Online Appendix

Oil, Equities, and the Zero Lower Bound

Deepa D. Datta, Benjamin K. Johannsen, Hannah Kwon, and Robert J. Vigfusson*

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Abstract

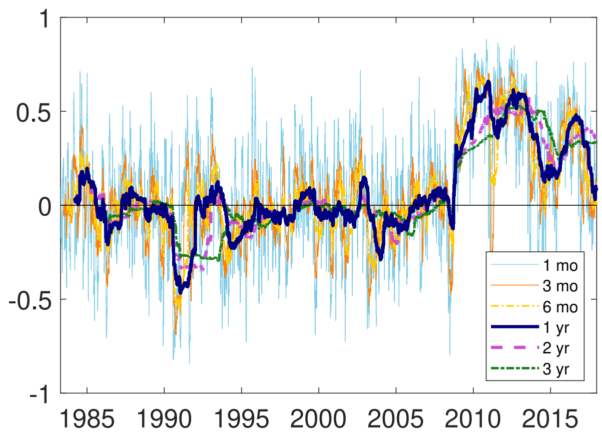
The following appendixes are supplementary material for our paper "Oil, Equities, and the Zero Lower Bound." Appendix A contains additional tables and figures from our empirical work. Appendix B contains details about our benchmark New Keynesian model with oil. Appendix C contains details about our two-country New Keynesian model with oil. Appendix D contains details on the power gains from using high frequency data. Appendix E provides details on covariance decomposition.

A Additional figures and tables

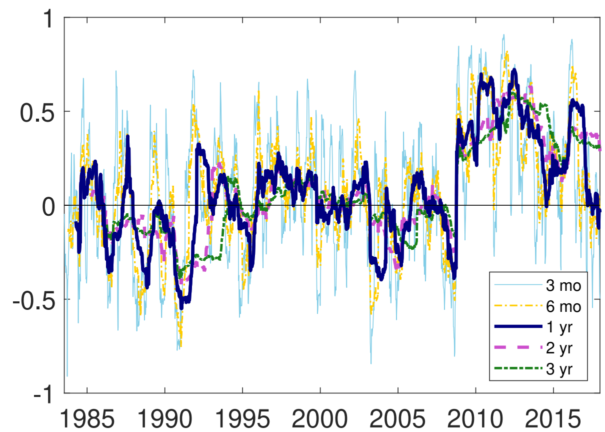
Figure A.1: Oil and Equity Correlation - Robustness

The rolling window correlations between oil and equity returns are presented here. The four panels illustrate the correlations for returns calculated over daily, weekly, monthly, and quarterly frequencies. The lines in each panel show the rolling windows of various lengths (1 month up to 3 years).

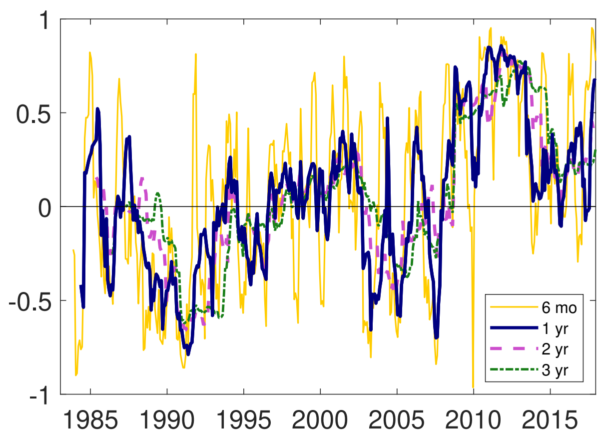
(a) Daily Returns



(b) Weekly Returns



(c) Monthly Returns



(d) Quarterly Returns

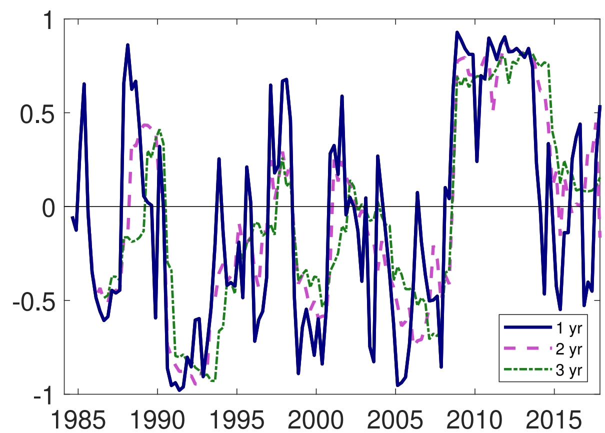
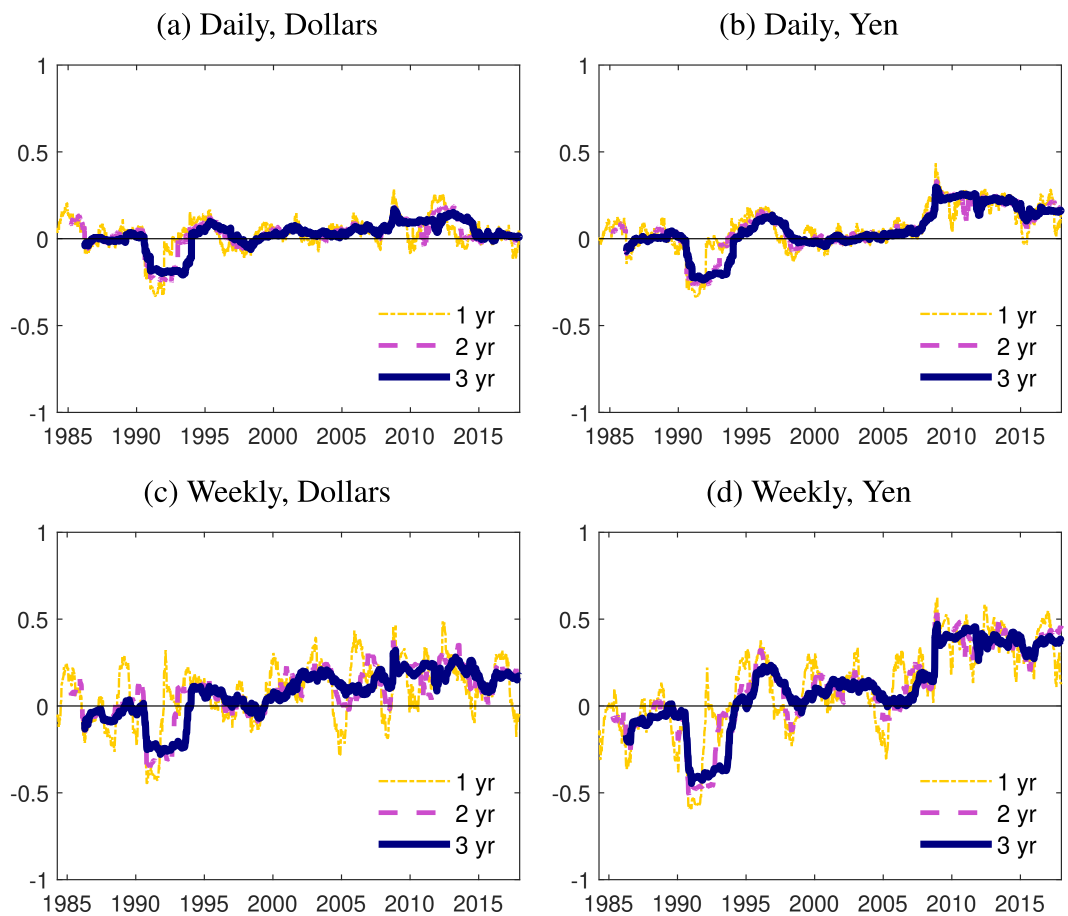
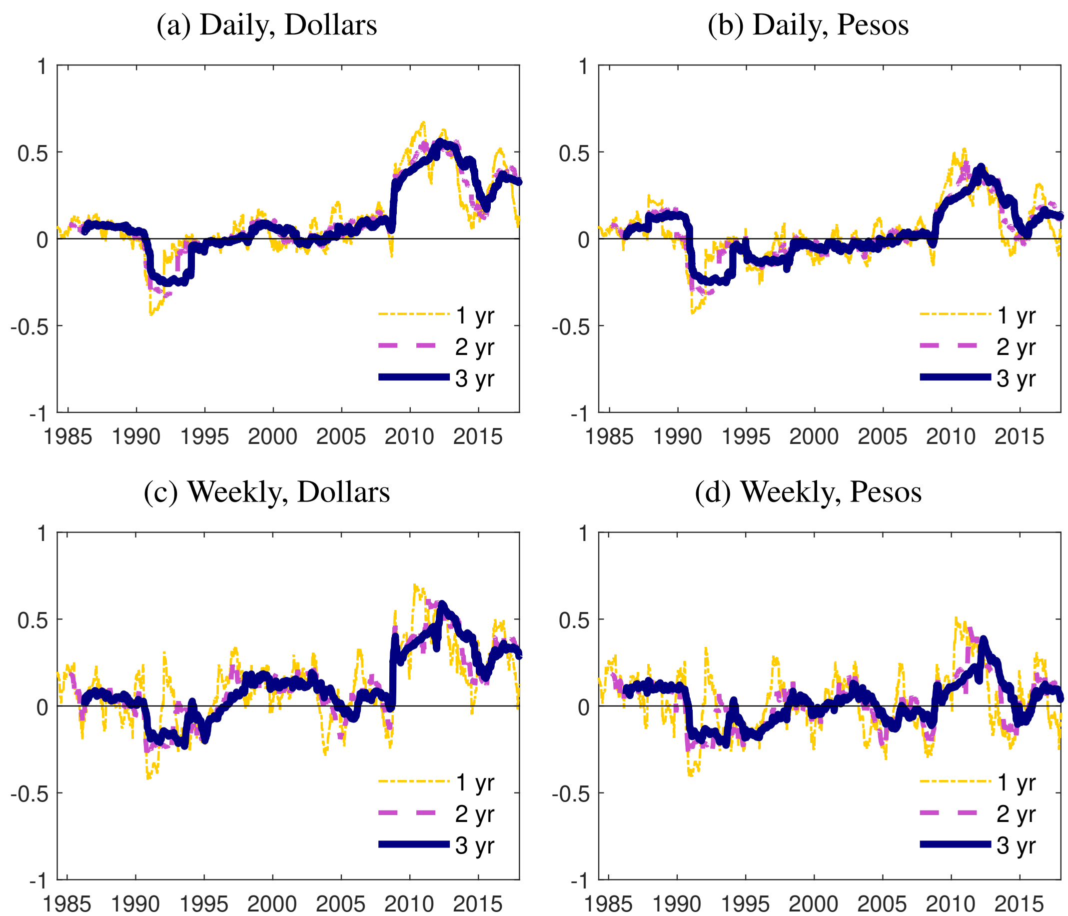


Figure A.2: Oil–Equity Rolling Correlation, Japan



Note: Legend labels correspond to length of rolling window. Correlations dated at end of rolling window. Currency conversion done using exchange rates from the H.10 release from the Federal Reserve. Period returns computed as log changes in equity index on last trading day of each period. Oil returns computed as log changes in oil price on last trading day of each period.

Figure A.3: Oil–Equity Rolling Correlation, Mexico



Note: Legend labels correspond to length of rolling window. Correlations dated at end of rolling window. Currency conversion done using exchange rates from the Bank of Mexico. Period returns computed as log changes in equity index on last trading day of each period. Oil returns computed as log changes in oil price on last trading day of each period.

Table A.1: Summary Statistics: Equity Sector Returns and Macroeconomic News Surprises

Variable	Obs.	Start	Mean	St.D.	Min.	Max.
Panel A: Equity Sector Returns						
Consumer nondurables	8584	1983-Apr-06	0.05	0.95	-18.67	8.83
Consumer durables	8584	1983-Apr-06	0.03	1.46	-20.27	9.12
Manufacturing	8584	1983-Apr-06	0.05	1.21	-22.61	9.55
Energy	8584	1983-Apr-06	0.04	1.46	-21.60	17.24
Chemicals	8584	1983-Apr-06	0.05	1.10	-21.33	9.40
Business equipment	8584	1983-Apr-06	0.04	1.54	-22.43	14.95
Telecommunications	8584	1983-Apr-06	0.04	1.23	-18.26	13.21
Utilities	8584	1983-Apr-06	0.04	0.97	-13.77	12.67
Shops	8584	1983-Apr-06	0.05	1.15	-18.32	10.43
Healthcare	8584	1983-Apr-06	0.05	1.15	-19.71	10.29
Finance	8584	1983-Apr-06	0.04	1.42	-16.08	15.62
Other	8584	1983-Apr-06	0.03	1.18	-18.13	9.43
Panel B: Macroeconomic News Surprises						
Capacity utilization (cu)	357	1988-Apr-18	-0.01	0.35	-1.57	1.40
Consumer confidence (con)	316	1991-Jul-30	0.25	5.12	-14.00	13.30
Core CPI (cpi)	341	1989-Aug-18	-0.01	0.11	-0.34	0.40
GDP advance (gdp)	123	1987-Apr-23	0.08	0.74	-1.68	1.80
Initial claims (clm)	1303	1991-Jul-18	0.05	18.08	-85.00	94.00
ISM manufacturing (ism)	333	1990-Feb-01	0.03	1.97	-6.30	7.40
Leading indicators (ind)	455	1980-Feb-29	0.02	0.31	-1.80	2.00
New home sales (nhs)	353	1988-Mar-29	5.43	56.77	-166.00	249.00
Nonfarm payrolls (nfp)	395	1985-Feb-01	-8.29	100.29	-328.00	408.50
Core PPI (ppi)	337	1989-Aug-11	-0.02	0.24	-1.20	1.07
Retail sales ex. autos (rtl)	454	1980-Feb-13	-0.03	0.66	-2.40	5.13
Unemployment rate (ur)	453	1980-Feb-07	0.04	0.16	-0.60	0.60

Note: In Panel (a), the 12 industry-specific equity returns series are obtained from the Fama-French data library, and are converted to levels. To calculate returns, we drop days with missing values for oil, metals, interest rates, or equities, and then calculate "daily" returns as the 100 times the log difference of these consecutive closing prices. For Panel (b) only, news surprises are defined as the difference between the announced realization of the macroeconomic aggregates and the survey expectations. Prior to use in regression analysis, each surprise is divided by the full sample standard deviation reported above. Following Beechey and Wright (2009), we flip the sign for unemployment and initial jobless claims announcements throughout the paper, so that positive surprises represent a stronger-than-expected economy.

Table A.2: VAR Decomposition of the Correlation between Oil and Equity Returns

			Contribution of			
		Corr.	Oil Supply	Agg. Demand	Oil Resid.	Equity Resid.
	Lags	$\rho_{pe}(h)$	$\frac{\sigma_{pe,1}(h)}{\sigma_p(h)\sigma_e(h)}$	$\frac{\sigma_{pe,2}(h)}{\sigma_p(h)\sigma_e(h)}$	$\frac{\sigma_{pe,3}(h)}{\sigma_p(h)\sigma_e(h)}$	$\frac{\sigma_{pe,4}(h)}{\sigma_p(h)\sigma_e(h)}$
Oil Price in Differences						
Jan. 1974 – Mar. 2009	**12**	**-0.102**	**0.002**	**0.029**	**-0.126**	**-0.006**
Jan. 1974 – Mar. 2009	24	-0.117	-0.000	0.015	-0.123	-0.009
Jan. 1974 – Dec. 2006	12	-0.172	-0.002	0.005	-0.166	-0.009
Jan. 1974 – Dec. 2006	24	-0.175	0.000	0.005	-0.171	-0.010
Apr. 2009 – Dec. 2017	**12**	**0.331**	**0.044**	**0.009**	**0.238**	**0.040**
Oil Price in Levels						
Jan. 1974 – Mar. 2009	12.000	-0.102	0.006	0.022	-0.126	-0.004
Jan. 1974 – Mar. 2009	24.000	-0.131	-0.001	0.006	-0.128	-0.007
Jan. 1974 – Dec. 2006	12.000	-0.172	0.001	0.004	-0.169	-0.007
Jan. 1974 – Dec. 2006	24.000	-0.182	-0.001	0.002	-0.176	-0.007
Apr. 2009 – Dec. 2017	12.000	0.336	0.049	0.007	0.247	0.033

Note: The table reports the decomposition of the correlation between monthly oil and equity returns based on the monthly VAR described in Section IV. The decomposition is based on Equation E.4. The VAR is estimated independently for each reported sample. When the VAR is estimated using the log-level of the oil price (instead of the log difference), we calculate the correlation and decompositions for oil and equity returns using the implied moving average representation for oil returns. The value of h is 1000. Bolded rows denote our benchmark results, as reported in the main text.

B Benchmark New Keynesian model

In this appendix, we describe our benchmark New Keynesian model. We use a medium-scale New Keynesian model and add endogenous oil demand along with exogenous oil supply along the lines of Bodenstein, Guerrieri and Gust (2013).

B.1 Household

The representative household maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\left(C_{t+j} - h\bar{C}_{t+j-1}\right)^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\phi} L_{t+j}^{1+\phi} + \log\left(\eta_{t+j}\right) V\left(\frac{B_{t+j}}{P_{C,t+j}}\right)\right) \tag{B.1}$$

where C_t is consumption, $\bar{C}_t$ is average aggregate consumption, L_t is hours worked, B_t is nominal bond holdings, and $P_{C,t}$ is the price of the consumption good. The stochastic variable η_t is a preference shifter than captures increased desire to hold safe nominal assets. The budget constraint is

$$B_t + P_{C,t}C_t + P_{Y,t}I_t = \left(1 + R_{t-1}\right)^{1/4} B_{t-1} + R_{K,t}K_t + W_tL_t + T_t, \tag{B.2}$$

where $P_{Y,t}$ is the price of non-oil output, R_t is the net annual nominal interest rate, W_t is the wage rate, $R_{K,t}$ is the rental rate on capital, K_t is the capital holdings of the household that is brought in from the previous period, I_t is investment, and T_t are lump-sum profits and taxes. The capital accumulation equation is

$$K_{t+1} = I_t \left(1 - \frac{\phi_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) + \left(1-\delta\right)K_t. \tag{B.3}$$

The definition of consumption is

$$C_t = \left(\omega_C^{1-\rho_C}\left(Y_{C,t}\right)^{\rho_C} + \left(1-\omega_C\right)^{1-\rho_C}\left(\frac{O_{C,t}}{\mu_{C,t}}\right)^{\rho_C}\right)^{\frac{1}{\rho_C}}. \tag{B.4}$$

The household creates the consumption good to minimize the cost of consumption. That is, the household solves

$$\min_{Y_{C,t},O_{C,t}} P_{Y,t}Y_{C,t} + P_{O,t}O_{C,t} \tag{B.5}$$

subject to the constraint that

$$\left(\omega_C^{1-\rho_C}\left(Y_{C,t}\right)^{\rho_C} + \left(1-\omega_C\right)^{1-\rho_C}\left(\frac{O_{C,t}}{\mu_{C,t}}\right)^{\rho_C}\right)^{\frac{1}{\rho_C}} \geq C_t. \tag{B.6}$$

Here $Y_{C,t}$ is non-oil output used for consumption, $O_{C,t}$ is oil that is consumed by the household, and $\mu_{C,t}$ is a preference-shifter similar to one studied in Bodenstein, Guerrieri and Gust (2013). The first-order conditions are

$$Y_{C,t} = \left(\frac{P_{Y,t}}{P_{C,t}}\right)^{\frac{1}{\rho_C-1}} \omega_C C_t, \tag{B.7}$$

$$O_{C,t} = \left(\frac{P_{O,t}}{P_{C,t}}\right)^{\frac{1}{\rho_C-1}} C_t\left(1-\omega_C\right)\mu_{C,t}^{\frac{\rho_C}{\rho_C-1}}. \tag{B.8}$$

The ideal price index for final consumption is given by

$$P_{C,t} = \left(\omega_C\left(P_{Y,t}\right)^{\frac{\rho_C}{\rho_C-1}} + \left(1-\omega_C\right)\left(P_{O,t}\mu_{C,t}\right)^{\frac{\rho_C}{\rho_C-1}}\right)^{\frac{\rho_C-1}{\rho_C}}. \tag{B.9}$$

The first-order conditions of the household are

$$\left(C_t - h\bar{C}_{t-1}\right)^{-\sigma} = \Lambda_t, \tag{B.10}$$

$$\Lambda_t W_t / P_{C,t} = \chi L_t^{\phi}, \tag{B.11}$$

$$\Lambda_t = \log\left(\eta_t\right) V'\left(\frac{B_t}{P_{C,t}}\right) + \beta\left(1+R_t\right)^{1/4} E_t \frac{\Lambda_{t+1}}{\pi_{C,t+1}}, \tag{B.12}$$

$$\frac{P_{Y,t}}{P_{C,t}}\Lambda_t = Q_t\left[\left(1-\frac{\phi_K}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right)-\frac{I_t}{I_{t-1}}\phi_K\left(\frac{I_t}{I_{t-1}}-1\right)\right] + \beta E_t Q_{t+1}\phi_K\left(\frac{I_{t+1}}{I_t}-1\right)\frac{I_{t+1}^2}{I_t^2}, \tag{B.13}$$

$$Q_t = \beta E_t\left[Q_{t+1}\left(1-\delta\right)+\Lambda_{t+1}\frac{R_{K,t+1}}{P_{C,t+1}}\right]. \tag{B.14}$$

Here, Λ_t and Q_t are Lagrange multipliers on the budget constraint and the capital accumulation equation, respectively.

B.2 Goods aggregators

Perfectly competitive firms aggregate intermediate inputs into non-oil output, Y_t. Non-oil output is a composite of goods purchased from monopolists. We denote the quantity purchased form monopolist i by $X_t\left(i\right)$. The intermediate inputs are aggregated according to

$$Y_t = \left(\int_0^1 X_t\left(i\right)^{\frac{\nu-1}{\nu}}di\right)^{\frac{\nu}{\nu-1}}. \tag{B.15}$$

Demand curves are then of the form

$$X_t\left(i\right) = \left(\frac{P_{X,t}\left(i\right)}{P_{Y,t}}\right)^{-\nu}Y_t. \tag{B.16}$$

Here, $P_{X,t}\left(i\right)$ is the price of $X_t\left(i\right)$. Perfect competition implies the ideal price index for Y_t is given by

$$P_{Y,t} = \left(\int_0^1 P_{X,t}\left(i\right)^{1-\nu}di\right)^{\frac{1}{1-\nu}}. \tag{B.17}$$

B.3 Monopolists

We introduce price stickiness as a Calvo-style price-setting friction. Monopolists are only able to optimize their price with probability ξ in each period. If monopolist i can update its price, it chooses

$\tilde{P}_{X,t}(i)$ to maximize

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \left(\frac{\tilde{P}_{X,t}(i)}{P_{C,t+j}} \tilde{X}_{t,j} (1+\tau_X) - MC_{t+j} \right) \left(\frac{\tilde{P}_{X,t}(i)}{P_{Y,t+j}} \tilde{X}_{t,j} \right)^{-\nu} Y_{t+j} \tag{B.18}$$

where τ_X is a subsidy that we use to offset monopoly distortions in steady state, MC_t is real marginal cost (made real by dividing by $P_{C,t}$), and

$$\tilde{X}_{t,j} = \begin{cases} 1 & j=1 \\ \pi_{Y,t} \times \pi_{Y,t+1} \times \cdots \times \pi_{Y,t+j-1} & else \end{cases}. \tag{B.19}$$

Here, $\tilde{X}$ captures indexation to past price changes. The first-order condition with respect to $\tilde{P}_{X,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \left[\frac{\tilde{P}_{X,t}}{P_{C,t}} \frac{P_{C,t}}{P_{C,t+j}} \tilde{X}_{t,j} \left(\frac{P_{Y,t}}{P_{Y,t+j}} \tilde{X}_{t,j} \right)^{-\nu} Y_{t+j} \right.$$
$$\left. - \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \left(\frac{P_{Y,t}}{P_{Y,t+j}} \tilde{X}_{t,j} \right)^{-\nu} Y_{t+j} \right] = 0. \tag{B.20}$$

Here we set $\tilde{P}_{X,t}(i) = \tilde{P}_{X,t}$ for all firms that can update their price because they all face the same problem. Then we have

$$F_{1,t} \tilde{p}_{X,t} = F_{2,t} \tag{B.21}$$

where $\tilde{p}_{X,t} \equiv \tilde{P}_{X,t}/P_{C,t}$ and $F_{1,t}$ and $F_{2,t}$ are defined as

$$F_{1,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{P_{C,t}}{P_{C,t+j}} \tilde{X}_{t,j}^{1-\nu} \left(\frac{P_{Y,t}}{P_{Y,t+j}} \right)^{-\nu} Y_{t+j} \tag{B.22}$$

$$F_{2,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \tilde{X}_{t,j}^{-\nu} \left(\frac{P_{Y,t}}{P_{Y,t+j}} \right)^{-\nu} Y_{t+j}. \tag{B.23}$$

The variables $F_{1,t}$ and $F_{2,t}$ can be expressed as

$$F_{1,t} = \Lambda_t Y_t + \beta \xi E_t \pi_{Y,t}^{1-\nu} \pi_{C,t+1}^{-1} \pi_{Y,t+1}^{\nu} F_{1,t+1}, \tag{B.24}$$

$$F_{2,t} = \Lambda_t \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_t Y_t + \beta \xi E_t \pi_{Y,t}^{-\nu} \pi_{Y,t+1}^{\nu} F_{2,t+1}, \tag{B.25}$$

where

$$\pi_{Y,t} \equiv P_{Y,t}/P_{Y,t-1} \tag{B.26}$$

and

$$\pi_{C,t} \equiv P_{C,t}/P_{C,t-1}. \tag{B.27}$$

The ideal price index for retail goods evolves according to

$$P_{Y,t} = \left((1-\xi) \tilde{P}_{X,t}^{1-\nu} + \xi \pi_{Y,t-1}^{1-\nu} P_{Y,t-1}^{1-\nu} \right)^{\frac{1}{1-\nu}}, \tag{B.28}$$

so that

$$p_{Y,t} = \left((1-\xi) \tilde{p}_{X,t}^{1-\nu} + \xi \pi_{Y,t-1}^{1-\nu} \frac{p_{Y,t-1}^{1-\nu}}{\pi_{C,t}^{1-\nu}} \right)^{\frac{1}{1-\nu}}. \tag{B.29}$$

B.4 Marginal cost

In this subsection we drop the i index from firm-specific quantities. The firm solves the following cost minimization problem when determining inputs to production:

$$\min_{K_t, L_t} W_t L_t + R_{K,t} K_t + P_{O,t} O_t \tag{B.30}$$

subject to the constraint that

$$\left((\omega_V)^{1-\rho_V} V_t^{\rho_V} + (1-\omega_V)^{1-\rho_V} \left(\frac{O_{X,t}}{\mu_{X,t}} \right)^{\rho_V} \right)^{\frac{1}{\rho_V}} \geq X_t. \tag{B.31}$$

Here, $\mu_{X,t}$ is a technology shifter that affects oil inputs specifically,

$$V_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{B.32}$$

and A_t is a stochastic process that represents aggregate technology. The first-order conditions are

$$\frac{R_{Kt}}{P_{Ct}} = MC_t \left(X_t\right)^{1-\rho_V} \left(\omega_V\right)^{1-\rho_V} V_t^{\rho_V - 1} \alpha A_t \left(\frac{L_t}{K_t}\right)^{1-\alpha}, \tag{B.33}$$

$$\frac{W_t}{P_{C,t}} = MC_t \left(X_t\right)^{1-\rho_V} \left(\omega_V\right)^{1-\rho_V} V_t^{\rho_V - 1} \left(1-\alpha\right) A_t \left(\frac{L_t}{K_t}\right)^{-\alpha}, \tag{B.34}$$

$$\frac{P_{O,t}}{P_{C,t}} = MC_t \left(X_t\right)^{1-\rho_V} \left(1-\omega_V\right)^{1-\rho_V} \left(\frac{O_{X,t}}{\mu_{X,t}}\right)^{\rho_V - 1} \frac{1}{\mu_{X,t}}. \tag{B.35}$$

B.5 Oil Market

There is an exogenous supply of oil, O_t. Oil-market clearing implies

$$O_{C,t} + O_{X,t} = O_t. \tag{B.36}$$

We assume that O_t is exogenous and strictly greater than zero.

B.6 Goods market clearing

We assume that oil is paid for to an external owner using non-oil output. So, goods market clearing implies

$$Y_{C,t} + G_t + I_t + \left(O_{C,t} + O_{X,t}\right) \frac{P_{O,t}}{P_{Y,t}} = Y_t. \tag{B.37}$$

B.7 Aggregation

Aggregating across firms yields

$$\int_0^1 \left(\frac{P_{X,t}(i)}{P_{Y,t}}\right)^{-\nu} Y_t di = \int_0^1 \left((\omega_V)^{1-\rho_V} V_t(i)^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{O_{X,t}(i)}{\mu_{X,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} di \quad \text{(B.38)}$$

$$= \int_0^1 \left((\omega_V)^{1-\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{O_{X,t}(i)}{V_t(i)\,\mu_{X,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} V_t(i)\, di \quad \text{(B.39)}$$

$$= \int_0^1 \left((\omega_V)^{1-\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{O_{X,t}(i)}{V_t(i)\,\mu_{X,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} A_t \left(\frac{K_t(i)}{L_t(i)}\right)^{\alpha} L_t(i)\, di. \quad \text{(B.40)}$$

From cost minimization and constant-returns-to-scale, the ratios $\frac{O_{X,t}(i)}{V_t(i)\mu_{X,t}}$ and $\frac{K_t(i)}{L_t(i)}$ are common across firms. Then

$$\int_0^1 \left(\frac{P_{X,t}(i)}{P_{Y,t}}\right)^{-\nu} Y_t di = \left((\omega_V)^{1-\rho_V} V_t^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{O_{X,t}}{\mu_{X,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}}, \quad \text{(B.41)}$$

so that

$$d_t^{-1} Y_t = \left((\omega_V)^{1-\rho_V} V_t^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{O_{X,t}}{\mu_{X,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}}. \quad \text{(B.42)}$$

The dispersion term, d_t^{-1}, represents the resource costs of price dispersion and can be written recursively as

$$d_t^{-1} = (1-\xi)\left(\tilde{p}_{X,t}/p_{Y,t}\right)^{-\nu} + \xi \pi_{Y,t-1}^{-\nu} \pi_{Y,t}^{\nu} d_{t-1}^{-1}. \quad \text{(B.43)}$$

B.8 Government

The monetary authority follows a truncated Taylor rule. The desired policy rate, $\tilde{R}_t$ evolves according to

$$\left[1+\tilde{R}_t\right]^{1/4} = \left(\left[1+\tilde{R}_{t-1}\right]^{1/4}\right)^{\gamma} \left(\left([1+R]^{1/4}\right)\left(\frac{\pi_{Y,t}}{\pi}\right)^{\theta_\pi}\left(\frac{Y_t}{Y_t^N}\right)^{\theta_Y}\right)^{1-\gamma} \text{ where } \theta_\pi > 1. \quad \text{(B.44)}$$

Here, R is the steady-state annualized net nominal interest rate, π is the target rate of inflation. The natural rate of output, Y_t^N is defined as the level of output that would prevail under flexible prices, given the entire history of shocks. The fiscal authority balances its budget with lump sum taxes so that $B_t = 0$. Government purchases, G_t, follows an $AR(1)$. To incorporate the zero lower bound,

$$R_t = \max\left\{0, \tilde{R}_t\right\}. \tag{B.45}$$

B.9 Equilibrium

A rational expectations equilibrium is a sequence of prices and quantities that have the property that the household and firm optimality conditions are satisfied, the goods market, labor market, and oil markets clear, and the nominal interest rate and government purchases evolve as specified. To solve for a rational expectations equilibrium, we solve for the following 24 endogenous objects: C_t, Λ_t, L_t, $w_t \equiv \frac{W_t}{P_{C,t}}$, Y_t, R_t, MC_t, $\pi_{C,t}$, K_t, I_t, Q_t, $r_{K,t} \equiv \frac{R_{K,t}}{P_{C,t}}$, $p_{Y,t} \equiv \frac{P_{Y,t}}{P_{C,t}}$, $\tilde{p}_{X,t}$, $F_{1,t}$, $F_{2,t}$, d_t, $\pi_{Y,t}$, V_t, $O_{X,t}$, $Y_{C,t}$, $O_{C,t}$, $p_{O,t} \equiv \frac{P_{O,t}}{P_{C,t}}$, $\tilde{R}_t$. To determine these variables, we require that the following 24 equations hold: (B.3), (B.7), (B.8), (B.9), (B.10), (B.11), (B.12), (B.13), (B.14), (B.37), (B.21), (B.24), (B.25), (B.26), (B.29), (B.32), (B.33), (B.34), (B.35), (B.36), (B.42), (B.43), (B.44), (B.45). The budget constraint of the household clears by Walras' law. We linearize the model around non-stochastic steady state. We incorporate the zero lower bound using the methodology of Guerrieri and Iacoviello (2015). We utilize the OccBin solver from Guerrieri and Iacoviello (2015).

B.10 Calibration of parameters not related to oil

For the parameters of our model not related to oil, we use the following values. We set the parameter governing consumption habit, h, to 0.7, in line with Boldrin, Christiano and Fisher (2001). We set $\delta = 0.025$, as in Christiano, Eichenbaum and Evans (2005). The parameter α is set to 0.33 so that the steady-state labor share of payments to labor and capital is roughly 0.67. We set $\phi_K = 3$, in line with Bodenstein, Guerrieri and Gust (2013). The value $1-\xi$ governs how often firms can update their prices optimally. We set $\xi = 0.75$. This value is slightly higher than the value implied by evidence in Nakamura and Steinsson (2008) but slightly lower than the value implied by estimates in Gust et al. (2017). As in Christiano, Eichenbaum and Evans (2005), we set $\sigma = 1$, $\varphi = 1$, and we normalize steady-state labor supply to be 1. We set $\beta = 0.9975$ to imply a steady-state risk-free real interest rate of 1 percent. The parameter ν governs substitution between different monopolists' output. We set $\nu = 7$, which is within the range of values considered in Altig et al. (2011), and implies steady-state markups of 15 percent. We calibrate steady-state government purchases to be 20 percent of steady-state output.

B.11 Steady state

To determine steady state, we assume that target inflation is π. So, $\pi_C = \pi_Y = \pi$. The intertemporal Euler equation determines $(1 + \tilde{R})^{1/4} = (1 + R)^{1/4} = \pi\beta^{-1}$. We normaliz $L = 1$. Firm optimality and symmetry of the equilibrium imply $\tilde{p}_X = 1$. Because of our indexation assumption, there is no price dispersion in steady state, so $d = 1$. We will normalize the price of oil to be $p_O = 1$ (we have to find O instead). As a result, $p_Y = 1$, meaning $Q = \Lambda$. Marginal cost is given by

$$MC = \frac{\nu - 1}{\nu} (1 + \tau_X) = 1 \tag{B.46}$$

From pricing optimality

$$F_1 = F_2 = (1 - \beta\xi)^{-1} \Lambda Y \tag{B.47}$$

The rental rate of capital is

$$r_K = \frac{1 - \beta (1 - \delta)}{\beta} \tag{B.48}$$

From our normalization of p_O

$$O_C = (1 - \omega_C) C \tag{B.49}$$

and

$$Y_C = \omega_C C \tag{B.50}$$

The marginal utility of consumption gives

$$([1 - h] C)^{-\sigma} = \Lambda \tag{B.51}$$

Note that

$$I = \delta K \tag{B.52}$$

and

$$Y = I + Y_C + G + O_C + O_X \tag{B.53}$$

From the definition of V we have

$$V = K^{\alpha} \tag{B.54}$$

This means

$$\delta K + C + G + O_X = \left((\omega_V)^{1-\rho_V} \left(K^{\alpha}\right)^{\rho_V} + (1-\omega_V)^{1-\rho_V} \left(O_X\right)^{\rho_V} \right)^{\frac{1}{\rho_V}} \tag{B.55}$$

We know that cost minimization implies

$$r_K = MC \left((\omega_V)^{1-\rho_V} V^{\rho_V} + (1-\omega_V)^{1-\rho_V} \left(O_X\right)^{\rho_V} \right)^{\frac{1-\rho_V}{\rho_V}} (\omega)^{1-\rho_V} V^{\rho_V - 1} \alpha \left(\frac{1}{K}\right)^{1-\alpha} \tag{B.56}$$

$$w = MC \left((\omega_V)^{1-\rho_V} V^{\rho_V} + (1-\omega_V)^{1-\rho_V} \left(O_X\right)^{\rho_V} \right)^{\frac{1-\rho_V}{\rho_V}} \left(\frac{\omega_V}{V}\right)^{1-\rho_V} (1-\alpha) \left(\frac{1}{K}\right)^{-\alpha} \tag{B.57}$$

$$1 = MC \left((\omega_V)^{1-\rho_V} V^{\rho_V} + (1-\omega_Y)^{1-\rho_V} \left(O_X\right)^{\rho_V} \right)^{\frac{1-\rho_V}{\rho_V}} (1-\omega_V)^{1-\rho_V} \left(O_X\right)^{\rho_V - 1} \tag{B.58}$$

Meaning

$$r_K = \left(\frac{\omega_V}{1-\omega_V}\right)^{1-\rho_V} \left(\frac{K^{\alpha}}{O_X}\right)^{\rho_V - 1} \alpha \left(\frac{1}{K}\right)^{1-\alpha} \tag{B.59}$$

and

$$O_X = r_K^{-\frac{1}{\rho_V - 1}} \left(\frac{\omega_V}{1-\omega_V}\right)^{-1} \left(K^{\alpha}\right) \alpha^{\frac{1}{\rho_V - 1}} \left(\frac{1}{K}\right)^{\frac{1-\alpha}{\rho_V - 1}} \tag{B.60}$$

So,

$$\begin{aligned} r_K = &\left((\omega_V)^{1-\rho_V} \left(K^{\alpha}\right)^{\rho_V} + (1-\omega_V)^{1-\rho_V} \left(r_K^{-\frac{1}{\rho_V - 1}} \left(\frac{1-\omega_V}{\omega_V}\right)^{-1} \left(K^{\alpha}\right) \alpha^{\frac{1}{\rho_V - 1}} \left(\frac{1}{K}\right)^{\frac{1-\alpha}{\rho_V - 1}} \right)^{\rho_V} \right)^{\frac{1-\rho_V}{\rho_V}} \\ &\times MC \left(1-\omega_V\right)^{1-\rho_V} \left(K^{\alpha}\right)^{\rho_V - 1} \alpha \left(\frac{1}{K}\right)^{1-\alpha} \end{aligned} \tag{B.61}$$

with r_K known and MC known, we can solve for K. With K we get O_X, V, and then w. With the intratemporal Euler equation, we get χ. We have Y from production technology, C from market clearing, O_C from $O_C = (1 - \omega_C)\, C$. With both O_X and O_C we have O. The rest follows easily.

C Two-country model

Here we extend our one-country model to a two-country environment. We assume that there are two countries, home and foreign. The home country is size $0 < n < 1$ and the foreign country is size $1-n$. We are only going to allow non-state-contingent home and foreign nominal bonds to be traded internationally. Our model features Calvo-style sticky prices and so-called "local-currency pricing." We add endogenous oil demand along with exogenous oil supply along the lines of Bodenstein, Guerrieri and Gust (2013).

C.1 Household

The representative household in the home country maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\left(C_{t+j} - h\bar{C}_{t+j-1}\right)^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\phi} L_{t+j}^{1+\phi} \right.$$
$$\left. + \log\left(\eta_{t+j}\right) V\left(\frac{B_{H,t+j}}{P_{C,+jt}}\right) + \log\left(\eta^*_{t+j}\right) V\left(\frac{B_{F,t+j} NER_{t+j}}{P_{C,t+j}}\right) \right). \tag{C.1}$$

Here C_t is per-capita consumption, $\bar{C}_t$ is average aggregate per-capita consumption, L_t is per-capita hours worked, $B_{H,t}$ is per-capita nominal home bond holdings, $B_{F,t}$ is per-capita nominal foreign bond holdings, $P_{C,t}$ is the price of the home consumption good in the home currency unit, and NER_t is the nominal exchange rate quoted as the price of the foreign currency unit. The stochastic variables η_t and η^*_t are preference shifters than capture the desire to hold safe nominal assets in the home and foreign currency. The budget constraint is

$$B_{H,t} + B_{F,t} NER_t + P_{C,t} C_t + P_{Y,t} I_t + \frac{\phi_b}{2} \left(\frac{B_{F,t} NER_t}{P_{C,t}}\right)^2 P_{Y,t} = \left(1 + R_{t-1}\right)^{1/4} B_{H,t-1} \tag{C.2}$$
$$+ \left(1 + R^*_{t-1}\right)^{1/4} B_{F,t-1} NER_t + R_{K,t} K_t + W_t L_t + T_t$$

where $P_{Y,t}$ is the price of non-oil output in the home country, R_t is the annualized net nominal interest rate on the home bond, R^*_t is the annualized net nominal interest rate on the foreign bond, W_t is the wage rate, $R_{K,t}$ is the rental rate on capital, K_t is per-capita capital holdings, I_t is per-capita investment, and T_t are per-capita lump-sum profits and taxes. The term $\frac{\phi_b}{2}\left(\frac{B_{F,t}NER_t}{P_{C,t}}\right)^2 P_{Y,t}$ is a carrying cost of holding the foreign-country bond. From a practical perspective, ϕ_b is set to a small number and this term ensures stationarity in the model. See Schmitt-Grohé and Uribe (2003). The capital accumulation equation is

$$K_{t+1} = I_t\left(1 - \frac{\phi_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) + (1-\delta)\,K_t. \tag{C.3}$$

The definition of consumption is

$$C_t = \left(\omega_C^{1-\rho_C}\left(Y_{C,t}\right)^{\rho_C} + (1-\omega_C)^{1-\rho_C}\left(\frac{O_{C,t}}{\mu_{O_C,t}}\right)^{\rho_C}\right)^{\frac{1}{\rho_C}}. \tag{C.4}$$

The household creates the consumption good to minimize costs

$$\min_{Y_{C,t},O_{C,t}} P_{Y,t}Y_{C,t} + P_{O,t}O_{C,t} \tag{C.5}$$

subject to the constraint that

$$\left(\omega_C^{1-\rho_C}\left(Y_{C,t}\right)^{\rho_C} + (1-\omega_C)^{1-\rho_C}\left(\frac{O_{C,t}}{\mu_{O_C,t}}\right)^{\rho_C}\right)^{\frac{1}{\rho_C}} \geq C_t \tag{C.6}$$

where $Y_{C,t}$ is non-oil output used for consumption and $O_{C,t}$ is oil that is consumed by the household. Then the first-order conditions are

$$Y_{C,t} = \left(\frac{P_{Y,t}}{P_{C,t}}\right)^{\frac{1}{\rho_C-1}} \omega_C C_t \tag{C.7}$$

$$O_{C,t} = \left(\frac{P_{O,t}}{P_{C,t}}\right)^{\frac{1}{\rho_C-1}} C_t\,(1-\omega_C)\,\mu_{O_C,t}^{\frac{\rho_C}{\rho_C-1}} \tag{C.8}$$

The ideal price index for final consumption is given by

$$P_{C,t} = \left(\omega_C \left(P_{Y,t}\right)^{\frac{\rho_C}{\rho_C - 1}} + \left(1-\omega_C\right)\left(P_{O,t}\mu_{O_{C,t}}\right)^{\frac{\rho_C}{\rho_C-1}}\right)^{\frac{\rho_C - 1}{\rho_C}}. \tag{C.9}$$

The household-wide first-order conditions are

$$\left(C_t - h\bar{C}_{t-1}\right)^{-\sigma} = \Lambda_t \tag{C.10}$$

$$\Lambda_t W_t / P_{C,t} = \chi L_t^{\phi} \tag{C.11}$$

$$\Lambda_t = \log\left(\eta_t\right) V'\left(\frac{B_{H,t}}{P_{C,t}}\right) + \beta\left(1+R_t\right)^{1/4} E_t \frac{\Lambda_{t+1}}{\pi_{C,t+1}} \tag{C.12}$$

$$\begin{aligned} \Lambda_t + \phi_B \frac{B_{F,t}}{P_{C,t}} NER_t \frac{P_{Y,t}}{P_{C,t}} =& \log\left(\eta_t^*\right) V'\left(\frac{B_{F,t}}{P_{C,t}} NER_t\right) \\ &+ \beta\left(1+R_t^*\right)^{1/4} E_t \frac{\Lambda_{t+1}}{\pi_{C,t+1}} \frac{NER_{t+1}}{NER_t} \end{aligned} \tag{C.13}$$

$$\begin{aligned} \frac{P_{Y,t}}{P_{C,t}}\Lambda_t =& Q_t \left[\left(1 - \frac{\phi_K}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) - \frac{I_t}{I_{t-1}}\phi_K\left(\frac{I_t}{I_{t-1}} - 1\right)\right] \\ &+ \beta E_t Q_{t+1}\phi_K\left(\frac{I_{t+1}}{I_t} - 1\right)\frac{I_{t+1}^2}{I_t^2} \end{aligned} \tag{C.14}$$

$$Q_t = \beta E_t\left[Q_{t+1}\left(1-\delta\right) + \Lambda_{t+1}\frac{R_{K,t+1}}{P_{C,t+1}}\right]. \tag{C.15}$$

Here, Λ_t and Q_t are the Lagrange multipliers on the budget constraint and the capital accumulation equation, respectively.

The representative foreign household maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{\left(C^*_{t+j} - h\bar{C}^*_{t+j-1}\right)^{1-\sigma}}{1-\sigma} - \frac{\chi}{1+\phi} \left(L^*_{t+j}\right)^{1+\phi} \right.$$
$$\left. + \log\left(\eta_{t+j}\right) V \left(\frac{B^*_{H,t+j}}{P^*_{C,t+j} NER_{t+j}} \right) + \log\left(\eta^*_{t+j}\right) V \left(\frac{B^*_{F,t+j}}{P^*_{C,t+j}} \right) \right) \tag{C.16}$$

where C^*_t is per-capita consumption, $\bar{C}^*_t$ is average aggregate per-capita consumption, L^*_t is per-capita hours worked, $B^*_{H,t}$ is per-capita home nominal bond holdings, $B^*_{F,t}$ is per-capita foreign nominal bonds, and $P^*_{C,t}$ is the price of the foreign consumption good in the foreign currency unit. Note that η_t and η^*_t are the same preference shifters as for the home household. In this way, we capture global demand for the desire to hold safe nominal assets in one currency or another. The budget constraint is

$$B^*_{F,t} + P^*_{C,t} C^*_t + P^*_{Y,t} I^*_t + B^*_{H,t} NER^{-1}_t + \frac{\phi_b}{2} \left(\frac{B^*_{H,t}}{P^*_{Ct} NER_t} \right)^2 P^*_{Y,t} =$$
$$\left(1 + R^*_{t-1}\right)^{1/4} B^*_{F,t-1} + \left(1 + R_{t-1}\right)^{1/4} B^*_{H,t-1} NER^{-1}_t + R^*_{K,t} K^*_t + W^*_t L^*_t + T^*_t \tag{C.17}$$

The term $\frac{\phi_b}{2} \left(\frac{B^*_{H,t}}{P^*_t NER_t} \right)^2 P^*_{C,t}$ is a carrying cost of holding the home-country bond. The capital accumulation equation is

$$K^*_{t+1} = I^*_t \left(1 - \frac{\phi_K}{2} \left(\frac{I^*_t}{I^*_{t-1}} - 1 \right)^2 \right) + (1-\delta) K^*_t. \tag{C.18}$$

The foreign household solves a similar cost minimization problem as the home household, and the associated first-order conditions are

$$Y^*_{C,t} = \left(\frac{P^*_{Y,t}}{P^*_{C,t}} \right)^{\frac{1}{\rho_C - 1}} \omega_C C^*_t, \tag{C.19}$$

$$O^*_{C,t} = \left(\frac{P^*_{O,t}}{P^*_{C,t}} \right)^{\frac{1}{\rho_C - 1}} C^*_t \left(1 - \omega_C\right) \left(\mu^*_{O_C,t}\right)^{\frac{\rho_C}{\rho_C - 1}}. \tag{C.20}$$

The ideal price index for final consumption is given by

$$P_{C,t}^{*}=\left(\omega_{C}\left(P_{Y,t}^{*}\right)^{\frac{\rho_{C}}{\rho_{C}-1}}+\left(1-\omega_{C}\right)\left(P_{O,t}^{*}\mu_{O_{C},t}^{*}\right)^{\frac{\rho_{C}}{\rho_{C}-1}}\right)^{\frac{\rho_{C}-1}{\rho_{C}}}. \tag{C.21}$$

The household-wide first-order conditions are

$$\left(C_{t}^{*}-h\bar{C}_{t-1}^{*}\right)^{-\sigma}=\Lambda_{t}^{*} \tag{C.22}$$

$$\Lambda_{t}^{*}W_{t}^{*}/P_{C,t}^{*}=\chi\left(L_{t}^{*}\right)^{\phi} \tag{C.23}$$

$$\Lambda_{t}^{*}+\phi_{b}\frac{B_{H,t}^{*}}{NER_{t}P_{C,t}^{*}}\frac{P_{Y,t}^{*}}{P_{C,t}^{*}}=\log\left(\eta_{t}\right)V'\left(\frac{B_{H,t}^{*}}{P_{C,t}^{*}}NER_{t}^{-1}\right)+\beta\left(1+R_{t}\right)^{1/4}E_{t}\frac{\Lambda_{t+1}^{*}}{\pi_{C,t+1}^{*}}\frac{NER_{t}}{NER_{t+1}} \tag{C.24}$$

$$\Lambda_{t}^{*}=\log\left(\eta_{t}^{*}\right)V'\left(\frac{B_{F,t}^{*}}{P_{C,t}^{*}}\right)+\beta\left(1+R_{t}^{*}\right)^{1/4}E_{t}\frac{\Lambda_{t+1}^{*}}{\pi_{C,t+1}^{*}} \tag{C.25}$$

$$\begin{aligned}\frac{P_{Y,t}^{*}}{P_{C,t}^{*}}\Lambda_{t}^{*}=&Q_{t}^{*}\left[\left(1-\frac{\phi_{K}}{2}\left(\frac{I_{t}^{*}}{I_{t-1}^{*}}-1\right)^{2}\right)-\frac{I_{t}^{*}}{I_{t-1}^{*}}\phi_{K}\left(\frac{I_{t}^{*}}{I_{t-1}^{*}}-1\right)\right]\\&+\beta E_{t}Q_{t+1}\phi_{K}\left(\frac{I_{t+1}^{*}}{I_{t}^{*}}-1\right)\left(\frac{I_{t+1}^{*}}{I_{t}^{*}}\right)^{2}\end{aligned} \tag{C.26}$$

$$Q_{t}^{*}=\beta E_{t}\left[Q_{t+1}^{*}\left(1-\delta\right)+\Lambda_{t+1}^{*}\frac{R_{K,t+1}^{*}}{P_{C,t+1}^{*}}\right]. \tag{C.27}$$

Here, Λ_t^* and Q_t^* are the Lagrange multipliers on the budget constraint and the capital accumulation equation, respectively. Note that we define the real exchange rate, RER_t, so that

$$RER_{t}=\frac{NER_{t}P_{C,t}^{*}}{P_{C,t}}. \tag{C.28}$$

C.2 Goods aggregators

In each country, perfectly competitive firms aggregate country-specific intermediate inputs into $Y_{H,t}$, $Y_{F,t}$, $Y_{H,t}^*$, and $Y_{F,t}^*$. The values $Y_{H,t}$ and $Y_{F,t}$ are composites of goods purchased from monopolists by perfectly competitive firms who produce using

$$Y_{H,t} = \left(\left(\frac{1}{n} \right)^{\frac{1}{\nu}} \int_0^n X_{H,t} \left(i \right)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}} \tag{C.29}$$

$$Y_{F,t} = \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\nu}} \int_0^{1-n} X_{F,t} \left(i \right)^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}} \tag{C.30}$$

Demand curves are then of the form

$$X_{H,t} \left(i \right) = \frac{1}{n} \left(\frac{P_{H,t} \left(i \right)}{P_{H,t}} \right)^{-\nu} Y_{H,t} \tag{C.31}$$

and

$$X_{F,t} \left(i \right) = \frac{1}{1-n} \left(\frac{P_{F,t} \left(i \right)}{P_{F,t}} \right)^{-\nu} Y_{F,t}. \tag{C.32}$$

The zero profit condition, along with these demand curves, implies the ideal price index is give by

$$P_{H,t} = \left(\frac{1}{n} \int_0^n P_{H,t} \left(i \right)^{1-\nu} di \right)^{\frac{1}{1-\nu}}. \tag{C.33}$$

Similarly,

$$P_{F,t} = \left(\frac{1}{1-n} \int_0^{1-n} P_{F,t} \left(i \right)^{1-\nu} di \right)^{\frac{1}{1-\nu}}. \tag{C.34}$$

The foreign country is symmetric. Demand curves are of the form

$$X^*_{H,t} \left(i \right) = \frac{1}{n} \left(\frac{P^*_{H,t} \left(i \right)}{P^*_{H,t}} \right)^{-\nu} Y^*_{H,t} \tag{C.35}$$

and

$$X^*_{F,t} \left(i \right) = \frac{1}{1-n} \left(\frac{P^*_{F,t} \left(i \right)}{P^*_{F,t}} \right)^{-\nu} Y^*_{F,t}. \tag{C.36}$$

The zero profit conditions, along with these demand curves, imply ideal price indexes

$$P^*_{H,t} = \left(\frac{1}{n} \int_0^n P^*_{H,t} \left(i \right)^{1-\nu} di \right)^{\frac{1}{1-\nu}} \tag{C.37}$$

and

$$P^*_{F,t} = \left(\frac{1}{1-n} \int_0^{1-n} P^*_{F,t}(i)^{1-\nu} \, di \right)^{\frac{1}{1-\nu}}. \tag{C.38}$$

C.3 Retailers

Non-oil output, Y_t, is created by combining goods from countries H and F ($Y_{H,t}$ and $Y_{F,t}$) using

$$Y_t = \left(\omega^{1-\rho} (Y_{H,t})^{\rho} + (1-\omega)^{1-\rho} (Y_{F,t})^{\rho} \right)^{\frac{1}{\rho}} \tag{C.39}$$

where $\omega \equiv 1 - (1-n)\,\Omega$. The value $0 < \Omega \leq 1$ captures home bias if it is less than one (see Faia and Monacelli (2008)). Profits are given by

$$P_{Y,t} \left(\omega^{1-\rho} (Y_{H,t})^{\rho} + (1-\omega)^{1-\rho} (Y_{F,t})^{\rho} \right)^{\frac{1}{\rho}} - P_{H,t} Y_{H,t} - P_{F,t} Y_{F,t} \tag{C.40}$$

where $P_{H,t}$ is the nominal price of $Y_{H,t}$, $P_{F,t}$ is the nominal price of $Y_{F,t}$. Demand curves are then

$$Y_{H,t} = \left(\frac{P_{H,t}}{P_{Y,t}} \right)^{\frac{1}{\rho-1}} \omega Y_t \tag{C.41}$$

and

$$Y_{F,t} = \left(\frac{P_{F,t}}{P_{Y,t}} \right)^{\frac{1}{\rho-1}} (1-\omega)\, Y_t. \tag{C.42}$$

There is free entry for retailers, so profits are zero. Substituting demand curves into the profits expression yields the ideal price index

$$P_{Y,t} = \left(\omega P_{H,t}^{\frac{\rho}{\rho-1}} + (1-\omega) (P_{F,t})^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}} \tag{C.43}$$

Non-oil output in the foreign country, Y^*_t, are created by combining goods for countries H and F ($Y^*_{H,t}$ and $Y^*_{F,t}$) using

$$Y^*_t = \left((\omega^*)^{1-\rho} \left(Y^*_{F,t} \right)^{\rho} + (1-\omega^*)^{1-\rho} \left(Y^*_{H,t} \right)^{\rho} \right)^{\frac{1}{\rho}} \tag{C.44}$$

where $\omega^* \equiv 1 - n\Omega^*$. The value $0 < \Omega^* \leq 1$ captures home bias if it is less that one. Profits are given by

$$P^*_{Y,t}\left(\left(\omega^*\right)^{1-\rho}\left(Y^*_{F,t}\right)^{\rho} + \left(1-\omega^*\right)^{1-\rho}\left(Y^*_{H,t}\right)^{\rho}\right)^{\frac{1}{\rho}} - P^*_{F,t}Y^*_{F,t} - P^*_{H,t}Y^*_{H,t} \tag{C.45}$$

where $P^*_{H,t}$ is the nominal price of $Y^*_{H,t}$, $P^*_{F,t}$ is the nominal price of $Y^*_{F,t}$. Demand curves are given by

$$Y^*_{H,t} = \left(\frac{P^*_{H,t}}{P^*_{Y,t}}\right)^{\frac{1}{\rho-1}}\left(1-\omega^*\right)Y^*_t \tag{C.46}$$

and

$$Y^*_{F,t} = \left(\frac{P^*_{F,t}}{P^*_{Y,t}}\right)^{\frac{1}{\rho-1}}\omega^* Y^*_t. \tag{C.47}$$

The ideal price index for Y^*_t is given by

$$P^*_{Y,t} = \left(\omega^*\left(P^*_{F,t}\right)^{\frac{\rho}{\rho-1}} + \left(1-\omega^*\right)\left(P^*_{H,t}\right)^{\frac{\rho}{\rho-1}}\right)^{\frac{\rho-1}{\rho}}. \tag{C.48}$$

We define,

$$\pi_{Y,t} \equiv P_{Y,t}/P_{Y,t-1} \tag{C.49}$$

and

$$\pi^*_{Y,t} \equiv P^*_{Y,t}/P^*_{Y,t-1}. \tag{C.50}$$

C.4 Monopolists

We introduce price stickiness as a Calvo-style price-setting friction. Monopolists set their price in the currency where their goods are sold (so-called “local-currency pricing”). Monopolists are only able to optimally update their price with probability ξ in each period. If monopolist i in the country H can optimally update its price, it chooses $\tilde{P}_{H,t}(i)$ and $\tilde{P}^*_{H,t}(i)$ to maximize

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \Bigg\{ \left(\frac{\tilde{P}_{H,t}(i)}{P_{C,t+j}} \tilde{X}_{H,t,j} (1+\tau_X) - MC_{t+j} \right) \left(\frac{\tilde{P}_{H,t}(i)}{P_{H,t+j}} \tilde{X}_{H,t,j} \right)^{-\nu} Y_{H,t+j}$$
$$+ \left(\frac{NER_{t+j} \tilde{P}^*_{H,t}(i)}{P_{C,t+j}} \tilde{X}^*_{H,t,j} (1+\tau_X) - MC_{t+j} \right) \left(\frac{\tilde{P}^*_{H,t}(i)}{P^*_{H,t+j}} \tilde{X}^*_{H,t,j} \right)^{-\nu} Y^*_{H,t+j} \Bigg\} \tag{C.51}$$

where

$$\tilde{X}_{H,t,j} = \begin{cases} 1 & j=1 \\ \pi_{H,t} \times \pi_{H,t+1} \times \cdots \times \pi_{H,t+j-1} & else \end{cases}, \tag{C.52}$$

and

$$\tilde{X}^*_{H,t,j} = \begin{cases} 1 & j=1 \\ \pi^*_{H,t} \times \pi^*_{H,t+1} \times \cdots \times \pi^*_{H,t+j-1} & else \end{cases}. \tag{C.53}$$

Here,

$$\pi_{H,t} \equiv P_{H,t}/P_{H,t-1} \tag{C.54}$$

and

$$\pi^*_{H,t} \equiv P^*_{H,t}/P^*_{H,t-1}. \tag{C.55}$$

The variables $\tilde{X}_{H,t,j}$ and $\tilde{X}^*_{H,t,j}$ capture indexation to past price changes. The first-order condition with respect to $\tilde{P}_{H,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \Bigg[\frac{\tilde{P}_{H,t}}{P_{C,t}} \frac{P_{C,t}}{P_{C,t+j}} \tilde{X}_{H,t,j}$$
$$- \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \Bigg] \left(\frac{P_{H,t}}{P_{H,t+j}} \tilde{X}_{H,t,j} \right)^{-\nu} Y_{H,t+j} = 0 \tag{C.56}$$

Then we have

$$F_{H,t} \tilde{p}_{H,t} = K_{H,t} \tag{C.57}$$

where $\tilde{p}_{H,t} \equiv \tilde{P}_{H,t}/P_{C,t}$ and $F_{H,t}$ and $K_{H,t}$ are given by

$$F_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{P_{C,t}}{P_{C,t+j}} \tilde{X}_{H,t,j} \left(\frac{P_{H,t}}{P_{H,t+j}} \tilde{X}_{H,t,j} \right)^{-\nu} Y_{H,t+j} \tag{C.58}$$

and

$$K_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \left(\frac{P_{H,t}}{P_{H,t+j}} \tilde{X}_{H,t,j} \right)^{-\nu} Y_{H,t+j}. \tag{C.59}$$

These can be written as

$$F_{H,t} = \Lambda_t Y_{H,t} + \beta\xi E_t \pi_{H,t}^{1-\nu} \pi_{C,t+1}^{-1} \pi_{H,t+1}^{\nu} F_{H,t+1} \tag{C.60}$$

and

$$K_{H,t} = \Lambda_t \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_t Y_{H,t} + \beta\xi E_t \pi_{H,t}^{-\nu} \pi_{H,t+1}^{\nu} K_{H,t+1} \tag{C.61}$$

The ideal price index for home goods in the home market is given by

$$P_{H,t} = \left((1-\xi) \tilde{P}_{H,t}^{1-\nu} + \xi \pi_{H,t-1}^{1-\nu} P_{H,t-1}^{1-\nu} \right)^{\frac{1}{1-\nu}}. \tag{C.62}$$

Then

$$p_{H,t} = \left((1-\xi) \tilde{p}_{H,t}^{1-\nu} + \xi \pi_{H,t-1}^{1-\nu} \frac{p_{H,t-1}^{1-\nu}}{\pi_{C,t}^{1-\nu}} \right)^{\frac{1}{1-\nu}}, \tag{C.63}$$

where $p_{H,t} \equiv P_{H,t}/P_{C,t}$. The first-order condition with respect to $\tilde{P}_{H,t}^{*}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \left[\frac{NER_{t+j}}{NER_t} \frac{NER_t P_{C,t}^{*}}{P_{C,t}} \tilde{p}_{H,t}^{*} \frac{P_{C,t}}{P_{C,t+j}} \tilde{X}_{H,t,j}^{*} - \frac{MC_{t+j}}{1+\tau_X} \frac{\nu}{\nu-1} \right] \left(\frac{P_{H,t}^{*}}{P_{H,t+j}^{*}} \tilde{X}_{H,t,j}^{*} \right)^{-\nu} Y_{H,t+j}^{*} = 0 \tag{C.64}$$

where $\tilde{p}_{H,t}^{*} \equiv \tilde{P}_{H,t}^{*}/P_{C,t}^{*}$. Then we have

$$F_{H,t}^{*} RER_t \tilde{p}_{H,t}^{*} = K_{H,t}^{*} \tag{C.65}$$

where

$$F^*_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{NER_{t+j}}{NER_t} \frac{P_{C,t}}{P_{C,t+j}} \tilde{X}^*_{H,t,j} \left(\frac{P^*_{H,t}}{P^*_{H,t+j}} \tilde{X}^*_{H,t,j} \right)^{-\nu} Y^*_{H,t+j} \tag{C.66}$$

$$K^*_{H,t} \equiv E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda_{t+j} \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_{t+j} \left(\frac{P^*_{H,t}}{P^*_{H,t+j}} \tilde{X}^*_{H,t,j} \right)^{-\nu} Y^*_{H,t+j}. \tag{C.67}$$

These variables can be written as

$$F^*_{H,t} = \Lambda_t Y^*_{H,t} + \beta\xi E_t \frac{NER_{t+1}}{NER_t} \left(\pi^*_{H,t}\right)^{1-\nu} \pi^{-1}_{C,t+1} \left(\pi^*_{H,t+1}\right)^{\nu} F^*_{H,t+1} \tag{C.68}$$

$$K^*_{H,t} = \Lambda_t \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC_t Y^*_{H,t} + \beta\xi E_t \left(\pi^*_{H,t}\right)^{-\nu} \left(\pi^*_{H,t+1}\right)^{\nu} K^*_{H,t+1} \tag{C.69}$$

The ideal price index for home goods in the foreign market is given by

$$P^*_{H,t} = \left((1-\xi) \left(\tilde{P}^*_{H,t}\right)^{1-\nu} + \xi \left(\pi^*_{H,t-1} P^*_{H,t-1}\right)^{1-\nu} \right)^{\frac{1}{1-\nu}} \tag{C.70}$$

so that

$$p^*_{H,t} = \left((1-\xi) \left(\tilde{p}^*_{H,t}\right)^{1-\nu} + \xi \left(\pi^*_{H,t-1}\right)^{1-\nu} \frac{\left(p^*_{H,t-1}\right)^{1-\nu}}{\left(\pi^*_{C,t}\right)^{1-\nu}} \right)^{\frac{1}{1-\nu}}, \tag{C.71}$$

where $p^*_{H,t} \equiv P^*_{H,t} / P^*_{C,t}$

The foreign firms are symmetric. If monopolist i can update its price, it chooses $\tilde{P}^*_{F,t}(i)$ and $\tilde{P}_{F,t}(i)$ to maximize

$$\begin{aligned} E_t \sum_{j=0}^{\infty} \Lambda^*_{t+j} \Bigg\{ & \left(\frac{\tilde{P}^*_{F,t}(i)}{P^*_{t+j}} \tilde{X}^*_{F,t,j} (1+\tau_X) - MC^*_{t+j} \right) \left(\frac{\tilde{P}^*_{F,t}(i)}{P^*_{F,t+j}} \tilde{X}^*_{F,t,j} \right)^{-\nu} Y^*_{F,t+j} \\ & + \left(\frac{\tilde{P}_{F,t}(i)}{NER_{t+j} P^*_{t+j}} \tilde{X}_{F,t,j} (1+\tau_X) - MC^*_{t+j} \right) \left(\frac{\tilde{P}_{F,t}(i)}{P_{F,t+j}} \tilde{X}_{F,t,j} \right)^{-\nu} Y_{F,t+j} \Bigg\} \end{aligned} \tag{C.72}$$

where

$$\tilde{X}_{F,t,j} = \begin{cases} 1 & j=1 \\ \pi_{F,t} \times \pi_{F,t+1} \times \cdots \times \pi_{F,t+j-1} & else \end{cases}, \tag{C.73}$$

and

$$\tilde{X}^*_{F,t,j} = \begin{cases} 1 & j = 1 \\ \pi^*_{F,t} \times \pi^*_{F,t+1} \times \cdots \times \pi^*_{F,t+j-1} & else \end{cases}. \tag{C.74}$$

where

$$\pi^*_{F,t} \equiv P^*_{F,t}/P^*_{F,t-1} \tag{C.75}$$

and

$$\pi_{F,t} \equiv P_{F,t}/P_{F,t-1}. \tag{C.76}$$

The first-order condition with respect to $\tilde{P}^*_{F,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda^*_{t+j} \left[\frac{\tilde{P}^*_{F,t}}{P^*_{C,t}} \frac{P^*_{C,t}}{P^*_{C,t+j}} \tilde{X}^*_{F,t,j} - \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC^*_{t+j} \right] \left(\frac{P^*_{F,t}}{P^*_{F,t+j}} \tilde{X}^*_{F,t,j} \right)^{-\nu} Y^*_{F,t+j} = 0. \tag{C.77}$$

We write this as

$$F^*_{F,t} \tilde{p}^*_{F,t} = K^*_{F,t} \tag{C.78}$$

where $\tilde{p}^*_{F,t} \equiv \tilde{P}^*_{F,t}/P^*_{C,t}$,

$$F^*_{F,t} = \Lambda^*_t Y^*_{F,t} + \beta\xi E_t \left(\pi^*_{F,t}\right)^{1-\nu} \left(\pi^*_{C,t+1}\right)^{-1} \left(\pi^*_{F,t+1}\right)^{\nu} F^*_{F,t+1} \tag{C.79}$$

and

$$K^*_{F,t} = \Lambda^*_t \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC^*_t Y^*_{F,t} + \beta\xi E_t \left(\pi^*_{F,t}\right)^{-\nu} \left(\pi^*_{F,t+1}\right)^{\nu} K^*_{F,t+1}. \tag{C.80}$$

The ideal price index implies

$$p^*_{F,t} = \left((1-\xi) \left(\tilde{p}^*_{F,t}\right)^{1-\nu} + \xi \left(\pi^*_{F,t-1}\right)^{1-\nu} \frac{\left(p^*_{F,t-1}\right)^{1-\nu}}{\left(\pi^*_{C,t}\right)^{1-\nu}} \right)^{\frac{1}{1-\nu}}. \tag{C.81}$$

where $p^*_{F,t} \equiv P^*_{F,t}/P^*_{C,t}$. The first-order condition with respect to $\tilde{P}_{F,t}(i)$ is

$$E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Lambda^*_{t+j} \left[\frac{NER_t}{NER_{t+j}} \frac{P_{C,t}}{NER_t P^*_{C,t}} \tilde{p}_{F,t} \frac{P^*_{C,t}}{P^*_{C,t+j}} \tilde{X}_{F,t,j} - \frac{MC^*_{t+j}}{1+\tau_X} \frac{\nu}{\nu-1} \right] \left(\frac{P_{F,t}}{P_{F,t+j}} \tilde{X}_{F,t,j} \right)^{-\nu} Y_{F,t+j} = 0. \quad \text{(C.82)}$$

We can write this as

$$F_{F,t} \frac{\tilde{p}_{F,t}}{RER_t} = K_{F,t} \quad \text{(C.83)}$$

where $\tilde{p}_{F,t} \equiv \tilde{P}_{F,t}/P_{C,t}$,

$$F_{F,t} = \Lambda^*_t Y_{F,t} + \beta\xi E_t \frac{NER_t}{NER_{t+1}} \pi^{1-\nu}_{F,t} \left(\pi^*_{C,t+1}\right)^{-1} \left(\pi_{F,t+1}\right)^{\nu} F_{F,t+1} \quad \text{(C.84)}$$

and

$$K_{F,t} = \Lambda^*_t \frac{1}{1+\tau_X} \frac{\nu}{\nu-1} MC^*_t Y_{F,t} + \beta\xi E_t \pi^{-\nu}_{F,t} \left(\pi_{F,t+1}\right)^{\nu} K_{F,t+1}. \quad \text{(C.85)}$$

The price index implies that

$$p_{F,t} = \left((1-\xi) \left(\tilde{p}_{F,t}\right)^{1-\nu} + \xi \left(\pi_{F,t-1}\right)^{1-\nu} \frac{\left(p_{F,t-1}\right)^{1-\nu}}{\left(\pi_{C,t}\right)^{1-\nu}} \right)^{\frac{1}{1-\nu}}. \quad \text{(C.86)}$$

C.5 Marginal cost

In this subsection we drop the i index because it should be understood that all quantities are the quantity purchased by firm i. The firm solves the following cost minimization problem

$$\min_{K_t, L_t} W_t L_t + R_{Kt} K_t + P_{O,t} O_t \quad \text{(C.87)}$$

subject to the constraint that

$$\left((\omega_V)^{1-\rho_V} V_t^{\rho_V} + (1-\omega_V)^{1-\rho_V} \left(\frac{V_{O,t}}{\mu_{V_{O,t}}} \right)^{\rho_V} \right)^{\frac{1}{\rho_V}} \geq X_t \quad \text{(C.88)}$$

where

$$V_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{C.89}$$

and A_t and A_t^* are stochastic processes. The first-order conditions for the home firms are

$$\frac{R_{Kt}}{P_{Ct}} = MC_t \left(X_t\right)^{1-\rho_V} \left(\omega_V\right)^{1-\rho_V} V_t^{\rho_V-1} \alpha A_t \left(\frac{L_t}{K_t}\right)^{1-\alpha} \tag{C.90}$$

$$\frac{W_t}{P_{C,t}} = MC_t \left(X_t\right)^{1-\rho_V} \left(\omega_V\right)^{1-\rho_V} V_t^{\rho_V-1} \left(1-\alpha\right) A_t \left(\frac{L_t}{K_t}\right)^{-\alpha} \tag{C.91}$$

$$\frac{P_{O,t}}{P_{C,t}} = MC_t \left(X_t\right)^{1-\rho_V} \left(1-\omega_V\right)^{1-\rho_V} \left(\frac{V_{O,t}}{\mu_{V_O,t}}\right)^{\rho_V-1} \frac{1}{\mu_{V_O,t}}. \tag{C.92}$$

Foreign firms minimize

$$\min_{K_t^*, L_t^*} W_t^* L_t^* + R_{Kt}^* K_t^* + P_{O,t}^* O_t^* \tag{C.93}$$

subject to the constraint that

$$\left(\left(\omega_V\right)^{1-\rho_V} \left(V_t^*\right)^{\rho_V} + \left(1-\omega_V\right)^{1-\rho_V} \left(\frac{V_{O,t}^*}{\mu_{V_O,t}^*}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} \geq X_t^* \tag{C.94}$$

where

$$V_t^* = A_t^* \left(K_t^*\right)^{\alpha} \left(L_t^*\right)^{1-\alpha}. \tag{C.95}$$

The first-order conditions for the foreign firms are

$$\frac{R_{Kt}^*}{P_{Ct}^*} = MC_t^* \left(X_t^*\right)^{1-\rho_V} \left(\omega_V\right)^{1-\rho_V} \left(V_t^*\right)^{\rho_V-1} \alpha A_t^* \left(\frac{L_t^*}{K_t^*}\right)^{1-\alpha} \tag{C.96}$$

$$\frac{W_t^*}{P_{C,t}^*} = MC_t^* \left(X_t^*\right)^{1-\rho_V} \left(\omega_V\right)^{1-\rho_V} \left(V_t^*\right)^{\rho_V-1} \left(1-\alpha\right) A_t^* \left(\frac{L_t^*}{K_t^*}\right)^{-\alpha} \tag{C.97}$$

$$\frac{P_{O,t}^*}{P_{C,t}^*} = MC_t^* \left(X_t^*\right)^{1-\rho_V} \left(1-\omega_V\right)^{1-\rho_V} \left(\frac{V_{O,t}^*}{\mu_{V_O,t}^*}\right)^{\rho_V-1} \frac{1}{\mu_{V_O,t}^*}. \tag{C.98}$$

C.6 Oil Market

There is an exogenous supply of oil, O_t. Oil-market clearing implies

$$n\left(V_{O,t}+O_{C,t}\right)+(1-n)\left(V_{O,t}^{*}+O_{C,t}^{*}\right)=O_t. \tag{C.99}$$

The price is set flexibly so that the market clears and

$$P_{O,t}=NER_tP_{O,t}^{*}. \tag{C.100}$$

C.7 Goods market clearing

We assume that oil is paid for to an outside owner using non-oil output. So, goods market clearing implies

$$Y_{C,t}+G_t+I_t+\left(O_{C,t}+V_{O,t}\right)\frac{P_{O,t}}{P_{Y,t}}+\frac{\phi_b}{2}\left(\frac{B_{F,t}NER_t}{P_{C,t}}\right)^2=Y_t \tag{C.101}$$

and

$$Y_{C,t}^{*}+G_t^{*}+I_t^{*}+\left(O_{C,t}^{*}+V_{O,t}^{*}\right)\frac{P_{O,t}^{*}}{P_{Y,t}^{*}}+\frac{\phi_b}{2}\left(\frac{B_{H,t}^{*}}{P_{C,t}^{*}NER_t}\right)^2=Y_t^{*}. \tag{C.102}$$

The quadratic costs of bond holdings show up in the resource constraint because we assume that non-oil output is used to pay those costs.

C.8 Bond market clearing

Define $b_{H,t}\equiv B_{H,t}/P_{C,t}$, $b_{H,t}^{*}\equiv B_{H,t}^{*}/P_{C,t}$, $b_{F,t}^{*}\equiv B_{F,t}^{*}/P_{C,t}^{*}$, and $b_{F,t}\equiv B_{F,t}/P_{C,t}^{*}$. We assume that only the home bond can be traded internationally and that both home and foreign bonds are in zero net supply. So,

$$nb_{H,t}+(1-n)\,b_{H,t}^{*}=0 \tag{C.103}$$

and

$$nb_{F,t}+(1-n)\,b_{F,t}^{*}=0. \tag{C.104}$$

C.9 Aggregation

Aggregating across home firms yields

$$
\begin{aligned}
& n\int_0^n \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\nu} Y_{H,t}di + (1-n)\int_0^n \frac{1}{n}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\nu} Y^*_{H,t}di \\
&= \int_0^n \left((\omega_V)^{1-\rho_V} V_t(i)^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{V_{O,t}(i)}{\mu_{V_O,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} di \qquad \text{(C.105)} \\
&= \int_0^n \left((\omega_V)^{1-\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{V_{O,t}(i)}{V_t(i)\,\mu_{V_O,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} V_t(i)\,di \qquad \text{(C.106)} \\
&= \int_0^n \left((\omega_V)^{1-\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{V_{O,t}(i)}{V_t(i)\,\mu_{V_O,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} A_t\left(\frac{K_t(i)}{L_t(i)}\right)^{\alpha} L_t(i)\,di. \qquad \text{(C.107)}
\end{aligned}
$$

Due to constant-returns-to-scale, the ratios $\frac{V_{O,t}(i)}{V_t(i)\mu_{V_O,t}}$ and $\frac{K_t(i)}{L_t(i)}$ are common across firms. Then

$$
\begin{aligned}
& n\int_0^n \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\nu} Y_{H,t}di + (1-n)\int_0^n \frac{1}{n}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\nu} Y^*_{H,t}di \\
&\qquad = n\left((\omega_V)^{1-\rho_V} V_t^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{V_{O,t}}{\mu_{V_O,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} \qquad \text{(C.108)}
\end{aligned}
$$

so that

$$
d_{H,t}Y_{H,t} + d^*_{H,t}\frac{1-n}{n}Y^*_{H,t} = \left((\omega_V)^{1-\rho_V} V_t^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{V_{O,t}}{\mu_{V_O,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} \qquad \text{(C.109)}
$$

where $d_{H,t}$ and $d^*_{H,t}$ are appropriately defined. Similarly,

$$
d_{F,t}\frac{n}{1-n}Y_{F,t} + d^*_{F,t}Y^*_{F,t} = \left((\omega_V)^{1-\rho_V} (V_t^*)^{\rho_V} + (1-\omega_V)^{1-\rho_V}\left(\frac{V^*_{O,t}}{\mu^*_{V_O,t}}\right)^{\rho_V}\right)^{\frac{1}{\rho_V}}. \qquad \text{(C.110)}
$$

Here, the dispersion terms can be written recursively as

$$
d_{H,t} = (1-\xi)\,p_{H,t}^{\nu}\,(\tilde{p}_{H,t})^{-\nu} + \xi\pi_{H,t-1}^{-\nu}\pi_{H,t}^{\nu}d_{H,t-1}, \qquad \text{(C.111)}
$$

$$
d^*_{H,t} = (1-\xi)\left(p^*_{H,t}\right)^{\nu}\left(\tilde{p}^*_{H,t}\right)^{-\nu} + \xi\left(\pi^*_{H,t-1}\right)^{-\nu}\left(\pi^*_{H,t}\right)^{\nu} d^*_{H,t-1}, \qquad \text{(C.112)}
$$

$$d_{F,t}^{*} = (1-\xi)\left(p_{F,t}^{*}\right)^{\nu}\left(\tilde{p}_{F,t}^{*}\right)^{-\nu} + \xi\left(\pi_{F,t-1}^{*}\right)^{-\nu}\left(\pi_{F,t}^{*}\right)^{\nu} d_{F,t-1}^{*}, \tag{C.113}$$

$$d_{F,t} = (1-\xi)\left(p_{F,t}\right)^{\nu}\left(\tilde{p}_{F,t}\right)^{-\nu} + \xi\pi_{F,t-1}^{-\nu}\left(\pi_{F,t}\right)^{\nu} d_{F,t-1}. \tag{C.114}$$

C.10 Government

In each country, the monetary authority follows a truncated Taylor rule. The desired policy rates, $\tilde{R}_t$ and $\tilde{R}_t^*$ evolves according to

$$\left(1+\tilde{R}_t\right)^{1/4} = \left(\left(1+\tilde{R}_{t-1}\right)^{1/4}\right)^{\gamma}\left((1+R)^{1/4}\left(\frac{\pi_{Y,t}}{\pi}\right)^{\theta_\pi}\left(\frac{Y_t}{Y_t^N}\right)^{\theta_Y}\right)^{1-\gamma} \tag{C.115}$$

where $\theta_\pi > 1$. Here, R is the steady-state annualized net nominal interest rate, π is the target rate of inflation. In the foreign country

$$\left(1+\tilde{R}_t^{\,*}\right)^{1/4} = \left(\left(1+\tilde{R}_t^{\,*}\right)^{1/4}\right)^{\gamma^*}\left((1+R^*)^{1/4}\left(\frac{\pi_{Y,t}^*}{\pi^*}\right)^{\theta_\pi^*}\left(\frac{Y_t^*}{Y_t^{N*}}\right)^{\theta_Y^*}\right)^{1-\gamma^*} \tag{C.116}$$

where $\theta_\pi > 1$. Here, R^* is the steady-state annualized net nominal interest rate, π^* is the target rate of inflation. The natural rates of output, Y_t^N and Y_t^{N*}, are defined as the levels of output that would prevail under flexible prices and constant prices, given the entire history of shocks. The fiscal authorities balances its budget with lump sum taxes so bonds are in zero net supply. Government purchases, G_t and G_t^*, follow independent $AR(1)$ processes. To incorporate the zero lower bound,

$$R_t = \max\left\{0, \tilde{R}_t\right\}. \tag{C.117}$$

For the foreign country, we ignore the zero lower bound, so that

$$R_t^* = \tilde{R}_t^*. \tag{C.118}$$

We ignore the zero lower bound for the foreign country because we want to study how a binding lower bound in the home country affects the foreign country.

C.11 Equilibrium

A rational expectations equilibrium is a sequence of prices and quantities that have the property that the household and firm optimality conditions are satisfied, the goods market, labor market, and oil markets clear, and the nominal interest rate and government purchases evolve as specified. To solve for a rational expectations equilibrium, we solve for the following 36 endogenous objects: C_t, Λ_t, L_t, $w_t \equiv \frac{W_t}{P_t}$, $Y_{H,t}$, $Y_{F,t}$, R_t, $\tilde{R}_t$, MC_t, $\pi_{C,t}$, K_t, I_t, Q_t, $r_{K,t} \equiv \frac{R_{K,t}}{P_{C,t}}$, Y_t, $p_{F,t} \equiv \frac{P_{F,t}}{P_{C,t}}$, $p_{H,t} \equiv \frac{P_{H,t}}{P_{C,t}}$, $\tilde{p}_{H,t}$, $F_{H,t}$, $K_{H,t}$, $d_{H,t}$, $\pi_{H,t}$, $\tilde{p}_{F,t}$, $F_{F,t}$, $K_{F,t}$, $d_{F,t}$, $\pi_{F,t}$, $b_{H,t}$, $b_{F,t}$, $p_{Y,t} \equiv \frac{P_{Y,t}}{P_{C,t}}$, V_t, $V_{O,t}$, $Y_{C,t}$, $O_{C,t}$, $p_{O,t} \equiv \frac{P_{O,t}}{P_{C,t}}$, $\pi_{Y,t}$, the 36 star versions, as well as $\Delta NER_t \equiv \frac{NER_t}{NER_{t-1}}$ and RER_t.

We linearize the model around non-stochastic steady state. Given parameter values, we study the unique bounded rational expectations equilibrium from the linearized model. To determine these variables, we require that the linearized versions following 74 equations hold: (C.3), (C.7), (C.8), (C.9), (C.10), (C.11), (C.12), (C.13), (C.14), (C.15), (C.18), (C.19), (C.20), (C.21), (C.22), (C.23), (C.24), (C.25), (C.26), (C.27), (C.28), (C.41), (C.42), (C.43), (C.46), (C.47), (C.48), (C.101), (C.102), (C.103), (C.104), (C.57), (C.60), (C.61), (C.54), (C.63), (C.65), (C.68), (C.69), (C.55), (C.71), (C.78), (C.79), (C.80), (C.75), (C.81), (C.83), (C.84), (C.85), (C.76), (C.86), (C.89), (C.90), (C.91), (C.92), (C.95), (C.96), (C.97), (C.98), (C.99), (C.100), (C.109), (C.110), (C.111), (C.112), (C.113), (C.114), (C.115), (C.116), (C.117), (C.118), (C.49), (C.50), (C.17). The home household budget constraint (C.2) clears by Walras' law. Note that to solve for the natural rate of output, we find the equilibrium of a similar economy where $\xi = 0$. We linearize the model around non-stochastic steady state. We incorporate the zero lower bound using the methodology of Guerrieri and Iacoviello (2015).

C.12 Steady State

We assume that government policy is symmetric between the home and foreign county and that the target inflation rate is π. So, $\pi_C = \pi_C^* = \pi_H = \pi_H^* = \pi_F = \pi_F^* = \pi_Y = \pi_Y^* = \pi$. The intertemporal Euler equations determine $(1+R)^{1/4} = (1+R^*)^{1/4} = \pi\beta^{-1}$. We normaliz $L = L^* = 1$ (we will have to find χ instead of L). From the definition of steady state, with symmetric inflation targets $\Delta NER = 1$. We define initial conditions so that $RER = 1$. In our steady state, there are no net home bond holdings in the foreign country because of the quadratic costs of holding them. Similarly, there are no net foreign bond holdings in the home country. From firm optimality and symmetry of the equilibrium, $p_H = p_H^* = p_F = p_F^* = 1$. This also gives us that $\tilde{p}_H = \tilde{p}_H^* = \tilde{p}_F = \tilde{p}_F^* = 1$. Because of our inflation indexation assumption, there is no price dispersion in steady state, so $d_H = d_F = d_H^* = d_F^* = 1$. We will normalize the price of oil to be $p_O = p_O^* = 1$ (we have to find O instead). As a result, $p_Y = p_Y^* = 1$, meaning $Q = \Lambda$ and $Q^* = \Lambda^*$. Marginal cost is given by

$$MC = MC^* = \frac{\nu - 1}{\nu} (1 + \tau_X). \tag{C.119}$$

From pricing optimality

$$F_F = K_F = (1 - \beta\xi)^{-1} \Lambda^* Y_F \tag{C.120}$$

$$F_F^* = K_F^* = (1 - \beta\xi)^{-1} \Lambda^* Y_F^* \tag{C.121}$$

$$F_H = K_H = (1 - \beta\xi)^{-1} \Lambda Y_H \tag{C.122}$$

$$F_H^* = K_H^* = (1 - \beta\xi)^{-1} \Lambda Y_H^*. \tag{C.123}$$

The rental rate of capital is

$$r_K = r_K^* = \frac{1 - \beta (1 - \delta)}{\beta}. \tag{C.124}$$

From our normalization of p_O and p_O^*,

$$O_C = (1 - \omega_C) C \tag{C.125}$$

$$O_C^* = (1 - \omega_C) C^* \tag{C.126}$$

and

$$Y_C = \omega_C C \tag{C.127}$$

$$Y_C^* = \omega_C C^*. \tag{C.128}$$

The marginal utility of consumption implies

$$(C[1-h])^{-\sigma} = \Lambda \tag{C.129}$$

$$(C^*[1-h])^{-\sigma} = \Lambda^* \tag{C.130}$$

Note that

$$Y_H = (1-(1-n)\Omega)Y \tag{C.131}$$

$$Y_F = (1-n)\Omega Y \tag{C.132}$$

$$Y_H^* = n\Omega^* Y^* \tag{C.133}$$

$$Y_F^* = (1-n\Omega^*)Y^* \tag{C.134}$$

and

$$I = \delta K \tag{C.135}$$

$$I^* = \delta K^* \tag{C.136}$$

$$Y = I + Y_C + G + O_C + V_O \tag{C.137}$$

$$Y^* = I^* + Y_C^* + G^* + O_C^* + V_O^*. \tag{C.138}$$

Our aggregate variables are expressed in per-capita terms, and we are going to consider a symmetric steady state where $Y = Y^*$. From

$$d_H Y_H + d_H^* \frac{1-n}{n} Y_H^* = \left((\omega_V)^{1-\rho_V} V^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O)^{\rho_V}\right)^{\frac{1}{\rho_V}} \tag{C.139}$$

we get

$$(1-(1-n)\Omega)Y + (1-n)\Omega^* Y^* = Y = Y^* \tag{C.140}$$

where

$$Y = \left((\omega_V)^{1-\rho_V} V^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O)^{\rho_V}\right)^{\frac{1}{\rho_V}}. \tag{C.141}$$

We can see this from

$$d_F \frac{n}{1-n} Y_F + d_F^* Y_F^* = \left((\omega_V)^{1-\rho_V} (V^*)^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O^*)^{\rho_V} \right)^{\frac{1}{\rho_V}} \tag{C.142}$$

which yields

$$n\Omega Y + (1 - n\Omega^*) Y^* = \left((\omega_V)^{1-\rho_V} (V^*)^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O^*)^{\rho_V} \right)^{\frac{1}{\rho_V}} \tag{C.143}$$

which means the equalities above hold. This means

$$\delta K + \omega_C C + G = \left((\omega_V)^{1-\rho_V} (K^\alpha)^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O)^{\rho_V} \right)^{\frac{1}{\rho_V}}. \tag{C.144}$$

From the definition of V we have

$$V = K^\alpha \tag{C.145}$$

and

$$V^* = (K^*)^\alpha. \tag{C.146}$$

Define

$$X \equiv \left((\omega_V)^{1-\rho_V} V^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O)^{\rho_V} \right)^{\frac{1}{\rho_V}}. \tag{C.147}$$

From cost minimization, we know that

$$r_K = MC\,(X)^{1-\rho_V} (\omega_V)^{1-\rho_V} V^{\rho_V - 1} \alpha \left(\frac{1}{K} \right)^{1-\alpha} \tag{C.148}$$

$$w = MC\,(X)^{1-\rho_V} (\omega_V)^{1-\rho_V} V^{\rho_V - 1} (1-\alpha) \left(\frac{1}{K} \right)^{-\alpha} \tag{C.149}$$

$$1 = MC\,(X)^{1-\rho_V} (1-\omega_V)^{1-\rho_V} (V_O)^{\rho_V - 1}. \tag{C.150}$$

Define

$$X^* \equiv \left((\omega_V)^{1-\rho_V} (V^*)^{\rho_V} + (1-\omega_V)^{1-\rho_V} (V_O^*)^{\rho_V} \right)^{\frac{1}{\rho_V}}. \tag{C.151}$$

From cost minimization, we know that

$$r_K^* = MC^* \left(X^*\right)^{1-\rho_V} \left(\omega_V\right)^{1-\rho_V} \left(V^*\right)^{\rho_V-1} \alpha \left(\frac{1}{K^*}\right)^{1-\alpha} \tag{C.152}$$

$$w^* = MC^* \left(X_t^*\right)^{1-\rho_V} \left(\omega_V\right)^{1-\rho_V} \left(V^*\right)^{\rho_V-1} (1-\alpha) \left(\frac{1}{K^*}\right)^{-\alpha} \tag{C.153}$$

$$1 = MC^* \left(X_t^*\right)^{1-\rho_V} \left(1-\omega_V\right)^{1-\rho_V} \left(V_O^*\right)^{\rho_V-1}. \tag{C.154}$$

Then

$$r_K = \left(\frac{\omega_V}{1-\omega_V}\right)^{1-\rho_V} \left(\frac{K^\alpha}{V_O}\right)^{\rho_V-1} \alpha \left(\frac{1}{K}\right)^{1-\alpha}. \tag{C.155}$$

Then

$$V_O = r_K^{-\frac{1}{\rho_V-1}} \left(\frac{\omega_V}{1-\omega_V}\right)^{-1} \left(K^\alpha\right) \alpha^{\frac{1}{\rho_V-1}} \left(\frac{1}{K}\right)^{\frac{1-\alpha}{\rho_V-1}}. \tag{C.156}$$

So

$$\begin{aligned} r_K = &\left(\left(\omega_V\right)^{1-\rho_V} \left(K^\alpha\right)^{\rho_V} + \left(1-\omega_V\right)^{1-\rho_V} \left(r_K^{-\frac{1}{\rho_V-1}} \left(\frac{\omega_V}{1-\omega_V}\right)^{-1} \left(K^\alpha\right) \alpha^{\frac{1}{\rho_V-1}} \left(\frac{1}{K}\right)^{\frac{1-\alpha}{\rho_V-1}}\right)^{\rho_V}\right)^{\frac{1-\rho_V}{\rho_V}} \\ &\times MC \left(\omega_V\right)^{1-\rho_V} \left(K^\alpha\right)^{\rho_V-1} \alpha \left(\frac{1}{K}\right)^{1-\alpha} \end{aligned} \tag{C.157}$$

with r_K known and MC known, we can solve for K. With K we get V and then w. With the household intratemporal Euler equation, we get χ. We have Y from

$$Y = \left(\left(\omega_V\right)^{1-\rho_V} V^{\rho_V} + \left(1-\omega_V\right)^{1-\rho_V} \left(V_O\right)^{\rho_V}\right)^{\frac{1}{\rho_V}} \tag{C.158}$$

We know

$$Y_C + G + I + \left(O_C + V_O\right) = Y \tag{C.159}$$

and $Y_C = \omega_C C$ and $O_C = \left(1-\omega_C\right) C$ meaning

$$C + G + I + V_O = Y \tag{C.160}$$

With C, we get Y_C and O_C. Combined with V_O (and the star versions), we get O. The rest follows easily.

C.13 Calibration and solution strategy

For parameters that are common with our one-country model, we use the same values as in our one-country model, which are specified in Section II.E. We set $n = 0.8$ so that the large country has size 0.8 and the small country has size 0.2. We assume that monetary policy is symmetric across the two countries and that the target level of inflation is 2 percent at an annualized rate. We set $\Omega = \Omega^* = 0.25$ to incorporate home bias. This value implies that in steady state 95 percent of non-oil expenditure in the big country is on goods from the big country. In the small country, in steady state 80 percent of non-oil expenditure is on goods from the small country, implying an openness parameter of 0.4 in steady state. We set $\rho = 1/3$ so that the elasticity between domestic and foreign goods is 1.5.

We compute the natural rate of output as the level of output under flexible prices in both countries. As in our one-country model, we solve the mode using the methodology of Guerrieri and Iacoviello (2015). Their solution strategy involves a first-order perturbation to the model, which is applied piecewise so as to accommodate the ZLB. We only ever impose the ZLB in one country or the other. The main advantage of using the methodology of Guerrieri and Iacoviello (2015) is that it is able to accommodate the number of state variables implied by medium-scale DSGE models. In our case, the number of state variables is even larger because of the second country.

D The power gains from high frequency data: A Monte Carlo study

We have claimed that, to identify the change caused by the ZLB, tests with daily data have greater statistical power than tests using quarterly data. To support this claim, this appendix presents a simple Monte Carlo study regarding how higher frequency data can increase statistical power.

Based on a data-generating process with a known break date and using parameter values estimated using the observed data on oil prices and equity returns, the test using daily data almost always rejects the false null of no break. In contrast, using quarterly data results in far fewer rejections. This improvement in statistical power does not come at any cost in statistical size. For a data-generating process with no change, tests done using either daily or quarterly data had the correct statistical size

Our Monte Carlo experiment is based on a stylized version of the regression that is reported in the paper's Table 2.

Oil price changes Δp_t are modeled as the following with equity returns Δs_t modeled as a random walk,

$$\begin{aligned} \Delta p_t &= \beta_1 \Delta s_t + \beta_2 D_t \Delta s_t + (\rho - 1) p_{t-1} + v_t \\ \Delta s_t &= u_t \end{aligned}$$

The effect of the ZLB is modeled through a dummy variable where D_t equals zero when the ZLB is not binding and equals one when the ZLB is binding. The error terms v_t and u_t are independent, normally distributed with variances σ_v^2 and σ_u^2, respectively.

The empirical test is to test for the null hypothesis that $\beta_{ZLB} = 0$, for the following estimated regression.

$$\Delta p_t = \alpha + \beta \Delta s_t + \beta_{ZLB} D_t \Delta s_t + e_t$$

Aggregation to quarterly frequency is done by cumulating all the price changes in a quarter, with the assumption of 65 business days per quarter.

$$\begin{aligned} \Delta p_j^q &= \sum_{t=65(j-1)+1}^{65j} \Delta p_t \\ \Delta s_j^q &= \sum_{t=65(j-1)+1}^{65j} \Delta s_t \end{aligned}$$

where j goes from 1 to $4T$, where T is the number of years.

and then we estimate the regression analogous to the daily one.

$$\Delta p_j^q = \alpha + \beta \Delta s_j^q + \beta_{ZLB} D_j \Delta s_j^q + e_j$$

To parameterize the data generating process, we estimated the following regression using the observed data,

$$\begin{aligned}\Delta p_t &= \beta_1 \Delta s_t + \beta_2 D_t \Delta s_t + (\rho - 1) p_{t-1} + v_t. \\ \Delta s_t &= u_t.\end{aligned}$$

The sample was from Jan. 1990 to December 2014, with the ZLB binding between April 2009 to December 2014. Based on the estimated empirical regression, the simulated data sets were created using the estimated model parameters of $\sigma_u = 1.09$, $\sigma_v = 2.27$, and $\beta_1 = -0.01$.

Four different sets of simulations were done. For each simulation, we generated data for 25 years of data with 260 business days per year, with 65 days per quarter. The value of D_t was set equal to 1 for the last 5 years of the simulation. For two sets of simulations, it was assumed that the null hypothesis of no change is true. In these simulations, the value of β_2 for the DGP equalled zero. In the other two sets of simulations, the value of β_2 was its estimated value of 0.82. Results are reported for two values of ρ: an estimated value of 0.91 and an alternative where ρ equals 1.

The Table reports the frequency of rejecting the null of no change. When the null hypothesis is true $\beta_{ZLB} = 0$, then both tests correctly reject the null hypothesis 5 percent of the time. When the null hypothesis is false (i.e. the data is generated with $\beta_2 = 0.82$), then using daily data, one always rejects the false null of no change. In contrast, in the quarterly data, one frequently fails to reject the false null of no change.

Fraction of Rejections of $\beta_{ZLB} = 0$

Value of ρ	$\rho = 0.91$		$\rho = 1$	
Observed Frequency of Data	Daily	Quarterly	Daily	Quarterly
Null Hypothesis is True	0.049	0.054	0.051	0.050
Null Hypothesis is False	1.000	0.115	1.000	0.374

Note: Results are based on 20000 simulations for each scenario.

Scenarios are simulated using the estimated coefficients as described in the text.

The performance of the test using quarterly data is particularly bad when $\rho = 0.91$. The transitory effects of permanent equity price changes on oil prices do not make a sufficient impact on the quarterly data to show up as statistically significant. However, even when ρ equals one, implying that permanent equity price changes have a permanent effect on oil price, the rejection rate is only 37 percent. One just does not have enough quarterly observations to get a rejection. If the number of years at the ZLB were to double to 10, then the rejection rate for quarterly data for $\rho = 1$ would increase to almost 49 percent. However, even with twice as many observations at the ZLB, the rejection rate for quarterly data when ρ equals 0.91 is still very poor, being only 12 percent.

E Oil and equity VAR

The estimated VAR implies the following moving average representation for the h-step ahead forecast errors,

$$y_{t+h} - y_{t+h|t} = \sum_{i=0}^{h-1} \Theta_i w_{t+h-i}, \tag{E.1}$$

where Θ_i is a 4-by-4 matrix of moving average coefficients implied by the estimated VAR and structural factorization of the variance-covariance matrix of the reduced form residuals.

The covariance calculation requires two kinds of matrices. The first is $MSPE(h)$ the value of the h-step ahead forecast variance-covariance matrix conditional on all shocks

$$MSPE(h) = E\left(\left(y_{t+h} - y_{t+h|t}\right)\left(y_{t+h} - y_{t+h|t}\right)'\right) = \sum_{i=0}^{h-1} \Theta_i I \Theta_i', \tag{E.2}$$

where I is the identity matrix. The second is $MSPE_j(h)$, the h-step ahead forecast variance-covariance matrix conditional on only the j-shock

$$MSPE_j(h) = \sum_{i=0}^{h-1} \Theta_i E_j \Theta_i^{'}, \tag{E.3}$$

where all elements of E_j are equal to zero except the j-th, j-th element, which equals one.

The correlation between oil and equity returns can be defined using terms from these two matrices. In particular, define $\sigma_p(h)$ as the square root of the 3,3 element of $MSPE(h)$, $\sigma_e(h)$ as the square root of the 4,4 element of $MSPE(h)$, and $\sigma_{pe}(h)$ as the 3,4 element of $MSPE(h)$. Furthermore, define $\sigma_{pe,j}(h)$, the covariance conditional just on shock j, as the 3,4 element of $MSPE_j(h)$. Having defined these terms, we can then write the correlation between oil and equity returns $\rho_{pe}(h)$ as the following equation

$$\rho_{pe}(h) = \frac{\sigma_{pe}(h)}{\sigma_p(h)\,\sigma_e(h)} = \sum_{j=1}^{4} \frac{\sigma_{pe,j}(h)}{\sigma_p(h)\,\sigma_e(h)}. \tag{E.4}$$

For any shock, a larger $\sigma_{pe,j}(h)$ indicates a larger contribution of the j-th shock to the overall correlation.

Table 8 reports our results for $h = 1000$, which is large enough that $\rho_{pe}(h)$ approximates well the correlation between oil and equity returns. Using a large h is a standard practice in the literature. See Hamilton (1994), page 324.

Using a large h is the natural object, given that we want to decompose the correlation between oil prices and equity prices, which is a function of the variance covariance matrix. For a smaller h, we would only be reporting the correlation for the h-step ahead forecast error, which could be very different from the object that is the focus of our paper. Using a large h should do a reasonable job of estimating the variance-covariance matrix of the underlying variables. Although, as discussed in Christiano, Eichenbaum and Vigfusson (2007), using a VAR to approximate a single point in the underlying spectrum (in particular the spectrum at frequency zero) can be concerning, the estimated VAR can do a reasonable job of fitting the overall spectral density of the underlying series, as is discussed in Christiano and Vigfusson (2003).

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