Quantitative Easing, Collateral Constraints, and Financial Spillovers

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ONLINE APPENDIX

A Appendix to the Main Text

A.1 A System of Non-linear Equations Used to Characterize the Two-Country Collateral Equilibrium with Monetary Policy

Consider the following set of equalities and inequalities:

First-order, budget and boundary conditions for agent h in AE:

First-order condition for c_0^h at date $t = 0$:

$$\partial_0 u^h(\bar{c}^h) - \bar{\delta}_0^h +_c \bar{\delta}_0^h = 0; \quad \forall h \in H \tag{1}$$

First-order condition for y_{AE}^h at date $t = 0$:

$$-\bar{\delta}_0^h \bar{\pi}_{AE} + \sum_{s \in S} d_s^{AE} \bar{\delta}_s^h +_{col} \bar{\delta}_{AE}^h +_y \bar{\delta}_{AE}^h = 0; \quad \forall h \in H \tag{2}$$

First-order condition for y_{EM}^h at date $t = 0$:

$$-\bar{\delta}_0^h \bar{\pi}_{EM} +_y \bar{\delta}_{EM}^h + \sum_{s \in S} d_s^{EM} \bar{\delta}_s^h = 0 \tag{3}$$

First-order condition for c_s^h at date $t = 1$, $\forall s \in S$:

$$\partial_s u^h(\bar{c}^h) - \bar{\delta}_s^h +_c \bar{\delta}_s^h = 0; \quad \forall s \in S; \forall h \in H \tag{4}$$

First-order condition for μ^h:

$$-\bar{\delta}_0^h \bar{q}_0 +_\mu \bar{\delta}^h + \sum_{s \in S} \bar{\delta}_s^h = 0; \quad \forall h \in H \tag{5}$$

First-order condition for ψ_j^h:

$$-\bar{\delta}_0^h \bar{q}_j +_\psi \bar{\delta}_j^h + \sum_{s \in S} \bar{\delta}_s^h r_{sj} = 0; \quad \forall h \in H; \forall j \in J \tag{6}$$

where $r_{sj} = min\{j, d_s^{AE}\}$.

First-order condition for φ_j^h:

$$\bar{\delta}_0^h \bar{q}_j +_{\varphi} \bar{\delta}_j^h - \sum_{s \in S} \bar{\delta}_s^h r_{sj} -_{col} \bar{\delta}_{AE}^h = 0; \quad \forall h \in H; \forall j \in J \tag{7}$$

Budget constraint at date $t = 0$:

$$\begin{aligned}(\bar{c}_0^h - e_{C_0}^h) + \bar{\pi}_{AE}(\bar{y}_{AE}^h - \bar{e}_{Y_{AE}}^h) + \bar{\pi}_{EM}\bar{y}_{EM}^h \\ +\bar{q} \cdot (\bar{\psi}^h - \bar{\varphi}^h) + \bar{q}_0(\bar{\mu}^h - e_B^h) = 0,\end{aligned} \tag{8}$$

Budget constraint at date $t = 1$:

$$\begin{aligned}(\bar{c}_s^h - e_{C_s}^h - d_s^{AE} y_{AE}^h - d_s^{EM} y_{EM}^h) - \sum_{j=1}^{S} (\bar{\psi}^h - \bar{\varphi}^h)\bar{r}_{sj} \\ +\bar{T}_s^h - \bar{\mu}^h = 0, \forall s \in S\end{aligned} \tag{9}$$

The boundary conditions:

$$_c\bar{\delta}_0^h \bar{c}_0^h = 0; \quad \forall h \in H \tag{10}$$

$$_y\bar{\delta}_{AE}^h \bar{y}_{AE}^h = 0; \quad \forall h \in H \tag{11}$$

$$_y\bar{\delta}_{EM}^h \bar{y}_{EM}^h = 0; \quad \forall h \in H \tag{12}$$

$$_c\bar{\delta}_s^h \bar{c}_s^h = 0; \quad \forall h \in H; \forall s \in S \tag{13}$$

$$_\mu\bar{\delta}_s^h \bar{\mu}_s^h = 0; \quad \forall h \in H; \forall s \in S \tag{14}$$

$$_\psi\bar{\delta}_j^h \bar{\psi}_j^h = 0; \quad \forall h \in H; \forall j \in J \tag{15}$$

$$_\varphi\bar{\delta}_j^h \bar{\varphi}_j^h = 0; \quad \forall h \in H; \forall j \in J \tag{16}$$

$$_{col}\bar{\delta}_{AE}^h (-\bar{y}_{AE}^h + \sum_{j \in J} \bar{\varphi}_j^h) = 0; \quad \forall h \in H \tag{17}$$

First-order, budget and boundary conditions for agent h^* in EM:

First-order condition for $c_0^{h^*}$ at date $t = 0$:

$$\partial_0 u^{h^*}(\bar{c}^{h^*}) - \bar{\delta}_0^{h^*} +_c \bar{\delta}_0^{h^*} = 0; \quad \forall h^* \in H^* \tag{18}$$

First-order condition for $y_{EM}^{h^*}$ at date $t = 0$:

$$-\bar{\delta}_0^{h^*} \bar{\pi}_{EM} + \sum_{s \in S} d_s^{EM} \bar{\delta}_s^{h^*} +_y \bar{\delta}_{EM}^{h^*} = 0; \quad \forall h^* \in H^* \tag{19}$$

First-order condition for $y_{AE}^{h^*}$ at date $t = 0$:

$$-\bar{\delta}_0^{h^*} \bar{\pi}_{AE} + \sum_{s \in S} d_s^{AE} \bar{\delta}_s^{h^*} +_{col} \bar{\delta}_{AE}^{h^*} +_y \bar{\delta}_{AE}^{h^*} = 0 \tag{20}$$

First-order condition for $c_s^{h^*}$ at date $t = 1$ $\forall s \in S$:

$$\partial_s u^{h^*}(\bar{c}^{h^*}) - \bar{\delta}_s^{h^*} +_c \bar{\delta}_s^{h^*} = 0; \quad \forall s \in S; \forall h^* \in H^* \tag{21}$$

First-order condition for μ^{h^*}:

$$-\bar{\delta}_0^{h^*} \bar{q}_0 +_\mu \bar{\delta}^{h^*} + \sum_{s \in S} \bar{\delta}_s^{h^*} = 0; \quad \forall h^* \in H^* \tag{22}$$

First-order condition for $\psi_j^{h^*}$:

$$-\bar{\delta}_0^{h^*} \bar{q}_j +_\psi \bar{\delta}_j^{h^*} + \sum_{s \in S} \bar{\delta}_s^{h} \bar{r}_{sj} = 0; \quad \forall h^* \in H^*; \forall j \in J \tag{23}$$

First-order condition for $\varphi_j^{h^*}$:

$$\bar{\delta}_0^{h^*} \bar{q}_j +_\varphi \bar{\delta}_j^{h^*} - \sum_{s \in S} \bar{\delta}_s^{h^*} \bar{r}_{sj} -_{col} \bar{\delta}_{AE}^{h^*} = 0; \quad \forall h^* \in H^*; \forall j \in J \tag{24}$$

Budget constraint at date $t = 0$:

$$\begin{aligned} (\bar{c}_0^{h^*} - e_{C_0}^{h^*}) + \bar{\pi}_{EM}(\bar{y}_{EM}^{h^*} - \bar{e}_{Y_{EM}}^{h^*}) + \bar{\pi}_{AE}\bar{y}_{AE}^{h^*} \\ + \bar{q} \cdot (\bar{\psi}^{h^*} - \bar{\varphi}^{h^*}) + \bar{q}_0(\bar{\mu}^{h^*} - e_B^{h^*}) = 0, \end{aligned} \tag{25}$$

Budget constraint at date $t = 1$:

$$\begin{aligned} (\bar{c}_s^{h^*} - e_{C_s}^{h^*} - y_{EM}^{h^*} d_s^{EM} - y_{AE}^{h^*} d_s^{AE}) - \sum_{j=1}^{S} (\bar{\psi}^{h^*} - \bar{\varphi}^{h^*}) \bar{r}_{sj} \\ + \bar{T}_s^{h^*} - \bar{\mu}^{h^*} = 0, \forall s \in S \end{aligned} \tag{26}$$

The boundary conditions:

$${}_c\bar{\delta}_0^{h^*} \bar{c}_0^{h^*} = 0; \quad \forall h^* \in H^* \tag{27}$$

$${}_y\bar{\delta}_{AE}^{h^*} \bar{y}_{AE}^{h^*} = 0; \quad \forall h^* \in H^* \tag{28}$$

$${}_y\bar{\delta}_{EM}^{h^*} \bar{y}_{EM}^{h^*} = 0; \quad \forall h^* \in H^* \tag{29}$$

$${}_c\bar{\delta}_s^{h^*} \bar{c}_s^{h^*} = 0; \quad \forall h^* \in H^*; \forall s \in S \tag{30}$$

$${}_\mu\bar{\delta}_s^{h^*} \bar{\mu}_s^{h^*} = 0; \quad \forall h^* \in H^*; \forall s \in S \tag{31}$$

$${}_\psi\bar{\delta}_j^{h^*} \bar{\psi}_j^{h^*} = 0; \quad \forall h^* \in H^*; \forall j \in J \tag{32}$$

$${}_\varphi\bar{\delta}_j^{h^*} \bar{\varphi}_j^{h^*} = 0; \quad \forall h^* \in H^*; \forall j \in J \tag{33}$$

$${}_{col}\bar{\delta}_{AE}^{h^*}(-\bar{y}_{AE}^{h^*} + \sum_{j \in J} \bar{\varphi}_j^{h^*}) = 0; \quad \forall h^* \in H^* \tag{34}$$

Inequality that r_{sj} must satisfy:

$$2\bar{r}_{sj} - j - d_s^{AE} \leq 0; \forall s \in S; \forall j \in J \tag{35}$$

Equality that r_{sj} must satisfy:

$$(\bar{r}_{sj} - j)(\bar{r}_{sj} - d_s^{AE}) = 0; \forall s \in S; \forall j \in J \tag{36}$$

Collateral constraints:

$$-y_{AE}^h + \sum_{j \in J} \bar{\varphi}_j^h \leq 0; \forall h \in H; \forall j \in J \tag{37}$$

$$-y_{AE}^{h^*} + \sum_{j \in J} \bar{\varphi}_j^{h^*} \leq 0; \forall h^* \in H^*; \forall j \in J \tag{38}$$

Portfolio conditions:

$$\bar{\psi}_j^h \bar{\varphi}_j^h = 0; \forall h \in H; \forall j \in J \tag{39}$$

$$\bar{\psi}_j^{h^*} \bar{\varphi}_j^{h^*} = 0; \forall h^* \in H^*; \forall j \in J \tag{40}$$

Finally, the model can be closed with the market clearing conditions presented in the main text. Note that by the No-Default Theorem, we can restrict our attention to the single, non-default financial claim j^* when solving the above system of equations.

List of variables for the numerical model and the matlab codes:

$x(1) : c_0^1; x(2) : y_{AE}^1; x(3) : y_{EM}^1; x(4) : \mu^1; x(5) : \psi^1; x(6) : \varphi^1; x(7) : c_1^1; x(8) : c_2^1; x(9) : \delta_0^1; x(10) : \delta_1^1; x(11) : \delta_2^1; x(12) :_{col} \delta_{AE}^1; x(13) :_c \delta_0^1; x(14) :_c \delta_1^1; x(15) :_c \delta_2^1; x(16) :_\psi \delta^1; x(17) :_\varphi \delta^1; x(18) :_\mu \delta^1; x(19) :_y \delta_{AE}^1; x(20) :_y \delta_{EM}^1; x(21) : c_0^2; x(22) : y_{AE}^2; x(23) : y_{EM}^2; x(24) : \mu^2; x(25) : \psi^2; x(26) : \varphi^2; x(27) : c_1^2; x(28) : c_2^2; x(29) : \delta_0^2; x(30) : \delta_1^2; x(31) : \delta_2^2; x(32) :_{col} \delta_{AE}^2; x(33) :_c \delta_0^2; x(34) :_c \delta_1^2; x(35) :_c \delta_2^2; x(36) :_\psi \delta^2; x(37) :_\varphi \delta^2; x(38)_\mu \delta^2; x(39) :_y \delta_{AE}^2; x(40) :_y \delta_{EM}^2; x(41) : c_0^3; x(42) : y_{AE}^3; x(43) : y_{EM}^3; x(44) : \mu^3; x(45) : \psi^3; x(46) : \varphi^3; x(47) : c_1^3; x(48) : c_2^3; x(49) : \delta_0^3; x(50) : \delta_1^3; x(51) : \delta_2^3; x(52) :_{col} \delta_{AE}^3; x(53) :_c \delta_0^3; x(54) :_c \delta_1^3; x(55) :_c \delta_2^3; x(56) :_\psi \delta^3; x(57) :_\varphi \delta^3; x(58)_\mu \delta^3; x(59) :_y \delta_{AE}^3; x(60) :_y \delta_{EM}^3; x(61) : c_0^4; x(62) : y_{AE}^4; x(63) : y_{EM}^4; x(64) : \mu^4; x(65) : \psi^4; x(66) : \varphi^4; x(67) : c_1^4; x(68) : c_2^4; x(69) : \delta_0^4; x(70) : \delta_1^4; x(71) : \delta_2^4; x(72) :_{col} \delta_{AE}^4; x(73) :_c \delta_0^4; x(74) :_c \delta_1^4; x(75) :_c \delta_2^4; x(76) :_\psi \delta^4; x(77) :_\varphi \delta^4; x(78)_\mu \delta^4; x(79) :_y \delta_{AE}^4; x(80) :_y \delta_{EM}^4; x(81) : \pi_{AE}; x(82) : \pi_{EM}; x(83) : q_0; x(84) : d_U^{AE}; x(85) : d_D^{AE}$