However, there are two differences between these data that I would like to highlight:

1. The SIPP provides more wage observations per worker than the CPS -up to 12 observations in the SIPP (one every 4 months) but *at most* 2 observations in the

CPS (one per year).[12]

2. The CPS provides a continuous sample period, while the SIPP has some gaps. In particular, between 1990 and 2013, we do not have observations for the following months in the SIPP: 96m1, 96m2, 00m1,... 00m12, 08m1,... 08m7, 13m9, 13m12.

Sample selection and sample period Gertler et al. focus on white prime-age males, while I focus on prime-age workers. Hence, I also include females and non-whites.[13] The sample period in this paper is 1979 to 2015, while the sample period in Gertler et al. (2019) is 1990 to 2013 (with gaps, as explained before).

Econometric model The final difference between my empirical methodology and Gertler et al. (2019) concerns the econometric model. In this paper, I computed aggregate wage series for different groups of workers based on a Mincer equation. As explained in section D.1 of this online appendix, my wage series are the coefficients of time fixed effects in a Mincer equation. I compute wage elasticities with respect to productivity by regressing these aggregate series on aggregate productivity (and other controls). In contrast, Gertler et al. (2019) compute wage semi-elasticities with respect to the unemployment rate by including a linear trend, the unemployment rate, and interactions of the unemployment rate with new hires dummies in a Mincer equation. The econometric model of Gertler et al. (2019) consist of estimating the following Mincer equation in either first difference or fixed effects:

$$\begin{aligned}\Delta^m log(w_{it}) =& \Delta^m x_{it} \cdot b_x + b_u \cdot \Delta^m U_t \\ &+ I_{EEit} \cdot \left[b_n^{EE} + b_{nu}^{EE} \cdot \Delta^m U_t\right] \\ &+ I_{ENEit} \cdot \left[b_n^{ENE} + b_n^{ENE} \cdot \Delta^m U_t\right] + e_{it}; \qquad m = \{fd, md\}\end{aligned} \tag{60}$$

where I_{EEit} and I_{ENEit} are indicator variables equal to 1 if the worker is a job changer or a new hire from non-employment, respectively. Δ^{fd} denotes first differences, and Δ^{md} denotes mean differences. Both approaches rely on previous empirical studies cited by Pissarides (2009).

[12] Even though the SIPP provides up to 48 observations per-individual, most researches (including Gertler et al., 2019) use only a fraction of those observations to avoid the "seam effect."

[13] Notice that I control for sex and race dummies in my Mincer equation.

D.3.2 Comparing Results

Given all the differences between the two empirical approaches listed before, I use the CPS microdata to estimate the Mincer equation (60) using a similar sample as in Gertler et al. (2019). In other words, I include only prime-age white males from 1990 through 2013 excluding the months for which there are not observations in the SIPP dataset. Given that the CPS provides, at most, two observations per worker, I estimate equation (60) in levels, but I control for as many individual characteristics as I can.[14] Because we can identify job changers only since 1994 in the CPS, I estimate equation (60) assuming that $I_{EEit} = 0$ before 1994. Given that I find that wages for job changers are more procyclical than wages for job stayers and new employees, this assumption will bias my estimates in favor of Gertler, et al (2019). I briefly review this assumption at the end of this section and find that it has minor-to-no effects on my conclusions, which is expected given that job changers represent a small fraction of all workers.

Table 9: Wage Cyclicalty a la Gertler, Huckfeldt and Trigari (2019)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
U_t	-0.750	-0.787	-0.799	0.165	0.062	-0.480	-0.584
	(0.033)	(0.025)	(0.023)	(0.026)	(0.018)	(0.027)	(0.041)
$U_t \cdot I_{EEit}$	-1.007	-0.907	-0.887	-1.278	-1.035	-0.546	-0.596
	(0.197)	(0.147)	(0.139)	(0.188)	(0.129)	(0.130)	(0.190)
$U_t \cdot I_{ENEit}$	0.088	-0.068	-0.107	-0.390	-0.450	-0.196	-0.140
	(0.162)	(0.107)	(0.103)	(0.133)	(0.082)	(0.113)	(0.177)
Sample	1990-2013	1990-2013	1990-2013	1979-2015	1979-2015	1994-2015	1994-2015
Gaps	YES	YES	NO	NO	NO	NO	NO
Females	NO	YES	YES	NO	YES	YES	NO
Observations	1'263,656	2'388,418	2'569,087	1'822,561	3'979,533	2'322,313	1'031,767

Notes: This table presents wage semi-elasticities with respect to the unemployment rate estimated based on equation 60 for different samples and time periods. Columns for which Gaps=YES indicate that the regression did not include the months for which the SIPP dataset does not provide data. Columns for which Females=NO indicate that the regression did not include females and non-white worker in the estimation. Robust standard errors are presented in parenthesis.

Column (1) in Table 9 presents the results of that estimation. It is worth pointing out that Gertler et al. (2019) estimate b_u to be between -0.42 and -0.14, $b_u + b_{nu}^{EE}$ to be between -2 and -1.6 and $b_{nu}^{ENE} = 0$. The results presented in column (1) are close to those

[14] In particular, I control for education, a fourth polynomial in experience, and dummy variables for race, sex, marital status, state, occupation, and industry.

Table 10: Wage Cyclicality a la Gertler, Huckfeldt and Trigari (2019)
No Job Changers

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
U_t	-0.757	-0.796	-0.810	0.172	0.063	-0.504	-0.611
	(0.033)	(0.024)	(0.023)	(0.026)	(0.018)	(0.027)	(0.040)
$U_t \cdot I_{ENEit}$	0.077	-0.075	-0.114	-0.399	-0.453	-0.196	-0.145
	(0.162)	(0.107)	(0.103)	(0.133)	(0.082)	(0.113)	(0.177)
Sample	1990-2013	1990-2013	1990-2013	1979-2015	1979-2015	1994-2015	1994-2015
Gaps	YES	YES	NO	NO	NO	NO	NO
Females	NO	YES	YES	NO	YES	YES	NO
Observations	1'263,656	2'388,418	2'569,087	1'822,561	3'979,533	2'322,313	1'031,767

Notes: This table presents wage semi-elasticities with respect to the unemployment rate estimated based on equation 60 for different samples and time periods, but excluding job changers. Columns for which Gaps=YES indicate that the regression did not include the months for which the SIPP dataset does not provide data. Columns for which Females=NO indicate that the regression did not include females and non-white workers in the estimation. Robust standard errors are presented in parenthesis.

numbers, even though my point estimate for b_u is larger than in Gertler et al. (2019). For this particular sample, the CPS microdata indicate that wages for job changers are more pro-cyclical than wages for job stayers and that wages for new employees are as cyclical as wages for job stayers. Now, column (5) presents the results of estimating equation (60) using the sample of this paper (a sample that includes females, non-white workers, and a larger sample period than that of Gertler et al., 2019). The results of column (5) are in line with the main results of this paper: Wages for job changers and new employees are more pro-cyclical than wages for job stayers. However, according to this empirical approach, wages for job changers are more pro-cyclical than wages for new employees.

Comparing column (2) with column (1) and column (4) with column (5) suggests that including females and non-white workers has little effect on the estimated semi-elasticities. Similarly, comparing column (3) with column (2) suggests that the gaps in the SIPP do not have a significant effect on the estimated semi-elasticity for new employees. However, this table suggests that the wage semi-elasticities with respect to the unemployment rate are very sensitive to the sample period: The sample period 1979-2015 delivers larger semi-elasticities, but the sample period 1994-2015 delivers smaller semi-elasticities than the sample period 1990-2013.

As mentioned before, the estimation results of Table 9 assume that $I_{EEit} = 0$ before

Table 11: Wage Cyclicality a la Gertler,
Huckfeldt and Trigari (2019)
Sample Period 1994 -

	(1)	(2)	(3)
U_t	-1.120	-1.162	-1.144
	(0.043)	(0.032)	(0.030)
$U_t \cdot I_{EEit}$	-0.645	-0.536	-0.544
	(0.199)	(0.148)	(0.140)
$U_t \cdot I_{ENEit}$	-0.058	-0.189	-0.206
	(0.201)	(0.132)	(0.126)
Sample	1994-2013	1994-2013	1994-2013
Gaps	YES	YES	NO
Females	NO	YES	YES
Observations	1'013,142	1'934,490	2'115,159

Notes: This table presents wage semi-elasticities with respect to the unemployment rate estimated based on equation 60 for different samples and time periods, excluding job changers. Columns for which Gaps=YES indicate that the regression did not include the months for which the SIPP dataset does not provide data. Columns for which Females=NO indicate that the regression did not include females and non-white workers in the estimation. Robust standard errors are presented in parenthesis.

1994. Tables 10 and 11 show that my conclusions are not sensitive to that assumption. Table 10 reports the results of estimating equation (60) without job changers. The estimated wage semi-elasticities of Table 10 are very close to those of Table 9. Similarly, Table 11 presents the results of re-estimating columns 1, 2, and 3 of Table 9 for the sample period 1994-2013.[15] The sample period in Table 11 is relatively similar to the sample period in Gertler, at al. (2019), and the results presented in that table suggest that wages for new employees are more procyclical than wages for job stayers. Tables 10 and 11 further suggest that the lack of differential wage cyclicality for new-employees in Gertler et al. (2019) could be driven by their sample period.

Finally, Table 12 assesses how sensitive the wage elasticities with respect to productivity are. Even though those elasticities change with sample selection, they tend to be more stable than the wage semi-elasticities with respect to unemployment and

[15]Note that the analogous exercise for columns 4 and 5 of Table 9 are presented in the last two columns of that table.

Table 12: Wage Elasticity with Respect to Productivity
Gertler, Huckfeldt and Trigari (2019) Sample

	(1)	(2)	(3)	(4)
All Workers	0.643	0.706	0.515	0.492
	(0.240)	(0.298)	(0.229)	(0.203)
Job Changers		1.519	0.865	1.007
		(0.759)	(0.599)	(0.467)
New Employees	1.370	1.455	1.170	0.974
	(0.716)	(0.850)	(0.667)	(0.473)
Sample	1990-2013	1994-2013	1994-2015	1994-2015
Gaps	YES	YES	NO	NO
Females	NO	NO	NO	YES
Observations	87	71	88	88

Notes: This table presents wage elasticities with respect to productivity estimated for different samples and time periods. Columns for which Gaps=YES indicate that the regression did not include the months for which the SIPP dataset does not provide data. Columns for which Females=NO indicate that the regression did not include females and non-white worker in the estimation. Robust standard errors are presented in parenthesis.

suggest that wages for job changers and new employees are equally cyclical. The business cycle relationship between unemployment and labor productivity has changed in recent decades (e.g., Hall, 2017; Gali & van Rens, 2019), but my results suggest that the relationship between wages and labor productivity has been relatively stable.

In sum, I conclude that wage semi-elasticities with respect to unemployment computed based on CPS microdata also suggest that wages for job changers and new employees are more pro-cyclical than wages for job stayers, even though wages for job changers tend to be more pro-cyclical than wages for new employees. The results presented in this online appendix suggest that lack of cyclicality in wages for new employees in Gertler et al. (2019) could be driven by their sample period.

E Movement Down the Wage-Ladder

I design two experiments to assess the robustness of my results to considering job changers that move to lower paying firms. In both experiments the main results of the paper continue to hold: Wages within job are sluggish amplifying the aggregate responses to productivity shocks, and wage elasticities for new hires are significantly higher than wages for job stayers because of employment composition. However, in both experiments, the reallocation of employment from low to high-paying firms is "weaker" than in the data, according to the indicators of Haltiwanger et al. (2018). Hence, in this extension, less workers move up the wage ladder in expansions than suggested by the data, which works against and not in favor of my results.

E.1 Experiment 1: Exogenous probability to accept a lower-paying job

In the first experiment, I assume that with exogenous probability $\hat{\chi}_t$, conditional on a match, a worker accepts a lower paying job. Based on empirical evidence, I assume the following reduced form specification for the evolution of this parameter:

$$\hat{\chi}_t = \hat{\chi} + \gamma_\chi(u_t - \bar{u}) \tag{61}$$

I calibrate $\hat{\chi}$ such that 40% of the job-to-job transitions are to lower paying firms in steady state, which is a conservative value given that Sorkin (2018) estimates that 40% of movements down the wage ladder can be explained by compensating differentials. following Gertler et al. (2019), I set γ_χ such that the simulated semi-elasticity of the fraction of job movements up the wage-ladder with respect to the unemployment rate is equal to -1.4. Gertler et al. (2019) find that a significant fraction of job-to-job transitions are to lower paying firms, but that fraction is countercyclical. They estimate that a 1 percentage point decline in the unemployment rate is associated with an increase of 1.4 percentage points in the share of workers that move to a higher paying job. Haltiwanger et al. (2018) also document a similar fact using a different data source and sample, and they state: *"We find that it is a decline in the propensity to move up the ladder that accounts for most of the cyclical variation in the firm wage ladder."* (page, 55). Because higher values of $\hat{\chi}$ tend to make high-productive firms smaller, the model requires a very small value for χ in order to keep matching the relative size of large firms. Given that low values for χ tend to affect the stability of the model and to degenerate the distribution of firms, I choose to target the average firm size in the economy instead of the relative size of the largest firms. Additionally, I set workers bargaining power to 0.5 in this extension to better match the elasticity of the FOCE with respect to productivity.[16] All other parameters in my model are recalibrated to target the same moments as in the baseline model.

E.1.1 Main Changes to the Model Equations

The value of employment becomes:

$$\begin{aligned}(W_j - U) = \quad & w_j - z_j \\ & + E\{Q((1-\delta_h)\left[1-\bar{i}q(F_j + (1-F_j)\hat{\chi})\right](W_j' - U') \\ & + (1-\delta_h)\bar{i}qF_j(\tilde{W}_j' - U') \\ & + (1-\delta_h)\bar{i}q(1-F_j)\hat{\chi}(\hat{W}_j' - U') \\ & - q\left(\bar{W}' - U'\right))\}\end{aligned} \tag{62}$$

where new variable $\hat{W}$ is the expected value of employment at a lower paying firm conditional on a match:

$$(1-F_j)\hat{W}_j' = \int_0^j W_x' \left(\frac{v_x}{v}\right) dx \tag{63}$$

Similarly, the evolution of employment and value of a filled vacancy at firm j are given by:

$$h_j' = (1-\delta_h)\left[1-\bar{i}q(F_j + (1-F_j)\hat{\chi})\right] h_j + \tilde{q}_j v_j \tag{64}$$

$$J_j = p_j - w_j + E\left[Q \cdot (1-\delta_h)\left[1-\bar{i}q(F_j + (1-F_j)\hat{\chi})\right] \cdot J_j'\right] \tag{65}$$

[16] Matching that elasticity only with $\tilde{z}$ turns out to be very complicated, given that movements down the wage ladder increase the volatility of employment at low-paying firms. If I kept workers bargaining power at 0.7, the elasticity of FOCE with respect to productivity would be around 2 instead of 1. In this extension, firms do not have incentives to pay higher wages in equilibrium with ϑ=0.5, given that movements down the wage ladder reduce those incentives.

These equations imply the following match surplus:

$$\begin{aligned} S_j = \quad & p_j - z_j \\ & + E\{Q((1-\delta_h)\left[1-\bar{i}q(F_j+(1-F_j)\hat{\chi})\right]S'_j \\ & + (1-\delta_h)\bar{i}qF_j\tilde{S}'_j \\ & + (1-\delta_h)\bar{i}q(1-F_j)\hat{\chi}\hat{S}'_j \\ & - q\bar{S}')\} \end{aligned} \tag{66}$$

Also, the probability of filling a vacancy with a job changer becomes:

$$\tilde{q}^c_j = \tilde{q}\cdot\left[\left(\int_0^j \frac{(1-\delta_h)\bar{i}h_x}{s}dx\right)+\left(\int_j^\infty \frac{(1-\delta_h)\bar{i}\hat{\chi}h_x}{s}dx\right)\right] \tag{67}$$

Unilateral Deviations in Equilibrium Following the same logic outline in online appendix 4, it can be verified that, if firms posted wages to poach and retain workers, the optimality condition for W'_j, in this case, would be given by:

$$E\left[-Q\tilde{q}(W'_j)v_j - QJ'h_j(1-\delta_h)\bar{i}q(1-\hat{\chi})\frac{\partial F(W'_j)}{\partial W'_j}\right] \leq 0 \tag{68}$$

As before, equation (68) holds with equality if $W'_j > \vartheta \cdot E_{\mathcal{I}_h}\left[S'_j\right]$. It can also be verified that, given my calibration strategy, firms do not have incentives to offer higher wages if ϑ is greater or equal to $\bar{\vartheta}$, where $\bar{\vartheta}$ solves:

$$\begin{aligned} & -1+(1-\delta_h)\left(1-\bar{i}q\left(F\left(e^{\frac{\bar{\vartheta}}{1-\bar{\vartheta}}\sigma^2_W}\right)+(1-F\left(e^{\frac{\bar{\vartheta}}{1-\bar{\vartheta}}\sigma^2_W}\right))\hat{\chi}\right)\right) \\ & \quad +\left(\frac{1-\bar{\vartheta}}{\bar{\vartheta}}\right)(1-\delta_h)\bar{i}q(1-\hat{\chi})\frac{1}{\sigma_W\sqrt{2\pi}}e^{-\frac{ln\left(e^{\frac{\bar{\vartheta}}{1-\bar{\vartheta}}\sigma^2_W}\right)^2}{2\sigma^2_W}} = 0 \end{aligned} \tag{69}$$

Notice that $\bar{\vartheta}$ is decreasing with $\hat{\chi}$. Hence, in this extension of the model and under my baseline calibration, firms do not have incentives to pay higher wages.

E.2 Experiment 2: Lower job-to-job transitions

In the second experiment, I re-compute all my results calibrating $\bar{i}$ to match a fraction of job changers in steady state of 1% instead of 2%. The main idea behind this experiment is to focus exclusively on job transition up the wage ladder, but recognizing that, in the data, not all job-to-job transitions are in that direction. For the same reasons as in the previous experiment, I target the average firm size in the economy instead of the relative size of the largest firms. All other parameters are recalibrated to target the same moments.

E.3 Results

Figure 7 compares the IRFs generated by this model and those generated by the baseline model. Similarly, Tables 13, 14 and 15 present the simulated elasticitites for wages and FOCE, the simulated differential employment flows from low to high-paying firms, and the simulated business cycle moments. The main results of this paper are robust to accounting for movements down the wage ladder. However, this extension shows that the cyclicality of high-paying firms relative to low-paying firms becomes weaker than in the data, which works against the results of this paper. The smaller cyclicality for high-paying firms in this extension is expected, given that low-paying firms have more incentives to growth when they can poach workers from high-paying employers.

As shown in Table 16, the first extension improves the model prediction significantly for relative-wages in steady state (forth row of Table 16). Given that low-paying firms can effectively poach workers from high-paying firms, the firm-size distribution shifts to the left compared with the baseline model. Hence, in the first extension, the relative wage for new employees is higher than in the baseline model because high-paying firms rely more on the pool of unemployment to fill a vacancy. The second extension also makes the relative wage for new employees go up (last row of Table 16), but the relative wage for job changers becomes larger than in the data. In the second extension, high-paying firms have to rely more on the pool of unemployment to fill a vacancy as well, increasing wages for new employees. However, in contrast to the first experiment, job changers always move up the wage ladder and, as a consequence, the relative difference between new-employees and job changers changes little.

Table 13: Wage and FOCE Elasticity with Respect to Labor Productivity

	Wage			$\tilde{z}$	$\int z_j \frac{h_j}{h} dj$
	All Workers	New Employees	Job Changers		
Data	0.35	0.94	1.03	1.11	1.11
Baseline model	0.47	1.00	1.04	1.15	0.94
Model with $\hat{\chi}$	0.45	0.96	1.13	1.06	0.76
Model with lower $\bar{i}$	0.47	1.03	1.41	1.52	1.11

Notes: Statistics for quarterly data. Wage elasticities are the result of estimating ϵ^x based on equation 16. The elasticities for $\tilde{z}$ and $\int z_j \frac{h_j}{h} dj$ are computed as in CRK16: these elasticities result from regressing the log-HP deviations of these variables on the log-HP deviations of output per worker, which is instrumented using the HP deviations of a. Data elasticities for $\tilde{z}$ and $\int z_j \frac{h_j}{h} dj$ are based on CRK16.

Table 14: Differential Net Flows, Coefficient on Cyclical Variable
High Wage minus Low Wage

	Deviation from HP trend	First Difference
U.S. Data	-0.269	-0.557
Baseline model	-0.448	-0.432
Model with $\hat{\chi}$	0.974	0.974
Model with lower $\bar{i}$	0.383	0.383

Note: Data for the first column are from Haltiwanger, et al. (2018) Table 1. Each model was used to generate artificial data over a time horizon of 55 quarters, which is consistent with the sample size of Haltiwanger, et al. (2018). Each model was simulated 1,000 times. The coefficient on the cyclical variable was computed for each artificial series, and the theoretical coefficient was estimated by averaging across the 1,000 simulations.

Table 15: Business Cycle Moments U.S. Data Versus Models

	Standard Deviation									
	u	v	v/u	y	c	Inv	w^a	w^u	w^c	a
U.S. data	0.19	0.19	0.38	0.02	0.02	0.10	0.02	0.03	0.03	0.02
Baseline model	0.17	0.20	0.36	0.03	0.02	0.14	0.02	0.03	0.02	0.02
Model with $\hat{\chi}$	0.28	0.28	0.53	0.03	0.02	0.15	0.02	0.06	0.05	0.02
Model with lower $\bar{\imath}$	0.16	0.16	0.31	0.03	0.02	0.12	0.02	0.03	0.02	0.02
	Autocorrelation									
	u	v	v/u	y	c	Inv	w^a	w^u	w^c	a
U.S. data	0.98	0.96	0.97	0.94	0.96	0.93	0.91	0.70	0.71	0.89
Baseline model	0.92	0.84	0.90	0.93	0.97	0.91	0.94	0.84	0.79	0.89
Model with $\hat{\chi}$	0.91	0.79	0.89	0.93	0.97	0.90	0.93	0.87	0.80	0.89
Model with lower $\bar{\imath}$	0.91	0.76	0.88	0.92	0.97	0.89	0.93	0.82	0.76	0.89
	Correlation with unemployment									
	u	v	v/u	y	c	Inv	w^a	w^u	w^c	a
U.S. data	1.00	-0.92	-0.98	-0.80	-0.63	-0.82	-0.14	-0.16	-0.12	-0.47
Baseline model	1.00	-0.87	-0.96	-0.79	-0.28	-0.88	-0.45	0.60	0.42	-0.62
Model with $\hat{\chi}$	1.00	-0.81	-0.95	-0.79	-0.25	-0.87	-0.20	0.85	0.79	-0.58
Model with lower $\bar{\imath}$	1.00	-0.79	-0.94	-0.74	-0.23	-0.83	-0.20	0.71	0.24	-0.62

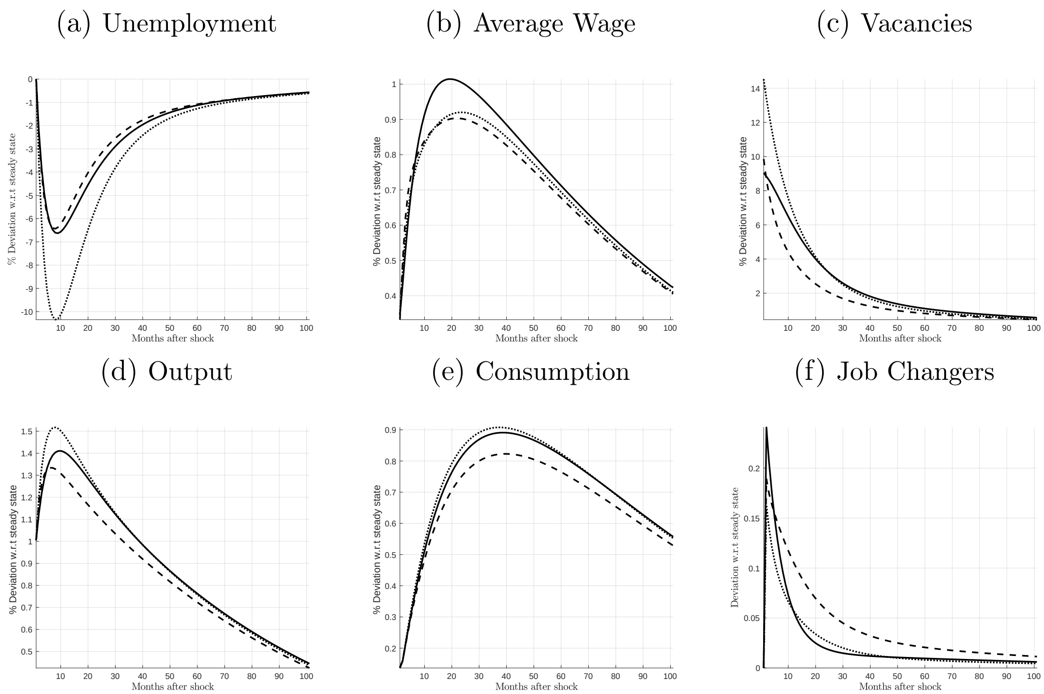
Note: *u:* Unemployment level. *v:* Vacancies *v/u:* Vacancy-unemployment ratio. *y:* Output. *c:* Consumption. *Inv:* Investment. w^a*:* Average wage in the economy. w^u*:* Average wage for new employees. w^c*:* Average wage for job changers. *a:* Aggregate TFP. All series are seasonally adjusted, logged, and detrended with the HP filter with a smoothing parameter of 100,000.

Table 16: Average Wages in Steady State

	All Workers	Job Stayers	New Employees	Job Changers	New Hires
Data	1.00	1.01	0.89	0.97	0.93
Model Information Frictions	1.00	1.01	0.51	0.94	0.75
Model Full Information	1.00	1.01	0.59	0.96	0.80
Model with $\hat{\chi}$	1.00	1.00	0.89	0.99	0.94
Model with lower $\bar{\imath}$	1.00	1.00	0.67	1.07	0.83

Notes: All wages are expressed as a fraction of the average wage for all workers in the economy. The first row presents data average from 1979 to 2015. The second and third rows of this table report the average wage for different groups of workers in steady state in the model with information frictions and full information, respectively. The last two rows represent the two experiments described in this Appendix.

Figure 7: Impulse Response Function to a 1% Increase in Aggregate Productivity



Note: This figure plots model Impulse Response Functions (IRFs) to a 1% increase in aggregate TFP. Solid black lines are the IRFs for the baseline model in which workers face information frictions. Dashed lines are the IRFs generated by a model in which workers face information frictions and accept a lower-paying job with exogenous probability equal to $\hat{\chi}_t$. Dotted lines are the IRFs generated by a calibrated model in which workers face information frictions but in which $\bar{i}$ is set to match a job-to-job transition rate of 1% per month instead of 2% as in the baseline calibration.

F Unilateral Deviations in Equilibrium (Details)

Nothing in my model prevents firms from paying a higher wage in equilibrium. Even though higher wages increase firms' payrolls, offering a higher wage can reduce the fraction of workers who are poached by other firms and, as a consequence, increase profits. When workers' bargaining power is low, firms' incentives to offer higher wages could be large. However, in my calibration with a bargaining power equal to 0.7, no firm has incentives to offer a higher wage. Suppose that firm j is considering offering a higher wage to its employees in order to reduce its separation rate and increase its profits. Hence, following Moscarini and Postel-Vinay (2013), the problem for firm j would be given by:

$$\begin{aligned}
\Pi_j(h_j, \bar{W}, \Omega) = &\max_{v_j, k_j, w_j, W_j'} e^{a_j + a} k_j^{\alpha} h_j^{1-\alpha} - w_j h_j - r k_j - \frac{\kappa}{1+\chi} (\tilde{q}_j v_j)^{1+\chi} \\
&+ E\left[Q \Pi_j(h_j', W_j', \Omega')\right] \qquad (70) \\
&\text{s.t.} \\
h_j' &= (1-\delta_h)(1 - \bar{i} q F(W_j')) h_j + \tilde{q}(W_j') v_j \qquad (71) \\
E_{\mathcal{I}_h}\left[W_j\right] &\geq \vartheta \cdot E_{\mathcal{I}_h}\left[S_j\right] \qquad (72) \\
E_{\mathcal{I}_h}\left[W_j'\right] &\geq \vartheta \cdot E_{\mathcal{I}_h}\left[S_j'\right] \qquad (73) \\
E_{\mathcal{I}_h}\left[W_j\right] &\geq \bar{W} \qquad (74) \\
\Omega' &= \lambda^f(\Omega) \qquad (75) \\
v_j, \quad k_j &\geq 0 \qquad (76)
\end{aligned}$$

where restrictions (72) and (73) imply that the wage offer cannot be lower than what workers would get otherwise, and restriction (74) implies that the continuation value for workers cannot be lower than what was promised at the end of the previous period. Given that the optimality conditions for k_j and v_j do not change, let me concentrate on W_j'. Given that $E_{\mathcal{I}_h}\left[W_j\right] = max\{\vartheta \cdot E_{\mathcal{I}_h}\left[S_j\right], \bar{W}\}$, the optimality condition for W_j' is:

$$E\left[-Q\tilde{q}(W_j')v_j - QJ' h_j (1-\delta_h) \bar{i} q \frac{\partial F(W_j')}{\partial W_j'}\right] \leq 0 \qquad (77)$$

Equation (77) holds with equality if $W_j' > \vartheta \cdot E_{\mathcal{I}_h}\left[S_j'\right]$. If firm j increase its wage in one unit, its wage bill will increase by $\tilde{q}(W'_j)v_j$ next period, but it will retain an additional fraction $(1-\delta_h)\bar{i}q\frac{\partial F(W'_j)}{\partial W'_j}$ of its current employees, each one of which will increase firm's profits by J'_j.[17] Hence, assuming that the employment value offers distribute log normal with mean equal to zero and standard deviation equal to σ_W, as I do in my calibration strategy, if equation (73) holds with equality in steady state, then for all j:

$$J_j = \frac{1-\vartheta}{\vartheta}W_j \tag{78}$$

$$\frac{\partial F(W'_j)}{\partial W'_j} = -\frac{1}{W_j\sigma_W\sqrt{2\pi}}e^{-\frac{ln(W_j)^2}{2\sigma_W}} \tag{79}$$

$$\frac{\tilde{q}_j v_j}{h_j} = 1-(1-\delta_h)(1-\bar{i}qF(W_j)) \tag{80}$$

Therefore, in steady state, firms do not have incentives to offer higher wages if for all j:

$$\underline{\Pi}_j = -1+(1-\delta_h)(1-\bar{i}qF(W_j)) + \left(\frac{1-\vartheta}{\vartheta}\right)W_j(1-\delta_h)\bar{i}q\frac{1}{W_j\sigma_W\sqrt{2\pi}}e^{-\frac{ln(W_j)^2}{2\sigma_W}} \leq 0 \tag{81}$$

Additionally, it can be verified that equation (81) has a global maximum at W^*, where:

$$W^* = e^{\frac{\vartheta}{1-\vartheta}\sigma_W^2} \tag{82}$$

Therefore, firms do not have incentives to offer higher wages if ϑ is greater or equal to $\bar{\vartheta}$, where $\bar{\vartheta}$ solves:

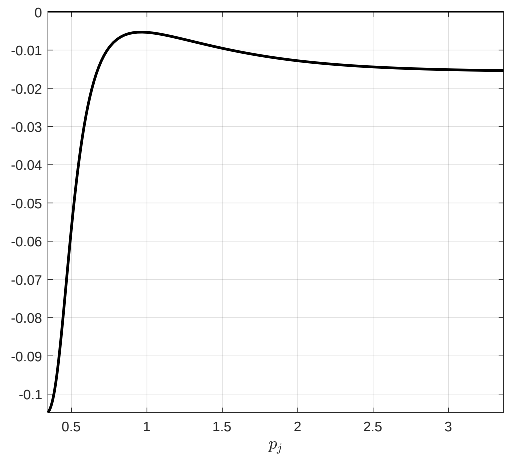
$$-1+(1-\delta_h)\left(1-\bar{i}qF\left(e^{\frac{\bar{\vartheta}}{1-\bar{\vartheta}}\sigma_W^2}\right)\right)+\left(\frac{1-\bar{\vartheta}}{\bar{\vartheta}}\right)(1-\delta_h)\bar{i}q\frac{1}{\sigma_W\sqrt{2\pi}}e^{-\frac{ln\left(e^{\frac{\bar{\vartheta}}{1-\bar{\vartheta}}\sigma_W^2}\right)^2}{2\sigma_W}} = 0 \tag{83}$$

Given my calibration strategy, $\bar{\vartheta} = 0.6635$. Hence, given that I set $\vartheta = 0.7$, firms

[17] Moscarini and Postel-Vinay (2013) show that the wage bill effect (the first term in equation (77)) does not depend on the firm's initial size (h_j). This is because firms are indifferent about the timing of wages paid to deliver a utility label $W_j = max\{\vartheta \cdot S_j, \bar{W}\}$, since workers and firms share the same discount factor.

do not have incentives to offer higher wages in equilibrium. Figure 8 shows that $\underline{\Pi}_j$ is negative for all firms in steady state.

Figure 8: $\underline{\Pi}_j$ For All Firms in Steady State



G Firms Face Information Frictions

Assuming that firms and workers face information frictions is not a straightforward exercise in my model. Depending on how this new friction is introduced in the benchmark model, hiring decisions and the unemployment response can become larger or smaller. However, as long as workers face information frictions regarding aggregate conditions, and assuming that the workers' information set is always a subset of the firms' information set, wages will have fewer pressures to increase in booms because the value of the outside option (the value of unemployment) is underestimated. Notice that the two equations governing the dynamics of wages and hiring decisions in this model are given by:

$$E_{\mathcal{I}_h}\left[\overrightarrow{W}_j(w_j)\right] = \vartheta \cdot E_{\mathcal{I}_h}\left[S_j\right] + E_{\mathcal{I}_h}\left[U\right] \tag{84}$$

$$\kappa(\tilde{q}_j v_j)^{\chi} = E_{\mathcal{I}_j}\left[Q \cdot J_j'\right] \tag{85}$$

where $\mathcal{I}_j$ is the information set of firm j. Hence, *wages depend on workers' expectations, while hiring decisions depend on firms' beliefs*. In the benchmark model, workers underestimate changes in S_j and U reducing the volatility of wages. This rises the volatility of Jj, which increases firms' hiring decision. Now let me consider the following three variations of my benchmark model:

Model 2: Firms and workers have the same information set. Aggregate productivity (a) is never observed, and all agents have to form expectations based on the public and aggregate signal $\hat{a}$.

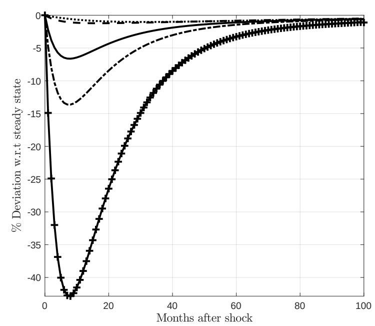
Model 3: Firms and workers have the same information set. Employers and workers at firm j observe their overall productivity ($a_j + a$) at all times but cannot decompose unexpected changes into aggregate and idiosyncratic shocks.

Model 4: Workers' information set is as described in section 2.3. Firms observe their overall productivity ($a_j + a$) at all times but cannot decompose unexpected changes into aggregate and idiosyncratic shocks. Firms also observe aggregate public signal $\hat{a}$.

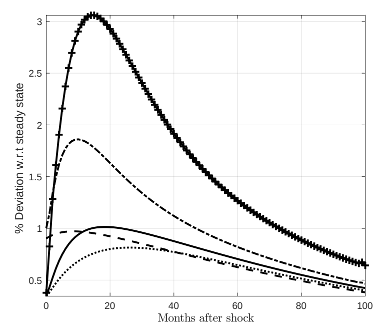
In Models 2 and 4, wages will continue to be very sluggish because workers under-

Figure 9: Impulse Response Functions to a 1% Increase in Aggregate Productivity
Comparing Different Information Structures

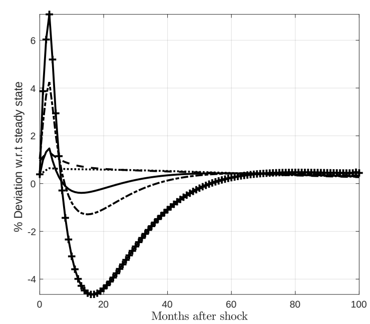
(a) Unemployment



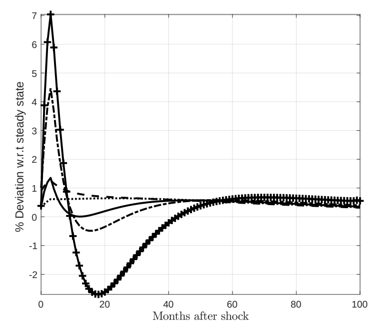
(b) Average Wage All Workers



(c) Average Wage New Employees



(d) Average Wage Job Changers



Note: This figure plots the impulse response function of unemployment and wages to a 1% increase in aggregate TFP. Solid black lines are the IRFs of a model in wihch workers face information frictions and firms have perfect information. Dashed black lines are the IRFs generated by a model in which all agents have perfect information. Models 2 (dotted black lines), 3 (dash-dotted black lines), and 4 (solid black lines with a plus mark) assume that firms and workers face information frictions as described in section 4.

estimate the responses of S_j and U to TFP shocks. However, in Model 3, workers' expectations about S_j will tend to be more volatile than in the benchmark model while changes in U will continue to be undervalued. Hence, wages will tend to be more procyclical in Model 3 than in my bench mark model.

When looking at hiring decisions in each one of these models, we should observe a lower hiring response to TFP shocks in Model 2 than in my benchmark model, as firms' expectations about J_j do not significantly change. Nonetheless, hiring responses in Model 3 should be larger than in Model 2 as firms' expectations regarding the value of an additional worker (equation (85)) become more volatile. In Model 3, firms anticipate a larger productivity in the future but do not expect a significant increase in their separation rate and wages in response to TFP shocks, which makes firms hire more workers in booms. Finally, hiring responses in Model 4 should be even larger than in Model 3. In Model 4, in response to a positive TFP shock, firms expect a lower change in wages than in Model 3 because workers' expectations about S_j are lower in Model 4 than in Model 3.

Even though hiring decisions in my benchmark model should be larger than in Model 2 but lower than in Model 4, the relationship between Model 3 and my benchmark model is not clear. In Model 3, firms expect a lower increase in their separation rate but a larger increase in their wages than in my benchmark model. However, both models should display larger unemployment responses than a model with full information.

Figure 9 plots the IRFs for unemployment and wages generated by these models. For these responses, I assumed that the persistence of idiosyncratic productivity shocks is equal to ρ_a.

As expected, Models 2 and 4 generate smaller and larger unemployment responses, respectively, than the benchmark model. However, for this particular calibration, Model 3 displays larger unemployment responses to TFP shocks than my benchmark model. In contrast, wage responses are larger in Models 3 and 4, but lower in Model 2, than in the benchmark model. As long as workers have more information (Model 3) or unemployment is more sensitive to the business cycle (Model 4), workers will demand higher wages in response to TFP shocks. It is worth pointing out that all models with information frictions display hump-shaped wage responses because workers demand higher wages as they learn that the economy is in an expansion and that the value of the outside option is greater than they thought. However, these models generate wage elasticities (Table 17) and differential growth rates (Table 18) that are not consistent with the data.

Table 17: Wage Elasticities with Respect to Labor Productivity
Model Simulated Data

	Data	Benchmark	Model 2	Model 3	Model 4
All Workers	(0.35-0.49)	0.47	0.39	1.16	0.98
New Employees	(0.94-0.97)	1.00	0.48	2.68	5.04
Job Changers	0.97	1.04	0.47	3.24	6.59

Note: This table reports the theoretical wage elasticities with respect to the unemployment rate based on equation (**??**). In the benchmark model only workers face information frictions. In models 2, 3, and 4 firms and workers face information frictions as described in this appendix.

Table 18: Differential Net Job Flows, Coefficient on Cyclical Variable.
Model Simulated Data

	Data	Benchmark	Model 2	Model 3	Model 4
Deviation from HP Trend	-0.269	-0.448	-0.296	-0.711	-1.025
First Difference	-0.557	-0.432	-14.842	-4.325	-4.618

Note: This table reports the theoretical differential net job flows as discuss in the paper. In the benchmark model only workers face information frictions. In models 2, 3 and 4 firms and workers face information frictions as described in this Appendix.

H Other Tables

Table 19: Parameter Values

Parameter	Information Frictions	Full Information	Description
Externally Calibrated			
σ	1.00	1.00	Intertemporal elasticity of substitution
η	1.50	1.50	Inverse of Frisch elasticity
α	0.33	0.33	Output elasticity of labor
ρ_a	0.98	0.98	Persistence of productivity shocks
ϑ	0.70	0.70	Workers' bargaining power in steady state
β	0.99	0.99	Discount factor.
δ_k	0.01	0.01	Capital depreciation rate.
$\mathcal{T}$	100	-	Information lag.
Internally Calibrated			
Parameter	Information Frictions	Full Information	Description
ς_n	$2 \cdot \varsigma_a$	0	Standard deviation of noise shocks.
ρ_n	0.84	0.00	Persistence of noise shocks.
δ_h	0.02	0.02	Exogenous separation rate.
Ψ	0.09	0.31	Disutility of labor parameter.
b	0.07	8.07	Unemployment benefits.
l	1.84	1.84	Matching function parameter.
$\bar{i}$	0.34	0.34	Relative search intensity of employed workers.
ξ	0.45	0.22	Elasticity of substitution between jobs.
χ	0.27	0.27	Hiring cost function convexity.
κ	0.97	0.97	Constant of hiring cost function.
σ_W	0.38	0.38	Dispersion of log net value of employment offers.

Note: Note: This table summarizes the parameterization of the model. Details are reported in section 3.2.

Table 20: Wage Distribution

Wage Quintile	Fraction of Firms	Monthly Earnings	Earnings (% of Lowest)		
			Data	Model	
				Inf. Frictions	Full Inf.
Lowest	20%	1842.16	1.00	1.00	1.00
2^{sd}	20%	2754.87	1.50	1.10	1.19
2^{sd}	20%	3458.19	1.88	1.24	1.42
2^{sd}	20%	4354.70	2.36	1.47	1.75
Highest	20%	6665.13	3.62	3.62	3.62
All	100%	3815.01	2.07	3.38	3.41

Note: This table reports average monthly earnings by pay quintile. Average earnings are weighted by employment. Data source is Kahn and McEntarfer (2014).

Table 21: Firm Size Distribution

Firm Size	Fraction of Firms	Average Size	Size (% of Smallest)		
			Data	Model	
				Inf. Frictions	Full Inf.
1 to 4	55.11%	2.23	1.00	1.00	1.00
5 to 9	21.12%	6.66	2.99	4.76	4.76
10 to 19	12.18%	13.70	6.14	15.00	15.00
20 to 49	7.39%	30.75	13.79	47.41	47.41
50 to 99	2.29%	69.85	31.32	123.28	123.28
100 to 249	1.21%	152.78	68.51	258.14	258.14
250+	0.71%	1648.26	739.13	740.30	740.30
All	100.00%	21.64	9.70	19.25	19.25

Note: This table reports firm size statistics for the United States. for the period 1977 to 2014. Firm size is defined as the number of employees per firm. Average size is computed as the total number of employees over the total number of firms. Data source is Business Dynamics Statistics.

Table 22: Distribution of Log-Wages: Data Versus Model

	Percentile									Std.
	10	20	30	40	50	60	70	80	90	
Model with information frictions	-0.66	-0.47	-0.30	-0.16	-0.02	0.09	0.25	0.40	0.58	0.47
Model with full information	-0.54	-0.34	-0.19	-0.08	0.03	0.11	0.21	0.30	0.42	0.37
Data	-0.50	-0.32	-0.20	-0.09	0.00	0.10	0.21	0.34	0.51	0.43

Note: Statistics for the U.S. economy are based on Current Population Survey microdata.

Table 23: Simulated Business Cycle
Model with Information Frictions

		u	v	v/u	y	c	Inv	w^a	w^u	w^c	a
Standard deviation		0.17	0.20	0.36	0.03	0.02	0.14	0.02	0.03	0.02	0.02
Autocorrelation		0.92	0.84	0.90	0.93	0.97	0.91	0.94	0.84	0.79	0.89
Correlation matrix	u	1	-0.87	-0.96	-0.79	-0.28	-0.88	-0.45	0.60	0.42	-0.62
	v		1	0.97	0.71	0.08	0.87	0.24	-0.35	-0.20	0.63
	v/u			1	0.77	0.18	0.90	0.35	-0.48	-0.31	0.65
	y				1	0.68	0.88	0.85	-0.15	0.08	0.94
	c					1	0.26	0.89	0.05	0.23	0.57
	Inv						1	0.55	-0.26	-0.08	0.86
	w^a							1	0.02	0.23	0.81
	w^u								1	0.97	0.10
	w^c									1	0.30
	a										1

Note: statistics for the simulated economy under information frictions. u*:* Unemployment level. v*:* Vacancies v/u*:* Vacancy-unemployment ratio. y*:* Output. c*:* Consumption. Inv*:* Investment. w^a*:* Average wage in the economy. w^u*:* Average wage for new employees. w^c*:* Average wage for job changers. a*:* Aggregate TFP. All series are seasonally adjusted, logged, and detrended with the HP filter with a smoothing parameter of 100,000.

Table 24: Simulated Business Cycle
Model with Full Information

		u	v	v/u	y	c	Inv	w^a	w^u	w^c	a
Standard deviation		0.10	0.13	0.21	0.02	0.01	0.09	0.02	0.01	0.01	0.02
Autocorrelation		0.88	0.62	0.82	0.91	0.96	0.88	0.92	0.66	0.78	0.89
Correlation matrix	u	1	-0.78	-0.93	-0.91	-0.64	-0.99	-0.86	0.27	-0.42	-0.96
	v		1	0.96	0.75	0.46	0.88	0.70	0.01	0.48	0.84
	v/u			1	0.87	0.57	0.98	0.82	-0.12	0.48	0.94
	y				1	0.89	0.91	0.99	0.09	0.73	0.99
	c					1	0.62	0.94	0.41	0.90	0.80
	Inv						1	0.85	-0.21	0.45	0.96
	w^a							1	0.19	0.80	0.96
	w^u								1	0.73	0.00
	w^c									1	0.65
	a										1

Note: statistics for the simulated economy under full information. *u:* Unemployment level. *v:* Vacancies *v/u:* Vacancy-unemployment ratio. *y:* Output. *c:* Consumption. *Inv:* Investment. *w^a:* Average wage in the economy. *w^u:* Average wage for new employees. *w^c:* Average wage for job changers. *a:* Aggregate TFP. All series are seasonally adjusted, logged, and detrended with the HP filter with a smoothing parameter of 100,000.

Table 25: Business Cycle Statistics, U.S. Economy, 1979:Q1 to 2015:Q4

		u	v	v/u	y	c	Inv	w^a	w^u	w^c	a
Standard deviation		0.19	0.19	0.38	0.02	0.02	0.10	0.02	0.03	0.03	0.02
Autocorrelation		0.98	0.96	0.97	0.94	0.96	0.93	0.91	0.70	0.71	0.89
Correlation matrix	u	1	-0.92	-0.98	-0.80	-0.63	-0.82	-0.14	-0.16	-0.12	-0.47
	v		1	0.98	0.76	0.56	0.85	-0.05	-0.06	-0.01	0.50
	v/u			1	0.79	0.60	0.86	0.05	0.05	0.06	0.49
	y				1	0.91	0.82	0.47	0.35	0.53	0.81
	c					1	0.61	0.58	0.44	0.61	0.76
	Inv						1	0.19	0.05	0.26	0.67
	w^a							1	0.79	0.82	0.44
	w^u								1	0.75	0.27
	w^c									1	0.48
	a										1

Note: Statistics for the U.S. economy are based on: *u:* Unemployment level. *v:* Help-wanted index (Barnichon, 2010). *v/u:* Vacancy-unemployment ratio. *y:* Real output in the nonfarm business sector. *c:* Consumption of non-durable goods and services. *Inv:* Real private domestic investment. *w^a:* Average wage in the economy. *w^u:* Average wage for new employees. *w^c:* Average wage for job changers. *a:* Solow residual. All series are seasonally adjusted, logged, and detrended via the HP filter with a smoothing parameter of 100,000.

Table 26: Simulated Business Cycle
Model with Information Frictions (ς =0)

		u	v	v/u	y	c	Inv	w^a	w^u	w^c	a
Standard deviation		0.02	0.03	0.05	0.02	0.01	0.08	0.02	0.02	0.02	0.02
Autocorrelation		0.96	0.93	0.96	0.92	0.96	0.90	0.92	0.85	0.86	0.89
Correlation matrix	u	1	-0.94	-0.98	-0.92	-0.94	-0.74	-0.93	-0.77	-0.80	-0.83
	v		1	0.99	1.00	0.91	0.90	1.00	0.93	0.95	0.96
	v/u			1	0.98	0.94	0.84	0.98	0.87	0.89	0.91
	y				1	0.90	0.91	1.00	0.95	0.97	0.97
	c					1	0.64	0.90	0.76	0.79	0.78
	Inv						1	0.91	0.95	0.94	0.98
	w^a							1	0.94	0.96	0.97
	w^u								1	1.00	0.98
	w^c									1	0.98
	a										1

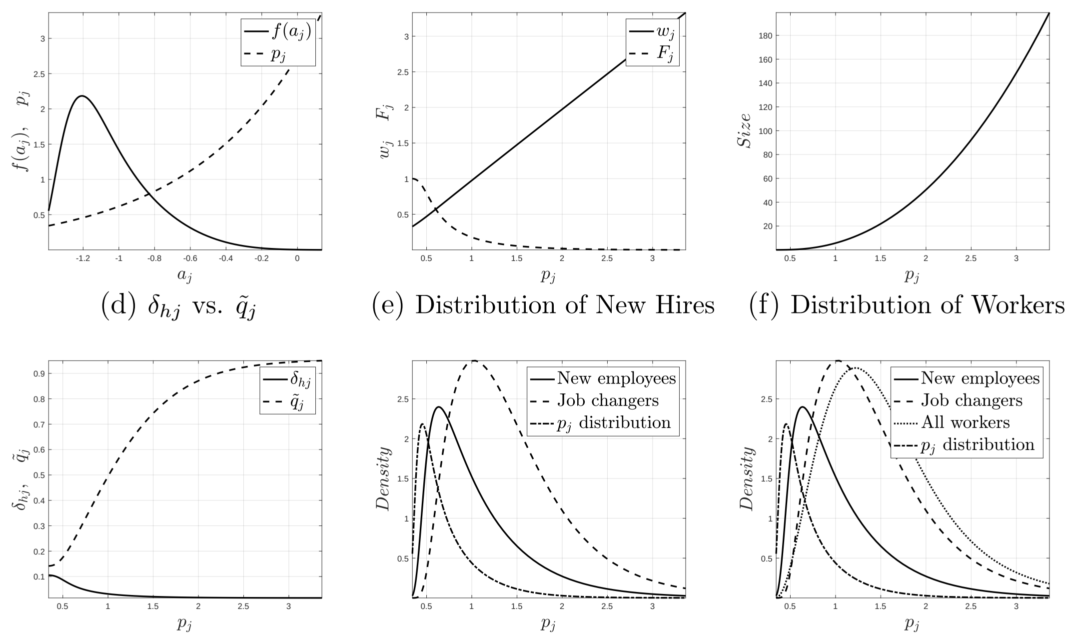
Note: statistics for the simulated economy under information frictions but with ς =0. u: Unemployment level. v: Vacancies v/u: Vacancy-unemployment ratio. y: Output. c: Consumption. Inv: Investment. w^a: Average wage in the economy. w^u: Average wage for new employees. w^c: Average wage for job changers. a: Aggregate TFP. All series are seasonally adjusted, logged, and detrended with the HP filter with a smoothing parameter of 100,000.

I Other Figures

Figure 10: Firm and Employment Distribution in Steady State

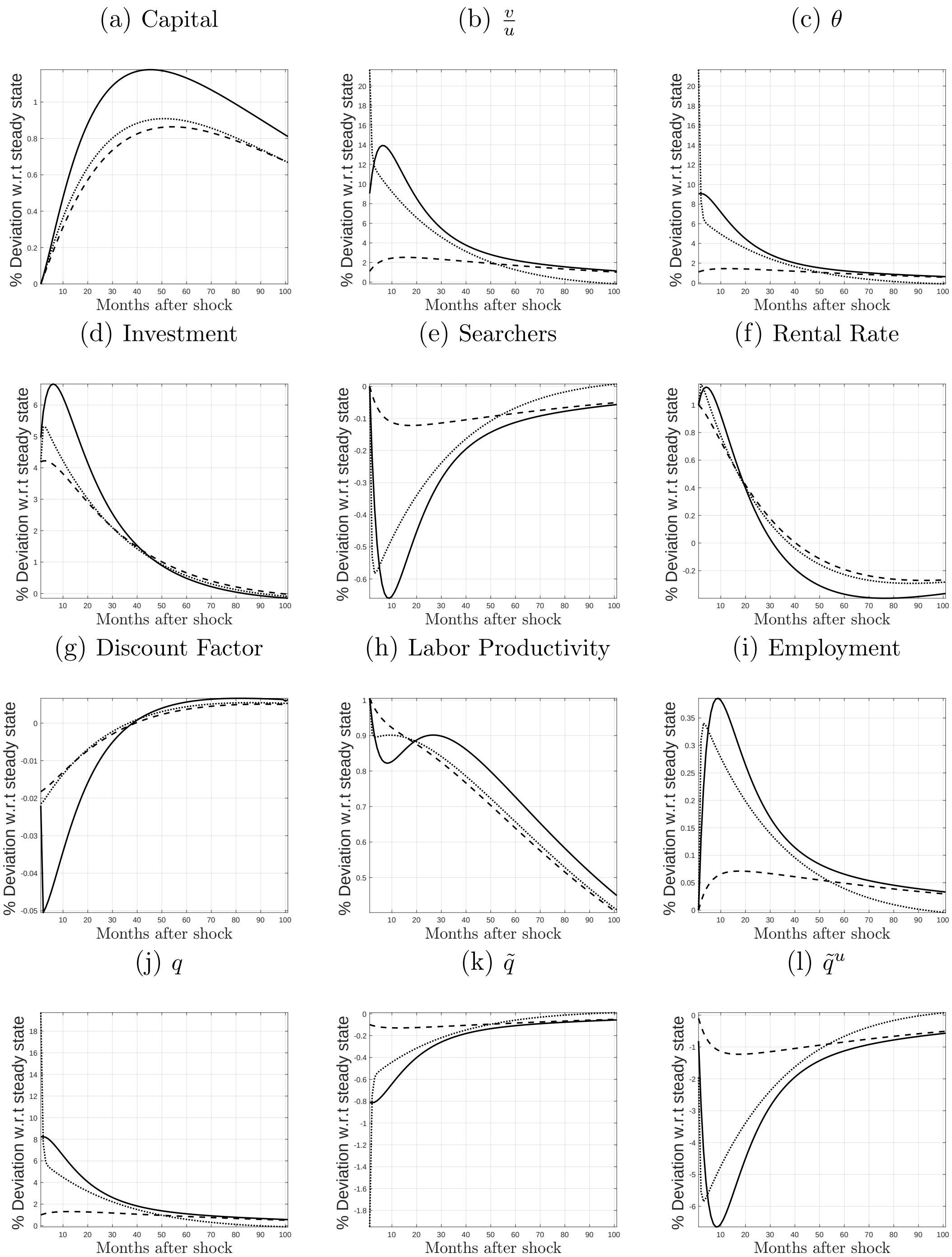
(a) p_j vs. a_j (b) w_j vs F_j (c) Firm Size

(d) δ_{hj} vs. $\tilde{q}_j$ (e) Distribution of New Hires (f) Distribution of Workers



Note: Panel 10a plots the distribution of idiosyncratic TFP across firms ($f(a_j)$) and the marginal labor productivity associated with each a_j. Panel 10b shows the wage rate (w_j) and the probability of finding a better job conditional on a match for employed workers (F_j). Panel 3a shows the average firm size as a function of the firm's labor productivity p. Panel 10d plots the separation rate (δ_{hj}) and the job filling rate ($\tilde{q}_j$) associated with each level of labor productivity. Panels 10e and 10f, plot the distribution of new employees (solid black lines) and job changers (dashed black lines) along with the labor productivity distribution (dashed-dotted line in panel 10e) and the distribution of all workers (dotted black line in Panel 10f).

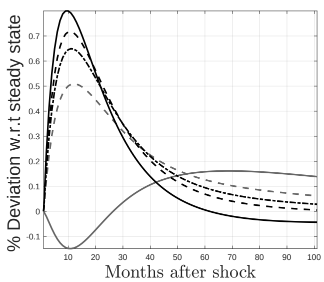
Figure 11: Impulse Response Function to a 1% Increase in Aggregate Productivity



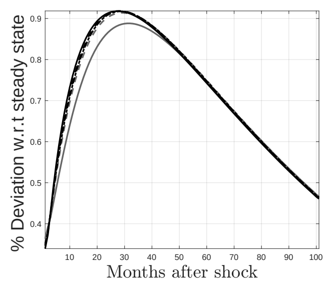
Note: This figure plots model impulse response functions (IRFs) to a 1% increase in aggregate TFP. Solid black lines are the IRFs for a model in which workers face information frictions. Dashed lines are the IRFs generated by my baseline calibration with no information frictions ($\varsigma_n = 0$). Dotted lines are the IRFs generated by a re-calibrated model in which all agents have full information.

Figure 12: Distributional Dynamics to a 1% Increase in Aggregate Productivity
Model with Information Frictions

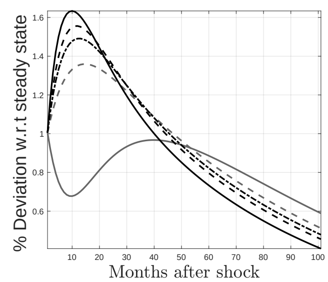
(a) Employment



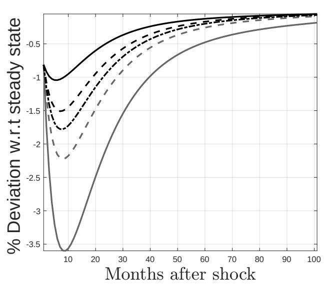
(b) Wages



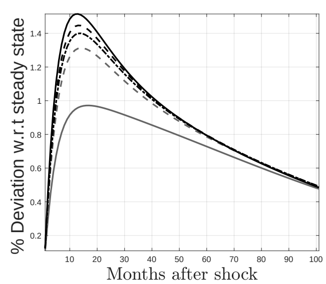
(c) Output



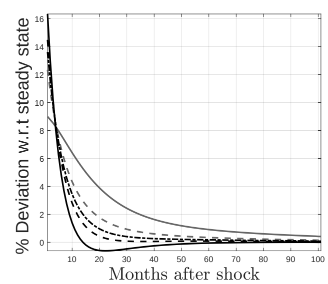
(d) $\tilde{q}_j$



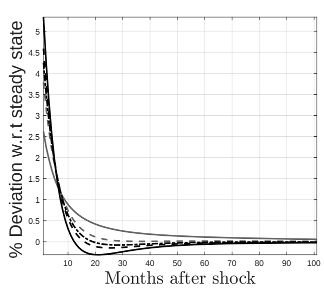
(e) z_j



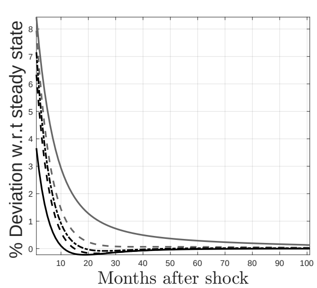
(f) v_j



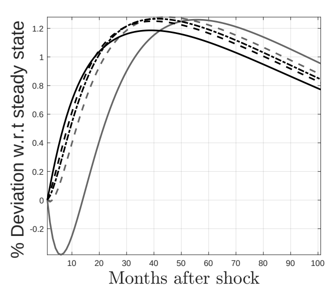
(g) J_j



(h) δ_j

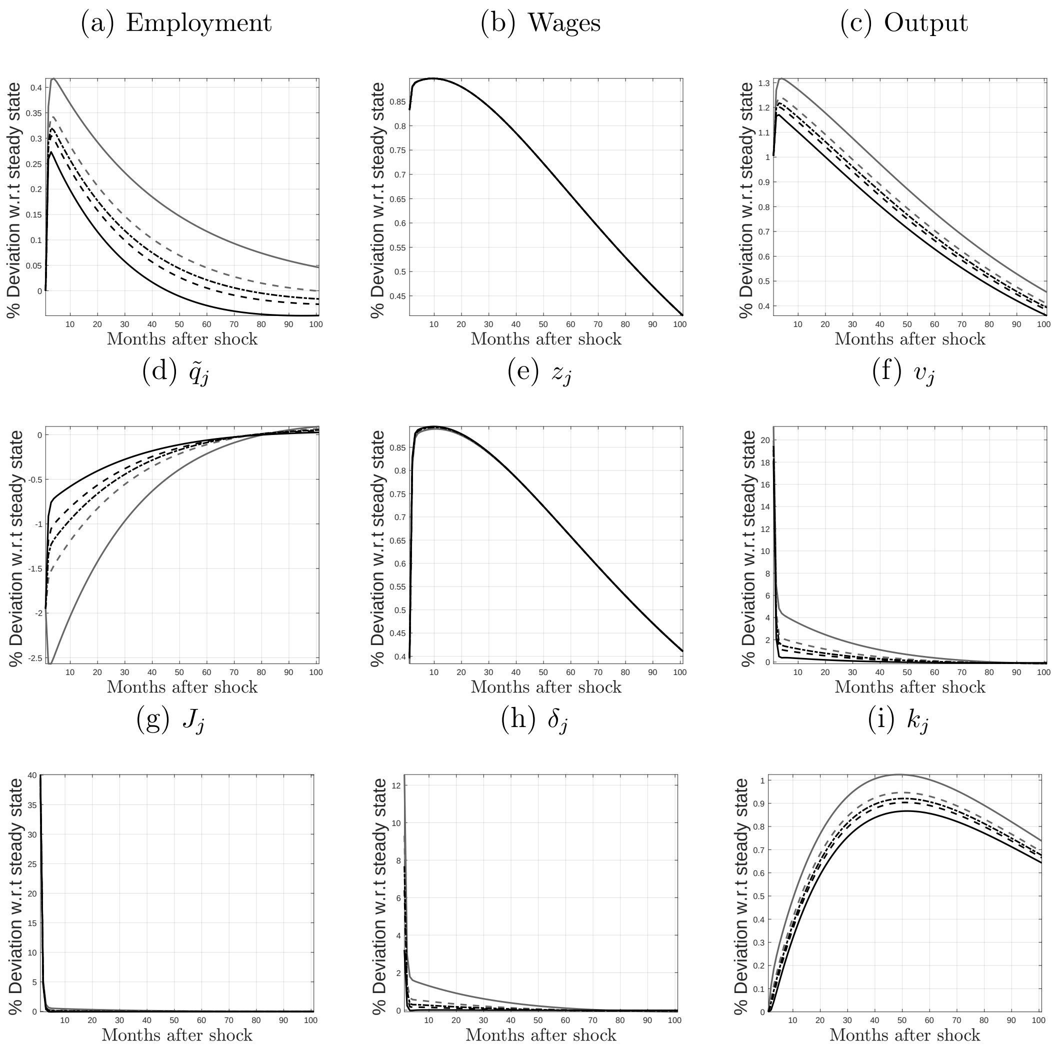


(i) k_j



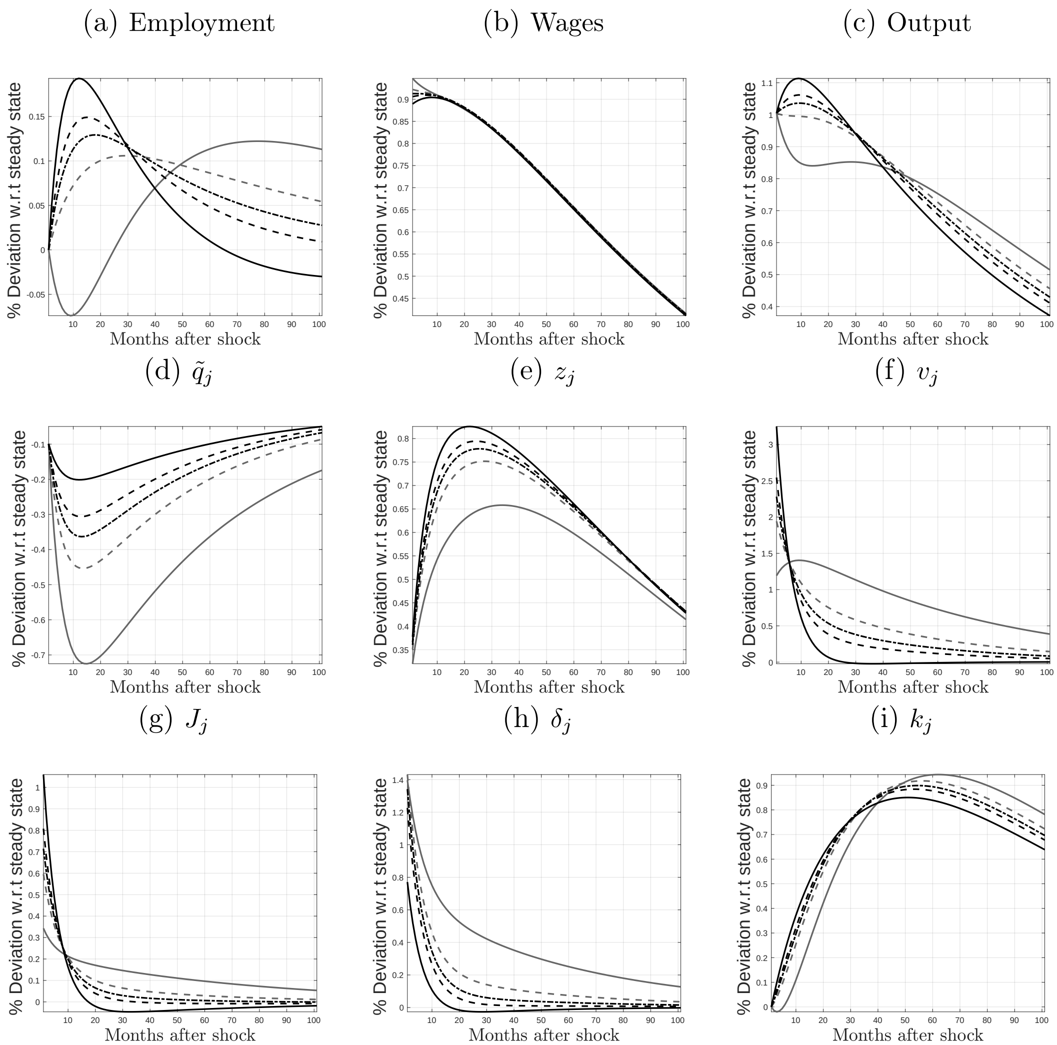
Note: This figure plots the IRFs to a 1% increase in aggregate TFP for a model in which agents face information frictions. Solid gray lines are the IRFs for firms at the 20th percentile of wage distribution. Dashed gray lines are the IRFs for firms at the 40th percentile. Dashed-dotted black lines are the IRF for firms art the 50th percentile. Dashed black lines are the IRF for firms at the 60th percentile. Solid black lines are the IRFs for firms at the 80th percentile. All IRFs are weighted by employment. z_j denotes the FOCE for firm j.

Figure 13: Distributional Dynamics to a 1% Increase in Aggregate Productivity
Model with Full Information



Note: This figure plots the IRFs to a 1% increase in aggregate TFP for a model in which agents have full information. Solid gray lines are the IRFs for firms at the 20th percentile of wage distribution. Dashed gray lines are the IRFs for firms at the 40th percentile. Dashed-dotted black lines are the IRF for firms art the 50th percentile. Dashed black lines are the IRF for firms at the 60th percentile. Solid black lines are the IRFs for firms at the 80th percentile. All IRFs are weighted by employment. z_j denotes the FOCE for firm j.

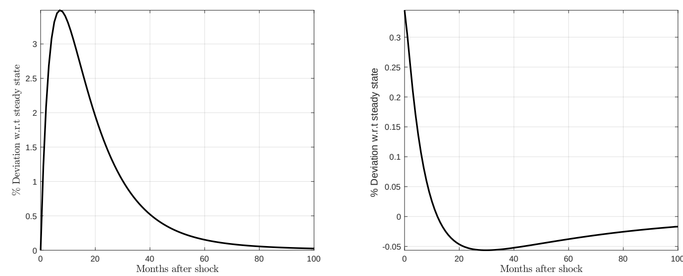
Figure 14: Distributional Dynamics to a 1% Increase in Aggregate Productivity
Model with Information Frictions (ς=0)



Note: This figure plots the IRFs to a 1% increase in aggregate TFP for a model in which agents face information frictions but seting ς=0. Solid gray lines are the IRFs for firms at the 20th percentile of wage distribution. Dashed gray lines are the IRFs for firms at the 40th percentile. Dashed-dotted black lines are the IRF for firms art the 50th percentile. Dashed black lines are the IRF for firms at the 60th percentile. Solid black lines are the IRFs for firms at the 80th percentile. All IRFs are weighted by employment. z_j denotes the FOCE for firm j.

Figure 15: Unemployment and Wage Responses to a 1% Increase in Noise Shock

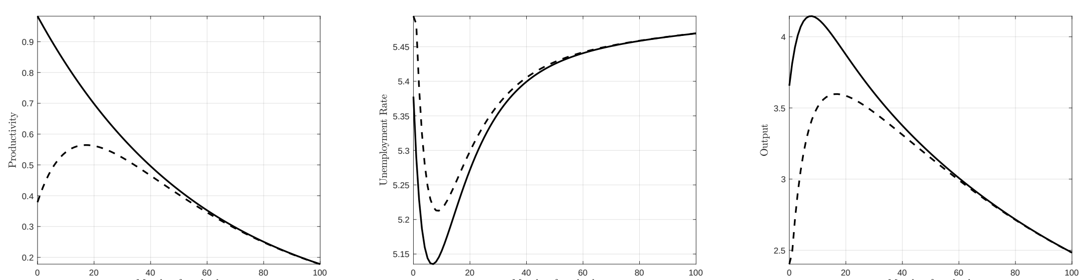
(a) Unemployment (b) Average Wage



Note: This figure plots the IRFs to a 1% increase in the signal noise (n). Lines represent percent deviation with respect to its steady state value.

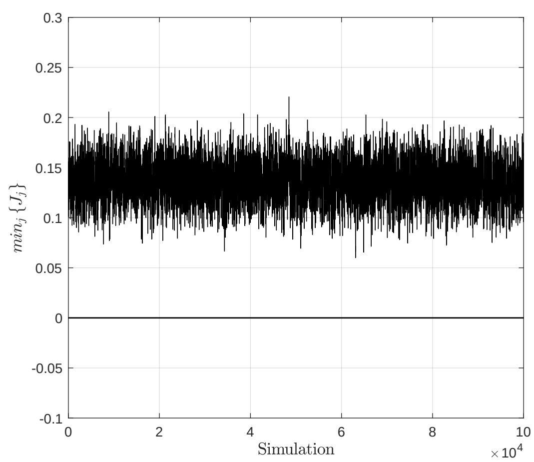
Figure 16: One Period Ahead Forecast in Response to a 1% in TFP

(a) Aggregate TFP (b) Unemployment Rate (c) Output



Note: This figure plots the one period ahead forecast for the aggregate TFP, unemployment rate and output (dashed lines) versus the realized value for those variables (solid lines), which are equal to the forecast made by a perfectly informed agent.

Figure 17: Simulated $mJ = min_j\{J_j\}$

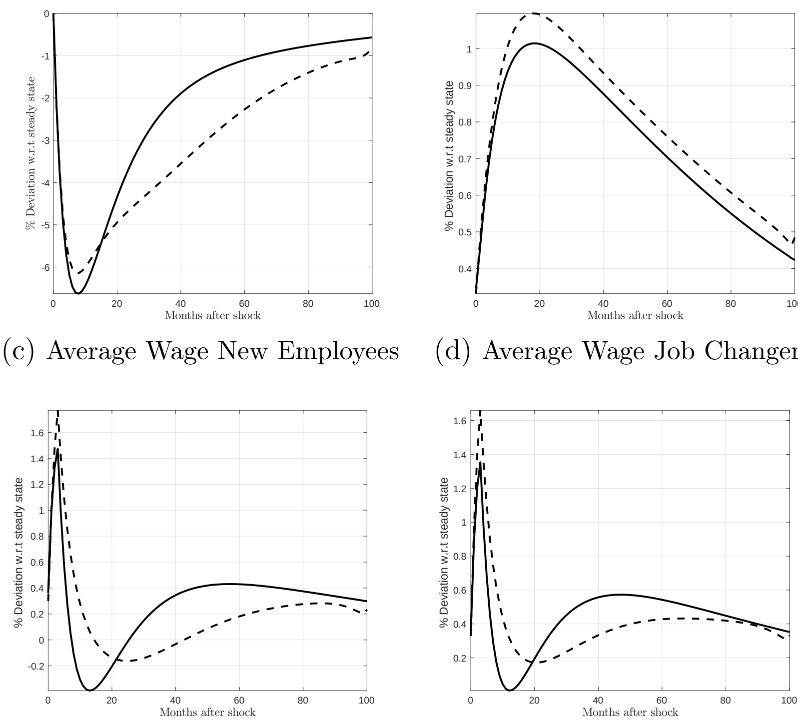


Note: This figure plots the evolution of $mJ = min_j\{J_j\}$ indicating that Assumption 1 is never violated.

Figure 18: Impulse Response Functions to a 1% Increase in Aggregate Productivity
Comparing Different Information Sets

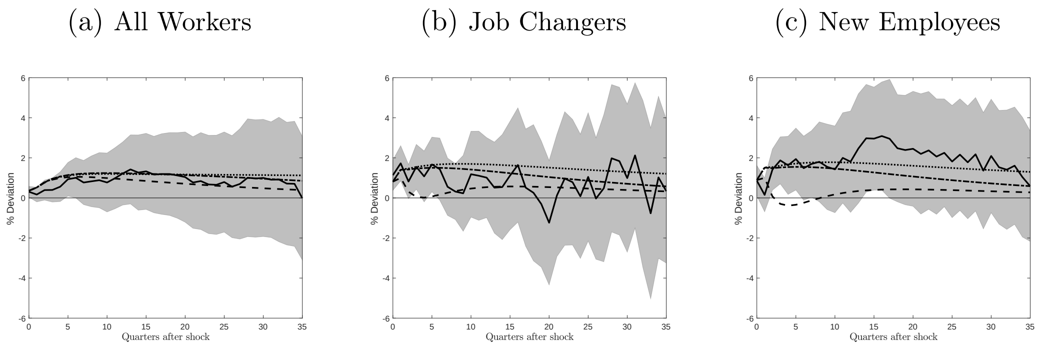
(a) Unemployment (b) Average Wage All Workers

(c) Average Wage New Employees (d) Average Wage Job Changers



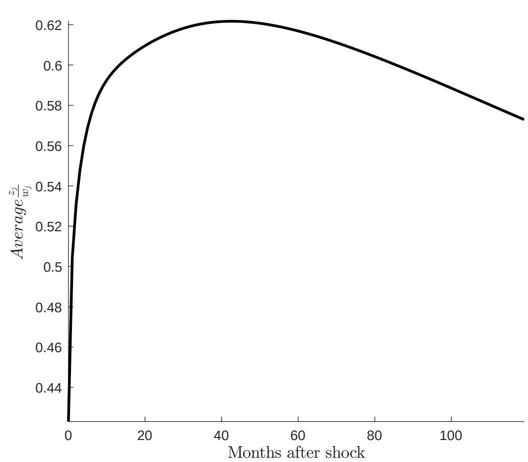
Note: This figure plots the impulse response function of unemployment and wages to a 1% increase in aggregate TFP. Solid black lines are the IRFs of my baseline model. Dashed black lines are the IRFs generated by a re-calibrated model in which workers have a larger information set as explained in section 4.

Figure 19: Wage Responses to a 1% Increase in Aggregate Productivity



Notes: This figure reports the empirical wage responses to a 1% increase in aggregate productivity (solid black lines) based on local projections. Shaded areas represent a 90% confidence interval based on heteroskedasticity-robust errors. Dashed lines represent the wage responses in the model with information frictions. Dotted lines represent the wage responses if one assumes that the flow of workers to each firm remains at the level observed at the end of the first quarter. Dash-dotted lines represent the wage responses if one assumes that the flow of workers returns to its steady-state level with a quarterly persistence of 95% after the first quarter.

Figure 20: Resonse of Average $\frac{z_j}{w_j}$ Across Firms
Model With Information Frictions



J Detailed Household's Problem

This online appendix presents the household's problem in recursive form and the complete derivation of the employment and unemployment functions. The household's utility function is given by:

$$\mathbb{U} = \frac{c^{1-\sigma}}{1-\sigma} - \Psi\frac{\tilde{h}^{1+\eta}}{1+\eta} + \beta E\left[\mathbb{U}\right] \tag{86}$$

Hence, the household's problem is:

$$\max_{c,k',\{h'_j\}_{j=0}^{1}} E_{\mathcal{I}_h}\left\{\mathbb{U}(\omega,\Omega)\right\} \tag{87}$$

subject to the budget constraint, the law of motion of labor, and the perceived law of motion of the economy:

$$c + k' = (r+1-\delta_k)k + \int_0^1 w_j h_j dj + \int_0^1 \pi_j dj + b\cdot u - T \tag{88}$$

$$h'_j = (1-\delta_h)(1-q\bar{i}F_j)h_j + q\left(\frac{v_j}{v}\right)u + \int_0^j q\bar{i}\left(\frac{v_j}{v}\right)(1-\delta_h)h_x dx \tag{89}$$

$$\tilde{h} = \left(\int_0^1 h_j^{1+\xi} dj\right)^{\frac{1}{1+\xi}} \tag{90}$$

$$u = \int_0^1 (1-h_j)dj \tag{91}$$

$$\Omega' = \lambda^h\left(\Omega\right) \tag{92}$$

where $E_{\mathcal{I}_h}[\cdot]$ is the expectation conditional on the household information set $\mathcal{I}_h$, and Ω is a vector that summarizes the aggregate state of the economy. Letting ϕ_c and ϕ_j denote the Lagrange multipliers for equations (88) and (89), the first-order conditions are given by:

$$
\begin{aligned}
c: \quad & E_{\mathcal{I}_h}\left\{c^{-\sigma}-\phi_c\right\} && =0 && (93)\\
k': \quad & E_{\mathcal{I}_h}\left\{-\phi_c+\beta\phi_c'(r'+1-\delta_k)\right\} && =0 && (94)\\
h_j': \quad & E_{\mathcal{I}_h}\{-\phi_j - E\{\beta\Psi\tilde{h}'^{\eta-\xi}{h_j'}^{\xi}+\beta\phi_c'\left(w_j'-b\right) && && \\
& +(1-\delta_h)(1-q'\bar{i}F_j')\beta\phi_j' - q'\int_0^1\beta\phi_x'\left(\frac{v_x'}{v'}\right)dx && && \\
& +(1-\delta_h)q'\bar{i}\int_j^1\beta\phi_x'\left(\frac{v_x'}{v}\right)dx\}\} && =0 && (95)
\end{aligned}
$$

Hence, combining (93) and (95) and lagging one period:

$$
\begin{aligned}
E_{\mathcal{I}_h}\{(W_j-U)\} = & E_{\mathcal{I}_h}\{w_j - z_j \\
& + E\{Q((1-\delta_h)(1-q\bar{i}F_j)(W_j-U) \\
& + (1-\delta_h)q\bar{i}F_j(\tilde{W}_j-U) \\
& - q\left(\bar{W}-U\right))\}\}
\end{aligned} \tag{96}
$$

where:

$$
(W_j-U)=\frac{\phi_j}{\beta\phi_c'} \tag{97}
$$

Also from the first-order conditions, we can verify that the optimality conditions for c is given by:

$$
c^{-\sigma}=\beta E_{\mathcal{I}_h}\left[(1-\delta+r')c'^{-\sigma}\right] \tag{98}
$$

K Recursive Competitive Equilibrium (Equations)

This online appendix presents the equations that characterize the recursive competitive equilibrium.

$$c^{-\sigma} = \beta E_{\mathcal{I}_h}\left[(1-\delta+r')c'^{-\sigma}\right] \tag{99}$$

$$\kappa(\tilde{q}_j v_j)^{\chi} = E\left[Q \cdot J_j'\right] \tag{100}$$

$$r = \quad p_j \left(\frac{h_j}{k_j}\right)\left(\frac{\alpha}{1-\alpha}\right) \tag{101}$$

$$h_j' = (1-\delta_h)(1-\bar{i}qF_j)h_j + \tilde{q}_j v_j \tag{102}$$

$$c + k' = (r+1-\delta_k)k + \int_0^1 \left[w_j h_j + \pi_j\right] dj \tag{103}$$

$$z_j = b + \Psi c^{\sigma} \tilde{h}_j^{\eta} h_j^{\xi} \tag{104}$$

$$\pi_j = y_j - w_j h_j - r k_j - \kappa(\tilde{q}_j v_j) \tag{105}$$

$$Q = \beta\left(\frac{c'}{c}\right)^{-\sigma} \tag{106}$$

$$\theta = \left(\frac{v}{s}\right) \tag{107}$$

$$y_j = e^{a_j + a} k_j^{\alpha} h_j^{1-\alpha} \tag{108}$$

$$F_j = \int_j^1 \frac{v_x}{v} dx \tag{109}$$

$$\tilde{q}_j = \tilde{q}^u + \tilde{q}_j^c \tag{110}$$

$$\tilde{q}^u = \tilde{q} \cdot \left(\frac{u}{s}\right) \tag{111}$$

$$\tilde{q}_j^c = \tilde{q} \cdot \left(\int_0^j \frac{(1-\delta_h)\bar{i}h_x}{s} dx\right) \tag{112}$$

$$q = m(v,s)/s \tag{113}$$

$$v = \int_0^1 v_j dj \tag{114}$$

$$y = \int_0^1 y_j dj \tag{115}$$

$$s = u + \int_0^1 \bar{i} h_j dj \tag{116}$$

$$u = \int_0^1 (1-h_j) dj \tag{117}$$

$$k = \int_0^1 k_j dj \tag{118}$$

$$\vartheta \cdot E_{\mathcal{I}_h}\left[S_j\right] = E_{\mathcal{I}_h}\left[\overrightarrow{W}_j(w_j) - U\right] \tag{119}$$

$$\mathbb{U} = \frac{c^{1-\sigma}}{1-\sigma} - \Psi\frac{\tilde{h}^{1+\eta}}{1+\eta} + \beta E\left[\mathbb{U}\right] \tag{120}$$

$$\tilde{h} = \left(\int_0^1 h_j^{1+\xi} dj\right)^{\frac{1}{1+\xi}} \tag{121}$$

$$U = b + E\left\{Q\left((1-q)\cdot U + q\cdot \int_0^1 W_x \frac{v_x}{v} dx\right)\right\} \tag{122}$$

$$\begin{aligned}(W_j - U) &= w_j - z_j \\ &+ E\{Q((1-\delta_h)(1-q\bar{i}F_j)(W_j - U) \\ &+ (1-\delta_h)q\bar{i}F_j(\tilde{W}_j - U) \\ &- q\left(\bar{W} - U\right))\}\end{aligned} \tag{123}$$

$$\tilde{W}_j = \int_j^1 W_x \frac{v_x}{v_t} \cdot F_j^{-1} dx \tag{124}$$

$$\bar{W} = \int_0^1 W_x \frac{v_x}{v} dx \tag{125}$$

$$\Pi_j = \pi_j + E\left[Q\Pi_j\right] \tag{126}$$

$$J_j = p_j - w_j + E\left[Q\cdot(1-\delta_h)(1-\bar{i}qF_j)\cdot J_j'\right] \tag{127}$$

$$S_j = J_j + W_j - U \tag{128}$$

$$\hat{a} = a + n \tag{129}$$

$$a' = \rho_a \cdot a + e_a' \tag{130}$$

$$n' = \rho_n \cdot n + e_n' \tag{131}$$

L Additional Model Properties

In this online appendix, I show the necessary condition for my model to be rank preserving: Wages, values of employment, and firm size are increasing in firms' own productivity a_j (Lemma 5). I also characterize sufficient conditions for the FOCE to productivity ratio $\left(\frac{z_j}{p_j}\right)$ to be increasing in firms' productivity (Lemma 6). Finally, I show the implications of these results for my model.

In this online appendix, I make two assumptions: First, the economy is in steady state. Hence, aggregate variables (variables not indexed by j) are constant. Second, in equilibrium, net values of employment offers distribute log normally with associated mean equal to 0 and associated standard deviation equal to σ_W.

L.1 Preliminary definitions

Given the assumed distribution of net values of employment offers, the following notation will be useful to proof the Lemmas discussed next. First, define the density function of net values of employment offers:

$$f^v(W_j) = f_j^v = \frac{1}{\sqrt{2\pi}\sigma_W}\frac{1}{W_j}e^{-\frac{log\left(W_j\right)^2}{2\sigma_W^2}} \tag{132}$$

Hence, we can rewrite the following model variables:

$$F(W_j) = F_j = \int_{W_j}^{\infty} f_x^v dx \tag{133}$$

$$\tilde{W}_j = F_j^{-1}\int_{W_j}^{\infty} x f_x^v dx \tag{134}$$

Then, use the job creation condition (10), the law of motion for employment, and the fact that in steady state $J_j = \frac{(1-\vartheta)}{\vartheta}W_j$, so that firm size in steady state is given by:

$$h_j = \frac{\left(\frac{\beta(1-\vartheta)}{\vartheta\kappa}W_j\right)^{\frac{1}{\chi}}}{[1-(1-\delta_s)(1-\bar{i}qF_j)]} \tag{135}$$

Plugging this equation in the definition of FOCE:

$$z_j = b + \Psi \tilde{h}^{\eta-\xi} c^{\sigma} \left(\frac{\left(\frac{\beta(1-\vartheta)}{\vartheta\kappa} W_j\right)^{\frac{1}{\chi}}}{[1-(1-\delta_s)(1-\bar{i}qF_j)]} \right)^{\xi} \tag{136}$$

Now, notice that the value of match surplus in steady state can be written as:

$$S_j = p_j - z_j + \beta(1-\delta_s)(1-\bar{i}qF_j)S_j + (1-\delta_s)\bar{i}q \int_{W_j}^{\infty} x f_x^v dx - \beta q \bar{W} \tag{137}$$

Finally, it is useful to note that, in equilibrium:

$$p_j = e^{a_j+a}(1-\alpha)\left(\frac{k_j}{h_j}\right)^{\alpha} \tag{138}$$

$$p_j = \left(e^{a_j+a}(1-\alpha)\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} \tag{139}$$

$$\frac{\partial p_j}{\partial a_j} > 0 \tag{140}$$

L.2 Lemmas

Lemma 5. *If net values of employment offers distribute log-normally with associated mean equal to 0 and associated standard deviation equal to σ_W, in equilibrium and in steady state, firm size (h_j) and net values of employment (W_j) are strictly increasing in firms' own productivity (a_j) if:*

$$\mathcal{Y}_j = 1 - \beta(1-\delta_s)\left[1 - \bar{i}qF(W^{RP}) + \bar{i}qf^v(W^{RP})(1-\vartheta)W^{RP}\right] > 0 \tag{141}$$

where $W^{RP} = e^{\frac{\sigma_W^2}{2(1-\vartheta)}}$. Additionally, if $\xi > 0$ and condition (141) is satisfied in steady state, z_j is increasing in firms' own productivity in steady state.

Proof. To proof this Lemma, I proceed in two steps. First, I will proof that h_j and z_j (provided that $\xi > 0$) are always increasing in W_j. Then, I will proof that W_j is increasing in p_j, and as a consequence, in a_j (140), if condition (141) is satisfied. The idea is to show that if condition (141) holds in steady state, W_j is increasing in a_j and, as a consequence, so are h_j and z_j given that those variables are strictly increasing in W_j.

First, to proof that h_j and z_j are increasing in W_j, I take the first derivative of (135) and (136) with respect to W_j:

$$\frac{\partial h_j}{\partial W_j} = h_j \left[\frac{1}{\chi W_j} + \frac{(1-\delta_s)\bar{i}qf_j^v}{[1-(1-\delta_s)(1-\bar{i}qF_j)]} \right] > 0 \tag{142}$$

$$\frac{\partial z_j}{\partial W_j} = (z_j - b)\xi \left[\frac{1}{\chi W_j} + \frac{(1-\delta_s)\bar{i}qf_j^v}{[1-(1-\delta_s)(1-\bar{i}qF_j)]} \right] \geq 0 \tag{143}$$

Second, to proof that $\frac{\partial W_j}{\partial p_j} > 0$ if condition (141) is satisfied, I proceed to find an expression for p_j in terms of W_j. Then, I compute $\frac{\partial p_j}{\partial W_j}$. Now, solving for p_j using the match surplus in steady state (137):

$$p_j = z_j + [1-\beta(1-\delta_s)(1-\bar{i}qF_j)]\, S_j + \beta q\bar{W} - \beta(1-\delta_s)\bar{i}q \int_{W_j}^{\infty} x f_x^v dx \tag{144}$$

$$p_j = z_j + [1-\beta(1-\delta_s)(1-\bar{i}qF_j)]\, \vartheta^{-1} W_j + \beta q\bar{W} - \beta(1-\delta_s)\bar{i}q \int_{W_j}^{\infty} x f_x^v dx \tag{145}$$

and computing $\frac{\partial p_j}{\partial W_j}$ based on this equation:

$$\frac{\partial p_j}{\partial W_j} = \frac{\partial z_j}{\partial W_j} + [1-\beta(1-\delta_s)(1-\bar{i}qF_j)]\, \vartheta^{-1} - \beta(1-\delta_s)\bar{i}qf_j^v \vartheta^{-1}(1-\vartheta)W_j \tag{146}$$

Hence, for any value of ξ, $\frac{\partial p_j}{\partial W_j} > 0$ if:

$$\mathcal{Y}_j = [1-\beta(1-\delta_s)(1-\bar{i}qF_j)] - \beta(1-\delta_s)\bar{i}qf_j^v(1-\vartheta)W_j \qquad > 0 \tag{147}$$

$$\mathcal{Y}_j = 1-\beta(1-\delta_s)\left[1-\bar{i}qF_j + \bar{i}qf_j^v(1-\vartheta)W_j\right] \qquad > 0 \tag{148}$$

Now, by taking the derivative of (148) with respect to W_j and given that net values of employment offers distribute log-normally, we can confirm that $\mathcal{Y}_j$ has a global minimum at $W = e^{\frac{\sigma_W^2}{(1-\vartheta)}} = W^{RP}$. As a consequence, $\frac{\partial p_j}{\partial W_j}$ is always greater than 0 for any value of ξ if $\mathcal{Y}_j$ evaluated at $W = W^{RP}$ is greater than 0

$$\mathcal{Y}\left(W^{RP}\right) = 1-\beta(1-\delta_s)\left[1-\bar{i}qF(W^{RP}) + \bar{i}qf^v(W^{RP})(1-\vartheta)W^{RP}\right] > 0 \tag{149}$$

$\square$

Lemma 6. *Assuming that net values of employment offers distribute log-normally with* *associated mean equal to zero and associated standard deviation equal to σ_W, assuming that $\xi > 0$, and assuming that condition (141) holds (meaning that W_j, h_j, and z_j are increasing in j); in steady state:*

- *If $\xi > \chi$, there is a productivity threshold ($\underline{a}^{zp}$) such that for all $a_j \geq \underline{a}^{zp}$ the ratio $\frac{z_j}{p_j}$ is increasing in j (increasing in firms' own productivity). If $b = 0$, $\underline{a}^{zp} = 0$ and the ratio $\frac{z_j}{p_j}$ is always increasing in j. Additionally, if $\underline{W}^{zp} \leq 1$, the productivity threshold $\underline{a}^{zp}$ is strictly increasing in b, where $\underline{W}^{zp}$ solves:*

$$b + \Psi \tilde{h}^{\eta-\xi} c^{\sigma} \left(\frac{\left(\frac{\beta(1-\vartheta)}{\vartheta\kappa} \underline{W}^{zp} \right)^{\frac{1}{\chi}}}{[1 - (1-\delta_s)(1 - \bar{i}qF(\underline{W}^{zp}))]} \right)^{\xi} = b \left(\frac{\xi}{\xi - \chi} \right) \tag{150}$$

- *If $\xi < \chi$, there is a productivity threshold ($\bar{a}^{zp}$) such that for all $a_j \leq \bar{a}^{zp}$ the ratio $\frac{z_j}{p_j}$ is increasing in j (increasing in firms' own productivity).*

Proof. To proof this Lemma, I proceed as follows: (1) I compute the derivative of $\frac{z_j}{p_j}$ with respect to W_j. (2), I show that $\frac{z_j}{p_j}$ is always increasing in W_j, and as consequence in j by Lemma 5, if $b = 0$ and $\xi > \chi$. (3) I show that if $b > 0$, $\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j}$ converges to a negative number when W_j converges to 0 (and therefore a_j is small) if ξ is greater than χ, and $\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j}$ converges to a negative number when W_j (and a_j) converges to ∞ if ξ is smaller than χ. Hence, I cannot claim that the ratio $\frac{z_j}{p_j}$ is always increasing in j if $b > 0$. (4) I show the existence of the productivity threshold $\underline{a}^{zp}$ and its aforementioned properties. Finally, (5) I show the existence of the productivity threshold $\bar{a}^{zp}$.

Computing $\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j}$

Given that I am assuming that condition (141) holds, by Lemma 5, if $\frac{z_j}{p_j}$ is increasing in W_j so is in j. Hence, let me find under which conditions, the ratio $\frac{z_j}{p_j}$ is increasing in W_j.

$$\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j} = \frac{1}{p_j}\left[\frac{\partial z_j}{\partial W_j} - \left(\frac{z_j}{p_j}\right)\frac{\partial p_j}{\partial W_j}\right] \tag{151}$$

Substituting in equation (146) and organizing:

$$\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j} = \left(\frac{1}{p_j^2}\right)\left\{ (p_j - z_j)\frac{\partial z_j}{\partial W_j} \right.$$
$$\left. - z_j\left[\left[1-\beta(1-\delta_s)(1-\bar{i}qF_j)\right]\vartheta^{-1} - \beta(1-\delta_s)\bar{i}qf_j^vW_j\left(\frac{1-\vartheta}{\vartheta}\right)\right]\right\} \quad (152)$$

using (145) to replace the first parenthesis:

$$\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j} = \left(\frac{1}{p_j^2}\right)\left\{ \left[1-\beta(1-\delta_s)(1-\bar{i}qF_j)\right]\vartheta^{-1}z_j\left(\frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j} - 1\right) \right.$$
$$\left. + \beta q\Omega_j\frac{\partial z_j}{\partial W_j} + z_j\beta(1-\delta_s)\bar{i}qf_j^vW_j\left(\frac{1-\vartheta}{\vartheta}\right)\right\} \quad (153)$$

where $\Omega_j = \bar{W} - (1-\delta_s)\bar{i}\int_{W_j}^{\infty} xf_x^v dx > 0$, and $\frac{\partial\Omega_j}{\partial W_j} > 0$.

If $\xi > \chi$ and $b = 0$, the ratio $\frac{z_j}{p_j}$ is always increasing in j:

Notice that $\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j}$ is always positive if the elasticity of FOCE with respect to W_j $\left(\frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j}\right)$ is greater than or equal to 1. This would be always the case if $b = 0$ and $\xi > \chi$. One can use (143) to see this:

$$\frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j} = \left(\frac{\xi}{\chi}\right)\left(1 - \frac{b}{z_j}\right)\left[1 + \chi\frac{(1-\delta_s)\bar{i}qf_j^vW_j}{[1-(1-\delta_s)(1-\bar{i}qF_j)]}\right] \quad (154)$$
$$> \left(\frac{\xi}{\chi}\right)\left(1 - \frac{b}{z_j}\right) \quad (155)$$

As a result, if $b = 0$ and $\xi > \chi$, $\left(\frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j}\right) > 1$ and the ratio $\frac{z_j}{p_j}$ is always increasing in firms' own productivity (given that W_j is increasing in a_j) because the vale for FOCE grows faster than labor productivity.

if $b > 0$, $\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j}$ converges to a negative number when W_j converges to 0 (and therefore a_j is small) if ξ is greater than χ, and $\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j}$ converges to a negative number when W_j (and a_j) converges to ∞ if ξ is smaller than χ

To make this point, I will take the limits of $\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j}$ when W_j converges to 0 and when W_j converges to ∞ to show that that derivative converges to a positive or negative number depending on the relationship between ξ and χ.

First, the limit of p_j, z_j, Ω_j, and $\frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j}$ when W_j converges to 0 are given by:

$$\lim_{W_j\to 0} p_j = b + \beta q\bar{W}\left[1-(1-\delta_s)\bar{i}\right] \tag{156}$$

$$\lim_{W_j\to 0} z_j = b \tag{157}$$

$$\lim_{W_j\to 0} \Omega_j = \bar{W}\left[1-(1-\delta_s)\bar{i}\right] \tag{158}$$

$$\lim_{W_j\to 0} \frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j} = 0 \tag{159}$$

The same limits when W_j converges to ∞ are given by:

$$\lim_{W_j\to \infty} p_j = \infty \tag{160}$$

$$\lim_{W_j\to \infty} z_j = \infty \tag{161}$$

$$\lim_{W_j\to \infty} \Omega_j = \bar{W} \tag{162}$$

$$\lim_{W_j\to \infty} \frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j} = \frac{\xi}{\chi} \tag{163}$$

To find the respective limits for $\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j}$, we are missing the limits of $\frac{\partial z_j}{\partial W_j}$. To that end, notice that after some manipulation, $\frac{\partial z_j}{\partial W_j}$ can be written as:

$$\frac{\partial z_j}{\partial W_j} = \underbrace{\Psi\tilde{h}^{\eta-\xi}c^{\sigma}\left(\frac{\left(\frac{\beta(1-\vartheta)}{\vartheta}\right)^{\frac{\xi}{\chi}}}{\left[1-(1-\delta_s)(1-\bar{i}qF_j)\right]^{\xi}}\right)W_j^{\frac{\xi}{\chi}}}_{(z_j-b)}\left(\frac{\xi}{\chi}\right)\left(\frac{1}{W_j}+\frac{(1-\delta_s)\bar{i}qf_j^v}{\left[1-(1-\delta_s)(1-\bar{i}qF_j)\right]}\right) \tag{164}$$

$$\frac{\partial z_j}{\partial W_j} = \left\{\Psi\tilde{h}^{\eta-\xi}c^{\sigma}\left(\frac{\left(\frac{\beta(1-\vartheta)}{\vartheta}\right)^{\frac{\xi}{\chi}}}{\left[1-(1-\delta_s)(1-\bar{i}qF_j)\right]^{\xi}}\right)\left(\frac{\xi}{\chi}\right)\left(1+\frac{(1-\delta_s)\bar{i}qf_j^vW_j}{\left[1-(1-\delta_s)(1-\bar{i}qF_j)\right]}\right)\right\}W_j^{\frac{\xi}{\chi}-1} \tag{165}$$

Notice that the term within braces converges to a positive and finite number when

W_j converges to 0 and when W_j converges to ∞. Hence, the limit of $\frac{\partial z_j}{\partial W_j}$ will depend on the relationship between ξ and χ:

$$\lim_{W_j \to 0} \frac{\partial z_j}{\partial W_j} = \begin{cases} 0 & if \quad \frac{\xi}{\chi} > 1 \\ \Psi \tilde{h}^{\eta-\xi} c^{\sigma} \left(\frac{\left(\frac{\beta(1-\vartheta)}{\vartheta}\right)^{\frac{\xi}{\chi}}}{[1-(1-\delta_s)(1-\bar{i}q)]^{\xi}} \right) & if \quad \frac{\xi}{\chi} = 1 \\ \infty & if \quad \frac{\xi}{\chi} < 1 \end{cases} \tag{166}$$

Similarly:

$$\lim_{W_j \to \infty} \frac{\partial z_j}{\partial W_j} = \begin{cases} \infty & if \quad \frac{\xi}{\chi} > 1 \\ \Psi \tilde{h}^{\eta-\xi} c^{\sigma} \left(\frac{\left(\frac{\beta(1-\vartheta)}{\vartheta}\right)^{\frac{\xi}{\chi}}}{\delta_s^{\xi}} \right) & if \quad \frac{\xi}{\chi} = 1 \\ 0 & if \quad \frac{\xi}{\chi} < 1 \end{cases} \tag{167}$$

Hence, going back to equation (153):

$$\lim_{W_j \to 0} \frac{\partial \left(\frac{z_j}{p_j}\right)}{\partial W_j} = \begin{cases} \left(\frac{-[1-\beta(1-\delta_s)(1-\bar{i}q)]\vartheta^{-1} b}{\left(b+\beta q \bar{W}[1-(1-\delta_s)\bar{i}]\right)^2} \right) < 0 & if \quad \xi > \chi \\ \left(\frac{-[1-\beta(1-\delta_s)(1-\bar{i}q)]\vartheta^{-1} b + \beta q \bar{W}[1-(1-\delta_s)\bar{i}] \Psi \tilde{h}^{\eta-\xi} c^{\sigma} \left(\frac{\left(\frac{\beta(1-\vartheta)}{\vartheta}\right)^{\frac{\xi}{\chi}}}{[1-(1-\delta_s)(1-\bar{i}q)]^{\xi}} \right)}{\left(b+\beta q \bar{W}[1-(1-\delta_s)\bar{i}]\right)^2} \right) & if \quad \xi = \chi \\ \infty & if \quad \xi < \chi \end{cases} \tag{168}$$

The $\lim_{W_j \to \infty} \frac{\partial \left(\frac{z_j}{p_j}\right)}{\partial W_j}$ is more cumbersome to compute. Given that I am interested only in the sign of the limit (positive or negative), and given that $p_j > 0$, I can rely on the term within braces in equation (153) to conclude that: Similarly:

$$\lim_{W_j \to \infty} \frac{\partial \left(\frac{z_j}{p_j}\right)}{\partial W_j} = \begin{cases} dz1 \geq 0 & if \quad \xi > \chi \\ 0 & if \quad \xi = \chi \\ dz2 \leq 0 & if \quad \xi < \chi \end{cases} \tag{169}$$

where $dz1$ and $dz2$ are positive and negative numbers (possibly infinitive), respectively.

Existence of productivity threshold $\underline{a}^{zp}$ and its properties

Now, let me show that there is a productivity value ($\underline{a}^{zp}$) above which the ratio $\frac{z_j}{p_j}$ is always increasing in j. Based on equation (155), if:

$$\left(\frac{\xi}{\chi}\right)\left(1-\frac{b}{z_j}\right) > 1, \tag{170}$$

then $\left(\frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j}\right)$ is greater than or equal to 1 and, as a consequence, the ratio $\frac{z_j}{p_j}$ is increasing in j. (170) is true if:

$$z_j \geq b\left(\frac{\xi}{\xi-\chi}\right) \tag{171}$$

Hence, given that condition(141) holds, z_j is increasing in j. Therefore, there is a productivity value above which (171) is satisfied. As a consequence, there is a productivity value $\underline{a}^{ZP}$ such that for all $a_j \geq \underline{a}^{zp}$ $\frac{z_j}{p_j}$ is increasing in j. Now, we can rewrite (153) as follows:

$$\begin{aligned}\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j} = &\left(\frac{\left[1-\beta(1-\delta_s)(1-\bar{i}qF_j)\right]\vartheta^{-1}}{p_j^2}\frac{z_j}{W_j}\right)\left\{W_j\left(\frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j}-1\right)\right.\\ &+\frac{\beta q\Omega_j}{\left[1-\beta(1-\delta_s)(1-\bar{i}qF_j)\right]}\frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j}\vartheta^{-1}\\ &\left.+\frac{\beta(1-\delta_s)\bar{i}qf_j^vW_j^2}{\left[1-\beta(1-\delta_s)(1-\bar{i}qF_j)\right]}(1-\vartheta)\right\}\end{aligned} \tag{172}$$

Hence, $\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j} > 0$ if the term between the braces, which I define as $\mathcal{X}_j$, is positive:

$$\begin{aligned}\mathcal{X}_j = W_j\left(\frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j}-1\right)+&\frac{\beta q\Omega_j}{\left[1-\beta(1-\delta_s)(1-\bar{i}qF_j)\right]}\frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j}\vartheta^{-1}\\ &+\frac{\beta(1-\delta_s)\bar{i}qf_j^vW_j^2}{\left[1-\beta(1-\delta_s)(1-\bar{i}qF_j)\right]}(1-\vartheta) > 0\end{aligned} \tag{173}$$

Based on equation (155), the FOCE elasticity with respect to W_j is increasing in W_j (and as a consequence in a_j) if $W_j \leq 1$ because f_j^v has a global maximum at 1.

For the same reason, we can claim that the last term in equation (173) is increasing in W_j if $W_j \leq 1$. As a consequence, we can conclude that $\mathcal{X}_j$ is strictly increasing in W_j for all net values of employments less than 1. Now, define $\underline{W}^{zp}$ as the net value of employment that makes (171) hold:

$$b + \Psi \tilde{h}^{\eta-\xi} c^{\sigma} \left(\frac{\left(\frac{\beta(1-\vartheta)}{\vartheta\kappa} W^{zp}\right)^{\frac{1}{\chi}}}{\left[1-(1-\delta_s)(1-\bar{i}qF(W^{zp}))\right]} \right)^{\xi} = b\left(\frac{\xi}{\xi-\chi}\right) \tag{174}$$

Given that $\mathcal{X}_j$ is strictly increasing in W_j for all values less than 1, if $\underline{W}^{zp}$ is less than 1, we can claim that the productivity threshold ($\underline{a}^{zp}$) is strictly decreasing in b. Based on the data, this case is the most likely because b is small and represents only 6% of labor productivity.

Existence of productivity threshold $\bar{a}^{zp}$

Going back and rewriting equation (153):

$$\begin{aligned} \frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j} &= \left(\frac{1}{p_j^2}\right)\left\{ \left[1-\beta(1-\delta_s)(1-\bar{i}qF_j)\right]\vartheta^{-1} z_j \left(\frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j} - 1\right) \right. \\ &\quad \left. + \beta q \Omega_j \frac{\partial z_j}{\partial W_j} + z_j \beta(1-\delta_s)\bar{i}q f_j^v W_j \left(\frac{1-\vartheta}{\vartheta}\right) \right\} \end{aligned} \tag{175}$$

$$\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j} = \left(\frac{\vartheta^{-1}}{p_j^2}\right)\mathcal{Z}_j \tag{176}$$

where:

$$\begin{aligned} \mathcal{Z}_j &= \left[1-\beta(1-\delta_s)(1-\bar{i}qF_j)\right] z_j \left(\frac{\partial z_j}{\partial W_j}\frac{W_j}{z_j} - 1\right) \\ &\quad + \beta q \vartheta \Omega_j \frac{\partial z_j}{\partial W_j} + z_j\beta(1-\delta_s)\bar{i}q f_j^v W_j (1-\vartheta) \end{aligned} \tag{177}$$

$$\tag{178}$$

Hence, $\frac{\partial\left(\frac{z_j}{p_j}\right)}{\partial W_j} > 0$ if $\mathcal{Z}_j > 0$. Notice that:

$$\mathcal{Z}_j > -z_j + \beta q \Omega_j \frac{\partial z_j}{\partial W_j} + z_j \vartheta \beta (1-\delta_s) \bar{i} q f_j^v W_j (1-\vartheta) \tag{179}$$

$$\mathcal{Z}_j > -z_j + \Omega_j \frac{\partial z_j}{\partial W_j} \tag{180}$$

$$\mathcal{Z}_j > -b - \Psi \tilde{h}^{\eta-\xi} c^\sigma \left(\frac{\left(\frac{\beta(1-\vartheta)}{\vartheta}\right)^{\frac{\xi}{\chi}}}{[1-(1-\delta_s)(1-\bar{i}qF_j)]^{\xi}} \right) W_j^{\frac{\xi}{\chi}}$$
$$\left\{ \Psi \tilde{h}^{\eta-\xi} c^\sigma \left(\frac{\left(\frac{\beta(1-\vartheta)}{\vartheta}\right)^{\frac{\xi}{\chi}}}{[1-(1-\delta_s)(1-\bar{i}qF_j)]^{\xi}} \right) \left(\frac{\xi}{\chi}\right) \left(1 + \frac{(1-\delta_s)\bar{i}qf_j^v W_j}{[1-(1-\delta_s)(1-\bar{i}qF_j)]}\right) \right\} W_j^{\frac{\xi}{\chi}-1} \tag{181}$$

$$\mathcal{Z}_j > -b - \Psi \tilde{h}^{\eta-\xi} c^\sigma \left(\frac{\left(\frac{\beta(1-\vartheta)}{\vartheta}\right)^{\frac{\xi}{\chi}}}{[1-(1-\delta_s)(1-\bar{i}qF_j)]^{\xi}} \right) W_j^{\frac{\xi}{\chi}} + \Psi \tilde{h}^{\eta-\xi} c^\sigma \left(\frac{\beta(1-\vartheta)}{\vartheta}\right)^{\frac{\xi}{\chi}} \left(\frac{\xi}{\chi}\right) W_j^{\frac{\xi}{\chi}-1} \tag{182}$$

Given that $\xi < \chi$ there is a unique value for W_j below which the previous term is always positive. Therefore, there exists a productivity threshold $\bar{a}^{zp}$ below which the ratio $\frac{z_j}{p_j}$ is increasing in j. □

L.3 Implications

In my baseline model with information frictions, condition (141) holds. In that model, $b > 0$ and $\xi > \chi$. Hence, for small productivity values, the ratio $\frac{z_j}{p_j}$ is decreasing in j. However, given my calibration, the fraction of firms with a productivity value below $\underline{a}^{zp}$ is indistinguishable from 0. In my baseline model with full information, condition (141) also holds. However, in that calibrated model $b > 0$ and $\xi < \chi$. Hence, for large productivity values the ratio $\frac{z_j}{p_j}$ is decreasing in j. However, given my calibration, the fraction of firms with a productivity value above $\underline{a}^{zp}$ is indistinguishable from 0. Table 27 presents those results.

The second column shows the value for $\mathcal{Y}(W^{RP})$ for the calibrated model with information frictions (first row) and full information (second row). In both cases, $\mathcal{Y}(W^{RP})$ is positive, which implies that in equilibrium W_j, h_j and z_j are increasing in j.

The third and fourth columns present the values of the productivity thresholds $\underline{a}^{zp}$ and $\bar{a}^{zp}$ as described in Lemma 6. Given that $\xi > \chi$ in the calibrated model with information frictions, the value of $\bar{a}^{zp}$ is ∞ in that case. Similarly, given that $\xi < \chi$ in the calibrated model with full information, the value of $\underline{a} = 0$ in that case.

The fifth and sixth columns present the net values of employment associated with the corresponding productivity thresholds $\underline{a}^{zp}$ and $\bar{a}^{zp}$. Finally, the last two columns present the probability of receiving a job offer with associated net value of employment below $W\left(\underline{a}^{zp}\right)$ and above $W\left(\bar{a}^{zp}\right)$, respectively.[18] In particular, the last two columns report:

$$Pr\left[W < W\left(\underline{a}^{zp}\right)\right] = \int_0^{W\left(\underline{a}^{zp}\right)} f_x^v dx \tag{183}$$

$$Pr\left[W > W\left(\bar{a}^{zp}\right)\right] = \int_{W\left(\bar{a}^{zp}\right)}^{\infty} f_x^v dx \tag{184}$$

Table 27: Model Properties in Steady State

Model	$\mathcal{Y}(W^{RP})$	$\underline{a}^{zp}$	$\bar{a}^{zp}$	$W\left(\underline{a}^{zp}\right)$	$W\left(\bar{a}^{zp}\right)$	$Pr\left[W < W\left(\underline{a}^{zp}\right)\right]$	$Pr\left[W > W\left(\bar{a}^{zp}\right)\right]$
Informatin Frictions	0.02	0.12	∞	0.08	∞	0.00	0.00
Full Information	0.02	0.00	876.71	0.00	35.97	0.00	0.00

Notes: Notes: The second columns report the value of $\mathcal{Y}(W^{RP})$ as described in Lemma 5. The remaining columns report the productivity thresholds $\underline{a}^{zp}$ and $\bar{a}^{zp}$ as described in Lemma 6, their associated values of employment, and mass of firms offering net values of employment below and above those values. The first row reports numbers for the baseline model with information frictions. The second row reports values for the baseline model with full information.

[18]This measure is a proxy for the mass of firms with productivity values below (above) $\underline{a}^{zp}$ ($\bar{a}^{zp}$). In my calibrated models, the productivity thresholds lie outside of my grid points, which is not surprising given that they are extreme points. Using the calibrated distribution of firms to approximate the mass of firms below and above those productivity thresholds results in the same answer: The mass is indistinguishable from 0.